

Algoritmos Numéricos

Exercício 02

1. Soluções dos Sistemas $Ax = b$

a)

$A = \begin{bmatrix} 10 & 2 & -3; \\ 1 & 8 & -1; \\ 2 & -1 & -5; \end{bmatrix}$
 $x = [x_1; x_2; x_3];$
 $b = [48; 4; -11];$

Jacobi	Gauss-Seidel
<pre> n = 3 A = 10 2 -3 1 8 -1 2 -1 -5 b = 48 4 -11 Toler = 1.0000e-06 IterMax = 100 Solução de sistema linear pelo método de Jacobi Iter = 1, x = [4.8 0.5 2.2], NormaRel = 1 Iter = 2, x = [5.36 0.175 4.02], NormaRel = 0.339552 Iter = 3, x = [5.971 0.3325 4.309], NormaRel = 0.102328 Iter = 4, x = [6.0262 0.29225 4.5219], NormaRel = 0.0353291 Iter = 5, x = [6.09812 0.311963 4.55203], NormaRel = 0.0117938 Iter = 6, x = [6.10322 0.306739 4.57686], NormaRel = 0.00406761 Iter = 7, x = [6.11171 0.309205 4.57994], NormaRel = 0.00138953 Iter = 8, x = [6.11214 0.308529 4.58284], NormaRel = 0.000475077 Iter = 9, x = [6.11315 0.308938 4.58315], NormaRel = 0.00016462 Iter = 10, x = [6.11318 0.30875 4.58349], NormaRel = 5.57386e-05 Iter = 11, x = [6.1133 0.308789 4.58352], NormaRel = 1.95773e-05 Iter = 12, x = [6.1133 0.308778 4.58356], NormaRel = 6.56257e-06 Iter = 13, x = [6.11331 0.308783 4.58356], NormaRel = 2.33677e-06 Iter = 14, x = [6.11331 0.308781 4.58357], NormaRel = 7.75363e-07 x = 6.1133 0.3088 4.5836 Iter = 14 Info = 0 r = 1.7114e-05 4.9008e-06 -1.1253e-06 raphael@raphael-ubuntu:~/Downloads/AlgNum/Ex02\$ </pre>	<pre> n = 3 A = 10 2 -3 1 8 -1 2 -1 -5 b = 48 4 -11 Toler = 1.0000e-06 IterMax = 100 Solução de sistema linear pelo método de Gauss-Seidel Iter = 1, x = [4.8 -0.1 4.14], NormaRel = 1 Iter = 2, x = [6.062 0.25975 4.57285], NormaRel = 0.208182 Iter = 3, x = [6.11991 0.306618 4.58664], NormaRel = 0.00946175 Iter = 4, x = [6.11467 0.308996 4.58407], NormaRel = 0.000856484 Iter = 5, x = [6.11342 0.308831 4.5836], NormaRel = 0.000203942 Iter = 6, x = [6.11331 0.308786 4.58357], NormaRel = 1.74363e-05 Iter = 7, x = [6.11331 0.308782 4.58357], NormaRel = 6.65329e-07 x = 6.1133 0.3088 4.5836 Iter = 7 Info = 0 r = 9.2277e-06 3.6433e-07 -1.7764e-15 </pre>

b)

$A = \begin{bmatrix} 10 & 2 & -3 & 5; \\ 1 & 8 & -1 & 2; \\ 2 & -1 & -5 & 1; \\ -1 & 2 & 3 & 20; \end{bmatrix}$
 $x = [x_1; x_2; x_3; x_4];$
 $b = [48; 4; -11; 150];$

Jacobi	Gauss-Seidel
<pre> n = 4 A = 10 2 -3 5 1 8 -1 2 2 -1 -5 1 -1 2 3 20 b = 48 4 -11 150 Toler = 1.0000e-06 IterMax = 100 Solução de sistema linear pelo método de Jacobi Iter = 1, x = [4.8 0.5 2.2 7.5], NormaRel = 1 Iter = 2, x = [1.61 -1.7 5.52 7.36], NormaRel = 0.451007 Iter = 3, x = [3.116 -0.85125 4.656 6.9225], NormaRel = 0.217551 Iter = 4, x = [2.9058 -1.03012 5.00115 7.04253], NormaRel = 0.0490094 Iter = 5, x = [2.90671 -0.998713 4.97845 6.99893], NormaRel = 0.01156 Iter = 6, x = [2.99381 -1.00076 4.99421 7.00244], NormaRel = 0.00225086 Iter = 7, x = [2.9972 -1.00056 4.99817 7.00064], NormaRel = 0.000564843 Iter = 8, x = [2.99924 -1.00004 4.99912 7.00019], NormaRel = 0.000292449 Iter = 9, x = [2.99965 -1.00006 4.99974 7.0001], NormaRel = 8.9365e-05 Iter = 10, x = [2.99989 -1.00001 4.99989 7.00003], NormaRel = 3.41655e-05 Iter = 11, x = [2.99996 -1.00001 4.99996 7.00001], NormaRel = 1.01798e-05 Iter = 12, x = [2.99998 -1 4.99999 7], NormaRel = 3.967e-06 Iter = 13, x = [2.99999 -1 4.99999 7], NormaRel = 1.45463e-06 Iter = 14, x = [3 -1 5 7], NormaRel = 5.15385e-07 x = 3.0000 -1.0000 5.0000 7.0000 Iter = 14 Info = 0 r = 1.3641e-05 1.5502e-06 -5.8149e-06 -7.1347e-06 </pre>	<pre> n = 4 A = 10 2 -3 5 1 8 -1 2 2 -1 -5 1 -1 2 3 20 b = 48 4 -11 150 Toler = 1.0000e-06 IterMax = 100 Solução de sistema linear pelo método de Gauss-Seidel Iter = 1, x = [4.8 -0.1 4.14 7.129], NormaRel = 1 Iter = 2, x = [2.4975 -1.07694 4.84019 7.00654], NormaRel = 0.328622 Iter = 3, x = [2.96417 -1.01713 4.9904 7.00136], NormaRel = 0.0666547 Iter = 4, x = [2.99987 -1.00152 5.00052 7.00007], NormaRel = 0.00509907 Iter = 5, x = [3.00043 -1 5.00019 6.99999], NormaRel = 0.000216914 Iter = 6, x = [3.00006 -0.999983 5.00002 7], NormaRel = 5.26551e-05 Iter = 7, x = [3 -0.999998 5 7], NormaRel = 8.07429e-06 Iter = 8, x = [3 -1 5 7], NormaRel = 4.86311e-07 x = 3.0000 -1.0000 5.0000 7.0000 Iter = 8 Info = 0 r = 2.0015e-06 -9.0938e-07 -1.5013e-07 0 </pre>

2. Comparação dos Números de Iterações Gastas e dos Vetores Resíduos

Sistema	Número k de iterações gastas		Norma de Máxima Coluna do Vetor Resíduo, $ r ^\infty$	
	Jacobi	Gauss-Seidel	Jacobi	Gauss-Seidel
a	14	7	1.7114e-5	9.2277e-6
b	14	8	1.3641e-5	2.0015e-6