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Bar-Ilan University
אוניברסיטת בר־אילן

Array Signal Processing

Lab Course Exercises

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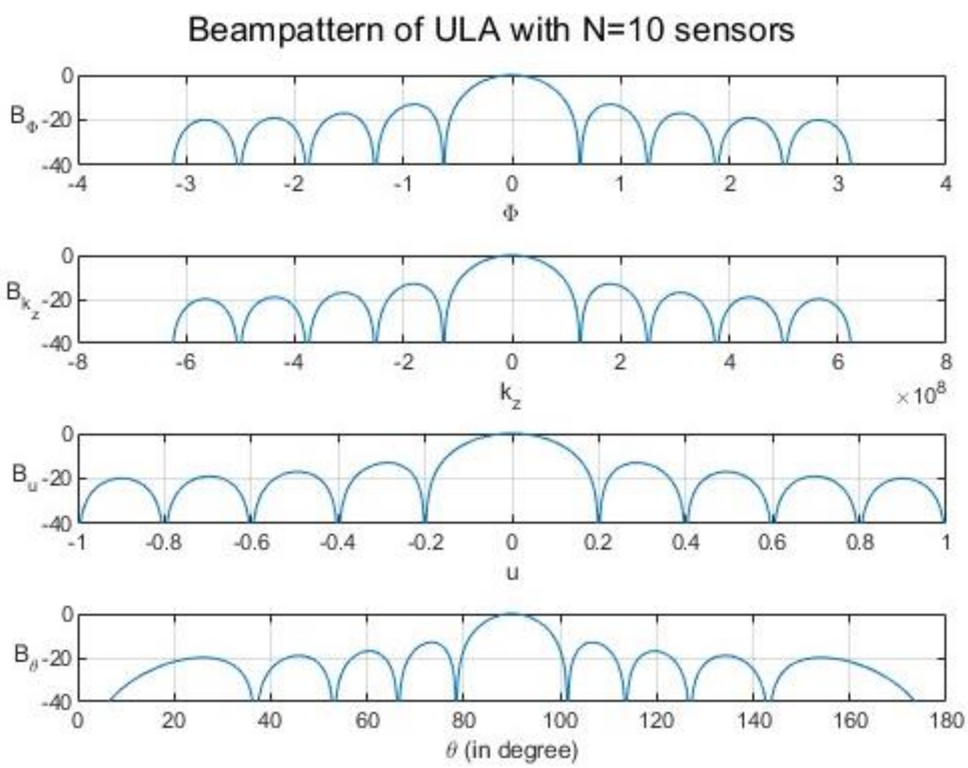
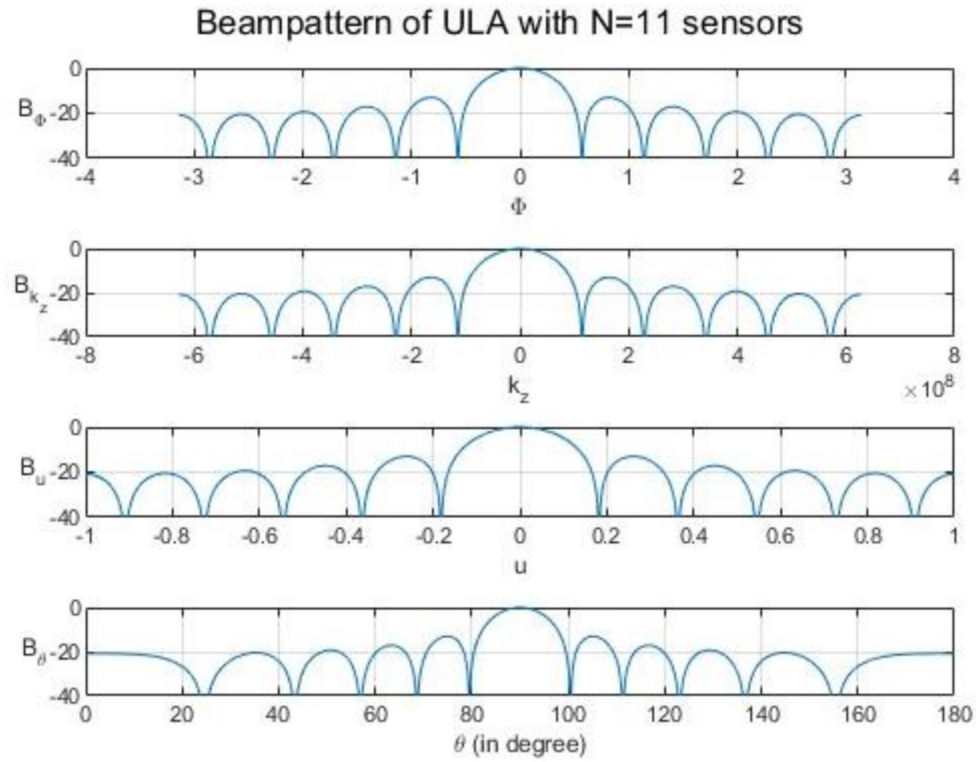
Exercise 1: Beampattern of ULA

A uniform linear array (ULA) with $N = 10$ sensors, uniform weights, $w_n = \frac{1}{N}$ and sensor spacing $d = \frac{\lambda}{2}$ is considered. The beampattern in the Θ-domain is given by:

a) Magnitude (in dB) of the beampattern for an even number of sensors:

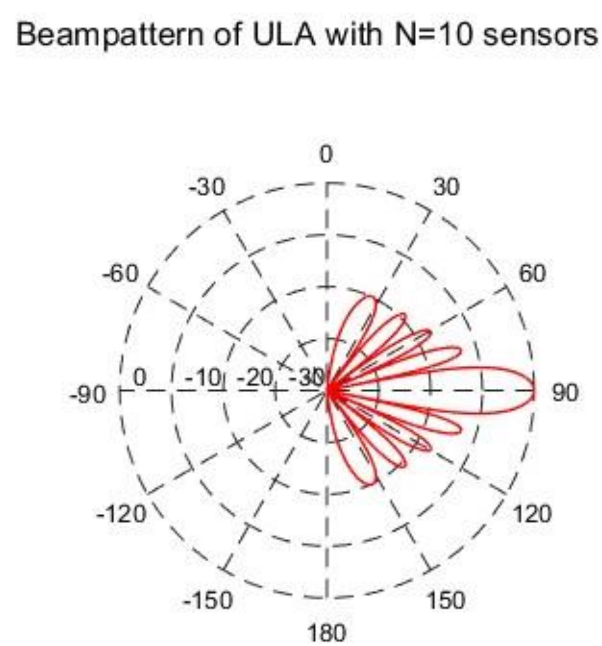


Magnitude (in dB) of the beampattern for an odd number of sensors:

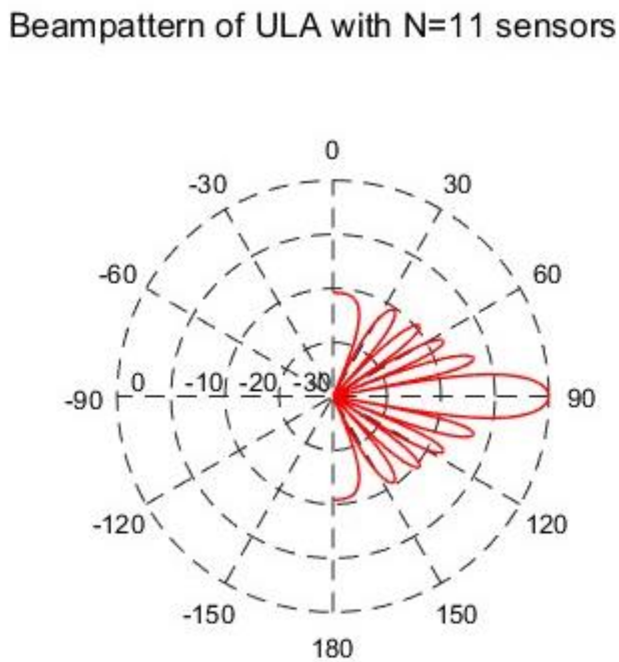


In a ULA with an even number of sensors, there is a null directly at endfire (0°) of the array, because each sensor cancels another one by one. Furthermore, it can be observed that the main lobe is narrower with 11 sensors, and this is because the more sensors we have, narrower the main lobe will be.

b) Polar plot of the powerpattern for $N = 10$:

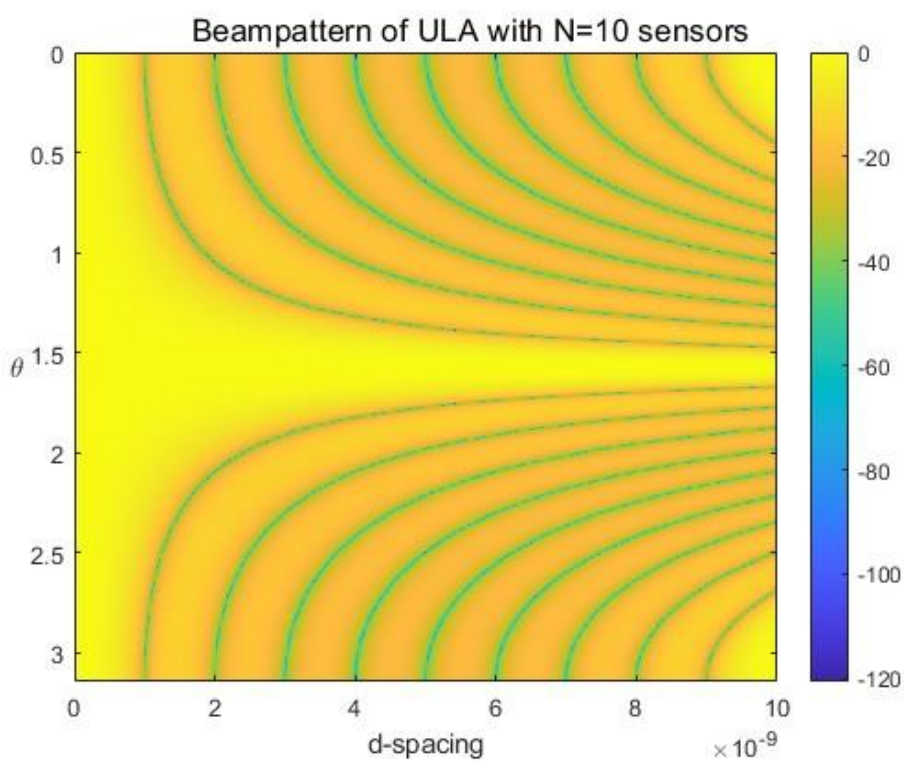


Polar plot of the powerpattern for $N = 11$:



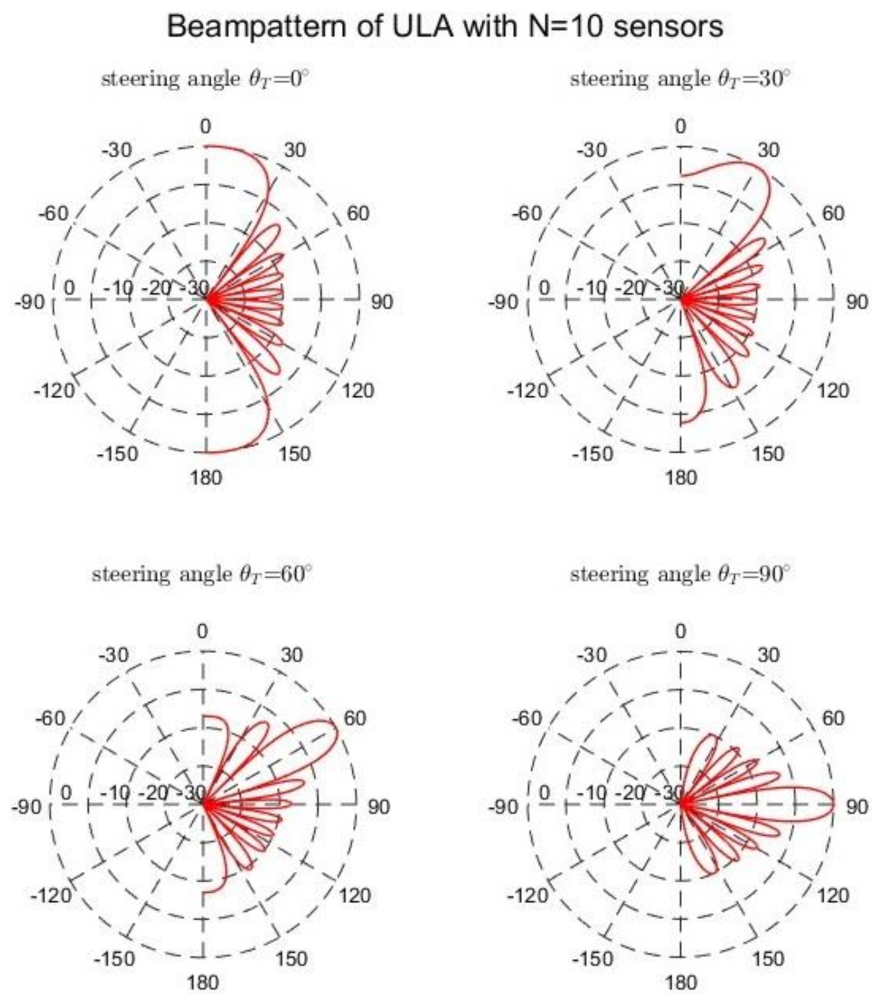
It can be observed that there are more side lobes for $N = 11$ sensors (5 instead of 4), thus the sidelobes becomes narrower. There is also a null at endfire direction as we mentioned before.

c) Powerpattern in Θ-space depending on d and θ for $0.001\lambda \leq d \leq \lambda$:



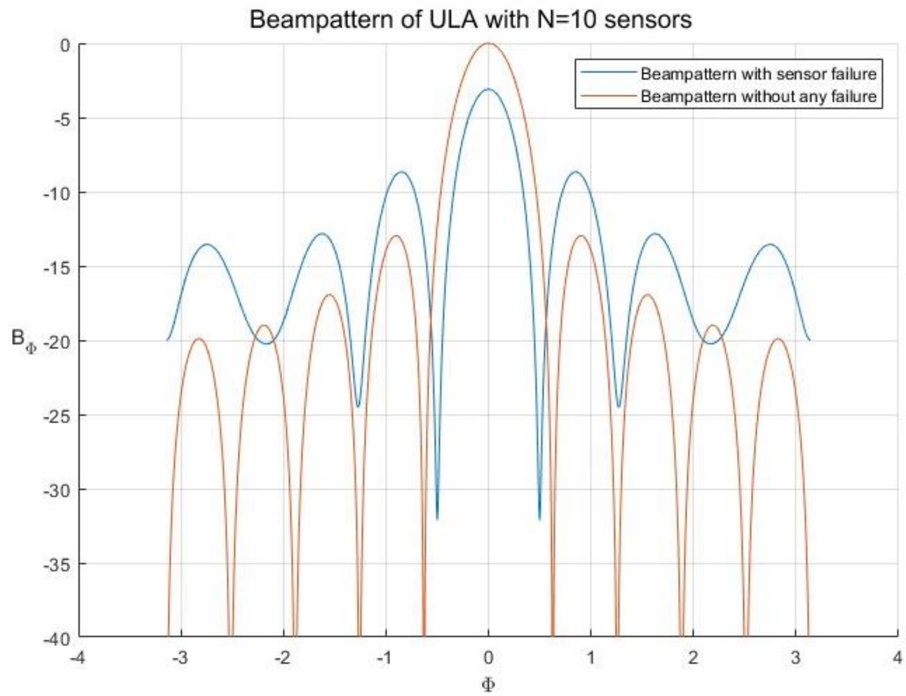
At broadside orientation, the more the distance between sensors becomes smaller, the more the main lobe becomes wider. The number of nulls and side lobes also drops with smaller d.

d) Powerpattern in Θ-space for steering angles $\Theta_T = 0°, 30°, 60°, 90°$:



It can be observed that the main beam becomes wider the more the array is steered.

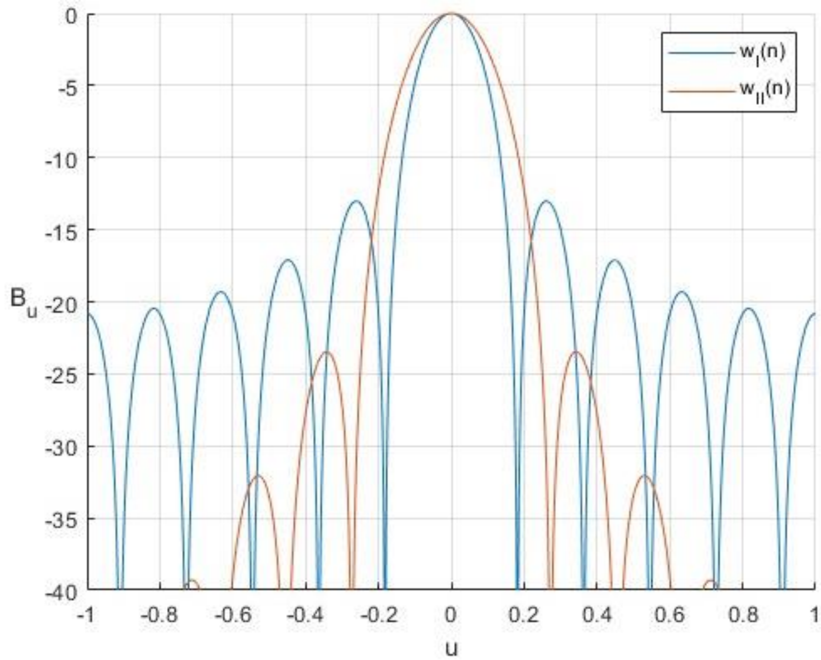
e) Powerpattern if the Φ-space with $w_n = 0$ for $n = 3, 5, 6$:



The sensor failure degrades the array beamforming. We can observe that the main lobe becomes lower compared to the sidelobes, which ones becomes higher. The nulls have disappeared, and the number of side lobes have decreased too.

Exercise 2: Non-uniform Weighting

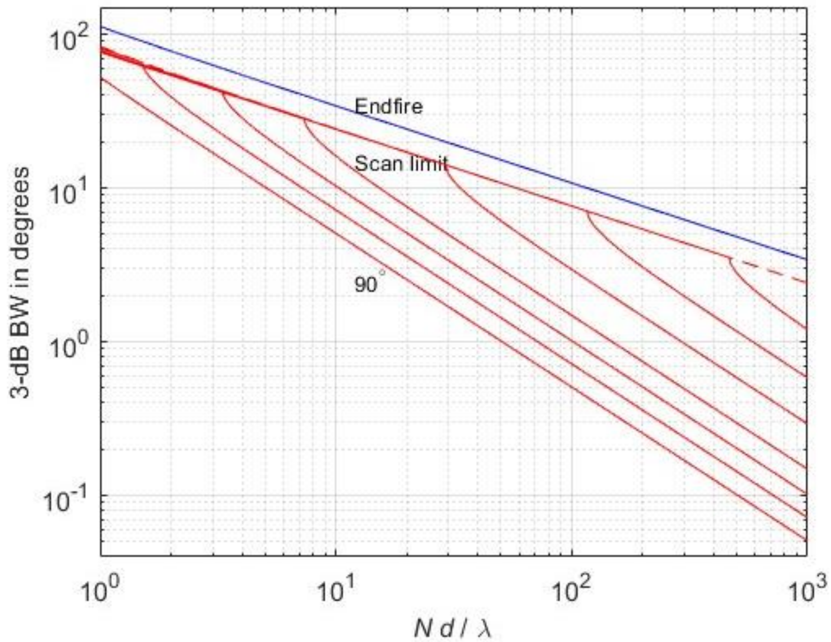
a) Magnitude (in dB) of the beampattern in the u-space with weights $w_I(n) = \frac{1}{N}$ and $w_{II}(n) = \sin\left(\frac{\pi}{2N}\right)\cos\left(\frac{\pi n}{N}\right)$:



It can be observed that the sidelobes have been reduced but the main lobe is wider.

Exercise 3: Half-Power Bandwidth

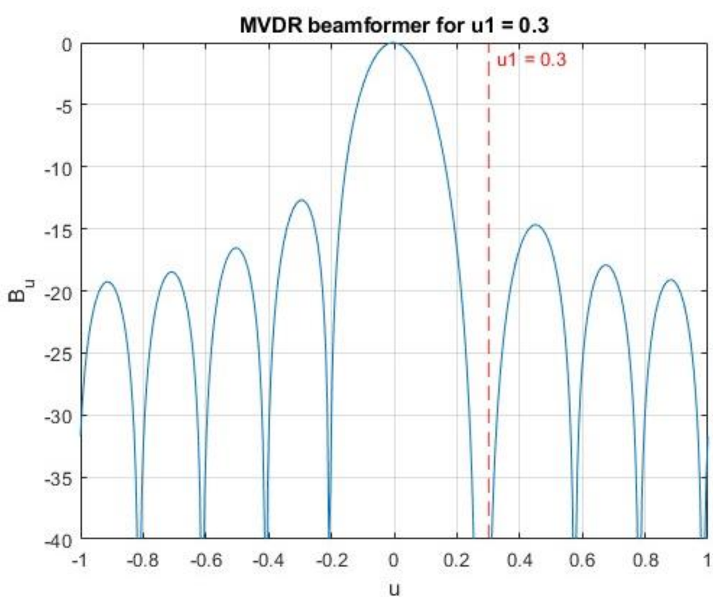
b) Half-Power beamwidth for the scan limit and steering angles from 2.5° to 90°.

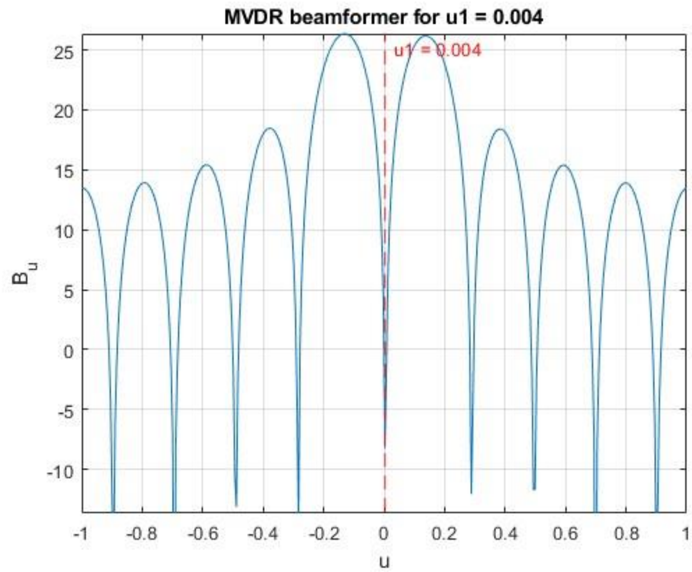


When the beam is not steered ($\theta = 90°$), the half-power beamwidth will be the smaller. The half-power beamwidth is getting bigger the more the array is steered and the more N or d is getting smaller.

Exercise 4: Beampattern of MVDR Beamformer

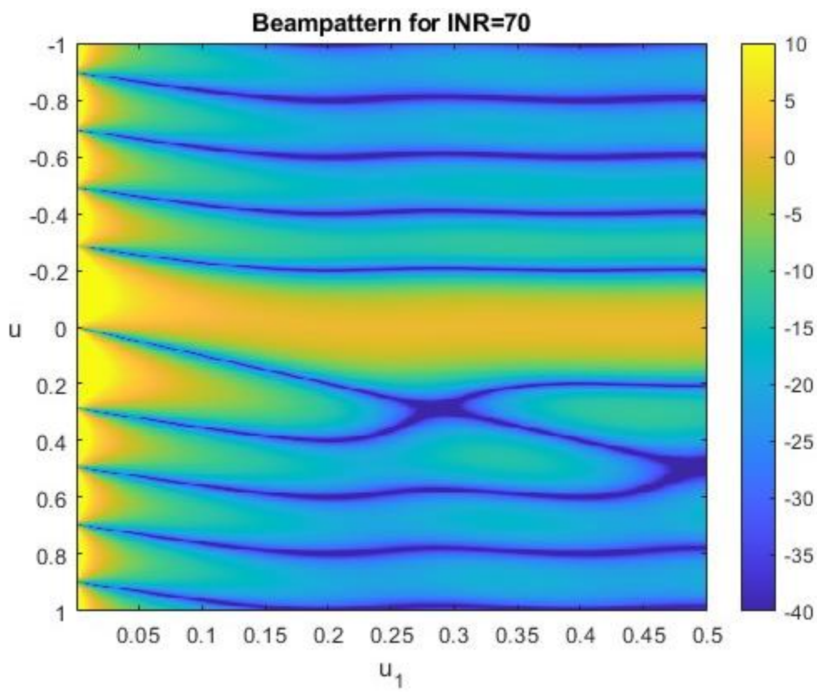
a) Plot of the beampattern in the u-domain for $u_1 = 0.3$ and $u_1 = 0.004$, for $INR = 70\ dB$:

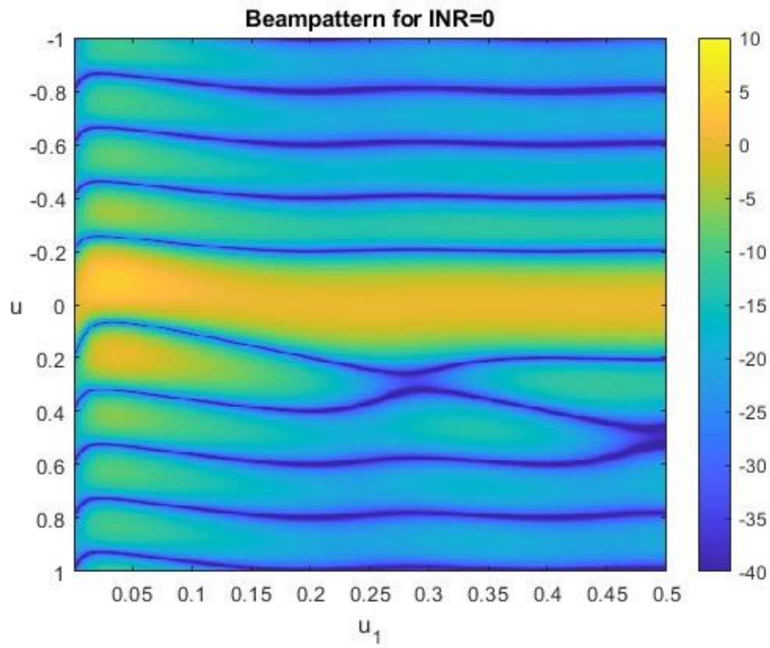




We must mention that for $u_1 \approx u_0$ (steering direction), the power pattern is exploding ($> 25dB$): the constraint we defined necessitates the beam to satisfy two conditions simultaneously: $w^H v_s = 1$ and $w^H v_1 = 0$.

b) Plot of the powerpattern with $0.001 \leq u_1 \leq 0.5$, for $INR = 70\ dB$ and $INR = 0\ dB$:





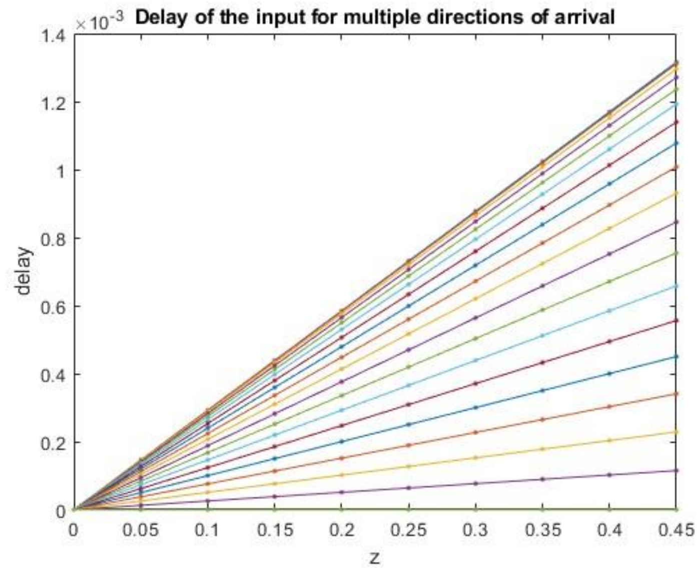
The direction of the interferer influences the beampattern by bending all the sidelobes in the u-space: We've got wave-like lines instead of straight lines.

For $INR = 0$, the interference isn't "strong enough" to break the main lobe (there is no null at zero for $u_1 = 0$), whereas for $INR = 70$, the blue line of the interference is not influenced by the isotropic noise (it's a straight diagonal line).

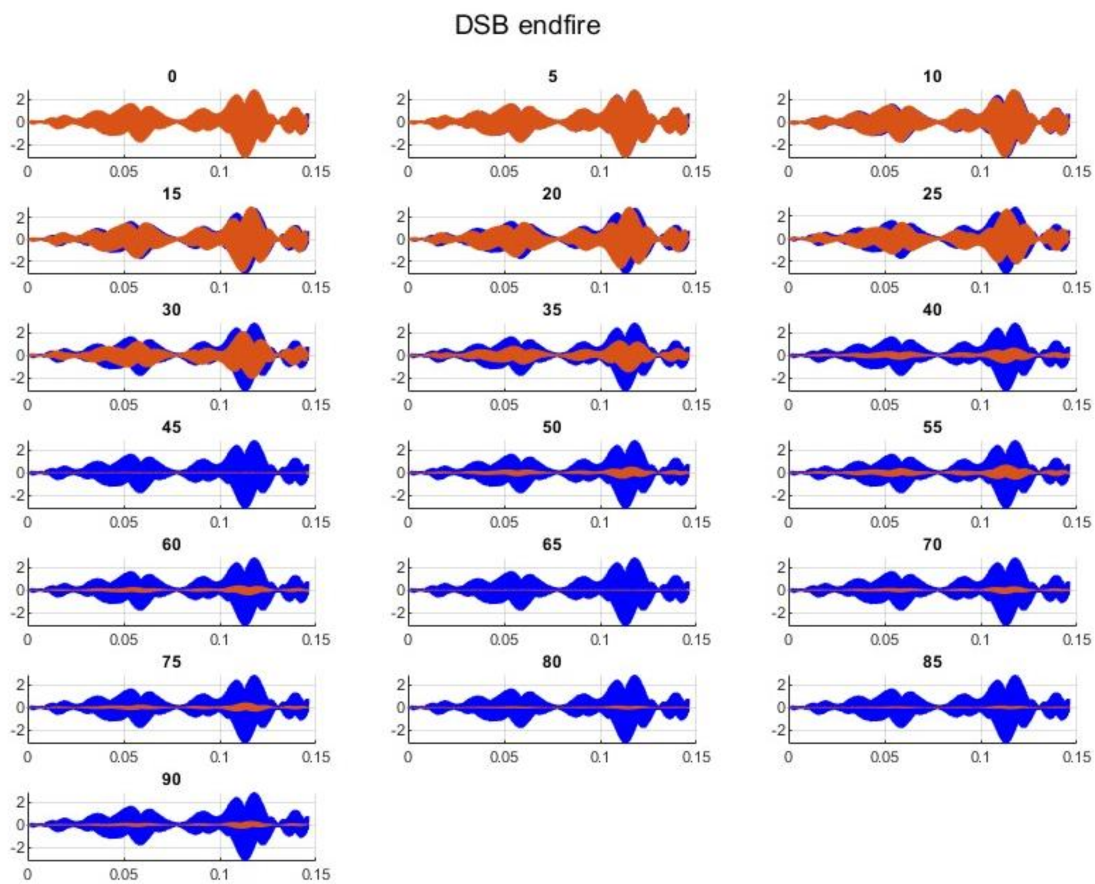
Exercise 5: Delay-and-Sum Beamformer for Narrowband Signals

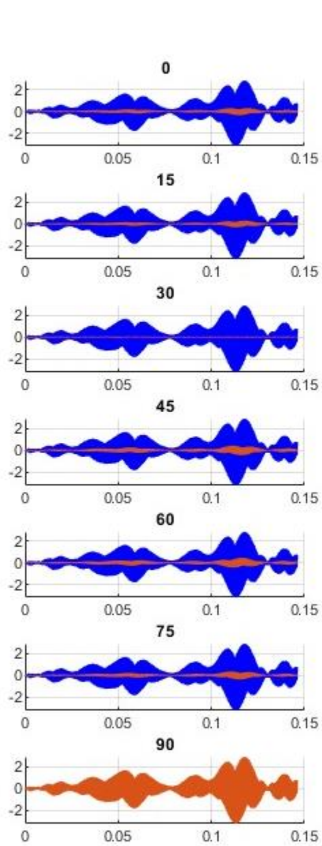
For this exercise, we'll use the provided functions `NB_signal` and `polardb`, but we are not using the provided function `frac_delay` to delay the signals. Instead, we'll use the integrated MATLAB function `delayseq` (same results, much better performance).

a) We generate the delays as function of θ and z (the position of the sensor). You can see the different delays below.

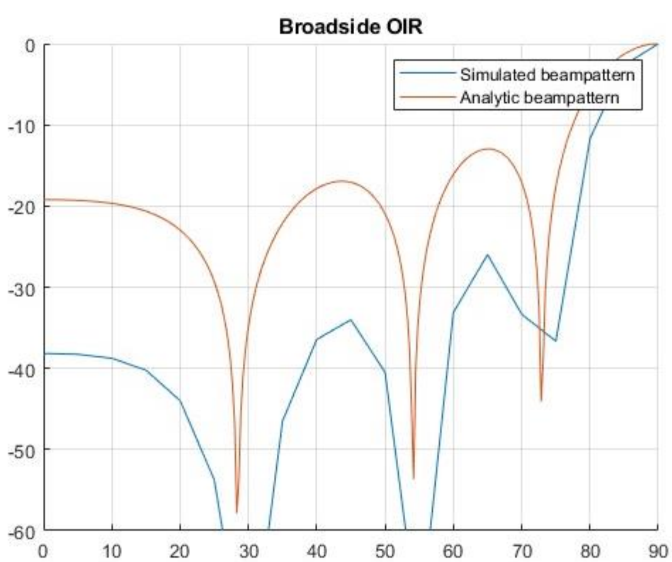


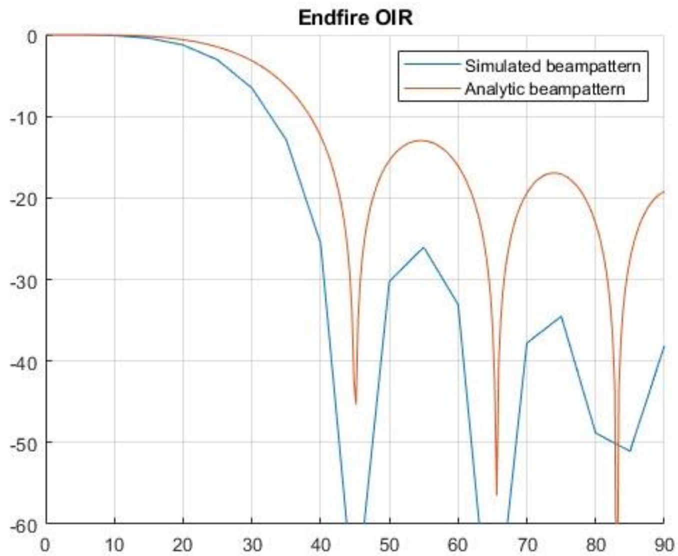
b) For two steering angles (endfire and broadside), we're computing the DSB for directions-of-arrival from 0° to 90°. The original signal and the DSP are drawn in blue and orange respectively.





c) We are using the ratio of the simulated signals to compute the OIR and will compare it with the analytic beampattern computed in exercise 1.





We can clearly observe that the simulated beampattern is following the same pattern as the analytically computed.