Comparing standard and GARCH-type models to estimate value-at-risk on ASEAN stock market returns

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Edgar John S. Dy
Christian Cedrick C. Olmon
Raphael Yvan S. Tan
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Acceptance Page

The Faculty of the Department of Mathematics of Ateneo de Manila University accepts the undergraduate thesis entitled

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Summary of the Thesis

Stock indices around the world have experienced a lot of spikes in terms of changes in prices due to the volatility caused by extreme phenomena such as the global financial crisis in 2008 and the COVID-19 pandemic. Most studies assume that the standard approaches to estimate market risk, specifically Value-at-Risk or VaR, are not sufficient under these conditions. Thus, time series models are usually used to estimate VaR accurately. This thesis then aims to present to its readers the comparison of standard approaches and GARCH-type models to estimate VaR using the data of five ASEAN stock indices on a time frame of 15 years – January 1, 2006 to January 1, 2022. Specifically, the study aims to compare the three standard models which are the Delta-Normal (DN), Historical Simulation (HS), and Boudoukh-Richardson-Whitelaw (BRW) Hybrid approaches to the GARCH-type models which are the Generalized ARCH, Exponential GARCH, Integrated GARCH, and Fractionally Integrated GARCH. Initial tests on the constant means, serial correlations, and normality of the data were done to properly prepare the data for the models. Afterwards, the standard approaches and the GARCH-type models were used to estimate a one-day ahead VaR for the ASEAN stock indices under two types of windows – fixed and rolling window – using the 5%, 1%, and 0.01% level of significance (LoS). Backtesting methods which are the VaR violations, Kupiec test, and Christoffersen test, and hypothesis testing, which is the Diebold-Mariano test, were used to verify the results of the comparisons which showed that the GARCH-type models generally perform better than the standard models for all levels of significance tested. The results also suggest that the HS approach performs better than the other standard approaches under the VaR violation test. For a longer time frame such as 15 years of data, the EGARCH model has shown to be the best model based on the backtesting methods done while the FIGARCH model comes second. Lastly, sensitivity analyses were also performed on the LoS and time frame of the log returns of the data. Results from the sensitivity analyses show that using a strict LoS of 0.01% on the chosen approaches and models underestimate the log returns of the stock indices. Additionally, using a five-year time frame is effective enough to estimate VaR accurately using the BRW approach and the FIGARCH model for all ASEAN stock indices.

Anti-Plagiarism Declaration

I declare that I have authored this thesis independently, that I have not used materials other than the declared sources or resources, and that I have explicitly marked all materials which have been quoted either literally or by content from the used sources.

Edgar John S. Dy Student ID No. 181776	Date
Christian Cedrick C. Olmon Student ID No. 183593	Date
Raphael Yvan S. Tan Student ID No. 184839	Date

Chapter 1

Introduction

Risk is one of the main factors that influences the decisions of an individual. This is especially true for risk analysts and investors since they must gauge how much risk they are willing to take for their investments. Now, individuals each have their own perception of risk, implying that risk is subjective. However, that is not always the case.

In financial risk management, risk measure is a main key point in making everyday decisions. More specifically, market measure of risk allows a better understanding of the changes in the market such as stock price, interest rates, and exchange rates. The main problem of measuring risk as stated above would be the subjectivity of risk measure. However, modern mathematics has been able to assign objective values to these different risks. This is done through the use of different statistical models which pattern themselves to match real world scenarios by giving a sufficient prediction on future outcomes of risk and creating objective conclusions based on observed data.

J.P. Morgan and Reuters™ (1996) came up with a document, RiskMetrics, that first introduced the concept of Value-at-Risk or VaR, which is commonly used as an alternative term to market risk [14]. The technical document contains information on the methods to compute VaR, the data that the company could distribute to market participants, and the software program that they developed for easier reporting and calculation. Morgan and Reuters simplified the term VaR to a question of how much value can an investor lose with xth percent probability in a given time period. This market risk metric has then changed the finance industry across the world as to how investment managers handle their portfolios [16].

Market risk is one of the most common forms of risk and to measure the amount of market risk taken by any individual would be to calculate the amount of VaR which focuses on the volatility of the assets.

Thus, there is a need to quantitatively measure said risk. The simplest of the form are the standard approaches in calculating VaR. This simplicity comes from the use of some form of parametric distributions and assumptions to better handle the data. Some common parametric approaches would be (1) Delta-Normal Approach (DN), (2) Historical Simulation (HS), and (3) Boudoukh-Richardson-Whitelaw (BRW) Hybrid Approach. First, the DN approach uses a parametric approach which has a key assumption that data must be normally distributed. Second, HS uses a nonparametric approach by assuming a uniform distribution and uses past data as the key assumption is 'history repeats itself'. Lastly, the BRW approach uses a semiparametric approach by creating weights that are dependent on some λ parameter and assigns them to past data [11].

These approaches, however, have some restrictions such as the assumption of a market where data is normally distributed (DN approach) or the need to have an extensive amount of past data to obtain an accurate estimation of VaR (HS and BRW approach). In reality, stock markets around the world rarely fall under normal market conditions. Therefore, more complex time-series models such as the Autoregressive Conditional Heteroskedasticity (ARCH), the Generalized ARCH (GARCH), or extensions of the GARCH-type models are used to model more erratic time periods to create more accurate estimates on VaR through the use of regression. This leads to the question on whether the differences of the VaR obtained using a standard approach has a significant difference to apply the GARCH-type models to obtain the VaR.

1.1 Review of Related Literature

The question posed by the group stems from Omari et al.'s article, *Forecasting Value-at-Risk of Financial Markets under the Global Pandemic of COVID-19 Using Conditional Extreme Value Theory* [16]. The article discussed the use of GARCH-type models and its extensions (e.g., Exponential GARCH (EGARCH), Glosten-Jagannathan-Runkle-GARCH (GJR-GARCH)) to estimate VaR. Data used in the article are the returns of blue chip stocks in the US from the time period June 2017 to July 2020.

Omari et al.'s goal is to quantitatively show that GARCH-Extreme Value Theory (GARCH-EVT), an extension of the GARCH model, is a more reliable model compared to simpler GARCH-type models such as the standard GARCH, EGARCH, and many more. They also compared the models amongst one another through the use of backtesting, mainly through the Kupiec Test which tests the fit of the model and the Christoffersen test which tests the independence of the data from each other. Some sensitivity analysis was also done by observing the differences in the obtained VaR when changing the used distribution from Gaussian to a skew student-t.

Omari et al. extended the study by discussing possible investment management to properly diversify against unstable market conditions and an exploration into another extension of GARCH, MSGARCH-EVT-Copula. A key assumption assumption in the article is the unviability of the standard approaches on estimating VaR especially in an unstable market condition. Thus, the group sees it fit to compare the standard approach with GARCH-type models to substantiate the claim. It follows that literature regarding estimating VaR on both the standard approaches and the GARCH-type models will be required.

A dissertation, entitled *A Study of the Delta Normal Method of Measuring VaR*, written by Rajesh Kondapaneni (2005) [13] discussed the basic fundamentals and scenarios every standard approach may be best used. The goal of the research was to properly educate the reader on the three approaches, DN approach, HS approach, and Monte Carlo Simulation. The research begun by discussing some forms of risk analysis such as mean variance analysis. Afterwards, Kondapaneni discussed VaR, its significance, and its criticisms. This is followed by a mathematical implementation of VaR through the programming language, MatLab, the researcher's programming language of choice. Majority of the research focused on ten stocks over 400 time periods starting from October 2001 to May 2003. Afterwards, the Kondapaneni reviewed the advantages and disadvantages of the said approach supported by observed data. A conclusion of using the DN approach exceeds the HS Approach when market conditions are normal while the opposite is true for the HS Approach. No further analysis was done on The Monte Carlo Simulation.

Another study done by Saša Žiković & Zdenko Prohaska entitled *Optimisation of Decay Factor in Time Weighted (BRW) Simulation: Implications* *for Var Performance in Mediterranean Countries* focused on estimating VaR in Mediterranean countries using the BRW Hybrid approach [22]. A similar approach was done as the previous paper where both VaR and BRW was defined followed by an extensive methodology on optimizing the BRW decay parameter. The paper focused on nine Mediterranean countries over a four year period including the Great Recession in 2008. The research used the Lopez function for optimization and compared them with standard BRW decay parameters, 0.97 and 0.99. Žiković and Prohaska then compared the optimal parameters and the standard BRW decay parameters by forecasting VaR and applying backtesting methods (Kupiec for unconditional coverage and Christoffersen for independence). The researchers concluded that an optimal decay parameter be set between 0.992 and 0.998.

Moving into the time-series models, Engle's *GARCH 101: The Use of ARCH/GARCH Models in Applied Econometrics* [8], discussed the basic foundations of the ARCH and GARCH models. Both the ARCH and GARCH model are patterned to the least-squares method (LSM) to estimate parameters that can properly capture the event. The use of LSM is to avoid the possibility of overfitting the data as it would lead to both an impractical and useless model. It follows that one key difference between Engle's ARCH model and Bollerslev's GARCH model is the distribution of weights on the data used. The ARCH model allows the weights of the squared shocks a_t to be equal to 0 as data further away from the present may have little to no effect in comparison with data yesterday or the day before that.

The GARCH model, on the other hand, assigns strictly decreasing weights on the squared shocks a_t^2 which never go to 0. From this, the GARCH model can be seen to continuously take in newer data and update older data as compared to the ARCH model which places a weight of 0 to older data as it takes in new data. Engle then discussed the applications of the models in real life applications. Estimating VaR was the main example used in the paper by showing how the GARCH model was able to capture the data and fit itself to obtain a forecasted volatility which was used to solve VaR. This then gave a basic idea on how to estimate VaR using the ARCH and GARCH models. However, the paper also raised questions discussing the extensions of the GARCH-type models such as EGARCH, TGARCH, and GARCH-M and how they can be applied with the concept of multivariate modeling with a goal of knowing whether these extensions would affect not only the predicted volatilities of the assets, but also their correlations.

Tayefi and Ramanathan published a paper entitled *An Overview of FIGARCH and Related Time Series Models*, claiming the increased efficacy compared to other conditional heteroskedastic models such as the ARCH and the base GARCH model [19]. The paper began by discussing the importance and significance of the fractionally integrated GARCH model in the context of risk. This is followed by a breakdown on the construction of the FIGARCH model by reviewing its predecessors such as the ARCH model and the GARCH model and on how the FIGARCH model was constructed. Afterwards, an explanation of the parameters of the FIGARCH model and their derivations was given.

The paper then continued on model application and extensive comparison of the FIGARCH model with siilar models such as the adaptive-FIGARCH (A-FIGARCH), hyperbolic GARCH (HYGARCH), and other models. Relating this to the research, the researchers chose to use the FIGARCH model as one of their extended GARCH-type models. This is because the FIGARCH model is shown in the paper of Tayefi and Ramannathan as an improvement of the model. Thus, comparing the GARCH model with the FIGARCH model may yield interesting results as the information surrounding the concept of the FIGARCH are shown to be as late as 2012.

FIGARCH, in terms of estimating VaR, has also been studied. Degiannakis et al. (2013) [7] *Forecasting Value-at-Risk and Expected Shortfall using Fractionally Integrated Models of Conditional Volatility: International Evidence* compared both the standard GARCH model to the FIGARCH model in estimating VaR and expect shortfall. The paper uses a 20-year time period (1989-2009) of over 20 stock indices worldwide. He also assumed two different distributions when estimating VaR, the skewed student-t distribution and the Normal Inverse Gaussian (NIG) distribution. Just like the other papers in the literature, to validate the accuracy of the predictions of the models, Degiannakis et al. applied the same backtesting methods (Kupiec and Chrsitoffersen Test) to test the model's validity. It is also noted that forecast were done are 1-steps, 10-steps, 20-steps. The conclusion reached by the authors stated that no conclusive evidence of applying fractional integrability to the model will make the estimation statistically significant, rather, the time horizon may play as a factor on why this is true. Thus, an increase in studies on the time horizon may benefit the research.

On the other hand, Zargar and Kumar (2018) [21] in their study entitled *Forecasting Value-at-Risk (VaR) in the Major Asian Economies* applied the concept of time series modelling to estimating VaR. Their dataset comprised equities on major Asian economies of Singapore, Malaysia, Hong Kong, Indonesia, South Korea, Philippines, Thailand, China, Taiwan, and India. The study wanted to compare various time series models in estimating VaR. Zargar and Kumar came up with the conclusion that for the given dataset, the FIGARCH model appears to be the most effective model in calculating VaR, with a success rate of 43.3% in the unconditional testing and 31.6% in the conditional testing. The authors then recommended to use the model to calculate VaR for other economic markets such as derivatives and bonds.

Other studies such as one study conducted by Smolvic et al [18] entitled *GARCH models in value at risk estimation: empirical evidence from the Montenegrin stock exchange* considered various GARCH-type models to measure VaR using the Montenegrin market.

The main problem of the study was to quantify and manage the risk of the emerging Montenegrin market. Similar with the study of Omari et al., backtesting using the Kupiec and Christoffersen tests were done to compare and contrast the various models. It was found that ARMA(1,2)-TS GARCH(1,1), ARMA(1,2)-T GARCH(1,1) model with a Student-t distribution of residuals and ARMA(1,2)-EGARCH(1,1) with a reparameterized unbounded Johnson distribution are the three models that they considered accurate in estimating VaR.

Another would be Boonyakunakorn et al (2019) [5] in their study entitled *Value at Risk of the stock market in ASEAN-5* analyzed the VaR for the five different ASEAN stock indices, Indonesia, Malaysia, Philippines, Thailand, and Singapore. The paper used Bayesian MSGARCH models to estimate

VaR. It was shown that the PSE is preferred for risky investors and KLSE is preferred for risk-averse investors.

All these studies have one common assumption, that the GARCH-type models are superior in terms of accurately estimating VaR. The empirical evidence is left out as a main assumption and it is in the interest of this study to fill that gap and come to a conclusion of which model or approach will accurately estimate VaR on the chosen dataset.

1.2 Methodology, Scope, and Limitation

1.2.1 Methodology

The research aims to create a quantitative comparison between the standard approaches which are the (1) Delta-normal Approach, (2) Historical Simulation, and (3) BRW Hybrid Approach and the time-series models which are the (1) GARCH model and (2) extensions of the GARCH model (Exponential GARCH (EGARCH), Integrated GARCH (IGARCH) and Fractionally Integrated GARCH (FIGARCH)) to forecast VaR and see whether there exists a statistically significant difference in the VaRs obtained from the different approaches. The choice of a comparative approach for the research is motivated by the research gap posed by the article written by Omari et al., as there were no quantitative data to support the claim that the time series GARCH-type models are better than standard approaches in estimating VaR.

The paper is split into six chapters: (1) Introduction, (2) Preliminaries, (3) Results and Discussion, (4) Extensions, (5) Conclusions and Recommendations, and (6) Appendix. Chapter 1, as seen above, begins by stating related literature with the research. Discussions on several papers such as Engle, Omari et al., and Aridi et al., which explores concepts of VaR and the other models are provided.

Chapter 2 discusses preliminary definitions and data that will be used in the research. The chapter begins with the chosen data obtained from Yahoo Finance, MarketWatch, Investing.com, and other similar websites which are extracted and prepared through the use Pandas.DataReader, a Python package. This is followed by a brief discussion on the chosen assets used for the research and an in depth discussion on the concept of VaR, the standard approaches, the standard GARCH model, the Exponential GARCH (EGARCH), the Integrated GARCH (IGARCH) the Fractionally Integrated GARCH (FIGARCH), and three methods of backtesting namely the VaR violations, the Kupiec test, and the Christoffersen test, and the use of the Diebold-Mariano test to test the models from one another which will be used to analyze the main problem of this paper.

Chapter 3 focuses on obtaining the VaR using the stated approaches. First, the chosen datasets will go through preparatory test such as the Shapiro-Wilk test, the Ljung-Box test, and the t-test. For both the standard approaches and the GARCH-type models, the use of the programming language Python will be used to obtain the forecasted VaR. The research will only assume a normal distribution when estimating VaR given the large datasets used. Afterwards, backtesting methods will be applied on both standard approaches and GARCH-type models to see whether the models are effective in capturing the data. Lastly, two-sample hypothesis testing through the use of the Diebold-Mariano test will be done to conclude whether the differences in VaR are statistically significant. This chapter will contain the needed data visualizations to properly accommodate the data.

Chapter 4 comprises of an extension of the study where a sensitivity analysis will be conducted by changing parameters such as the time window of the data to observe differences with the main results in Chapter 3. Chapter 5 concludes the results and the discussion followed by possible extensions and recommendations that can be done from this research. Lastly, Chapter 6 contains a compilation of the data visualizations and the link of the GitHub repository which contains the codes used to obtain said visualizations and results..

1.2.2 Scope and Limitation

The scope of the research will focus on five (5) ASEAN stock indices within a time period of 15-years (January 2006 to December 2021) and the estimated 1-day forecast of their VaR at the significance level of 5%, 1%, and 0.01%. Both the standard approaches, the GARCH model and its extensions are used in obtaining these estimates. Three extended GARCH-type models will also be used to estimate VaR, specifically the EGARCH, IGARCH and the FIGARCH. A discussion will also be done to explain the forecasted VaR obtained from the different approaches, to test the viability of these models, and to show whether the differences are significant enough to choose a different approach.. The research also aims to explore the variability of VaR when changing certain parameters such as altering the length of time period to see any significant changes in the output. This leads to creating a quantitative comparison on the standard approaches, the standard GARCH model, and its extensions.

The limitations of the research is the inability to capture the market as a whole and create an optimized portfolio based from the forecasted volatility and VaR as forecasts done are heavily reliant on information currently available in the market. Given the data that will be used will be the market index of different ASEAN countries, some data may be withheld and may skew the data. The researchers are also not stating that the chosen models are the best in practice, but rather, they will be quantitatively compared relative to one another to come to a decision on which model can sufficiently capture a stock indices' log returns consistently. The chosen assets may also lead to sampling bias as blue chip stocks may be more stable in comparison to lesser known stocks in more volatile market conditions.

There are also some delimitations that were set by the researchers. The researchers did not choose to have more than three extended GARCH-type models as the main goal would be to compare the GARCH model with standard approaches in obtaining VaR. It follows that the researchers also chose to use ASEAN stock indices. The researchers also choose to use the programming language Python to conduct the test and to analyze and visualize the data. The researchers did not also compare the standard approaches to estimate VaR to models with Extreme Value Theory integrated into them as derived by Omari et al. given the constraints on time to review the concept in depth. This could be a future research question that can be answered.

Chapter 2

Preliminaries

In this chapter, a discussion on the definitions and fundamental models will be needed to prove the necessary claims of this paper. The definition of returns and value at risk allows for a better understanding of the following concepts.

Definition 2.1 (Continuously Compounded Returns)**.** [20]

To obtain the continuously compounded or log returns, R_t, at time t, where t is a positive integer, the following formula is used

$$R_t = \ln \frac{S_t}{S_{t-d}}$$

where S_t denotes the closing price of the stock at time t and S_{t-d} is the closing price of the stock at time $t - d$ where d is a positive integer.

2.1 Value-at-Risk

Value-at-risk or VaR for short is one of the main focuses in financial risk management. VaR focuses on estimating the worst possible amount of loss over some time frame of an asset (such as a single stock, bonds, or a portfolio that comprises risk-free and risky assets). VaR can either be expressed in terms of currency or in percentage of market returns. Throughout the paper, VaR will be in terms of percentages. Three of the main parameters in estimating the amount of loss in stocks or stock indices are (1) the forecast horizon, d, (2) the volatility of the asset, σ, and (3) the level of significance

$100p\%$ where $0 < p < 1$. The general formula, as defined by Kelepouris D. and Kelepouris I. (2019) [12], in estimating a confidence level $(1-p)\%$ of a 1-day VaR of a single asset or stock given a normal market is denoted as

$$\text{VaR}(1-p) = \mu + \sigma N^{-1}(1-p)$$

where μ denotes the mean of the log returns of an asset or portfolio, σ as the volatility of the asset, and $N^{-1}(1-p)$ is the cumulative distribution (CDF) of a normal distribution with parameter $(1-p)$ which denotes the confidence level. The standard approaches use the general formula to use VaR.

However, as discussed in the review of related literature of the ARCH and GARCH-type models, these models do not necessarily assume a normal market, thus there is the need to have some slight adjustments to the general formula for VaR by changing the distribution from a normal distribution to a type of leptokurtic distribution, such as the Poisson distribution, Student-t distribution, and Exponential distribution, which all have a feature of having fatter tails compared to the normal distribution. The formula of a ARCH/GARCH model is as follows

$$\text{VaR}(1-p) = \mu + \sigma_{t|t-1} T^{-1}(1-p)$$

Once again, μ denotes the log returns of a single asset. On the other hand, $\sigma_{t|t-1}$ also describes the volatility of the asset, however, the volatility is based of past information at time $t-1$, thus making $\sigma_{t|t-1}$ the conditional variance of the model.

Lastly, $T^{-1}(p-1)$ is the CDF of a Student-t distribution. The use of this distribution rather than the standard normal distribution is because of the existence of fat tails in unstable market conditions. However, if the time period is sufficiently long enough, the values of the Student-t distribution also converge to the normal distribution. Therefore, the use of a normal distribution instead of a Student-t distribution for ARCH/GARCH models is acceptable for sufficiently long time periods.

2.2 Standard Approaches

The standard approaches to estimate VaR are usually the ones easier to work with (Alexander, 2009) [1]. Therefore, given the ease of the standard models, they set a benchmark for estimating VaR. The common standard approaches are (1) the Delta-Normal (DN) approach, (2) the Historical Simulation (HS) approach, and (3) the Boudoukh-Richardson-Whitelaw (BRW) approach. These approaches are commonly used to estimate the n-day VaR.

There are benefits to using the standard approaches. The DN approach is effective at estimating even at 400 time periods as revealed in the study of Kondapaneni (2005) [13]. The Historical Simulation approach is still widely used given its simplicity as there is no need to assume any distribution (Alexander, 2009) [1]. The combined parametric and historical simulation approach of the BRW model estimates the VaR more accurately as revealed in the study of Boudoukh, Richardson, and Whitelaw [6].

2.2.1 Delta-Normal Approach

The Delta-Normal approach for calculating VaR is one of the standard approaches to estimate the worst possible loss over a given time frame. This is because the DN approach only has two main requirements to obtain the VaR: (1) variance of the log returns of the asset and (2) the covariances between different assets if there exist more than one asset in the portfolio. This removes the need to have a large archive historical data to forecast VaR. This is based on the key assumptions that the Delta-Normal approach places the log returns of an asset under a parametric distribution (normal distribution with a mean of 0 and a variance, σ^2, $N \sim (0, \sigma^2)$). The formula is as follows

$$\text{VaR}(1-p) = \mu + \sigma N^{-1}(1-p) = \sigma N^{-1}(1-p)$$

This leads to two different ways to calculate VaR using the Delta-Normal approach: The undiversified Delta-Normal Approach and the diversified Delta-Normal Approach. However, this is beyond the scope of the research as each stock index will be taken as a portfolio contain the said stock index.

2.2.2 Historical Simulation Approach

Historical Simulation takes a different approach compared to the DN approach since the log returns of an asset are not assumed to be under some parametric distribution, rather the log returns are assumed to be uniformly distributed. It follows that the main focus of the Historical Simulation would be the historical values of the asset. From this, the key assumption of the approach would be the idea that "history repeats itself." To obtain the VaR using Historical Simulation, it starts by taking the log returns of the asset [15]. It follows that the change in portfolio value is obtained using the following formula

$$\Delta P = P_1 - P_0 = NS_1 - NS_0$$

where P_1 is the portfolio value in the next time period, P_0 is the current portfolio value , N is the number of shares of the asset, S_0 is the current closing price of the asset, and S_1 is the closing price of the next time period. S_1 can be written as $S_1 = S_0 e^{R_0} \approx S_0(1 + R_0)$ when t is sufficiently small. From here, the periodicity that is used will be daily. Thus, the change in portfolio value can be written as

$$\Delta P_j = NS_{1,j} - NS_{0,j} \approx N(S_{0,j}(1 + R_{0,j}) - S_{0,j}) = NS_{0,j}R_{0,j}$$

where $S_{1,j}$ is the closing price tomorrow based on time j, $S_{0,j}$ is the closing price today based on time j, and $R_{0,j}$ is the return of the asset at time j. Lastly, the values are then sorted from the worst to the possible amount of loss. Since the values are normally distributed, the $100p$th percentile of the change in portfolio values is then the estimated VaR where p is the level of significance.

For the DN approach, the values have the assumption of being i.i.d. and normally distributed. Thus, this assumption may falsely overestimate or underestimate the log returns of the assets. On the other hand, the HS approach greatly relies on the past data. The less data available the more overestimated or underestimated the VaR will be. It follows that the BRW approach works on the disadvantages of both the DN approach and the HS approach.

2.2.3 BRW Hybrid Approach

The BRW Approach takes a combination of the two approaches where it weighs the historical data based on the BRW parameter λ[6]. This is to compensate for the possibility that there may be a lack of data. To obtain the VaR using the BRW approach, the BRW approach patterns the Historical Simulation by taking the change in portfolio values. Afterwards, it has an additional step which designated weights on these changes.

The weight w_i, can be solved using the following formula

$$w_i = \frac{1-\lambda}{1-\lambda^M}\lambda^i$$

where λ is the BRW parameter such that $0 < \lambda < 1$, M is ith number of log returns which can be taken by getting the difference between the number of entries for the closing day price by the d-day forecast such that M is a positive integer. From this, the change in portfolio values are then arranged from greatest to least possible loss. Afterwards, the cumulative sum of the weights are taken for each entry. This is written as follows

$$\Psi_i = \sum_{i=0}^{M-1} w_i$$

where Ψ_i is the cumulative weight given the change in value ΔP_i. Lastly, the BRW approach patterns the HS approach by obtaining the pth percentile to estimate VaR. This is only applicable though if p is an element of the sequenceΨ_i. If not, there is a need for linear interpolation to obtain the VaR under the BRW approach.

2.3 GARCH-type Models

GARCH-type models are commonly used time-series models to measure volatility. One of the main advantage of a GARCH-type model is that it can produce volatility clusters, which can be used to model stock returns better [20]. GARCH-type models are used to model financial data, much like how Riskmetrics uses an IGARCH(1,1) model for estimating VaR [14] [20]. The study will then compare the standard approaches to GARCH-type models to assess the performance of each model for the chosen dataset.

2.3.1 The ARMA Model

Some major key assumptions with the standard approaches is the fact that the log returns of the assets are placed under some forms of restrictions such as being normally distributed with mean 0 and variance σ^2 as seen in the DN Approach or being uniformly distributed as seen from both the HS. Due to these assumptions, these standard approaches fail to model more dynamic and unstable market conditions as fitting the data may lead to fatter tails or multiple peaks.

This then requires a model that uses past data to obtain forecasted data. This leads to two of the most basic time-series models, the Autoregressive (AR) Model, which focuses on the log returns of the asset and the Moving Average (MA) model, which focuses on the volatility of an asset.

These two models, however, fail to model unstable market conditions as it leads to creating an inefficient model with an unreasonable number of parameters. Thus, the creation of the Autoregressive Moving Average (ARMA) model happened. A key assumption in the ARMA model is the fact that the variables must be weakly stationary, meaning, that the mean and variance are time invariant in multiple subperiods [20].

The general formula of the ARMA(p, q) model is as follows:

$$x_t = \Phi_0 + \sum_{i=1}^{p} \Phi_i x_{t-i} + a_t - \sum_{i=1}^{q} \theta_i a_{t-i}$$

where $\{a_t\}$ is a white noise series, Φ_i and θ_i are parameters where $\Phi_i \neq \theta_i$, and x_{t-i} being the past returns of the asset where t and i are positive integers.

Since the ARMA model is a combination of the two models, then it can be broken down into its AR components and its MA components.

AR component

The AR component focuses on the past returns of the asset to predict the future returns of the asset. This is done by taking p parameters to estimate the returns based on past returns. It is also noted that the weights an individual past return will decay to 0 as p increases. This implies that effects of past returns are permanent [20].

The formula for the AR component is as follows:

$$x_t = \Phi_0 + \sum_{i=1}^{p} \Phi_i x_{t-i}$$

Therefore, for the AR model to properly capture the data, especially under unstable market conditions, there is a need to increase the number of parameters, however, it is unrealistic to have an infinite number of parameters. The MA component of the ARMA model solves the possibility of having an infinite number of parameters.

MA component

The Moving Average (MA) component of the ARMA model focuses on the error terms or residuals $\{a_t\}$ based on time t. The error terms help visualize how accurate the prediction is to the log returns and correct the forecasted returns of the AR component. Compared to the AR component, the weights held by the residuals become 0 as q becomes sufficiently large, therefore implying that past residuals do not have a permanent effect in the future returns of an asset [20].

The formula for the MA component of the ARMA model is as follows:

$$x_t = c_0 + a_t - \sum_{i=1}^{q} \theta_i a_{t-i}$$

where c_0 is some constant.

The use of the autocorrelation function (ACF) is used in order to obtain the order q of the MA component.

Connecting this to VaR, the ARMA model does not assume that the returns are under a normal distribution. Thus, it takes into account the possibility of fat tails that may happen. Consequently, the ARMA model does not factor in the possible conditionality of the returns of an asset with one another, having the need to implement the ARCH model.

Lag or Backshift Operators

Lag operators dictate the shift in time of an element in a time series. This is usually denoted by L^k where k is an integer which indicates the number of periods before the chosen element. An example is as follows:

Let a_t be an element in a time series at time t, then $L\alpha_t = a_{t-1}$. Generalizing the case, $L^k\alpha_t = a_{t-k}$.

From this, it is possible to write the general formula of the ARMA model in terms of the lag operators [20].

The general formula of the ARMA model in terms of the lag operators is as follows:

$$(1 - \phi_1 L - \cdots - \phi_p L^p)x_t = \phi_0 + (1 - \theta_1 L - \cdots - \theta_q L^q)a_t$$

which can be further simplified to as

$$(1 - \phi_p(L))x_t = \phi_0 + (1 - \theta_q(L)))a_t$$

where $\alpha_n(L) = \sum_{i=1}^{n} L^i \alpha_i$

2.3.2 ARCH Effects

The Autoregressive Conditional Heteroskedasticity (ARCH) model of Engle (1982) [20] can be understood by breaking down each term of the ARMA model. The *autoregressive* portion discusses the use of the AR(p) model in estimating future risk and volatility while the *conditional* portion implies the use of past information in the model.

In contrast to time-invariant time series models (AR, MA, ARMA), the ARCH model does not assume a time-invariant system, explaining the *heteroskedasticity* portion. It follows that to use the ARCH model, the data must display ARCH effects. Two tests can be applied to draw an appropriate conclusion, (1) the Ljung-Box Test and (2) the Lagrange multiplier test of Engle [20].

Ljung-Box Test

The Ljung-Box test [20] focuses on the existence of serial correlations in the data. The test has the following null hypothesis, H_0: The model has a proper fit on the data, in other words, the data does not exhibit any serial correlations. On the other hand, it has an alternative hypothesis, H_a: The model does not properly fit the data, in other words, the data is serially correlated. That is,

$$H_0 : \rho_1 = \rho_2 = \cdots = \rho_m$$
$$H_a : \rho_j \neq \rho_k, \text{ for } j \neq k$$

Rejection of H_0 is done when the test statistic Q is greater than $\chi^2_{1-\alpha,h}$ or if the p-value obtained is less than the level of significance α. The test statistic Q can be written as follows:

$$Q = n(n+2)\sum_{k=1}^{h} \frac{\hat{\rho}_k^2}{n-k}$$

where n, k, h are elements of the set of all natural numbers and n is the sample size of the data, $\hat{\rho}_k$ is the sample autocorrelation at lag k, and h is the number of lags being tested.

The Lagrange Multiplier Test

The Lagrange multiplier test focuses on the conditional heteroskedasticity. The test has the following null hypothesis, H_0: The residuals of the data is equivalent to 0. The residuals can be understood as white noise. The alternative hypothesis is given as, H_a: The residuals are statistically significant thus not equal to 0. That is,

$$H_0 : \alpha_0 = \alpha_1 = \cdots = \alpha_m = 0$$
$$H_a : \alpha_k = 0 \text{ for } k \in N \text{ and } k \leq m$$

Rejection of H_0 happens when the test statistic $mF > \chi^2_m$, where m is the number of lags, or the p-value of mF is less than the level of significance. The test statistic F is as follows:

$$F = \frac{(SSR_0 - SSR_1)/m}{SSR_1/(T - 2m - 1)}, \quad SSR_0 = \sum_{t=m+1}^{T} (a_t^2 - \bar{\omega})^2, \quad SSR_1 = \sum_{t=m+1}^{T} \hat{e}_t^2$$

where $\bar{\omega} = \frac{1}{T} \sum_{t=1}^{T} a_t^2$, or the sample mean of a_t^2 and $\hat{e}_t^2$ is the least squares residual of the prior linear regressions.

2.3.3 The ARCH Model

From the two tests, the two ARCH effects needed to show is that the data is serially correlated, that is, the null hypothesis of the Ljung-Box test is rejected, and the existence of residuals not equal to 0, that is, the null hypothesis of the Lagrange multiplier test is also rejected.

The ARCH model then uses the following equations to obtain the residuals, the conditional volatility [20]. The equations are as follows:

$$a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_m a_{t-m}^2,$$

where $\{\varepsilon_t\}$ is the sequence of independently and identically distributed random variables with a mean of 0 and variance of 1. It can be noticed that the ARCH model patterns AR model. Therefore, the partial ACF (PACF) is used to obtain the order of the model p. Placing this in the context of VaR, the general formula from Kelepouris and Kelepouris (2019) [12] is

$$\mathrm{VaR}(1-p) = \mu + \sigma N^{-1}(1-p)$$

However, since the ARCH model does not necessarily assume a normal distribution, it is possible that other distributions can be employed. More specifically, the Student-t distribution, the skew Student-t distribution, or the Generalized Error distribution (GED). Thus, the function for the VaR can be rewritten as,

$$\mathrm{VaR}(1-p) = \mu + \sigma_i D^{-1}(1-p)$$

where σ_i is the conditional volatility and D is the type of distribution used. Since the ARCH model is not limited to following a fixed distribution, there are several advantages that the model has,

1. The model can produce volatility clusters.

2. The shocks a_t of the model have heavy tails.

On the other hand, the model also has its disadvantages such as,

1. The model assumes that positive and negative shocks have the same effect on the volatility.

2. The restrictiveness of the ARCH model.

3. The model does not discuss any possible hypotheses on why the behavior of the volatility is what can be observed.

4. The model overpredicts the volatility as it responds slowly to large changes in the shocks.

It will be noted that the use of the Gaussian distribution will be used for the paper given the large dataset used. Thus, leading to the implementation of the Generalized ARCH (GARCH) model, which is an improvement from the ARCH model [20].

2.3.4 The GARCH Model

As stated in the previous section, the ARCH model has a disadvantage of being too restrictive, thus creating models that contain an unpractical number of parameters. An improvement on the restrictiveness of the ARCH Model is the Generalized ARCH (GARCH) model. Unlike the ARCH model where forecasts are dependent on the past residuals of the data, the GARCH model extends by factoring the past volatilities as another parameter alongside the past residuals to forecast volatility [20].

Since the GARCH model is an improvement of the ARCH model, the data used under the GARCH model must also exhibit ARCH effects as stated in the previous section. Similar to the ARCH model, the GARCH model can forecast the residuals and conditional volatilities [20].

The equations for the respective outputs of a GARCH(m, s) model are as follows,

$$a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{s} \alpha_i a_{t-i}^2 + \sum_{j=1}^{m} \beta_j \sigma_{t-j}^2$$

where α, β are parameters obtained from fitting the data into the GARCH model, and $\{\sigma_{t-j}\}$ is the sequence of past volatilities of data. The other variables are similar to those in the ARCH model. Compared to the ARCH model, the inclusion of past volatility as a factor to forecast future volatility pattern closely to the ARMA model. However, just like the ARCH model, the GARCH model does not assume a time-invariant system. This leads to the existence of conditionality and dependency on past data.

The GARCH model can then be written in the form of the ARMA model of squared residuals a_t^2 and also in terms of lag operators respectively.

$$a_t^2 = \alpha_0 + \sum_{i=1}^{s} \alpha_i a_{t-i}^2 + \sum_{j=1}^{m} \beta_j \sigma_{t-j}^2 - \sum_{j=1}^{m} \beta_j v_{t-j} + v_t$$

$$[1 - \alpha_s(L) - \beta_m(L)] a_t^2 = \alpha_0 + [1 - \beta_m(L)] v_t$$

where $v_t = a_t^2 - \sigma_t = (\varepsilon_t^2 - 1)\sigma_t$.

Placing this in the context of VaR, the GARCH model has a similar approach to the ARCH model in calculating the VaR as both the volatility and distributions are the main variables in estimating the VaR. However, the GARCH model also suffers from a similar problem as the ARCH model, where both models respond equally to positive and negative shocks and the tails of the models are too short which may create false connotations on the kurtosis — whether or not a distribution has a heavy-tail, that is, there are more data points that exist on the ends of the distribution, or has a light tail, there are less data points on the ends of the distribution. Therefore, it is appropriate to seek for a further improvement of the model.

2.3.5 The EGARCH Model

The GARCH model is a foundational model which could be used to derive other time series models. This means that the GARCH model could be improved to better fit the time series data that it wants to model. The exponential GARCH or EGARCH model improves upon the GARCH model and its inability to create any distinction between the effect of positive and negative shocks. The EGARCH model recognizes the shocks, both positive and negative as asymmetric. In the study of Omari et al. (2020), the EGARCH model proved to be an appropriate model to forecasting VaR as it outperforms the standard approaches and the GARCH model [16].

Developed by Daniel B. Nelson in 1991, a weighted innovation was developed to allow for asymmetric effects of both positive and negative shocks [20]. This is defined by function g which is defined as follows,

$$g(\varepsilon_t) = \theta\varepsilon_t + \gamma[|\varepsilon_t| - E(|\varepsilon_t|)]$$

where θ and γ are real constraints obtained when fitting the data into the model and ε_t is the residuals at time t. This then can be rewritten as a system of equations where the residuals are nonnegative or negative.

$$
\begin{aligned}
&\text{Case } 1 : \varepsilon_t \geq 0 \\
&\qquad g(\varepsilon_t) = (\theta + \gamma)\varepsilon_t = \gamma E(|\varepsilon_t|) \\
&\text{Case } 2 : \varepsilon < 0 \\
&\qquad g(\varepsilon_t) = (\theta - \gamma)\varepsilon_t - \gamma E(|e_t|)
\end{aligned}
$$

From this, the equation for the residuals and conditional volatility can be obtained respectively under the EGARCH model:

$$
a_t = \sigma_t \varepsilon_t, \quad \ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{m} \alpha_i \frac{|a_{t-i}| + \gamma a_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{s} \beta_j \ln(\sigma_{t-j}^2)
$$

where α_i, γ_i, β_j are obtained through model fitting, a_{t-i} and σ_{t-j} are past residuals and past volatilities respectively. Similar with both the ARCH and GARCH models, the general VaR formula will be used where the distribution D will be manipulated. The formula is as follows,

$$
\text{VaR}(1-p)_i = \mu + \sigma_i D^{-1}(1-p)
$$

where σ_i is the conditional volatility at time i, and D is the chosen time distribution.

2.3.6 The IGARCH Model

The Integrated GARCH (IGARCH) model does not deviate too far from the standard GARCH model. This is the model that J.P. Morgan and Reuters$^{\text{TM}}$

(1996) [14] used to derive their methodology to calculate VaR. The IGARCH model is also important to discuss before understanding it's more complex model, the Fractionally Integrated GARCH (FIGARCH) model.

Intuitively, the IGARCH model is a more restrictive model compared to the GARCH model as it is just the unit root GARCH model where the summation of the derived parameters must equal to 1 [20].

The given conditions of the IGARCH model can be written mathematically as

$$\sum_{i=1}^{s} \alpha_i + \sum_{j=1}^{m} \beta_j = 1$$

This can also be compared to simpler time-series models, that is, the ARMA model and the Autoregressive Integrated Moving Average model or the ARIMA model. Similar to the ARIMA model, the key feature of the IGARCH model is that the impact of the past shocks $\eta_{t-i} = a_{t-i}^2 - \sigma_{t-i}^2$ for $i > 0$ on a_t^2 is persistent. The general formula for the residuals and the conditional variance are as follows,

$$a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{s} \alpha_i + \sum_{j=1}^{m} \beta_j$$

Since the model is derived from the GARCH model, this can also be written in terms of lag operators where the most significant change is the addition of the differencing operator, $(1 - L)$.

$$[1 - \alpha_s(L) - \beta_m(L)](1 - L)a_t^2 = \alpha_0 + [1 - \beta_m(L)]v_t$$

The parameters of the IGARCH model are similar to the GARCH model, however, because of condition that the derived parameters must be equal to

1, the model tends to be more restrictive compared to the standard GARCH model. A major weakness of the IGARCH model is its restrictiveness as it implies an infinite persistence of shocks.

2.3.7 The FIGARCH Model

The FIGARCH model [3] is directly derived from the IGARCH model. This is done by applying a change on the differencing operator of the IGARCH model when in terms of lag operators. The change consist of making the first differencing operator $(1-L)$ into a fractional differencing operator $(1-L)^d$ where d is in between 0 and 1 [3]. Bentes and Ferreira (2014) also made the observation that the FIGARCH model is a derivation for both the GARCH model and the IGARCH model for special cases when $d = 0$ and $d = 1$ respectively [4].

The FIGARCH model can be written as,

$$[1-\alpha_s(L)-\beta_m(L)](1-L)^d a_t^2 = \alpha_0 + [1-\beta_m(L)]v_t$$

Just as the former models have a general to obtain the conditional variance and residuals, the FIGARCH(p, d, q) also has a general formula to obtain the following,

$$a_t^2 = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \alpha_0[1-\beta(1)]^{-1}+\alpha_s(L)[1-\beta_m(L)]^{-1}a_t^2 \equiv \alpha[1-\beta(1)]^{-1}+\lambda(L)a_t^2$$

where the roots of $[1-\alpha_s(L)-\beta_m(L)]$ and $[1-\beta_m(L)]$ are constrained to lie outside the unit circle.

Baillie and Bollerslev [3] also proposed an optimal value for d. According to their study using the data of daily returns from 1953 to 1990 of Standard and Poor's 500 composite stock index, the estimated fractional differencing parameter d is 0.633. This also comes with a 0.063 asymptotic standard error which makes sure that the model takes into account past volatility with high significance.

2.4 Backtesting

Given the stated models above, it is important to apply backtesting methods to the said models. Backtesting is the general method to examine whether a given model or approach is sufficient enough to capture said data. Intuitively, backtesting is done through the use of historical data to see whether the actual returns line up with the predicted values, which comes to a conclusion on the viability of the model. Two main approaches will be used to check the sufficiency of the models: *(1) The Unconditional Coverage Property* and *(2) The Independence Property.* Both properties must be exhibited by the model to conclude that the model is sufficient in capturing the data. This is consistent with the backtesting methods done by Omari et al. (2020) [16] in their study.

The Unconditional Coverage Property

The unconditional coverage property uses VaR violations to observe the validity of the model. The following is a recommended standard to observe the unconditional coverage property. A *VaR Violation* can be understood as the number of times that the actual loss exceeds the predicted worst possible loss or VaR. Written mathematically, let y be the number of predictions and let the VaR violations, N, be the number of times the predicted VaR is greater than the actual loss. It follows that the violation ratio can be taken as the ratio between the number of violations over the number of predictions. The equation is as follows:

$$\text{Violation Ratio} = \frac{N}{y}$$

The Violation Ratio then can give a percentage of the number of VaR violations compared to the total predicted losses. It follows that the two conclusions can be deduced from the violation ratio. Given a level of significance, α, if the Violation Ratio is greater than α, then the model understates the chosen data, thus concluding an exaggerated model. The other scenario shows that if the Violation Ratio is less than α, then the model overstates the data, thus concluding a conservative model. However, implementing a binomial distribution, $\text{Bi}(y+1, \alpha)$ with the null hypothesis H_0 is the observed violation rate $\frac{N}{y}$ is statistically equal to the level of significance α. The alternative hypothesis, H_a, is that the violation rate $\frac{N}{y}$ is not statistically equal to α. The test statistic X and the confidence interval can be written as the following, Let VR be the Violation Ratio,

$$X = \frac{VR - \alpha y}{\sqrt{\alpha y(1-y)}},$$

$$(1-\alpha)\%CI = \left(VR \mp Z_{\frac{\alpha}{2}} \sqrt{\frac{VR(1-VR)}{y}} \right).$$

H_0 is rejected when the test statistic X is less than $Z_{\frac{\alpha}{2}}$ or greater than $-Z_{\frac{\alpha}{2}}$ where Z is the standard normal distribution or if N does not fall within the specified confidence interval. The study will be using the confidence interval to show whether the models overestimate or underestimate the data.

Kupiec Test

Another test that is as effective as the standard test for unconditional coverage was devised by Paul H. Kupiec [17] with a similar goal to show that the Violation Ratio is statistically equal to α. Both tests hold the same null and alternative hypothesis where H_0 states that the observed violation rate $\frac{N}{y}$ is statistically equal to the level of significance α and H_a states that the violation rate $\frac{N}{y}$ is not statistically equal to α. However, compared to the previous unconditional convergence test, the Kupiec test uses a chi-squared distribution with 1 degree of freedom, χ_1^2, as its critical value. The test statistic, LR_{uc}, is given as follows,

$$LR_{uc} = -2\ln\frac{\alpha^N(1-\alpha)^{y-N}}{(1-VR)^{y-N}VR^N}.$$

The null hypothesis is then rejected when LR_{uc} is greater than $(\chi^2)^{-1}_{\alpha,1}$.

The Independence Property

The independence property [17] focuses on the independence on the occurrence of the VaR violations. Thus, given two random instances of VaR violations, the two said instances must not have any conditional relationship between one another, therefore they are independent of one another. To put it simply, the two instances of VaR violations must not convey any information about any future VaR violation, in order words, the failure to exhibit the Markov Property at the first order. Mathematically, this can be written as the following:

Let $1^t = 1_{\{L_t < VaR_{\alpha,t}\}}$ where L_t is the actual losses at time t, then to show the failure of the Markov Property at the first moment,

$$P(1^t = |1^{t-1} = 0) = P(1^t = 0|1^{t-1} = 1) = \alpha$$

Suppose the opposite where the two instances show some form of causation, then the model will fail to adequately measure the VaR as the future predicted VaR may be greater or less, thus may skew the data in a way where there may be an excess or lack of VaR violations. Peter F. Christoffersen [17] proposed a way to test the independence of the VaR violations, thus the introduction of the Christoffersen Test.

The Christoffersen Test

The Christoffersen Test has the goal of showing independence of the VaR violations through the use of a chi-squared distribution. The null hypothesis H_0 states that the VaR violations are independent from one another while the alternative hypothesis H_a states that the VaR violations are not independent from one another. Christoffersen introduces the likelihood ratio of independence, LR_{ind}, and the test statistic can be written mathematically as follows,

$$LR_{ind} = -2\ln\frac{(1-\pi)^{n00+n10}\pi^{n01+n11}}{(1-\pi_0)^{n00}\pi_0^{n01}(1-\pi_1)^{n10}\pi_1^{n11}}$$

where $n00$is the number of periods with no VaR violations followed by another with no violations, $n01$ is the number of periods having no VaR violations followed by a VaR violation, $n10$ is the number of periods with a VaR violation followed by one with no VaR violations, and $n11$ is the number of periods with a VaR violation followed by another VaR violation. $\pi_0, \pi_1, and \pi$ are the probabilities that a violation occurs given a certain condition, π_0 is the probability that a violation occurred in the following period given that the previous period had no VaR violation. This can be written as follows,

$$\pi_0 = \frac{n01}{n00 + n01}$$

π_1 is the probability that a VaR violation occurred in the following period given that the previous period was also a VaR violation. This can be written as the following,

$$\pi_1 = \frac{n11}{n10 + n11}$$

Lastly, π is the probability that a VaR violation occurred given that the previous period was either a VaR violation or not. It can be written as follows,

$$\pi = \frac{n01 + n11}{n00 + n01 + n10 + n11}$$

As stated above, the use of a chi-squared distribution is done to determine the independence of VaR violations. Thus, using the test statistic LR_{ind}, H_0 is rejected if LR_{ind} is greater than $(\chi^2_{\alpha,1})$ or if the $p - value$ of LR_{ind} is less than α.

The Conditional Coverage Property

The conditional coverage property was proposed by Christoffersen as an improvement on the two properties by combining both the unconditional coverage and the independence property into one. The likelihood ratio of the conditional convergence test is $LR_c c$ which can be obtained by taking the sum of $LR_{cc} = LR_{ind} + LR_{uc}$ which then follows a chi-squared distribution with 2 degrees of freedom. The equation is as follows,

$$LR_{cc} = LR_{ind} + LR_{u,c}$$

The test then has a null hypothesis H_0 which states that the Violation Ratio, VR is statistically equal to α and that the VaR violations are independent from one another. The alternative hypothesis H_a states the complement where the VR is not statistically equal to α or the VaR violations are dependent on another, H_0 is then rejected when the test statistic LR_{cc} is greater than $(\chi^2_{\alpha,2})^{-1}$.

2.5 Hypothesis Testing

After concluding the models' validity on the data set. The researchers may move the main rationale of the research. Hypothesis testing will be on the models comparing the standard approaches with the GARCH model and its extensions. The method of testing that will be applied is the Diebold-Mariano Test.

Diebold-Mariano Test

Francis X. Diebold and Roberto S. Mariano [10] developed a method of testing the predictive accuracy of statistical model. The model uses two different forecasts and compares them to the actual data with a goal of seeing whether the difference between the two is statistically significant. The test statistic of the Diebold-Mariano test follows a standard normal distribution and can be shown mathematically as

$$DM \sim N(0,1)$$

DM can be obtain by taking the differences of the actual data and the forecasted data or residuals, e_i and r_i, followed by taking the squared differences of the residuals for each scenario i or the difference of the absolute values of the residuals. This then gives the loss-differential d_i. Afterwards, it is required to take the mean squared error (MSE) and mean absolute error (MAE) of the statistic respectively. Thus, giving

$$\bar{d} = \frac{1}{n}\sum_{i=1}^{n}(d_i) or \mu = \mathbb{E}[d_i]$$

$$\gamma_k = \frac{1}{n}\sum_{i=k+1}^{n}(d_i - \bar{d})(d_{i-k} - \bar{d})$$

where $\bar{d}$ is the mean of the sequence d_i and γ_k is the autocovariance function at lag k. Now, DM can be written as the following:

For $h \geq 1$,

$$DM = \frac{\bar{d}}{\sqrt{[\gamma_0 + 2\sum_{k=1}^{h-1} \gamma_k]n}}$$

where it is sufficient to use $h = n^{\frac{1}{3}} + 1$. Finally, the null hypothesis, H_0, states that the two forecasts of the two models are statistically equal while the alternative hypothesis, H_a, states that the two models' forecasts are statistically different. It is mathematically written as follows:

$$H_0 : |DM| < Z_{crit,\alpha}, e_i = r_i H_a : |DM| > Z_{crit,\alpha}, e_i \neq r_i$$

where DM is the test statistic and Z is the critical value of the standard normal distribution while *crit* is the method used such as the MSE or the MAE. The researchers chose to use the MSE for the DM test.

Chapter 3

Main Results and Discussion

3.1 Introduction

Chapter 3 will first discuss what data was used, why said data was chosen, and how the data will be employed in the research. Afterwards, initial tests will be conducted on the data. Specifically, the t-test, the Ljung-Box Test, and the Shapiro-Wilk Test to analyze the mean, correlations, and the distribution of the data. A secondary Ljung-Box test and the Lagrange Multiplier test on the squared residuals of the data, specifically for the GARCH-type models will also be tested to account for ARCH effects. Completing these initial tests will now allow the data to be experimented on by the standard models and the GARCH model and their extensions by using out-sampling to obtain future estimates. To check the validity of the models utilized, two methods of backtesting will be applied. Lastly, to compare the models with one another, the Diebold-Mariano Test will be adopted. An analysis on the results will be provided after applying the data in said tests or fit the data in the specified model.

3.2 ASEAN Stocks

The base data used throughout the research will be on the index funds of the stock exchanges from several ASEAN countries: (1) PSEi Index (PSEI.PS), (2) FTSE Bursa Malaysia KLCI (KLSE), (3) Hanoi Stock Exchange (HNX), (4) Stock Exchange of Thailand (SET), and (5) FTSE Straits Times Index (STI). The reason for choosing only five out of the 10 ASEAN countries is the inaccessibility of information on the country's stock exchange index such as Cambodia which established their stock exchanges in the mid 2012, the nonexistence of a stock exchange in the country (e.g Brunei), and a choice of limiting the data to only 5 countries.

Data will be loaded into Python either through a .csv file or through Pandas.DataReader depending on whether said stock index exists in the database of Yahoo Finance, Investing.com, MarketWatch, and similar sites that contain the needed data. The time frame that will be observed in these market indices will be from January 1, 2006 to January 1, 2022. The raw data taken from either method will contain the following columns and entries as seen in Figure 3.1. An example of the stock prices of the PSEi from January 4, 2010 to December 29, 2020 is shown.

	Open	High	Low	Close	Adj Close	Volume
Date						
2020-12-29	7123.120117	7173.270020	7120.819824	7139.709961	7139.709961	118100.0
2020-12-28	7142.950195	7142.950195	7058.660156	7122.250000	7122.250000	89200.0
2020-12-24	7169.049805	7206.459961	7075.180176	7204.379883	7204.379883	0.0
2020-12-23	7169.049805	7206.459961	7075.180176	7204.379883	7204.379883	99900.0
2020-12-22	7206.330078	7206.540039	7081.140137	7202.390137	7202.390137	172600.0
...	...	...	...	...	...	...
2016-01-08	6588.080078	6651.000000	6555.450195	6575.430176	6573.739746	133300.0
2016-01-07	6823.370117	6823.370117	6618.879883	6618.879883	6617.178223	0.0
2016-01-06	6846.990234	6870.560059	6813.899902	6813.899902	6812.148438	0.0
2016-01-05	6823.919922	6863.750000	6785.720215	6835.129883	6833.372559	0.0
2016-01-04	6954.270020	6957.240234	6833.419922	6833.419922	6831.663086	0.0

Figure 3.1: Philippine Stock Exchange Index (PSEI.PS) stock prices from January 4, 2010 to December 29, 2021

The data that will specifically focused on is the 'Close' price of the country's stock index. As this will be used to normalize the data between the countries. Normalization of the data is done by taking the logarithmic returns (log returns) of the countries.

To obtain the log returns, the researchers used Definition 2.1 with an assumption the log returns always uses the previous closing price of the index, that is $d = 1$. Afterwards, a visual representation of the log returns is shown to better grasp the data throughout the years. Figure 3.2 is used to visualize the data above.

	Open	High	Low	Close	Adj Close	Volume	Previous	Returns
Date								
2020-12-29	7123.120117	7173.270020	7120.819824	7139.709961	7139.709961	118100.0	7122.250000	0.002448
2020-12-28	7142.950195	7142.950195	7058.660156	7122.250000	7122.250000	89200.0	7204.379883	-0.011465
2020-12-24	7169.049805	7206.459961	7075.180176	7204.379883	7204.379883	0.0	7204.379883	0.000000
2020-12-23	7169.049805	7206.459961	7075.180176	7204.379883	7204.379883	99900.0	7202.390137	0.000276
2020-12-22	7206.330078	7206.540039	7081.140137	7202.390137	7202.390137	172600.0	7224.890137	-0.003119
...	...	...	...	...	...	...	...	...
2016-01-08	6588.080078	6651.000000	6555.450195	6575.430176	6573.739746	133300.0	6618.879883	-0.006586
2016-01-07	6823.370117	6823.370117	6618.879883	6618.879883	6617.178223	0.0	6813.899902	-0.029038
2016-01-06	6846.990234	6870.560059	6813.899902	6813.899902	6812.148438	0.0	6835.129883	-0.003111
2016-01-05	6823.919922	6863.750000	6785.720215	6835.129883	6833.372559	0.0	6833.419922	0.000250
2016-01-04	6954.270020	6957.240234	6833.419922	6833.419922	6831.663086	0.0	NaN	NaN

Figure 3.2: PSEI.PS log returns from January 2010 to December 2020

The total number of entries for the Philippines, Malaysia, Singapore, Thailand, and Vietnam are 3903, 4054, 4014, 3903, 3955 respectively.

3.3 Initial Tests

3.3.1 *t*-test

The first test applied is the t-test to check the data on whether it has a constant mean. The null hypothesis H_0 of the test is the claim that the constant mean $mu_i = 0$ where the variable i represents one of the countries.

Observing the results from 3.1, it can be seen that each p-value is greater than the significance level of 5%. Thus, concluding that the null hypothesis is accepted and that the data that will be used has a constant mean of 0. The significance of showing that each country uses a constant mean allows the VaR formula to be adjusted where $\mu = 0$. Thus, the following VaR formula will be used for Chapter 3.

$$\text{VaR}(1-p) = \sigma N^{-1}(1-p)$$

t-test		
Index	Test Statistic	p-value
PSEi	1.4888	0.1366
ˆKLSE	1.1937	0.2327
STI	0.4403	0.6598
SETi	1.0707	0.2844
HNX	1.4423	0.1493

Table 3.1: t-test's test statistics and p-values on the 5 countries

3.3.2 Ljung-Box Test

The second test applied is the Ljung-Box test which tests the existence of autocorrelations between the log returns. Restating the null hypothesis from Chapter 2, the null hypothesis H_0 claims that the correlation between any two data entries is equal to 0. The number lags used for the data is 20 lags as this was a standard value used in the Ljung-Box Test. Therefore, the stock indices' returns will be tested on a chi-squared distribution with 20 degrees of freedom, chi^2_20. As seen from 3.2, the p-value for the log returns of each country is shown to reject H_0 as all p-values are shown to be equal to 0 up to the fifth decimal place. Therefore, the data must exhibit serial correlations among different entries. The importance of showing serial correlations in the data is a criteria needed for the GARCH-type models.

Ljung-Box test		
Index	Test Statistic	p-value
PSEi	71.7142	9.5381e-08
ˆKLSE	63.9440	2e-06
STI	62.5176	3e-06
SETi	80.8869	2.7743e-09
HNX	210.9214	7.6804e-34

Table 3.2: Ljung-Box test's test statistics and p-values on the 5 countries

3.3.3 Shapiro-Wilk Test

Lastly, the Shapiro-Wilk test determines whether the data is normally distributed. The null hypothesis H_0 is that the data is normally distributed. As seen from 3.3, the p-values are all shown to be very close to 0. It can be concluded that the data is not normally distributed. Thus, substantiating the claim that the stock indices' returns are not normally distributed. The non-normality of data creates an environment where the models' predictive accuracy are tested since the formula used to estimate VaR uses a normal distribution.

Shapiro-Wilk test		
Index	Test Statistic	p-value
PSEi	0.9153	2.9119e-42
ˆKLSE	0.9075	2.9427e-44
STI	0.9146	7.4549e-43
SETi	0.8728	0.0
HNX	0.9258	1.8320e-40

Table 3.3: Shapiro-Wilk test's test statistic and p-value for the 5 countries

3.4 ARCH Effects

The second test that needs to be applied on the data is to verify whether ARCH effects can be observed in the data. To verify the existence of ARCH effects, two test will be done. The first test applied will be the Ljung-Box Test on the squared residuals on the chosen GARCH-type models with lag being equal to 20. The second test applied will be the Lagrange Multiplier test also on the squared residuals. For the log returns to exhibit ARCH effects, both results must reject their null hypothesis as stated in Chapter 2.

3.4.1 Ljung-Box Test

Applying the Ljung-Box Test to the squared residuals of the data shows that for the chosen model, the p-value obtained is close to 0. Therefore, the squared residuals of the countries reject the null hypothesis and the claim that serial correlations exists among the residuals is true.

ARCH Effects					
	Ljung-Box Test's p-value				
Index	GARCH	EGARCH	FIGARCH	IGARCH	Opt-FIGARCH
PSEi	7.13e-216	8.73e-264	6.27e-264	6.27e-264	6.27e-264
ˆKLSE	9.91e-147	9.98e-147	9.81e-147	9.81e-147	9.81e-147
STI	0	0	0	0	0
SETi	4.18e-222	4.32e-222	4.10e-222	4.10e-222	4.10e-222
HNX	0	0	0	0	0

Table 3.4: The Ljung-Box Test's p-value obtained from the squared residuals of the chosen countries.

3.4.2 Lagrange Multiplier Test

The ARCH LM test, developed by Engle (1982), is used on the log returns of the five different countries to identify if the daily log return of the different indices confirm strong ARCH effects. The critical value used will be a chi-squared distribution with 20 degrees of freedom. This is to keep the consistency with the lags used in the Ljung-Box test. In the table below, it can be concluded that the daily log returns of the five different ASEAN indices confirm strong ARCH effects since all of them reject the null hypothesis of the ARCH LM test, that is, that the data does not exhibit ARCH effects.

ARCH LM Test			
Index	F-statistic	mF	p-value
PSEi	59.02	1180.20	0
ˆKLSE	37.28	745.60	0
STI	165.20	3304.00	0
SETi	52.60	1052.00	0
HNX	90.44	1808.80	0

Table 3.5: F-statistic, mF, and p-values of the indices of the chosen countries ($m = 20$).

3.5 Parameters Used for the GARCH-type models

Using the 'arch' package from Python, Table 3.6 contains all the obtained parameters for the following models. Entries in red signify that the parameter has a p-value greater than 5%, thus, the said parameter does not necessarily improve the model. It can also be noted that for the GARCH model, all the parameters are shown to be significant. For the FIGARCH, the ϕ parameter is shown to be insignificant, especially when tested under the Malaysian stock. This may cause the model to improperly capture the Malaysian stock.

Lastly, both the IGARCH and the Opt-FIGARCH take the same parameters as the FIGARCH with the exception that the d parameter is adjusted to 1 and 0.633 for the IGARCH and Opt-FIGARCH respectively.

Parameters for the GARCH(1,1) model					
Parameter	PSEi	ˆKLSE	STI	SETi	HNX
α_0	0.0683	0.0092	0.0141	0.0157	0.0402
α_1	0.1591	0.1294	0.1123	0.1318	0.2003
β_1	0.8056	0.8585	0.8757	0.8660	0.7997
Parameters for the EGARCH(1,1) model					
α_0	0.0204	-0.0081	0.0014	0.0112	0.0322
α_1	0.2782	0.2348	0.2126	0.2467	0.3706
β_1	0.9626	0.9787	0.9856	0.9823	0.9730
Parameters for the FIGARCH(1, d, 1) model					
α_0	0.0796	0.0148	0.0124	0.0184	0.0730
α_1	0.0579	0.2035	0.0395	4.0199e-19	0.0536
d	0.6143	0.5930	0.9100	0.9300	0.5735
β_1	0.5240	0.6075	0.8393	0.8248	0.4182
Parameters for the IGARCH(1, 1, 1) model					
α_0	0.0796	0.0148	0.0124	0.0184	0.0730
α_1	0.0579	0.2035	0.0395	4.0199e-19	0.0536
d	1	1	1	1	1
β_1	0.5240	0.6075	0.8393	0.8248	0.4182
Parameters for the Opt-FIGARCH(1, 0.633, 1) model					
α_0	0.0796	0.0148	0.0124	0.0184	0.0730
α_1	0.0579	0.2035	0.0395	4.0199e-19	0.0536
d	0.633	0.633	0.633	0.633	0.633
β_1	0.5240	0.6075	0.8393	0.8248	0.4182

Table 3.6: Parameters obtained using the given models.

3.6 Predictions 20%

For the main section of the research, the researchers will focus on the whole dataset of 15 years. The data will be split into two groups: the 'training' set and the 'testing' set. The group then applies the Pareto principle split the data that is 80% of the data will be used training to train the models while 20% will be used to test the models. After splitting the data, the training data will then be subjected to two different methods: (1) A Fixed Window and (2) A Rolling Window.

Definition 3.1. The fixed window can be defined as follows:

Given some set r_t where t = 1, 2, ..., n; a fixed window can be defined as a subset of r_t, f where f is equal to r_{i-j}, r_{i-j+1}, r_{i-j+2}, ..., r_i where i and j are positive integers, i is the entry prior the forecasted entry and the number of entries f is equal to at least 80%.

Definition 3.2. The rolling window can be defined as follows:

Given some set r_t where t = 1, 2, ..., n; a rolling window can be defined as a subset of r_t, f where f is equal to r_1, r_2, r_3, ..., r_i where i is positive integers, and the entry prior the forecasted entry.

Graphs of the estimates with respects to the actual log returns will be provided for each respective level of significance with respect to the said method of training and the model used to predict the data. However, for conciseness, only the data of the Philippines will be shown while the other countries will be available in the Appendix.

The visualizations will begin with the fixed window followed by the rolling window. Afterwards, they will first be arranged by the level of significance – 5%, 1%, and 0.01%. The visualizations will also be grouped under two categories: Standard models and the GARCH-type models. The visualizations are as follows:

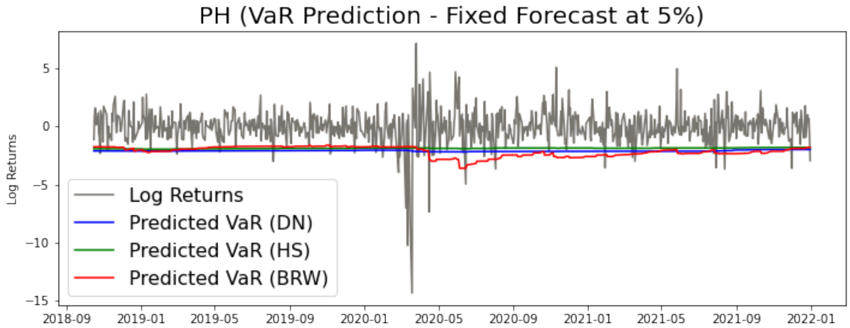

Figure 3.3: PH Log Returns and Estimated VaR using the standard models with 5% LoS

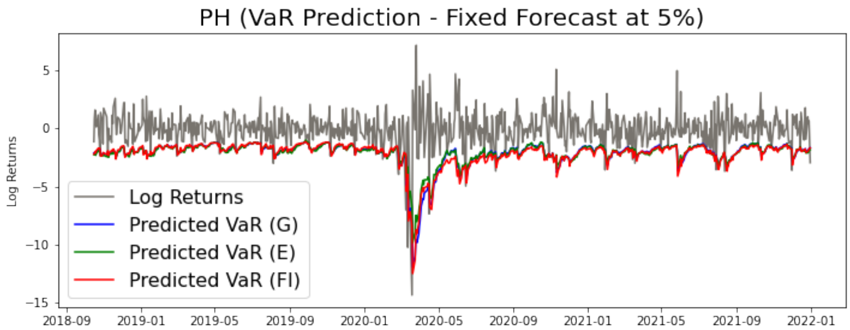

Figure 3.4: PH Log Returns and Estimated VaR using the GARCH-type models with 5% LoS

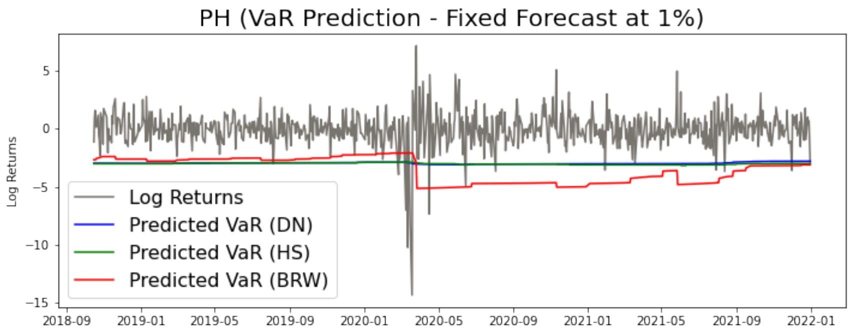

Figure 3.5: PH Log Returns and Estimated VaR using the standard models with 1% LoS

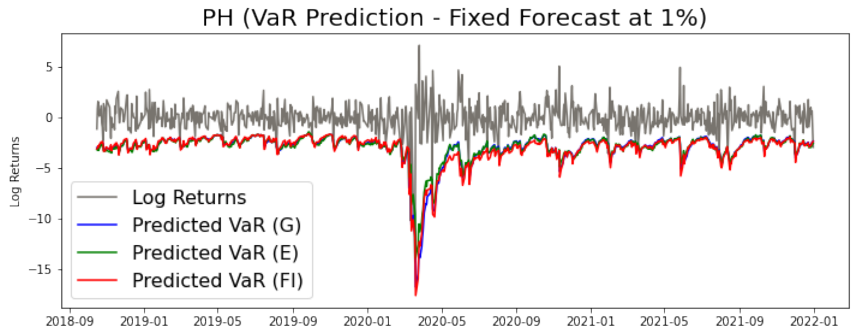

Figure 3.6: PH Log Returns and Estimated VaR using the GARCH-type models with 1% LoS

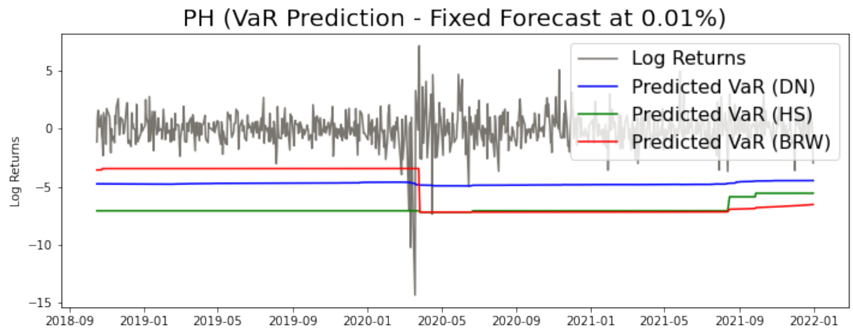

Figure 3.7: PH Log Returns and Estimated VaR using the standard models with 0.01% LoS

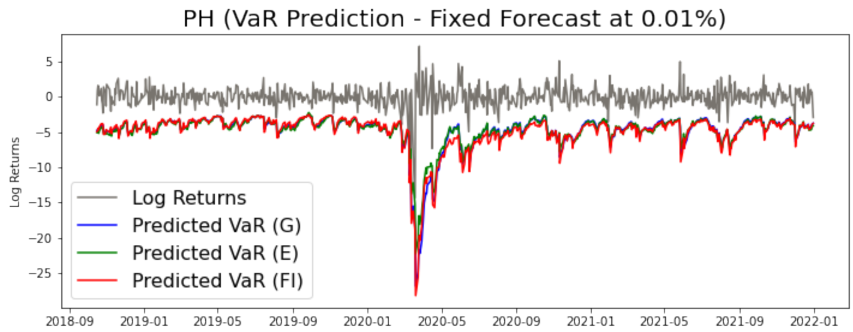

Figure 3.8: PH Log Returns and Estimated VaR using the GARCH-type models with 0.01% LoS

Comparing the fixed window from the rolling window shows minimal differences when compared on the same level of significance. Visually, the

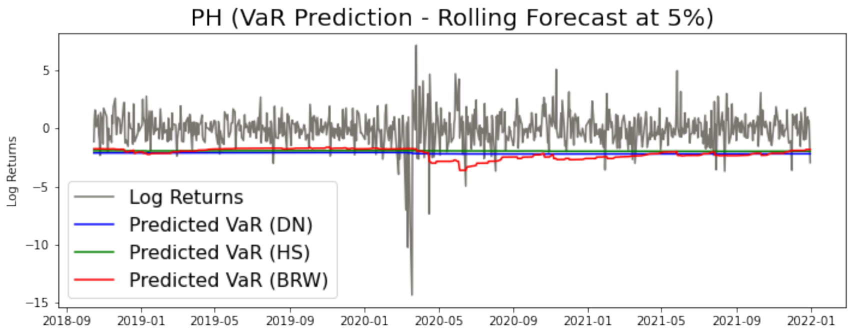

Figure 3.9: PH Log Returns and Estimated VaR using the standard models with 5% LoS

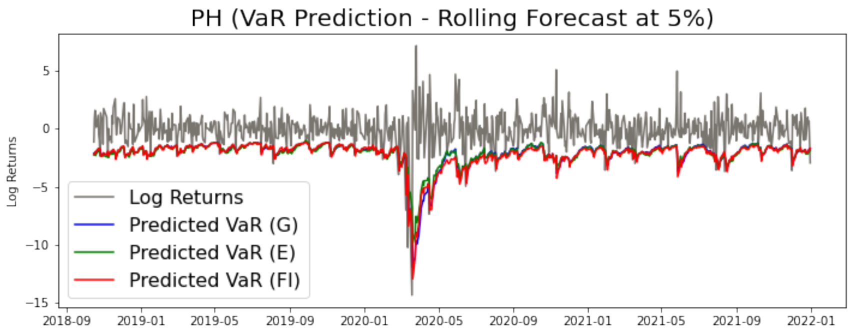

Figure 3.10: PH Log Returns and Estimated VaR using the GARCH-type models with 5% LoS

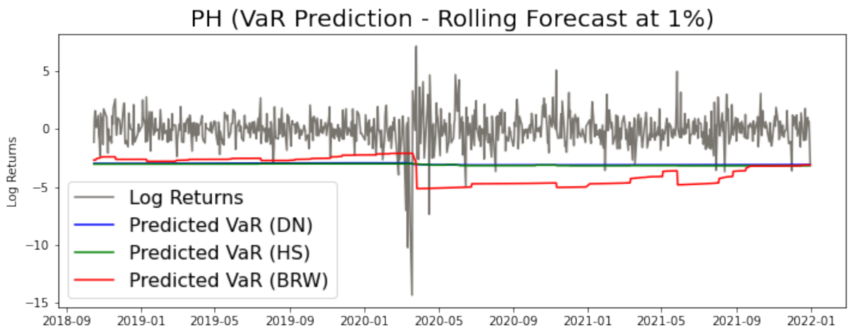

Figure 3.11: PH Log Returns and Estimated VaR using the standard models with 1% LoS

changes can be seen after the spike between the months of January and June of 2020 for the standard models. However, for the GARCH-type models

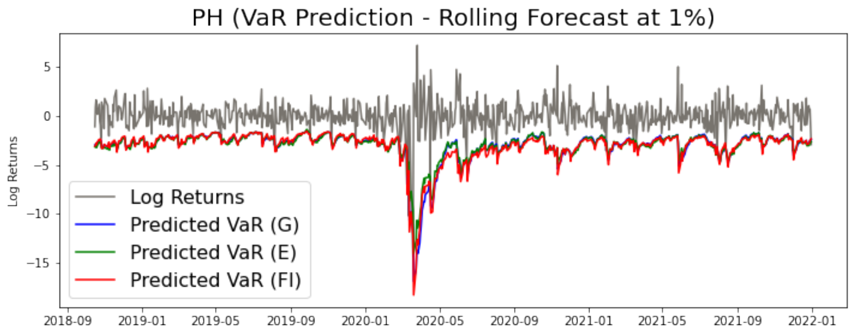

Figure 3.12: PH Log Returns and Estimated VaR using the GARCH-type models with 1% LoS

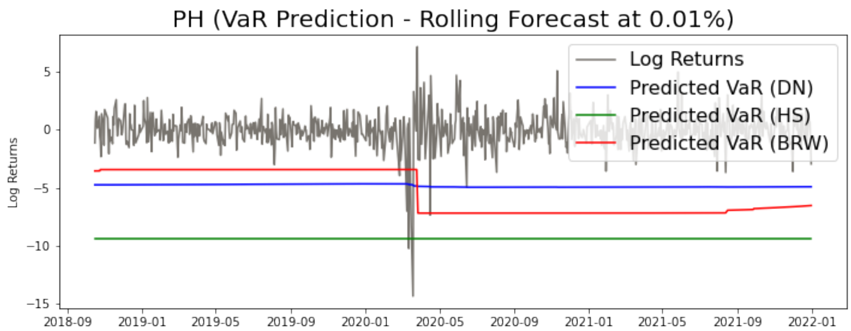

Figure 3.13: PH Log Returns and Estimated VaR using the standard models with 0.01% LoS

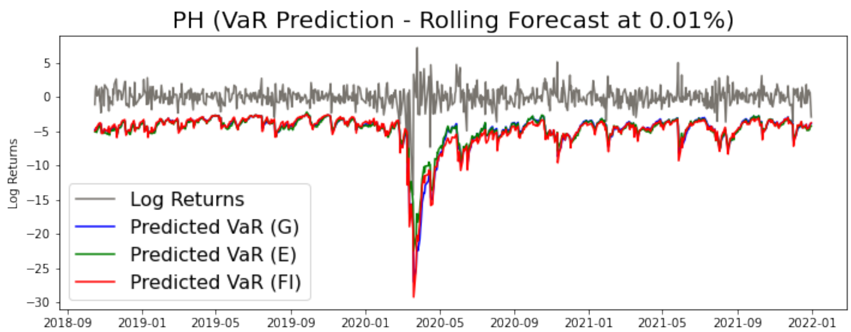

Figure 3.14: PH Log Returns and Estimated VaR using the GARCH-type models with 0.01% LoS

and its extensions, most of them follow the same shape and pattern despite changing the level of significance. What is noted on the changes seen when comparing them by the level of significance is the size of the graphs. This is evidently seen when comparing the 5% level of significance to the 0.01%. Visually, the standard models and the GARCH-type models are different. However, it is still required to objectively note their differences. This leads to the three methods of backtesting – the unconditional coverage test, the Kupiec test, and the Christoffersen test.

3.7 Backtesting Methods

3.7.1 VaR Violations

The first backtesting method use will be the unconditional coverage test which will be examining the number of VaR violations that occur. This is done using a characteristic function where is the actual returns of the on a specified date is less than the estimated VaR will be equal to one violation. The null hypothesis, H_0, is that the model used is appropriate and sufficiently captures the data. A confidence interval for the number of violations is also provided as to verify whether the model is not overestimating or underestimating the data. The confidence intervals – obtained using a binomial distribution – of Philippines, Malaysia, Singapore, Thailand, and Vietnam are (28,51), (29,53), (29,53), (28,51), (28,52) for the 5% LoS, (1, 15), (1, 16), (1, 16), (1, 15), (1, 16) for the 1% LoS, and [0, 2) for all countries for the 0.01% LoS, respectively.

Six tables are made to summarize the results from all the graphs above for both the fixed and rolling window. The tables will contain the number of VaR violations that have occurred using said model under the chosen dataset. The color of the text will also signify whether the number of violations occurred is within (black) or outside (red or blue) the confidence interval. Red signifies whether the model overestimates the data, that is, the number of VaR violations is less than the confidence interval while blue signifies whether the model is underestimating the data, that is, the number of VaR violations is greater than the confidence interval.

Fixed Window						
	VaR Violations with 5% LoS					
Index	DN	HS	BRW	GARCH	EGARCH	FIGARCH
PSEi	35	40	32	38	36	37
ˆKLSE	51	58	54	55	47	52
STI	21	33	43	46	45	44
SETi	21	32	37	42	38	32
HNX	23	25	39	40	44	43

Table 3.7: Number of VaR violations under different models on a fixed window with 5% LoS

Rolling Window						
	VaR Violations with 5% LoS					
Index	DN	HS	BRW	GARCH	EGARCH	FIGARCH
PSEi	35	39	32	38	37	36
ˆKLSE	45	55	54	52	46	52
STI	20	30	43	45	45	44
SETi	20	28	37	35	33	31
HNX	19	20	39	40	42	42

Table 3.8: Number of VaR violations under different models on a rolling window with 5% LoS

Under the 5% LoS, the VaR violations mainly occur under the standard models where the model may overestimate or underestimate the log returns. It is also noted that the DN approach overestimates the log returns more often than other models. The differences that occur by applying a fixed or rolling window is the decrease in number of VaR violations that occur for any approach chosen. The VaR violations either decrease or stays at the current number of VaR violations when applying the rolling window. In this case, the GARCH-type models show to be sufficient models under the 5% LoS compared to the standard models.

Fixed Window						
	VaR Violations with 1% LoS					
Index	DN	HS	BRW	GARCH	EGARCH	FIGARCH
PSEi	14	13	13	20	20	19
ˆKLSE	19	17	23	19	17	16
STI	12	10	17	12	13	10
SETi	11	8	14	15	17	14
HNX	10	6	13	17	16	18

Table 3.9: Number of VaR violations under different models on a fixed window with 1% LoS

Rolling Window						
	VaR Violations with 1% LoS					
Index	DN	HS	BRW	GARCH	EGARCH	FIGARCH
PSEi	13	13	13	19	20	19
ˆKLSE	19	17	23	19	17	16
STI	12	9	17	12	12	10
SETi	11	8	14	12	13	14
HNX	7	3	13	16	15	17

Table 3.10: Number of VaR violations under different models on a rolling window with 0.01% LoS

Moving to the 1% LoS, the standard models exceed the GARCH-type models in capturing the log returns of the countries. It will be noticed that in both the fixed and rolling window the standard models have the sufficient amount of VaR violations almost every country to accept the null hypothesis with the exception being Malaysia. Two important observations to note is the fact that all the models failed when tested under the Malaysia stock index and the GARCH-type models all failed to capture the Philippines stock index's log returns. Thus, it can be seen that the standard models prove to be more sufficient approaches compared to the GARCH-type models under 1% LoS.

Fixed Window						
	VaR Violations with 0.01% LoS					
Index	DN	HS	BRW	GARCH	EGARCH	FIGARCH
PSEi	6	4	6	2	2	2
ˆKLSE	7	3	11	4	4	4
STI	5	1	7	4	4	4
SETi	6	4	8	5	7	5
HNX	2	1	4	2	3	2

Table 3.11: Number of VaR violations under different models on a fixed window with 0.01% LoS

Rolling Window						
	VaR Violations with 0.01% LoS					
Index	DN	HS	BRW	GARCH	EGARCH	FIGARCH
PSEi	6	2	6	2	2	2
ˆKLSE	7	2	11	4	4	3
STI	5	1	7	4	4	4
SETi	6	1	8	5	7	6
HNX	1	0	4	2	2	2

Table 3.12: Number of VaR violations under different models on a rolling window with 0.01% LoS

Lastly, the 0.01% LoS VaR violations show that failure of almost every model under most of the countries. Once again, the fixed window and the rolling window have the same observation as what was seen in the 5% LoS. The GARCH-type models have shown to underestimate every country as seen from Table 3.11 and 3.12. However, the standard models, more specifically the HS, are able to capture the log returns despite the stricter parameters. An observation that can be noted is that under the 0.01% LoS, the VaR violations under a rolling window may be larger when compared to the fixed window counterpart. This is noticeable on the SETi VaR violations under the FIGARCH model of the fixed window and rolling window giving 5 and 6 violations respectively.

Summarizing the results, it can be observed that the VaR violations that the GARCH model and its extensions tend to be more sufficient compared to the standard models in terms of the a larger α. However, as α decrease a significant change can be seen especially in when α is equal to 1%. Here, the standard models are shown to be better approaches to estimate VaR compared to the time-series models. Lastly, for the last case where α is equal to 0.01%, the VaR violations for almost every model exceeds the desired amount except for the case of the Historical Simulation where the model shows to be sufficient in two of the countries. Another observation that can be noticed is that the GARCH-type models are shown to underestimate the number of VaR violation while the standard models may either overestimate or underestimate the data as seen when α is 5% and 1%. Lastly, the number of VaR violations that fall in the specified confidence interval is shown to increase when applying a rolling window rather than a fixed window. The Kupiec Test will also be applied in the next section to further verify the results from this test.

3.7.2 Kupiec Test

A second unconditional coverage test will be applied for on the VaR violations as both tests test for the sufficiency of the model, therefore both test having the same null hypothesis. Compared to the VaR violation test, the Kupiec Test will use a test statistic rather than a confidence interval to decide on the outcome of the test. The critical value is obtained from the inverse chi-squared distribution (assuming it is right-tailed) with 1 degree of freedom, $\chi^2_{\alpha,1}$ where α is the level of significance. Given α is the 5%, 1%, and 0.01%, the critical values are 3.8414, 6.6349, and 15.1367 respectively. If the test statistic is greater than the critical value for the appropriate level of significance, the test rejects the null hypothesis of being a sufficient model. Similar to the previous backtesting method, the tables will contain the test statistics obtain from the Kupiec Test. The entries of the table will then be coloured black if H_0 is accepted and red if H_0 is rejected.

Fixed Window						
	Kupiec Test with 5% LoS					
Index	DN	HS	BRW	GARCH	EGARCH	FIGARCH
PSEi	0.4574	0.0241	1.4239	0.03	0.2572	0.1152
ˆKLSE	2.6299	7.0155	4.2726	4.9004	1.0297	3.1366
STI	11.5566	1.4232	0.2083	0.8587	0.5945	0.3774
SETi	10.4821	1.4239	0.1152	0.2292	0.3	1.4239
HNX	8.5266	6.4454	0.0081	0.0054	0.5093	0.3084

Table 3.13: Kupiec Test's test statistic under different models on a fixed window with 5% LoS

For the 5% LoS, the results from the Kupiec Test shows the effectivity of the GARCH-type models compared to the standard models where almost all the GARCH-type models accept the null hypothesis for all the countries

Rolling Window						
	Kupiec Test with 5% LoS					
Index	DN	HS	BRW	GARCH	EGARCH	FIGARCH
PSEi	0.4574	0.0001	1.4239	0.03	0.1152	0.2572
ˆKLSE	0.4972	4.9004	4.2726	3.1366	0.1152	3.1366
STI	12.952	2.949	0.2083	0.5945	0.5945	0.3774
SETi	11.8206	3.636	0.1152	0.4574	1.0391	1.874
HNX	13.7981	12.3309	0.0081	0.0054	0.1567	0.1567

Table 3.14: Kupiec Test's test statistic under different models on a rolling window with 5% LoS

while the standard models fail to accept the null hypothesis with exception to the BRW approach which may be on par with the GARCH-type models at 5% LoS. The fixed window and rolling window at this LoS show little difference where the main difference is that under the rolling window the GARCH model sufficiently captures the Malaysian stock index.

Fixed Window						
	Kupiec Test with 1% LoS					
Index	DN	HS	BRW	GARCH	EGARCH	FIGARCH
PSEi	4.012	2.9031	2.9031	13.4263	13.4263	11.566
ˆKLSE	10.7193	7.4827	18.4481	10.7193	7.4827	6.0415
STI	1.7212	0.4529	7.6626	1.7212	2.617	0.4529
SETi	1.168	0.0046	4.012	5.2664	8.1752	4.012
HNX	0.5147	0.5082	2.7705	7.9388	6.4466	9.5514

Table 3.15: Kupiec Test's test statistic under different models on a fixed window with 1% LoS

For the 1% LoS, a similar observation can be noticed in Table 3.9 and 3.10 where all the models struggle to properly capture the Malaysian stock index. The fixed window under the 1% LoS shows the GARCH-type models failing to capture the log returns of most countries compared to the standard models which are able to sufficiently capture them with the exception of

Rolling Window						
	Kupiec Test with 1% LoS					
Index	DN	HS	BRW	GARCH	EGARCH	FIGARCH
PSEi	2.9031	2.9031	2.9031	11.566	13.4263	11.566
ˆKLSE	10.7193	7.4827	18.4481	10.7193	7.4827	6.0415
STI	1.7212	0.1139	7.6626	1.7212	1.7212	0.4529
SETi	1.168	0.0046	4.012	1.9508	2.9031	4.012
HNX	0.11	4.0336	2.7705	6.4466	5.0821	7.9388

Table 3.16: Kupiec Test's test statistic under different models on a rolling window with 1% LoS

Malaysia. However, applying the rolling window allows the GARCH-type models to perform almost par with the standard models as seen in Table 3.16 when compared to Table 3.15. Nonetheless, the standard models show to be more suffiecnt compared to the GARCH-type models at 1% LoS.

Fixed Window						
	Kupiec Test with 0.01% LoS					
Index	DN	HS	BRW	GARCH	EGARCH	FIGARCH
PSEi	40.2995	23.6644	40.2995	9.1326	9.1326	9.1326
ˆKLSE	48.6332	15.83687	86.3292	23.3681	23.3681	23.3681
STI	31.505	3.2056	48.7709	23.446	23.446	23.446
SETi	40.2995	23.6644	58.3041	31.7793	49.1572	31.7793
HNX	9.0836	3.2334	23.5644	9.0836	15.9829	9.0836

Table 3.17: Kupiec Test's test statistic under different models on a fixed window with 0.01% LoS

Lastly, at the 0.01% LoS, both the fixed window and the rolling window output different result. For the fixed window, GARCH-type models are able to capture at least 1 country, mainly the Philippines, while the standard models fall short where only the HS is able to capture a country not captured by the GARCH-type models, Singapore. The rolling window, on the other hand, shows that the HS approach is able capture all the countries while

Rolling Window						
	Kupiec Test with 0.01% LoS					
Index	DN	HS	BRW	GARCH	EGARCH	FIGARCH
PSEi	40.2995	9.1326	40.2995	9.1326	9.1326	9.1326
^KLSE	48.6332	8.9876	86.3292	23.3681	23.3681	15.8368
STI	31.505	3.2056	48.7709	23.446	23.446	23.446
SETi	40.2995	3.2568	58.3041	31.7793	49.1572	40.2995
HNX	3.2334	0.1582	23.5644	9.0836	9.0836	9.0836

Table 3.18: Kupiec Test's test statistic under different models on a rolling window with 0.01% LoS

the other models mainly the GARCH-type models only have little change. Interestingly, all three (3) GARCH-type models fail to properly capture the Malaysian, Singaporean, and Thailand stock index.

As seen from the tables above, the GARCH-type models are performing better in the Kupiec Test compared to their results in the standard unconditional coverage test. Just as the standard unconditional coverage test, the GARCH-type models perform exceptionally by accepting the null hypothesis in almost all the countries. A similar observation from the unconditional coverage test is also noticed in the Kupiec Test results. A rolling window is shown to increase the number of models that sufficiently capture the data. A major difference with the standard unconditional coverage test data is that the Historical Simulation under a rolling window with 0.01% LoS is seen to be sufficient in capturing the log returns from each country exceeding the GARCH model and its extensions. Another similarity with the unconditional coverage test is that the GARCH-type models performed poorly against the standard models under the 1% LoS for both fixed and rolling window. Lastly, the applying a rolling window does not necessarily indicate an increase or decrease in the test statistic for the Kupiec Test. The last backtesting method will be applied to test whether these VaR violations incurred by the models are independent from one another.

3.7.3 Christoffersen Test

The last backtesting method applied is the Christoffersen test which will test the sufficiency of the model once again, but also the independence of the VaR violations from one another. The null hypothesis H_0 is the claim that the model is sufficient in capturing the log returns of the country and that the VaR violations incurred by the models are independent from one another. The critical value of the model is tested by a inverse chi-squared distribution with 2 degrees of freedom, $\chi^2_{\alpha,2}$, where it is assumed to be a right-tailed distribution. For the 5%, 1%, and the 0.01% LoS, the critical values are as follows: 5.9915, 9.2103, and 18.4207 respectively. The tables will be order in the same order as the previous backtesting methods used and entries in red denoted the rejection of the null hypothesis under the stated models in a given country. The tables are as follows:

Fixed Window						
	Christoffersen Test with 5% LoS					
Index	DN	HS	BRW	GARCH	EGARCH	FIGARCH
PSEi	1.7947	0.6175	3.4061	0.153	0.4457	0.2649
ˆKLSE	4.8968	8.9486	7.268	5.0772	1.268	3.8394
STI	11.8727	19.5823	14.0075	2.8039	2.7964	2.8248
SETi	11.5363	1.7334	0.0732	1.0283	0.0441	3.9942
HNX	17.2262	13.8972	6.4524	1.715	0.6392	1.4181

Table 3.19: Christoffersen Test's test statistic under different models on a fixed window with 5% LoS

At 5% LoS, it is a similar result as the past two backtesting methods

Rolling Window						
	Christoffersen Test with 5% LoS					
Index	DN	HS	BRW	GARCH	EGARCH	FIGARCH
PSEi	1.7947	0.7117	3.4061	0.153	0.2649	0.4457
ˆKLSE	4.6127	7.6057	7.268	3.177	0.9144	3.8394
STI	13.3671	15.0923	14.0075	3.177	2.7964	2.8248
SETi	12.7707	4.4843	0.0732	0.5918	1.0653	4.2779
HNX	12.7707	23.2293	6.4524	1.715	1.4503	1.4503

Table 3.20: Christoffersen Test's test statistic under different models on a rolling window with 5% LoS

can be observed. The GARCH-type models greatly outperform the standard models. It can be noticed that only two out of the 5 countries are shown to be captured by the standard models under the Christoffersen test while the GARCH-type models are able to capture all the countries at the 5% LoS. Another thing that can be noted is that the only country that every model was able to capture was the Philippines (PSEi) with Thailand (SETi) coming in second where it is captured by five of the 6 models. Similarly, the rolling window and the fixed window do not gravitate towards a specific trend of increase of decreasing the test value.

Fixed Window						
	Christoffersen Test with 1% LoS					
Index	DN	HS	BRW	GARCH	EGARCH	FIGARCH
PSEi	5.5167	4.494	8.7533	13.9191	16.3523	12.177
ˆKLSE	17.7977	11.4305	37.1966	11.6322	8.3084	6.6864
STI	2.0858	0.7054	27.5807	2.0858	8.5698	0.7054
SETi	1.4827	0.1704	4.4533	5.8547	8.8463	0.7054
HNX	3.0619	0.6001	4.3821	8.7312	7.4082	10.1947

Table 3.21: Christoffersen Test's test statistic under different models on a fixed window with 1% LoS

Under the 1% LoS, a shift from the standard models under-performing

Rolling Window						
	Christoffersen Test with 1% LoS					
Index	DN	HS	BRW	GARCH	EGARCH	FIGARCH
PSEi	4.494	4.494	8.7533	12.177	16.3523	12.177
ˆKLSE	17.7977	11.4305	37.1966	11.6322	8.3084	6.6864
STI	2.0858	0.7054	27.5807	2.0858	2.0858	0.7054
SETi	1.4827	0.3182	4.4533	2.3258	3.2786	4.5237
HNX	0.2352	4.0565	4.3821	7.4082	6.2349	8.7312

Table 3.22: Christoffersen Test's test statistic under different models on a rolling window with 1% LoS

compared to the GARCH-type models to the standard models performing on par or better compared to the GARCH-type models. The test statistic shows that the standard models are able to capture three to 4 countries (leaning towards 4) while the GARCH-type models are also able to capture 3 to 4 models. However, the GARCH-type models may only capture 3 countries rather than 4. It follows that the GARCH-type models are unable to capture the Philippine stock index while the standard models and the GARCH model is unable to capture the Malaysian stock index.

Fixed Window						
	Christoffersen Test with 0.01% LoS					
Index	DN	HS	BRW	GARCH	EGARCH	FIGARCH
PSEi	44.868	30.0075	44.868	9.1429	9.1429	9.1429
ˆKLSE	52.6324	23.6014	108.0768	23.4078	23.4078	23.4078
STI	31.5678	3.2081	48.8942	23.4861	23.4861	23.4861
SETi	40.3925	23.7056	58.4699	31.8438	49.284	31.8438
HNX	9.0938	3.2359	23.6051	9.0938	23.6977	9.0938

Table 3.23: Christoffersen Test's test statistic under different models on a fixed window with 0.01% LoS

Under the 0.01% LoS, both the standard models and the GARCH-type models seem to fail on most countries where only at most 2 countries are

Rolling Window						
	Christoffersen Test with 0.01% LoS					
Index	DN	HS	BRW	GARCH	EGARCH	FIGARCH
PSEi	44.868	9.1429	44.868	9.1429	9.1429	9.1429
ˆKLSE	52.6324	18.8378	108.0768	23.4078	23.4078	15.8591
STI	31.5678	3.2081	48.8942	23.4861	23.4861	23.4861
SETi	40.3925	3.2594	58.4699	31.8438	49.284	40.3925
HNX	3.2359	∞	23.6051	9.0938	9.0938	9.0938

Table 3.24: Christoffersen Test's test statistic under different models on a rolling window with 0.01% LoS

captured by the models with the least being 0, which can be seem under the BRW approach for both windows. Applying the rolling window shows better results compared to using a fixed window. This can be seen from the Historical Simulation where the fixed window shows that the HS only has 2 countries where the model is sufficient while the rolling window shows 3 countries. This is also observed on the GARCH-type models. However, compared to the standard models, the each GARCH model is able to capture at least 2 countries. Again, it shows that the K̂SLE is difficult to capture with most of the models.

3.8 GARCH, IGARCH, and FIGARCH

As seen from the previous sections, the FIGARCH model can be transformed into the GARCH model and the IGARCH model by setting the differencing factor d to 0 and 1 respectively. This is seen when writing the FIGARCH model using lag operators. The equations of the FIGARCH, GARCH, and IGARCH are as follows:

$$[1 - \alpha_s(L) - \beta_m(L)](1 - L)^d a_t^2 = \alpha_0 + [1 - \beta_m(L)]v_t$$

$$[1 - \alpha_s(L) - \beta_m(L)](1-L)^0 a_t^2 = \alpha_0 + [1-\beta_m(L)]v_t$$

$$[1 - \alpha_s(L) - \beta_m(L)](1-L)^1 a_t^2 = \alpha_0 + [1-\beta_m(L)]v_t$$

A comparison will be done on the GARCH, IGARCH, FIGARCH, and FIGARCH with an optimized values of 0.6333 (Opt-FIGARCH). A visualization of the 4 models followed by backtesting method will be applied to said models. The visualization is only be using the 4 models under the PSEi log returns.

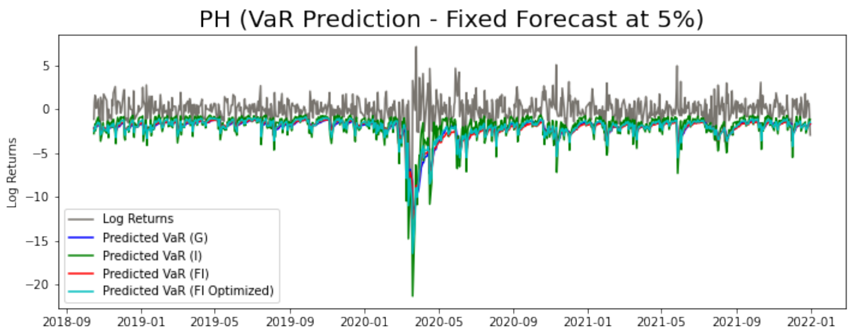

Figure 3.15: PH Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and OFIGARCH with 5% LoS

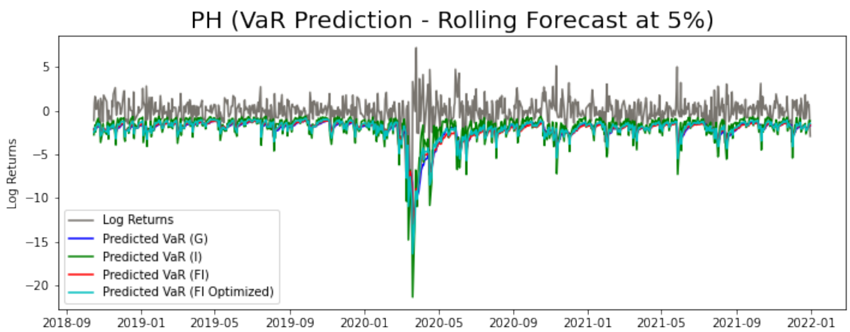

Figure 3.16: PH Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and OFIGARCH with 5% LoS

Similar to the other GARCH-type models used, the fixed and rolling window show little differences when compared on the same LoS. At the 5% LoS, what can be noted is the forecasted VaR of the IGARCH as it can be seen that compared to the other models it is more erratic and volatile. It can be seen that large spike in the volatility causes the IGARCH top have a large reaction as seen from the multiple spike throughout the time window. The optimized FIGARCH model, on the other hand, shows itself to have a similar forecast to the GARCH and the FIGARCH, but during times of instability the optimized FIGARCH is shown to predict a lower VaR compared to the GARCH and FIGARCH.

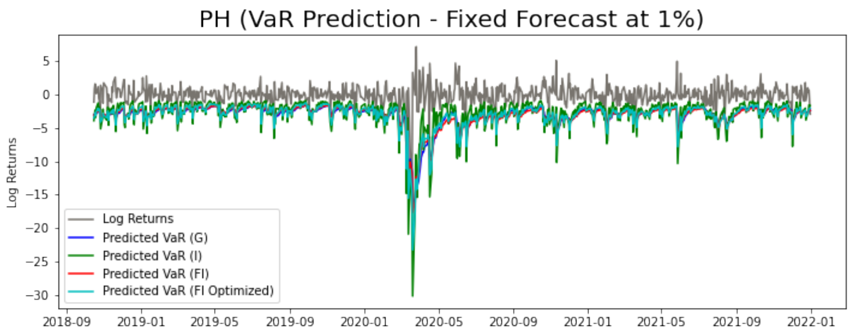

Figure 3.17: PH Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and OFIGARCH with 1% LoS

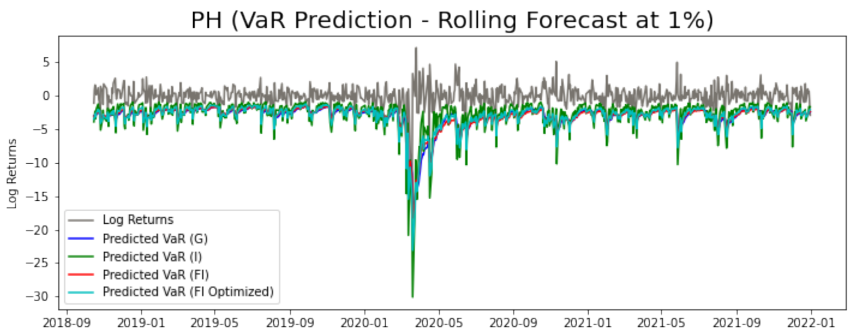

Figure 3.18: PH Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and OFIGARCH with 1% LoS

Under the 1% LoS, a similar observation is made as seen that the IGARCH model is still shown to have the large forecasted VaR during periods of instability while having a small forecasted VaR during periods of stability. Interestingly, comparing the 5% LoS and the 1% LoS shows that the IGARCH values do not seem to vary visually compared to the other GARCH-type models which estimate a greater VaR when decreasing the LoS. However, upon closer inspection, the IGARCH's forecasted values do increase when decreasing the LoS, but during period of stability the forecasted VaR seems to be similar to the 5% LoS.

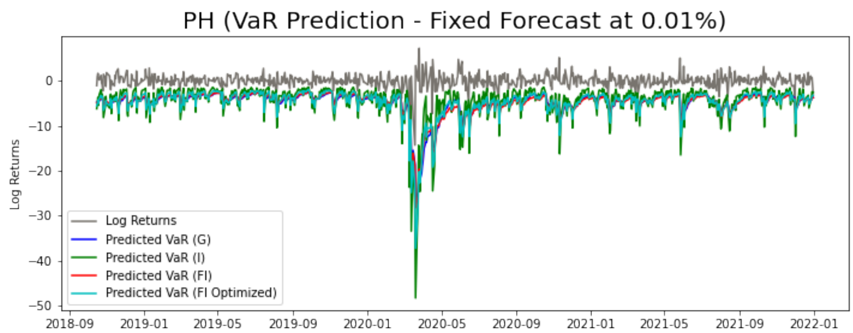

Figure 3.19: PH Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and OFIGARCH with 0.01% LoS

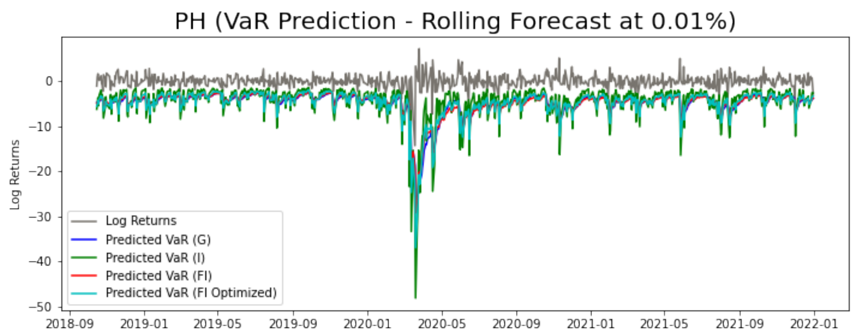

Figure 3.20: PH Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and OFIGARCH with 0.01% LoS

Lastly, at the 0.01%, the IGARCH follows the same patterns as the past two LoS. However, something to note may be the value estimated of the IGARCH model as under the 5% the largest VaR estimated by the IGARCH is around 20% by 0.01%, the IGARCH then estimates a VaR of around 50%. Another thing to note is that the other GARCH-type models at the 5% LoS estimate a VaR of around 15% and almost 30% at 0.01% LoS.

Summarizing the visualizations, the IGARCH model may overestimate the log returns of the country as seen from the forecasted VaR. The optimized FIGARCH model where d is equal to 0.6333 seems to show similar results to the GARCH and the FIGARCH model. Backtesting method will be applied on these models to develop more objective analysis. The VaR violations of the models are as follows and the text colors used will follow the previous tables:

Fixed Window				
	VaR Violations with 5% LoS			
Index	GARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	38	78	37	50
ˆKLSE	55	73	52	55
STI	46	57	44	45
SETi	42	67	32	39
HNX	40	57	43	48

Table 3.25: Number of VaR violations under GARCH, IGARCH, FIGARCH, and Opt-FIGARCH on a fixed window with 5% LoS

Under the 5% LoS, the IGARCH model is shown to underestimate all the chosen stock indices in both the fixed window and the rolling window while the Opt-FIGARCH model is able to capture the log returns of most countries and faces a similar problem as the GARCH model in the fixed window.

Rolling Window				
	VaR Violations with 5% LoS			
Index	GARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	38	75	36	48
ˆKLSE	52	74	52	56
STI	45	56	44	45
SETi	35	62	31	37
HNX	40	53	42	44

Table 3.26: Number of VaR violations under GARCH, IGARCH, FIGARCH, and Opt-FIGARCH on a rolling window with 5% LoS

Fixed Window				
	VaR Violations with 1% LoS			
Index	GARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	20	40	19	19
ˆKLSE	19	38	16	23
STI	12	21	10	11
SETi	15	21	14	16
HNX	17	34	18	20

Table 3.27: Number of VaR violations under GARCH, IGARCH, FIGARCH, and Opt-FIGARCH on a fixed window with 1% LoS

Rolling Window				
	VaR Violations with 1% LoS			
Index	GARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	19	38	19	19
ˆKLSE	19	37	16	23
STI	12	16	10	11
SETi	12	21	14	12
HNX	16	26	17	20

Table 3.28: Number of VaR violations under GARCH, IGARCH, FIGARCH, and Opt-FIGARCH on a rolling window with 1% LoS

Under the 1% LoS, the IGARCH model is still unable to capture the log returns of the countries as seen that its model underestimates all the log returns of the countries. The Opt-FIGARCH model is shown to be insufficient for most countries under the fixed window, but under the rolling window, the Opt-FIGARCH model is on par with both the GARCH and the FIGARCH model. Something to note however, is that all the chosen models fail the countries the Philippines, Malaysia, and Vietnam.

Fixed Window				
	VaR Violations with 0.01% LoS			
Index	GARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	2	11	2	3
^KLSE	4	17	4	4
STI	4	5	4	4
SETi	5	6	5	5
HNX	2	9	2	2

Table 3.29: Number of VaR violations under GARCH, IGARCH, FIGARCH, and Opt-FIGARCH on a fixed window with 0.01% LoS

Rolling Window				
	VaR Violations with 0.01% LoS			
Index	GARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	2	11	2	3
^KLSE	4	14	3	4
STI	4	5	4	4
SETi	5	7	6	6
HNX	2	7	2	2

Table 3.30: Number of VaR violations under GARCH, IGARCH, FIGARCH, and Opt-FIGARCH on a rolling window with 0.01% LoS

Under the 0.01%, a similar observations as the FIGARCH model can be seen with the IGARCH and the Opt-FIGARCH model where the rolling window may have more VaR violations compared to the fixed window counterpart. It can also be seen that all the models fail to properly capture the data. However further testing is required.

Summarizing the data, the IGARCH model is shown to be the least effective model in cpaturing the data since it repeatedly underestimates the log returns. The Opt-FIGARCH, on the other hand, shows great similarities with the FIGARCH and the GARCH model in terms of the number of violations. The Kupiec test will then be applied to further verify the sufficiency of the models.

Fixed Window				
	Kupiec Test with 5% LoS			
Index	GARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	0.03	32.1126	0.1152	2.9804
ˆKLSE	4.9004	22.3232	3.1366	4.9004
STI	0.8587	6.6238	0.3774	0.5945
SETi	0.2292	17.5062	1.4239	0.0001
HNX	0.0054	7.1737	0.3084	1.7844

Table 3.31: Kupiec Test's test statistic under GARCH, IGARCH, FIGARCH, and Opt-FIGARCH on a fixed window with 5% LoS

Rolling Window				
	Kupiec Test with 5% LoS			
Index	GARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	0.03	27.7675	0.2572	2.0188
ˆKLSE	3.1366	23.6001	3.1366	5.5673
STI	0.5945	5.8973	0.3774	0.5945
SETi	0.4574	12.1415	1.874	0.1152
HNX	0.0054	4.3712	0.1567	0.5093

Table 3.32: Kupiec Test's test statistic under GARCH, IGARCH, FIGARCH, and Opt-FIGARCH on a rolling window with 5% LoS

Under the 5% LoS, the fixed and rolling window have little effect on the output of the Kupiec test's test statistic. The IGARCH model continues to underperform and fails to capture the data while the Opt-FIGARCH is able to capture most of the countries and be on par with both the GARCH and FIGARCH model. Among the 4 models the FIGARCH model is shown to be the best of the four.

Fixed Window				
	Kupiec Test with 1% LoS			
Index	GARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	13.4263	67.6571	11.566	11.566
^KLSE	10.7193	58.7279	6.0415	18.4481
STI	1.7212	14.649	0.4529	0.9947
SETi	5.2664	15.3892	4.012	6.6569
HNX	7.9388	47.8589	9.5514	13.1118

Table 3.33: Kupiec Test's test statistic under GARCH, IGARCH, FIGARCH, and Opt-FIGARCH on a fixed window with 1% LoS

Rolling Window				
	Kupiec Test with 1% LoS			
Index	GARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	11.566	61.0602	11.566	11.566
^KLSE	10.7193	55.5909	6.0415	18.4481
STI	1.7212	6.2011	0.4529	0.9947
SETi	1.9508	15.3892	4.012	1.9508
HNX	6.4466	26.1195	7.9388	13.1118

Table 3.34: Kupiec Test's test statistic under GARCH, IGARCH, FIGARCH, and Opt-FIGARCH on a rolling window with 1% LoS

Under the 1% LoS, a similar observation can be seen where at 1% LoS, none of the GARCH-type models are able to capture the Philippine's log returns. The differences between the fixed and rolling window can be distinguished as seen that the FIGARCH model is the best under the fixed window while under the rolling window both the FIGARCH and the GARCH model performed well. The Opt-FIGARCH at 1% LoS is shown to be underperforming compared to both the GARCH and the FIGARCH.

Fixed Window				
	Kupiec Test with 0.01% LoS			
Index	GARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	9.1326	87.1582	9.1326	16.0574
ˆKLSE	23.3681	148.2574	23.3681	23.3681
STI	23.446	31.505	23.446	23.446
SETi	31.7793	40.2995	31.7793	31.7793
HNX	9.0836	67.476	9.0836	9.0836

Table 3.35: Kupiec Test's test statistic under GARCH, IGARCH, FIGARCH, and Opt-FIGARCH on a fixed window with 0.01% LoS

Rolling Window				
	Kupiec Test with 0.01% LoS			
Index	GARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	9.1326	87.1582	9.1326	16.0574
ˆKLSE	23.3681	148.2574	15.8368	23.3681
STI	23.446	31.505	23.446	23.446
SETi	31.7793	49.1572	40.2995	40.2995
HNX	9.0836	48.9803	9.0836	9.0836

Table 3.36: Kupiec Test's test statistic under GARCH, IGARCH, FIGARCH, and Opt-FIGARCH on a rolling window with 0.01% LoS

Under the 0.01% LoS, most of the models have difficulty capturing the log returns of the different countries. The IGARCH model, once again, is underperforming in both the fixed and rolling window while the Opt-FIGARCH model is able to capture at least 1 country. However, the GARCH and FIGARCH model are able to capture more countries than the Opt-FIGARCH. Finally, the models will be test using the Christoffersen Test.

At 5% LoS, again the IGARCH model is shown to reject the null hypothesis for most countries. However, the Opt-FIGARCH model is shown to

Fixed Window				
	Christoffersen Test with 5% LoS			
Index	GARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	0.153	33.6233	0.2649	3.1202
ˆKLSE	5.0772	27.3669	3.8394	8.0316
STI	2.8039	10.0214	2.8248	1.4317
SETi	1.0283	20.9550	3.9942	0.0941
HNX	1.715	8.0729	1.4181	2.1989

Table 3.37: Christoffersen Test's test statistic under GARCH, IGARCH, FIGARCH, and Opt-FIGARCH on a fixed window with 5% LoS

Rolling Window				
	Christoffersen Test with 5% LoS			
Index	GARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	0.153	28.8107	0.4457	2.1503
ˆKLSE	3.177	26.6804	3.8394	8.9257
STI	3.177	9.0633	2.8248	2.7794
SETi	0.5918	14.4279	4.2779	3.6052
HNX	1.715	4.4328	1.4503	1.4506

Table 3.38: Christoffersen Test's test statistic under GARCH, IGARCH, FIGARCH, and Opt-FIGARCH on a rolling window with 5% LoS

capture the log returns of the countries, however, compared to the GARCH and the FIGARCH model, the Opt-FIGARCH is lacking. It is also noticed that the country that the Opt-FIGARCH is the Malaysian index.

Fixed Window				
	Christoffersen Test with 1% LoS			
Index	GARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	13.9191	68.4304	12.177	12.5137
ˆKLSE	11.6322	62.4704	6.6864	19.7926
STI	2.0858	15.7784	0.7054	1.3007
SETi	5.8547	16.5514	0.7054	7.3271
HNX	8.7312	48.0375	10.1947	13.5111

Table 3.39: Christoffersen Test's test statistic under GARCH, IGARCH, FIGARCH, and Opt-FIGARCH on a fixed window with 1% LoS

Rolling Window				
	Christoffersen Test with 1% LoS			
Index	GARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	12.177	61.6236	12.177	12.5137
ˆKLSE	11.6322	59.1343	6.6864	19.7926
STI	2.0858	6.8526	0.7054	1.3007
SETi	2.3258	16.5514	4.5237	2.3258
HNX	7.4082	27.8895	8.7312	13.5111

Table 3.40: Christoffersen Test's test statistic under GARCH, IGARCH, FIGARCH, and Opt-FIGARCH on a rolling window with 1% LoS

At the 1% LoS, the GARCH-type models are shown to struggle to capture the log returns of the Philippines. It can be seen that the IGARCH model is able to capture at least 1 country under the rolling window while under the fixed window is still an insufficient model. The Opt-FIGARCH is also shown to be lacking compared to the FIGARCH model and may only be comparable to the GARCH model.

Fixed Window				
	Christoffersen Test with 0.01% LoS			
Index	GARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	9.1429	87.4725	9.1429	16.0806
ˆKLSE	23.4078	148.9863	23.4078	23.4078
STI	23.4861	31.5678	23.4861	23.4861
SETi	31.8438	40.3925	31.8438	31.8438
HNX	9.0938	67.6834	9.0938	9.0938

Table 3.41: Christoffersen Test's test statistic under GARCH, IGARCH, FIGARCH, and Opt-FIGARCH on a fixed window with 0.01% LoS

Lastly, at the 0.01% LoS, the IGARCH model shows the same problem as when it was under the past LoS, the Opt-FIGARCH model is inferior compared to the FIGARCH model based on the number of countries that are captured by the two models.

Rolling Window				
	Christoffersen Test with 0.01% LoS			
Index	GARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	9.1429	87.4725	9.1429	16.0806
ˆKLSE	23.4078	117.1266	15.8591	23.4078
STI	23.4861	31.5678	23.4861	23.4861
SETi	31.8438	49.2840	40.3925	40.3925
HNX	9.0938	49.1054	9.0938	9.0938

Table 3.42: Christoffersen Test's test statistic under GARCH, IGARCH, FIGARCH, and Opt-FIGARCH on a rolling window with 0.01% LoS

Summarizing the data from the tables above, the IGARCH model is not a sufficient model to capture the data as it is either overestimating or underestimating the data. The Opt-FIGARCH model is shown to mimic the FIGARCH and GARCH model at large value of LoS. However, when decrease the LoS, the Opt-FIGARCH often underestimates the data.

3.9 Diebold-Mariano Test

The final test that the we implemented was the Diebold-Mariano test which examines and compares the forecast error loss differential of two sets of predicted values. DM-test's null hypothesis is that the loss differential between two sets of forecasted values is covariance-stationary, that is, the two sets of values have equal predictive accuracy.

The researchers compared the forecasted VaR values of the standard models against the GARCH-type models as well as compared the GARCH-type models against other GARCH-type models. These comparisons were completed for each of the ASEAN stock indices (PSEi, KLSE, STI, SETi, and HNX) for both forecast windows (Fixed and Rolling). Consequently, we used different levels of alphas (0.01 %, 1%, 5%) to compare to the resulted p-values from the DM test. For concisenessonly the table for the Philippines's stock index will be shown. The rest of the tables for the other countries can be found in Chapter 6: Appendix. Each of the tables will contain the test statistic and the p-values of the two compared models. An asterisk, '*', will be placed on the end of the p-values to signify the following:

- '***' H_0 is true at the 0.01% LoS.
- '**' H_0 is true at the 1% LoS.
- '*' H_0 is true at the 5% LoS.
- ' ' H_0 is rejected.

PSEi - Fixed Window (0.01%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	3.5183	3.0047	2.5242	4.1219	3.4807
	4.34E-04***	2.66E-03***	1.16E-02**	3.7575E-05	5.00E-04***
HS	8.8155	15.537	3.7569	8.535	5.5918
	1.1919E-18	1.9475E-54	1.72E-04***	1.4021E-17	2.2474E-08
BRW	1.6006	4.4496	0.1027	1.1568	0.3111
	1.09E-01*	8.6028E-06	9.18E-01*	2.47E-01*	7.56E-01*
GARCH		4.3156	1.0716	2.9045	2.0981
		1.5919E-05	2.84E-01*	3.68E-03***	3.59E-02**
EGARCH			1.9942	5.9873	3.5285
			4.61E-02**	2.1338E-09	4.18E-04***
IGARCH				0.7512	0.2856
				4.53E-01*	7.75E-01*
FIGARCH					1.4632
					1.43E-01*

Table 3.43: PSEi DM test statistics and p-values on Fixed Window (0.01%).

PSEi - Fixed Window (1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	3.4785	2.9543	2.516	4.075	3.4446
	5.04E-04***	3.13E-03***	1.19E-02**	4.6007E-05	5.72E-04***
HS	3.117	2.4142	2.339	3.703	3.1836
	1.83E-03***	1.58E-02**	1.93E-02**	2.13E-04***	1.45E-03***
BRW	2.5451	5.8161	0.5344	2.1037	0.9606
	1.09E-02**	6.0245E-09	5.93E-01*	3.54E-02**	3.37E-01*
GARCH		4.3005	1.1179	2.997	2.1365
		1.7045E-05	2.64E-01*	2.73E-03***	3.26E-02**
EGARCH			2.0166	5.9558	3.5187
			4.37E-02**	2.5883E-09	4.34E-04***
IGARCH				0.7911	0.3318
				4.29E-01*	7.40E-01*
FIGARCH					1.4862
					1.37E-01*

Table 3.44: PSEi DM test statistics and p-values on Fixed Window (1.00%).

PSEi - Fixed Window (5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	3.4258	2.8893	2.5035	4.0136	3.3991
	6.13E-04***	3.86E-03***	1.23E-02**	5.9797E-05	6.76E-04***
HS	5.5702	6.0938	3.5804	6.1779	4.9474
	2.5447E-08	1.1025E-09	3.43E-04***	6.4975E-10	7.5225E-07
BRW	2.4519	1.4398	2.0162	3.0471	2.6968
	1.42E-02**	1.50E-01*	4.38E-02**	2.31E-03***	7.00E-03***
GARCH		4.2771	1.166	3.0857	2.1739
		1.8936E-05	2.44E-01*	2.03E-03***	2.97E-02**
EGARCH			2.0381	5.9091	3.5034
			4.15E-02**	3.4391E-09	4.59E-04***
IGARCH				0.8327	0.3812
				4.05E-01*	7.03E-01*
FIGARCH					1.5085
					1.31E-01*

Table 3.45: PSEi DM test statistics and p-values on Fixed Window (5.00%).

PSEi - Rolling Window (0.01%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	3.3663	2.7079	2.3487	3.7188	3.2568
	7.62E-04***	6.77E-03***	1.88E-02**	2.00E-04***	1.13E-03***
HS	26.5309	41.3283	13.54	26.756	19.7358
	4.2712E-155	0.0000E+00	9.0799E-42	1.0527E-157	1.0627E-86
BRW	1.228	4.0082	0.0637	0.9712	0.255
	2.19E-01*	6.1173E-05	9.49E-01*	3.31E-01*	7.99E-01*
GARCH		4.4293	0.8667	1.5394	1.6775
		9.4543E-06	3.86E-01*	1.24E-01*	9.34E-02*
EGARCH			1.876	5.3293	3.3982
			6.07E-02*	9.8595E-08	6.78E-04***
IGARCH				0.7182	0.279
				4.73E-01*	7.80E-01*
FIGARCH					1.4939
					1.35E-01*

Table 3.46: PSEi DM test statistics and p-values on Rolling Window (0.01%).

PSEi - Rolling Window (1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	3.3316	2.6676	2.344	3.6828	3.2272
	8.63E-04***	7.64E-03***	1.91E-02**	2.31E-04***	1.25E-03***
HS	2.901	2.0204	2.1248	3.2456	2.9045
	3.72E-03***	4.33E-02**	3.36E-02**	1.17E-03***	3.68E-03***
BRW	2.1585	5.3603	0.5008	1.8896	0.9169
	3.09E-02**	8.3105E-08	6.16E-01*	5.88E-02*	3.59E-01*
GARCH		4.4139	0.9136	1.6414	1.7237
		1.0154E-05	3.61E-01*	1.01E-01*	8.48E-02*
EGARCH			1.8982	5.3059	3.3906
			5.77E-02*	1.1214E-07	6.97E-04***
IGARCH				0.7555	0.3224
				4.50E-01*	7.47E-01*
FIGARCH					1.5137
					1.30E-01*

Table 3.47: PSEi DM test statistics and p-values on Rolling Window (1.00%).

PSEi - Rolling Window (5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	3.2848	2.6143	2.3354	3.6343	3.1888
	1.02E-03***	8.94E-03***	1.95E-02**	2.79E-04***	1.43E-03***
HS	5.2581	5.5734	3.349	5.6152	4.6623
	1.4554E-07	2.4979E-08	8.11E-04***	1.9629E-08	3.1273E-06
BRW	2.7555	1.8255	2.0604	3.1191	2.7919
	5.86E-03***	6.79E-02*	3.94E-02**	1.81E-03***	5.24E-03***
GARCH		4.39	0.9627	1.7471	1.77
		1.1335E-05	3.36E-01*	8.06E-02*	7.67E-02*
EGARCH			1.9196	5.2707	3.3776
			5.49E-02*	1.3587E-07	7.31E-04***
IGARCH				0.7946	0.3689
				4.27E-01*	7.12E-01*
FIGARCH					1.5323
					1.25E-01*

Table 3.48: PSEi DM test statistics and p-values on Rolling Window (5.00%).

3.10 Summary of Results of the 20% Predictions

A compilation of the data obtained from Chapter 3 starting from the initial tests followed by test to verify the existence of ARCH effects. Lastly, tables will be provided for the summary of the backtesting methods and the hypothesis testing is provided to summarize the data. Each table contains the number of times the null hypothesis of the given test is accepted at the given

LoS under both the fixed and rolling window – the maximum being 10 and the minimum being 0. The tables are as follow: (1) the VaR Violations, (2) the Kupiec Test, and (3) the Christoffersen Test.

From the initial tests, it can be concluded that the claim a constant mean for log returns each country is equal to 0 is true, the log returns of each the countries exhibit auto correlation and the log returns of the each country are shown to not exhibit a normal distribution. Testing the squared residuals under each GARCH-type model shows that all squared residuals exhibit autocorrelation, thus, the rejection of both the Ljung-Box test and the Lagrange Multiplier test confirms the existence of ARCH effects.The tables containing the results from the backtesting methods in Chapter 3 are as follows:

3.10.1 Summary of the VaR Violations

Number of Countries Sufficiently Captured by the Model								
	VaR Violations							
LoS	DN	HS	BRW	G	E	FI	I	Opt-FI
5%	4	5	8	9	10	10	0	8
1%	8	8	6	3	4	5	0	3
0.01%	1	5	0	0	0	0	0	0

Table 3.49: The total number of countries sufficiently captured by the different models under both the fixed and rolling window

The sufficiency of the models using the VaR violations as a test statistic shows that the GARCH-type models excluding the IGARCH model is most effective at the 5% LoS, more specifically the EGARCH and the FIGARCH model. However, lowering the LoS shows a great change where the standard models outperform the GARCH-type models where the DN and the HS at the 1% and the HS at the 0.01% prove to be more sufficient compare to the GARCH-type models chosen. Verifying the results above, an analysis of the compiled data of the Kupiec test will be shown.

3.10.2 Summary of the Kupiec Test

Number of Countries Sufficiently Captured by the Model								
	Kupiec Test							
LoS	DN	HS	BRW	G	E	FI	I	Opt-FI
5%	4	6	8	9	10	10	0	8
1%	8	8	6	5	5	6	1	3
0.01%	2	7	0	4	3	4	0	2

Table 3.50: The total number of countries sufficiently captured by the different models under both the fixed and rolling window

Testing the sufficiency of the models for a second time using the Kupiec test shows a similar result under the 5% LoS. The GARCH-type models, mainly the GARCH and the FIGARCH model, excluding the IGARCH model is shown to be superior in capturing the log returns of the countries compared to the standard models. However, under the 1% LoS, the GARCH-type models do not perform as poor as seen in the VaR violations test as the GARCH-type models were able to capture at least 50% of the chosen countries. Nonetheless, comparing the GARCH-type models to the standard models at the 1% LoS, the standard models still prove to be superior. At the 0.01% LoS, all GARCH-type models are able to perform, except the IGARCH, and capture the data of some of the chosen countries while the one of the 3 standard approaches are unable to capture the data of any country, that is, the BRW approach. The HS is shown to be the best candidate under the 0.01% LoS as it was able to capture more than 50% of the chosen countries. Lastly, the Christoffersen test will analyze the sufficiency, once again, and the independence of occurrence the VaR violations.

3.10.3 Summary of the Christoffersen Test

Number of Countries Sufficiently Captured by the Model								
	Christoffersen Test							
LoS	DN	HS	BRW	G	E	FI	I	Opt-FI
5%	4	4	4	10	10	10	0	8
1%	8	8	6	6	8	7	1	5
0.01%	2	5	0	4	5	5	0	4

Table 3.51: The total number of countries sufficiently captured by the different models under both the fixed and rolling window

As seen from the results, the GARCH model perform exceptionally at the 5% LoS as they are able to capture all the countries while the standard models are unable to capture 50% of the countries. From the past table, the standard models are shown to exceed the GARCH-type models at the 1% LoS, however, the GARCH-type models are shown to be on par with the standard models. the EGARCH is comparable against the DN and HS where all three models where able to capture eight of the 10 countries. The FIGARCH and the GARCH-type models captured 7 and 6 countries respectively. At the 0.01% LoS, the all the GARCH-type models are shown to capture the countries' data as well as the HS approach which was shown to be a sufficient model in the past LoS and the past backtesting methods.

3.10.4 Analysis of the First Three Backtesting Methods

A general observation taken from the all three backtesting methods is the minimal differences and major similarities obtained when applying a fixed or rolling window on the log returns of the different indices. A common trend that can be noticed when applying a rolling window rather than a fixed window is that the rolling window, under any LoS, allows the models to become a suitable fit for the given data. This can be observed in Table 3.17 and Table 3.18. Another would the general increase in the number of suitable models when applying a more robust test. This is seen when comparing the summary of all three backtesting methods against one another with the exception being the HS and BRW approach under the Christoffersen test.

Moving to the each of the three backtesting methods, the VaR violations and the Kupiec Test have similar results as seen from Table 3.49 and Table 3.50. As both backtesting methods are claiming the sufficiency of the model, it is not surprising that the results should be almost identical. However, the major difference of the two models is at the 0.01% LoS. This major differences from the results may be the distributions used in testing the data as the binomial distribution tends to focus on the an exact number, as seen with the use of confidence intervals to accept or reject the claims, while the chi-squared distribution is able to use an approximate, thus being more flexible. Therefore the Kupiec test compared to the standard VaR violations test shows improvement in the models' sufficiency. The Christoffersen test, while being the last of the test, gives an important insight as the independence of the occurrence of the VaR violations must be noted as this may skew the sufficiency of the model. This is the only instance that the GARCH-type models perform as well (1% LoS) or even outperform (5% and 0.01% LoS) the standard models. This may stem from the fact that the independence of the occurrence VaR violations is also taken into account. From this, the results from the Christoffersen test will be one of the main objective factors that will be used in the comparison of the models.

Moving to the models themselves, starting with the DN approach, the among the three test, the results have been consistent from the first backtesting method used to the last backtesting method used. Compared to the other standard approaches the HS is the most erratic where its best performance is under the Kupiec test (Table 3.50) and worst under the Christoffersen test (Table 3.51). The BRW, on the other hand, is relatively consistent up until the Christoffersen test where the BRW approach greatly underperformed under the 5% LoS where it performed exceptionally well in the past two backtesting methods. Interpreting the data, the researchers give the following claims based on the data:

- Under the DN approach, the VaR violations backtesting method may be sufficient as a stand-alone backtesting metric.
- Under both the HS and BRW approach, the Christoffersen test would be an appropriate backtesting method as the independence of the occurrence may be correlated in these two models.

Moving the the GARCH-type models, the GARCH model compared to the other GARCH-type models (EGARCH and FIGARCH) does not perform as well as the other two models. However, compared to the (IGARCH and the Opt-FIGARCH), the GARCH model greatly outperforms the two models. This may come from the differencing parameter, d used in the IGARCH and the Opt-FIGARCH model as the two models have both d set to 1 and 0.633 respectively. It is can also be noted when compared to the FIGARCH model, d is not necessarily fixed. However, despite the d values being close as seen when under the Philippine stock index (0.6143 for FIGARCH and 0.633 for Opt-FIGARCH), the Opt-FIGARCH still underperforms. Thus, the use of a fixed 'optimized' differencing parameter may not be suitable. This claim of a fixed d is further supported by the outputs of the IGARCH model as seen that for almost all test, the IGARCH model is not able to perform. The EGARCH and the FIGARCH models maybe prove to be better models since both models are build on the foundation of the GARCH model. However, the EGARCH model is shown to outperfrom the FIGARCH model under the Christoffersen test where as the FIGARCH is able to outperform the EGARCH model under the VaR violations and the Kupiec test. The effects of asymmetrically weighing the positive and negative shocks may be show better outputs compared to applying a fractional differencing factor. The following claims are based on the data:

- Applying a fixed 'optimized' fractional differencing factor is not suitable for the data as it underperform when compared to using the derived optimized parameter.
- The GARCH model is an effective standard model, but is outperformed by its successors (EGARCH and FIGARCH).

- Despite the FIGARCH model outperforming the EGARCH under the VaR violations and the Kupiec test, the EGARCH is the best model when the independence of the occurrences of the violations are taken into account.

3.10.5 Summary of the Diebold-Mariano Test

From the collection of p-values from the tables derived, the first observation that can be made between the standard models and the GARCH-type models is that the standard models tend to have the same predictive accuracy with the GARCH-type models when the LoS is minimized. At the 0.01% and 1% LoS, the BRW approach is shown to be have similar predictions to the GARCH-type models as seen that in three of the 5 countries (the Philippines, Malaysia, and Vietnam), the BRW approach has shown to have the same predictive accuracy as the GARCH-type models even at 5% LoS. The DN approach comes in second as having the same predictive accuracy with the GARCH-type models in 2 of the 5 countries (Singapore and Thailand).

Comparing the GARCH-type models to one another, The EGARCH model has the most number of instances where its forecasted values are significantly different from the other GARCH-type models, instances where they are the two models give the same predictive accuracy is under the Malaysian stock index where EGARCH and the other GARCH-type models have the same predictive accuracy, however, this is at 0.01%LoS. The FIGARCH model also shows similar insights as the EGARCH model as majority of the GARCH-type models are unable to produce a statistically equal prediction as the FIGARCH model. Instances that the other GARCH-type models are able to have the same predictive accuracy is also under a 0.01% or 1% LoS. However, under the Malaysian stock index, the other GARCH-type models are able to have a similar predictive accuracy as the FIGARCH model at 5% LoS. On the other hand, the IGARCH model has the least amount of instances wherein its forecasted values are significantly different when compared to other models, thus, this means that the IGARCH model's forecasted VaR values are similar to that of the other standard and GARCH-type models. When comparing the tables between the fixed window against the rolling window, it is also observed that the rolling window data has less number of instances of forecasted VaR values being significantly different when compared to another model's forecasted VaR values. A summary of the Diebold-Mariano Test follows:

Model	PSEi	ˆKLSE	STI	SETi	HNX	Total
DN	9	2	10	10	0	**31**
HS	2	0	10	4	0	16
BRW	10	2	3	1	8	24

Table 3.52: Number of instances where the standard models' forecasted VaR values have similar predictive accuracy when compared to the GARCH models under a 5.00% LoS.

Model	PSEi	ˆKLSE	STI	SETi	HNX	Total
DN	9	2	9	10	0	30
HS	10	10	0	8	0	28
BRW	8	10	0	8	8	**34**

Table 3.53: Number of instances where the standard models' forecasted VaR values have similar predictive accuracy when compared to the GARCH models under a 1.00% LoS.

Model	PSEi	ˆKLSE	STI	SETi	HNX	Total
DN	9	2	9	10	0	**30**
HS	1	0	0	0	0	1
BRW	8	0	0	1	10	19

Table 3.54: Number of instances where the standard models' forecasted VaR values have similar predictive accuracy when compared to the GARCH models under a 0.01% LoS.

3.10.6 Analysis of the Diebold-Mariano Test

To start the discussion on the Diebold-Mariano test, a similar pattern is also seen with the fixed and rolling window where the differences between the two are minimal. The standard approaches compared to the GARCH-type models, the closest of the three models to the GARCH-type models is the BRW approach followed by the DN with the HS approach as model with the least number of models to have the same predictive accuracy as the GARCH-type models. As seen from the analysis done on the backtesting methods, the HS is shown to be the best of the standard approach in terms of sufficiency, however, the accuracy to the GARCH-type models are shown to be significantly different. The BRW, on the other hand, is shown to be closer to the GARCH-type models. This may be from the use of a semiparametric approach. The DN approach follows the BRW as being the second best standard approach to have the same predictive accuracy as the GARCH-type models. The use of some sort of parametric model may be the reason on why the DN and the BRW approach is able to have a significantly equal predictive accuracy as the GARCH-type models. This is best seen when the LoS is small (1% and 0.01% LoS). In this case, the BRW and the DN approach is best able to capture the FIGARCH model which is shown (from the backtesting methods used above) to be one of the better models. Both models, however, fail to capture the EGARCH model for most of the stock indices used with the exception of Thailand where the DN approach is able to have the same predictive accuracy as the EGARCH model. The following claims are then made from the obtained data:

- The BRW and the DN approach may be used instead of the GARCH-type models given a lower LoS (1% and 0.01%) as it able to have the same predictive accuracy as the FIGARCH model. Thus, the choice of a more parsimonious model.

- The HS approach has instances where it is able to have the same predictive accuracy as the GARCH-type models. However, the consistency of the BRW and the DN approach outclass the HS.

With the GARCH-type models, the GARCH model is able to have the same predictive accuracy as the FIGARCH model. Similarly, this observation is also seen in the IGARCH and the Opt-FIGARCH model. This may stem from the fact that the GARCH, IGARCH, and the FIGARCH model differ on the differencing factor d having d equal to 0, 1 and the derive factor respectively. The EGARCH, on the other hand, is the most unique model incomparison to the other GARCH-type models. This may, once again, be from the property that the EGARCH model asymmetrically weighs the positive and negative shocks differently. The following claims are then made from the data:

- The GARCH model is the preferred model to the FIGARCH and IGARCH model as a more parsimonious model is always preferred.

- The EGARCH model is unique compared to the other GARCH-type models chosen.

Chapter 4

Sensitivity Analysis of standard and GARCH-type models on estimating VaR

We now conduct sensitivity analyses to the chosen models for estimating VaR. In order to support the results discussed in Chapter 3 and draw out more conclusions on the models, a sensitivity analysis on level of significance and time frame will be conducted.

4.1 Sensitivity Analysis on Level of Significance

The first parameter that we will manipulate to estimate VaR for each of the models is the level of significance. The range of the level of significance, which we denote as α, that we used for this experiment is from 0 to 0.10, with steps of 0.11. This gives us enough information to see how the estimated VaR performs through a range of confidence levels. We conduct this analysis using the estimated VaR data of the Philippines or PSEi.

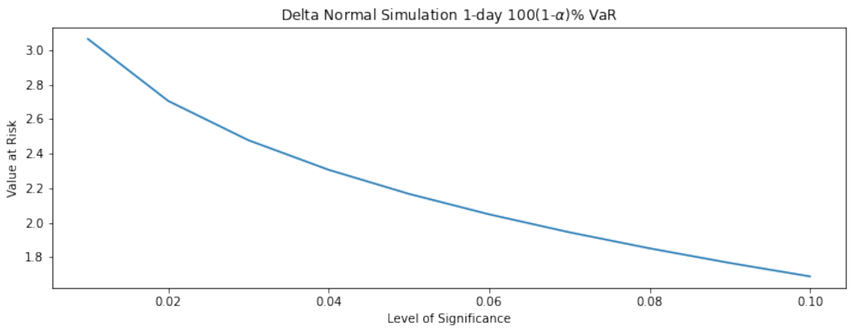

Figure 4.1: PH Estimated VaR using the DN Approach from 0% to 10% LoS

For all the models, we could observe that as the confidence level increases,

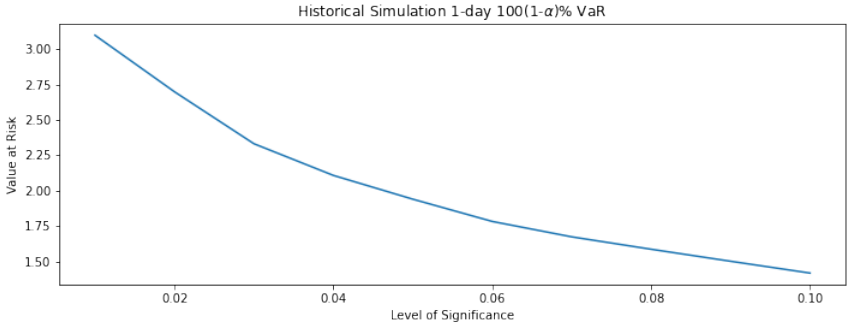

Figure 4.2: PH Estimated VaR using the HS Approach from 0% to 10% LoS

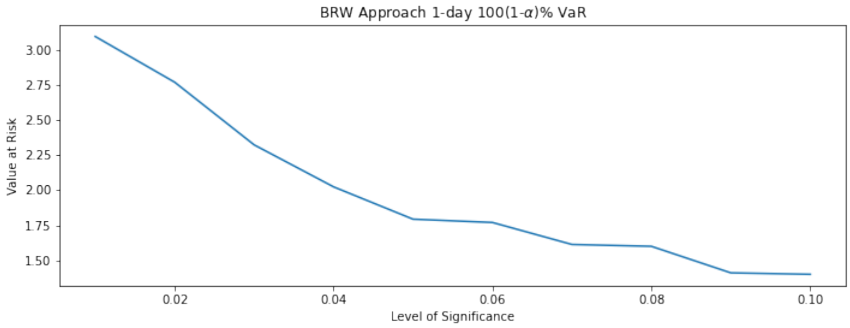

Figure 4.3: PH Estimated VaR using the BRW Approach from 0% to 10% LoS

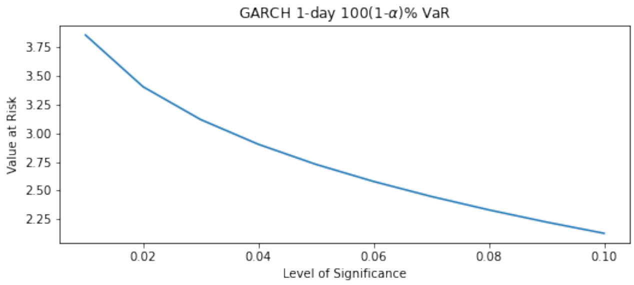

Figure 4.4: PH Estimated VaR using the GARCH Model from 0% to 10% LoS

the estimated VaR decreases. Conversely, we observe that when the confidence level decreases, the estimated VaR increases. Given this information, we see from the previous chapter that if we set the level of significance to

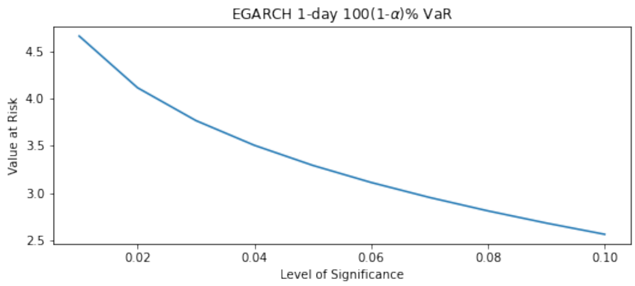

Figure 4.5: PH Estimated VaR using the EGARCH Model from 0% to 10% LoS

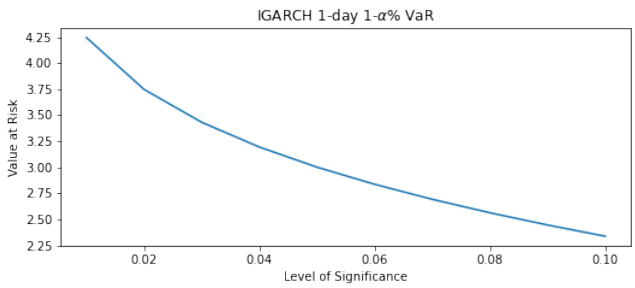

Figure 4.6: PH Estimated VaR using the IGARCH Model from 0% to 10% LoS

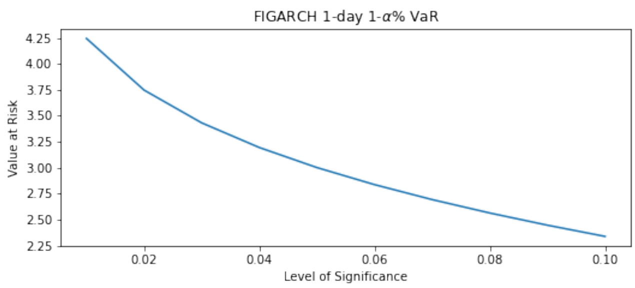

Figure 4.7: PH Estimated VaR using the FIGARCH Model from 0% to 10% LoS

0.01%, almost all the models underestimate the data. This is because the VaR estimated may be more than the actual VaR. Thus, the 0.01% LoS is too strict for the chosen models of this study.

The Opt-FIGARCH model is also not included anymore for conciseness as it is merely a derivation of the FIGARCH model and would attain the same result either way.

It is interesting to note that among all models, only the BRW approach has a more jagged curve in terms of estimated VaR. The reason behind this may be because of the weights of being put into more recent data in terms of log returns. Although there is not much difference, all other models have smoother curves.

4.2 Sensitivity Analysis on Data Time Frame

The other key information that we will manipulate is the time frame of the data, specifically the log returns. We are interested as well to come up with results as to how the performance of the models are affected when we have a relatively smaller time frame compared to that was used in Chapter 3. From this analysis, we aim to attain a conclusion on which models still perform well in estimating VaR on two different time frames. Specifically, we will be using a 10-year data set, from 2011-2021, and 5-year data set, from 2016-2021.

Similar to Chapter 3, the Pareto method will still be used to do the VaR predictions. For conciseness, The other initial tests are assumed to be the same for the data set in Chapter 3 as the data set for both manipulated time frames will have the same results, especially for the normality of the data and the ARCH effects.

4.2.1 10-year Data Time Frame

We begin with fixing the log returns, or our data, to 10 years, specifically from 2011-2021. The visualizations are arranged starting from the 5% LoS then the 1% LoS for both fixed and rolling forecast windows. The 0.01% LoS is not included in further analyses as it has been established in the previous section that it is too strict for the chosen approaches and models to estimate VaR. The visualizations will only feature the PSEi for conciseness and all other graphs for the other stock indices could be found in the Appendix.

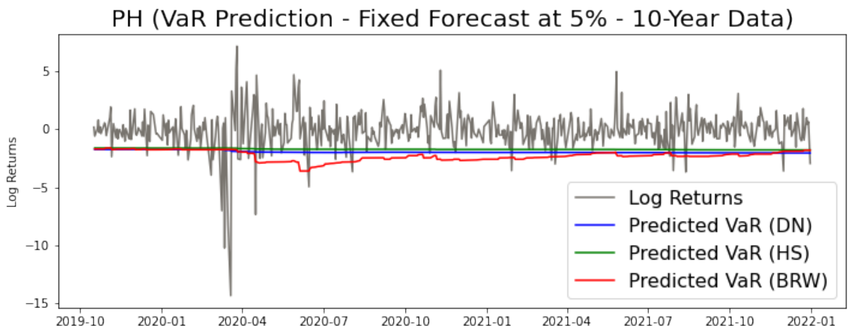

Figure 4.8: PH Log Returns and Estimated VaR using the standard models on a fixed window with 5% LoS for a 10-year Time Frame

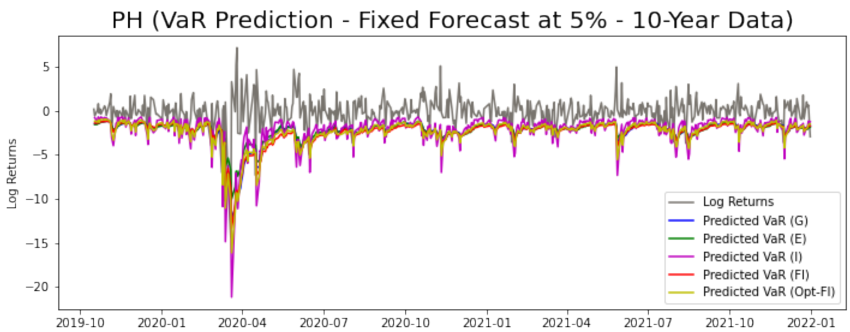

Figure 4.9: PH Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 5% LoS for a 10-year Time Frame

At the 5% LoS, for the standard models, the DN and HS approach have almost constant VaR estimates, whereas the BRW approach has some minor spikes. These spikes mainly occur between April to July 2020. Visually,

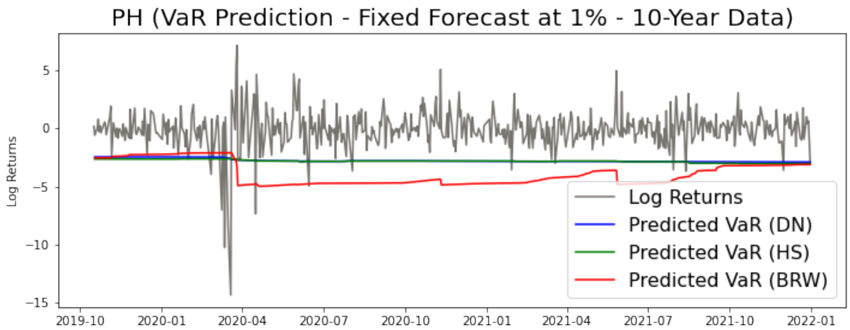

Figure 4.10: PH Log Returns and Estimated VaR using the standard models on a fixed window with 1% LoS for a 10-year Time Frame

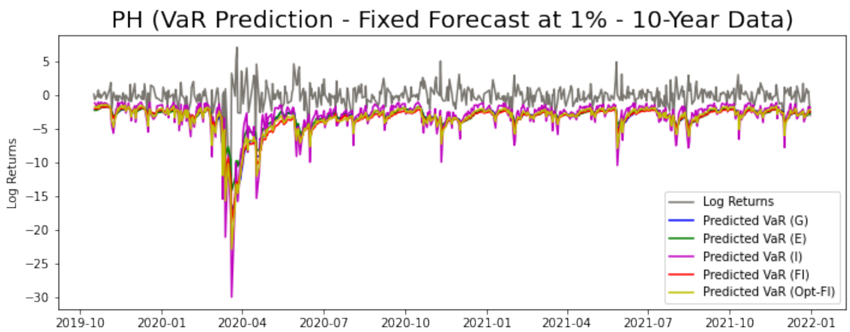

Figure 4.11: PH Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 1% LoS for a 10-year Time Frame

we could observe that the standard models couldn't really capture the log returns of the PSEi data.

As for the GARCH-type models, we could observe that there seems to be more spiked in the VaR estimates which could mean that it captures the log returns better. It is also a good thing to note that a major spike in the VaR and log returns happened around January to May of 2020.

Under the 1% LoS, the only main difference from the 5% LoS is that the BRW approach seems to have more spikes from April to October of 2021. The GARCH-type models look similar to that of the 5% LoS graphs for the fixed forecast window.

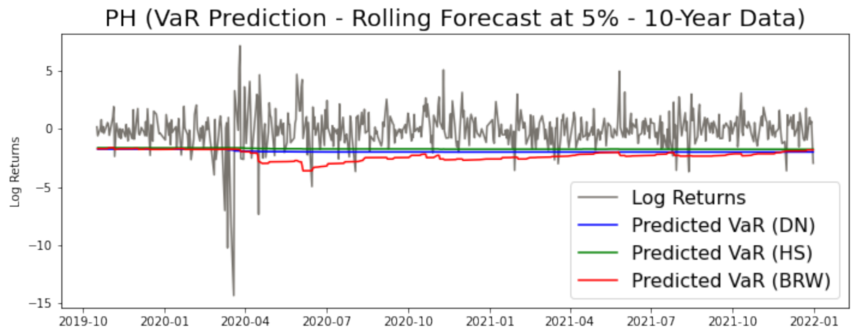

Figure 4.12: PH Log Returns and Estimated VaR using the standard models on a rolling window with 5% LoS for a 10-year Time Frame

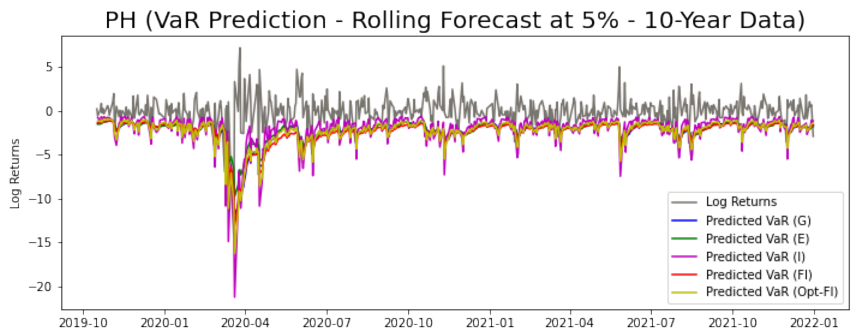

Figure 4.13: PH Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 5% LoS for a 10-year Time Frame

There is not much difference in the graphs for the fixed window and the rolling window. Similar interpretations could be made. However, it is important to note that for both windows and across all levels of significance, the BRW approach shows more spikes than the other two standard approaches and the IGARCH model seems to underestimate the data for the GARCH-type models. Similar to what was done in Chapter 3, we now test the estimated VaR using the Kupiec, Christoffersen, and Diebold-Mariano tests.

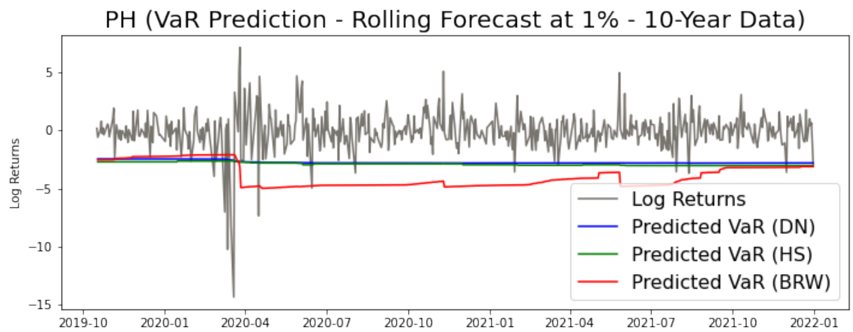

Figure 4.14: PH Log Returns and Estimated VaR using the standard models on a rolling window with 1% LoS for a 10-year Time Frame

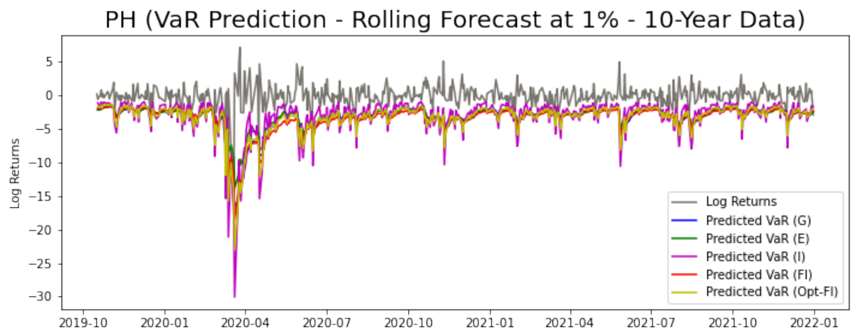

Figure 4.15: PH Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 1% LoS for a 10-year Time Frame

Backtesting Methods

VaR Violations

The following tables summarize the results from all the graphs above for both the fixed and rolling window for the 10-year data time frame. The tables will contain the number of VaR violations that have occurred using said model under the chosen dataset. A similar method is used as in Chapter 3 on tabulating the VaR violations using confidence intervals. The confidence intervals – obtained using a binomial distribution – of Philippines, Malaysia, Singapore, Thailand, and Vietnam are (17,37), (18,39), (18,38), (17,37), (18,38) for the 5% LoS and (0, 12) for all countries for the 1% LoS, respectively.

Fixed Window - 10-Year Time Frame								
	VaR Violations with 5% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	34	41	25	31	30	55	27	33
ˆKLSE	52	63	37	34	33	51	31	35
STI	33	46	34	40	40	48	34	35
SETi	29	45	28	34	32	45	25	27
HNX	34	37	36	33	37	39	33	34

Table 4.1: Number of VaR violations under different models on a fixed window with 5% LoS for a 10-year Time Frame

Rolling Window - 10-Year Time Frame								
	VaR Violations with 5% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	34	42	25	31	32	60	27	36
ˆKLSE	52	62	37	35	33	54	32	36
STI	33	40	34	39	39	48	36	36
SETi	27	42	28	34	31	50	25	28
HNX	31	35	36	33	36	39	34	35

Table 4.2: Number of VaR violations under different models on a rolling window with 5% LoS for a 10-year Time Frame

Fixed Window - 10-Year Time Frame								
	VaR Violations with 1% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	17	15	13	17	18	28	17	16
ˆKLSE	26	27	17	17	17	27	13	19
STI	16	15	13	9	10	22	8	7
SETi	11	11	10	14	14	24	17	18
HNX	23	18	14	17	19	24	17	18

Table 4.3: Number of VaR violations under different models on a fixed window with 1% LoS for a 10-year Time Frame

Rolling Window - 10-Year Time Frame								
	VaR Violations with 1% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	17	13	13	17	18	33	17	17
^KLSE	26	25	17	17	16	28	13	19
STI	16	15	13	9	11	24	9	9
SETi	11	11	10	14	14	18	13	13
HNX	20	15	13	17	18	26	17	17

Table 4.4: Number of VaR violations under different models on a rolling window with 1% LoS for a 10-year Time Frame

Under the 5% LoS, the VaR violations mainly occur under the HS approach and the IGARCH model where the models underestimate the data. The BRW Approach, FIGARCH and Opt-FIGARCH models seem to be promising as the violations are within the confidence interval for all countries. The violations between the fixed and rolling windows are almost exactly the same. It is also good to note that for Vietnam and Thailand, most of the models are able to capture the log returns of the index. In comparison to the full data from Chapter 3, the DN model performs better as it is able to accurately capture the log returns of Singapore, Thailand, and Vietnam.

For the 1% LoS, almost all the models in all stock indices underestimate the data. This means that the VaR violations exceed the confidence interval of the VaR violations. Although, for Thailand, the standard models are able to capture the log returns accurately. For Singapore, the GARCH-type models, with the exception of IGARCH, perform well in capturing the log returns. It is also worth noting that with only having 10 years of the data, the standard models do not accurately capture the log returns of Vietnam compared to the dataset in Chapter 3. There is no difference between the forecast windows as well. We then take a look at the Kupiec test to verify these results.

Kupiec Test

The methods for the Kupiec tests in this chapter will be similar to Chapter 3. We will still use the color scheme to determine whether the null hypothesis of the test is rejected or not.

Fixed Window - 10-Year Time Frame								
	Kupiec Test with 5% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	23.6606	6.8073	0.1372	0.661	0.3879	24.2738	0.0016	1.4108
ˆKLSE	65.8672	34.2608	2.7102	1.2256	0.8538	15.994	0.3053	1.6601
STI	20.583	10.8493	1.4595	5.1806	5.1806	13.1291	1.4595	1.9322
SETi	14.4284	10.8292	0.0512	1.8551	0.9823	10.8292	0.1372	0.0009
HNX	22.6737	3.1319	2.531	1.1126	3.1688	4.5509	1.1126	1.5342

Table 4.5: Kupiec Test's test statistic under different models on a fixed window with 5% LoS for a 10-year Time Frame

Rolling Window - 10-Year Time Frame								
	Kupiec Test with 5% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	23.6606	7.7364	0.1372	0.661	1.0027	32.5264	0.0016	3.0157
ˆKLSE	65.8672	32.5288	2.7102	1.6601	0.8538	20.0244	0.5464	2.1554
STI	20.583	5.1806	1.4595	4.4178	4.4178	13.1291	2.4659	2.4659
SETi	11.2618	7.7364	0.0512	1.8551	0.6446	16.9425	0.1372	0.0512
HNX	17.0677	1.9895	2.531	1.1126	2.5639	4.5509	1.5342	2.0186

Table 4.6: Kupiec Test's test statistic under different models on a rolling window with 5% LoS for a 10-year Time Frame

Under the 5% LoS, for the Kupiec test, the DN and HS approaches as well as the IGARCH model fail to accept the null hypothesis thus the number of VaR violations may not be within 5% LoS. It could not properly capture all

Fixed Window - 10-Year Time Frame								
	Kupiec Test with 1% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	16.1774	11.7322	7.8372	16.2217	18.6342	48.2815	16.2217	13.9308
^KLSE	39.6476	42.8256	15.1083	15.1083	15.1083	42.8256	7.1424	19.8525
STI	13.2970	11.1956	7.4137	1.8614	2.961	28.3793	0.9883	0.3695
SETi	4.5753	4.5753	3.2156	9.7113	9.7113	13.8903	6.1209	7.8372
HNX	31.3829	17.9716	9.294	15.6552	20.496	34.4237	15.6552	18.0181

Table 4.7: Kupiec Test's test statistic under different models on a fixed window with 1% LoS for a 10-year Time Frame

Rolling Window - 10-Year Time Frame								
	Kupiec Test with 1% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	16.1774	7.8372	7.8372	16.2217	18.6342	66.1433	16.2217	16.2217
^KLSE	39.6476	36.5502	15.1083	15.1083	12.9165	46.0814	7.1424	19.8525
STI	13.2970	11.1956	7.4137	1.8614	4.2645	34.2165	1.8614	1.8614
SETi	4.5753	4.5753	3.2156	9.7113	9.7113	18.586	7.8372	7.8372
HNX	23.029	11.2658	7.469	15.6552	18.0181	40.6326	15.6552	15.6552

Table 4.8: Kupiec Test's test statistic under different models on a rolling window with 1% LoS for a 10-year Time Frame

the five stock indices. The BRW Approach, FIGARCH, and Opt-FIGARCH models are still performing well for all countries. It is also worth noting that for Vietnam, most of the models are able to properly capture the VaR violations within the LoS. There is also no difference in terms of the forecast window for the Kupiec test.

On the other hand, under the 1% LoS, with the Kupiec test, it further strengthens the claim that the standard approaches accurately capture the log returns for Thailand. In the case of Singapore, the GARCH-type models, with the exception of IGARCH, fail to reject the null hypothesis. This means that for these cases, the models accurately captured the VaR violations within the LoS. There is also minimal difference in terms of the two forecasting windows.

We then further verify these results for the independence of the VaR violations using the Christoffersen test.

Christoffersen Test

Similar to the Kupiec test, all the Christoffersen tests in this chapter will follow the same methods found in Chapter 3. We will still use the color scheme to determine whether the null hypothesis of the test is rejected or not.

Fixed Window - 10-Year Time Frame								
	Christoffersen Test with 5% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	24.1921	7.2792	2.7048	0.8229	0.5937	26.1776	0.4543	2.0696
^KLSE	66.1976	34.2618	7.1722	1.9728	1.4678	18.2132	0.6902	6.3213
STI	33.4878	15.1587	17.1942	6.6732	6.6732	17.1040	3.0568	2.874
SETi	33.4878	15.1587	0.2505	2.7173	0.9869	14.1103	0.1644	0.1163
HNX	28.4742	7.4192	7.2912	4.938	5.4889	6.2773	4.938	3.1081

Table 4.9: Christoffersen Test's test statistic under different models on a fixed window with 5% LoS for a 10-year Time Frame

Rolling Window - 10-Year Time Frame								
	Kupiec Test with 5% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	24.1921	8.1124	2.7048	0.8229	1.1387	35.7668	0.4543	4.2747
^KLSE	66.1976	32.5328	7.1722	2.5525	1.4678	23.0323	1.0396	7.0964
STI	33.4878	12.7267	17.1942	6.1724	6.1724	17.1040	3.5878	3.5705
SETi	11.5574	7.9064	0.2505	2.7173	0.6707	21.7985	0.1644	0.1644
HNX	19.5455	5.0181	7.2912	4.938	3.6653	6.2773	4.941	3.3444

Table 4.10: Christoffersen Test's test statistic under different models on a rolling window with 5% LoS for a 10-year Time Frame

Fixed Window - 10-Year Time Frame								
	Christoffersen Test with 1% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	19.0351	12.3553	12.3316	16.6871	21.1062	48.537	16.6871	14.9156
^KLSE	47.5414	50.0471	31.6017	15.4945	15.4945	45.5571	7.7592	21.1848
STI	13.7972	11.8529	29.9826	2.1604	3.3307	28.3968	1.224	0.5496
SETi	5.0363	5.0363	3.5958	10.4623	10.4623	14.8751	6.6706	8.4835
HNX	37.5936	23.6193	10.1304	16.0174	25.5452	34.4264	16.0174	18.2714

Table 4.11: Christoffersen Test's test statistic under different models on a fixed window with 1% LoS for a 10-year Time Frame

Rolling Window - 10-Year Time Frame								
	Kupiec Test with 1% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	19.0351	8.8525	12.3316	16.6871	21.1062	66.9218	16.6871	17.3356
^KLSE	47.5414	45.161	31.6017	15.4945	13.4359	49.0246	7.7592	21.1848
STI	13.7972	11.8529	29.9826	2.1604	4.7127	34.2187	2.1604	2.1604
SETi	5.0363	5.0363	3.5958	10.4623	10.4623	19.8373	8.4835	8.4835
HNX	31.31	14.7908	8.5189	16.0174	23.6564	40.6845	16.0174	16.0174

Table 4.12: Christoffersen Test's test statistic under different models on a rolling window with 1% LoS for a 10-year Time Frame

The Christoffersen test under the 5% LoS makes the overall analysis of the VaR violations. It was already well established in the Kupiec test that the DN, HS, and IGARCH models fail to capture the VaR violations within the LoS. Taking a look at the BRW approach, three out of the five stock indices rejected the null hypothesis of the test which means that the VaR violations of the BRW approach were not independent of each other. All other models, especially the FIGARCH model, are able to capture the log returns accurately.

Table 4.12 of the Christoffersen test contemplates the two aforementioned analyses with the VaR violations and Kupiec test for the 1% LoS. It further shows that the VaR violations are within the LoS and are independent with each other. Both forecasts show similar results as well.

For a better analysis on the comparison across models, we will use the Diebold-Mariano test.

Diebold-Mariano Test

The Diebold-Mariano test then compared the loss differential of the forecasted VaR values of the standard models against the GARCH-type models as well as the GARCH-type models against each other under 10-year timeframe. Under this test, we examine if two models have the same predictive accuracy, or if there is a significant difference between the model's forecasts.

PSEi - Fixed Window (10-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	5.0609	5.2405	3.6224	6.1172	5.2136
	4.1719E-07	1.6013E-07	2.92E-04***	9.5253E-10	1.8523E-07
HS	4.8102	4.8335	3.5016	5.8724	5.0301
	1.5077E-06	1.3412E-06	4.62E-04***	4.2942E-09	4.9023E-07
BRW	2.3756	6.3241	0.1181	1.0824	0.3322
	1.75E-02**	2.5473E-10	9.06E-01*	2.79E-01*	7.40E-01*
GARCH		4.5232	1.6663	7.6934	3.8411
		6.0923E-06	9.57E-02*	1.4326E-14	1.22E-04***
EGARCH			2.6696	7.1608	4.9412
			7.59E-03***	8.0205E-13	7.7652E-07
IGARCH				0.7603	0.2653
				4.47E-01*	7.91E-01*
FIGARCH					1.7478
					8.05E-02*

Table 4.13: PSEi DM test statistics and p-values on a 10-Year Fixed Window (1.00%).

PSEi - Fixed Window (10-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	5.0059	5.1552	3.5956	6.0427	5.1462
	5.5603E-07	2.5332E-07	3.24E-04***	1.5153E-09	2.6579E-07
HS	6.4732	7.4757	4.3259	7.4281	6.2192
	9.5933E-11	7.6816E-14	1.5193E-05	1.1019E-13	4.9969E-10
BRW	2.2383	0.7554	2.2312	3.4091	3.12
	2.52E-02**	4.50E-01*	2.57E-02**	6.52E-04***	1.81E-03***
GARCH		4.4957	1.6998	7.6241	3.8281
		6.9341E-06	8.92E-02*	2.4575E-14	1.29E-04***
EGARCH			2.6716	7.1002	4.8892
			7.55E-03***	1.2461E-12	1.0124E-06
IGARCH				0.7999	0.3101
				4.24E-01*	7.56E-01*
FIGARCH					1.7617
					7.81E-02*

Table 4.14: PSEi DM test statistics and p-values on a 10-Year Fixed Window (5.00%).

PSEi - Rolling Window (10-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	5.0609	5.2405	3.6224	6.1172	5.2136
	4.1719E-07	1.6013E-07	2.92E-04***	9.5253E-10	1.8523E-07
HS	4.8102	4.8335	3.5016	5.8724	5.0301
	1.5077E-06	1.3412E-06	4.62E-04***	4.2942E-09	4.9023E-07
BRW	2.3756	6.3241	0.1181	1.0824	0.3322
	1.75E-02**	2.5473E-10	9.06E-01*	2.79E-01*	7.40E-01*
GARCH		4.5232	1.6663	7.6934	3.8411
		6.0923E-06	9.57E-02*	1.4326E-14	1.22E-04***
EGARCH			2.6696	7.1608	4.9412
			7.59E-03***	8.0205E-13	7.7652E-07
IGARCH				0.7603	0.2653
				4.47E-01*	7.91E-01*
FIGARCH					1.7478
					8.05E-02*

Table 4.15: PSEi DM test statistics and p-values on a 10-Year Rolling Window (1.00%).

PSEi - Rolling Window (10-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	5.1439	5.2943	3.6063	6.2062	5.0973
	2.6915E-07	1.1949E-07	3.11E-04***	5.4273E-10	3.4461E-07
HS	6.5607	7.5381	4.2845	7.5318	6.0894
	5.3543E-11	4.7701E-14	1.8311E-05	5.0052E-14	1.1330E-09
BRW	2.1303	0.4964	2.1799	3.3599	2.9873
	3.31E-02**	6.20E-01*	2.93E-02**	7.80E-04***	2.81E-03***
GARCH		4.6193	1.7224	7.636	3.5736
		3.8510E-06	8.50E-02*	2.2399E-14	3.52E-04***
EGARCH			2.6733	7.2381	4.6996
			7.51E-03***	4.5487E-13	2.6065E-06
IGARCH				0.8318	0.3871
				4.06E-01*	6.99E-01*
FIGARCH					1.6045
					1.09E-01*

Table 4.16: PSEi DM test statistics and p-values on a 10-Year Rolling Window (5.00%).

Under the 10-year timeframe, we have observed that the standard models, particularly the Historical Simulation (HS) approach, has the most number of instances where its forecasted VaR values are significantly different than that of the GARCH-type models. Following the HS approach is the DN approach then the BRW approach. On the other hand, we note that for the GARCH-type models, a pattern is formed on where a specific model is different and where it is similar. For GARCH and EGARCH, it is different when compared to FIGARCH and OPT-FIGARCH and vice versa, whereas IGARCH stands in the middle, remaining somewhat similar to the four GARCH-type models.

4.2.2 5-year Data Time Frame

We then further shorten the time frame for our data to 5 years, specifically from 2016-2021. We will begin with the visualization of the models, then employ the same backtesting methods we did for the 10-year time frame. The visualization is limited to PSEi for conciseness. All other graphs can be found in the Appendix.

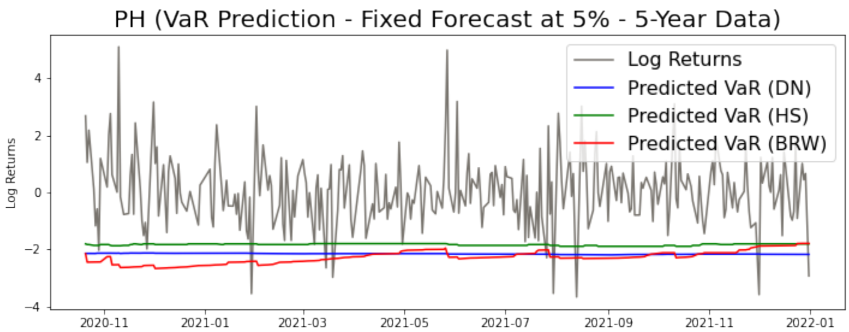

Figure 4.16: PH Log Returns and Estimated VaR using the standard models on a fixed window with 5% LoS for a 5-year Time Frame

For the fixed forecast, we can observe from the graphs of the different approaches and models to be more stable in terms of the spikes in log returns and VaR estimates. It also appears that the standard models seem to have a constant VaR estimate under the 5% LoS, but under the 1% LoS, the BRW approach has more spikes in VaR estimates. As for the GARCH-type models, the IGARCH model seem to underestimate the data for both LoS.

For the rolling window, there is not much difference from the graphs of

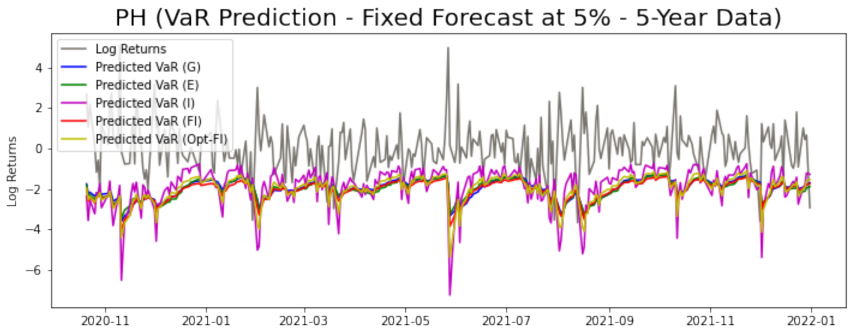

Figure 4.17: PH Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 5% LoS for a 5-year Time Frame

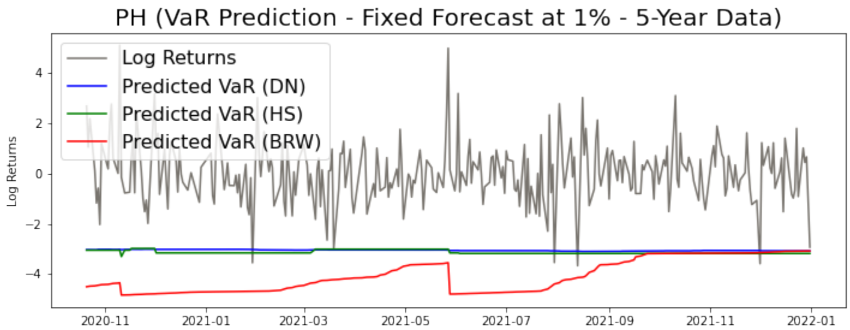

Figure 4.18: PH Log Returns and Estimated VaR using the standard models on a fixed window with 1% LoS for a 5-year Time Frame

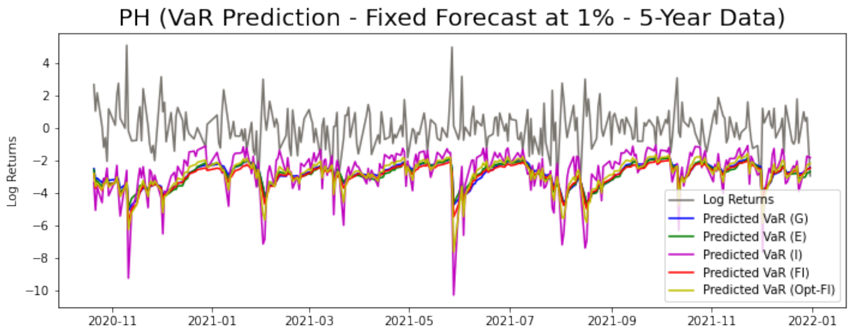

Figure 4.19: PH Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 1% LoS for a 5-year Time Frame

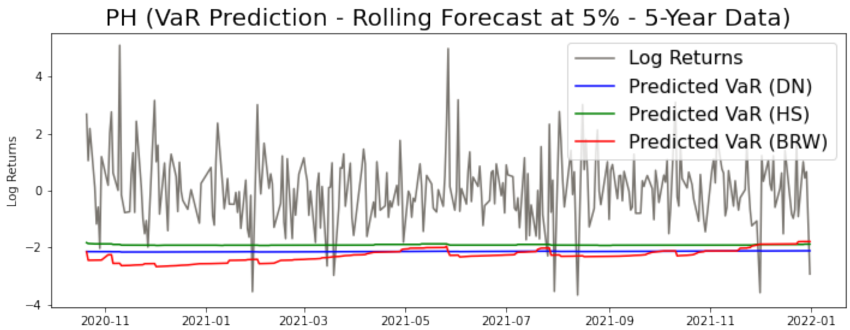

Figure 4.20: PH Log Returns and Estimated VaR using the standard models on a rolling window with 5% LoS for a 5-year Time Frame

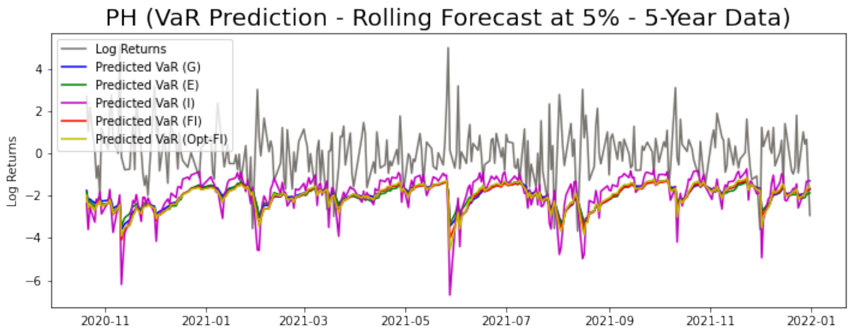

Figure 4.21: PH Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 5% LoS for a 5-year Time Frame

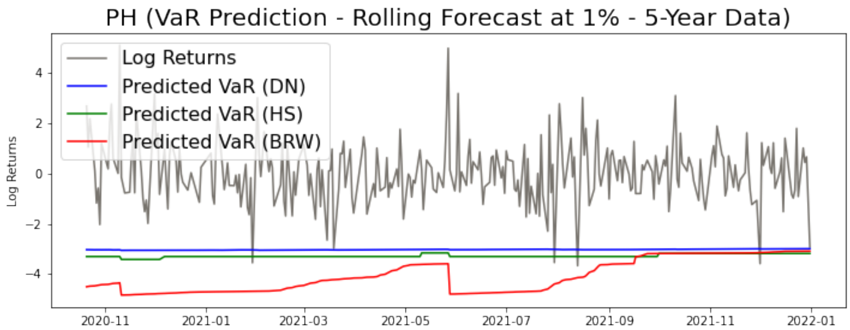

Figure 4.22: PH Log Returns and Estimated VaR using the standard models on a rolling window with 1% LoS for a 5-year Time Frame

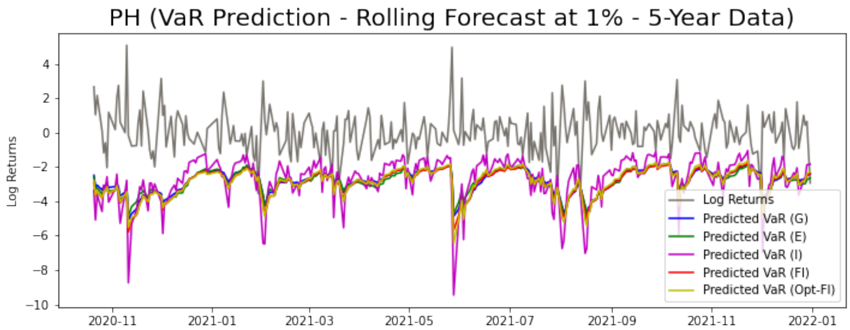

Figure 4.23: PH Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 1% LoS for a 5-year Time Frame

the fixed window forecast. The spikes in log returns still look stable and the same conclusions can be made for the BRW Approach at the 1% LoS and for the IGARCH model for both levels of significance.

We then proceed to test the data using the VaR violations, Kupiec test, Christoffersen test, and Diebold-Mariano test.

Backtesting Methods

VaR Violations

The following tables summarize the results from all the graphs above for both the fixed and rolling window for the 5-year data time frame. The tables will contain the number of VaR violations that have occurred using said model under the chosen dataset. A similar method is used as in the previous section on tabulating the VaR violations using confidence intervals. The confidence intervals – obtained using a binomial distribution – of Philippines, Malaysia, Singapore, Thailand, and Vietnam are (8,22), (8,23), (8,23), (8,22), (8,23) for the 5% LoS and [0, 8) for all countries for the 1% LoS, respectively.

Fixed Window - 5-Year Time Frame								
	VaR Violations with 5% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	8	12	8	13	12	25	11	15
ˆKLSE	17	21	9	12	12	28	13	15
STI	6	11	8	20	18	22	16	19
SETi	8	16	9	14	15	14	14	12
HNX	18	21	15	19	19	19	18	18

Table 4.17: Number of VaR violations under different models on a fixed window with 5% LoS for a 5-year Time Frame

Rolling Window - 5-Year Time Frame								
	VaR Violations with 5% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	8	10	8	13	14	24	12	15
ˆKLSE	18	21	9	13	13	29	14	15
STI	6	8	8	20	18	21	18	17
SETi	8	17	9	14	15	14	14	13
HNX	20	21	15	19	19	19	18	18

Table 4.18: Number of VaR violations under different models on a rolling window with 5% LoS for a 5-year Time Frame

Fixed Window - 5-Year Time Frame								
	VaR Violations with 1% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	4	4	1	7	7	12	7	7
ˆKLSE	4	4	2	5	6	11	5	7
STI	2	2	1	3	4	9	3	3
SETi	1	1	1	6	6	7	6	8
HNX	12	8	4	9	8	11	8	7

Table 4.19: Number of VaR violations under different models on a fixed window with 1% LoS for a 5-year Time Frame

Under the 5% LoS, we can easily observe that most of the VaR violations are within the confidence intervals of each stock index. The DN and BRW approaches however overestimated the stock indices of Philippines and Singapore, while the HS approach only overestimated Singapore. In comparison

Rolling Window - 5-Year Time Frame								
	VaR Violations with 1% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	4	4	1	7	7	10	7	7
ˆKLSE	4	4	2	5	6	12	5	7
STI	2	1	1	3	3	8	3	4
SETi	1	1	1	6	6	6	6	10
HNX	13	6	4	9	9	10	8	7

Table 4.20: Number of VaR violations under different models on a rolling window with 1% LoS for a 5-year Time Frame

to the 10-year data analysis, the IGARCH model now underestimated lesser stock indices, which means that it captures the log returns better for the three other indices. It is also worth noting that for Vietnam, all approaches and models capture the log returns accurately.

As for the 1% LoS, most of the models could accurately capture the log returns of the 5-year data as most of the VaR violations are within the confidence interval. Although the IGARCH model fails to capture most of the stock indices, most of the standard and GARCH-type models perform well. It is also worth mentioning that for the fixed window, only the BRW approach and the Opt-FIGARCH could accurately capture the VaR estimates of Vietnam. This is in contrast to the rolling window as the HS approach and the FIGARCH model perform well. As for Thailand, almost all models perform well except for the Opt-FIGARCH model.

We now turn to the Kupiec test to verify these results.

Kupiec Test

We perform the similar methods used in the previous sections for the Kupiec test for the 5-year time frame.

Fixed Window - 5-Year Time Frame								
	Kupiec Test with 5% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	3.7776	0.5363	3.7776	0.203	0.5363	6.4115	1.0437	0.0087
ˆKLSE	0.2292	2.1336	3.0565	0.7401	0.7401	9.2778	0.3367	0.0016
STI	7.4104	1.2887	4.2102	1.5604	0.5743	2.9755	0.0619	1.0111
SETi	3.7776	1.273	2.644	0.0263	0.0114	0.0263	0.0263	0.5175
HNX	0.5743	2.2167	0.0002	1.0111	1.0111	1.0111	0.5743	0.5743

Table 4.21: Kupiec Test's test statistic under different models on a fixed window with 5% LoS for a 5-year Time Frame

Rolling Window - 5-Year Time Frame								
	Kupiec Test with 5% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	3.7776	1.7401	3.7776	0.203	0.0308	5.3111	0.5363	0.0087
ˆKLSE	0.5337	2.1336	3.0565	0.3367	0.3367	10.6365	0.0942	0.0016
STI	7.4104	4.2102	4.2102	1.5604	0.5743	2.2167	0.5743	0.2557
SETi	3.7776	0.3782	2.644	0.0263	0.0114	0.0263	0.0263	0.1913
HNX	1.5604	2.2167	0.0002	1.0111	1.0111	1.0111	0.5743	0.5743

Table 4.22: Kupiec Test's test statistic under different models on a rolling window with 5% LoS for a 5-year Time Frame

From the Kupiec test for the 5% LoS, we see that most of the models fail to reject the null hypothesis, which means that the VaR violations are within the LoS. We also note that the results for the standard approaches on Singapore rejected the null hypothesis, which is consistent with the fact that the VaR violations exceed the confidence interval, and thus it underestimates the data. There is also not much difference between the two forecast windows.

Fixed Window - 5-Year Time Frame								
	Kupiec Test with 1% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	0.3543	0.3543	1.7228	4.1101	4.1101	15.9843	4.1101	4.1101
ˆKLSE	0.285	0.285	0.4019	1.0817	2.2879	12.639	1.0817	3.8356
STI	0.395	0.395	1.8431	0	0.2981	7.8563	0	0
SETi	1.7228	1.7228	1.7228	2.515	2.515	4.1384	2.515	6.0555
HNX	15.4852	5.7441	0.2981	7.8563	5.7441	12.7472	5.7441	3.8892

Table 4.23: Kupiec Test's test statistic under different models on a fixed window with 1% LoS for a 5-year Time Frame

Rolling Window - 5-Year Time Frame								
	Kupiec Test with 1% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	0.3543	0.3543	1.7228	4.1101	4.1101	10.5854	4.1101	4.1101
ˆKLSE	0.285	0.285	0.4019	1.0817	2.2879	15.3632	1.0817	3.8356
STI	0.395	0.395	1.8431	0	0	5.7441	0	0.2981
SETi	1.7228	1.7228	1.7228	2.515	2.515	2.515	2.515	10.6349
HNX	18.397	2.3279	0.2981	7.8563	7.8563	10.1982	5.7441	3.8892

Table 4.24: Kupiec Test's test statistic under different models on a rolling window with 1% LoS for a 5-year Time Frame

Meanwhile, for the Kupiec test for the 1% LoS, most of the models for all countries capture the VaR violations within the LoS. For Thailand, the Opt-FIGARCH also rejects the null hypothesis, which means that the VaR violations are not within the range of the 1% confidence level. For Vietnam, it backs up the analysis that only the HS, BRW, Opt-FIGARCH and FIGARCH capture the VaR violations accurately with respect to the LoS. Interestingly, for Singapore, the Kupiec test concludes that all models accurately accounted for all the VaR violations within the LoS.

We now proceed to further verify these results using the Christoffersen test.

Christoffersen Test

We perform the similar methods used in the previous sections for the Christoffersen test for the 5-year time frame.

Fixed Window - 5-Year Time Frame								
	Christoffersen Test with 5% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	4.2269	1.5617	4.2269	1.4107	1.5617	7.3619	1.9022	1.6284
ˆKLSE	2.2585	2.3192	3.6095	1.7337	1.7335	9.4557	1.5066	1.5702
STI	7.6545	5.0278	10.5038	3.4101	3.3313	6.4612	0.0887	1.0524
SETi	4.2284	1.9834	3.2166	0.147	4.7248	0.147	0.147	1.5501
HNX	3.3313	8.8071	4.5878	3.2868	3.2868	1.5248	3.3313	3.3313

Table 4.25: Christoffersen Test's test statistic under different models on a fixed window with 5% LoS for a 5-year Time Frame

Rolling Window - 5-Year Time Frame								
	Christoffersen Test with 5% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	4.2269	2.4471	4.2269	1.4107	1.4366	6.0655	1.5617	1.6284
ˆKLSE	2.8169	5.2753	3.6094	1.5066	1.5066	10.6564	1.4558	1.5702
STI	7.6545	10.5038	10.5038	3.4101	3.3313	5.381	3.3313	0.2573
SETi	4.2284	2.4813	3.2166	0.147	4.7248	0.147	0.147	2.6442
HNX	5.8441	8.8071	4.5878	3.2868	3.2868	1.5248	3.3313	3.3313

Table 4.26: Christoffersen Test's test statistic under different models on a rolling window with 5% LoS for a 5-year Time Frame

Under the 5% LoS, for the Christoffersen test, similar analysis could be done with the Kupiec test. In addition to the standard models unable to capture the log returns for Singapore, the VaR violations are also not independent with one another, thus making it insufficient to estimate VaR.

Additionally, for the 1% LoS, most of the models capture the VaR violations, accounting its independence with the other violations, with the

Fixed Window - 5-Year Time Frame								
	Christoffersen Test with 1% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	0.4654	0.4654	1.7297	4.4529	4.4529	17.0097	4.4529	4.4529
ˆKLSE	0.3924	0.3924	0.4285	1.2501	2.5311	13.4708	1.2501	4.1679
STI	0.4218	0.4218	1.8498	0.0606	0.4062	8.4131	0.0606	0.0606
SETi	1.7297	1.7297	1.7297	2.7677	2.7677	4.4835	2.7677	6.5079
HNX	22.6566	12.0251	0.4062	8.4131	7.4273	13.5848	6.1825	4.2237

Table 4.27: Christoffersen Test's test statistic under different models on a fixed window with 1% LoS for a 5-year Time Frame

Rolling Window - 5-Year Time Frame								
	Christoffersen Test with 1% LoS							
Index	DN	HS	BRW	GARCH	EGARCH	IGARCH	FIGARCH	Opt-FIGARCH
PSEi	0.4654	0.4654	1.7297	4.4529	4.4529	11.2924	4.4529	4.4529
ˆKLSE	0.3924	0.3924	0.4285	1.2501	2.5311	16.3566	1.2501	4.1679
STI	0.4218	1.8498	1.8498	0.0606	0.0606	6.1825	0.0606	0.4062
SETi	1.7297	1.7297	1.7297	2.7677	2.7677	2.7677	2.7677	14.9813
HNX	24.6043	5.0888	0.4062	8.4131	13.1529	10.888	6.1825	4.2237

Table 4.28: Christoffersen Test's test statistic under different models on a rolling window with 1% LoS for a 5-year Time Frame

exception of the IGARCH model. For Singapore, since the VaR violations are independent with one another, we can say that all models, except the IGARCH model from the VaR violations table, are sufficient in terms of estimating VaR. This is the same case with the Philippines, Malaysia, and Thailand. As for Vietnam, the BRW approach, GARCH, FIGARCH, and Opt-FIGARCH is seen to be sufficient enough to estimate VaR at 1% LoS for a 5-year dataset.

Overall, most of the models captured the VaR violations, accounting its independence with the other violations, with the exception of the IGARCH model. Additionally, based on both forecast windows, the BRW approach and the FIGARCH model performed best in a 5-year dataset for the 1% LoS.

Diebold-Mariano Test

PSEi - Fixed Window (5-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	3.1627	2.7083	0.1912	0.4301	0.3033
	1.56E-03***	6.76E-03***	8.48E-01*	6.67E-01*	7.62E-01*
HS	4.7595	4.3816	0.7571	1.9234	1.2659
	1.9412E-06	1.1780E-05	4.49E-01*	5.44E-02*	2.06E-01*
BRW	18.927	18.905	9.0977	17.6147	13.7425
	6.8320E-80	1.0377E-79	9.2238E-20	1.8998E-69	5.6456E-43
GARCH		2.4421	1.1531	10.3579	2.8344
		1.46E-02**	2.49E-01*	3.8545E-25	4.59E-03***
EGARCH			0.8945	6.4394	2.1467
			3.71E-01*	1.1994E-10	3.18E-02**
IGARCH				0.0398	0.0265
				9.68E-01*	9.79E-01*
FIGARCH					0.0613
					9.51E-01*

Table 4.29: PSEi DM test statistics and p-values on a 5-Year Fixed Window (1.00%).

PSEi - Fixed Window (5-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	2.6828	2.2713	0.0116	0.1707	0.0466
	7.30E-03***	2.31E-02**	9.91E-01*	8.64E-01*	9.63E-01*
HS	5.8131	6.4405	3.0147	7.4843	4.9863
	6.1322E-09	1.1904E-10	2.57E-03***	7.1919E-14	6.1558E-07
BRW	6.4653	6.1898	1.3464	3.8267	2.3056
	1.0110E-10	6.0252E-10	1.78E-01*	1.30E-04***	2.11E-02**
GARCH		2.1989	1.1968	9.4124	2.8047
		2.79E-02**	2.31E-01*	4.8493E-21	5.04E-03***
EGARCH			0.9518	6.0246	2.1564
			3.41E-01*	1.6951E-09	3.11E-02**
IGARCH				0.073	0.0335
				9.42E-01*	9.73E-01*
FIGARCH					0.1388
					8.90E-01*

Table 4.30: PSEi DM test statistics and p-values on a 5-Year Fixed Window (5.00%).

PSEi - Rolling Window (5-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	2.9009	2.5226	0.0674	0.1173	0.077
	3.72E-03***	1.16E-02**	9.46E-01*	9.07E-01*	9.39E-01*
HS	8.1074	8.1391	2.2932	4.7685	4.2078
	5.1702E-16	3.9838E-16	2.18E-02**	1.8562E-06	2.5788E-05
BRW	18.8664	19.061	10.1384	16.8365	15.8177
	2.1567E-79	5.3293E-81	3.7333E-24	1.3183E-63	2.3481E-56
GARCH		2.2472	1.5221	10.3655	6.1811
		2.46E-02**	1.28E-01*	3.5585E-25	6.3644E-10
EGARCH			1.2116	6.1245	4.3195
			2.26E-01*	9.0977E-10	1.5640E-05
IGARCH				0.0184	0.046
				9.85E-01*	9.63E-01*
FIGARCH					0.1541
					8.78E-01*

Table 4.31: PSEi DM test statistics and p-values on a 5-Year Rolling Window (1.00%).

PSEi - Rolling Window (5-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	2.43	2.0873	0.1201	0.1185	0.1717
	1.51E-02**	3.69E-02**	9.04E-01*	9.06E-01*	8.64E-01*
HS	3.6155	4.2994	2.5864	5.2364	4.7032
	3.00E-04***	1.7124E-05	9.70E-03***	1.6378E-07	2.5617E-06
BRW	6.5322	6.4276	1.5941	3.5576	3.0483
	6.4807E-11	1.2959E-10	1.11E-01*	3.74E-04***	2.30E-03***
GARCH		2.0066	1.559	9.6742	5.9154
		4.48E-02**	1.19E-01*	3.8821E-22	3.3097E-09
EGARCH			1.264	5.8298	4.1776
			2.06E-01*	5.5493E-09	2.9460E-05
IGARCH				0.1004	0.049
				9.20E-01*	9.61E-01*
FIGARCH					0.3843
					7.01E-01*

Table 4.32: PSEi DM test statistics and p-values on a 5-Year Rolling Window (5.00%).

With a shorter time frame of 5 years, the Diebold-Mariano test shows more of instances where the predictive accuracy of two compared models' forecasted VaR values are similar. This 5 year span, however, still showcases some models' forecasts that are significantly different when compared to the GARCH-type models. Among these models are the HS approach followed by the DN approach, which, among the five indices, still had the most amount of instances where its forecast is significantly different when compared to the GARCH-type models.

As an overall conclusion, lessening the time frame makes the other models perform better, especially for the standard approaches which are mostly parametric in nature. For the GARCH-type models, the IGARCH model performs better in estimating VaR for Thailand compared to the larger dataset we used in Chapter 3. Out of all the approaches and models, the BRW Approach and FIGARCH perform best, especially at the 1% LoS. All other GARCH-type models still perform well for other countries, even after changing the LoS and the time frame.

Chapter 5

Conclusion and Recommendations

This chapter gives the overall conclusions and key insights as well as the recommendations for future extensions of the study. Returning back to the main focus of the paper, objectively quantifying risk is considered to be a challenge and applying multiple models may be tedious and ultimately create more confusion.

5.1 Conclusion

The use of the standard models has been seen as subpar in comparison with the GARCH-type models as evidenced by the results found in Chapter 3. Thus, the objective of the paper was to quantify the standard models (DN, HS, and BRW approach) and the GARCH-type models (GARCH, EGARCH, IGARCH, and FIGARCH) by applying backtesting methods such as the VaR violations, Kupiec test, and Christoffersen test to check the sufficiency and the independence of the occurrence of the violations and comparing the forecasted VaR values of each model with one another using the Diebold-Mariano test.

A conclusion that can be drawn is that the EGARCH has shown to be the most sufficient model for a 15-year time frame while also having the most instances where the occurrences VaR violations are independent. The standard models are also shown to be effective alternative to the GARCH-type models as shown in the Diebold-Mariano test above. The standard models are also shown to be an effective alternative to the GARCH-type models as shown in the Diebold-Mariano test above under the following LoS:

- 5.00%: DN Approach
- 1.00%: BRW Approach
- 0.01%: DN Approach

In summary, the study had:

1. Verified that the log returns of the five ASEAN stock indices, or the data, have constant means using the *t*-test.
2. Verified that the data exhibit serial correlations, a criterion for GARCH-type models.
3. Verified that the data is not normally distributed using the Shapiro-Wilk test.
4. Verified that the data used observes ARCH effects using the Lagrange Multiplier test.
5. Visualized the log returns with the VaR estimates of each model for each of the stock indices.
6. Compared the eight different approaches and models to estimate VaR, namely DN, HS, BRW, GARCH, EGARCH, IGARCH, FIGARCH, an Opt-FIGARCH, using backtesting methods by performing the VaR Violations, Kupiec, Christoffersen, and Diebold-Mariano tests.
7. Identified which models are effective to use for each stock index.

- Under the VaR violation method, the HS approach is the most sufficient model with the FIGARCH model coming in second.
- Under the Kupiec test, the HS approach is shown to be the most sufficient model followed by the FIGARCH model.
- The EGARCH model is shown to be the best model followed by the FIGARCH model being the second best model.

8. Performed sensitivity analyses on level of significance and data time frame.

    - Results show that the use of the 0.01% LoS on the chosen approaches and models to estimate VaR underestimate the data of the five ASEAN stock indices.
    - Under the 5% and 1% LoS, the BRW approach and FIGARCH model perform well in accurately estimating VaR in a five-year data time frame.

5.2 Recommendations

1. Given the large number of data points available to the researchers, the effects of applying different distributions, more specifically the Leptokurtic distributions, may have an effect on a smaller sample dataset and may show the responsiveness of the GARCH-type models.

2. Based on the findings from Chapter 3, the FIGARCH model has shown to be the most consistent model in estimating VaR as it has shown to be both sufficient in capturing the data while showing the indepen-

dence of the VaR violations from one another. Further research by comparing the FIGARCH model with the model proposed by Omari et al., the GARCH-EVT, would also provide insights to the FIGARCH's efficiency.

3. The BRW parameter of 0.992 may not have the optimal parameter in the case of the countries used. Thus, a further study on adjusting the BRW parameter may be done.

4. From the results in Chapter 3 and the sensitivity analysis on the level of significance in Chapter 4, the 0.01% LoS was found to be too strict for all the eight models used. Thus, this level of significance may be too small to use for all models across the five ASEAN stock indices.

5. The use of the Harvey, Leybourne and Newbold (HLN) test, which is a modification of the DM test, is recommended for small-sample data since the HLN test uses an approximately unbiased estimate of the variance of the mean loss differential, which then addresses the instance of small-sample data to frequently reject the null hypothesis under the DM test.

6. Changing the time frame, specifically a five-year time frame, of the data made the other approaches and models to perform better under the 1% LoS. Thus, it will be beneficial to use a shorter time frame as this costs less computing time and more accurate VaR estimation.

7. Since there is no major difference in terms of the forecast windows in changing the forecast window to a five-year time frame, we can just use

the fixed forecast window to estimate VaR.

8. Further studies on changing the other parameters of the models, such as the order of the GARCH-type models, could be done.

Chapter 6

Appendix

6.1 Log Returns and Estimated VaR of the Chosen Countries [20% Predictions]

6.1.1 Standard Models and GARCH, EGARCH, and FIGARCH Models

Malaysia

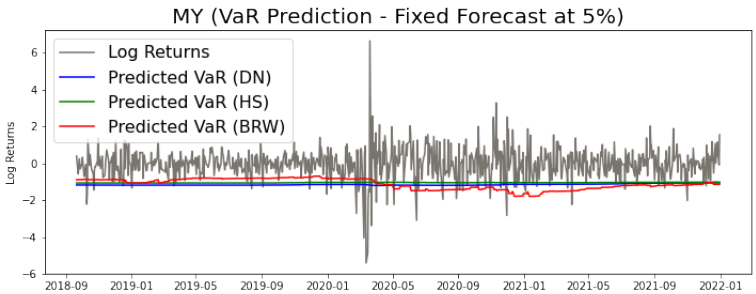

Figure 6.1: MY Log Returns and Estimated VaR using the standard models with 5% LoS

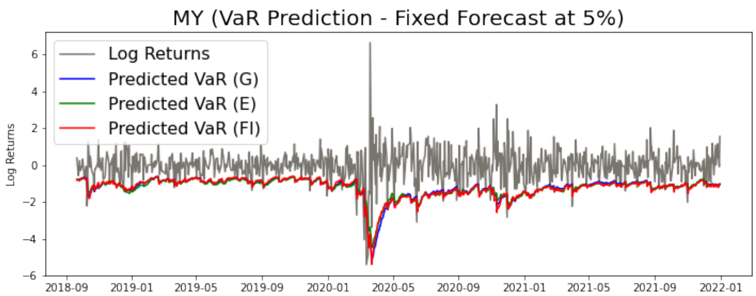

Figure 6.2: MY Log Returns and Estimated VaR using the standard models with 5% LoS

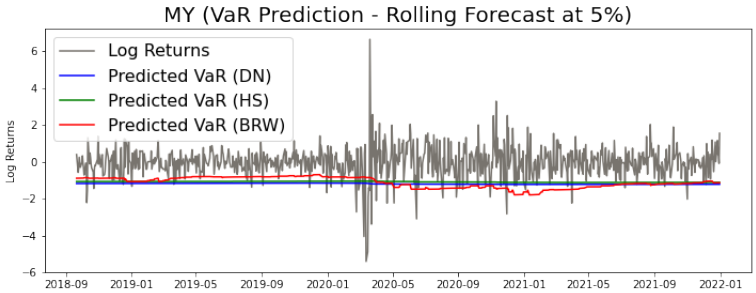

Figure 6.3: MY Log Returns and Estimated VaR using the standard models with 5% LoS

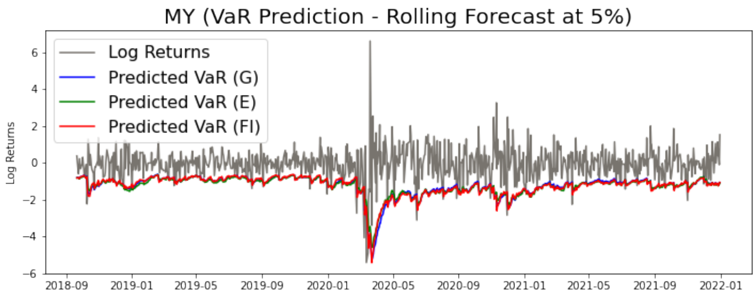

Figure 6.4: MY Log Returns and Estimated VaR using the standard models with 5% LoS

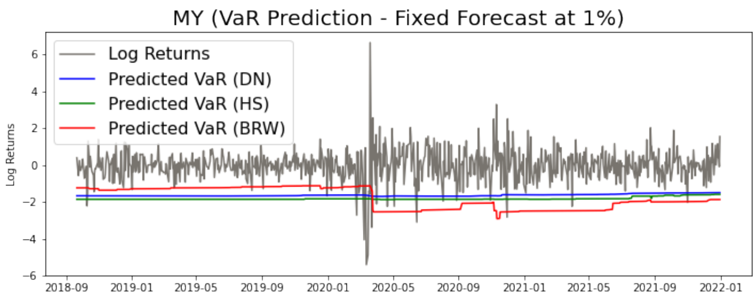

Figure 6.5: MY Log Returns and Estimated VaR using the standard models with 1% LoS

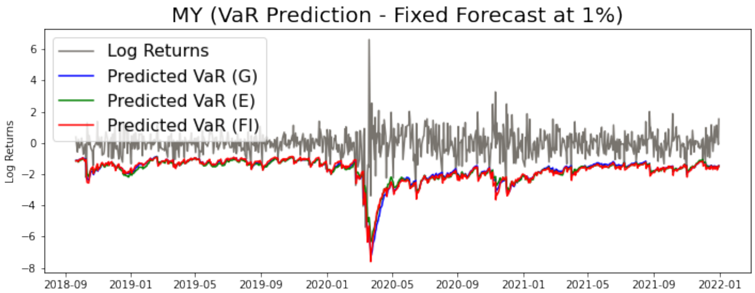

Figure 6.6: MY Log Returns and Estimated VaR using the standard models with 1% LoS

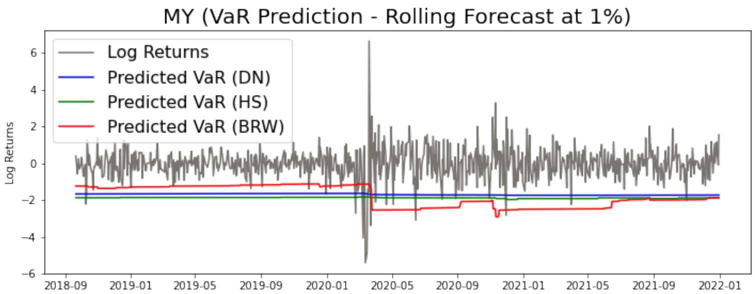

Figure 6.7: MY Log Returns and Estimated VaR using the standard models with 1% LoS

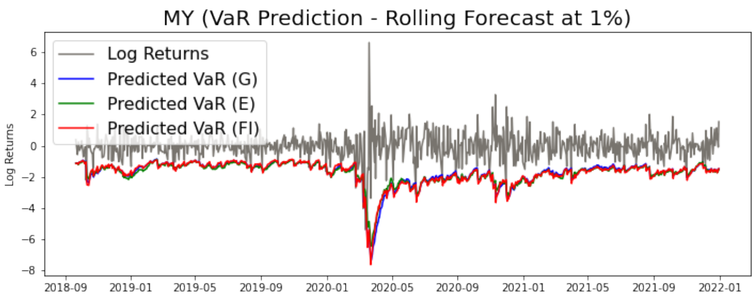

Figure 6.8: MY Log Returns and Estimated VaR using the standard models with 1% LoS

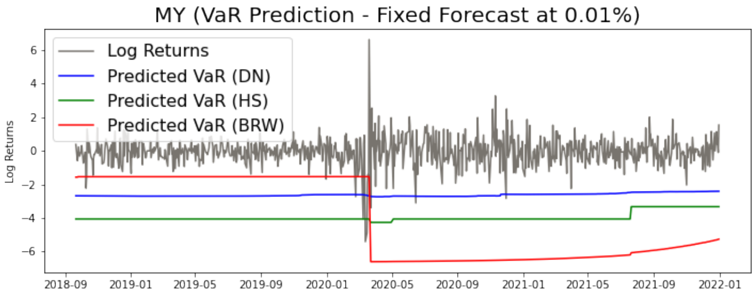

Figure 6.9: MY Log Returns and Estimated VaR using the standard models with 0.01% LoS

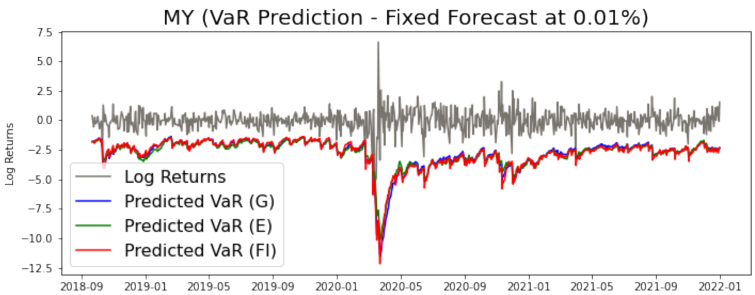

Figure 6.10: MY Log Returns and Estimated VaR using the standard models with 0.01% LoS

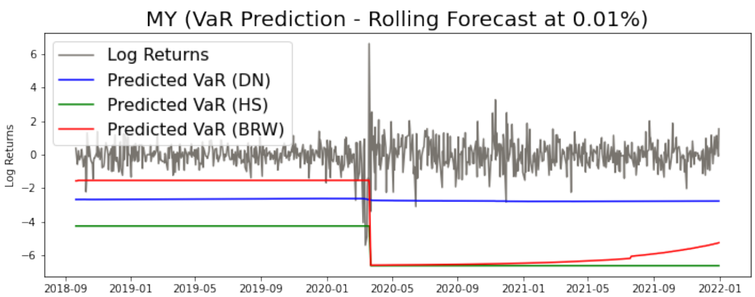

Figure 6.11: MY Log Returns and Estimated VaR using the standard models with 0.01% LoS

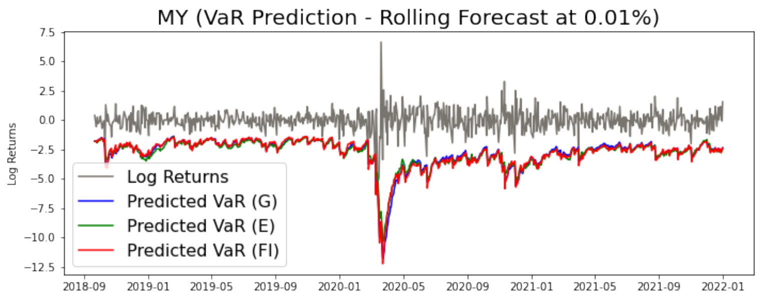

Figure 6.12: MY Log Returns and Estimated VaR using the standard models with 0.01% LoS

Singapore

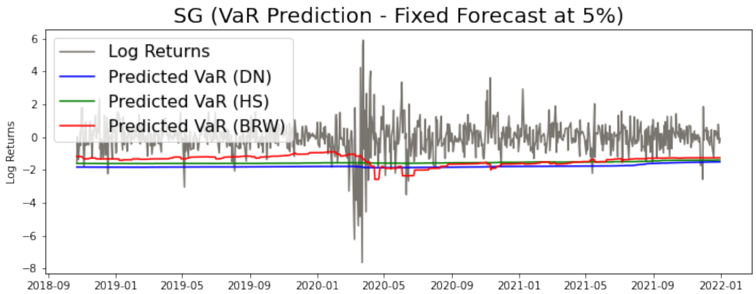

Figure 6.13: SG Log Returns and Estimated VaR using the standard models with 5% LoS

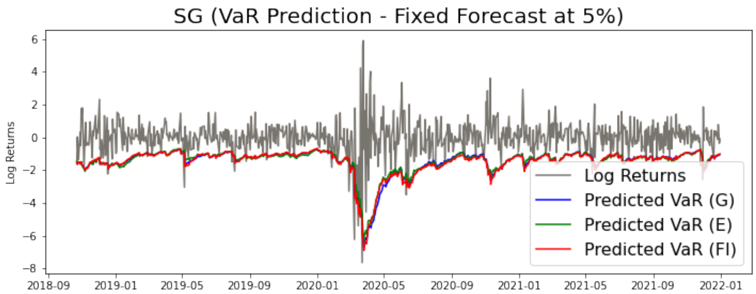

Figure 6.14: SG Log Returns and Estimated VaR using the standard models with 5% LoS

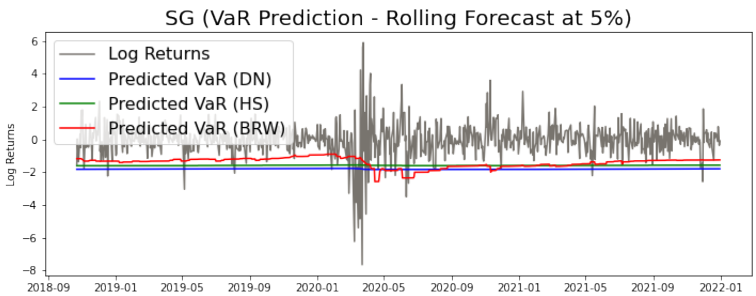

Figure 6.15: SG Log Returns and Estimated VaR using the standard models with 5% LoS

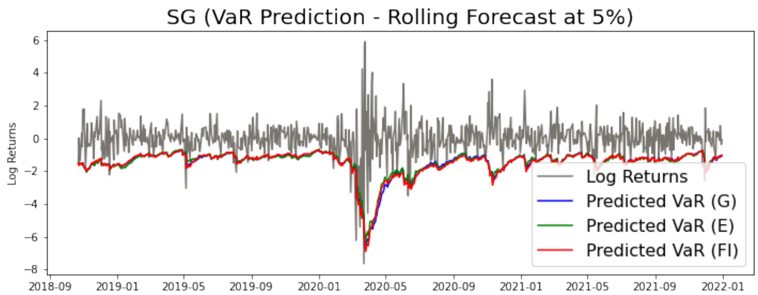

Figure 6.16: SG Log Returns and Estimated VaR using the standard models with 5% LoS

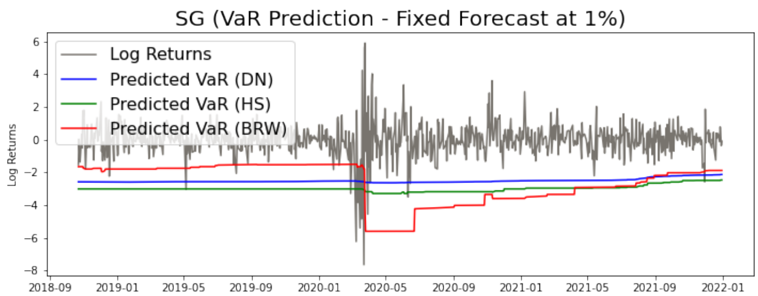

Figure 6.17: SG Log Returns and Estimated VaR using the standard models with 1% LoS

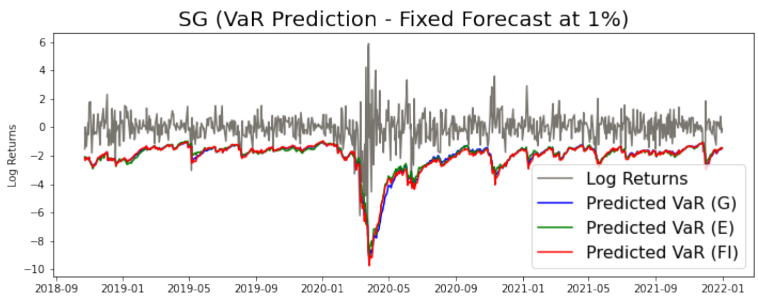

Figure 6.18: SG Log Returns and Estimated VaR using the standard models with 1% LoS

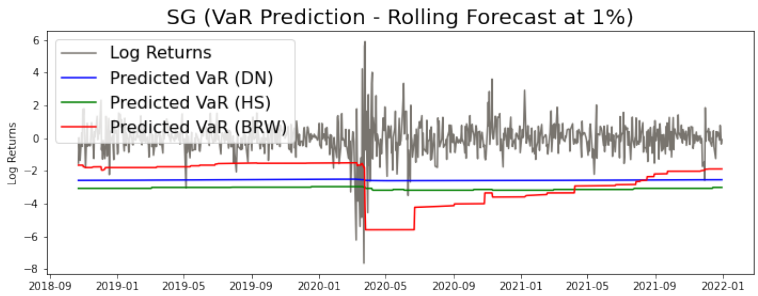

Figure 6.19: SG Log Returns and Estimated VaR using the standard models with 1% LoS

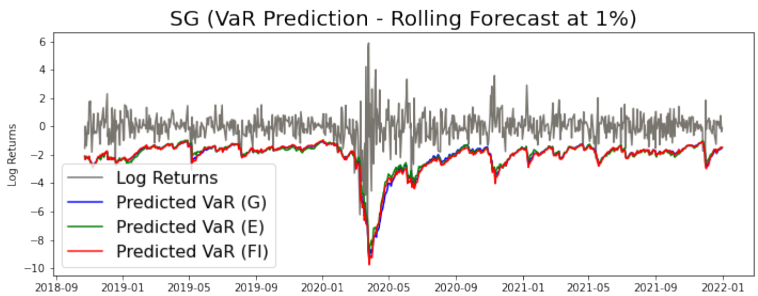

Figure 6.20: SG Log Returns and Estimated VaR using the standard models with 1% LoS

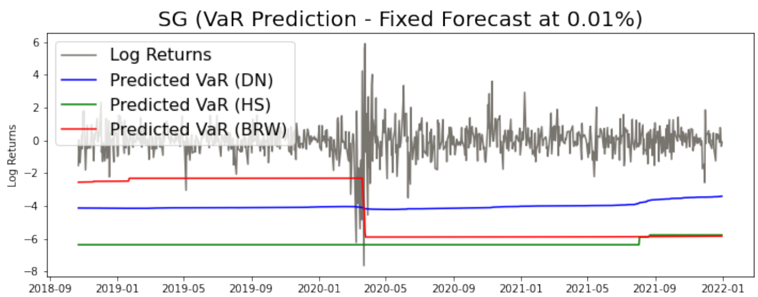

Figure 6.21: SG Log Returns and Estimated VaR using the standard models with 0.01% LoS

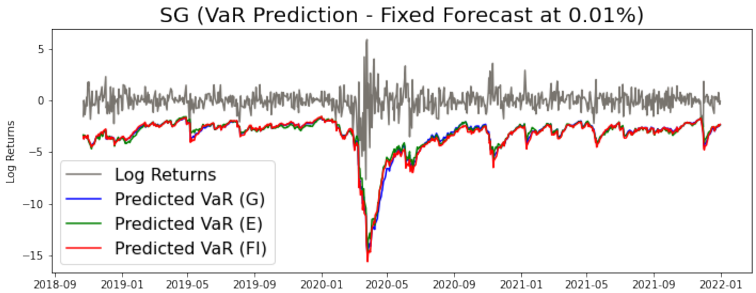

Figure 6.22: SG Log Returns and Estimated VaR using the standard models with 0.01% LoS

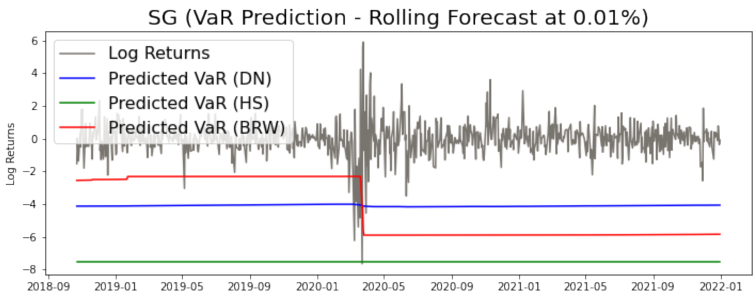

Figure 6.23: SG Log Returns and Estimated VaR using the standard models with 0.01% LoS

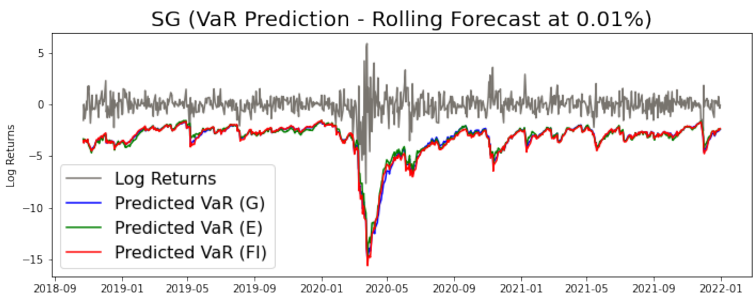

Figure 6.24: SG Log Returns and Estimated VaR using the standard models with 0.01% LoS

Thailand

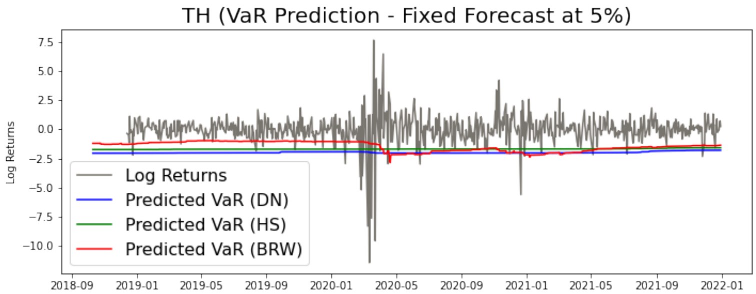

Figure 6.25: TH Log Returns and Estimated VaR using the standard models with 5% LoS

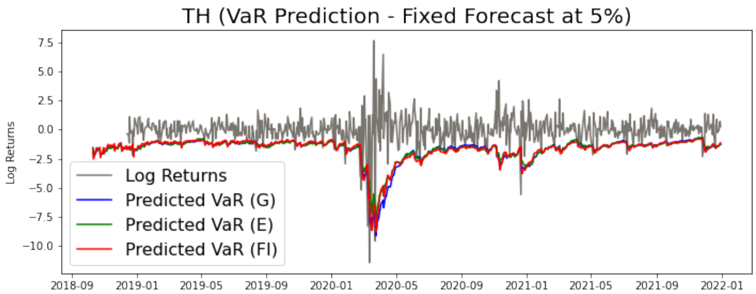

Figure 6.26: TH Log Returns and Estimated VaR using the standard models with 5% LoS

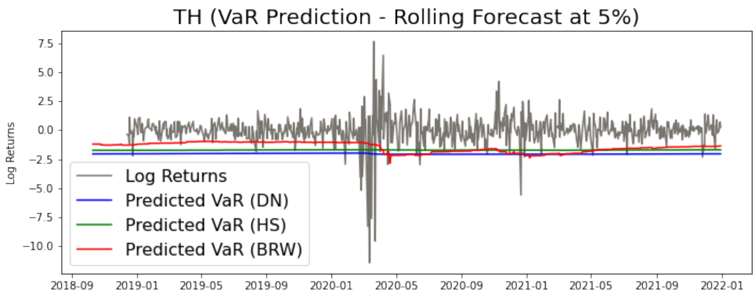

Figure 6.27: TH Log Returns and Estimated VaR using the standard models with 5% LoS

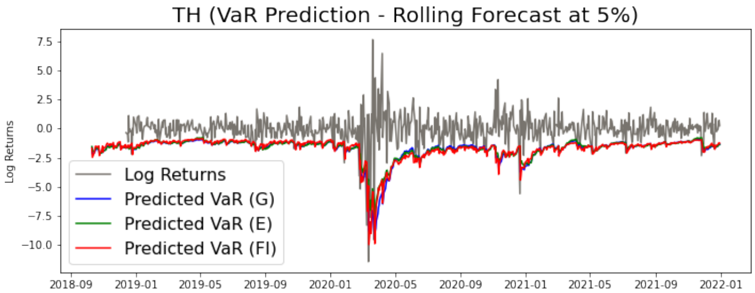

Figure 6.28: TH Log Returns and Estimated VaR using the standard models with 5% LoS

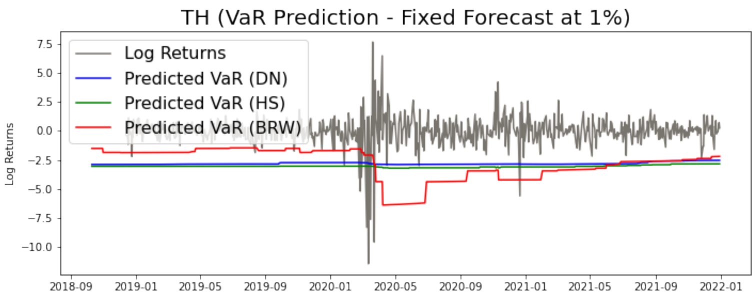

Figure 6.29: TH Log Returns and Estimated VaR using the standard models with 1% LoS

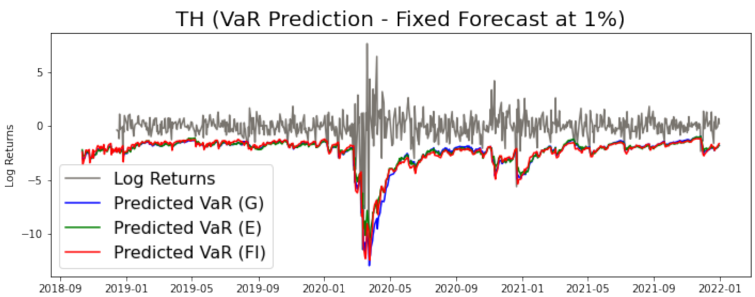

Figure 6.30: TH Log Returns and Estimated VaR using the standard models with 1% LoS

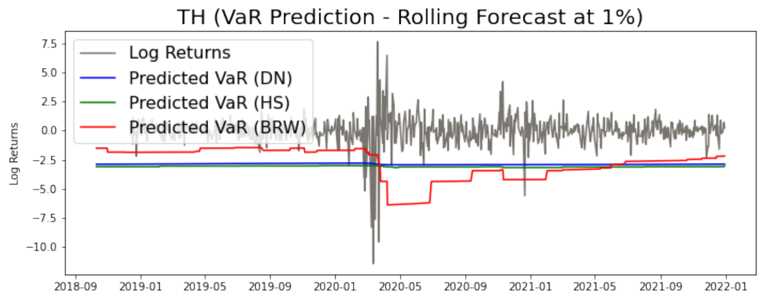

Figure 6.31: TH Log Returns and Estimated VaR using the standard models with 1% LoS

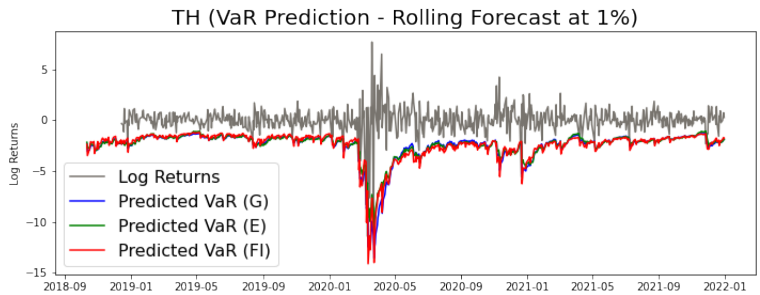

Figure 6.32: TH Log Returns and Estimated VaR using the standard models with 1% LoS

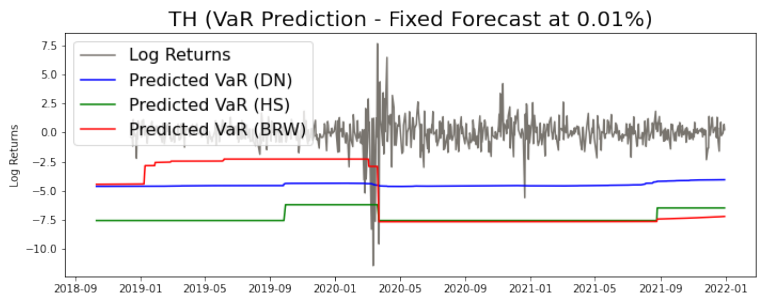

Figure 6.33: TH Log Returns and Estimated VaR using the standard models with 0.01% LoS

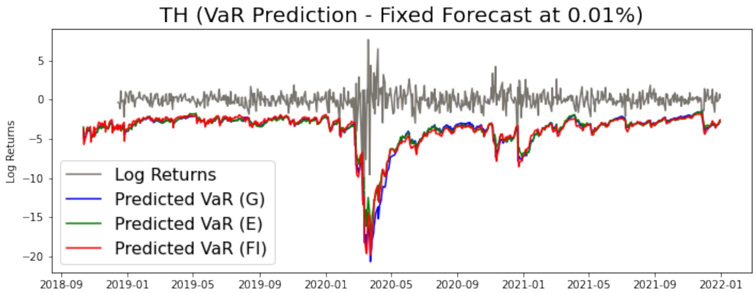

Figure 6.34: TH Log Returns and Estimated VaR using the standard models with 0.01% LoS

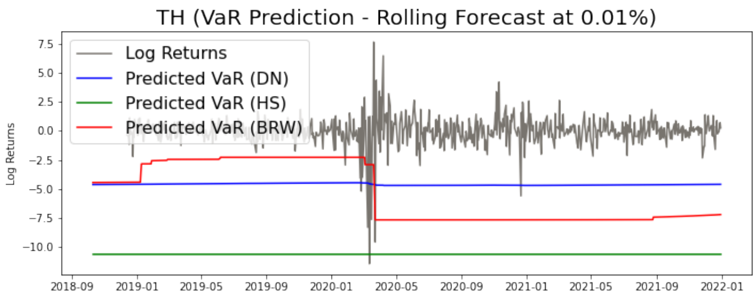

Figure 6.35: TH Log Returns and Estimated VaR using the standard models with 0.01% LoS

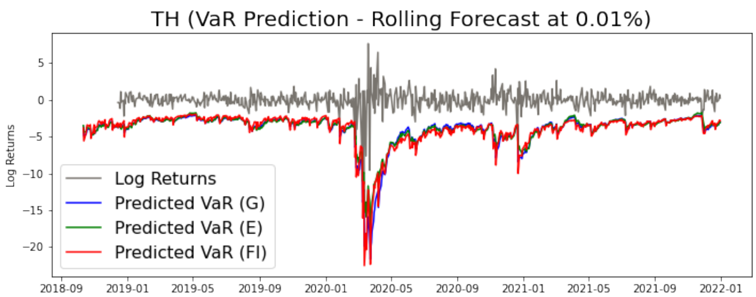

Figure 6.36: TH Log Returns and Estimated VaR using the standard models with 0.01% LoS

Vietnam

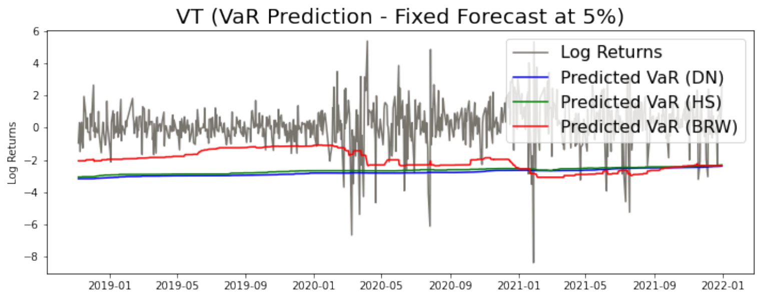

Figure 6.37: VT Log Returns and Estimated VaR using the standard models with 5% LoS

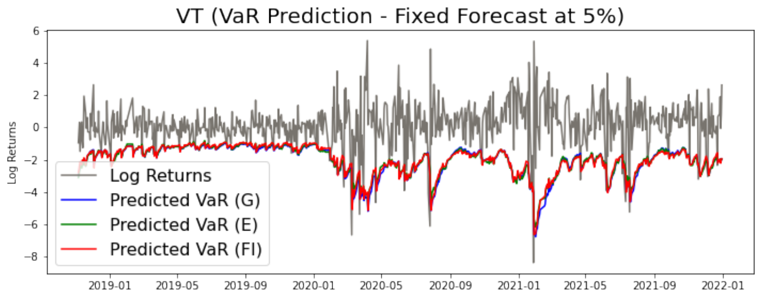

Figure 6.38: VT Log Returns and Estimated VaR using the standard models with 5% LoS

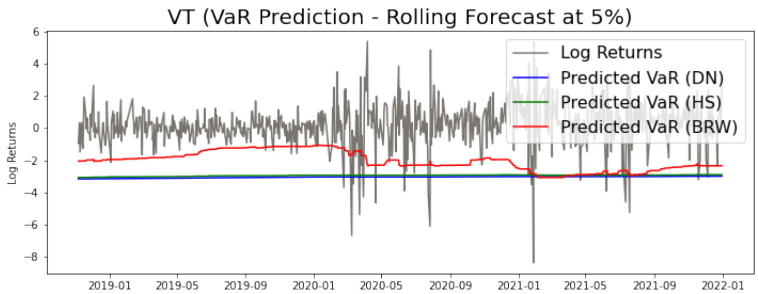

Figure 6.39: VT Log Returns and Estimated VaR using the standard models with 5% LoS

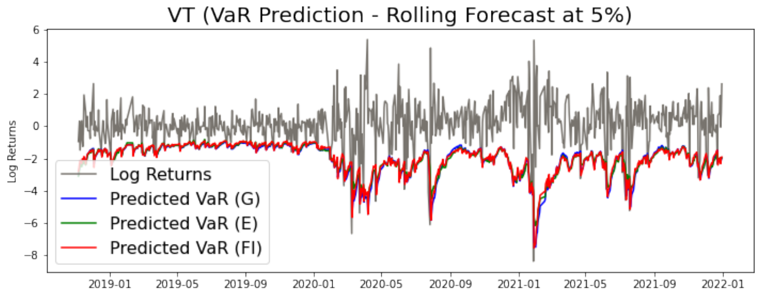

Figure 6.40: VT Log Returns and Estimated VaR using the standard models with 5% LoS

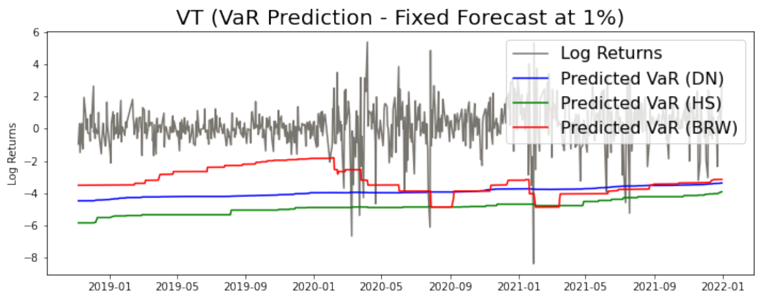

Figure 6.41: VT Log Returns and Estimated VaR using the standard models with 1% LoS

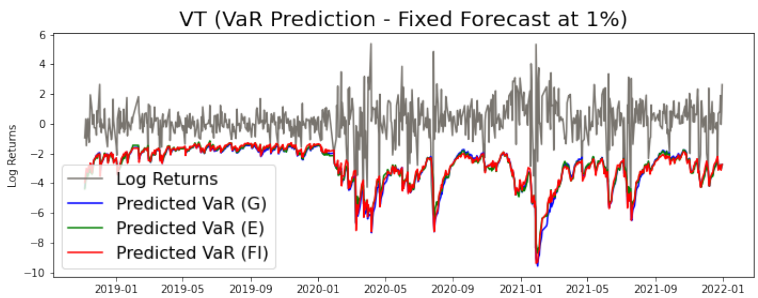

Figure 6.42: VT Log Returns and Estimated VaR using the standard models with 1% LoS

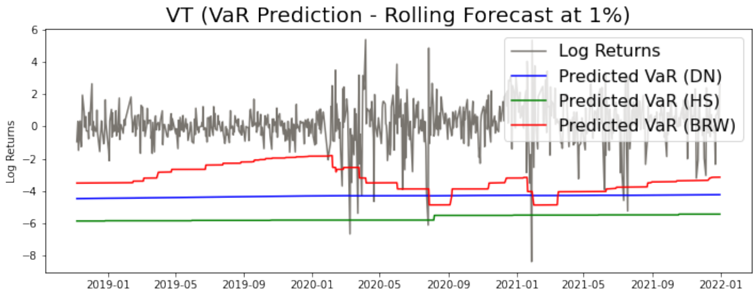

Figure 6.43: VT Log Returns and Estimated VaR using the standard models with 1% LoS

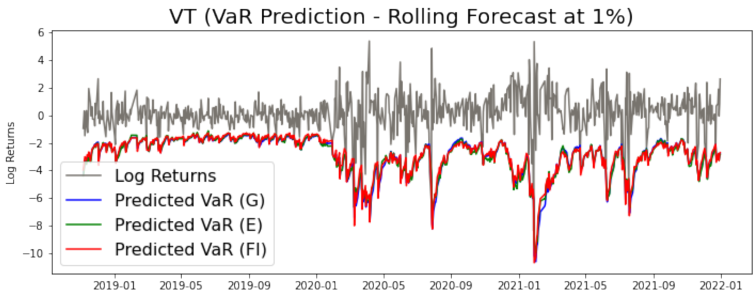

Figure 6.44: VT Log Returns and Estimated VaR using the standard models with 1% LoS

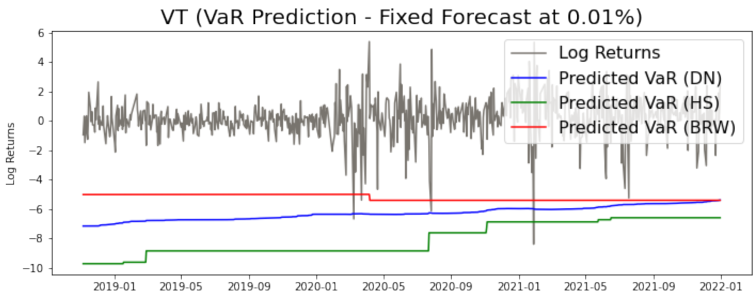

Figure 6.45: VT Log Returns and Estimated VaR using the standard models with 0.01% LoS

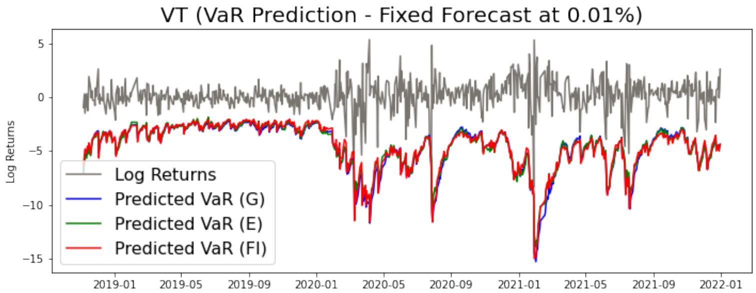

Figure 6.46: VT Log Returns and Estimated VaR using the standard models with 0.01% LoS

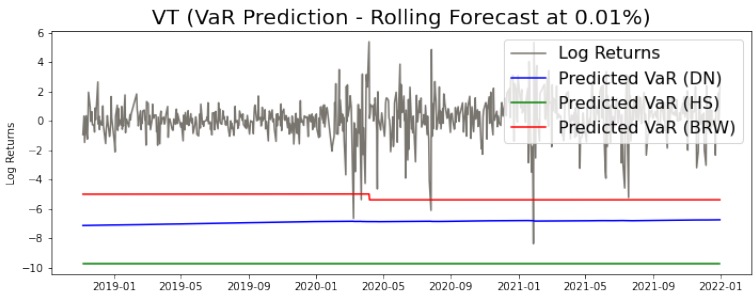

Figure 6.47: VT Log Returns and Estimated VaR using the standard models with 0.01% LoS

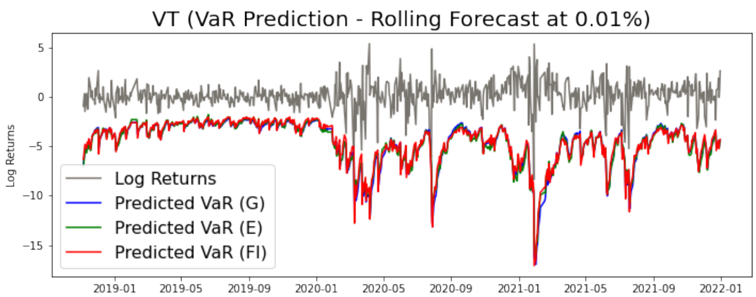

Figure 6.48: VT Log Returns and Estimated VaR using the standard models with 0.01% LoS

6.1.2 GARCH, IGARCH, FIGARCH, and Opt-FIGARCH Models

Malaysia

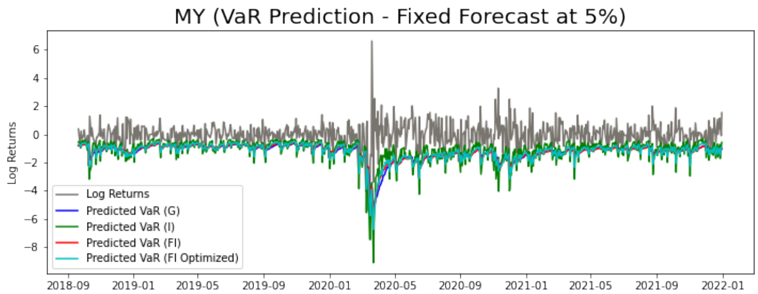

Figure 6.49: MY Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 5% LoS

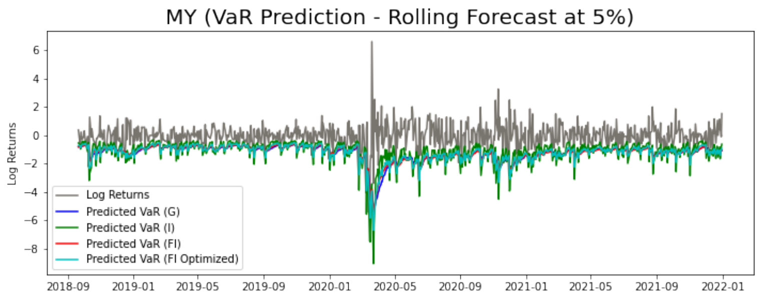

Figure 6.50: MY Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 5% LoS

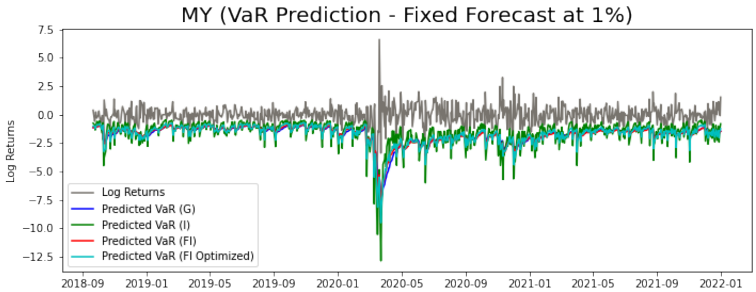

Figure 6.51: MY Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 1% LoS

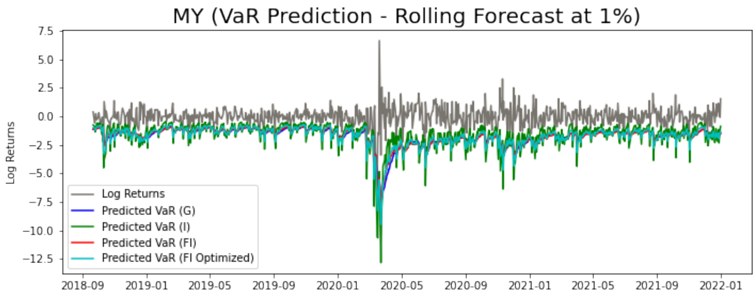

Figure 6.52: MY Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 1% LoS

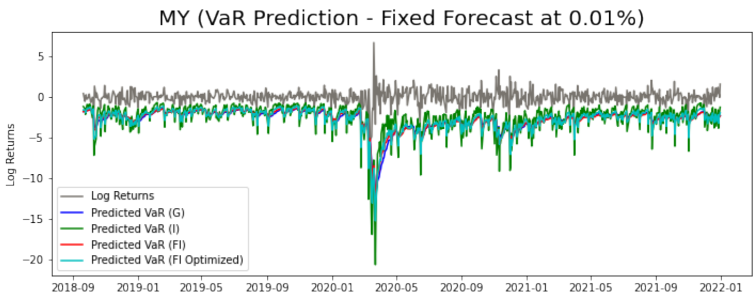

Figure 6.53: MY Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 0.01% LoS

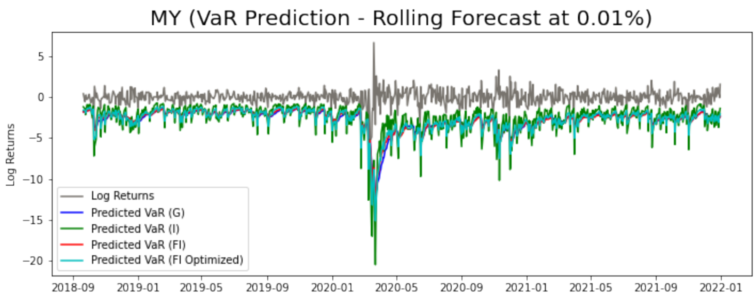

Figure 6.54: MY Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 0.01% LoS

Singapore

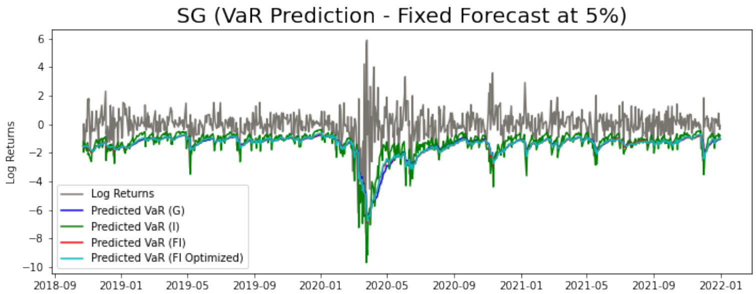

Figure 6.55: SG Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 5% LoS

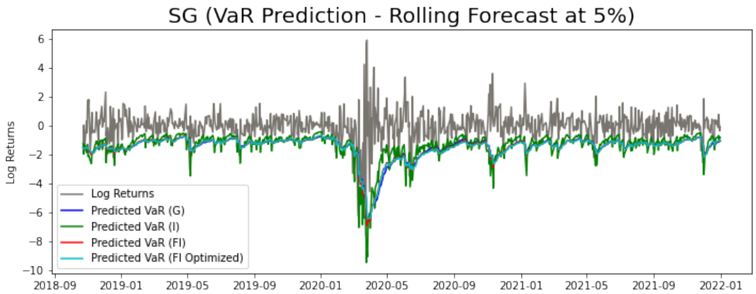

Figure 6.56: SG Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 5% LoS

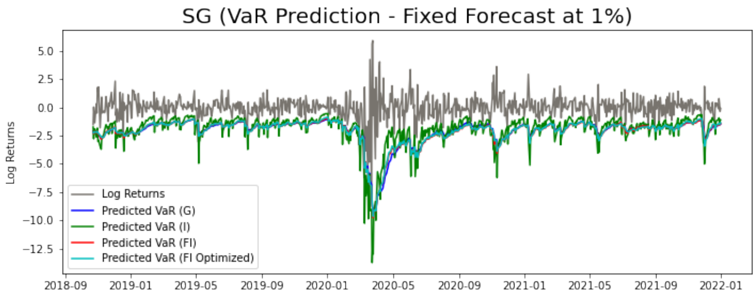

Figure 6.57: SG Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 1% LoS

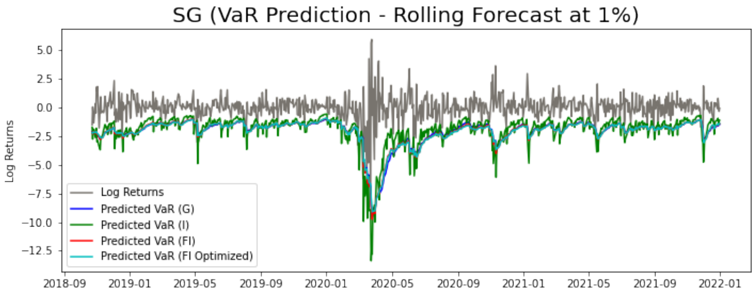

Figure 6.58: SG Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 1% LoS

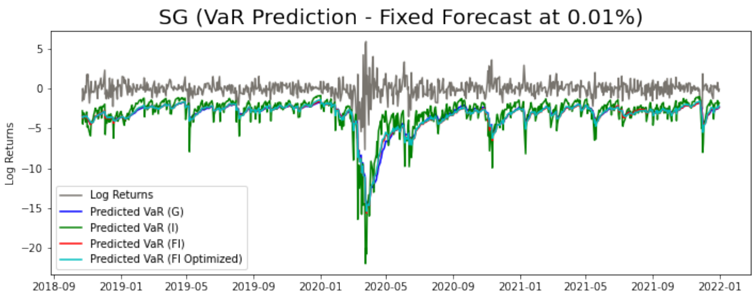

Figure 6.59: SG Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 0.01% LoS

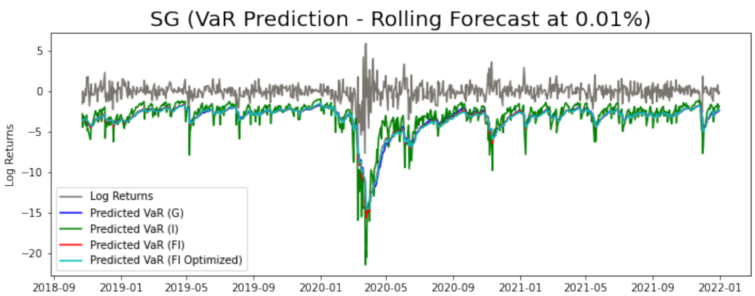

Figure 6.60: SG Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 0.01% LoS

Thailand

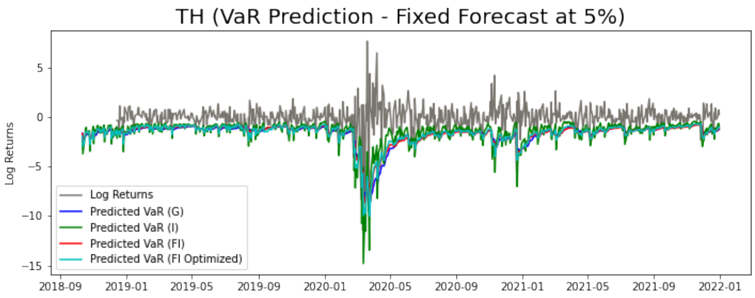

Figure 6.61: TH Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 5% LoS

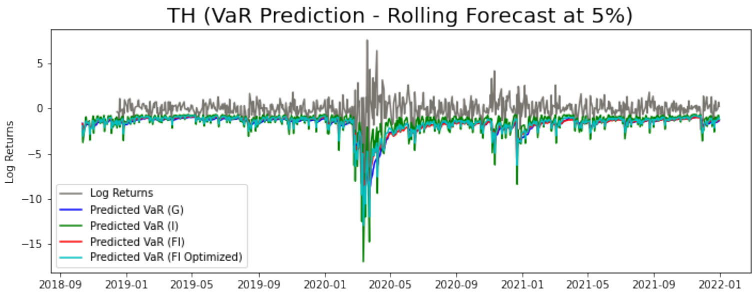

Figure 6.62: TH Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 5% LoS

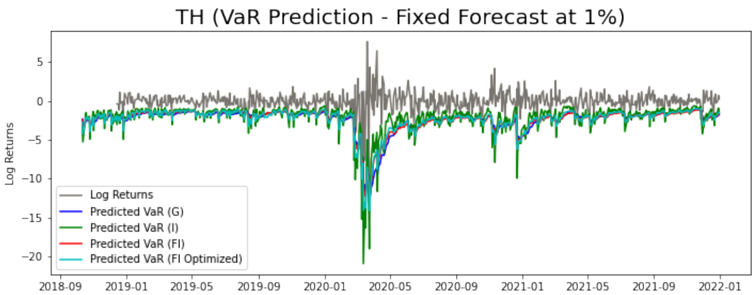

Figure 6.63: TH Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 1% LoS

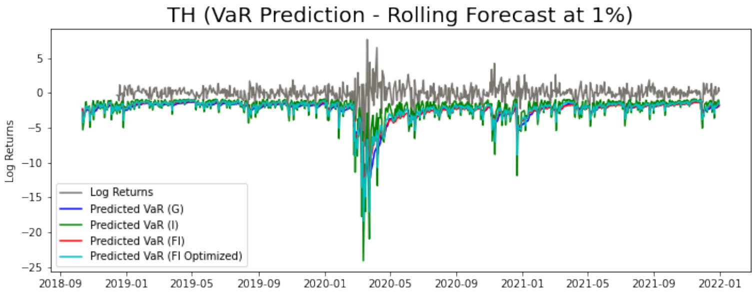

Figure 6.64: TH Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 1% LoS

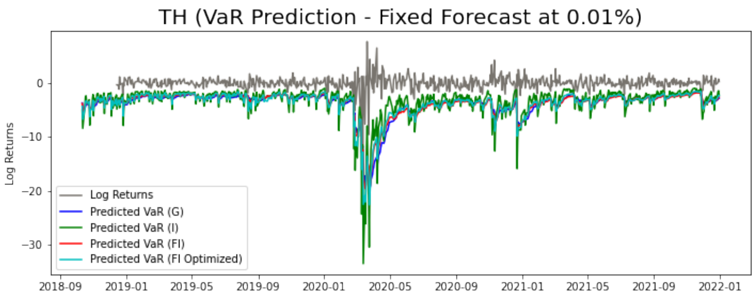

Figure 6.65: TH Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 0.01% LoS

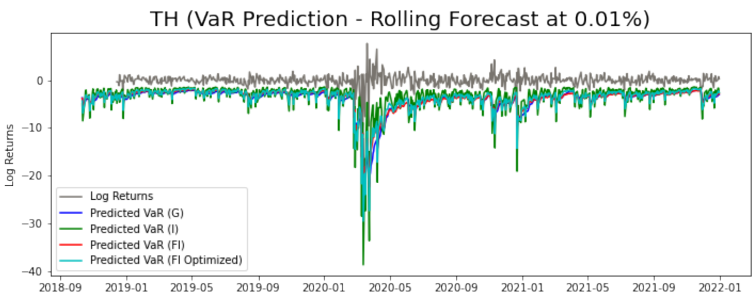

Figure 6.66: TH Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 0.01% LoS

Vietnam

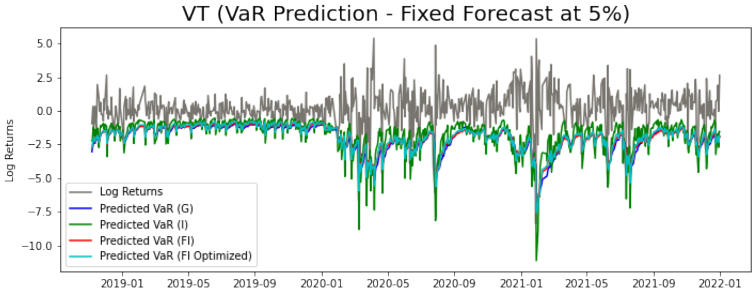

Figure 6.67: VT Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 5% LoS

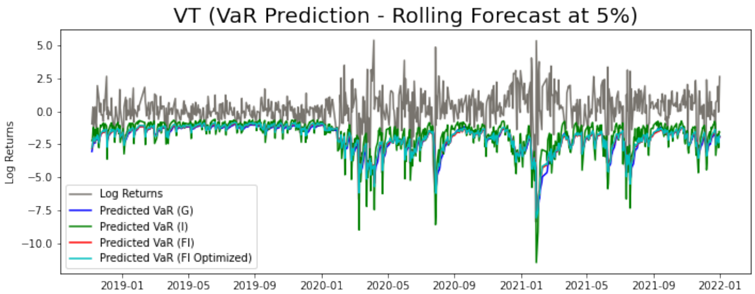

Figure 6.68: VT Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 5% LoS

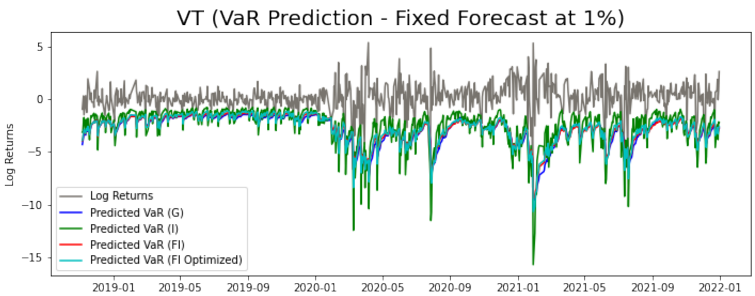

Figure 6.69: VT Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 1% LoS

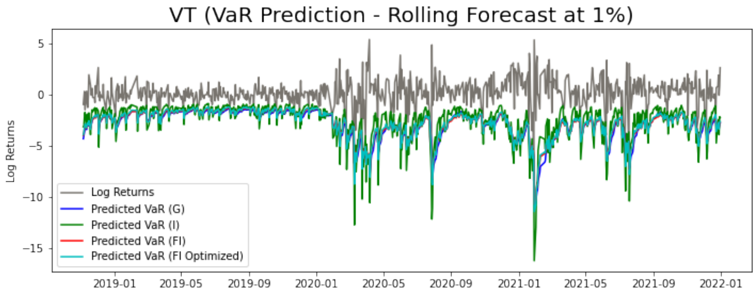

Figure 6.70: VT Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 1% LoS

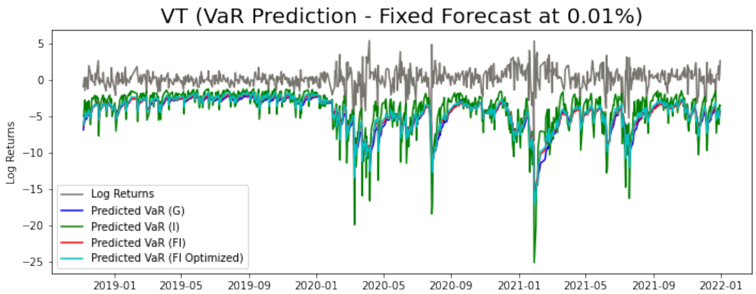

Figure 6.71: VT Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 0.01% LoS

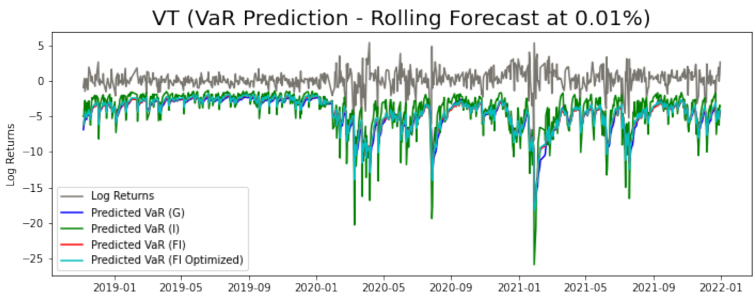

Figure 6.72: VT Log Returns and Estimated VaR under the GARCH, IGARCH, FIGARCH, and Opt-FIGARCH with 0.01% LoS

6.2 Log Returns and Estimated VaR of the Chosen Countries for a 10-year Data Time Frame

Malaysia

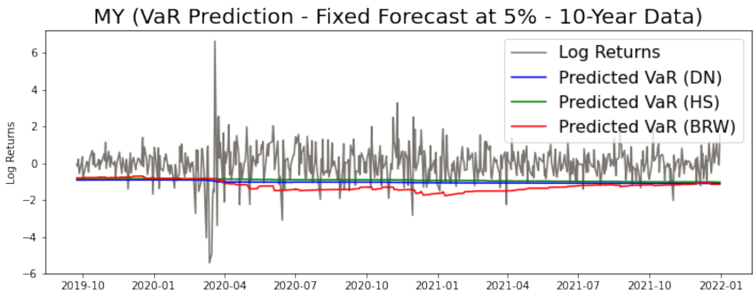

Figure 6.73: MY Log Returns and Estimated VaR using the standard models on a fixed window with 5% LoS for a 10-year Time Frame

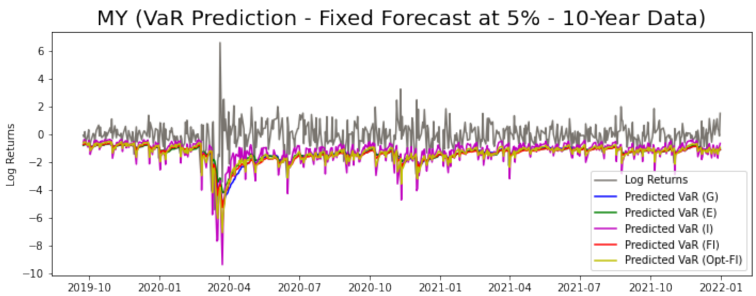

Figure 6.74: MY Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 5% LoS for a 10-year Time Frame

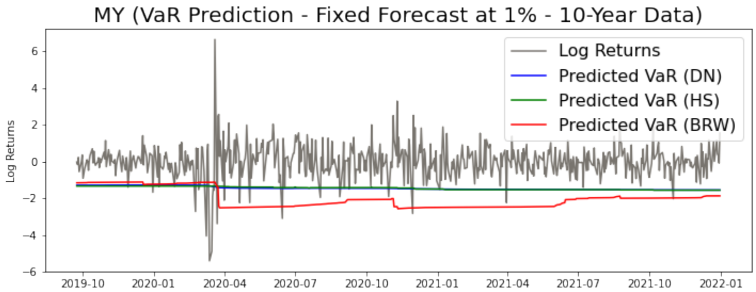

Figure 6.75: MY Log Returns and Estimated VaR using the standard models on a fixed window with 1% LoS for a 10-year Time Frame

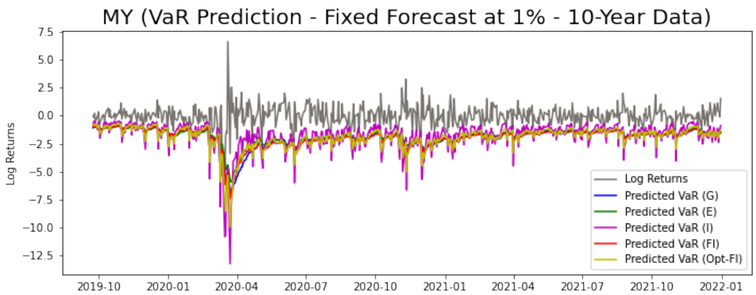

Figure 6.76: MY Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 1% LoS for a 10-year Time Frame

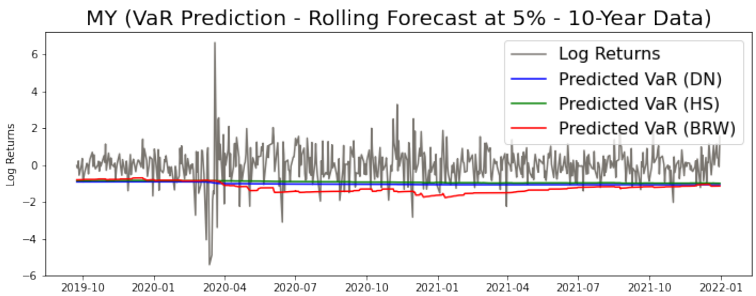

Figure 6.77: MY Log Returns and Estimated VaR using the standard models on a rolling window with 5% LoS for a 10-year Time Frame

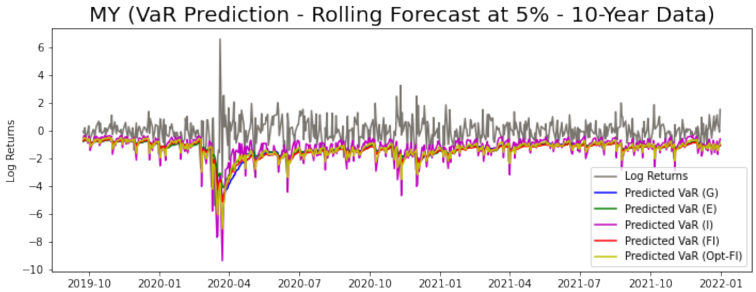

Figure 6.78: MY Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 5% LoS for a 10-year Time Frame

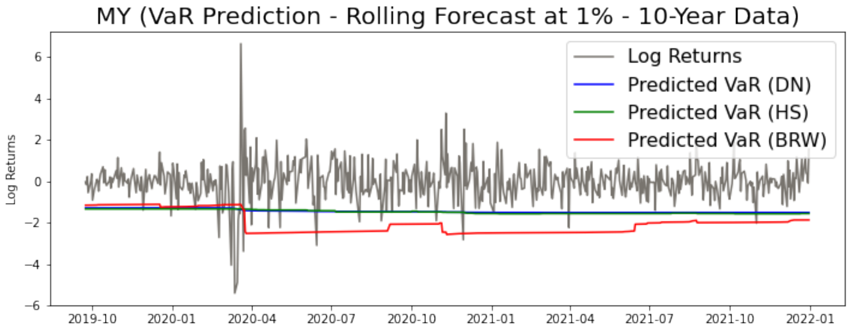

Figure 6.79: MY Log Returns and Estimated VaR using the standard models on a rolling window with 1% LoS for a 10-year Time Frame

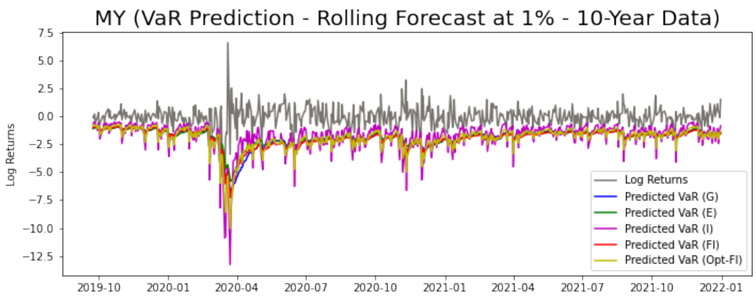

Figure 6.80: MY Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 1% LoS for a 10-year Time Frame

Singapore

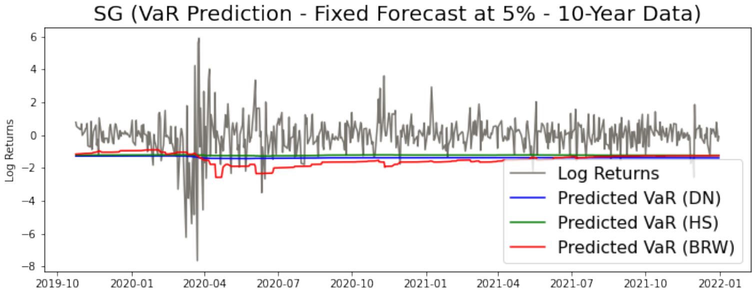

Figure 6.81: SG Log Returns and Estimated VaR using the standard models on a fixed window with 5% LoS for a 10-year Time Frame

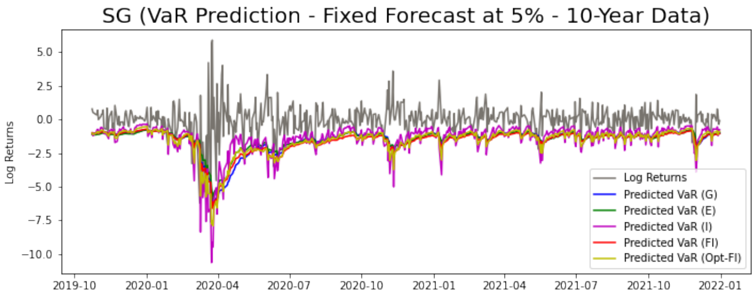

Figure 6.82: SG Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 5% LoS for a 10-year Time Frame

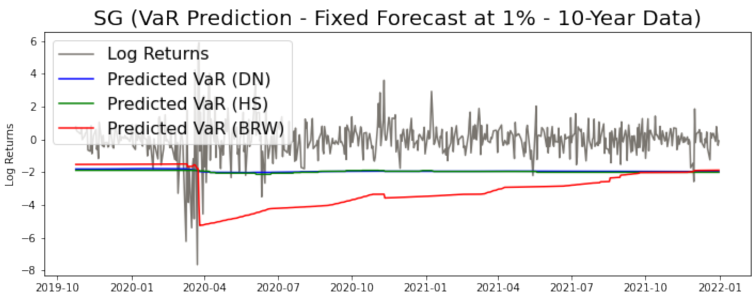

Figure 6.83: SG Log Returns and Estimated VaR using the standard models on a fixed window with 1% LoS for a 10-year Time Frame

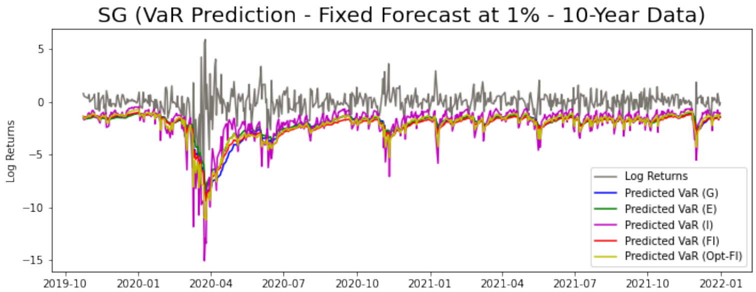

Figure 6.84: SG Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 1% LoS for a 10-year Time Frame

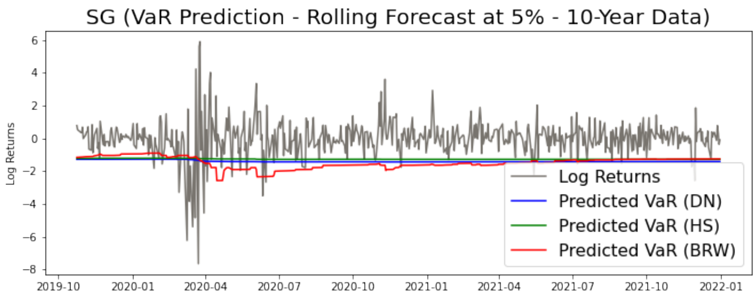

Figure 6.85: SG Log Returns and Estimated VaR using the standard models on a rolling window with 5% LoS for a 10-year Time Frame

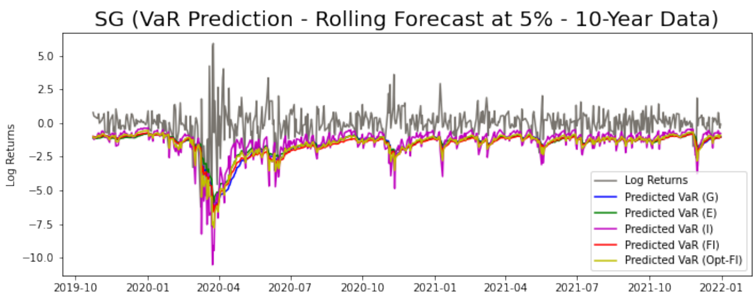

Figure 6.86: SG Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 5% LoS for a 10-year Time Frame

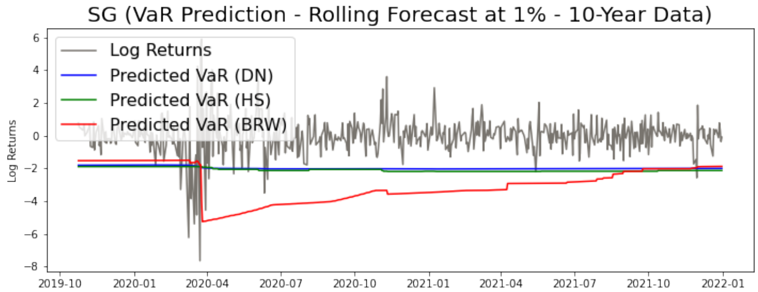

Figure 6.87: SG Log Returns and Estimated VaR using the standard models on a rolling window with 1% LoS for a 10-year Time Frame

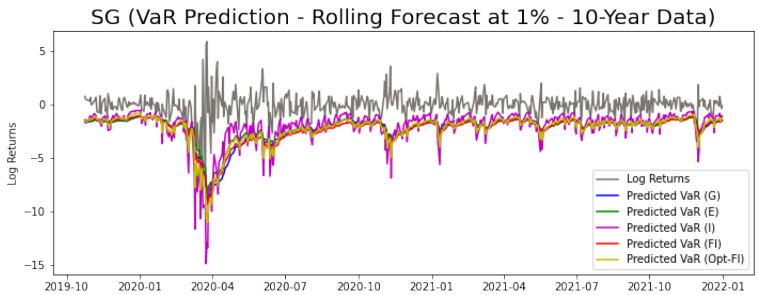

Figure 6.88: SG Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 1% LoS for a 10-year Time Frame

Thailand

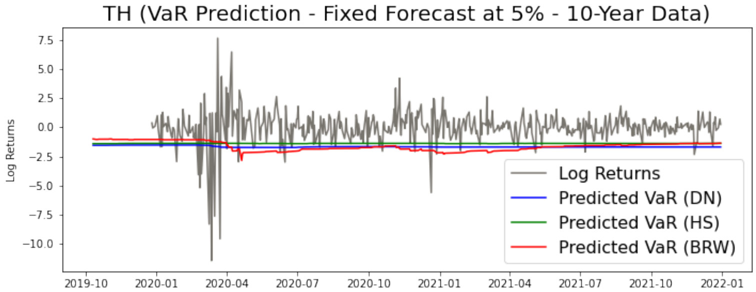

Figure 6.89: TH Log Returns and Estimated VaR using the standard models on a fixed window with 5% LoS for a 10-year Time Frame

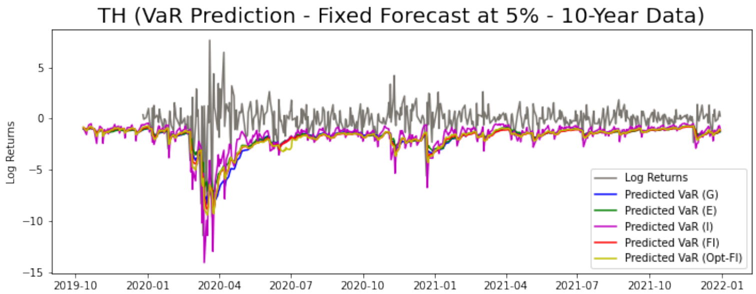

Figure 6.90: TH Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 5% LoS for a 10-year Time Frame

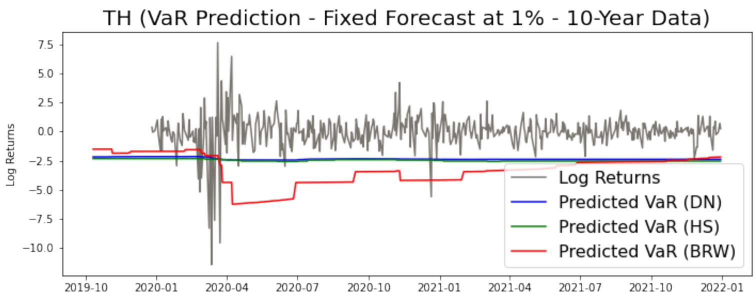

Figure 6.91: TH Log Returns and Estimated VaR using the standard models on a fixed window with 1% LoS for a 10-year Time Frame

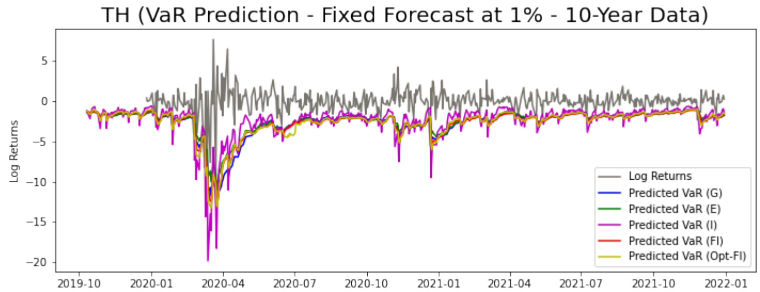

Figure 6.92: TH Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 1% LoS for a 10-year Time Frame

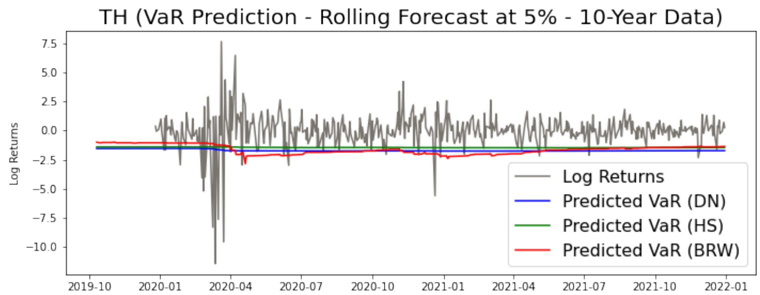

Figure 6.93: TH Log Returns and Estimated VaR using the standard models on a rolling window with 5% LoS for a 10-year Time Frame

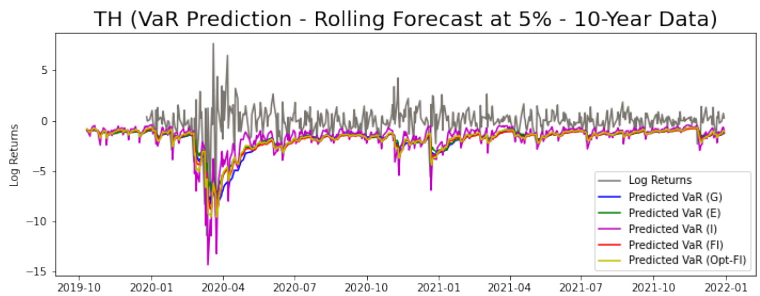

Figure 6.94: TH Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 5% LoS for a 10-year Time Frame

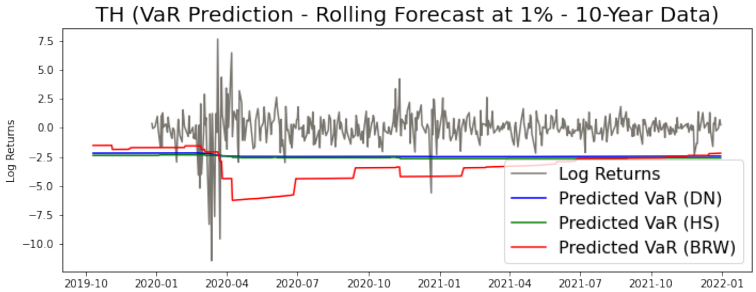

Figure 6.95: TH Log Returns and Estimated VaR using the standard models on a rolling window with 1% LoS for a 10-year Time Frame

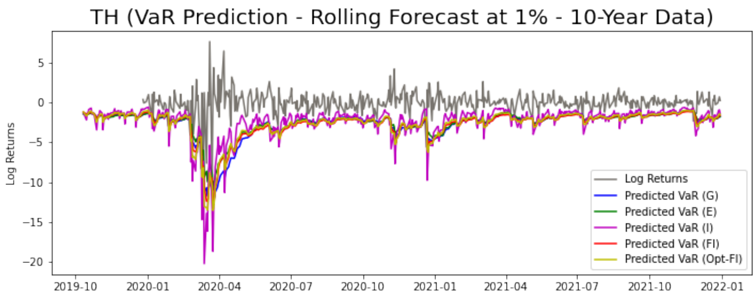

Figure 6.96: TH Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 1% LoS for a 10-year Time Frame

Vietnam

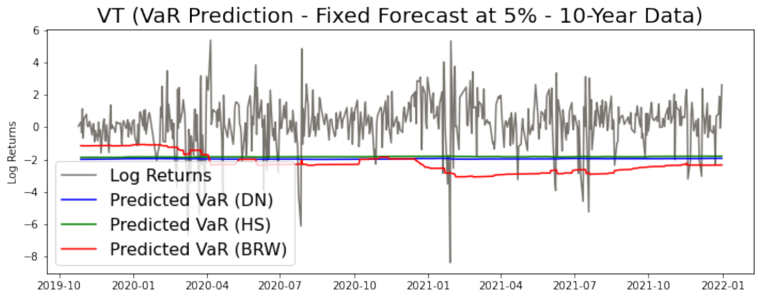

Figure 6.97: VT Log Returns and Estimated VaR using the standard models on a fixed window with 5% LoS for a 10-year Time Frame

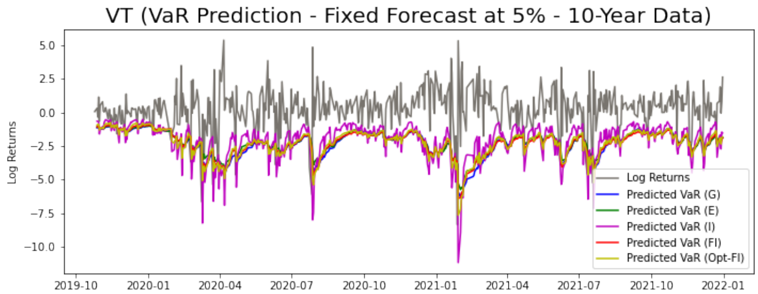

Figure 6.98: VT Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 5% LoS for a 10-year Time Frame

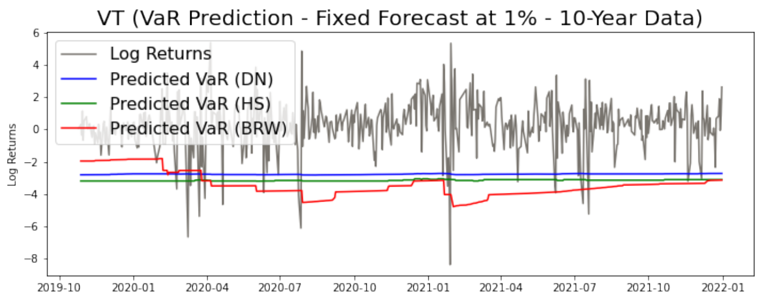

Figure 6.99: VT Log Returns and Estimated VaR using the standard models on a fixed window with 1% LoS for a 10-year Time Frame

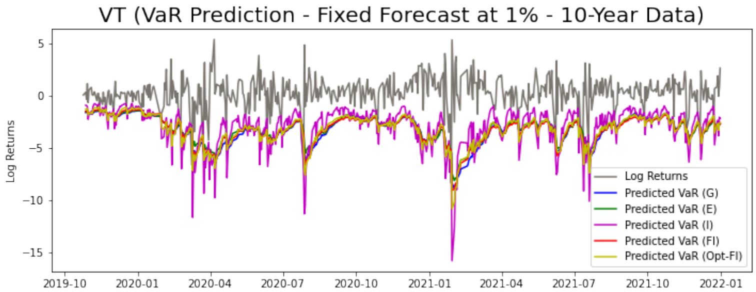

Figure 6.100: VT Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 1% LoS for a 10-year Time Frame

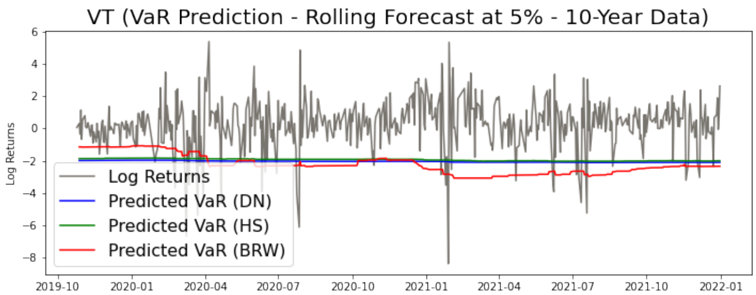

Figure 6.101: VT Log Returns and Estimated VaR using the standard models on a rolling window with 5% LoS for a 10-year Time Frame

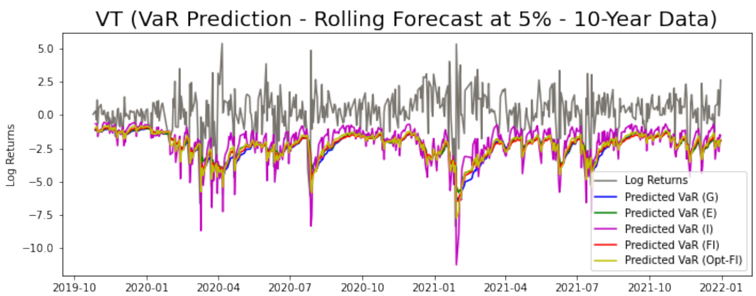

Figure 6.102: VT Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 5% LoS for a 10-year Time Frame

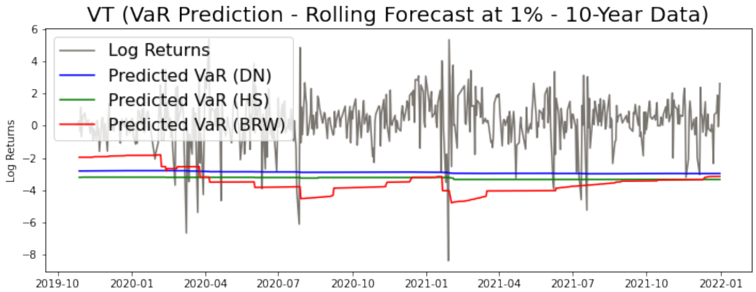

Figure 6.103: VT Log Returns and Estimated VaR using the standard models on a rolling window with 1% LoS for a 10-year Time Frame

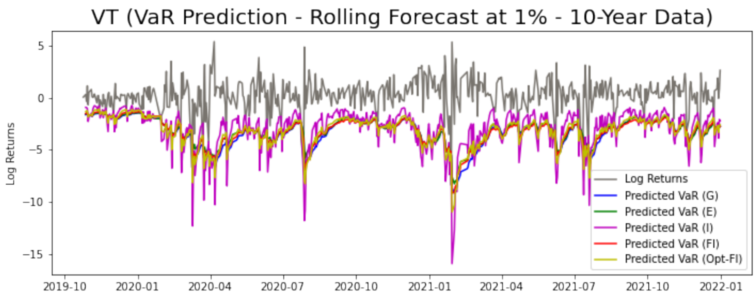

Figure 6.104: VT Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 1% LoS for a 10-year Time Frame

6.3 Log Returns and Estimated VaR of the Chosen Countries for a 5-year Data Time Frame

Malaysia

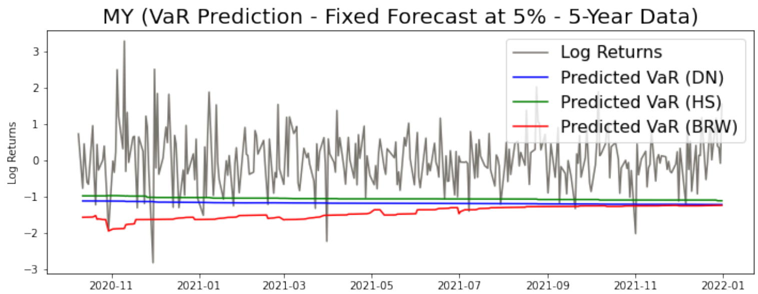

Figure 6.105: MY Log Returns and Estimated VaR using the standard models on a fixed window with 5% LoS for a 5-year Time Frame

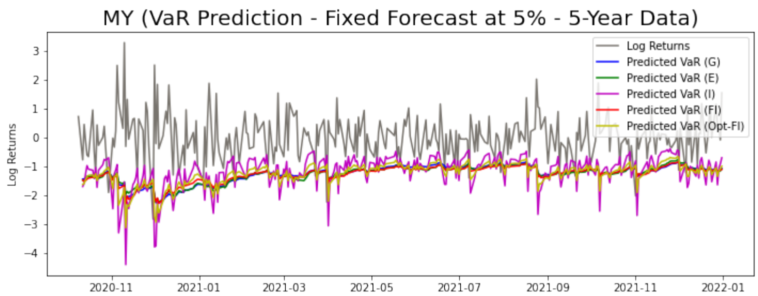

Figure 6.106: MY Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 5% LoS for a 5-year Time Frame

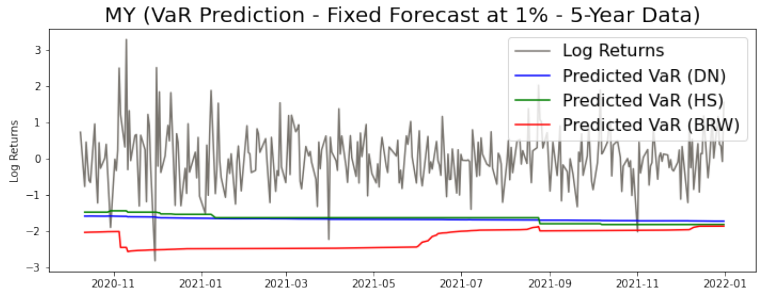

Figure 6.107: MY Log Returns and Estimated VaR using the standard models on a fixed window with 1% LoS for a 5-year Time Frame

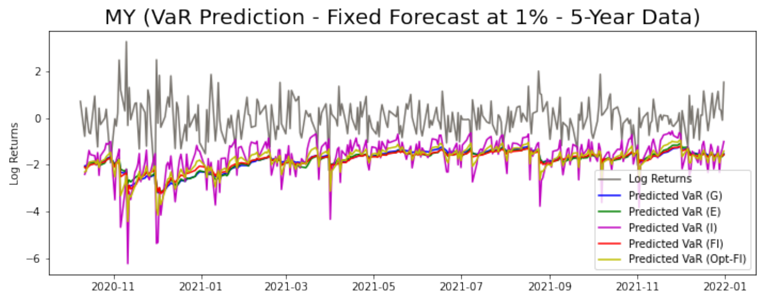

Figure 6.108: MY Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 1% LoS for a 5-year Time Frame

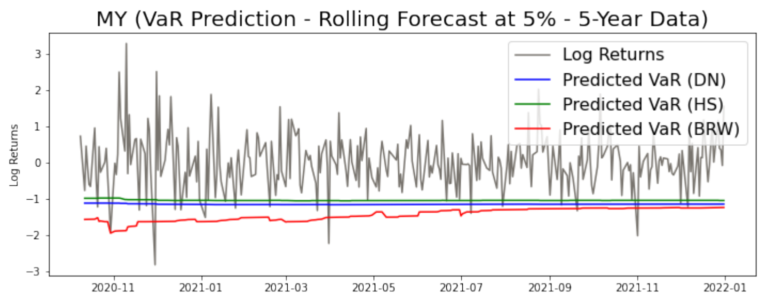

Figure 6.109: MY Log Returns and Estimated VaR using the standard models on a rolling window with 5% LoS for a 5-year Time Frame

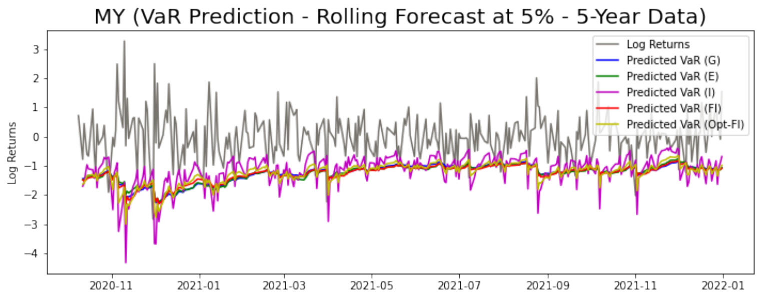

Figure 6.110: MY Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 5% LoS for a 5-year Time Frame

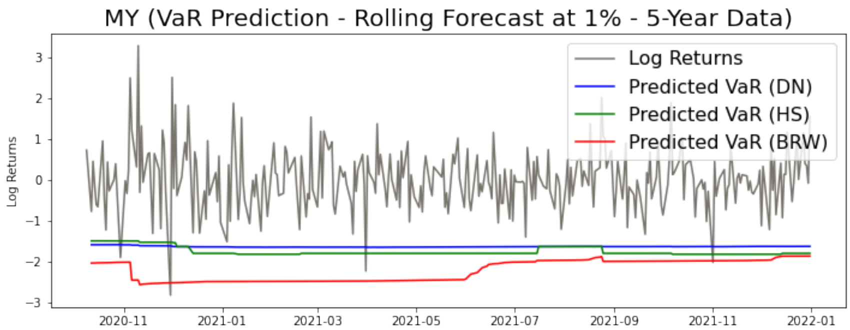

Figure 6.111: MY Log Returns and Estimated VaR using the standard models on a rolling window with 1% LoS for a 5-year Time Frame

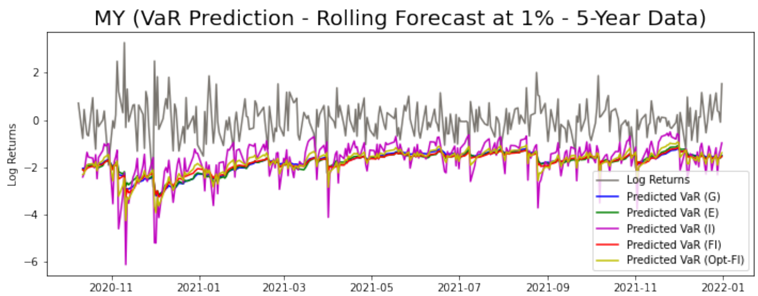

Figure 6.112: MY Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 1% LoS for a 5-year Time Frame

Singapore

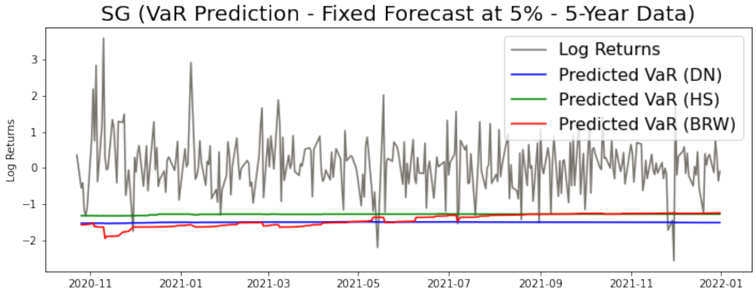

Figure 6.113: SG Log Returns and Estimated VaR using the standard models on a fixed window with 5% LoS for a 5-year Time Frame

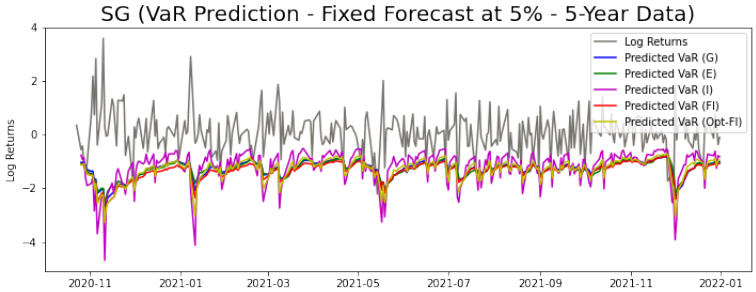

Figure 6.114: SG Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 5% LoS for a 5-year Time Frame

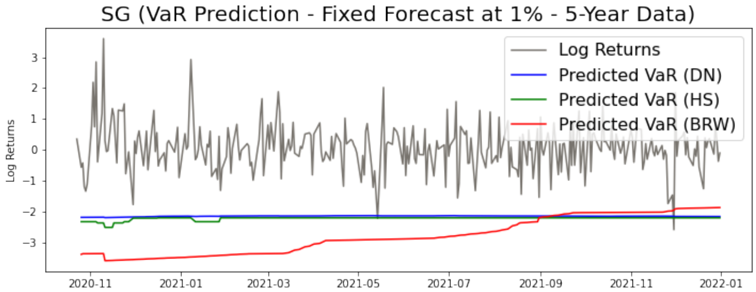

Figure 6.115: SG Log Returns and Estimated VaR using the standard models on a fixed window with 1% LoS for a 5-year Time Frame

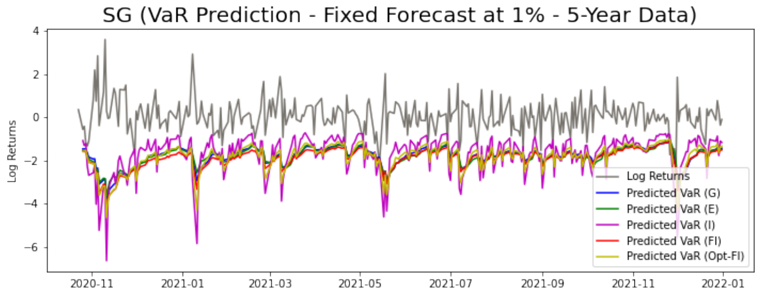

Figure 6.116: SG Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 1% LoS for a 5-year Time Frame

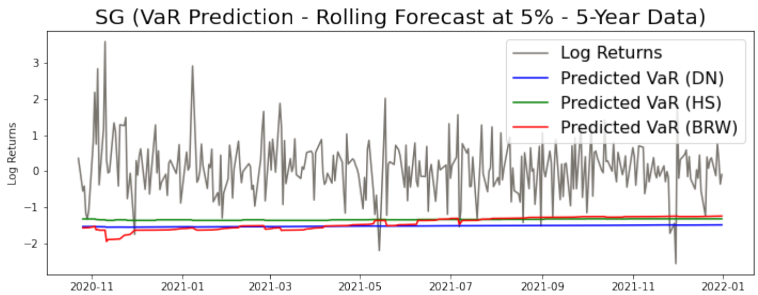

Figure 6.117: SG Log Returns and Estimated VaR using the standard models on a rolling window with 5% LoS for a 5-year Time Frame

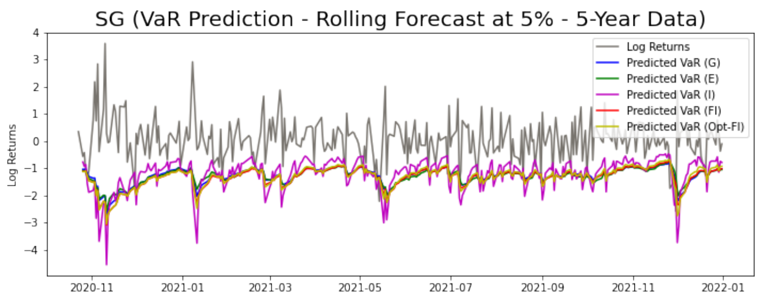

Figure 6.118: SG Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 5% LoS for a 5-year Time Frame

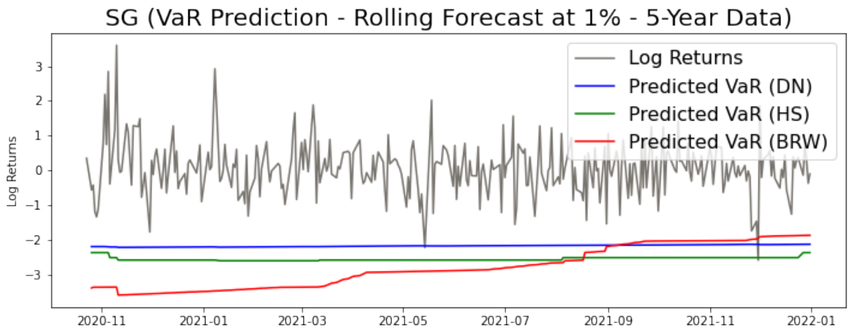

Figure 6.119: SG Log Returns and Estimated VaR using the standard models on a rolling window with 1% LoS for a 5-year Time Frame

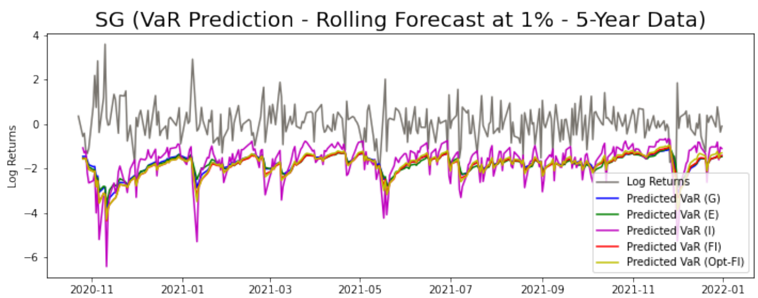

Figure 6.120: SG Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 1% LoS for a 5-year Time Frame

Thailand

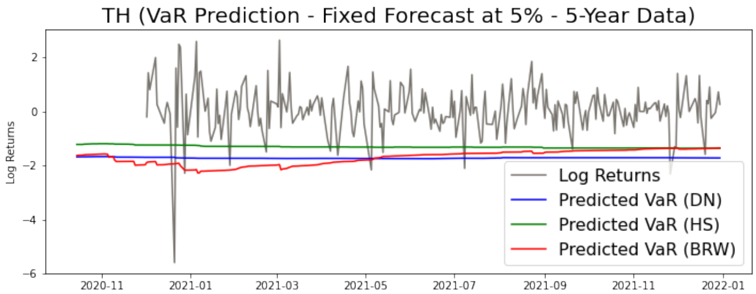

Figure 6.121: TH Log Returns and Estimated VaR using the standard models on a fixed window with 5% LoS for a 5-year Time Frame

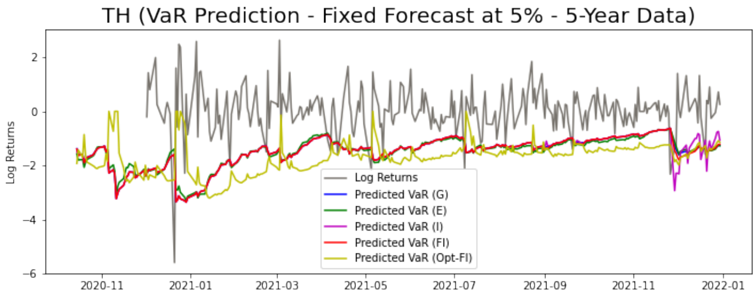

Figure 6.122: TH Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 5% LoS for a 5-year Time Frame

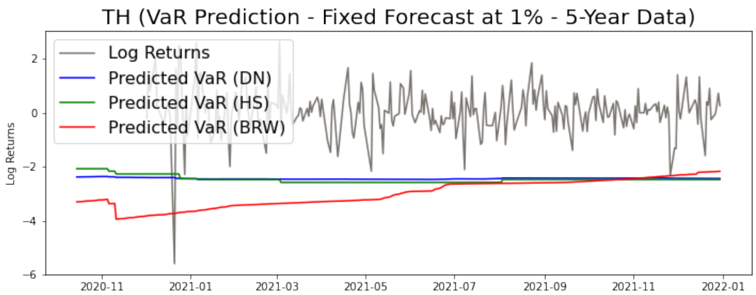

Figure 6.123: TH Log Returns and Estimated VaR using the standard models on a fixed window with 1% LoS for a 5-year Time Frame

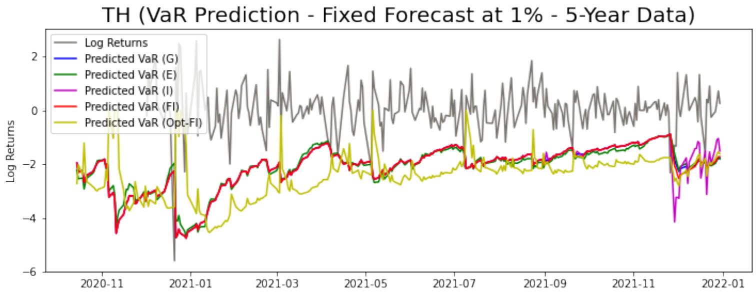

Figure 6.124: TH Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 1% LoS for a 5-year Time Frame

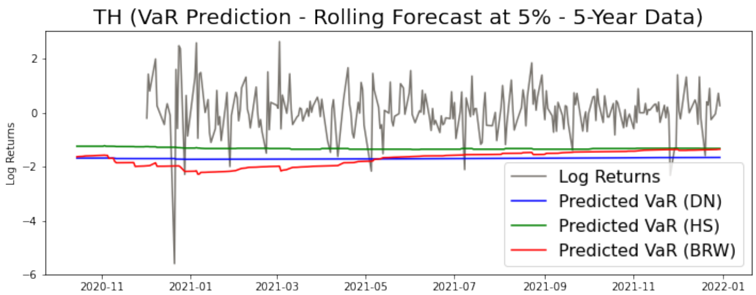

Figure 6.125: TH Log Returns and Estimated VaR using the standard models on a rolling window with 5% LoS for a 5-year Time Frame

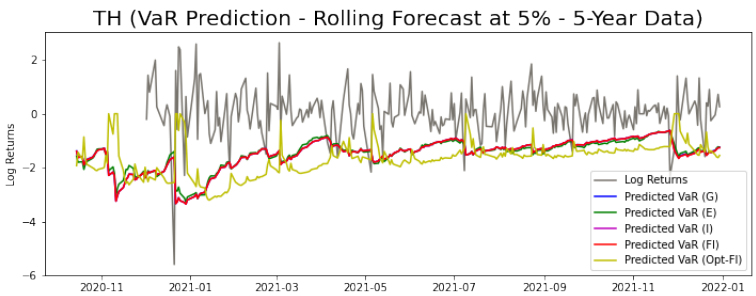

Figure 6.126: TH Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 5% LoS for a 5-year Time Frame

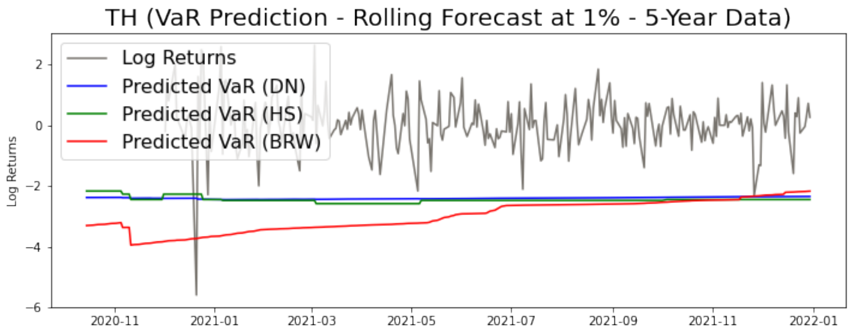

Figure 6.127: TH Log Returns and Estimated VaR using the standard models on a rolling window with 1% LoS for a 5-year Time Frame

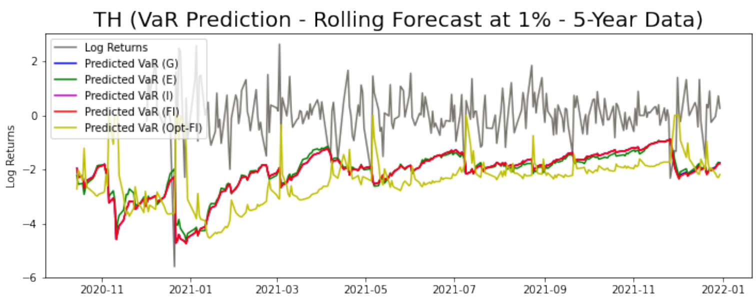

Figure 6.128: TH Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 1% LoS for a 5-year Time Frame

Vietnam

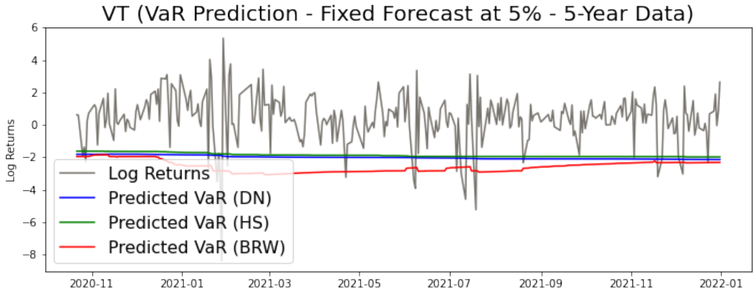

Figure 6.129: VT Log Returns and Estimated VaR using the standard models on a fixed window with 5% LoS for a 5-year Time Frame

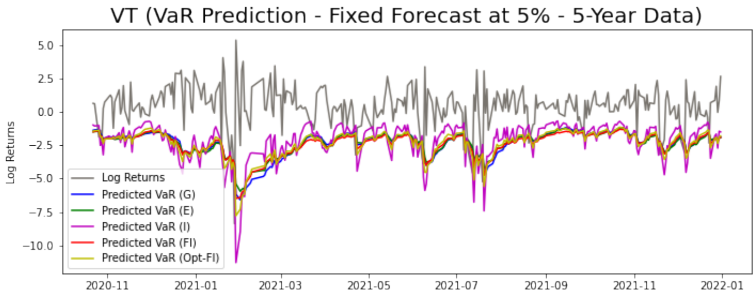

Figure 6.130: VT Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 5% LoS for a 5-year Time Frame

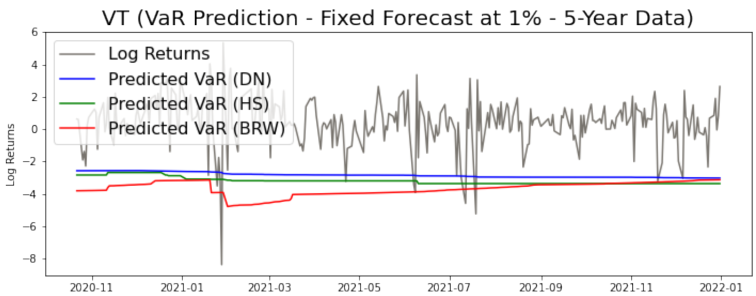

Figure 6.131: VT Log Returns and Estimated VaR using the standard models on a fixed window with 1% LoS for a 5-year Time Frame

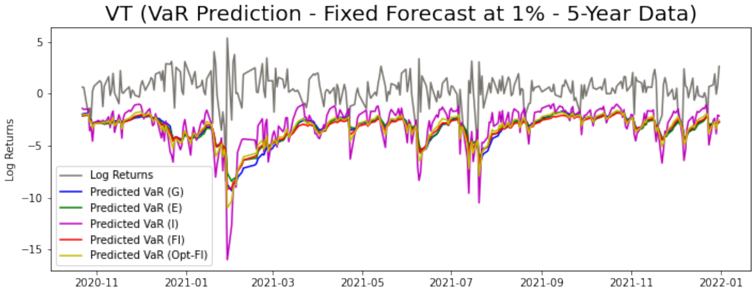

Figure 6.132: VT Log Returns and Estimated VaR using the GARCH-type models on a fixed window with 1% LoS for a 5-year Time Frame

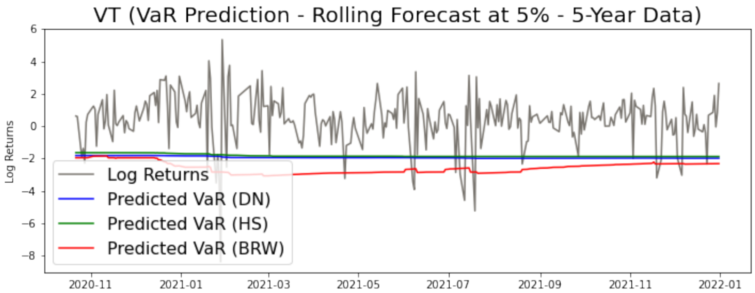

Figure 6.133: VT Log Returns and Estimated VaR using the standard models on a rolling window with 5% LoS for a 5-year Time Frame

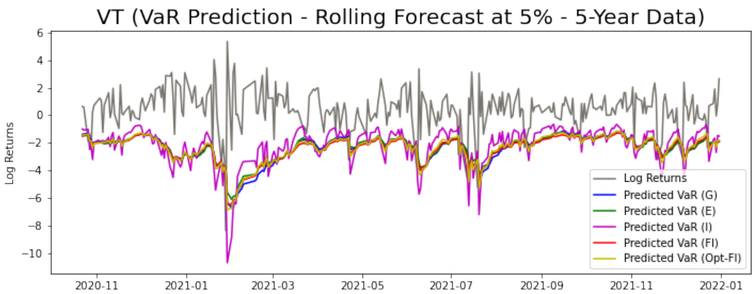

Figure 6.134: VT Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 5% LoS for a 5-year Time Frame

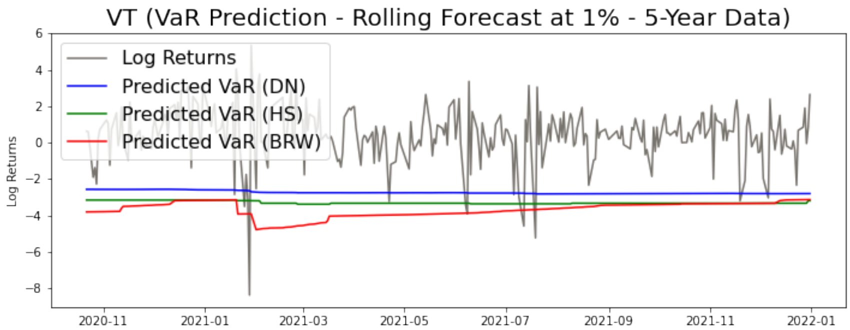

Figure 6.135: VT Log Returns and Estimated VaR using the standard models on a rolling window with 1% LoS for a 5-year Time Frame

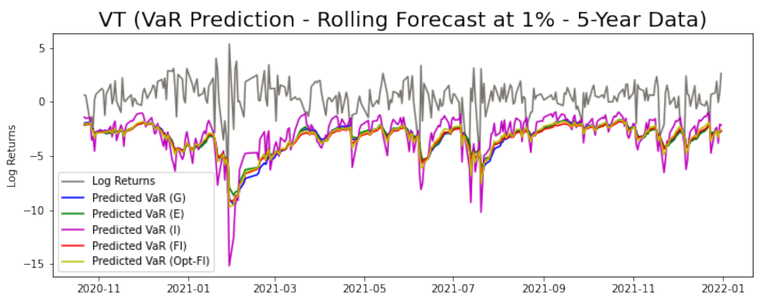

Figure 6.136: VT Log Returns and Estimated VaR using the GARCH-type models on a rolling window with 1% LoS for a 5-year Time Frame

6.4 Diebold Mariano Test Statistics and p-values from KLSE, STI, SETi, and HNX

6.4.1 Malaysia - KLSE

KLSE - Fixed Window (0.01%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	5.5288	6.2388	3.7515	5.9181	5.2313
	3.2239E-08	4.4107E-10	1.76E-04***	3.2561E-09	1.6829E-07
HS	12.4114	17.2585	6.117	13.0052	9.3103
	2.2676E-35	9.6554E-67	9.5356E-10	1.1437E-38	1.2742E-20
BRW	17.3688	19.6812	11.2929	17.9154	14.9234
	1.4217E-67	3.1239E-86	1.4234E-29	8.9423E-72	2.3206E-50
GARCH		2.9339	0.9825	0.4388	1.8188
		3.35E-03***	3.26E-01*	6.61E-01*	6.89E-02*
EGARCH			1.4599	3.3039	2.644
			1.44E-01*	9.53E-04***	8.19E-03***
IGARCH				1.0034	0.4353
				3.16E-01*	6.63E-01*
FIGARCH					2.0716
					3.83E-02**

Table 6.1: KLSE DM test statistics and p-values on Fixed Window (0.01%).

KLSE - Fixed Window (1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	5.4882	6.1922	3.7501	5.8642	5.1969
	4.0605E-08	5.9335E-10	1.77E-04***	4.5122E-09	2.0265E-07
HS	2.1918	1.8896	1.914	2.3905	2.5164
	2.84E-02**	5.88E-02*	5.56E-02*	1.68E-02**	1.19E-02**
BRW	1.1661	0.5363	1.3163	1.3189	1.6637
	2.44E-01*	5.92E-01*	1.88E-01*	1.87E-01*	9.62E-02*
GARCH		2.8641	0.98	0.4187	1.8132
		4.18E-03***	3.27E-01*	6.75E-01*	6.98E-02*
EGARCH			1.4494	3.2289	2.6144
			1.47E-01*	1.24E-03***	8.94E-03***
IGARCH				1.0027	0.4314
				3.16E-01*	6.66E-01*
FIGARCH					2.0676
					3.87E-02**

Table 6.2: KLSE DM test statistics and p-values on Fixed Window (1.00%).

KLSE - Fixed Window (5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	5.4269	6.1148	3.7376	5.7856	5.1431
	5.7357E-08	9.6673E-10	1.86E-04***	7.2244E-09	2.7026E-07
HS	7.6382	8.9457	5.0197	8.104	6.965
	2.2034E-14	3.6966E-19	5.1765E-07	5.3161E-16	3.2840E-12
BRW	5.2266	5.9375	3.6396	5.6414	4.9928
	1.7269E-07	2.8947E-09	2.73E-04***	1.6863E-08	5.9502E-07
GARCH		2.7806	0.9741	0.3951	1.8
		5.43E-03***	3.30E-01*	6.93E-01*	7.19E-02*
EGARCH			1.4338	3.139	2.5745
			1.52E-01*	1.70E-03***	1.00E-02**
IGARCH				0.9985	0.426
				3.18E-01*	6.70E-01*
FIGARCH					2.0544
					3.99E-02**

Table 6.3: KLSE DM test statistics and p-values on Fixed Window (5.00%).

KLSE - Rolling Window (0.01%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	4.7393	5.2352	3.2607	4.9551	4.5231
	2.1444E-06	1.6476E-07	1.11E-03***	7.2281E-07	6.0954E-06
HS	35.2111	41.9229	21.7718	36.7214	29.9766
	1.3505E-271	0.0000E+00	4.2953E-105	3.3311E-295	1.9821E-197
BRW	16.9867	19.3777	11.1982	17.5251	14.722
	1.0311E-64	1.1905E-83	4.1594E-29	9.2178E-69	4.6581E-49
GARCH		2.9146	0.8366	0.287	1.5817
		3.56E-03***	4.03E-01*	7.74E-01*	1.14E-01*
EGARCH			1.35	2.9246	2.512
			1.77E-01*	3.45E-03***	1.20E-02**
IGARCH				0.9441	0.369
				3.45E-01*	7.12E-01*
FIGARCH					2.0893
					3.67E-02**

Table 6.4: KLSE DM test statistics and p-values on Rolling Window (0.01%).

KLSE - Rolling Window (1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	4.7175	5.2198	3.2645	4.9243	4.5022
	2.3874E-06	1.7911E-07	1.10E-03***	8.4668E-07	6.7244E-06
HS	1.4988	0.9888	1.4302	1.5296	1.8403
	1.34E-01*	3.23E-01*	1.53E-01*	1.26E-01*	6.57E-02*
BRW	1.4449	0.9057	1.3688	1.4758	1.7745
	1.48E-01*	3.65E-01*	1.71E-01*	1.40E-01*	7.60E-02*
GARCH		2.839	0.8321	0.2997	1.5749
		4.53E-03***	4.05E-01*	7.64E-01*	1.15E-01*
EGARCH			1.3358	2.8395	2.4761
			1.82E-01*	4.52E-03***	1.33E-02**
IGARCH				0.9405	0.3641
				3.47E-01*	7.16E-01*
FIGARCH					2.0819
					3.73E-02**

Table 6.5: KLSE DM test statistics and p-values on Rolling Window (1.00%).

KLSE - Rolling Window (5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	4.6785	5.1801	3.2591	4.8743	4.4657
	2.8892E-06	2.2174E-07	1.12E-03***	1.0919E-06	7.9803E-06
HS	7.008	8.1963	4.6227	7.3183	6.4058
	2.4167E-12	2.4796E-16	3.7882E-06	2.5118E-13	1.4958E-10
BRW	5.3899	6.1854	3.6787	5.6627	5.0604
	7.0505E-08	6.1930E-10	2.34E-04***	1.4898E-08	4.1830E-07
GARCH		2.7494	0.8248	0.3123	1.5628
		5.97E-03***	4.10E-01*	7.55E-01*	1.18E-01*
EGARCH			1.3167	2.7401	2.4309
			1.88E-01*	6.14E-03***	1.51E-02**
IGARCH				0.9338	0.3578
				3.50E-01*	7.20E-01*
FIGARCH					2.0671
					3.87E-02**

Table 6.6: KLSE DM test statistics and p-values on Rolling Window (5.00%).

6.4.2 Singapore - STI

STI - Fixed Window (0.01%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	1.8054	3.2825	0.7828	1.4865	1.6621
	7.10E-02*	1.03E-03***	4.34E-01*	1.37E-01*	9.65E-02*
HS	26.8414	31.6728	18.0858	26.5748	26.9296
	1.0622E-158	3.6797E-220	4.1270E-73	1.3281E-155	9.8825E-160
BRW	7.3217	9.2018	4.6863	7.0626	7.2415
	2.4490E-13	3.5214E-20	2.7819E-06	1.6344E-12	4.4371E-13
GARCH		6.562	0.7635	2.5914	1.143
		5.3083E-11	4.45E-01*	9.56E-03***	2.53E-01*
EGARCH			1.9082	7.6527	6.6852
			5.64E-02*	1.9687E-14	2.3055E-11
IGARCH				0.416	0.6265
				6.77E-01*	5.31E-01*
FIGARCH					4.1204
					3.7829E-05

Table 6.7: STI DM test statistics and p-values on Fixed Window (0.01%).

STI - Fixed Window (1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	1.8429	3.3107	0.7568	1.5147	1.6799
	6.53E-02*	9.31E-04***	4.49E-01*	1.30E-01*	9.30E-02*
HS	9.3012	11.8093	5.7382	8.9755	9.1875
	1.3882E-20	3.4946E-32	9.5696E-09	2.8206E-19	4.0221E-20
BRW	8.3616	10.1675	4.7694	7.9466	8.0796
	6.1848E-17	2.7700E-24	1.8479E-06	1.9163E-15	6.4971E-16
GARCH		6.5566	0.8395	2.6646	1.2683
		5.5041E-11	4.01E-01*	7.71E-03***	2.05E-01*
EGARCH			1.9703	7.6952	6.7448
			4.88E-02**	1.4130E-14	1.5324E-11
IGARCH				0.4879	0.6865
				6.26E-01*	4.92E-01*
FIGARCH					3.6676
					2.45E-04***

Table 6.8: STI DM test statistics and p-values on Fixed Window (1.00%).

STI - Fixed Window (5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	1.8713	3.3139	0.7248	1.5354	1.6874
	6.13E-02*	9.20E-04***	4.69E-01*	1.25E-01*	9.15E-02*
HS	2.4415	1.6094	2.1308	2.7769	2.6542
	1.46E-02**	1.08E-01*	3.31E-02**	5.49E-03***	7.95E-03***
BRW	4.3889	3.7994	3.315	4.7255	4.6034
	1.1391E-05	1.45E-04***	9.16E-04***	2.2955E-06	4.1570E-06
GARCH		6.5259	0.9195	2.7348	1.3972
		6.7587E-11	3.58E-01*	6.24E-03***	1.62E-01*
EGARCH			2.0326	7.7131	6.7787
			4.21E-02**	1.2279E-14	1.2128E-11
IGARCH				0.5643	0.7499
				5.73E-01*	4.53E-01*
FIGARCH					3.1858
					1.44E-03***

Table 6.9: STI DM test statistics and p-values on Fixed Window (5.00%).

STI - Rolling Window (0.01%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	2.3021	3.9077	1.1853	1.976	2.2652
	2.13E-02**	9.3160E-05	2.36E-01*	4.81E-02**	2.35E-02**
HS	43.2401	50.1692	30.7596	42.6681	44.9474
	0.0000E+00	0.0000E+00	9.0963E-208	0.0000E+00	0.0000E+00
BRW	7.1344	9.075	4.7024	6.8279	7.3341
	9.7215E-13	1.1366E-19	2.5718E-06	8.6143E-12	2.2327E-13
GARCH		6.6725	0.7924	3.0916	1.3041
		2.5147E-11	4.28E-01*	1.99E-03***	1.92E-01*
EGARCH			2.0806	7.7888	8.4248
			3.75E-02**	6.7635E-15	3.6125E-17
IGARCH				0.4076	0.576
				6.84E-01*	5.65E-01*
FIGARCH					1.8129
					6.99E-02*

Table 6.10: STI DM test statistics and p-values on Rolling Window (0.01%).

STI - Rolling Window (1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	2.3272	3.9154	1.1542	1.9976	2.2898
	2.00E-02**	9.0248E-05	2.48E-01*	4.58E-02**	2.20E-02**
HS	10.3218	13.0023	6.7519	9.9453	10.6752
	5.6161E-25	1.1873E-38	1.4588E-11	2.6438E-23	1.3304E-26
BRW	8.1056	9.974	4.8123	7.7053	8.382
	5.2507E-16	1.9811E-23	1.4919E-06	1.3050E-14	5.2034E-17
GARCH		6.6654	0.8655	3.1391	1.3394
		2.6386E-11	3.87E-01*	1.69E-03***	1.80E-01*
EGARCH			2.137	7.828	8.47
			3.26E-02**	4.9578E-15	2.4537E-17
IGARCH				0.4801	0.6401
				6.31E-01*	5.22E-01*
FIGARCH					1.8027
					7.14E-02*

Table 6.11: STI DM test statistics and p-values on Rolling Window (1.00%).

STI - Rolling Window (5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	2.3377	3.8885	1.1143	2.007	2.2989
	1.94E-02**	1.01E-04***	2.65E-01*	4.47E-02**	2.15E-02**
HS	1.9055	0.9609	1.7979	2.2083	2.1646
	5.67E-02*	3.37E-01*	7.22E-02*	2.72E-02**	3.04E-02**
BRW	4.5202	3.9246	3.4768	4.8076	4.9521
	6.1782E-06	8.6874E-05	5.07E-04***	1.5276E-06	7.3435E-07
GARCH		6.6334	0.9423	3.179	1.3723
		3.2807E-11	3.46E-01*	1.48E-03***	1.70E-01*
EGARCH			2.1929	7.8435	8.47
			2.83E-02**	4.3829E-15	2.4547E-17
IGARCH				0.5573	0.7081
				5.77E-01*	4.79E-01*
FIGARCH					1.7899
					7.35E-02*

Table 6.12: STI DM test statistics and p-values on Rolling Window (5.00%).

6.4.3 Thailand - SETi

SETi - Fixed Window (0.01%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	1.1432	1.1686	0.6506	0.4992	0.8216
	2.53E-01*	2.43E-01*	5.15E-01*	6.18E-01*	4.11E-01*
HS	17.4986	27.1738	11.6827	19.7413	16.6439
	1.4682E-68	1.3270E-162	1.5630E-31	9.5276E-87	3.3514E-62
BRW	7.1022	11.6632	4.8842	8.2835	6.8566
	1.2277E-12	1.9651E-31	1.0387E-06	1.1963E-16	7.0515E-12
GARCH		6.0605	0.1479	3.1279	0.784
		1.3572E-09	8.82E-01*	1.76E-03***	4.33E-01*
EGARCH			1.6619	4.5493	3.6454
			9.65E-02*	5.3823E-06	2.67E-04***
IGARCH				0.5511	0.164
				5.82E-01*	8.70E-01*
FIGARCH					1.6377
					1.01E-01*

Table 6.13: SETi DM test statistics and p-values on Fixed Window (0.01%).

SETi - Fixed Window (1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	1.143	1.1639	0.664	0.5017	0.8303
	2.53E-01*	2.44E-01*	5.07E-01*	6.16E-01*	4.06E-01*
HS	1.0011	4.2389	0.7416	1.8474	1.1862
	3.17E-01*	2.2457E-05	4.58E-01*	6.47E-02*	2.36E-01*
BRW	1.467	4.6159	1.0104	2.2937	1.5654
	1.42E-01*	3.9139E-06	3.12E-01*	2.18E-02**	1.17E-01*
GARCH		6.0652	0.1299	3.1089	0.7575
		1.3179E-09	8.97E-01*	1.88E-03***	4.49E-01*
EGARCH			1.6825	4.5462	3.6698
			9.25E-02*	5.4616E-06	2.43E-04***
IGARCH				0.5674	0.1769
				5.70E-01*	8.60E-01*
FIGARCH					1.6623
					9.65E-02*

Table 6.14: SETi DM test statistics and p-values on Fixed Window (1.00%).

SETi - Fixed Window (5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	1.139	1.1511	0.6769	0.5026	0.837
	2.55E-01*	2.50E-01*	4.98E-01*	6.15E-01*	4.03E-01*
HS	4.588	3.8093	2.9436	4.2797	4.0861
	4.4750E-06	1.39E-04***	3.24E-03***	1.8718E-05	4.3878E-05
BRW	6.1272	6.0238	3.9135	5.9667	5.4975
	8.9442E-10	1.7041E-09	9.0957E-05	2.4215E-09	3.8524E-08
GARCH		6.0659	0.1097	3.0826	0.7269
		1.3120E-09	9.13E-01*	2.05E-03***	4.67E-01*
EGARCH			1.7032	4.5341	3.6929
			8.85E-02*	5.7854E-06	2.22E-04***
IGARCH				0.5845	0.1909
				5.59E-01*	8.49E-01*
FIGARCH					1.6855
					9.19E-02*

Table 6.15: SETi DM test statistics and p-values on Fixed Window (5.00%).

SETi - Rolling Window (0.01%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	0.9201	2.1456	0.5101	0.1871	0.6541
	3.58E-01*	3.19E-02**	6.10E-01*	8.52E-01*	5.13E-01*
HS	48.801	76.6046	28.4656	53.8448	40.1751
	0.0000E+00	0.0000E+00	3.1260E-178	0.0000E+00	0.0000E+00
BRW	6.5349	12.2796	3.8761	7.8262	5.4693
	6.3639E-11	1.1654E-34	1.06E-04***	5.0284E-15	4.5177E-08
GARCH		5.4584	0.0078	2.9294	0.1839
		4.8041E-08	9.94E-01*	3.40E-03***	8.54E-01*
EGARCH			1.5553	4.1172	2.7728
			1.20E-01*	3.8343E-05	5.56E-03***
IGARCH				0.6531	0.1354
				5.14E-01*	8.92E-01*
FIGARCH					1.4926
					1.36E-01*

Table 6.16: SETi DM test statistics and p-values on Rolling Window (0.01%).

SETi - Rolling Window (1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	0.9195	2.1266	0.5173	0.1924	0.6662
	3.58E-01*	3.35E-02**	6.05E-01*	8.47E-01*	5.05E-01*
HS	0.9131	5.0719	0.5141	1.8195	0.8146
	3.61E-01*	3.9381E-07	6.07E-01*	6.88E-02*	4.15E-01*
BRW	0.9853	4.7233	0.5549	1.8691	0.8652
	3.24E-01*	2.3201E-06	5.79E-01*	6.16E-02*	3.87E-01*
GARCH		5.4555	0.001	2.9013	0.1604
		4.8832E-08	9.99E-01*	3.72E-03***	8.73E-01*
EGARCH			1.5634	4.137	2.7944
			1.18E-01*	3.5183E-05	5.20E-03***
IGARCH				0.6583	0.132
				5.10E-01*	8.95E-01*
FIGARCH					1.5116
					1.31E-01*

Table 6.17: SETi DM test statistics and p-values on Rolling Window (1.00%).

SETi - Rolling Window (5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	0.9159	2.0896	0.5242	0.1976	0.6778
	3.60E-01*	3.67E-02**	6.00E-01*	8.43E-01*	4.98E-01*
HS	4.6327	3.9542	2.6147	4.2929	3.6857
	3.6094E-06	7.6805E-05	8.93E-03***	1.7639E-05	2.28E-04***
BRW	6.4172	6.9062	3.6034	6.2961	5.1157
	1.3884E-10	4.9771E-12	3.14E-04***	3.0531E-10	3.1256E-07
GARCH		5.4487	0.0109	2.8653	0.134
		5.0728E-08	9.91E-01*	4.17E-03***	8.93E-01*
EGARCH			1.5707	4.1536	2.816
			1.16E-01*	3.2728E-05	4.86E-03***
IGARCH				0.6631	0.1281
				5.07E-01*	8.98E-01*
FIGARCH					1.53
					1.26E-01*

Table 6.18: SETi DM test statistics and p-values on Rolling Window (5.00%).

6.4.4 Vietnam - HNX

HNX - Fixed Window (0.01%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	11.3935	15.159	7.837	13.815	12.1572
	4.5059E-30	6.6079E-52	4.6138E-15	2.0693E-43	5.2545E-34
HS	27.9389	32.1468	20.2658	30.3878	28.0564
	9.0005E-172	9.7968E-227	2.5759E-91	7.9501E-203	3.3400E-173
BRW	0.6073	2.9727	1.0293	2.4779	1.9429
	5.44E-01*	2.95E-03***	3.03E-01*	1.32E-02**	5.20E-02*
GARCH		8.8883	1.007	10.2377	5.0345
		6.2068E-19	3.14E-01*	1.3445E-24	4.7916E-07
EGARCH			0.6301	1.1025	1.23
			5.29E-01*	2.70E-01*	2.19E-01*
IGARCH				0.5221	0.362
				6.02E-01*	7.17E-01*
FIGARCH					1.1825
					2.37E-01*

Table 6.19: HNX DM test statistics and p-values on Fixed Window (0.01%).

HNX - Fixed Window (1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	10.7614	14.1729	7.4658	12.9596	11.4593
	5.2370E-27	1.3480E-45	8.2805E-14	2.0722E-38	2.1119E-30
HS	25.5124	29.8141	17.8239	28.0152	25.5079
	1.4370E-143	2.5679E-195	4.6137E-71	1.0610E-172	1.6130E-143
BRW	2.35	5.3393	1.9144	4.5438	3.6176
	1.88E-02**	9.3308E-08	5.56E-02*	5.5260E-06	2.97E-04***
GARCH		8.8659	0.9343	9.9544	4.8783
		7.5892E-19	3.50E-01*	2.4120E-23	1.0702E-06
EGARCH			0.6899	1.256	1.3066
			4.90E-01*	2.09E-01*	1.91E-01*
IGARCH				0.5624	0.4136
				5.74E-01*	6.79E-01*
FIGARCH					1.1739
					2.40E-01*

Table 6.20: HNX DM test statistics and p-values on Fixed Window (1.00%).

HNX - Fixed Window (5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	9.9257	12.8989	7.0003	11.8563	10.555
	3.2177E-23	4.5630E-38	2.5536E-12	1.9958E-32	4.8156E-26
HS	7.5295	10.3211	5.3814	9.3949	8.2861
	5.0926E-14	5.6601E-25	7.3899E-08	5.7247E-21	1.1702E-16
BRW	3.9937	2.3743	1.6146	2.4572	2.3255
	6.5060E-05	1.76E-02**	1.06E-01*	1.40E-02**	2.00E-02**
GARCH		8.7676	0.8528	9.5293	4.6675
		1.8259E-18	3.94E-01*	1.5835E-21	3.0482E-06
EGARCH			0.7495	1.4087	1.3808
			4.54E-01*	1.59E-01*	1.67E-01*
IGARCH				0.6018	0.4656
				5.47E-01*	6.41E-01*
FIGARCH					1.1569
					2.47E-01*

Table 6.21: HNX DM test statistics and p-values on Fixed Window (5.00%).

HNX - Rolling Window (0.01%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	16.7898	19.4741	11.7992	19.687	18.1775
	2.8982E-63	1.8226E-84	3.9383E-32	2.7860E-86	7.7772E-74
HS	53.648	59.6754	36.7577	58.6333	54.7262
	0.0000E+00	0.0000E+00	8.7514E-296	0.0000E+00	0.0000E+00
BRW	0.0966	0.6995	0.7602	1.6293	1.4029
	9.23E-01*	4.84E-01*	4.47E-01*	1.03E-01*	1.61E-01*
GARCH		4.2838	1.3453	9.0393	6.3787
		1.8376E-05	1.79E-01*	1.5773E-19	1.7856E-10
EGARCH			0.567	4.7508	2.8569
			5.71E-01*	2.0262E-06	4.28E-03***
IGARCH				0.3637	0.2839
				7.16E-01*	7.76E-01*
FIGARCH					0.8899
					3.74E-01*

Table 6.22: HNX DM test statistics and p-values on Rolling Window (0.01%).

HNX - Rolling Window (1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	15.7878	18.1574	11.1917	18.3447	16.9993
	3.7745E-56	1.1213E-73	4.4797E-29	3.6423E-75	8.3176E-65
HS	38.4068	41.8379	27.6249	41.4904	39.1654
	0.0000E+00	0.0000E+00	5.5885E-168	0.0000E+00	0.0000E+00
BRW	1.3757	2.4806	1.6053	3.3286	2.9111
	1.69E-01*	1.31E-02**	1.08E-01*	8.73E-04***	3.60E-03***
GARCH		4.4252	1.2571	8.7949	6.1552
		9.6338E-06	2.09E-01*	1.4316E-18	7.4972E-10
EGARCH			0.4642	4.4319	2.6201
			6.42E-01*	9.3386E-06	8.79E-03***
IGARCH				0.415	0.3351
				6.78E-01*	7.38E-01*
FIGARCH					0.9401
					3.47E-01*

Table 6.23: HNX DM test statistics and p-values on Rolling Window (1.00%).

HNX - Rolling Window (5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	14.4813	16.4733	10.4355	16.6534	15.5093
	1.5914E-47	5.7085E-61	1.7072E-25	2.8566E-62	3.0014E-54
HS	12.3645	14.2295	8.9618	14.4803	13.4487
	4.0664E-35	6.0100E-46	3.1933E-19	1.6135E-47	3.1353E-41
BRW	4.1624	3.8202	1.7774	2.7398	2.6062
	3.1495E-05	1.33E-04***	7.55E-02*	6.15E-03***	9.16E-03***
GARCH		4.5397	1.1575	8.4378	5.8649
		5.6321E-06	2.47E-01*	3.2331E-17	4.4928E-09
EGARCH			0.354	4.0642	2.3567
			7.23E-01*	4.8195E-05	1.84E-02**
IGARCH				0.4669	0.3872
				6.41E-01*	6.99E-01*
FIGARCH					0.9874
					3.23E-01*

Table 6.24: HNX DM test statistics and p-values on Rolling Window (5.00%).

6.5 Diebold Mariano Test Statistics and p-values from KLSE, STI, SETi, and HNX for 5-Year and 10-Year Timeframe

6.5.1 Malaysia - KLSE

KLSE - Fixed Window (5-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	5.4806	6.2907	2.4017	6.1669	4.3284
	4.2392E-08	3.1594E-10	1.63E-02**	6.9657E-10	1.5023E-05
HS	5.4686	6.1916	2.5436	6.0737	4.4389
	4.5355E-08	5.9538E-10	1.10E-02**	1.2499E-09	9.0428E-06
BRW	13.3865	14.2714	5.6471	14.036	9.5101
	7.2507E-41	3.3012E-46	1.6319E-08	9.3798E-45	1.9055E-21
GARCH		2.3767	0.0603	0.8765	0.4824
		1.75E-02**	9.52E-01*	3.81E-01*	6.30E-01*
EGARCH			0.2176	1.3047	0.229
			8.28E-01*	1.92E-01*	8.19E-01*
IGARCH				0.0417	0.2003
				9.67E-01*	8.41E-01*
FIGARCH					0.2572
					7.97E-01*

Table 6.25: KLSE DM test statistics and p-values on a 5-Year Fixed Window (1.00%).

KLSE - Fixed Window (10-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	8.8207	10.1703	5.5407	9.9901	7.8627
	1.1373E-18	2.6900E-24	3.0130E-08	1.6838E-23	3.7591E-15
HS	8.7308	10.0256	5.521	9.8859	7.8153
	2.5285E-18	1.1766E-23	3.3699E-08	4.7945E-23	5.4834E-15
BRW	0.2454	2.0663	1.2867	1.0705	1.6854
	8.06E-01*	3.88E-02**	1.98E-01*	2.84E-01*	9.19E-02*
GARCH		5.2398	1.5919	4.2481	3.1435
		1.6079E-07	1.11E-01*	2.1555E-05	1.67E-03***
EGARCH			2.5465	8.3303	4.6333
			1.09E-02**	8.0623E-17	3.5996E-06
IGARCH				1.1655	0.3121
				2.44E-01*	7.55E-01*
FIGARCH					2.3755
					1.75E-02**

Table 6.26: KLSE DM test statistics and p-values on a 10-Year Fixed Window (1.00%).

KLSE - Fixed Window (5-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	5.4005	6.1227	2.3745	6.0252	4.2158
	6.6446E-08	9.1991E-10	1.76E-02**	1.6892E-09	2.4887E-05
HS	9.5988	10.1005	4.3564	10.3282	7.4903
	8.0853E-22	5.4964E-24	1.3223E-05	5.2525E-25	6.8716E-14
BRW	7.0592	7.3246	2.6702	7.6136	4.6911
	1.6751E-12	2.3958E-13	7.58E-03***	2.6658E-14	2.7172E-06
GARCH		2.3167	0.1222	0.7397	0.4782
		2.05E-02**	9.03E-01*	4.60E-01*	6.33E-01*
EGARCH			0.1722	1.5024	0.265
			8.63E-01*	1.33E-01*	7.91E-01*
IGARCH				0.039	0.1032
				9.69E-01*	9.18E-01*
FIGARCH					0.2988
					7.65E-01*

Table 6.27: KLSE DM test statistics and p-values on a 5-Year Fixed Window (5.00%).

KLSE - Fixed Window (10-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	8.6823	9.9486	5.5166	9.7985	7.7669
	3.8781E-18	2.5585E-23	3.4554E-08	1.1429E-22	8.0416E-15
HS	10.1141	11.8578	6.2912	11.2366	8.836
	4.7849E-24	1.9608E-32	3.1492E-10	2.6957E-29	9.9224E-19
BRW	4.5908	4.2403	3.4929	5.5943	4.8289
	4.4156E-06	2.2326E-05	4.78E-04***	2.2153E-08	1.3731E-06
GARCH		5.1308	1.5791	4.1426	3.1211
		2.8845E-07	1.14E-01*	3.4343E-05	1.80E-03***
EGARCH			2.5216	8.1549	4.5779
			1.17E-02**	3.4949E-16	4.6970E-06
IGARCH				1.1583	0.3017
				2.47E-01*	7.63E-01*
FIGARCH					2.3659
					1.80E-02**

Table 6.28: KLSE DM test statistics and p-values on a 10-Year Fixed Window (5.00%).

KLSE - Rolling Window (5-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	6.4545	7.3273	2.8598	7.2744	5.3728
	1.0858E-10	2.3478E-13	4.24E-03***	3.4800E-13	7.7535E-08
HS	2.632	3.3229	1.2443	3.1861	2.398
	8.49E-03***	8.91E-04***	2.13E-01*	1.44E-03***	1.65E-02**
BRW	13.5834	14.651	6.0727	13.8828	10.3773
	5.0218E-42	1.3266E-48	1.2575E-09	8.0577E-44	3.1451E-25
GARCH		2.0943	0.0519	2.0324	0.6774
		3.62E-02**	9.59E-01*	4.21E-02**	4.98E-01*
EGARCH			0.2111	0.2623	0.0975
			8.33E-01*	7.93E-01*	9.22E-01*
IGARCH				0.1847	0.2627
				8.53E-01*	7.93E-01*
FIGARCH					0.0054
					9.96E-01*

Table 6.29: KLSE DM test statistics and p-values on a 5-Year Rolling Window (1.00%).

KLSE - Rolling Window (10-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	8.8207	10.1703	5.5407	9.9901	7.8627
	1.1373E-18	2.6900E-24	3.0130E-08	1.6838E-23	3.7591E-15
HS	8.7308	10.0256	5.521	9.8859	7.8153
	2.5285E-18	1.1766E-23	3.3699E-08	4.7945E-23	5.4834E-15
BRW	0.2454	2.0663	1.2867	1.0705	1.6854
	8.06E-01*	3.88E-02**	1.98E-01*	2.84E-01*	9.19E-02*
GARCH		5.2398	1.5919	4.2481	3.1435
		1.6079E-07	1.11E-01*	2.1555E-05	1.67E-03***
EGARCH			2.5465	8.3303	4.6333
			1.09E-02**	8.0623E-17	3.5996E-06
IGARCH				1.1655	0.3121
				2.44E-01*	7.55E-01*
FIGARCH					2.3755
					1.75E-02**

Table 6.30: KLSE DM test statistics and p-values on a 10-Year Rolling Window (1.00%).

KLSE - Rolling Window (5-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	6.3306	7.0842	2.8032	7.0748	5.2194
	2.4423E-10	1.3981E-12	5.06E-03***	1.4966E-12	1.7953E-07
HS	10.0028	10.5304	4.6044	10.699	8.2336
	1.4819E-23	6.2553E-26	4.1368E-06	1.0291E-26	1.8168E-16
BRW	7.3384	7.7837	2.9738	7.5661	5.3195
	2.1623E-13	7.0452E-15	2.94E-03***	3.8460E-14	1.0404E-07
GARCH		2.0923	0.1164	1.8384	0.6693
		3.64E-02**	9.07E-01*	6.60E-02*	5.03E-01*
EGARCH			0.169	0.5454	0.1596
			8.66E-01*	5.85E-01*	8.73E-01*
IGARCH				0.0973	0.1664
				9.23E-01*	8.68E-01*
FIGARCH					0.0663
					9.47E-01*

Table 6.31: KLSE DM test statistics and p-values on a 5-Year Rolling Window (5.00%).

KLSE - Rolling Window (10-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	8.8554	10.1521	5.4889	9.8925	7.7046
	8.3402E-19	3.2439E-24	4.0451E-08	4.4886E-23	1.3121E-14
HS	10.2432	12.006	6.2123	11.2807	8.7043
	1.2697E-24	3.3061E-33	5.2199E-10	1.6337E-29	3.1954E-18
BRW	4.5622	4.1716	3.4444	5.5045	4.7404
	5.0629E-06	3.0251E-05	5.72E-04***	3.7032E-08	2.1332E-06
GARCH		5.1775	1.6188	3.9608	3.0742
		2.2484E-07	1.05E-01*	7.4693E-05	2.11E-03***
EGARCH			2.5209	7.973	4.4544
			1.17E-02**	1.5483E-15	8.4143E-06
IGARCH				1.2506	0.3227
				2.11E-01*	7.47E-01*
FIGARCH					2.4572
					1.40E-02**

Table 6.32: KLSE DM test statistics and p-values on a 10-Year Rolling Window (5.00%).

6.5.2 Singapore - STI

STI - Fixed Window (5-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	7.0693	7.5144	2.0881	3.7591	3.2319
	1.5568E-12	5.7173E-14	3.68E-02**	1.71E-04***	1.23E-03***
HS	9.3417	9.9085	3.0295	6.0263	4.8194
	9.4801E-21	3.8251E-23	2.45E-03***	1.6780E-09	1.4400E-06
BRW	15.7243	15.896	9.7545	14.9213	13.0069
	1.0300E-55	6.7502E-57	1.7653E-22	2.3943E-50	1.1185E-38
GARCH		1.1177	1.2156	13.5228	3.891
		2.64E-01*	2.24E-01*	1.1471E-41	9.9841E-05
EGARCH			1.0273	8.114	2.9666
			3.04E-01*	4.8978E-16	3.01E-03***
IGARCH				0.67	0.2979
				5.03E-01*	7.66E-01*
FIGARCH					1.392
					1.64E-01*

Table 6.33: STI DM test statistics and p-values on a 5-Year Fixed Window (1.00%).

STI - Fixed Window (10-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	5.8246	5.1835	4.3563	6.8409	5.96
	5.7240E-09	2.1771E-07	1.3226E-05	7.8719E-12	2.5218E-09
HS	5.4884	4.747	4.1734	6.4924	5.6949
	4.0558E-08	2.0651E-06	3.0004E-05	8.4453E-11	1.2347E-08
BRW	8.1761	12.0538	3.0468	7.3887	4.7328
	2.9326E-16	1.8516E-33	2.31E-03***	1.4823E-13	2.2141E-06
GARCH		7.0927	1.6604	5.4036	3.4729
		1.3153E-12	9.68E-02*	6.5299E-08	5.15E-04***
EGARCH			3.026	11.2602	6.0215
			2.48E-03***	2.0626E-29	1.7285E-09
IGARCH				1.0351	0.3916
				3.01E-01*	6.95E-01*
FIGARCH					2.1647
					3.04E-02**

Table 6.34: STI DM test statistics and p-values on a 10-Year Fixed Window (1.00%).

STI - Fixed Window (5-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	5.6754	5.9301	1.7104	3.0315	2.5789
	1.3834E-08	3.0278E-09	8.72E-02*	2.43E-03***	9.91E-03***
HS	1.8738	2.2915	1.5785	4.7346	2.8907
	6.10E-02*	2.19E-02**	1.14E-01*	2.1945E-06	3.84E-03***
BRW	4.2526	4.3046	1.0907	1.6987	1.5801
	2.1129E-05	1.6731E-05	2.75E-01*	8.94E-02*	1.14E-01*
GARCH		0.9183	1.1563	11.6035	3.6487
		3.58E-01*	2.48E-01*	3.9565E-31	2.64E-04***
EGARCH			0.9958	7.4895	2.8468
			3.19E-01*	6.9162E-14	4.42E-03***
IGARCH				0.5751	0.2734
				5.65E-01*	7.85E-01*
FIGARCH					1.1291
					2.59E-01*

Table 6.35: STI DM test statistics and p-values on a 5-Year Fixed Window (5.00%).

STI - Fixed Window (10-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	5.8312	5.1822	4.3712	6.8543	5.9662
	5.5024E-09	2.1930E-07	1.2358E-05	7.1655E-12	2.4278E-09
HS	7.4409	7.2766	5.2626	8.4777	7.2281
	1.0002E-13	3.4238E-13	1.4205E-07	2.2970E-17	4.8973E-13
BRW	3.8732	2.5595	3.2292	4.8769	4.3815
	1.07E-04***	1.05E-02**	1.24E-03***	1.0777E-06	1.1787E-05
GARCH		7.0762	1.7394	5.4436	3.5559
		1.4816E-12	8.20E-02*	5.2225E-08	3.77E-04***
EGARCH			3.0811	11.1902	6.0476
			2.06E-03***	4.5538E-29	1.4704E-09
IGARCH				1.1093	0.4589
				2.67E-01*	6.46E-01*
FIGARCH					2.2358
					2.54E-02**

Table 6.36: STI DM test statistics and p-values on a 10-Year Fixed Window (5.00%).

STI - Rolling Window (5-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	8.0418	8.8627	2.7757	4.8848	4.5451
	8.8532E-16	7.8096E-19	5.51E-03***	1.0353E-06	5.4921E-06
HS	16.0625	17.0791	7.7604	13.2118	12.3176
	4.6765E-58	2.1234E-65	8.4646E-15	7.4968E-40	7.2844E-35
BRW	16.1136	16.378	10.5618	15.0104	14.3863
	2.0479E-58	2.7478E-60	4.4786E-26	6.2809E-51	6.3059E-47
GARCH		0.247	1.1428	10.4419	5.8868
		8.05E-01*	2.53E-01*	1.5955E-25	3.9373E-09
EGARCH			1.0467	6.1484	4.2015
			2.95E-01*	7.8287E-10	2.6510E-05
IGARCH				0.4937	0.4198
				6.22E-01*	6.75E-01*
FIGARCH					0.7791
					4.36E-01*

Table 6.37: STI DM test statistics and p-values on a 5-Year Rolling Window (1.00%).

STI - Rolling Window (10-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	5.8246	5.1835	4.3563	6.8409	5.96
	5.7240E-09	2.1771E-07	1.3226E-05	7.8719E-12	2.5218E-09
HS	5.4884	4.747	4.1734	6.4924	5.6949
	4.0558E-08	2.0651E-06	3.0004E-05	8.4453E-11	1.2347E-08
BRW	8.1761	12.0538	3.0468	7.3887	4.7328
	2.9326E-16	1.8516E-33	2.31E-03***	1.4823E-13	2.2141E-06
GARCH		7.0927	1.6604	5.4036	3.4729
		1.3153E-12	9.68E-02*	6.5299E-08	5.15E-04***
EGARCH			3.026	11.2602	6.0215
			2.48E-03***	2.0626E-29	1.7285E-09
IGARCH				1.0351	0.3916
				3.01E-01*	6.95E-01*
FIGARCH					2.1647
					3.04E-02**

Table 6.38: STI DM test statistics and p-values on a 10-Year Rolling Window (1.00%).

STI - Rolling Window (5-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	6.4803	7.0185	2.2681	3.9685	3.6566
	9.1555E-11	2.2427E-12	2.33E-02**	7.2335E-05	2.56E-04***
HS	0.3829	0.4048	0.6394	1.8108	1.5175
	7.02E-01*	6.86E-01*	5.23E-01*	7.02E-02*	1.29E-01*
BRW	4.5264	4.7931	1.3037	2.1956	2.0296
	5.9994E-06	1.6420E-06	1.92E-01*	2.81E-02**	4.24E-02**
GARCH		0.0853	1.1423	9.8894	5.6195
		9.32E-01*	2.53E-01*	4.6267E-23	1.9147E-08
EGARCH			1.0695	6.0403	4.1312
			2.85E-01*	1.5381E-09	3.6094E-05
IGARCH				0.3821	0.3393
				7.02E-01*	7.34E-01*
FIGARCH					0.5251
					5.99E-01*

Table 6.39: STI DM test statistics and p-values on a 5-Year Rolling Window (5.00%).

STI - Rolling Window (10-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	5.4301	4.6614	4.1561	6.3917	5.5929
	5.6316E-08	3.1413E-06	3.2379E-05	1.6406E-10	2.2330E-08
HS	7.0173	6.7448	5.04	7.9792	6.862
	2.2627E-12	1.5324E-11	4.6562E-07	1.4724E-15	6.7892E-12
BRW	3.9316	2.6071	3.252	4.9098	4.3455
	8.4400E-05	9.13E-03***	1.15E-03***	9.1162E-07	1.3895E-05
GARCH		7.0538	1.736	6.3575	3.4746
		1.7404E-12	8.26E-02*	2.0512E-10	5.12E-04***
EGARCH			3.1195	10.8409	6.108
			1.81E-03***	2.2039E-27	1.0091E-09
IGARCH				1.0515	0.6579
				2.93E-01*	5.11E-01*
FIGARCH					1.8088
					7.05E-02*

Table 6.40: STI DM test statistics and p-values on a 10-Year Rolling Window (5.00%).

6.5.3 Thailand - SETi

SETi - Fixed Window (5-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	1.9594	2.3071	1.5898	1.8744	2.6266
	5.01E-02*	2.10E-02**	1.12E-01*	6.09E-02*	8.62E-03***
HS	2.3351	2.6734	1.983	2.2556	2.035
	1.95E-02**	7.51E-03***	4.74E-02**	2.41E-02**	4.18E-02**
BRW	12.738	12.8686	11.9117	12.6266	11.0834
	3.6346E-37	6.7628E-38	1.0286E-32	1.5061E-36	1.5104E-28
GARCH		1.1163	1.1606	2.4519	3.6199
		2.64E-01*	2.46E-01*	1.42E-02**	2.95E-04***
EGARCH			1.5761	1.5584	3.8827
			1.15E-01*	1.19E-01*	1.03E-04***
IGARCH				0.9098	3.3194
				3.63E-01*	9.02E-04***
FIGARCH					3.5512
					3.83E-04***

Table 6.41: SETi DM test statistics and p-values on a 5-Year Fixed Window (1.00%).

SETi - Fixed Window (10-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	5.7683	5.2791	4.1148	5.7938	5.8521
	8.0073E-09	1.2981E-07	3.8759E-05	6.8792E-09	4.8528E-09
HS	5.0496	4.1842	3.6377	5.0434	5.1594
	4.4268E-07	2.8613E-05	2.75E-04***	4.5736E-07	2.4780E-07
BRW	1.5451	5.4231	0.8263	1.8285	1.2021
	1.22E-01*	5.8588E-08	4.09E-01*	6.75E-02*	2.29E-01*
GARCH		6.4665	0.3237	1.0505	0.9728
		1.0030E-10	7.46E-01*	2.94E-01*	3.31E-01*
EGARCH			2.5897	5.9266	5.9989
			9.61E-03***	3.0931E-09	1.9867E-09
IGARCH				0.5965	0.0226
				5.51E-01*	9.82E-01*
FIGARCH					3.2532
					1.14E-03***

Table 6.42: SETi DM test statistics and p-values on a 10-Year Fixed Window (1.00%).

SETi - Fixed Window (5-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	1.4929	1.7408	1.1343	1.4177	2.6058
	1.35E-01*	8.17E-02*	2.57E-01*	1.56E-01*	9.17E-03***
HS	6.8386	7.1698	6.8154	6.9169	8.6297
	7.9980E-12	7.5118E-13	9.3995E-12	4.6178E-12	6.1494E-18
BRW	2.0893	2.2864	1.6614	2.0064	2.7789
	3.67E-02**	2.22E-02**	9.66E-02*	4.48E-02**	5.45E-03***
GARCH		0.993	1.1132	2.1571	3.1123
		3.21E-01*	2.66E-01*	3.10E-02**	1.86E-03***
EGARCH			1.4676	1.3934	3.2698
			1.42E-01*	1.64E-01*	1.08E-03***
IGARCH				0.9075	2.8239
				3.64E-01*	4.74E-03***
FIGARCH					3.0536
					2.26E-03***

Table 6.43: SETi DM test statistics and p-values on a 5-Year Fixed Window (5.00%).

SETi - Fixed Window (10-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	5.8145	5.3286	4.1644	5.842	5.9026
	6.0819E-09	9.8951E-08	3.1224E-05	5.1588E-09	3.5785E-09
HS	7.6341	8.1265	5.4201	7.7421	7.6685
	2.2733E-14	4.4177E-16	5.9581E-08	9.7806E-15	1.7402E-14
BRW	5.5322	4.8838	3.9638	5.539	5.6094
	3.1620E-08	1.0407E-06	7.3774E-05	3.0412E-08	2.0303E-08
GARCH		6.473	0.3408	1.0351	0.9925
		9.6096E-11	7.33E-01*	3.01E-01*	3.21E-01*
EGARCH			2.6115	5.9091	6.0089
			9.01E-03***	3.4395E-09	1.8678E-09
IGARCH				0.6108	0.0339
				5.41E-01*	9.73E-01*
FIGARCH					3.2525
					1.14E-03***

Table 6.44: SETi DM test statistics and p-values on a 10-Year Fixed Window (5.00%).

SETi - Rolling Window (5-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	1.4067	2.1473	1.3507	1.3507	2.6641
	1.60E-01*	3.18E-02**	1.77E-01*	1.77E-01*	7.72E-03***
HS	2.2061	2.9385	2.1538	2.1538	1.737
	2.74E-02**	3.30E-03***	3.13E-02**	3.13E-02**	8.24E-02*
BRW	12.7916	13.1933	12.7269	12.7269	11.351
	1.8262E-37	9.5846E-40	4.1928E-37	4.1930E-37	7.3336E-30
GARCH		2.928	4.9102	4.9105	3.1729
		3.41E-03***	9.0986E-07	9.0857E-07	1.51E-03***
EGARCH			3.2185	3.2185	3.7016
			1.29E-03***	1.29E-03***	2.14E-04***
IGARCH				1.0147	3.1228
				3.10E-01*	1.79E-03***
FIGARCH					3.1228
					1.79E-03***

Table 6.45: SETi DM test statistics and p-values on a 5-Year Rolling Window (1.00%).

SETi - Rolling Window (10-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	5.7683	5.2791	4.1148	5.7938	5.8521
	8.0073E-09	1.2981E-07	3.8759E-05	6.8792E-09	4.8528E-09
HS	5.0496	4.1842	3.6377	5.0434	5.1594
	4.4268E-07	2.8613E-05	2.75E-04***	4.5736E-07	2.4780E-07
BRW	1.5451	5.4231	0.8263	1.8285	1.2021
	1.22E-01*	5.8588E-08	4.09E-01*	6.75E-02*	2.29E-01*
GARCH		6.4665	0.3237	1.0505	0.9728
		1.0030E-10	7.46E-01*	2.94E-01*	3.31E-01*
EGARCH			2.5897	5.9266	5.9989
			9.61E-03***	3.0931E-09	1.9867E-09
IGARCH				0.5965	0.0226
				5.51E-01*	9.82E-01*
FIGARCH					3.2532
					1.14E-03***

Table 6.46: SETi DM test statistics and p-values on a 10-Year Rolling Window (1.00%).

SETi - Rolling Window (5-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	0.9978	1.5992	0.9474	0.9474	2.5488
	3.18E-01*	1.10E-01*	3.43E-01*	3.43E-01*	1.08E-02**
HS	6.695	6.7779	6.745	6.745	8.017
	2.1562E-11	1.2192E-11	1.5305E-11	1.5305E-11	1.0832E-15
BRW	2.1504	2.6738	2.097	2.097	1.9805
	3.15E-02**	7.50E-03***	3.60E-02**	3.60E-02**	4.76E-02**
GARCH		2.6969	4.0359	4.0361	2.6898
		7.00E-03***	5.4396E-05	5.4342E-05	7.15E-03***
EGARCH			2.9616	2.9616	3.0795
			3.06E-03***	3.06E-03***	2.07E-03***
IGARCH				1.0135	2.646
				3.11E-01*	8.15E-03***
FIGARCH					2.646
					8.15E-03***

Table 6.47: SETi DM test statistics and p-values on a 5-Year Rolling Window (5.00%).

SETi - Rolling Window (10-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	5.5149	4.8138	3.7593	5.5087	5.197
	3.4906E-08	1.4810E-06	1.70E-04***	3.6144E-08	2.0253E-07
HS	7.2983	7.5628	4.9574	7.4065	6.9037
	2.9147E-13	3.9448E-14	7.1436E-07	1.2963E-13	5.0677E-12
BRW	5.5873	4.9091	3.7934	5.5748	5.2393
	2.3061E-08	9.1495E-07	1.49E-04***	2.4779E-08	1.6122E-07
GARCH		6.5765	0.1356	1.437	0.186
		4.8153E-11	8.92E-01*	1.51E-01*	8.52E-01*
EGARCH			2.3552	5.884	4.9734
			1.85E-02**	4.0048E-09	6.5793E-07
IGARCH				0.5087	0.2532
				6.11E-01*	8.00E-01*
FIGARCH					1.4096
					1.59E-01*

Table 6.48: SETi DM test statistics and p-values on a 10-Year Rolling Window (5.00%).

6.5.4 Vietnam - HNX

HNX - Fixed Window (5-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	6.3661	6.7089	4.0505	6.9145	6.002
	1.9387E-10	1.9604E-11	5.1100E-05	4.6946E-12	1.9494E-09
HS	3.888	3.6977	2.5981	4.1704	3.7429
	1.01E-04***	2.18E-04***	9.37E-03***	3.0408E-05	1.82E-04***
BRW	0.4446	1.7197	0.218	0.6807	0.1063
	6.57E-01*	8.55E-02*	8.27E-01*	4.96E-01*	9.15E-01*
GARCH		3.9763	0.6205	0.754	0.7249
		6.9992E-05	5.35E-01*	4.51E-01*	4.69E-01*
EGARCH			1.2332	3.7558	2.4722
			2.17E-01*	1.73E-04***	1.34E-02**
IGARCH				0.7928	0.5592
				4.28E-01*	5.76E-01*
FIGARCH					1.4389
					1.50E-01*

Table 6.49: HNX DM test statistics and p-values on a 5-Year Fixed Window (1.00%).

HNX - Fixed Window (10-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	9.5401	8.9171	5.6059	9.4489	8.5438
	1.4264E-21	4.7854E-19	2.0719E-08	3.4245E-21	1.2986E-17
HS	5.2589	3.5377	3.2704	5.0147	4.7128
	1.4495E-07	4.04E-04***	1.07E-03***	5.3104E-07	2.4438E-06
BRW	0.7146	2.508	0.8201	0.2668	0.6647
	4.75E-01*	1.21E-02**	4.12E-01*	7.90E-01*	5.06E-01*
GARCH		10.1759	0.651	2.4745	0.1598
		2.5392E-24	5.15E-01*	1.33E-02**	8.73E-01*
EGARCH			2.3446	8.5276	5.5008
			1.91E-02**	1.4936E-17	3.7815E-08
IGARCH				1.0204	0.8013
				3.08E-01*	4.23E-01*
FIGARCH					1.7607
					7.83E-02*

Table 6.50: HNX DM test statistics and p-values on a 10-Year Fixed Window (1.00%).

HNX - Fixed Window (5-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	6.0869	6.3721	3.997	6.5954	5.8065
	1.1511E-09	1.8644E-10	6.4142E-05	4.2397E-11	6.3779E-09
HS	7.1267	7.6088	4.6545	7.7301	6.7795
	1.0278E-12	2.7657E-14	3.2482E-06	1.0748E-14	1.2061E-11
BRW	0.0528	1.0666	0.4897	0.1414	0.3141
	9.58E-01*	2.86E-01*	6.24E-01*	8.88E-01*	7.53E-01*
GARCH		3.9223	0.6476	0.8155	0.7039
		8.7696E-05	5.17E-01*	4.15E-01*	4.82E-01*
EGARCH			1.2607	3.6323	2.4631
			2.07E-01*	2.81E-04***	1.38E-02**
IGARCH				0.8374	0.6068
				4.02E-01*	5.44E-01*
FIGARCH					1.4718
					1.41E-01*

Table 6.51: HNX DM test statistics and p-values on a 5-Year Fixed Window (5.00%).

HNX - Fixed Window (10-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	9.1028	8.4455	5.4789	8.9963	8.2139
	8.8069E-20	3.0287E-17	4.2798E-08	2.3350E-19	2.1411E-16
HS	10.7058	10.4453	6.4463	10.656	9.7017
	9.5648E-27	1.5397E-25	1.1460E-10	1.6339E-26	2.9661E-22
BRW	4.4421	2.3092	2.6732	4.1431	3.8624
	8.9096E-06	2.09E-02**	7.51E-03***	3.4262E-05	1.12E-04***
GARCH		9.948	0.6466	2.4982	0.0939
		2.5735E-23	5.18E-01*	1.25E-02**	9.25E-01*
EGARCH			2.3231	8.2896	5.3973
			2.02E-02**	1.1365E-16	6.7642E-08
IGARCH				1.0263	0.8205
				3.05E-01*	4.12E-01*
FIGARCH					1.7108
					8.71E-02*

Table 6.52: HNX DM test statistics and p-values on a 10-Year Fixed Window (5.00%).

HNX - Rolling Window (5-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	6.8293	7.1416	4.5872	7.4429	7.0014
	8.5327E-12	9.2265E-13	4.4931E-06	9.8529E-14	2.5343E-12
HS	2.9183	2.4032	2.1737	3.2139	3.091
	3.52E-03***	1.63E-02**	2.97E-02**	1.31E-03***	1.99E-03***
BRW	0.6565	2.0713	0.0883	0.6845	0.4529
	5.12E-01*	3.83E-02**	9.30E-01*	4.94E-01*	6.51E-01*
GARCH		4.4654	0.6783	0.2124	0.8175
		7.9936E-06	4.98E-01*	8.32E-01*	4.14E-01*
EGARCH			1.4558	5.8647	4.0657
			1.45E-01*	4.4983E-09	4.7890E-05
IGARCH				0.6794	0.6105
				4.97E-01*	5.42E-01*
FIGARCH					1.0249
					3.05E-01*

Table 6.53: HNX DM test statistics and p-values on a 5-Year Rolling Window (1.00%).

HNX - Rolling Window (10-Year, 1.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	9.5401	8.9171	5.6059	9.4489	8.5438
	1.4264E-21	4.7854E-19	2.0719E-08	3.4245E-21	1.2986E-17
HS	5.2589	3.5377	3.2704	5.0147	4.7128
	1.4495E-07	4.04E-04***	1.07E-03***	5.3104E-07	2.4438E-06
BRW	0.7146	2.508	0.8201	0.2668	0.6647
	4.75E-01*	1.21E-02**	4.12E-01*	7.90E-01*	5.06E-01*
GARCH		10.1759	0.651	2.4745	0.1598
		2.5392E-24	5.15E-01*	1.33E-02**	8.73E-01*
EGARCH			2.3446	8.5276	5.5008
			1.91E-02**	1.4936E-17	3.7815E-08
IGARCH				1.0204	0.8013
				3.08E-01*	4.23E-01*
FIGARCH					1.7607
					7.83E-02*

Table 6.54: HNX DM test statistics and p-values on a 10-Year Rolling Window (1.00%).

HNX - Rolling Window (5-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	6.4705	6.7162	4.4956	7.0334	6.6845
	9.7694E-11	1.8652E-11	6.9386E-06	2.0157E-12	2.3179E-11
HS	7.2505	7.639	5.029	7.8609	7.4697
	4.1510E-13	2.1888E-14	4.9316E-07	3.8124E-15	8.0374E-14
BRW	0.0884	1.3241	0.4263	0.0824	0.1081
	9.30E-01*	1.85E-01*	6.70E-01*	9.34E-01*	9.14E-01*
GARCH		4.373	0.7213	0.0884	0.8331
		1.2255E-05	4.71E-01*	9.30E-01*	4.05E-01*
EGARCH			1.4934	5.6639	4.094
			1.35E-01*	1.4793E-08	4.2391E-05
IGARCH				0.7409	0.6569
				4.59E-01*	5.11E-01*
FIGARCH					1.1801
					2.38E-01*

Table 6.55: HNX DM test statistics and p-values on a 5-Year Rolling Window (5.00%).

HNX - Rolling Window (10-Year, 5.00%)					
	GARCH	EGARCH	IGARCH	FIGARCH	OPT-FI
DN	8.1663	7.1923	4.7075	7.7923	7.0213
	3.1791E-16	6.3698E-13	2.5078E-06	6.5811E-15	2.1975E-12
HS	9.6102	9.0377	5.5206	9.3423	8.3078
	7.2443E-22	1.5994E-19	3.3791E-08	9.4233E-21	9.7454E-17
BRW	4.6297	2.3904	2.5891	3.9222	3.7562
	3.6617E-06	1.68E-02**	9.62E-03***	8.7745E-05	1.73E-04***
GARCH		10.1239	0.422	5.0875	0.3391
		4.3262E-24	6.73E-01*	3.6290E-07	7.35E-01*
EGARCH			2.1743	7.3598	4.7623
			2.97E-02**	1.8421E-13	1.9144E-06
IGARCH				1.1547	0.7412
				2.48E-01*	4.59E-01*
FIGARCH					2.3671
					1.79E-02**

Table 6.56: HNX DM test statistics and p-values on a 10-Year Rolling Window (5.00%).

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