Huffman Coding

From ASCII Coding to Huffman Coding

Many programming languages use ASCII coding for characters (ASCII stands for American Standard Code for Information Interchange). Some recent languages, e.g., Java, use UNICODE which, because it can encode a bigger set of characters, is more useful for languages like Japanese and Chinese which have a larger set of characters than are used in English.

We'll use ASCII encoding of characters as an example. In ASCII, every character is encoded with the same number of bits: 8 bits per character. Since there are 256 different values that can be encoded with 8 bits, there are potentially 256 different characters in the ASCII character set. The common characters, e.g., alphanumeric characters, punctuation, control characters, etc., use only 7 bits; there are 128 different characters that can be encoded with 7 bits. In C++ for example, the type *char* is divided into subtypes unsigned-char and (the default signed) char. As we'll see, Huffman coding compresses data by using fewer bits to encode more frequently occurring characters so that not all characters are encoded with 8 bits. In Java there are no unsigned types and *char* values use 16 bits (Unicode compared to ASCII). Substantial compression results regardless of the character-encoding used by a language or platform.

A Simple Coding Example

We'll look at how the string "go go gophers" is encoded in ASCII, how we might save bits using a simpler coding scheme, and how Huffman coding is used to compress the data resulting in still more savings.

With an ASCII encoding (8 bits per character) the 13 character string "go go gophers" requires 104 bits. The table below on the left shows how the coding works.

coding a message

ASCII coding

char	ASCII	binary
g	103	1100111
o	111	1101111
p	112	1110000
h	104	1101000
e	101	1100101
r	114	1110010
s	115	1110011
space	32	1000000

3-bit coding

char	code	binary
g	0	000
o	1	001
p	2	010
h	3	011
e	4	100
r	5	101
s	6	110
space	7	111

The string "go go gophers" would be written (coded numerically) as 103 111 32 103 111 32 103 111 112 104 101 114 115. Although not easily readable by humans, this would be written as the following stream of bits (the spaces would not be written, just the 0's and 1's)

1100111 1101111 1100000 1100111 1101111 1000000 1100111 1101111 1110000 1101000 1100101 1110010 1110011

Since there are only eight different characters in "go go gophers", it's possible to use only 3 bits to encode the different characters. We might, for example, use the encoding in the table on the right above, though other 3-bit encodings are possible.

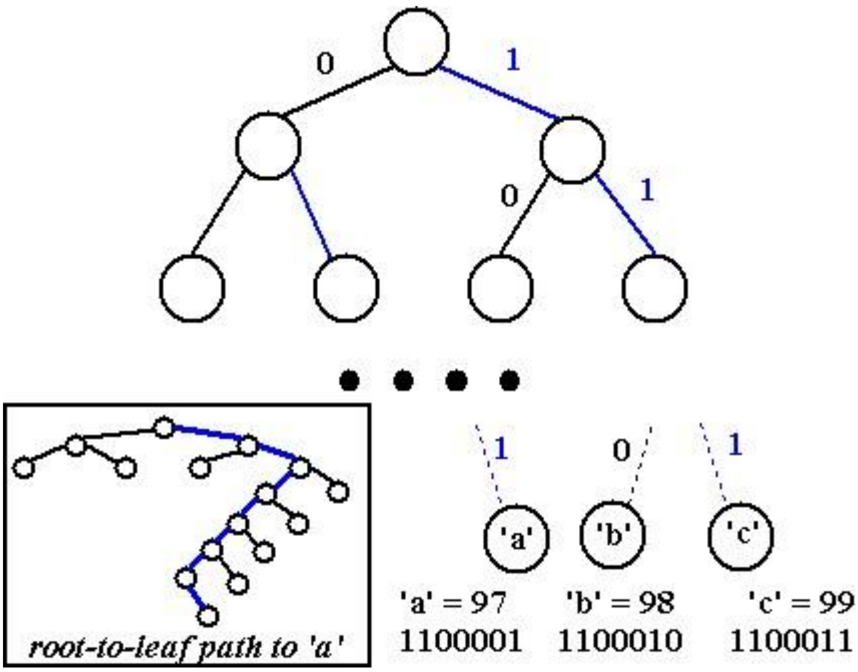
Now the string "go go gophers" would be encoded as 0 1 7 0 1 7 0 1 2 3 4 5 6 or, as bits:

000 001 111 000 001 111 000 001 010 011 100 101 110 111

By using three bits per character, the string "go go gophers" uses a total of 39 bits instead of 104 bits. More bits can be saved if we use fewer than three bits to encode characters like g, o, and space that occur frequently and more than three bits to encode characters like e, p, h, r, and s that occur less frequently in "go go gophers". This is the basic idea behind Huffman coding: to use fewer bits for more frequently occurring characters. We'll see how this is done using a tree that stores characters at the leaves, and whose root-to-leaf paths provide the bit sequence used to encode the characters.

Towards a Coding Tree

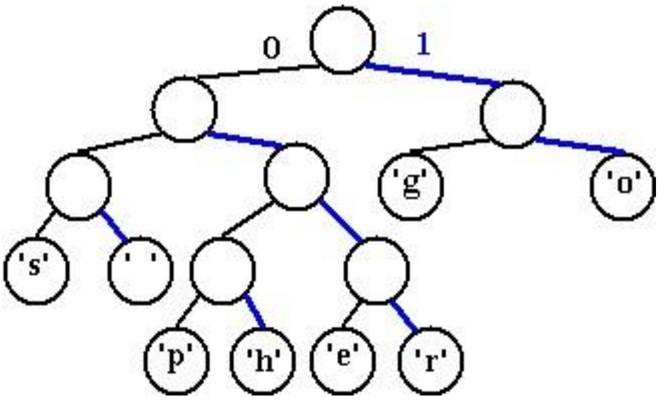
A tree view of the ASCII character set



Using a tree (actually a binary trie, more on that later) all characters are stored at the leaves of a complete tree. In the diagram to the right, the tree has eight levels meaning that the root-to-leaf path always has seven edges. A left-edge (black in the diagram) is numbered 0, a right-edge (blue in the diagram) is numbered 1. The ASCII code for any character/leaf is obtained by following the root-to-leaf path and catening the 0's and 1's. For example, the character 'a', which has ASCII value 97 (1100001 in binary), is shown with root-to-leaf path of *right-right-left-left-left-left-right*.

The structure of the tree can be used to determine the coding of any leaf by using the 0/1 edge convention described. If we use a different tree, we get a different coding. As an example, the tree below on the right yields the coding shown on the left.

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



Using this coding, "go go gophers" is encoded (spaces wouldn't appear in the bitstream) as:

10 11 001 10 11 001 10 11 0100 0101 0110 0111 000

This is a total of 37 bits, which saves two bits from the encoding in which each of the 8 characters has a 3-bit encoding that is shown above! The bits are saved by coding frequently occurring characters like 'g' and 'o' with fewer bits (here two bits) than characters that occur less frequently like 'p', 'h', 'e', and 'r'.

The character-encoding induced by the tree can be used to decode a stream of bits as well as encode a string into a stream of bits. You can try to decode the following bitstream; the answer with an explanation follows:

01010110011100100001000101011001110110001101101100000010101
011001110110

To decode the stream, start at the root of the encoding tree, and follow a left-branch for a 0, a right branch for a 1. When you reach a leaf, write the character stored at the leaf, and start again at the top of the tree. To start, the bits are 010101100111. This yields *left-right-left-right* to the letter 'h', followed (starting again at the root) with *left-right-right-left* to the letter 'e', followed by *left-right-right-right* to the letter 'r'. Continuing until all the bits are processed yields

her sphere goes here

Prefix codes and Huffman Codes

When all characters are stored in leaves, and every interior/(non-leaf) node has two children, the coding induced by the 0/1 convention outlined above has what is called the *prefix property*: no bit-sequence encoding of a character is the prefix of any other bit-sequence encoding. This makes it possible to decode a bitstream using the coding tree by following root-to-leaf paths. The tree shown above for "go go gophers" is an optimal tree: there are no other trees with the same characters that use fewer bits to encode the string "go go gophers". There are other trees that use 37 bits; for example you can simply swap any sibling nodes and get a different encoding that uses the same number of bits. We need an algorithm for constructing an optimal tree which in turn yields a minimal per-character encoding/compression. This algorithm is called Huffman coding, and was invented by D. Huffman in 1952. It is an example of a greedy algorithm.

Huffman Coding

We'll use Huffman's algorithm to construct a tree that is used for data compression. In the previous section we saw examples of how a stream of bits can be generated from an encoding, e.g., how "go go gophers" was written as 1011001101100110110100010101100111000. We also saw how the tree can be used to decode a stream of bits. We'll discuss how to construct the tree here.

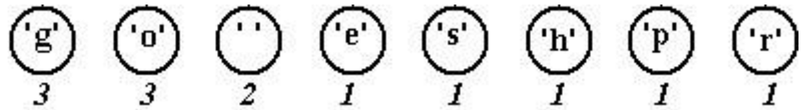
We'll assume that each character has an associated weight equal to the number of times the character occurs in a file, for example. In the "go go gophers" example, the characters 'g' and 'o' have weight 3, the space has weight 2, and the other characters have weight 1. When compressing a file we'll need to calculate these weights, we'll ignore this step for now and assume that all character weights have been calculated. Huffman's algorithm assumes that we're building a single tree from a group (or forest) of trees. Initially, all the trees have a single node with a character and the character's weight. Trees are combined by picking two trees, and making a new tree from the two trees. This decreases the number of trees by one at each step since two trees are combined into one tree. The algorithm is as follows:

1. Begin with a forest of trees. All trees are one node, with the weight of the tree equal to the weight of the character in the node. Characters that occur most frequently have the highest weights. Characters that occur least frequently have the smallest weights.
2. Repeat this step until there is only one tree:

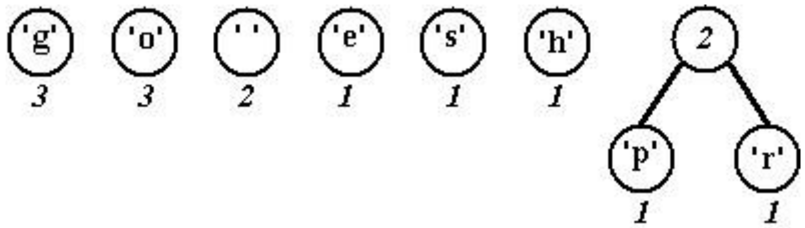
   Choose two trees with the smallest weights, call these trees T_1 and T_2. Create a new tree whose root has a weight equal to the sum of the weights $T_1 + T_2$ and whose left subtree is T_1 and whose right subtree is T_2.

3. The single tree left after the previous step is an optimal encoding tree.

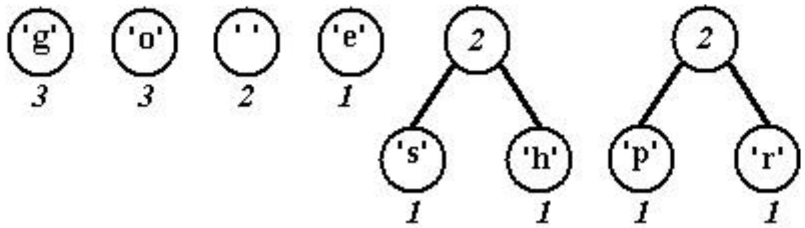
We'll use the string "go go gophers" as an example. Initially we have the forest shown below. The nodes are shown with a weight/count that represents the number of times the node's character occurs.



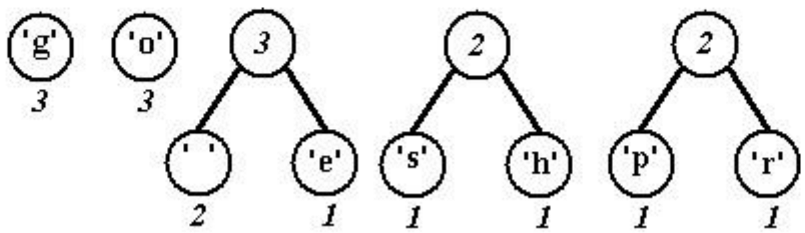
We pick two minimal nodes. There are five nodes with the minimal weight of one, it doesn't matter which two we pick. In a program, the deterministic aspects of the program will dictate which two are chosen, e.g., the first two in an array, or the elements returned by a priority queue implementation. We create a new tree whose root is weighted by the sum of the weights chosen. We now have a forest of seven trees as shown here:



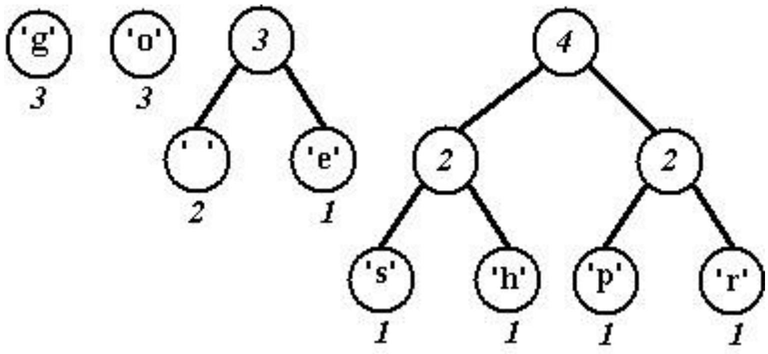
Choosing two minimal trees yields another tree with weight two as shown below. There are now six trees in the forest of trees that will eventually build an encoding tree.



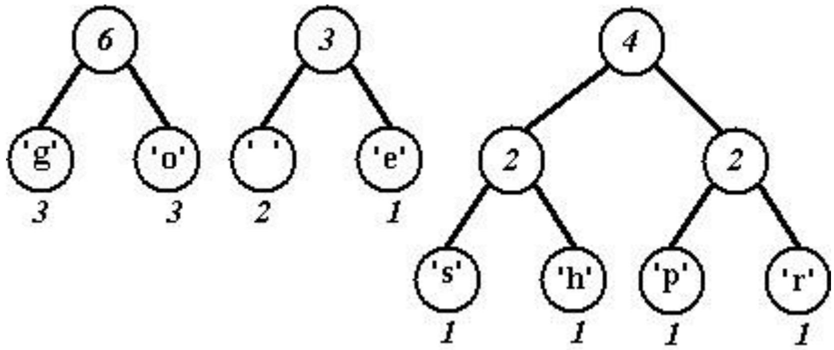
Again we must choose the two trees of minimal weight. The lowest weight is the 'e'-node/tree with weight equal to one. There are three trees with weight two, we can choose any of these to create a new tree whose weight will be three.



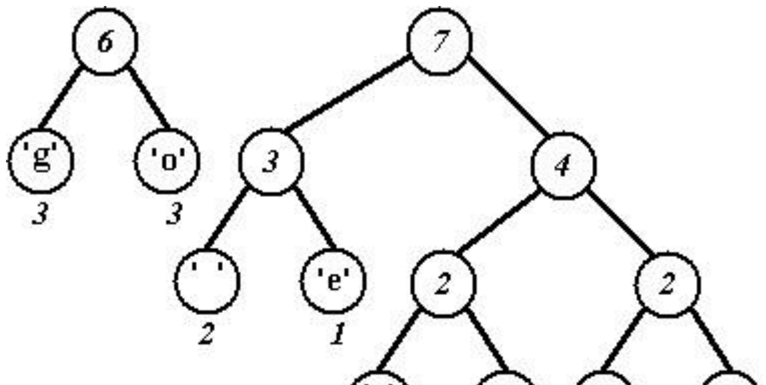
Now there are two trees with weight equal to two. These are joined into a new tree whose weight is four. There are four trees left, one whose weight is four and three with a weight of three.



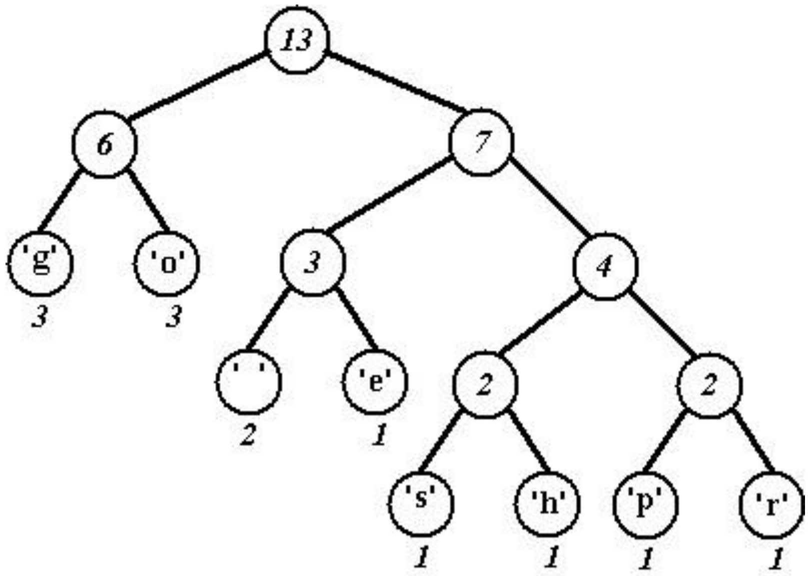
Two minimal (three weight) trees are joined into a tree whose weight is six. In the diagram below we choose the 'g' and 'o' trees (we could have chosen the 'g' tree and the space-'e' tree or the 'o' tree and the space-'e' tree.) There are three trees left.



The minimal trees have weights of three and four, these are joined into a tree whose weight is seven leaving two trees.



Finally, the last two trees are joined into a final tree whose weight is thirteen, the sum of the two weights six and seven. Note that this tree is different from the tree we used to illustrate Huffman coding above, and the bit patterns for each character are different, but the total number of bits used to encode "go go gophers" is the same.



The character encoding induced by the last tree is shown below where again, 0 is used for left edges and 1 for right edges.

char	binary
'g'	00
'o'	01
'p'	1110
'h'	1101
'e'	101
'r'	1111
's'	1100
' '	100

The string "go go gophers" would be encoded as shown (with spaces used for easier reading, the spaces wouldn't appear in the real encoding).

00 01 100 00 01 100 00 01 1110 1101 101 1111 1100

Once again, 37 bits are used to encode "go go gophers". There are several trees that yield an optimal 37-bit encoding of "go go gophers". The tree that actually results from a programmed implementation of Huffman's algorithm will be the same each time the program is run for the same weights (assuming no randomness is used in creating the tree).

Why is Huffman Coding Greedy?

Huffman's algorithm is an example of a **greedy algorithm**. It's called greedy because the two smallest nodes are chosen at each step, and this local decision results in a globally optimal encoding tree. In general, greedy algorithms use small-grained, or local minimal/maximal choices to result in a global minimum/maximum. Making change using U.S. money is another example of a greedy algorithm.

- Problem: give change in U.S. coins for any amount (say under $1.00) using the minimal number of coins.
- Solution (assuming coin denominations of $0.25, $0.10, $0.05, and $0.01, called quarters, dimes, nickels, and pennies, respectively): use the highest-value coin that you can, and give as many of these as you can. Repeat the process until the correct change is given.
- Example: make change for $0.91. Use 3 quarters (the highest coin we can use, and as many as we can use). This leaves $0.16. To make change use a dime (leaving $0.06), a nickel (leaving $0.01), and a penny. The total change for $0.91 is three quarters, a dime, a nickel, and a penny. This is a total of six coins, it is not possible to make change for $0.91 using fewer coins.

The solution/algorithm is greedy because the largest denomination coin is chosen to use at each step, and as many are used as possible. This locally optimal step leads to a globally optimal solution. Note that the algorithm **does not work with different denominations**. For example, if there are no nickels, the algorithm will make change for $0.31 using one quarter and six pennies, a total of seven coins. However, it's possible to use three dimes and one penny, a total of four coins. This shows that greedy algorithms are not always optimal algorithms.

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