Convex optimization : Homework 3

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1 Interior points Method

- The gradient of ϕ_t the objective function with a logarithmic barrier is:

$$\nabla \phi_t(x) = t(Qx + p) + \alpha(x)$$

where for any $k = 1, ..., d$,

$$\alpha_k(x) = \sum_{i=1}^{p} \frac{a_{ik}}{b_i - \sum_{j=1}^{d} a_{ij} x_j}. \tag{1}$$

- The hessian of ϕ_t is:

$$\nabla^2 \phi_t(x) = tQ + \beta(x)$$

where for any $k, m = 1, ..., d$,

$$\beta_{km}(x) = \sum_{i=1}^{p} \frac{a_{ik} a_{im}}{(b_i - \sum_{j=1}^{d} a_{ij} x_j)^2} \tag{2}$$

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2 Newton Method

See Matlab Code

3 Support Vector Machine Problem

1. For the primal problem, a possible strictly feasible point is $x = (w, z)^t$ where for , $i = 1, ..., n$, $z_i = 2$ and for $k = 1, ..., d$, $w_k = 0$. For the dual problem, a possible strictly feasible point is λ such that for any $i = 1, ..., n$, $\lambda_i = \frac{1}{2n\tau}$.

2. See Matlab Code

3. The results obtained with different values of τ are shown in Figure 1. It seems that the optimal score is obtained when the regularization parameter is around 10^{-1}.

4. The duality gap is computed here as the difference of the primal value and the dual value at a given number of Newton's method iteration. The duality gaps for different values of mu using damped Newton's method are shown on Figure 2. As expected the decreasing rate of the duality gap increases with μ . However It seems that this rate stabilizes above a certain value of μ.

   The duality gaps for the same values of mu, this time using backtracking line search, are shown on Figure 3. It seems that in this particular case the backtracking line search method needs less newton iterations to give solution at given precision, especially with $\mu = 50$.

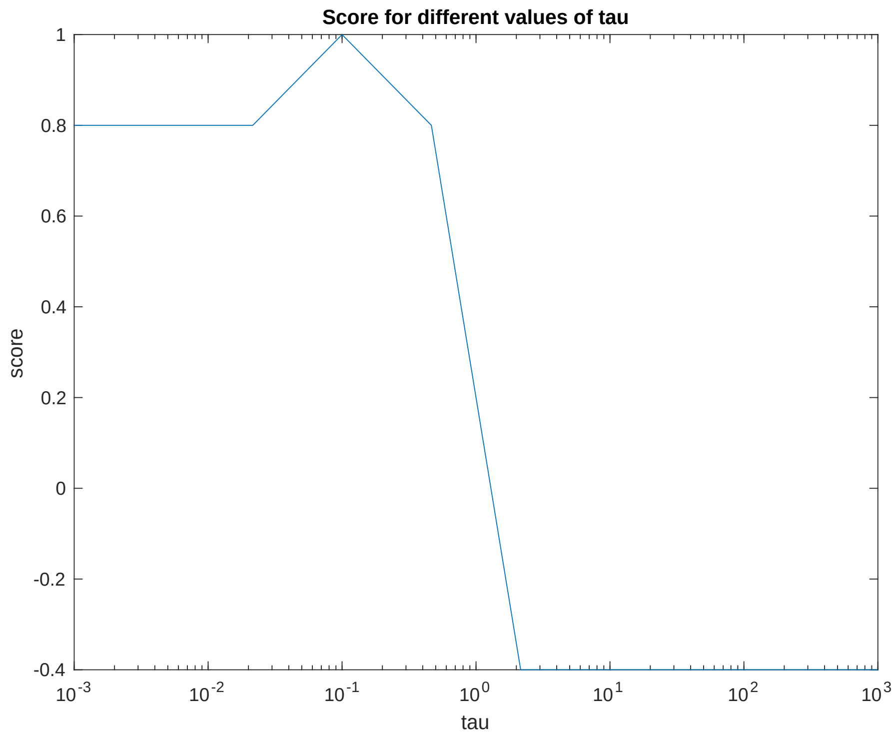

Figure 1: Scores obtained with different values of τ

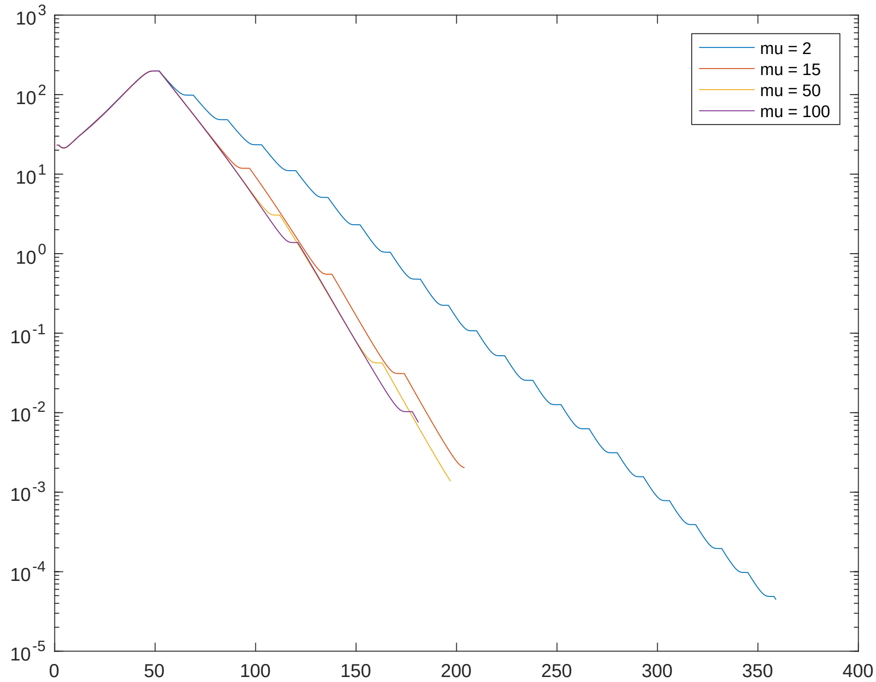

Figure 2: Duality gaps for different values of μ, using damped Newton method

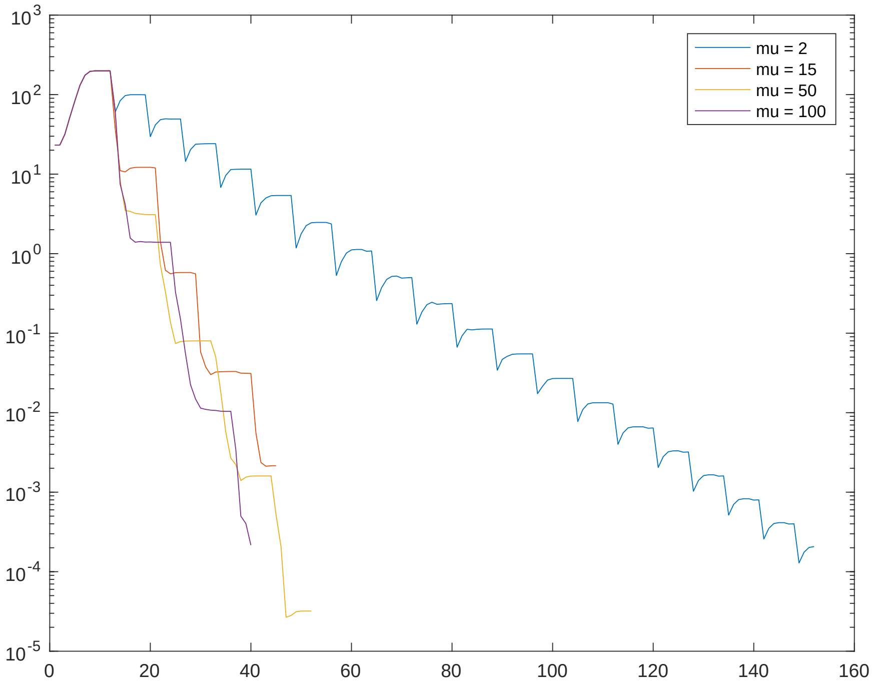

Figure 3: Duality gaps for different values of μ, using backtracking line search