Linear Regression

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Contents

Chapter 1

Linear Regression

1.1 What is linear regression ?

Linear regression - Predicts an *outcome* variable, aka. *dependent* variable - Predicts using a set of *input* variables, aka. *independent* variables

1.2 Example: Predicting prices of Bordeaux wines

- Dependent variable: price (1990/1991 auction)
- Independent variables:
  - Age (wines are more expensive, if older)
  - Weather conditions at harvest, winter, while growing
- Download wine.csv to follow code examples
  Source: Orley Ashenfelter (Princeton)

1.3 Wine data set - structure

```
wine = read.csv("../data/wine.csv")
str(wine)
```

```
## 'data.frame':    25 obs. of  7 variables:
##  $ Year       : int  1952 1953 1955 1957 1958 1959 1960 1961 1962 1963 ...
##  $ Price      : num  7.5 8.04 7.69 6.98 6.78 ...
##  $ WinterRain : int  600 690 502 420 582 485 763 830 697 608 ...
##  $ AGST       : num  17.1 16.7 17.1 16.1 16.4 ...
```

```
##  $ HarvestRain: int  160 80 130 110 187 187 290 38 52 155 ...
##  $ Age        : int  31 30 28 26 25 24 23 22 21 20 ...
##  $ FrancePop  : num  43184 43495 44218 45152 45654 ...
```

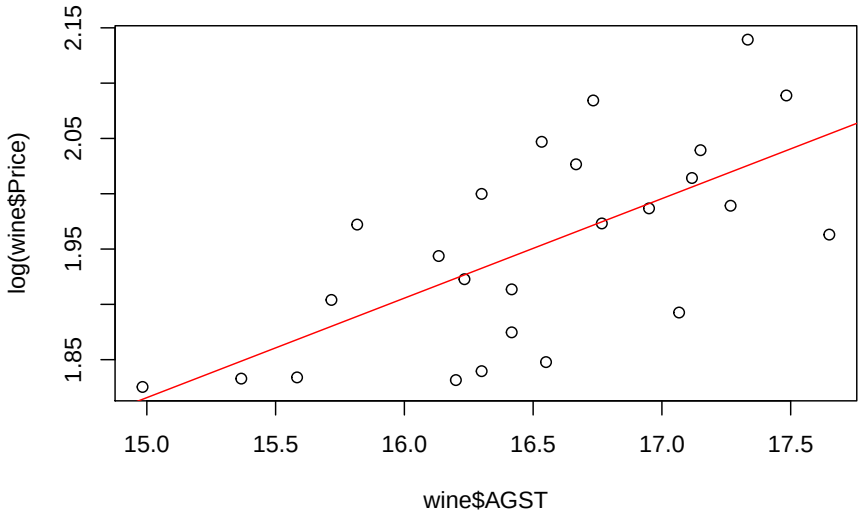
1.4 Bivariate prediction model (R)

Predict price based on one variable **AGST**

AGST = Average Growing Season Temperature

```
wine = read.csv("../data/wine.csv")
model1 = lm(Price ~ AGST, data=wine)
summary(model1)
```

1.5 Understanding the bivariate model (I)



1.6 Understanding the bivariate model (II)

```
##
## Call:
```

```
## lm(formula = Price ~ AGST, data = wine)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.78450 -0.23882 -0.03727  0.38992  0.90318
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -3.4178     2.4935  -1.371 0.183710
## AGST          0.6351     0.1509   4.208 0.000335 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4993 on 23 degrees of freedom
## Multiple R-squared:  0.435,  Adjusted R-squared:  0.4105
## F-statistic: 17.71 on 1 and 23 DF,  p-value: 0.000335
```

1.7 Understanding the bivariate model (III)

R model summary outputs 5 columns

- Coefficient estimate β
- standard error: measure of how much the coefficient is likely to vary from the estimate value.
- t value is the estimate divided by the standard error. Larger absolute value indicate more significance of the variable.
- Probability value: Plausibility of the estimate
- **Significance by indicated by up to 3 stars**

1.8 Bivariate Regression Models

$$y^i = \beta_0 + \beta_1 x^i + \epsilon^i$$

- y^i dependent variable (price) for the ith observation
- x^i independent variable (AGST) for the ith observation
- ϵ^i error term for the ith observation
- β_0 intercept coefficient
- β_1 regression coefficient for the independent variable

The **best model** makes the **smallest errors**

1.9 How to select the best model ?

- Based on a measure of **choice**
- Sum of Squared Errors:
  - $$SSE = \epsilon^{1^2} + \epsilon^{2^2} + ... + \epsilon^{n^2}$$
  - depends on n number of data points
  - unit hard to understand
- Root Mean Squared Error (RMSE)
  - $$RMSE = \sqrt{SSE/n}$$
  - units of dependent variable

1.10 R^2

- Compares best model to a baseline expectation
- Baseline expectation has no variables
- Baseline expectation is the average
- SST (total sum of squares) captures deviation of dependent variable against average μ

$$R^2 = 1 - (SSE/SST)$$

1.11 Interpreting R^2

$$0 \leq SSE \leq SST$$

- $R^2 = 0$: no improvement over baseline
- $R^2 = 1$: perfect prediction model
- Unitless and universally applicable
- Can compare between models
- Generally cannot compare between problems

1.12 Multivariate Regression Models

$$y^i = \beta_0 + \beta_1 x_1^i + \beta_2 x_2^i + ... + \beta_k x_k^i + \epsilon^i$$

1.13 Improving Model Quality by Adding Variables (I)

```
wine = read.csv("../data/wine.csv")
model4 = lm(Price ~ AGST + HarvestRain + WinterRain + Age, data=wine)
summary(model4)
```

1.14 Improving Model Quality by Adding Variables (II)

```
##
## Call:
## lm(formula = Price ~ AGST + HarvestRain + WinterRain + Age, data = wine)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.45470 -0.24273  0.00752  0.19773  0.53637
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3.4299802  1.7658975  -1.942 0.066311 .
## AGST         0.6072093  0.0987022   6.152  5.2e-06 ***
## HarvestRain -0.0039715  0.0008538  -4.652 0.000154 ***
## WinterRain   0.0010755  0.0005073   2.120 0.046694 *
## Age          0.0239308  0.0080969   2.956 0.007819 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.295 on 20 degrees of freedom
## Multiple R-squared:  0.8286, Adjusted R-squared:  0.7943
## F-statistic: 24.17 on 4 and 20 DF,  p-value: 2.036e-07
```

1.15 Improving Model Quality by Adding Variables (III)

Variable	R^2
Average Growth Season Temperature (AGST)	0.44
AGST, Harvest Rain	0.71
AGST, Harvest Rain, Age	0.79

Variable	R^2
AGST, Harvest Rain, Age, Winter Rain	0.83

- Adding variables can improve a model
- Dimishing returns as more variables are added

1.16 Selecting Variables

- Not all available variables should be used
- Each new variable requires more data
- Causes overfitting: high R2 on data used to create model, but bad performance on unseen data

1.17 Making predictions using the model

- Apply model on unknown data
- Use equation: Unknown x_j^i values multiplied with β_j of the model
- Available function in R: *predict*
  - Parameter 1: model
  - Parameter 2: new data
  - Result: Vector of predictions per observation

1.18 Making predictions using the model (in R)

```
wine = read.csv("../data/wine.csv")
model4 = lm(Price ~ AGST + HarvestRain + WinterRain + Age, data=wine)

wineTest = read.csv("wine_test.csv")
predictTest = predict(model4, newdata=wineTest)

# Compute R-squared
SSE = sum((wineTest$Price - predictTest)^2)
SST = sum((wineTest$Price - mean(wine$Price))^2)
1 - SSE/SST
```

```
## [1] 0.7944278
```

1.19 Out-of-sample performance

Variable	Training R2	Test R2
Average Growth Season Temperature (AGST)	0.44	0.79
AGST, Harvest Rain	0.71	-0.08
AGST, Harvest Rain, Age	0.79	0.53
AGST, Harvest Rain, Age, Winter Rain	0.83	0.79

- Better model R^2 does not mean better test R^2
- Out of sample R^2 can be negative

1.20 Real World Results for Bordeaux wines

- Robert Parker
  - 1986 is very good
  - **2000 greatest vintage ever produced**
- Orley Ashenfelter
  - 1986 is mediocre
  - 1989 is very good
  - 1990 better than 1989
  - **2000 is great**
  - 2003 is great
- Market Auctions
  - 1989 double price of 1986 wines

1.21 Summary

- Linear regression is a fairly simple prediction model
- Model is a linear equation
- Computer learns β coefficients from data
- Make prediction by inserting data into such equations

1.22 Logistic Regression

$$P(Y = 1) = 1/(1 + e^{-(\beta_0+\beta_1 x_1+\beta_2 x_2+...+\beta_n x_n)})$$

- Assume binomial not Gausian distribution when building model
- Limit range of predictions to numbers between 0 and 1
- Binary classification: Use threshold to interpret prediction as YES or NO
- Choose threshold based on Receiver Operator Curve (ROC)