Leonard De Vinci Graduate School Of Engineering

VBA Report

About Volatility

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1 Introduction

There are many ways to look at volatility.
The intuitive meaning of the word is that volatility measures the level of fluctuations for a particular price.

The way we measure it, the unit we use, the time-scale at which we are looking all have an impact and should be specified in order to transform a single volatility number into a solid understanding of how much level of fluctuation there is in that particular stock.
By taking a more academic approach based on statistics, one can argue that the value of the stock in one year is uncertain and assign a probability distribution to it. It could be desirable to use the width or standard deviation of this distribution to link to the volatility of the stock. This point of view is exactly what has become the market standard.
It is clear that the statistical approach is focused on the one-year horizon, whereas a trader, who wants to delta-hedge on a daily basis, is not so interested in knowing the uncertainty accumulated over the year. What he is really interested in is to understand how the uncertainty plays a role on a much smaller scale, such that piled up over the year it leads the same distribution as the statistician has presented.

In mathematical terms, knowing the distribution at one time (or multiple times) is not enough to complete the dynamic picture. One needs to know how the distribution changes over time. Clearly, on a very short time-scale, the uncertainty is very small and the distribution function should be sharply peaked about the current level of the stock. As the time horizon increases, the density should widen up.
One can show that at any time t, the solution of the Black-Scholes SDE, describing a model for the movement of the stock, is a random variable S(t) that behaves according to a lognormal distribution. So at any time t, we have a density that depends on the original parameters in the equation, being the the drift μ and the volatility parameter σ.
In a way, when people use the word volatility, they also agree on the underlying mathematical model !
A higher volatility means more uncertainty about the size of an asset's fluctuations and, as such, it can be considered a measurement of uncertainty.

Volatility is dynamic and changes a great deal over time. It experiences high and low regimes, but it also has a long-term mean to which it reverts. Also, as a stock market witnesses a large decline, volatility tends to shoot up: we therefore generally see a negative correlation between such assets and their volatilities.

2 Project and Data presentation

The project focuses on analyzing and working with financial data to study volatility. It is divided into five sheets, each with specific objectives and tasks related to the analysis of Credit Agricole (CA) and BNP Paribas (BNP) stock data, as well as implied volatility calculations for call options on these stocks. Additionally, the last sheet deals with implied volatility and volatility surface calculations for Amazon call and put options.

In the first sheet and second sheet, we import historical price data for Credit Agricole and BNP Paribas stocks over the past five years from Yahoo Finance. This data will serve as the foundation for our analysis in the subsequent sheets.

The third sheet focuses on analyzing the returns of Credit Agricole and BNP Paribas stocks. Key highlights include:

- Return Calculation: We calculate the returns of CA and BNP stocks based on the price data. Returns are an essential component in assessing volatility.
- Statistical Analysis: We conduct a preliminary statistical analysis of the data. This may include measures like mean, standard deviation, skewness, and kurtosis to gain insights into the data's distribution.
- Correlation Analysis: Notably, we identify a strong correlation between the returns of CA and BNP. This finding aligns with common market intuition, as these two stocks often move together.
- Volatility Calculation: We compute realized volatility for both CA and BNP. Additionally, we implement the Heston stochastic volatility model to estimate stochastic volatility.
- Implied Volatility Calculator: We provide buttons for calculating implied volatility for specified strike prices, spot prices, and maturities for call options on CA and BNP stocks.

The fourth and fifth sheet focus on implied volatility calculations and the construction of a volatility surface for Amazon call and put options. This involves:

- Data Import: We import real call and put option data for Amazon
- Implied Volatility Calculation: Using option pricing models like Black-Scholes, we calculate the implied volatility for the given options
- Volatility Surface: We construct a volatility surface, which displays implied volatility levels across various strike prices and maturities. This surface helps visualize implied volatility changes

3 Realized Volatility

This is probably the most common volatility measure.

One could imagine selecting a stock and a certain time period from the past, and trying to estimate the σ parameter in the Black-Scholes model based on this data.

This requires knowledge of Ito's formula, which allows us to transform the Black-Scholes equation into a more suitable format. The solution of this equation is following a lognormal distribution. So the logarithmic of the stock price follows a normal distribution.

If one wants to estimate the σ parameter of the stock, one can use the returns over one day and calculate the standard deviation from them. This would give the volatility over one day.

The first obvious question is how many days there are in a year. In your dataset, there are no quotes for weekends and holidays. So there is no volatility to observe there. Therefore a common approach is to use the number of trading days in a year.

To move from the standard deviation of daily returns to an annualized volatility therefore requires you to multiply by the daily volatility by $\sqrt{252}$.

So, that's what we did in sheet 3. For CA and BNP, we take the standard deviations and then multiply them by the square root of 252 and by 100 to obtain the annualized volatility in percentage terms.

E	F	G	H
Standard Deviation CA	Annualized Volatilty CA (in %)	Standard Deviation BNP	Annualized Volatility BNP(in %)
0,021382111	33,9430493	0,022519162	35,74806164

Figure 1: Results of realized volatility for CA and BNP returns

4 Implied Volatility

This is a more market-related volatility concept. This takes everything to the next step, taking into account the options market.

By observing the price of the option, one can back out the σ parameter one has to push into the formula in order to find that price. The market has adjusted for the shortcomings of the Black-Scholes model and the market-implied distribution is not lognormal anymore.

However, the beauty of the Black-Scholes formula is that you can tune your σ parameter such that you match this market price of the option.

"Implied volatility is the wrong number you put in the wrong formula to get the right price." Riccardo Rebonato

Vanilla options are quoted in terms of their implied volatilities since this, or a given price, amounts to the same information. The implied volatilities are in fact the market's consensus on the forward-looking volatility of the asset. This implied volatility incorporates the forward views on all market participants on the asset's volatility.

Sometimes analysts try to use implied volatility to defer conclusions about market direction. It is very tempting to think that this information is present in the option market, but the motivation for buying an OTM put does not have to be because the buyer is expecting a decrease in the stock price.

So, to calculate implied volatility, we need three functions that we have coded in VBA. The first one is a calculation method using the Newton-Raphson approach. The second is the classic Black-Scholes function, and the last one corresponds to Vega.
In sheet 3, we have used these functions and assigned macros to two buttons in order to obtain the implied volatility of a call option on CA and BNP with a given strike, spot price, and maturity.

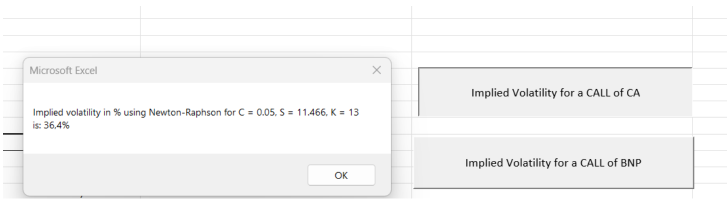

Figure 2: Implied Volatility for a call on CA

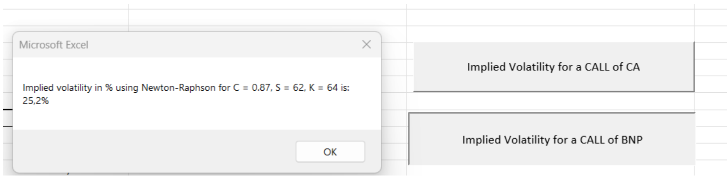

Figure 3: Implied Volatility for a call on BNP

5 Volatility Surface

Vanilla options are quoted in volatility terms.

For this to work, both counterparties have first to agree on the values of the inputs to the Black-Scholes equation. The volatility one must plug into the Black-Scholes formula to get the true market price of a vanilla option is called the implied volatility.

In liquid markets, brokers will quote fairly tight two-way prices for vanilla options at several strikes and several maturities.

Plotting the associated implied volatilities in a three-dimensional plot results in what is called the volatility surface.

In sheet 5, we collected call and put option data for Amazon with various strike prices and maturities. We then calculated implied volatility using two VBA functions: a slightly modified Black-Scholes function (which directly handles both puts and calls) and a bisection method. Finally, we calculated the "Implied Volatility Fit" using the Dumas, Fleming, and Whaley equation.

With this implied volatility data for multiple strikes and maturities, we were able to create a 3D graph that represents the associated volatility surface. The result is presented in the following figure.

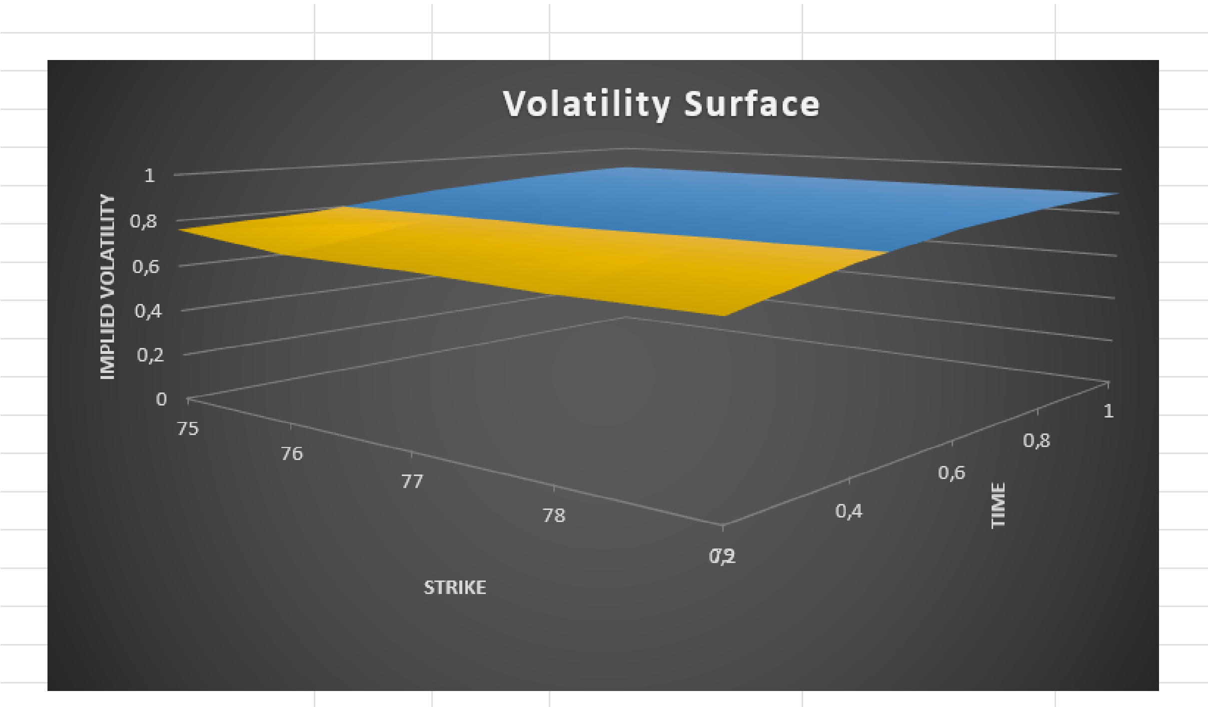

Figure 4: Volatility surface of Amazon Call for various strike and maturity

6 Local Volatility

The concept of local volatility, introduced by Emanuel Derman, Iraj Kani, and Dupire around 1994, represents a significant advancement in the modeling of financial derivatives. Local volatility is a deterministic function of the asset price and time, aiming to capture the true volatility dynamics of an underlying asset.

Basically, local volatility tries to figure out the real volatility in an option by looking at the strike price and when it expires. This is different from old-fashioned models that think the volatility stays the same no matter the strike price or expiration date. The old way of thinking doesn't always capture the true ups and downs in the market, especially when there are uneven patterns in volatility.

Local volatility, as proposed by Dupire, replaces the constant volatility function with a more dynamic one, resulting in a model that better aligns with observed market prices. By acknowledging that volatility is not constant but rather varies with the underlying asset's price and time, local volatility offers a more nuanced and realistic depiction of the financial markets.

One notable advantage of local volatility is its ability to provide more accurate results than implied volatility, particularly in scenarios where the market exhibits complex volatility patterns. Implied volatility, derived from option prices, may not always capture the true underlying volatility due to market uncertainties or inefficiencies.

To calculate the local volatility of stocks, Dupire's formula is employed.

$$\sigma_{\text{Dupire}}(K,T) = \sqrt{\frac{\frac{\partial C}{\partial T} + rK\frac{\partial C}{\partial K}}{\frac{1}{2}K^2\frac{\partial^2 C}{\partial K^2}}}$$

In our current context, we employed finite differences to calculate derivatives, applying the resulting values directly to our computations.

Local volatility in % using Dupire Formula for C = 0,05, S = 11,466, K = 13 is 37%

Figure 5: Implied Volatility for a call on CA

Local volatility in % using Dupire Formula for C = 0,87, S = 62, K = 65 is 30,1%

Figure 6: Implied Volatility for a call on BNP

7 Stochastic Volatility

The heteroskedasticity in stock returns makes it very tempting to express the volatility as a stochastic process.

Based on a body of work on stochastic volatility models (Scott (1987), Hull and White (1987), Stein and Stein (1991)), Heston (1993) advanced the first stochastic volatility model with a generalized solution. His model permits the capturing of essential features of stock markets, namely the leverage effect, the volatility clustering and the tail behavior of stock returns. However, it cannot yield realistic implied volatilities for short maturities.

The Heston Model commonly stands out among the stochastic volatility models for several reasons:

- It provides a closed-form solution for European Call options
- It allows the stock price to follow a non log-Normal probability distribution
- It expresses the volatility as a mean-reverting process
- It fits pretty well the implied volatility surface of option prices observed in the market
- It takes into account a possible correlation between the stock price and its volatility

In this context, we focused on a simplified approach to the Heston model. In sheet 3, we developed a VBA function that calculates stochastic volatility using the Heston model for the returns of CA and BNP.It's important to note that this implementation is a simplified version of the Heston model and does not involve complex numerical methods like Monte Carlo simulations or partial differential equations that are typically used to solve the full Heston model. Instead, it provides a basic estimate of the stochastic volatility based on historical return data and a few key model parameters.

I	J
Stochastic Volatility with Heston Model for CA (in %)	Stochastic Volatility with Heston Model for BNP (in %)
21,88802437	21,88574364

Figure 7: Results of stochastic volatility for CA and BNP returns