Associação de Ensino Superior do Piauí

Ciência da Computação

Circuitos Lógicos Digitais

Prof. Me. Athos Denis

Roteiro da aula

Postulados da Álgebra booleanas;

- **Expressões auxiliares**

Mapa de Karnaugh;

Revisão

QUADRO RESUMO			
POSTULADOS			
COMPLEMENTAÇÃO		ADIÇÃO	MULTIPLICAÇÃO
A = 0	A’ = 1	0 + 0 = 0	0 * 0 = 0
A = 1	A’ = 0	0 + 1 = 1	0 * 1 = 0
		1 + 0 = 1	1 * 0 = 0
		1 + 1 = 1	1 * 1 = 1

Revisão

IDENTIDADES:		
COMPLEMENTAÇÃO	ADIÇÃO	MULTIPLICAÇÃO
A’’ = A	A + 0 = A	A * 0 = 0
	A + 1 = 1	A * 1 = A
	A + A = A	A * A = A
	A + A’ = 1	A * A’ = 0

Revisão

PROPRIEDADES:	
COMUTATIVA	A + B = B + A
	A * B = B * A
ASSOCIATIVA	A + (B + C) = (A + B) + C = A + B + C
	A * (B * C) = (A * B) * C = A * B * C
DISTRIBUTIVA	
	A * (B + C) = (A * B) + (A * C) / A + (B * C) = (A + B) * (A + C)

Revisão

TEOREMA DE DE MORGAN
(A + B)’ = A’ * B‘
(A * B)’ = A’ + B’
IDENTIDADES AUXILIARES
A + (A*B) = A / A * (A+B) = A - Absorção
A + (A’ * B) = A + B
(A + B) * (A + C) = A + (B * C)

Atividade:

Expressões auxiliares

1. $A + A * B = A$?

→ LEI DA ABSORÇÃO
- $A + (A * B)$
- A
- OU SEJA:

$A + A * B = A$

2. $\overline{A} + A * \overline{B} = \overline{A} + \overline{B}$?

- $(\overline{A} + A) \cdot (\overline{A} + \overline{B})$
- $1 \cdot (\overline{A} + \overline{B})$
- $(\overline{A} + \overline{B})$

OU SEJA:

$\overline{A} + A * B = \overline{A} + \overline{B}$

3. $\overline{A} + \overline{A} * \overline{B} = \overline{A}$?

- $(\overline{A} + \overline{A}) * (\overline{A} + \overline{B})$
- $\overline{A} \cdot (\overline{A} + \overline{B})$ LEI DA ABSORÇÃO
- $\overline{A}$

OU SEJA

$\overline{A} + \overline{A} * \overline{B} = \overline{A}$

Atividade:

Expressões auxiliares

4. $A+\overline{A}*B=A+B$?

- $(A+\overline{A})*(A+B)$
- $1*(A+B)$
- $(A+B)$

ou seja:

$A+\overline{A}*B=A+B$

5. $\overline{A}+A*B=\overline{A}+B$?

- $(\overline{A}+A)*(\overline{A}+B)$
- $1\cdot(\overline{A}+B)$
- $(\overline{A}+B)$

ou seja:

$\overline{A}+A*B=\overline{A}+B$

6. $A+\overline{A}*\overline{B}=A+\overline{B}$?

- $(A+\overline{A})*(A+\overline{B})$
- $1\cdot(A+\overline{B})$
- $(A+\overline{B})$

ou seja:

$A+\overline{A}*\overline{B}=A+\overline{B}$

Atividade:

Expressões auxiliares

7. $\overline{A}+\overline{A}*B=\overline{A}$?

- $(\overline{A}+\overline{A})*(\overline{A}*B)$
- $\overline{A}*(\overline{A}*B)$ Lei ABS.
- $\overline{A}$

ou seja:

$\overline{A}+\overline{A}\cdot B=\overline{A}$

8. $A+A*\overline{B}=A$?

- $(A+A)*(A+\overline{B})$
- $A*(A+\overline{B})$
- A

ou seja:

$A+A*\overline{B}=A$

9. $(A+B)*(A+C)=A+B*C$?

- $A+(B*C)$

ou seja:

$(A+B)\cdot(A+C)=A+B\cdot C$

Atividade:

Exercício: Simplificar as expressões:

1. $(A+B+C)*(A+B+C)$

A • A

Logo:

$(A+B+C)$

2. $A*B+A(B+C)+B(B+C)$

$AB+AB+AC+BB+BC$ ID

$AB+AC+B+BC$ ABSORÇÃO

$AB+AC+B$ COMUTAÇÃO

$B+AB+AC$ ABSORÇÃO

$B+AC$

Atividade:

Exercício: Simplificar as expressões:

3. $A*B*C+A*\overline{C}+A*\overline{B}$ EVIDÊNCIA

$A(BC+\overline{C}+\overline{B})$ DE MORGAN

$A(BC+\overline{BC})$ ID.

$A(1)$

$\boxed{A}$

4. $(A+\overline{B})*(A+C)$

$AA+AC+\overline{B}A+\overline{B}C$ ID. $AA=A$

$(A+AC+\overline{B}A)+\overline{B}C$

$A(1+AC+\overline{B}A)+\overline{B}C$ ID. $1+A=1$

$A(1)+\overline{B}C$

$\boxed{A+\overline{B}C}$

Atividade:

Exercício: Simplificar as expressões:

5. $A*B*C(A*B+\overline{C}(B.C+A*C))$

$ABC(AB+(\overline{C}BC+\overline{C}AC))$ ID $A\bar{A}=0$

$ABC(AB+(B0+A0)$ ID $A\cdot 0=0$

$ABC(AB)$ ASSOCIATIVA

$AA\cdot BB\cdot C$ ID.

ABC

6. $(A+B)*\overline{(A+B)}$ IDENTIDADE

$A \cdot \bar{A}$

$S=0$

A	B	A+B	$\overline{A+B}$	S
0	0	0	1	0
0	1	1	0	0
1	0	1	0	0
1	1	1	0	0

Atividade:

Exercício: Simplificar as expressões:

07. $\overline{((A+B+C)*D)}$ DE MORGAN

$\overline{(A+B+C)} + \bar{D}$ DE MORGAN

$\bar{A}\bar{B}\bar{C} + \bar{D}$

8. $\overline{(A*B*C+D*E*F)}$ DE MORGAN

$\overline{(ABC)} \cdot \overline{(DEF)}$ DE MORGAN

$(\bar{A}+\bar{B}+\bar{C}) \cdot (\bar{D}+\bar{E}+\bar{F})$

Mapa de Karnaugh

A	B	S
0	0	**S1**
0	1	**S2**
1	0	**S3**
1	1	**S4**

A \ B	0	1
0	S1	S2
1	S3	S4

A	B	C	S
0	0	0	**S1**
0	0	1	**S2**
0	1	0	**S3**
0	1	1	**S4**
1	0	0	**S5**
1	0	1	**S6**
1	1	0	**S7**
1	1	1	**S8**

A \ BC	00	01	11	10
0	S1	S2	S4	S3
1	S5	S6	S8	S7

Mapa de Karnaugh

A \ BC	00	01	11	10
0	0	1	1	1
1	1	1	1	1

A	B	C	S
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**1**
1	0	0	**1**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

Para obter a menor expressão possível, pegamos o maior grupo possível de bits:

- **2 – Pares** **8 - Octetos**
- **4 - Quartetos**

Mapa de Karnaugh

A \ BC	00	01	11	10
0	0	1	1	1
1	1	1	1	1

A	B	C	S
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**1**
1	0	0	**1**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

Para obter a menor expressão possível, pegamos o maior grupo possível de bits:

- **2 – Pares** **8 - Octetos**
- **4 - Quartetos**

Mapa de Karnaugh

A \ BC	00	01	11	10
0	0	1	1	1
1	1	1	1	1

A	B	C	S
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**1**
1	0	0	**1**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

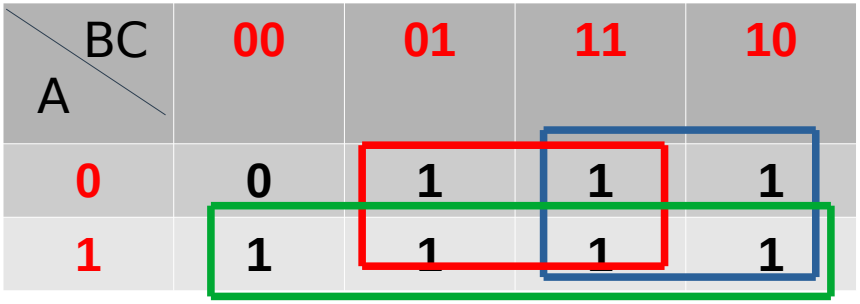
A = 0/1 B = 0/1 C = 1

A = 0/1 B = 1 C = 0/1

A = 1 B = 0/1 C = 0/1

Mapa de Karnaugh

A	B	C	S
0	0	0	**0**
0	0	1	**1**
0	1	0	**1**
0	1	1	**1**
1	0	0	**1**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**



~~A = 0/1~~ ~~B = 0/1~~ C = 1 C

~~A = 0/1~~ B = 1 ~~C = 0/1~~ B → A + B + C

A = 1 ~~B = 0/1~~ ~~C = 0/1~~ A

Mapa de Karnaugh

A \ BC	00	01	11	10
0	0	0	1	1
1	0	1	1	1

A	B	C	S
0	0	0	**0**
0	0	1	**0**
0	1	0	**1**
0	1	1	**1**
1	0	0	**0**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

Para obter a menor expressão possível, pegamos o maior grupo possível de bits:

- **2 – Pares** **8 - Octetos**
- **4 - Quartetos**

Mapa de Karnaugh

A \ BC	00	01	11	10
0	0	0	1	1
1	0	1	1	1

A	B	C	S
0	0	0	**0**
0	0	1	**0**
0	1	0	**1**
0	1	1	**1**
1	0	0	**0**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

A = 0/1 B = 1 C = 0/1

Mapa de Karnaugh

A \ BC	00	01	11	10
0	0	0	1	1
1	0	1	1	1

A	B	C	S
0	0	0	**0**
0	0	1	**0**
0	1	0	**1**
0	1	1	**1**
1	0	0	**0**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

A = 0/1 **B = 1** **C = 0/1**

A = 1 **B = 1/0** **C = 1**

Mapa de Karnaugh

A	B	C	S
0	0	0	**0**
0	0	1	**0**
0	1	0	**1**
0	1	1	**1**
1	0	0	**0**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

A \ BC	00	01	11	10
0	0	0	1	1
1	0	1	1	1

~~A = 0/1~~ B = 1 ~~C = 0/1~~ B

A = 1 ~~B = 1/0~~ C = 1 AC → B + AC

Mapa de Karnaugh

A \ BC	00	01	11	10
0	0	0	0	0
1	1	1	1	1

A	B	C	S
0	0	0	**0**
0	0	1	**0**
0	1	0	**0**
0	1	1	**0**
1	0	0	**1**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

Mapa de Karnaugh

A \ BC	00	01	11	10
0	0	0	0	0
1	1	1	1	1

A	B	C	S
0	0	0	**0**
0	0	1	**0**
0	1	0	**0**
0	1	1	**0**
1	0	0	**1**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

A = 1 **B = 0/1** **C = 0/1**

Mapa de Karnaugh

A \ BC	00	01	11	10
0	0	0	0	0
1	1	1	1	1

A	B	C	S
0	0	0	**0**
0	0	1	**0**
0	1	0	**0**
0	1	1	**0**
1	0	0	**1**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

A = 1 ~~**B = 0/1**~~ ~~**C = 0/1**~~ → **A**

Mapa de Karnaugh

A \ BC	00	01	11	10
0	0	1	0	0
1	1	1	1	1

A	B	C	S
0	0	0	**0**
0	0	1	**1**
0	1	0	**0**
0	1	1	**0**
1	0	0	**1**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

Mapa de Karnaugh

A	B	C	S
0	0	0	**0**
0	0	1	**1**
0	1	0	**0**
0	1	1	**0**
1	0	0	**1**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

A \ BC	00	01	11	10
0	0	1	0	0
1	1	1	1	1

A = 1 **~~B = 0/1~~** **~~C = 0/1~~** **A**

Mapa de Karnaugh

A	B	C	S
0	0	0	**0**
0	0	1	**1**
0	1	0	**0**
0	1	1	**0**
1	0	0	**1**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

A \ BC	00	01	11	10
0	0	1	0	0
1	1	1	1	1

A = 1 ~~**B = 0/1**~~ ~~**C = 0/1**~~ **A**

~~**A = 1/0**~~ **B = 0** **C = 1** **B’C**

Mapa de Karnaugh

A	B	C	S
0	0	0	**0**
0	0	1	**1**
0	1	0	**0**
0	1	1	**0**
1	0	0	**1**
1	0	1	**1**
1	1	0	**1**
1	1	1	**1**

A \ BC	00	01	11	10
0	0	1	0	0
1	1	1	1	1

A = 1 ~~**B = 0/1**~~ ~~**C = 0/1**~~ **A**

~~**A = 1/0**~~ **B = 0** **C = 1** **B'C** **A + B'C**

Mapa de Karnaugh

A \ BC	00	01	11	10
0	0	0	0	0
1	0	0	1	0

A	B	C	S
0	0	0	**0**
0	0	1	**0**
0	1	0	**0**
0	1	1	**0**
1	0	0	**0**
1	0	1	**0**
1	1	0	**0**
1	1	1	**1**

Mapa de Karnaugh

A \ BC	00	01	11	10
0	0	0	0	0
1	0	0	1	0

A	B	C	S
0	0	0	**0**
0	0	1	**0**
0	1	0	**0**
0	1	1	**0**
1	0	0	**0**
1	0	1	**0**
1	1	0	**0**
1	1	1	**1**

A = 1 **B = 1** **C =1** **ABC**

Mapa de Karnaugh

A	B	C	D	S
0	0	0	0	**0**
0	0	0	1	**0**
0	0	1	0	**0**
0	0	1	1	**1**
0	1	0	0	**1**
0	1	0	1	**1**
0	1	1	0	**0**
0	1	1	1	**0**
1	0	0	0	**1**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**0**
1	1	1	1	**1**

CD / AB	00	01	11	10
00	0	0	1	0
01	1	1	0	0
11	0	0	1	0
10	1	1	1	1

Mapa de Karnaugh

CD AB	00	01	11	10
00	0	0	1	0
01	1	1	0	0
11	0	0	1	0
10	1	1	1	1

A = 1 ~~B = 0/1~~ C =1 D = 1 → ACD

A	B	C	D	S
0	0	0	0	**0**
0	0	0	1	**0**
0	0	1	0	**0**
0	0	1	1	**1**
0	1	0	0	**1**
0	1	0	1	**1**
0	1	1	0	**0**
0	1	1	1	**0**
1	0	0	0	**1**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**0**
1	1	1	1	**1**

Mapa de Karnaugh

CD AB	00	01	11	10
00	0	0	1	0
01	1	1	0	0
11	0	0	1	0
10	1	1	1	1

A = 1 ~~B = 0/1~~ C =1 D = 1 → ACD

A = 1 B = 0 ~~C = 0/1~~ ~~D = 0/1~~ → AB'

A	B	C	D	S
0	0	0	0	**0**
0	0	0	1	**0**
0	0	1	0	**0**
0	0	1	1	**1**
0	1	0	0	**1**
0	1	0	1	**1**
0	1	1	0	**0**
0	1	1	1	**0**
1	0	0	0	**1**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**0**
1	1	1	1	**1**

Mapa de Karnaugh

AB \ CD	00	01	11	10
00	0	0	1	0
01	1	1	0	0
11	0	0	1	0
10	1	1	1	1

A = 1 B = ~~0/1~~ C =1 D = 1 → ACD

A = 1 B = 0 ~~C = 0/1~~ ~~D = 0/1~~ → AB'

A= 0 B = 1 C = 0 ~~D = 0/1~~ → A'BC

A	B	C	D	S
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

Mapa de Karnaugh

AB \ CD	00	01	11	10
00	0	0	1	0
01	1	1	0	0
11	0	0	1	0
10	1	1	1	1

A = 1 ~~B = 0/1~~ C =1 D = 1 → ACD

A = 1 B = 0 ~~C = 0/1~~ ~~D = 0/1~~ → AB'

A = 0 B = 1 C = 0 ~~D = 0/1~~ → A'BC'

~~A = 0/1~~ B = 0 C = 1 D = 1 → B'CD

A	B	C	D	S
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

Mapa de Karnaugh

CD / AB	00	01	11	10
00	0	0	1	0
01	1	1	0	0
11	0	0	1	0
10	1	1	1	1

A = 1 B = 0/1 C =1 D = 1 → ACD

A = 1 B = 0 C = 0/1 D = 0/1 → AB'

A = 0 B = 1 C = 0 D = 0/1 → A'BC'

A = 0/1 B = 0 C = 1 D = 1 → B'CD

ACD + AB' + A'BC' + B'CD

A	B	C	D	S
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1