EMAp Summer Course

Topological Data Analysis with Persistent Homology

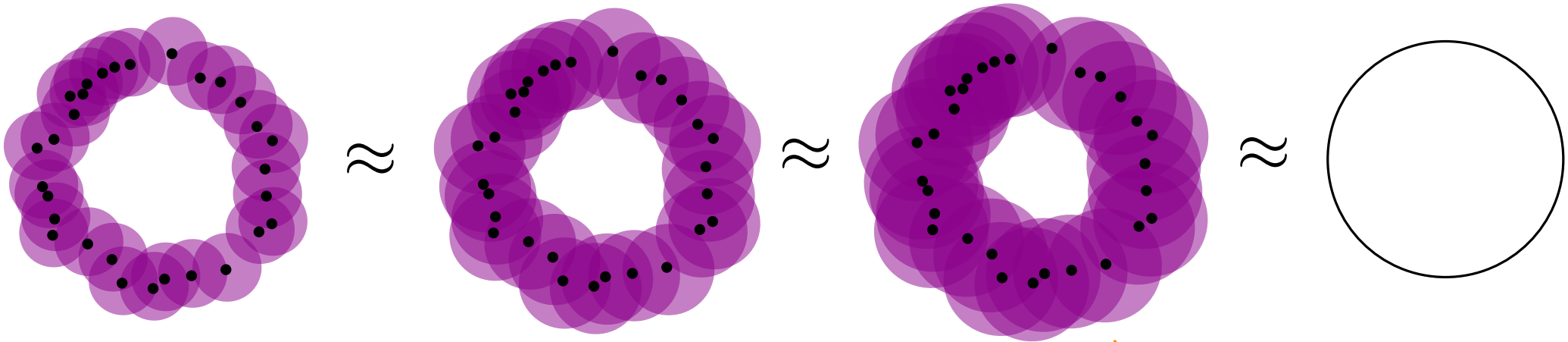
https://raphaeltinarrage.github.io/EMAp.html

Lesson 11: Persistent homology in practice

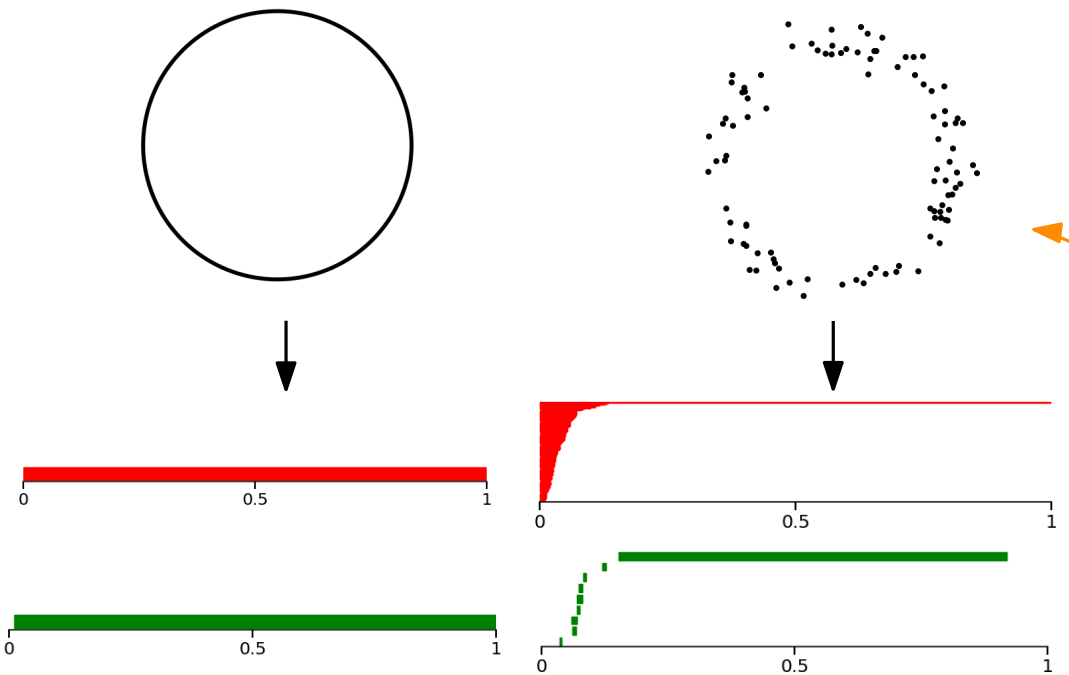
Last update: February 8, 2021

Introduction

We have studied the mathematical foundations of persistent homology.



consistency



stability

However, its use in practice requires some expertise.

I - Variations on persistent homology

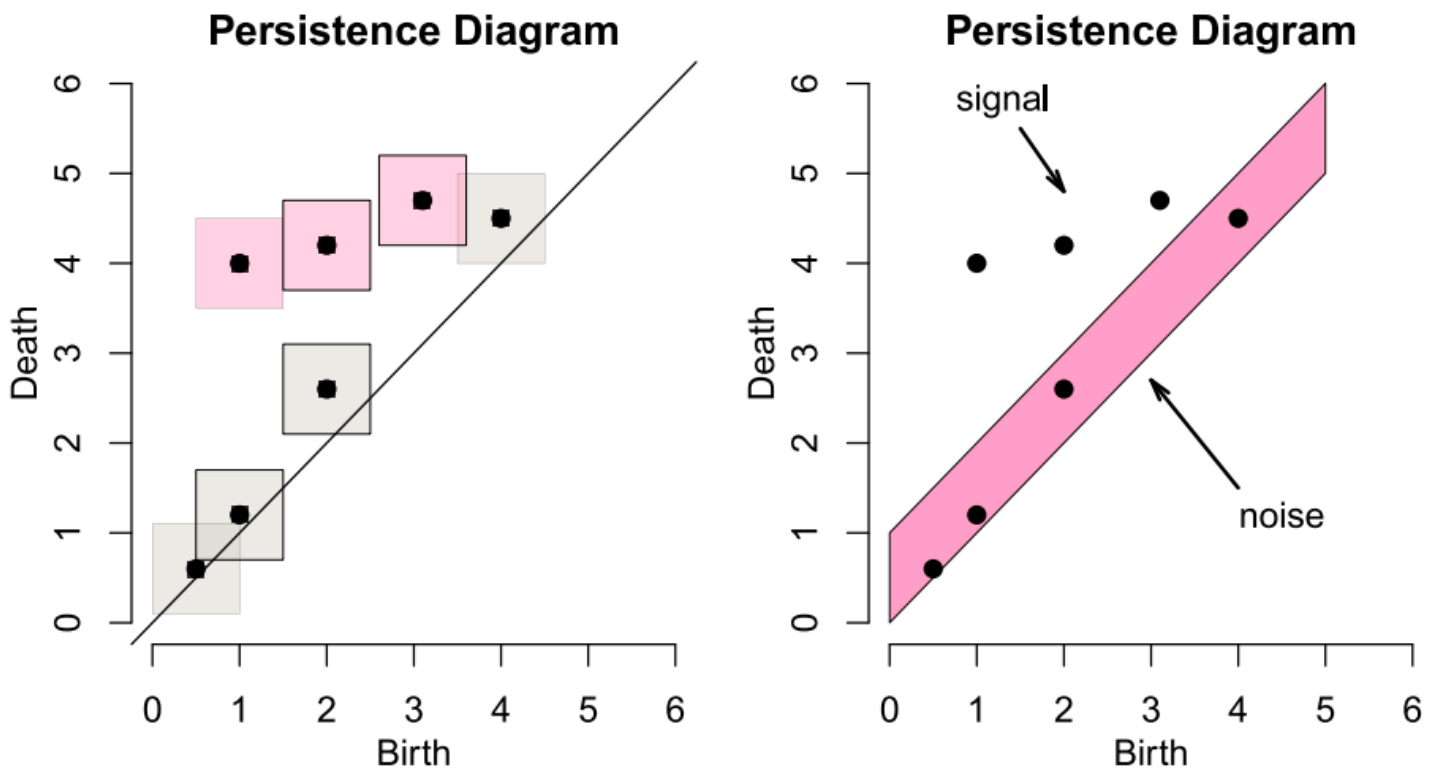
(II - Tutorial)

Statistical aspects of persistent homology

Brittany Terese Fasy, Fabrizio Lecci, Alessandro Rinaldo, Larry Wasserman, Sivaraman Balakrishnan and Aarti Singh, Confidence sets for persistence diagrams, 2014

https://arxiv.org/pdf/1303.7117.pdf

⟶ Given a barcode, how to determine statistically what is noise and what is not?

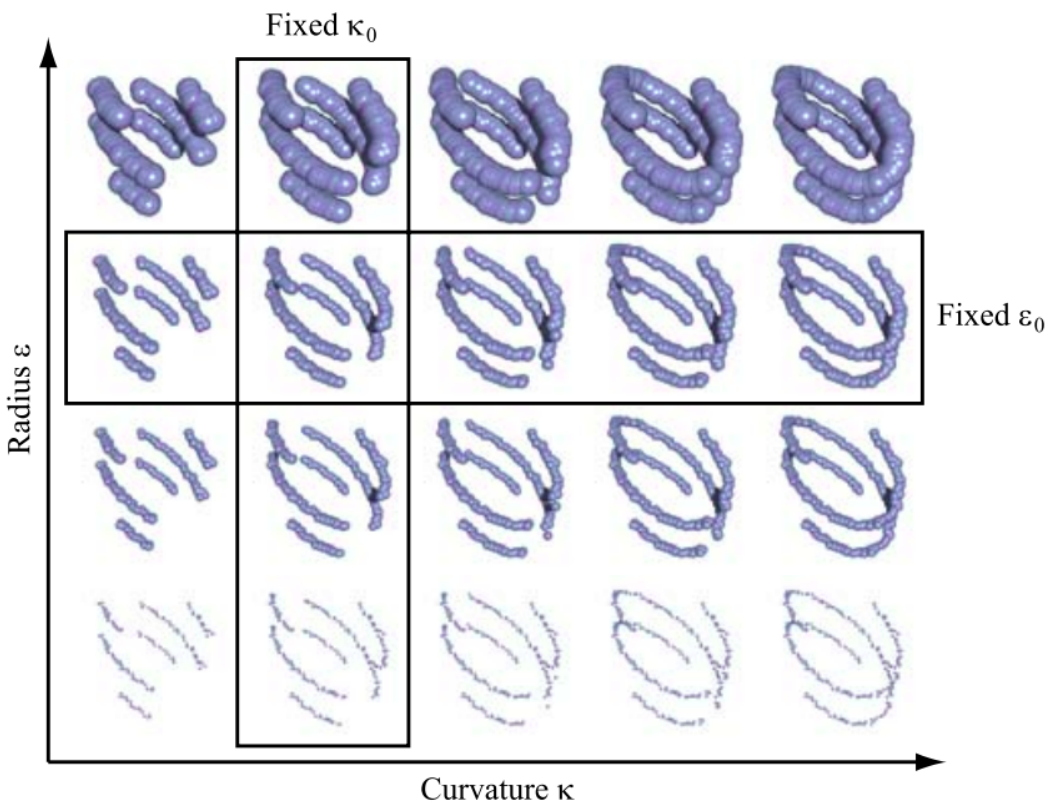


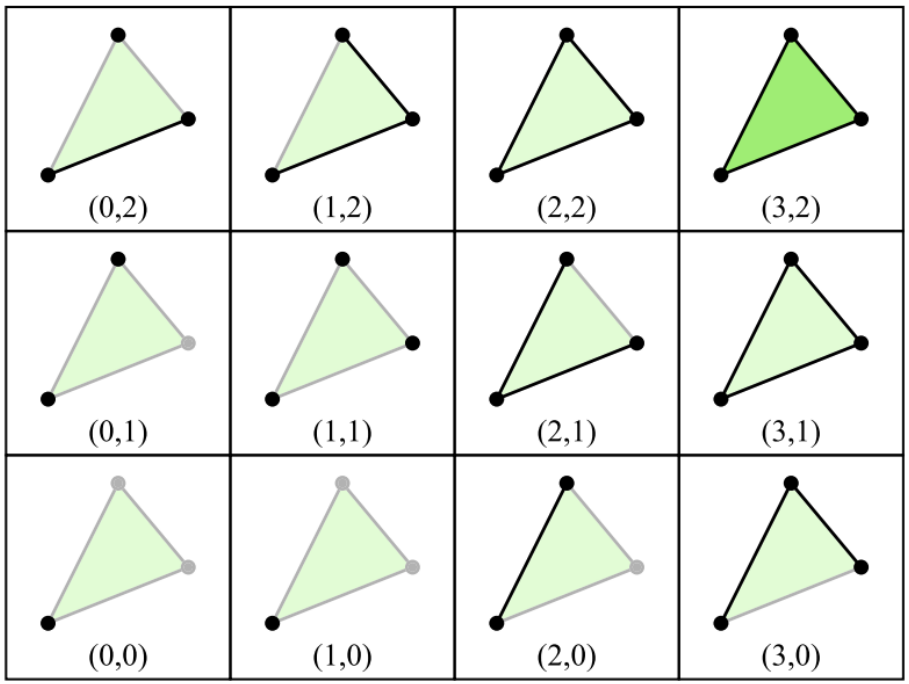
Higher-dimensional persistence

Gunnar Carlsson, Afra Zomorodian, The Theory of Multidimensional Persistence, 2009

https://link.springer.com/article/10.1007/s00454-009-9176-0

→ What if our filtration is not indexed only by $t \in \mathbb{R}^+$?



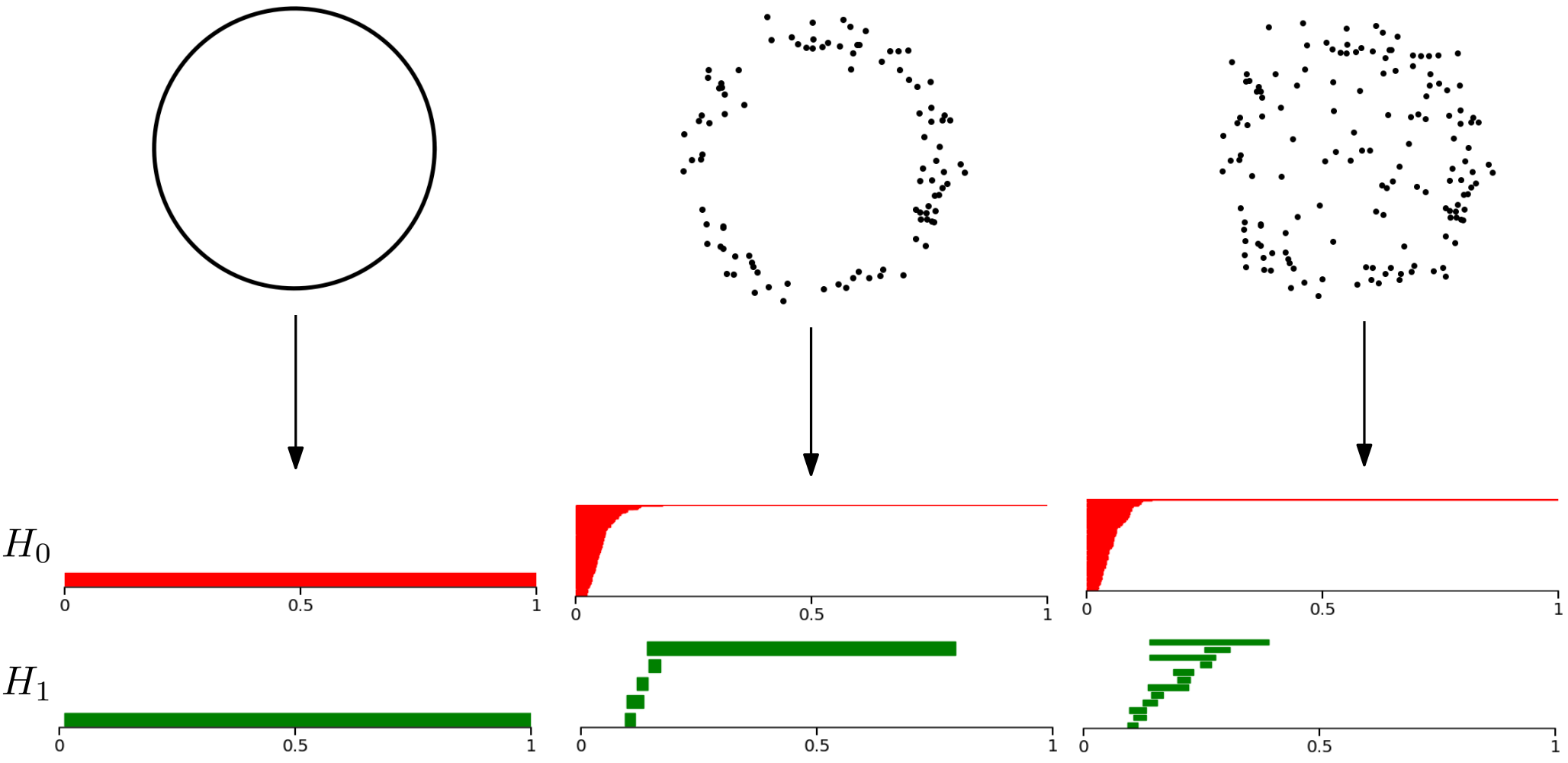


Wasserstein stability

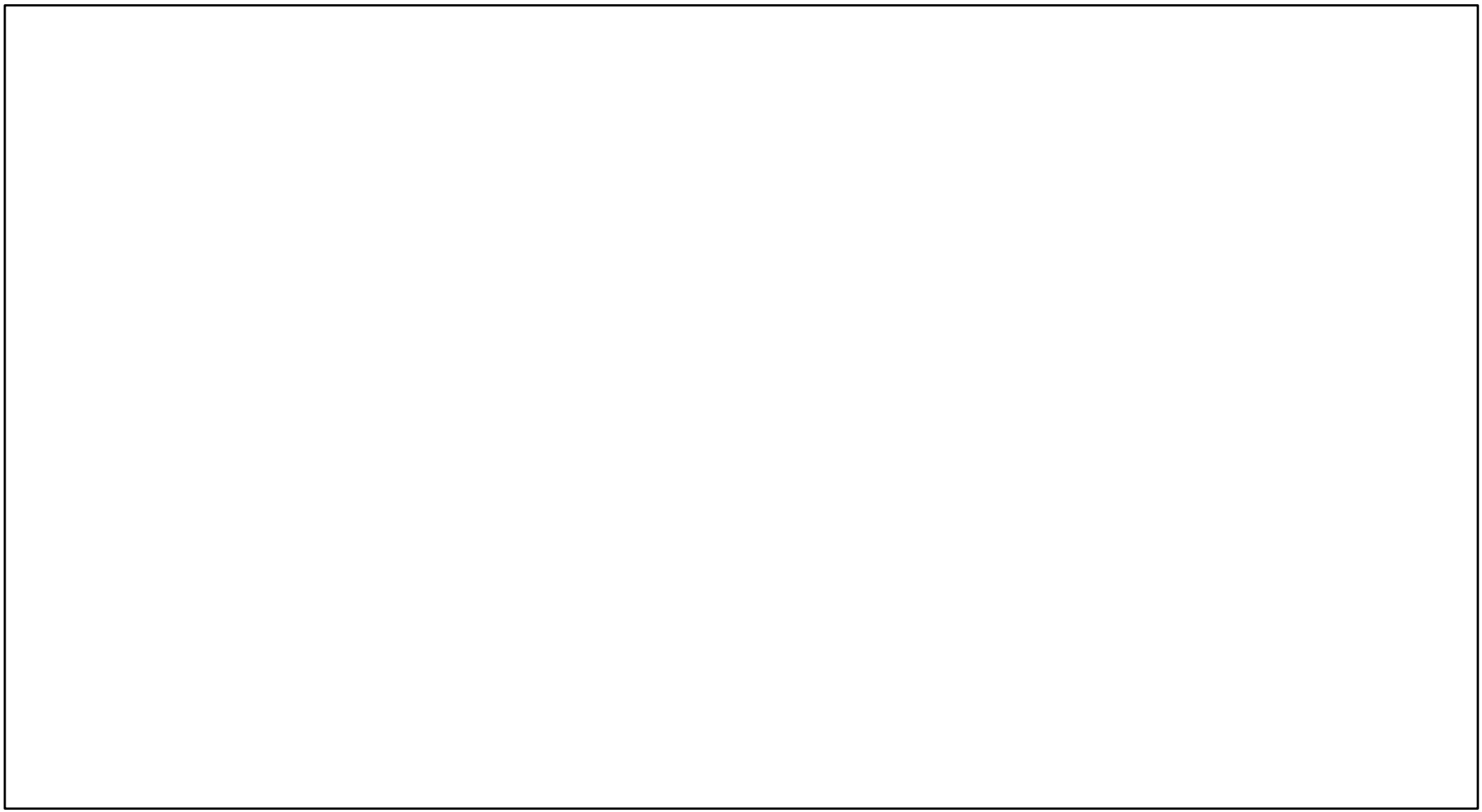
Hirokazu Anai, Frédéric Chazal, Marc Glisse, Yuichi Ike, Hiroya Inakoshi, Raphaël T., Yuhei Umeda, DTM-based filtrations, 2020

https://arxiv.org/abs/1811.04757

⟶ When our dataset is not close to an uderlying object in **Hausdorff distance**



Wasserstein stability



Kernels for machine learning

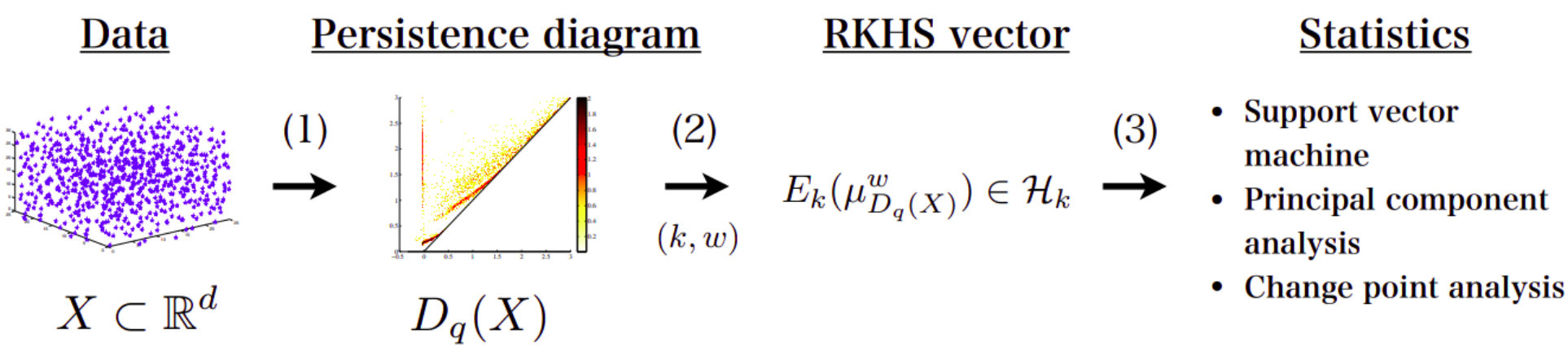
Mathieu Carrière, Marco Cuturi, Steve Oudot, Sliced Wasserstein Kernel for Persistence Diagrams, 2017

https://arxiv.org/abs/1706.03358

Genki Kusano, Kenji Fukumizu, Yasuaki Hiraoka, Kernel Method for Persistence Diagrams via Kernel Embedding and Weight Factor, 2018

https://www.jmlr.org/papers/volume18/17-317/17-317.pdf

⟶ Barcodes are not subsets of some Euclidean space, hence usual machine learning methods cannot be used directly



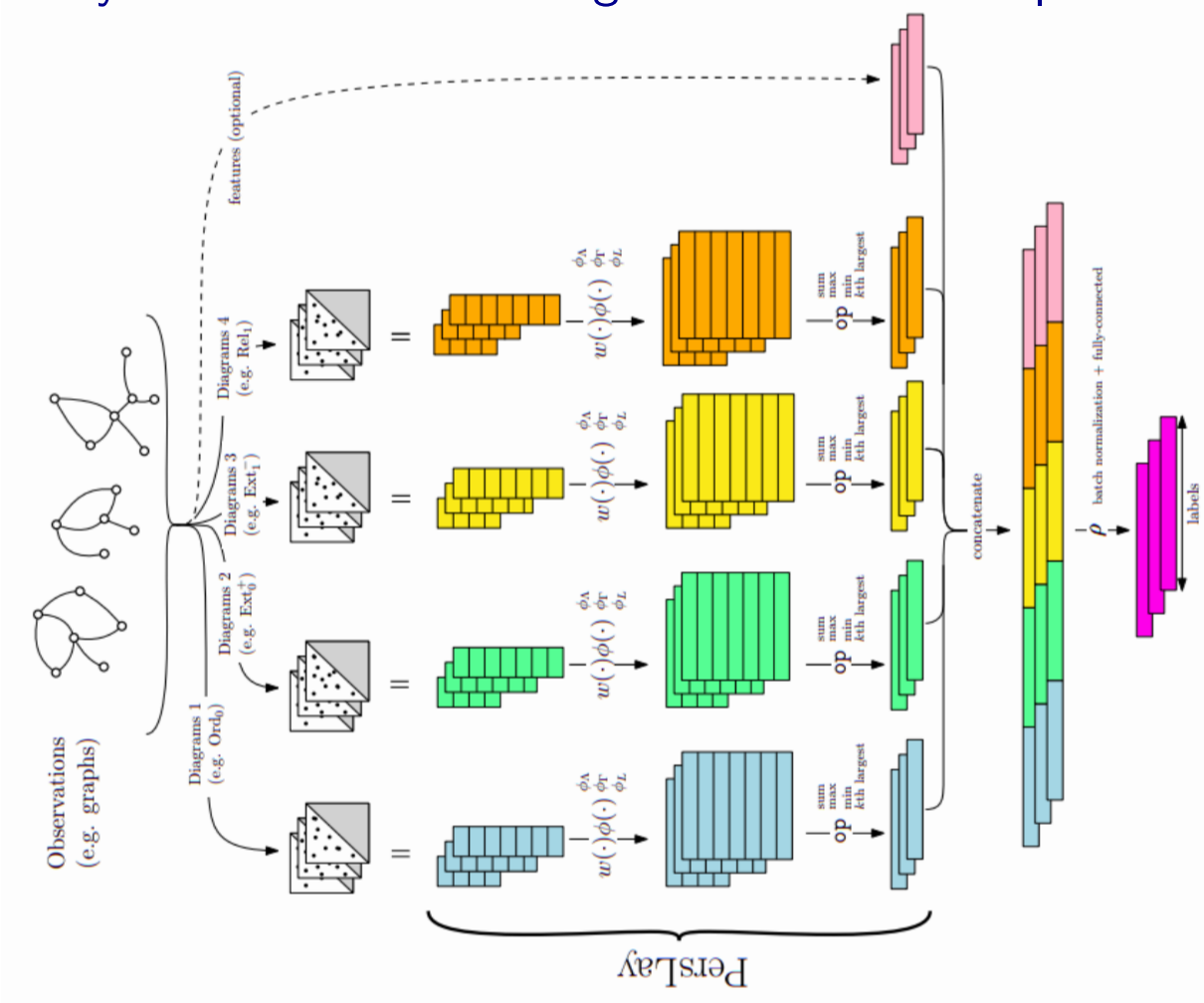
Topological layer in Neural Networks

Rickard Brüel-Gabrielsson, Bradley J. Nelson, Anjan Dwaraknath, Primoz Skraba, Leonidas J. Guibas, Gunnar Carlsson, A Topology Layer for Machine Learning, 2019

https://arxiv.org/abs/1905.12200

Mathieu Carrière, Frédéric Chazal, Yuichi Ike, Théo Lacombe, Martin Royer, Yuhei Umeda, PersLay: A Neural Network Layer for Persistence Diagrams and New Graph Topological Signatures, 2019

https://arxiv.org/abs/1904.09378

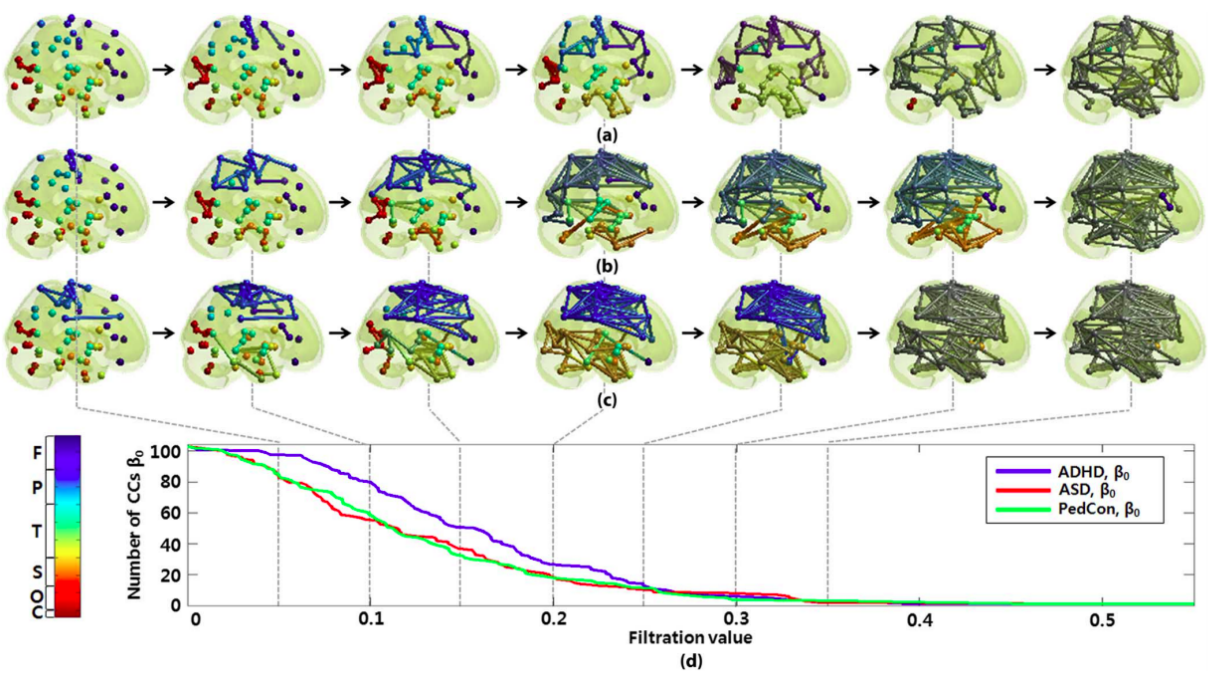


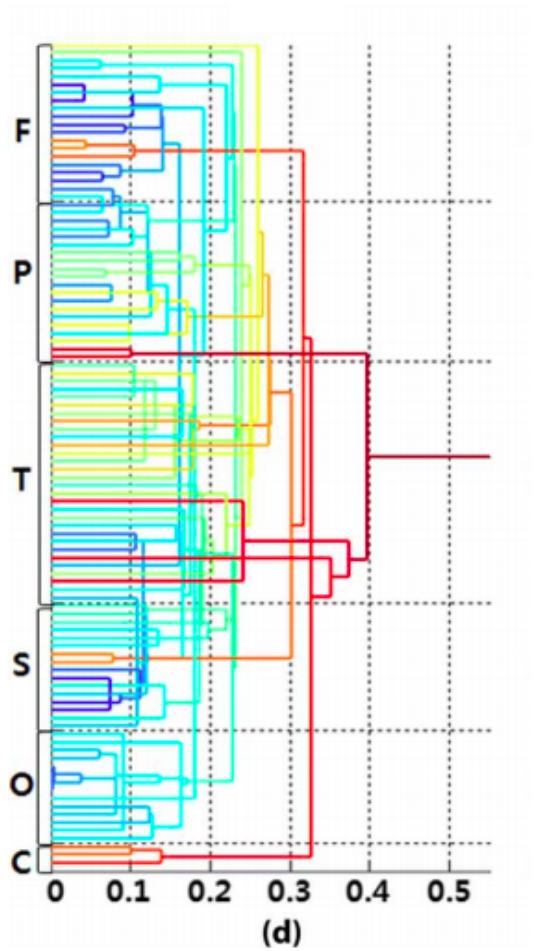
Hierarchical clustering

Hyekyoung Lee, Hyejin Kang, Moo K Chung, Bung-Nyun Kim, Dong Soo Lee, Persistent brain network homology from the perspective of dendrogram, 2012

http://pages.stat.wisc.edu/~mchung/papers/lee.2012.TMI.pdf

$\longrightarrow$ H_0-persistent homology induces a hierarchical clustering





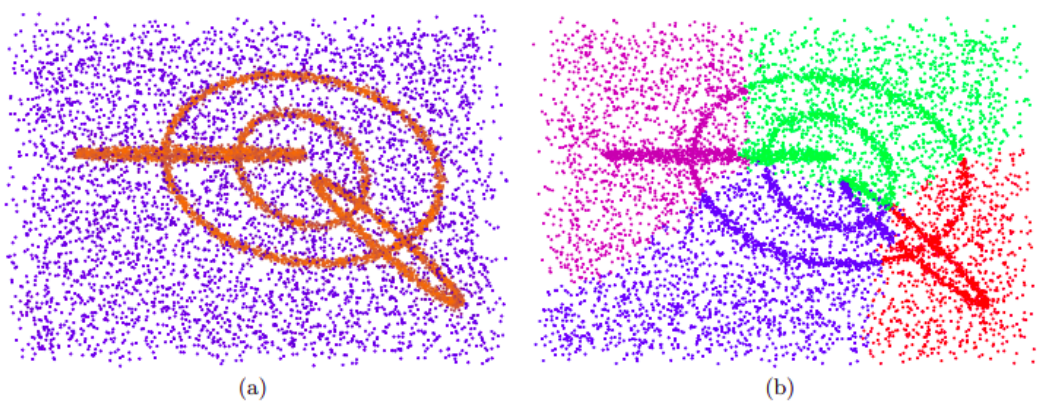
Classification

Frédéric Chazal, Steve Oudot, Primoz Skraba, Leonidas J. Guibas, Persistence-Based Clustering in Riemannian Manifolds, 2011

https://geometrica.saclay.inria.fr/team/Fred.Chazal/papers/cgos-pbc-09/cgos-pbcrm-11.pdf

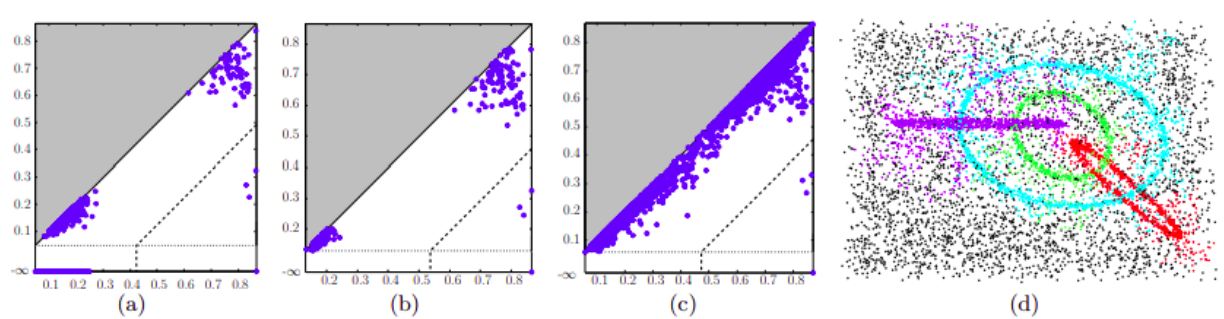
Chunyuan Li, Maks Ovsjanikov, Frederic Chazal, Persistence-based Structural Recognition, 2014

https://geometrica.saclay.inria.fr/team/Fred.Chazal/papers/loc-pbsr-14/CVPR2014.pdf

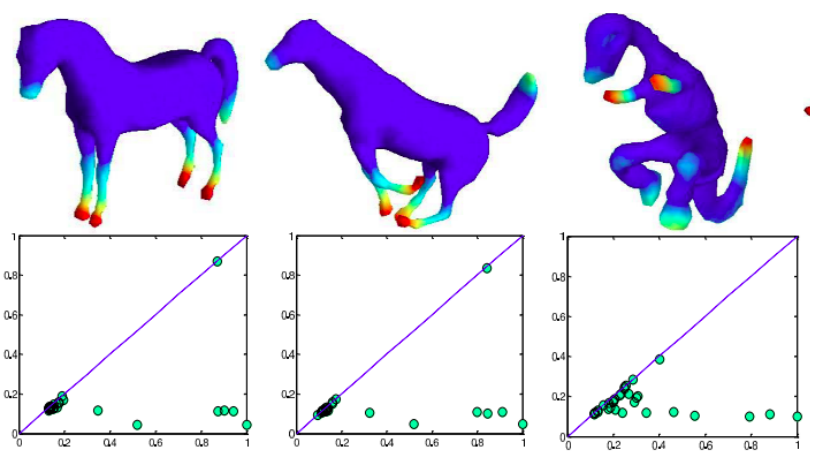


(a) (b)

Figure 7: (a) The rings data set with the estimated density function. (b) The result obtained using spectral clustering.



(a) (b) (c) (d)



Time series

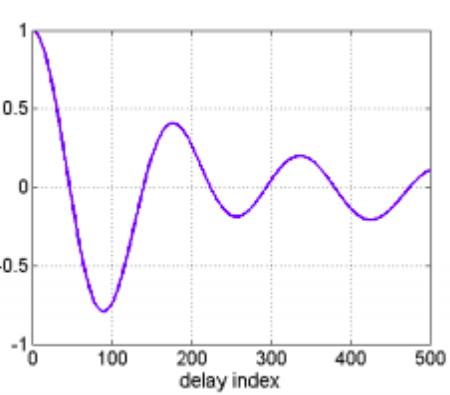
Saba Emrani, Thanos Gentimis, Hamid Krim Persistent Homology of Delay Embeddings and its Application to Wheeze Detection, 2014

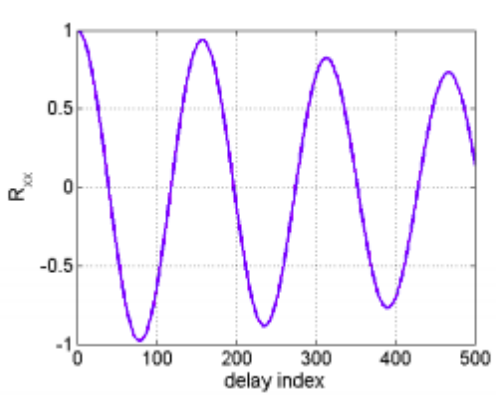
https://www.researchgate.net/publication/260523931_Persistent_Homology_of_Delay_Embeddings_and_its_Application_to_Wheeze_Detection

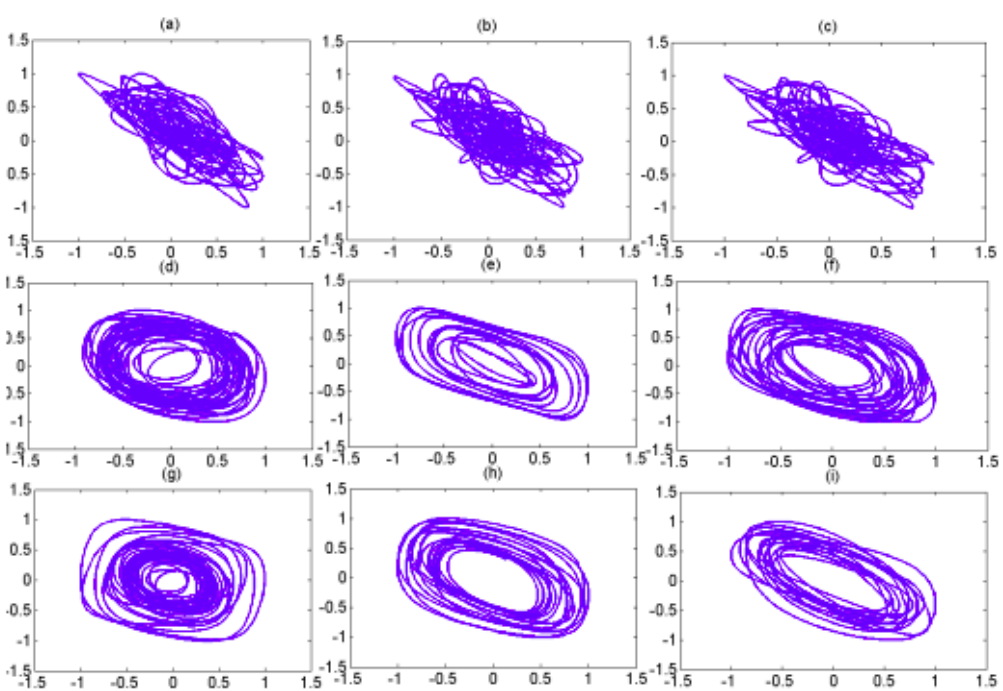
⟶ a time series $(x_1, x_2, x_3, \dots)$ does not contain topology...

turn it into a point cloud of $\mathbb{R}^n$ via **time delay embedding**!

$X = \{\overline{x}_1, \overline{x}_2, \overline{x}_3, \dots\} \subset \mathbb{R}^n$ where $\overline{x}_k = (x_k, x_{k+1}, ..., x_{k+n-1})$







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Tutorial

Download the notebook at

https://github.com/raphaeltinarrage/EMAp/blob/main/Tutorial3.ipynb

Conclusion

- algebraic topology
- persistent homology
- applications

REPÚBLICA FEDERATIVA DO BRASIL

MINISTÉRIO DA EDUCAÇÃO - MEC

UNIVERSIDADE FEDERAL DO PERSISTENT HOMOLOGY

O Reitor da UNIVERSIDADE FEDERAL DO RIO DE JANEIRO. no uso de suas atribuições e tendo em vista a conclusão do Curso de *Engenharia da Computação*, em 21/05/2002
confere o titulo de *Bacharel em Engenharia da Computação* a

Tu Nombre

cédula de identidade nº. 182910 (órgão expedidor) I.F.P. - RJ
nascido (a) a 13 de Dezembro de 1972 natural Rio de Janiero
e outorga - lhe o presente Diploma, a fim de que possa gozar de todos os direitos e prerrogativas legais.

Rio de Janeiro, 21 de Maio de 20 02

Reitor

Diretor

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