תרגיל בית 2 בלמידת מכונה

1. Show that $g'(z)=g(z)\big(1-g(z)\big)$

$$g'(z)=\frac{\partial}{\partial z}\left(\frac{1}{1+e^{-z}}\right)=-\frac{-e^{-z}}{(1+e^{-z})^2}=\frac{1}{1+e^{-z}}\frac{e^{-z}}{1+e^{-z}}=g(z)\frac{e^{-z}+1-1}{1+e^{-z}}$$
$$=g(z)\frac{(1+e^{-z})-1}{1+e^{-z}}=g(z)\big(1-g(z)\big)$$

2. Prove that $\frac{\partial l(w)}{\partial w}=\frac{1}{N}\sum_{t=1}^{N}\big(y_t-g(wx_t)\big)\vec{x}_t$

$$\frac{\partial l(w)}{\partial w}=\frac{\partial}{\partial w}\left(\frac{1}{N}\sum_{t=1}^{N}\log p(y_t|x_t)\right)=\frac{1}{N}\sum_{t=1}^{N}\frac{\partial}{\partial w}(\log p(y_t|x_t))$$
$$=\frac{1}{N}\sum_{t=1}^{N}\frac{\partial}{\partial w}\log\left(g(wx)^y\big(1-g(wx)\big)^{1-y}\right)$$

$$=\frac{\frac{\partial}{\partial w}\left(g(wx)^y\big(1-g(wx)\big)^{1-y}\right)}{g(wx)^y\big(1-g(wx)\big)^{1-y}}$$

$$=\frac{\left(\frac{\partial}{\partial w}(g(wx)^y)\big(1-g(wx)\big)^{1-y}+g(wx)^y\frac{\partial}{\partial w}\big(1-g(wx)\big)^{1-y}\right)}{g(wx)^y\big(1-g(wx)\big)^{1-y}}$$

$$=\frac{\big(yg(wx)^{y-1}g'(wx)\big)\big(1-g(wx)\big)^{1-y}+g(wx)^y(1-y)\big(1-g(wx)\big)^{-y}g'(wx)}{g(wx)^y\big(1-g(wx)\big)^{1-y}}$$

$$=\big(yg(wx)^{-1}g'(wx)\big)+(1-y)\big(1-g(wx)\big)^{-1}g'(wx)$$
$$=\left(yg(wx)^{-1}+(1-y)\big(1-g(wx)\big)^{-1}\right)g'(wx)$$
$$=\left(\frac{y}{g(wx)}+\frac{1-y}{1-g(wx)}\right)g'(wx)$$
$$=\left(\frac{y}{g(wx)}+\frac{1-y}{1-g(wx)}\right)x\left(g(wx)\big(1-g(wx)\big)\right)$$
$$=\left(y\big(1-g(wx)\big)+(1-y)g(wx)\right)x$$
$$=\big(y-yg(wx)+g(wx)-yg(wx)\big)x$$
$$=\big(y-2yg(wx)+g(wx)\big)x$$