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Dynamic Models for Building Energy Management
Smart Cities

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1. Introduction

The purpose of this study is to understand a building thermal network. In order to analyse the energy management of dynamic systems, the study presents a thermal circuit modelling, a steady-state response of the system of Differential Algebraic Equations (DAE), a step response and the integration with weather data.

2. Assignment 1: Model

a. Description of the building

The first step is to design a two zone building, which is composed of two rooms, with one window each, one door to the outside and one connecting both rooms.

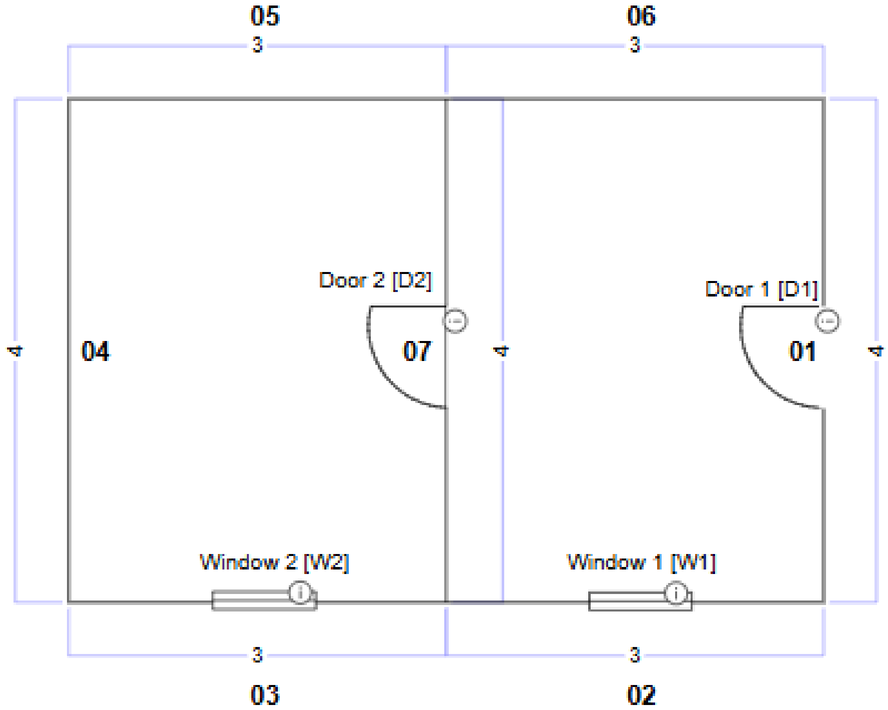


Fig. 1: Apartment floor plan with id and dimensions

id	Type	Material	Height (m)	Lenght (m)	Surface (m^2)
1	Wall	Concrete + Insulation	3	4	12
1	Door	Wood	2	1	2
2	Wall	Concrete + Insulation	3	3	9
2	Window	Double glazing	1	1	1
3	Wall	Concrete + Insulation	3	3	9
3	Window	Double glazing	1	1	1
4	Wall	Concrete + Insulation	3	4	12
5	Wall	Concrete + Insulation	3	3	9
6	Wall	Concrete + Insulation	3	3	9
7	Wall	Concrete	3	4	12
7	Door	Wood	2	1	2

Table 3: Parameters for each wall

	Thermal conductivity	Density	Specific heat	Width	Surface
	W/(mK)	kg/m^3	J/(kgK)	m	m^2
Concrete	1.4	2300	880	0.2	9 or 12
Insulation	0.04	16	1210	0.08	9 or 12
Window	1.4 (U-value)	2500	1210	0.04	1
Door	0.9 (U-value)	314	2380	0.044	2

Table 2: Materials parameters

b. Hypothesis

On boundary conditions, we take into account that walls 1,2 and 3 have solar irradiation. Moreover, walls 4, 5 and 6 are adiabatic, as if they are one that has no heat transfer. Doors 1 and 2 are closed. Along with that, the apartment is isolated so there is no heat transfer through the floor or ceiling. With all those hypotheses we succeed in planning the convection and conduction presence on each wall.

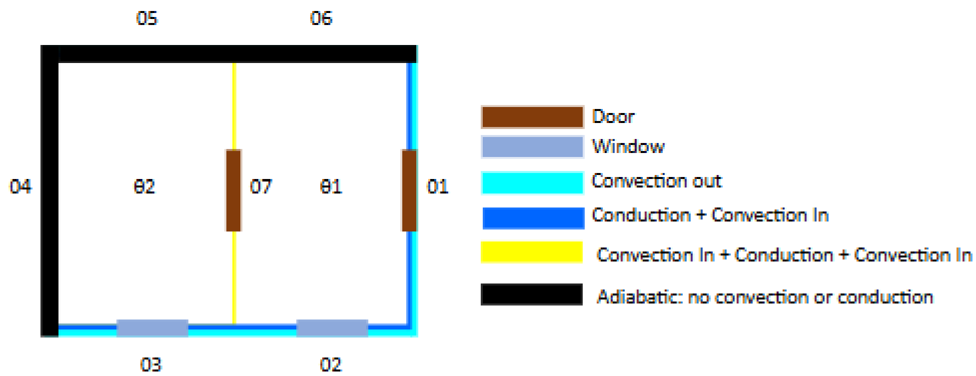


Fig. 3: Walls forming thermal conductances in the model

c. Thermal model

It is important to remember that conduction is the heat transfer through direct contact between materials, however convection is the phenomenon through the movement of fluids or gases. To represent the constitutive laws and energy balance we use the method of nodes for each temperature and discretization of heat equation for a plan wall from the application of finite element method.

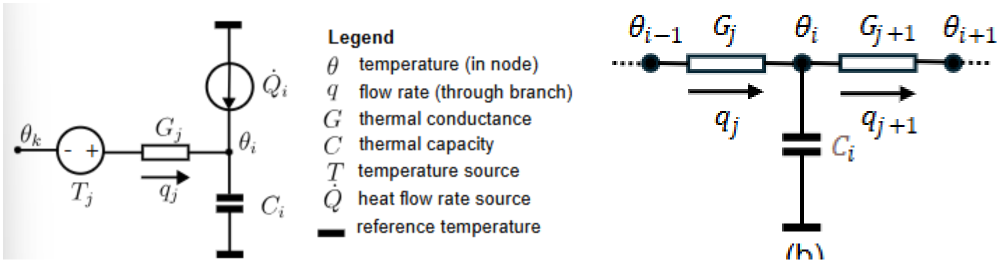


Fig. 4: Basic thermal network on the left and thermal circuit for plan wall on the right

Applying the theory for all the walls non adiabatic, we have the thermal circuit with temperature nodes, flow-rate paths, sources of temperature (in yellow), thermal conductances for conduction (in red), convection (in green), long-wave radiation (in blue) and P-controllers (in black).

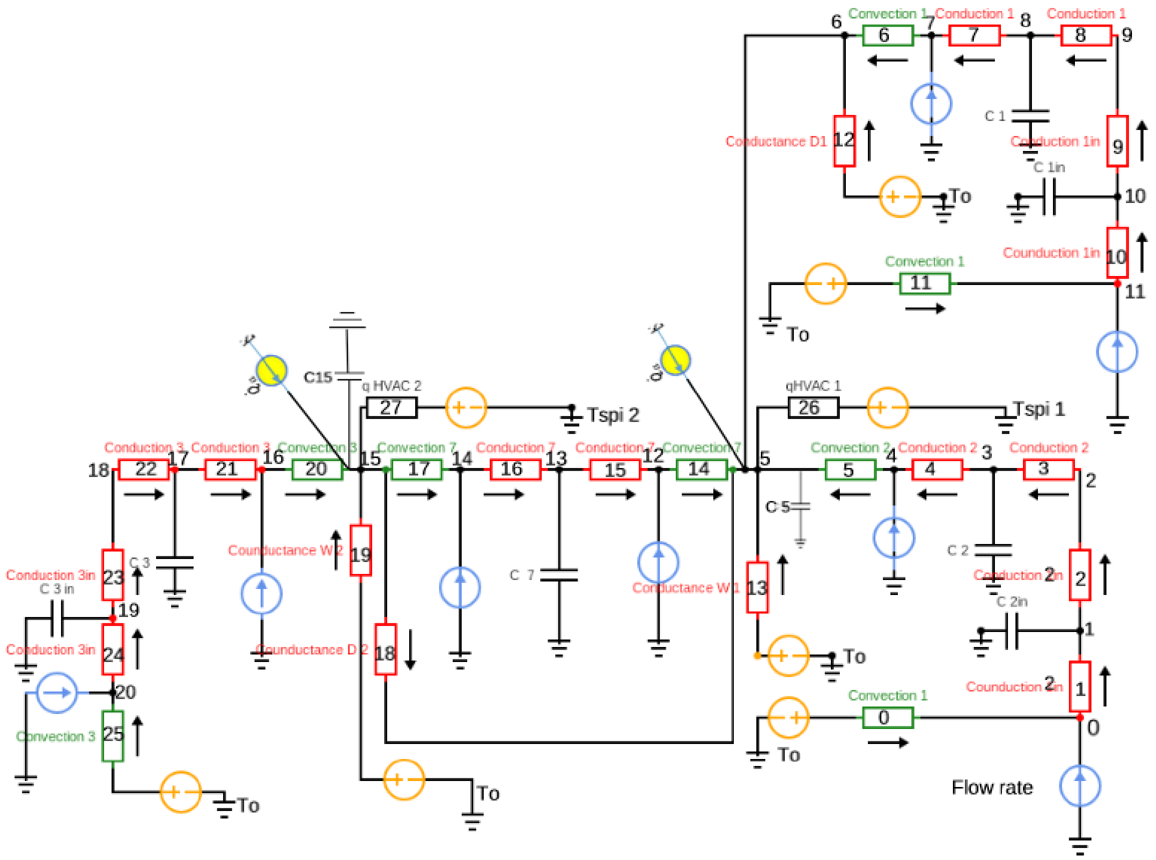


Fig. 5: Thermal network model for the building

For better understanding is valid to clarify that from nodes 0 to 5 [0,1,2,3,4,5] is wall/door 2, from nodes 5 to 11 [5,6,7,8,9,10,11] is wall/door 1, between nodes 5 and 15 [5,12,16,14,15] is the wall/door 7, and at last from 15 to 20 is wall/window 3.

For outdoor walls 1,2 and 3 we have a combination of insulation plus concrete while for indoor wall 7 we have only concrete. On all walls we have conduction and convection passing through, although for the cases of door and window we have an overall conductance represented for the U-value.

With the objective of energy management, there is one controller for each room to control the temperature. Those temperatures are on node 5 and node 15.

d. Mathematical model

After the thermal circuit was done, we passed to the calculation of thermal conductances and thermal capacities.

$$Conduction\ conductance\ =\ \frac{\lambda * S}{\omega}\ (W/K)$$

$$Convection\ conductance\ =\ h\ *\ S\ (W/K)$$

$$Thermal\ capacity\ =\ \rho\ *\ c\ *\ \omega\ *\ S\ (J/K)$$

$$Heat\ flow\ rate\ by\ advection\ =\ \rho_a\ *\ c_a\ *\ \frac{ACH * V_a}{3600}\ *\ (T_{out}\ -\ \theta_{in})\ (W)$$

$$Heat\ flow\ \ rate\ by\ controller\ =\ K_p\ *\ (T_{i,sp}\ -\ \theta_{in})\ (W)$$

$$Irradiation\ absorbed\ by\ external\ wall\ =\ \alpha\ *\ S\ *\ E\ (W)$$

With λ = thermal conductivity (W/m.K) , S = area (m²), ω = width of the wall (m), ρ = density (kg/m³), c = specific heat capacity (J/kg.K) and h = convection coefficient (W/m².K). With the h_indoor = 4 W/m.²K and h_outdoor = 10 W/m.²K. Moreover, ACH = air changes per hour (1/h), V_a = air volume (m³), K_p = proportional gain (W/K), α = absorptance and E = solar irradiance (W/m²).

To model our building, these equations will be used on the three matrices of the Algebraic Differential Equations (DAE): incidence A, conductance G and capacity C. Along with that two vectors: temperature sources b, flow-rate sources f and output vector y.

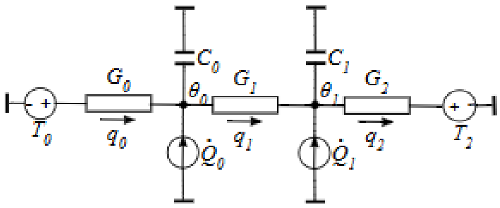
Matrix A [28x21] correlates the flow rates and nodes. Each line is a flow rate and each column is a node. A[i,j] is then the relation between flow i and node j, that can be:

- 0, if the flow is not connected to node
- +1, if the flow enters into node
- -1 if the flows gets out of the node

Matrix G [28x28] is the diagonal matrix that contains the conductances of each flow starting from node 0 that can be for the wall outdoor convection, conduction and indoor convection. As well as conductance for doors and windows. All the equations are explained above.

Matrix C [21x21] is the diagonal matrix containing the thermal capacities of each node starting from node 0.The thermal capacities of the materials air, concrete and insulation were calculated with 1 corresponding to walls 1 and 7; and 2 corresponding to walls 2 and 3. It is important to note that for steady-state the capacitances are considered zero.

Vector b [28,1] sets the temperature sources for each heat flow rate following the sign convention that is positive from low to high temperatures. The vector f [21,1] sets the flow rate sources for each node explained above. To conclude, the output vector y [21,1] indicates which nodes are outputs of the model, so the indoor temperature of both rooms.



$$\begin{array}{cc} & \begin{array}{cc} \theta_0 & \theta_1 \end{array} \\ \begin{array}{c} q_0 \\ q_1 \\ q_2 \end{array} & A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{bmatrix} ; G = \begin{bmatrix} G_0 & 0 & 0 \\ 0 & G_1 & 0 \\ 0 & 0 & G_2 \end{bmatrix} ; b = \begin{bmatrix} T_0 \\ 0 \\ -T_2 \end{bmatrix} \end{array}$$

$$C = \begin{bmatrix} C_0 & 0 \\ 0 & C_1 \end{bmatrix}$$

$$f = \begin{bmatrix} \dot{Q}_0 & \dot{Q}_1 \end{bmatrix}^T$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$$

Fig. 6: Example of a simple thermal network and the matrices for one wall

For testing the model we considered the vector f null, a hypothetical outside temperature of -5°C and the setpoint temperature for the controllers of 25°C (T_{sp}).

The analysis of a thermal circuit means to find the temperatures θ in all unknown nodes, and the heat flows q on the branches using the system of Differential-Algebraic Equations (DAE):

$$\begin{cases} C\dot{\theta} = -(A^T G A)\theta + A^T G b + f \\ q = G(-A\theta + b) \end{cases}$$

Fig. 7: DAE system

To conclude, for each room the controller has the gain K_p of 10^5 when it is on, while their gain goes to zero if the controller is off. To validate the model we first ran the simulation without controllers, and without vector f

3. Assignment 2: Steady-state

The second assignment is to implement our model on steady-state. So we solve the DAE with capacities considered zero, vector f null and no controllers on the first simulation.

$$\begin{cases} \theta = (A^T G A)^{-1}(A^T G b + f) \\ q = G(-A\theta + b) \end{cases}$$

Fig. 8: Steady-state solution

After testing and validating the model in steady-state, we tried it with perfect controllers. The error corresponds to the difference between the setpoint temperature for a certain node and the measured temperature there. We were aiming for a very small error, so it was necessary to set a very high Kp, this characterises our controller as a near perfect one, since the error is the smallest possible. The gain is calculated by multiplying Kp and error, so none of those values can be zero. The values we found for each controller are reasonable and coherent to what was expected.

```
The temperatures on the nodes are:
[-4.3148687   9.38775732 23.09038334 23.18825924 23.28613514 24.9989634
 -5.         -5.         -5.         -5.         -5.         -5.
 24.9989634  24.9989634  24.9989634  24.9989634  23.28613514 23.18825924
 23.09038334  9.38775732 -4.3148687 ]

The heat flow on each resistance is:
[-6.16618171e+01 -6.16618171e+01 -6.16618171e+01 -6.16618171e+01
 -6.16618171e+01 -6.16618171e+01  1.27897692e-13  0.00000000e+00
  0.00000000e+00 -3.30402372e-13 -3.30402372e-13 -4.26325641e-13
 -9.99200722e-14 -4.19985488e+01 -3.41060513e-13  2.98427949e-12
 -2.98427949e-12  3.41060513e-13  0.00000000e+00 -4.19985488e+01
 -6.16618171e+01 -6.16618171e+01 -6.16618171e+01 -6.16618171e+01
 -6.16618171e+01 -6.16618171e+01  1.03660366e+02  1.03660366e+02]

The error of controller 1 is: 0.0010366036585054417 and for controller 2 is: 0.0010366036585054417
The heat flow rate of controller 1 is: 103.66036585054417 and for controller 2 is: 103.66036585054417
```

a. Steady-state results

i. State-space representation

The differential-algebraic system was then transformed in the following state-space representation, with the first equation being the state equation and the second one the observation one:

$$\begin{cases} \dot{\theta} = A_s\theta + B_s u \\ y = C_s\theta + D_s u \end{cases}$$

Where A_s is the state matrix; B_s the input matrix; C_s the output matrix and D_s the feedforward matrix. The vector u is equal to the number of sources of temperature and heat flows (that are nonzero), and y is the vector of outputs that contain the temperature nodes that we are interested in.

$$\begin{bmatrix} 0 & 0 \\ 0 & C_C \end{bmatrix} \begin{bmatrix} \dot{\theta}_0 \\ \dot{\theta}_C \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_C \end{bmatrix} + \begin{bmatrix} K_{b1} \\ K_{b2} \end{bmatrix} b + \begin{bmatrix} I_{11} & 0 \\ 0 & I_{22} \end{bmatrix} \begin{bmatrix} f_0 \\ f_C \end{bmatrix}$$

b. Dimensioning the system

To dimension our HVAC system we ran simulations in four different scenarios, that are the combination of two different variables, the season of the year: summer or winter, and the occupation of each room, that could be 0 or 2 (four persons in total, two in each room).

The seasons influence the temperature outside (24ºC during summer and -5ºC during winter), the setpoint temperature chosen for unoccupied buildings (20ºC during summer and 16ºC during winter), and the solar irradiance (E), which is 100 for summer and 0 for winter.

The setpoint temperature for the occupied building is always 22ºC.

Vector f now is no longer empty, it contains information on the heat sources. In the outside walls there's the heat from the sun, that is calculated by multiplying the surface of the wall by E and alfa, solar irradiance and absorptance of the painting respectively.

The inside of the walls also receive new values in the f vector, related to the sun heat that comes to the wall and is then transmitted, by irradiance. It is calculated by multiplying the surface of the wall by E and it's thermal transmittance, and then divide it by the total surface of the room's walls.

At last, when there is occupation, we also consider the heat produced by people. we consider that one person produces 0.1kW, and then we multiply this value by the number of people in the room.

i. Summer unoccupied

The first simulation ran was for summer and the empty building, unoccupied. We can see that the heat flow is negative, since it's summer and the controllers are cooling the rooms.

```
Case 1: summer unnocupied

The temperatures on the nodes are:
[33.19427292 27.0797313  20.96518968 20.92151439 20.87783909 20.00042615
 32.5574738  32.71792644 32.72063257 32.72333871 33.10219788 33.48105704
 20.11352139 20.11352139 20.11352139 20.00042615 20.87783909 20.92151439
 20.96518968 27.0797313  33.19427292]

The heat flow on each resistance is:
[-8.27484563e+02  2.75154373e+01  2.75154373e+01  2.75154373e+01
  2.75154373e+01  3.15868659e+01  7.70172642e+00  2.27315499e+00
  2.27315499e+00  2.27315499e+00  2.27315499e+00 -1.13772685e+03
 -7.70172642e+00  5.59940339e+00  5.42857143e+00  2.98427949e-12
 -2.98427949e-12 -5.42857143e+00  0.00000000e+00  5.59940339e+00
  3.15868659e+01  2.75154373e+01  2.75154373e+01  2.75154373e+01
  2.75154373e+01 -8.27484563e+02 -4.26148407e+01 -4.26148407e+01]

The error of controller 1 is: -0.00042614840672428045 and for controller 2 is: -0.00042614840672428045
The heat flow rate of controller 1 is: -42.614840672428045 and for controller 2 is: -42.614840672428045
```

ii. Summer with occupation

We now consider that there are four persons in the building, two in each room, which is reasonable since they are very small: each has $14m^2$. Since the rooms are occupied, the vector f is now different, it has to consider for node 5 and 15 (the interior of each room) a new heat source, coming from the people inside them.
We consider the heat provided by one person as 0.1kW, therefore the heat generated inside each room is 0.2kW.

```
Case 2: summer with occupation

The temperatures on the nodes are:
[33.23994834 28.03891512 22.83788191 22.80073167 22.76358143 22.00035704
 32.5574738  32.71792644 32.72063257 32.72333871 33.10219788 33.48105704
 22.11345228 22.11345228 22.11345228 22.00035704 22.76358143 22.80073167
 22.83788191 28.03891512 33.23994834]

The heat flow on each resistance is:
[ -831.59535053    23.40464947    23.40464947    23.40464947
    23.40464947    27.47607805     7.70172642     2.27315499
     2.27315499     2.27315499     2.27315499 -1137.72684501
    -7.70172642     2.79950014     5.42857143     0.
     0.            -5.42857143     0.             2.79950014
    27.47607805    23.40464947    23.40464947    23.40464947
    23.40464947  -831.59535053   -35.70414962   -35.70414962]

The error of controller 1 is: -0.0003570414961586721 and for controller 2 is: -0.0003570414961586721
The heat flow rate of controller 1 is: -35.70414961586721 and for controller 2 is: -35.70414961586721
```

The heat flow for the controllers is still negative, since it's still cooling down the building, but smaller then when the building is occupied. This happens because the setpoint temperature is now 2ºC higher, and since the building is small and can't host that many people, the heat generated by the four people is not big enough to cause problems, it is more than compensated by the rise in the setpoint temperature.

iii. Winter unoccupied

For winter, we consider the solar irradiance as 0, since it's the worst case scenario. Now the heat flow rate is positive, since we are heating the room. The values are reasonable for the room size and temperatures needed.

```
Case 3: winter unnocupied

The temperatures on the nodes are:
[-4.52040809  5.07143013 14.66326834 14.73178147 14.8002946  15.99927438
 -5.         -5.         -5.         -5.         -5.         -5.
 15.99927438 15.99927438 15.99927438 15.99927438 14.8002946  14.73178147
 14.66326834  5.07143013 -4.52040809]

The heat flow on each resistance is:
[-4.31632720e+01 -4.31632720e+01 -4.31632720e+01 -4.31632720e+01
 -4.31632720e+01 -4.31632720e+01  1.27897692e-13  0.00000000e+00
  0.00000000e+00 -3.30402372e-13 -3.30402372e-13 -4.26325641e-13
 -9.99200722e-14 -2.93989841e+01 -8.52651283e-14  0.00000000e+00
 -2.98427949e-12  2.55795385e-13  0.00000000e+00 -2.93989841e+01
 -4.31632720e+01 -4.31632720e+01 -4.31632720e+01 -4.31632720e+01
 -4.31632720e+01 -4.31632720e+01  7.25622561e+01  7.25622561e+01]

The error of controller 1 is: 0.0007256225609513223 and for controller 2 is: 0.0007256225609513223
The heat flow rate of controller 1 is: 72.56225609513223 and for controller 2 is: 72.56225609513223
```

iv. Winter with occupation

For winter with occupation the necessary heating is bigger than without occupation. This happens because the setpoint temperature is higher, 22ºC instead of 20ºC, but since the room is small and the maximum occupation is a small number of people, they don't produce enough heat to compensate the raise of setpoint temperature, which has to be done by the HVAC system

```
Case 4: winter with occupation

The temperatures on the nodes are:
[-4.38338183  7.94898159 20.28134501 20.36943332 20.45752163 21.99906706
 -5.         -5.         -5.         -5.         -5.         -5.
 21.99906706 21.99906706 21.99906706 21.99906706 20.45752163 20.36943332
 20.28134501  7.94898159 -4.38338183]

The heat flow on each resistance is:
[-5.54956354e+01 -5.54956354e+01 -5.54956354e+01 -5.54956354e+01
 -5.54956354e+01 -5.54956354e+01  1.27897692e-13  0.00000000e+00
  0.00000000e+00 -3.30402372e-13 -3.30402372e-13 -4.26325641e-13
 -9.99200722e-14 -3.77986939e+01 -1.70530257e-13  0.00000000e+00
  0.00000000e+00  1.70530257e-13  0.00000000e+00 -3.77986939e+01
 -5.54956354e+01 -5.54956354e+01 -5.54956354e+01 -5.54956354e+01
 -5.54956354e+01 -5.54956354e+01  9.32943293e+01  9.32943293e+01]

The error of controller 1 is: 0.0009329432926570291 and for controller 2 is: 0.0009329432926570291
The heat flow rate of controller 1 is: 93.29432926570291 and for controller 2 is: 93.29432926570291
```

C. State-space representation

To evaluate the thermal behavior of the system, the state-space equations were solved using a numerical integration method. The simulation yielded the temperature evolution of the internal nodes capacitive temperatures and the output temperatures over the defined time period.

Using the resulting temperature values, the heat transfer flow through each thermal resistance is calculated.

```
[-5.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0. -5. -5. -5.  0.  0.  0.  0.
  0. -5.  0.  0.  0.  0.  0. -5. 25. 25.  0.  0.  0.  0.  0.  0.  0.  0.
  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
The heat flow on each resistance is:
[-3.68790744e+01 -4.54888507e+01 -6.18621185e+00  1.57389790e+03
 -1.15817210e+02  3.75730408e-02  2.97506349e+02 -1.86517468e-12
 -7.62427605e+02 -8.24828247e+00 -6.06518009e+01 -4.91720992e+01
 -3.66496318e-01 -6.99046671e+00 -2.33159082e+01  1.61863794e+03
  4.11580744e+04 -1.19869972e+03  3.30294256e-03 -6.99560462e+00
 -1.86285960e+01  1.83747105e+03  3.02803973e+04 -2.20489684e+02
 -8.67127469e+00 -4.05669818e+02  2.50068095e+06  2.50031396e+06]
```

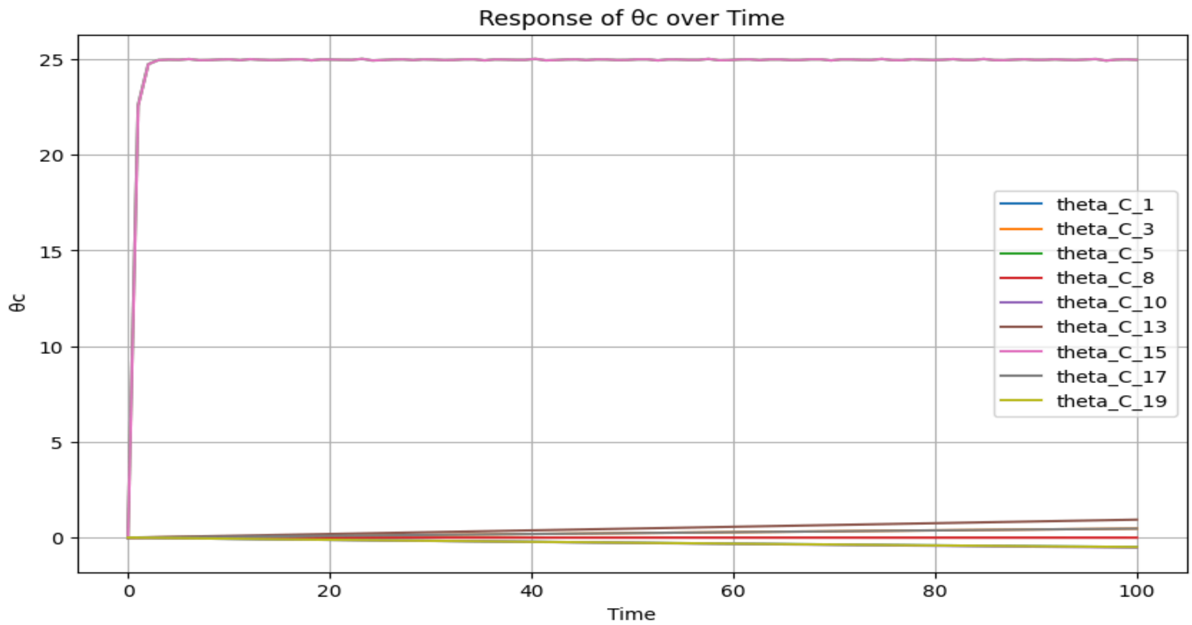
D. The difference between the state space and differential equation

The comparison showed a difference in the heat flow at the final time step, indicating a numerical deviation between the two methods.

```
Difference between ststae-space solution and normal DAE q:
[ 7.73143758e+01  6.87045995e+01  1.08007238e+02  1.68809135e+03
 -1.62376029e+00  1.14231023e+02  2.97506349e+02 -1.86517468e-12
 -7.62427605e+02 -8.24828247e+00 -6.06518009e+01 -4.91720992e+01
 -3.66496318e-01  3.50073466e+01 -2.33159082e+01  1.61863794e+03
  4.11580744e+04 -1.19869972e+03  3.30294256e-03  3.50022087e+01
  9.55648541e+01  1.95166450e+03  3.03945908e+04 -1.06296234e+02
  1.05522175e+02 -2.91476368e+02  2.50052476e+06  2.50015776e+06]
```

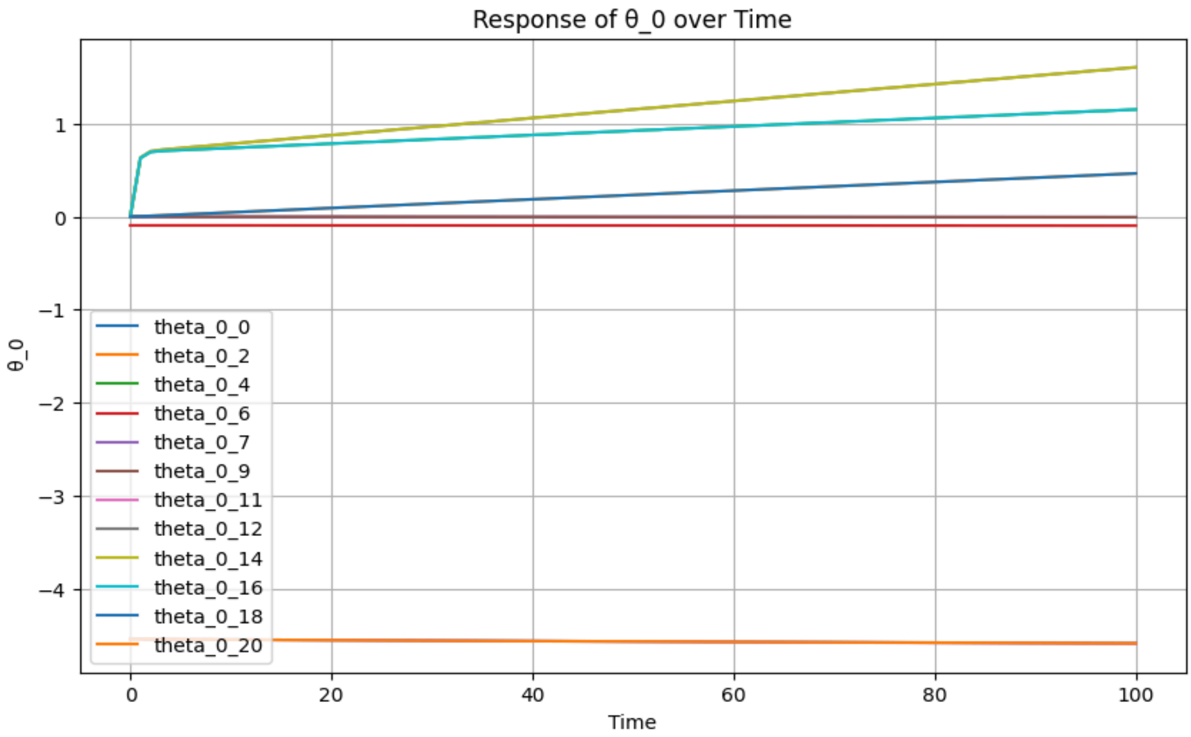
The time response of internal node temperatures (θc):

Illustrates how the temperatures at the capacitive elements of the system evolve over time in response to the applied inputs.



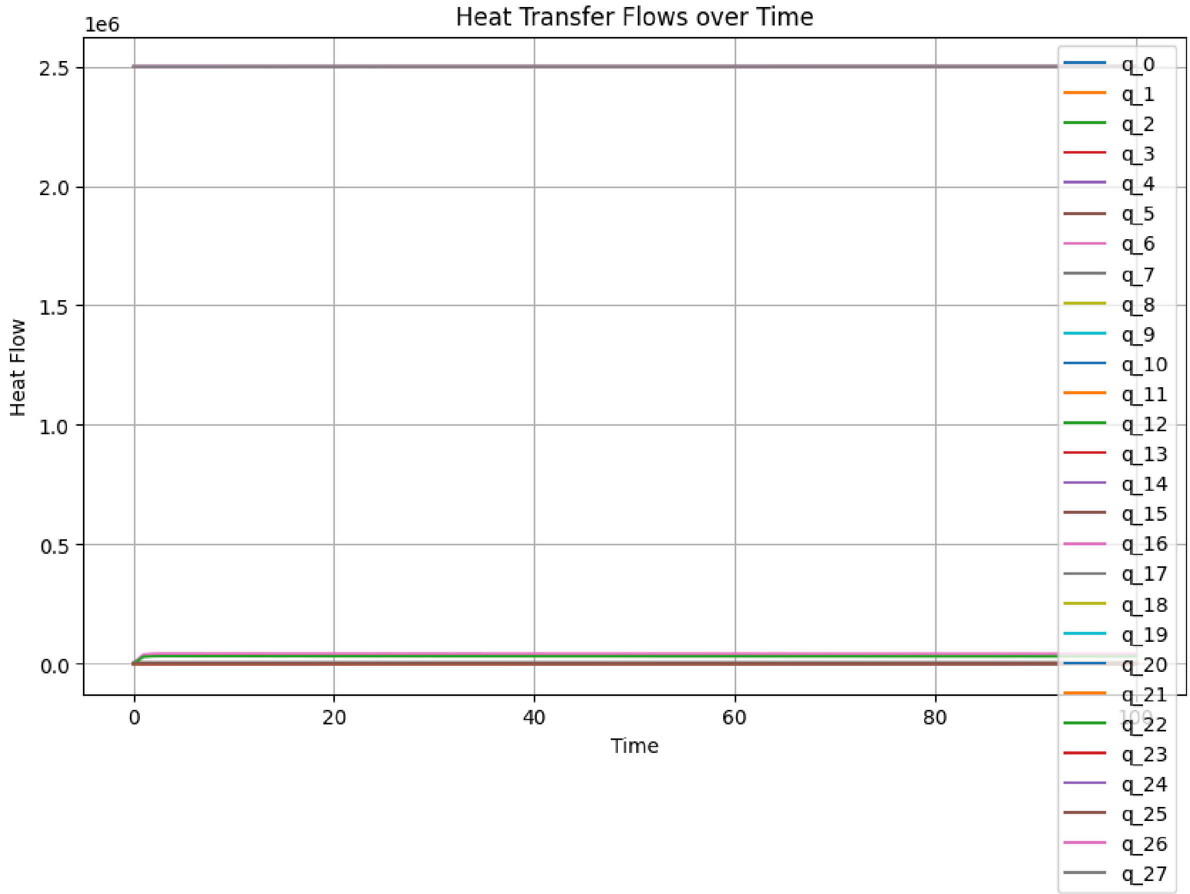
The output temperatures (θ_0):

Represent the temperatures at key observable points in the system, such as indoor air or surface temperatures.These outputs are directly influenced by both the internal state (θc) and external inputs like outdoor temperature and internal heat flow.



Heat transfer flows (q) over time:

The heat transfer flows (q) over time represent the amount of thermal energy exchanged through each thermal resistance in the system. These flows indicate how heat moves between different parts of the model



4. Assignment 3: Simulate step response

A. Determine the time step and the settling time.

The_ time step represents the amount of time that elapses between two consecutive calculations in a numerical integration method in numerical integration.We choose as time step a value which is smaller than the maximum admitted for numerical stability of Euler explicit method and is a multiple of 60 s. we get max time step 0.86s and settling time 143.5h.

```
Time constants:
 [4.3e-01 4.3e-01 5.4e+03 1.0e+04 1.0e+04 1.6e+03 1.6e+03 1.3e+05 1.6e+03] s

Max time step Δt_max = 0.86 s
Selected Δt = 1.0 s
Settling time: 516599.08 s = 143.50 h
```

B. Number of time step

The number of time steps depends on the settling time and the time step we get this result.

	time
0	0.0
1	1.0
2	2.0
3	3.0
4	4.0
...	...
149247	149247.0
149248	149248.0
149249	149249.0
149250	149250.0
149251	149251.0

149252 rows × 1 columns

C. Give the input vector u

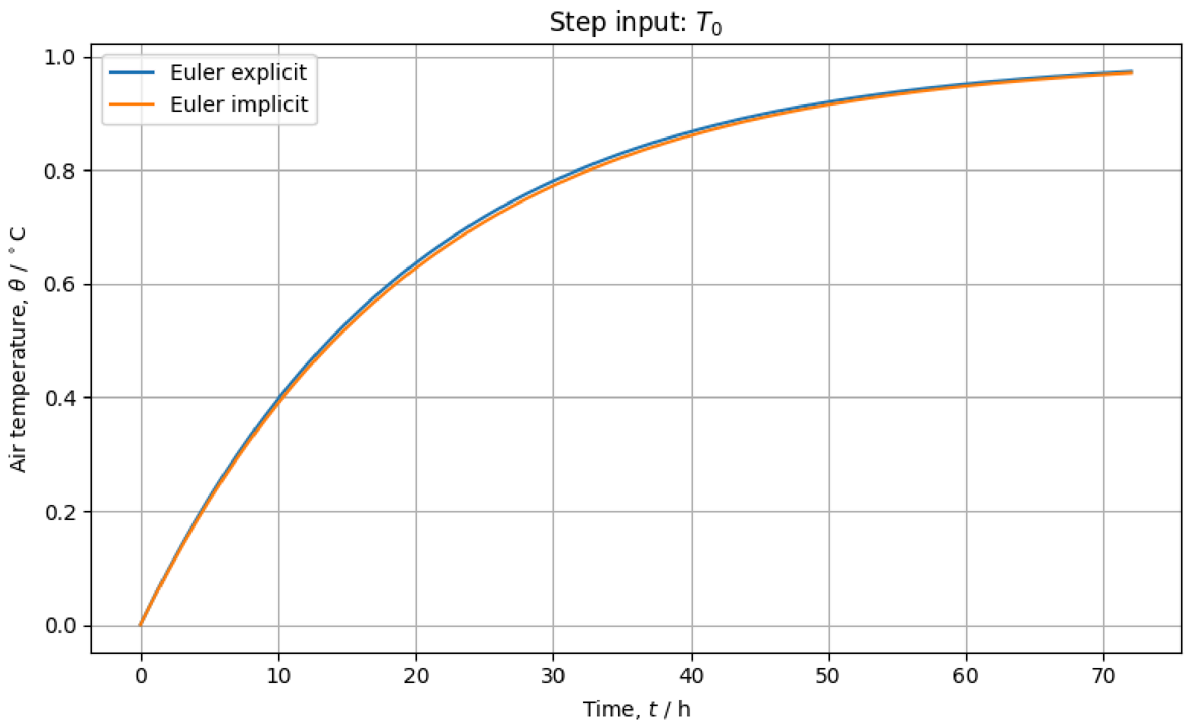
We assume that the outdoor temperature undergoes a step variation of 1°C, while the indoor heat flow is considered to be zero during this condition.

	0	1	2	3	4	5	6	7	8	9	...	149242	149243	149244	149245	149246	149247	149248	149249	149250	149251
0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	...	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	...	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	...	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
3	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	...	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
4	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	...	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

9 rows × 149252 columns

D. Step response to outdoor temperature obtained by explicit and implicit Euler integration of the state-space model

The step response to outdoor temperature is computed using both explicit and implicit Euler integration methods applied to the state-space model. When the time steps are small, the results from both methods are identical.



E. Indoor heat flow rate

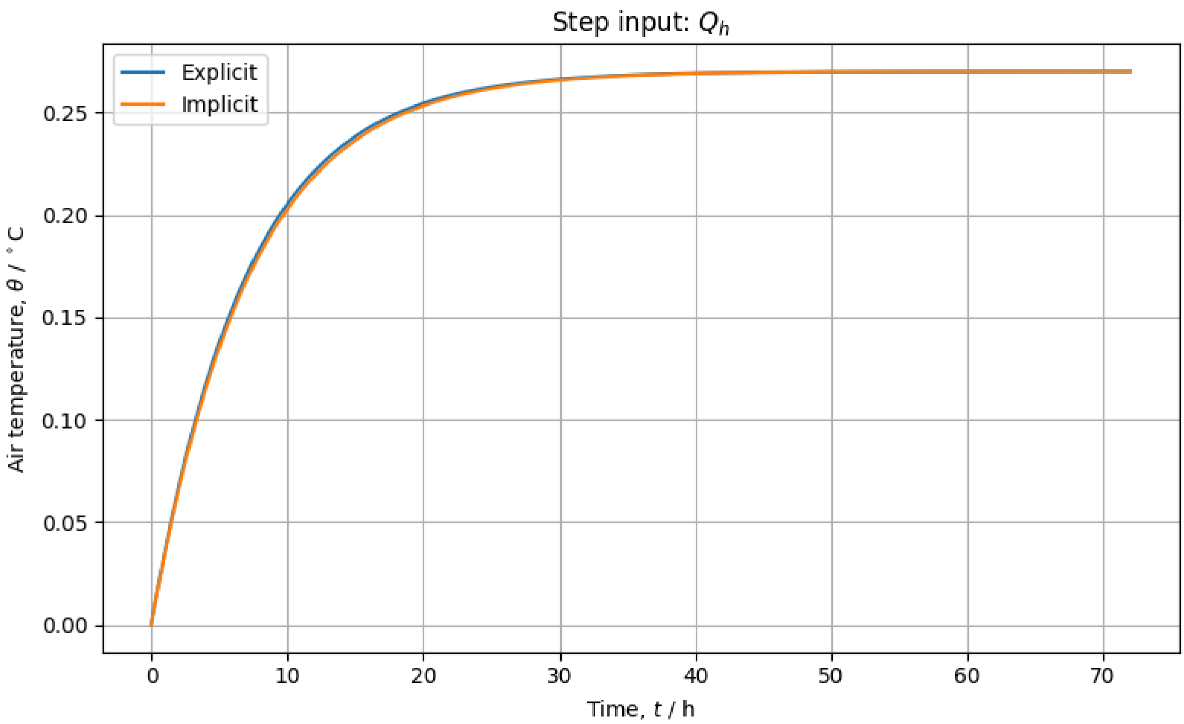
We consider that the indoor heat flow rate has a step variation of 1 W the outdoor temperature is zero, The output is a table with two rows and n columns. The first row represents the outdoor temperature values, which are all zeroes. The second row represents the indoor heat flow values, which are all ones. Each column corresponds to a time step or sample, so for every time step, the outdoor temperature input is zero and the indoor heat flow input is one.

	0	1	2	3	4	5	6	7	8	9	...	149242	149243	149244	149245	149246	149247	149248	149249	149250	149251
0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	...	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	...	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	...	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
3	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	...	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
4	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	...	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

9 rows × 149252 columns

F. Step response to indoor heat flow rate obtained by explicit and implicit Euler integration of the state-space model.

The graph shows the indoor air temperature response to a step change in indoor heat flow, calculated using explicit and implicit Euler methods. Both methods produce very similar temperature curves over time, confirming their accuracy in simulating the system's behavior.



5. Simulation with outdoor temperature from weather data

We extract outdoor air temperature data for Lyon from a weather file. To ensure consistency, we set all dates to the year 2000 and then select data between '2000-04-10 (start_date) and 2000-05-15 (end_date). This filtered temperature data is then used as the input for the analysis.

	temp_air
2000-04-10 00:00:00+01:00	5.1
2000-04-10 01:00:00+01:00	4.5
2000-04-10 02:00:00+01:00	3.8
2000-04-10 03:00:00+01:00	3.2
2000-04-10 04:00:00+01:00	3.1
...	...
2000-05-15 19:00:00+01:00	14.0
2000-05-15 20:00:00+01:00	14.0
2000-05-15 21:00:00+01:00	12.6
2000-05-15 22:00:00+01:00	12.0
2000-05-15 23:00:00+01:00	12.0

864 rows × 1 columns

A. Time vector for weather data at 1 h time step

A time vector tw was generated to represent the timeline of the weather data at uniform 1-hour intervals. This vector starts at 0 seconds and increments by 3600 seconds (1 hour) for each data point, covering the entire duration of the dataset. The resulting time vector aligns with the number of weather data entries and provides a consistent time reference for further analysis.

	0
0	0
1	3600
2	7200
3	10800
4	14400
...	...
859	3092400
860	3096000
861	3099600
862	3103200
863	3106800

864 rows × 1 columns

B. Resampled outdoor temperature

The original outdoor temperature data has a time step of 1 hour, represented by the vector tw. For simulation purposes, a new time vector t is created with a different time step Δt. Since the simulation time step may differ from the original data's 1-hour interval, the outdoor temperature data is resampled to match this new time scale.

This resampling is done using linear interpolation , which estimates the outdoor temperature values at the new time points in t based on the original data at times tw. The result is a temperature vector 'To' aligned with the simulation's time step, ensuring consistent input data for the model. The resampled data is organized into a DataFrame indexed by the new time vector for easy use and visualization.

	θ °C
0	5.1
3600	4.5
7200	3.8
10800	3.2
14400	3.1
...	...
3092400	14.0
3096000	14.0
3099600	12.6
3103200	12.0
3106800	12.0

864 rows × 1 columns

C. Input vector

The input vector u is constructed by combining the outdoor temperature To and the indoor heat flow Qh into a single block vector. This vector stacks the two input signals vertically, with the first row representing the outdoor temperature and the second row representing the indoor heat flow. Organizing the inputs this way simplifies their use in the state-space model, ensuring both variables are handled simultaneously. The combined input vector is then converted into a DataFrame with labeled rows for clarity and easier analysis.

	0	1	2	3	4	5	6	7	8	9	...	854	855	856	857	858	859	860	861	862	863
To	5.1	4.5	3.8	3.2	3.1	3.0	2.9	4.4	5.8	7.3	...	19.0	18.5	19.0	17.0	15.4	14.0	14.0	12.6	12.0	12.0
Qh	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

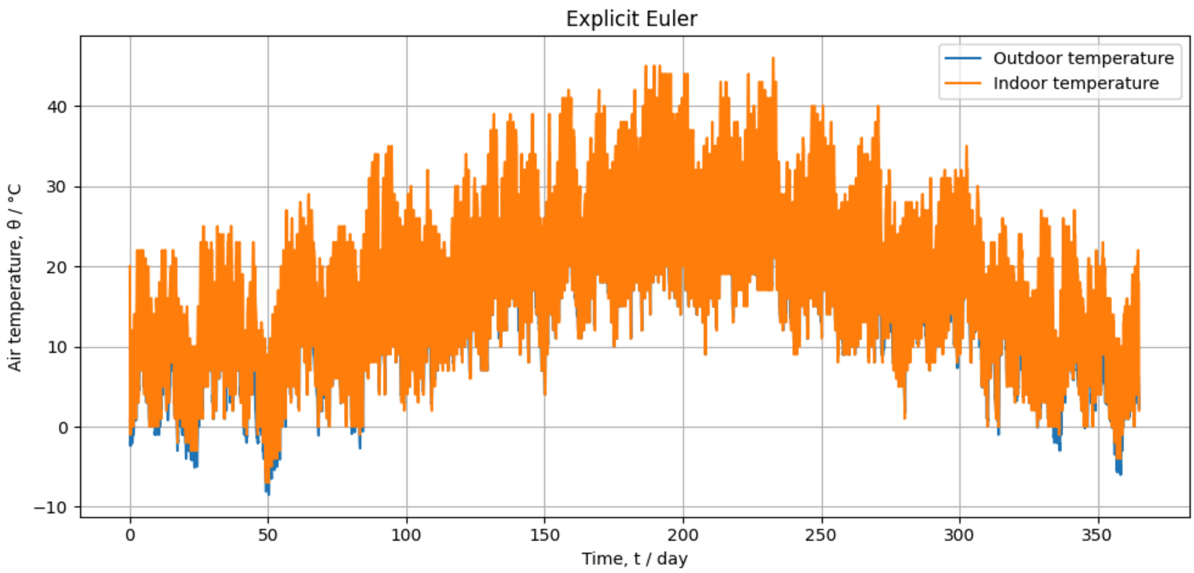
2 rows × 864 columns

D. Time integration

The temperature responses were computed by initializing the temperature vectors for both explicit and implicit Euler methods with an initial value of . The system was then integrated over time using the state-space matrices, updating the temperature at each time step based on the previous state and current inputs. For the explicit Euler method, the next temperature state is calculated directly from the current state and inputs.
For the implicit Euler method, the next state is found by solving an equation involving the future state, providing improved numerical stability.

The results are visualized by plotting the outdoor temperature alongside the indoor temperature (from the explicit Euler solution) over time, expressed in days. This comparison highlights how indoor temperature evolves in response to outdoor conditions during the simulation period.

The output results from weather data for explicit Euler and implicit are not satisfactory, likely due to inconsistencies between the input data and the system parameters. This mismatch may have affected the accuracy of the simulation, leading to unrealistic temperature responses.



6. **Link to the code:**
   a. **Github classroom:** https://github.com/dm4bem/model-and-steady-state-group6_dm4bem?tab=readme-ov-file
   b. **Public github with launch binder:** https://github.com/raphaellabelo/raphaellabelo-dm4bem_smartcities_group6/tree/main