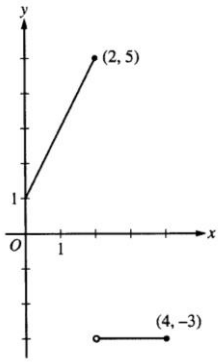
2012 MULTIPLE CHOICE (Odd Problems) ASSIGNMENT #1
No calculator is allowed on this portion.

1. If $y = \sin^3 x$, then $\frac{dy}{dx} =$

(A) $\cos^3 x$ (B) $3\cos^2 x$ (C) $3\sin^2 x$ (D) $-3\sin^2 x \cos x$ (E) $3\sin^2 x \cos x$

3. The graph of f is shown for $0 \le x \le 4$. What is the value of $\int_0^4 f(x)\,dx$?



Graph of f

(A) -1 (B) 0 (C) 2 (D) 6 (E) 12

5. The Maclaurin series for the function f is given by $f(x) = \sum_{n=0}^{\infty} \left(-\frac{x}{4}\right)^n$. What is the value of $f(3)$?

(A) -3 (B) $-\frac{3}{7}$ (C) $\frac{4}{7}$ (D) $\frac{13}{16}$ (E) 4

7. If $\arcsin x = \ln y$, then $\frac{dy}{dx} =$

(A) $\frac{y}{\sqrt{1-x^2}}$

(B) $\frac{xy}{\sqrt{1-x^2}}$

(C) $\frac{y}{1+x^2}$

(D) $e^{\arcsin x}$

(E) $\frac{e^{\arcsin x}}{1+x^2}$

9. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{8^n}{n!}$ II. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$ III. $\sum_{n=1}^{\infty} \frac{n+1}{(n)(n+2)(n+3)}$

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

11. Let f be the function defined by $f(x) = \sqrt{|x-2|}$ for all x. Which of the following statements is true?

(A) f is continuous but not differentiable at $x = 2$.

(B) f is differentiable at $x = 2$.

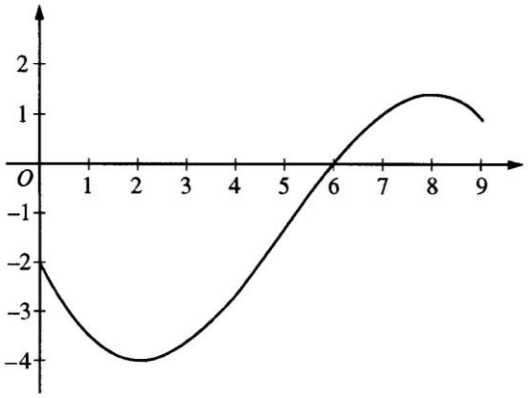
(C) f is not continuous at $x = 2$.

(D) $\lim_{x \to 2} f(x) \neq 0$

(E) $x = 2$ is a vertical asymptote of the graph of f.

13. What is the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(x-4)^{2n}}{3^n}$?

(A) $2\sqrt{3}$ (B) 3 (C) $\sqrt{3}$ (D) $\frac{\sqrt{3}}{2}$ (E) 0



Graph of f

15. The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t)\,dt$, which of the following is true?

(A) $h(6) < h'(6) < h''(6)$

(B) $h(6) < h''(6) < h'(6)$

(C) $h'(6) < h(6) < h''(6)$

(D) $h''(6) < h(6) < h'(6)$

(E) $h''(6) < h'(6) < h(6)$

17. For $x > 0$, the power series $1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + (-1)^n \frac{x^{2n}}{(2n+1)!} + \cdots$ converges to which of the following?

(A) $\cos x$ (B) $\sin x$ (C) $\frac{\sin x}{x}$ (D) $e^x - e^{x^2}$ (E) $1 + e^x - e^{x^2}$

19. The function f is defined by $f(x) = \dfrac{x}{x+2}$. What points (x,y) on the graph of f have the property that the line tangent to f at (x,y) has slope $\dfrac{1}{2}$?

(A) $(0,0)$ only

(B) $\left(\dfrac{1}{2},\dfrac{1}{5}\right)$ only

(C) $(0,0)$ and $(-4,2)$

(D) $(0,0)$ and $\left(4,\dfrac{2}{3}\right)$

(E) There are no such points.

21. The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?

(A) $y = \dfrac{\sin(5x)}{x}$ (B) $y = 5x$ (C) $y = \dfrac{1}{x-5}$ (D) $y = \dfrac{5x}{1-x}$ (E) $y = \dfrac{20x^2 - x}{1+4x^2}$

23. If $P(t)$ is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population?

(A) $\dfrac{dP}{dt} = 200$

(B) $\dfrac{dP}{dt} = 200t$

(C) $\dfrac{dP}{dt} = 100t^2$

(D) $\dfrac{dP}{dt} = 200P$

(E) $\dfrac{dP}{dt} = 100P^2$

25. $\displaystyle\int_1^{\infty} xe^{-x^2}\,dx$ is

(A) $-\dfrac{1}{e}$ (B) $\dfrac{1}{2e}$ (C) $\dfrac{1}{e}$ (D) $\dfrac{2}{e}$ (E) divergent

27. For what values of p will both series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^{2p}}$ and $\displaystyle\sum_{n=1}^{\infty}\left(\frac{p}{2}\right)^n$ converge?

(A) $-2 < p < 2$ only

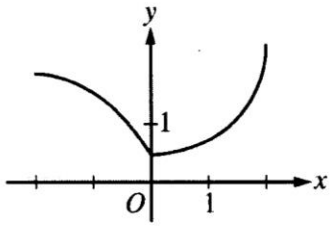
(B) $-\dfrac{1}{2} < p < \dfrac{1}{2}$ only

(C) $\dfrac{1}{2} < p < 2$ only

(D) $p < \dfrac{1}{2}$ and $p > 2$

(E) There are no such values of p.

You may use a graphing calculator for the remaining questions.



Graph of f

29. The function f, whose graph is shown above, is defined on the interval $-2 \le x \le 2$. Which of the following statements about f is false?

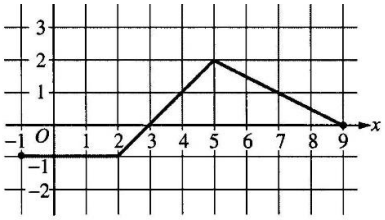
(A) f is continuous at $x = 0$.

(B) f is differentiable at $x = 0$.

(C) f has a critical point at $x = 0$.

(D) f has an absolute minimum at $x = 0$.

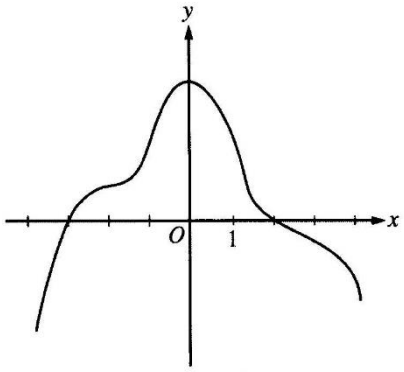
(E) The concavity of the graph of f changes at $x = 0$.



Graph of f

31. The graph of the piecewise linear function f is shown above. What is the value of $\int_{-1}^{9} (3f(x) + 2)\,dx$?

(A) 7.5 (B) 9.5 (C) 27.5 (D) 47 (E) 48.5



Graph of f'

33. The graph of f', the derivative of the function f, is shown above. Which of the following statements must be true?

I. f has a relative minimum at $x = -3$.

II. The graph of f has a point of inflection at $x = -2$.

III. The graph of f is concave down for $0 < x < 4$.

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

35. What is the average value of $y = \sqrt{\cos x}$ on the interval $0 \le x \le \frac{\pi}{2}$?

(A) −0.637 (B) 0.500 (C) 0.763 (D) 1.198 (E) 1.882

37. For $-1.5 < x < 1.5$, let f be a function with first derivative given by $f'(x) = e^{\left(x^4 - 2x^2 + 1\right)} - 2$. Which of the following are all intervals on which the graph of f is concave down?

(A) $(-0.418, 0.418)$ only

(B) $(-1, 1)$

(C) $(-1.354, -0.409)$ and $(0.409, 1.354)$

(D) $(-1.5, -1)$ and $(0, 1)$

(E) $(-1.5, -1.354)$, $(-0.409, 0)$, and $(1.354, 1.5)$

39. If $f'(x) > 0$ for all real numbers x and $\int_4^7 f(t)dt = 0$, which of the following could be a table of values for the function f?

(A)

x	$f(x)$
4	−4
5	−3
7	0

(B)

x	$f(x)$
4	−4
5	−2
7	5

(C)

x	$f(x)$
4	−4
5	6
7	3

(D)

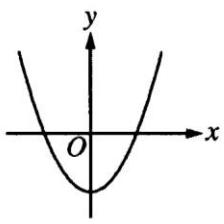
x	$f(x)$
4	0
5	0
7	0

(E)

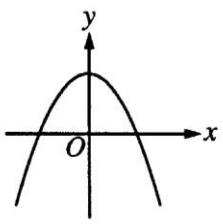
x	$f(x)$
4	0
5	4
7	6

41. The derivative of a function f is increasing for $x < 0$ and decreasing for $x > 0$. Which of the following could be the graph of f?

(A)



(B)



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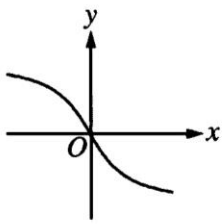
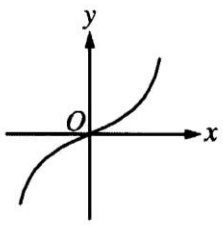
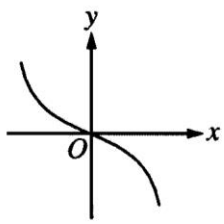


(D)



(E)



43. If the series $\sum_{n=1}^{\infty} a_n$ converges and $a_n > 0$ for all n, which of the following must be true?

(A) $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 0$

(B) $|a_n| < 1$ for all n

(C) $\sum_{n=1}^{\infty} a_n = 0$

(D) $\sum_{n=1}^{\infty} n a_n$ diverges.

(E) $\sum_{n=1}^{\infty} \frac{a_n}{n}$ converges.

45. The function h is differentiable, and for all values of x, $h(x) = h(2 - x)$. Which of the following statements must be true?

I. $\int_0^2 h(x)\,dx > 0$

II. $h'(1) = 0$

III. $h'(0) = h'(2) = 1$

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III