AER501 Assignment 2: Vibrations

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1 Objective

The objective of this assignment is to write a program finite element based free-vibration analysis of a cantilevered rod. This will use linear shape functions to approximate the displacement within each element, as well as, study the convergence of the first 5 natural frequencies and mode shapes of the structure when the finite element spatial mesh is refined.

2 Code Structure and Inputs

This assignment is written in **Python** Code using *Python 2.7*. The given MATLAB files are converted into Python by Hayden Lau. All meshes are in file, `mesh.py` and the pre-made element matrix are in `element_mats.py` The description of the code structure and input formats are written following coding standards for document generators. Please see the block comments in each function.

3 Part A

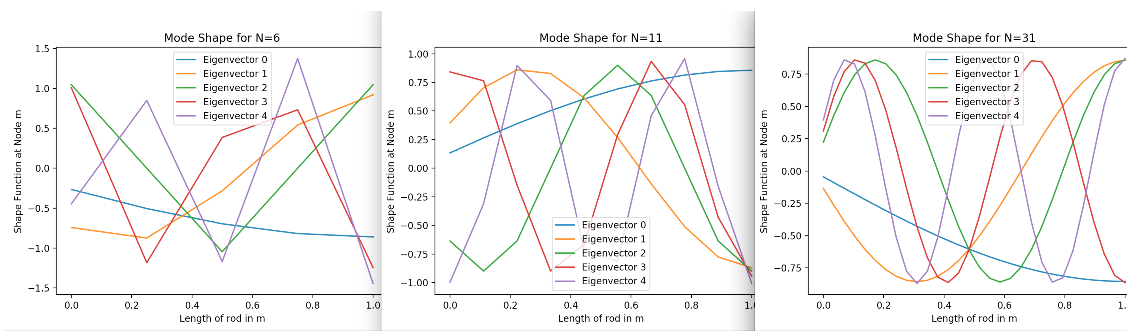


Figure 1: Mode Shapes for N=6, 11, and 31

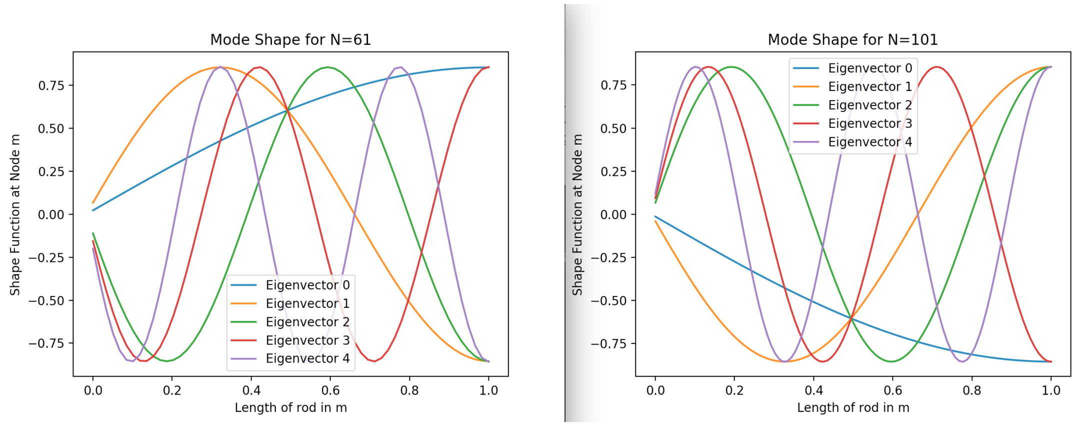


Figure 2: Mode Shapes for N=61, 101

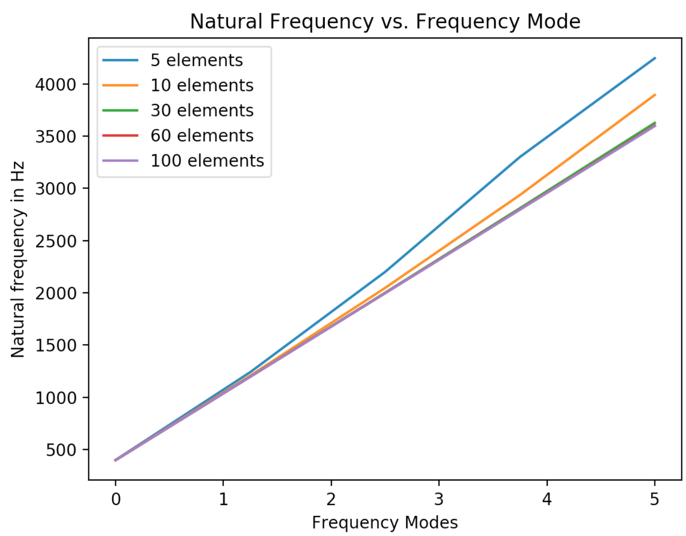


Figure 3: Natural Frequency

As can be seen in Figures 1 and 2, the eigenvector (shape function) becomes more and more sinusoidal as the number of N (nodes) are increased.

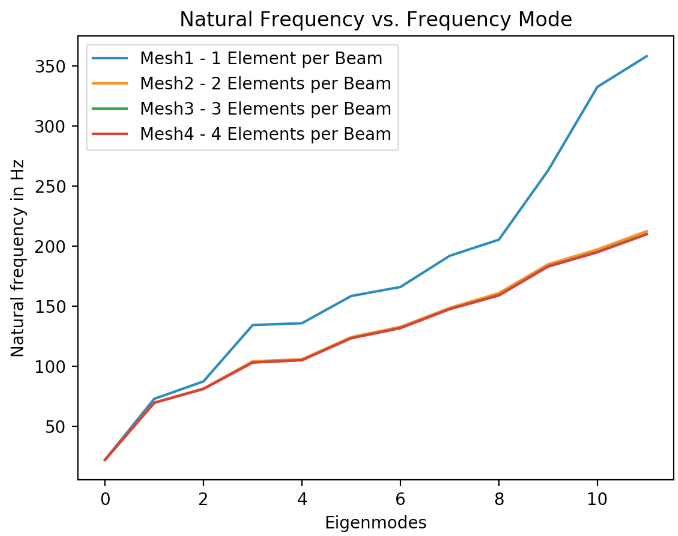
Figure 3 shows the first 5 natural frequencies for various numbers of nodes (or elements which is N-1). As the number of nodes increase (more comlpex and finer meshes), we can see the natural frequency converging to a particular value per frequency mode. The graph where there are 5 elements and 10 elements seem very far off and are slow at converging for their frequency and shape function. After this, elements=30, 60, and 100, converge to a sinusoidal shape function and similar frequencies per mode (values can be read off the plot).

4 Part B

4.1 Question 1

See Section 2 for the Code Structure or the block comments in the code structure itself. A documentation generator (e.g. doxygen) can also be used for details on individual functions.

4.2 Question 2

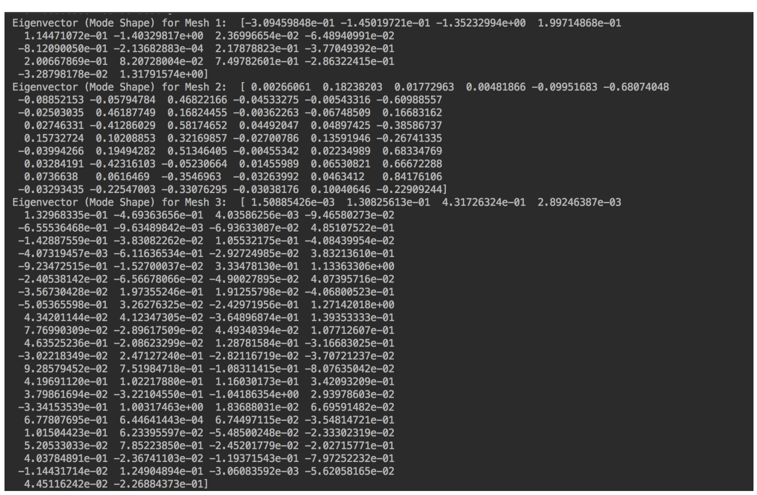


Frequency.png

Figure 4: Natural Frequency for Truss

Note, that Mesh 2 and Mesh 3 are very hard to be seen since their numbers are so close (have converged) with/to Mesh 4. By investigating the first twelve natural frequencies, as shown in Figure 4, we can see that mesh refinements beyond Mesh 2 does not change the natural frequencies of the truss structure being analyzed. The natural frequencies converge at Mesh 2 with 2 elements per beam and thus are around the same values for the first 12 frequencies.

Figures 5 and 6 shows the eigenvector (mode shape) outputs calculated from the Python Console. You may also run the file and read the output from your console.



```
Eigenvector (Mode Shape) for Mesh 1:  [-3.09459848e-01 -1.45019721e-01 -1.35232994e+00  1.99714868e-01
  1.14471072e-01 -1.40329817e+00  2.36996654e-02 -6.48940991e-02
 -8.12090050e-01 -2.13682883e-04  2.17878823e-01 -3.77049392e-01
  2.00667869e-01  8.20728004e-02  7.49782601e-01 -2.86322415e-01
 -3.28798178e-02  1.31791574e+00]
Eigenvector (Mode Shape) for Mesh 2:  [ 0.00266061  0.18238203  0.01772963  0.00481866 -0.09951683 -0.68074048
 -0.08852153 -0.05794784  0.46822166 -0.04533275 -0.00543316 -0.60988557
 -0.02503035  0.46187749  0.16824455 -0.00362263 -0.06748509  0.16683162
  0.02746331 -0.41286029  0.58174652  0.04492047  0.04897425 -0.38586737
  0.15732724  0.10208853  0.32169857 -0.02700786  0.13591946 -0.26741335
 -0.03994266  0.19494282  0.51346405 -0.00455342  0.02234989  0.68334769
  0.03284191 -0.42316103 -0.05230664  0.01455989  0.06530821  0.66672288
  0.0736638   0.0616469  -0.3546963  -0.03263992  0.0463412   0.84176106
 -0.03293435 -0.22547003 -0.33076295 -0.03038176  0.10040646 -0.22909244]
Eigenvector (Mode Shape) for Mesh 3:  [ 1.50885426e-03  1.30825613e-01  4.31726324e-01  2.89246387e-03
  1.32968335e-01 -4.69363656e-01  4.03586256e-03 -9.46580273e-02
 -6.55536468e-01 -9.63489842e-03 -6.93633087e-02  4.85107522e-01
 -1.42887559e-01 -3.83082262e-02  1.05532175e-01 -4.08439954e-02
 -4.07319457e-03 -6.11636534e-01 -2.92724985e-02  3.83213610e-01
 -9.23472515e-01 -1.52700037e-02  3.33478130e-01  1.13363306e+00
 -2.40538142e-02 -6.56678066e-02 -4.90027895e-02  4.07395716e-02
 -3.56730428e-02  1.97355246e-01  1.91255798e-02 -4.06800523e-01
 -5.05365598e-01  3.26276325e-02 -2.42971956e-01  1.27142018e+00
  4.34201144e-02  4.12347305e-02 -3.64896874e-01  1.39353333e-01
  7.76990309e-02 -2.89617509e-02  4.49340394e-02  1.07712607e-01
  4.63525236e-01 -2.08623299e-02  1.28781584e-01 -3.16683025e-01
 -3.02218349e-02  2.47127240e-01 -2.82116719e-02 -3.70721237e-02
  9.28579452e-02  7.51984718e-01 -1.08311415e-01 -8.07635042e-02
  4.19691120e-01  1.02217880e-01  1.16030173e-01  3.42093209e-01
  3.79861694e-02 -3.22104550e-01 -1.04186354e+00  2.93978603e-02
 -3.34153539e-01  1.00317463e+00  1.83688031e-02  6.69591482e-02
  6.77807695e-01  6.44641443e-04  6.74497115e-02 -3.54814721e-01
  1.01504423e-01  6.23395597e-02 -5.48500248e-02 -2.33302319e-02
  5.20533033e-02  7.85223850e-01 -2.45201779e-02 -2.02715771e-01
  4.03784891e-01 -2.36741103e-02 -1.19371543e-01 -7.97252232e-01
 -1.14431714e-02  1.24904894e-01 -3.06083592e-03 -5.62058165e-02
  4.45116242e-02 -2.26884373e-01]
```

Figure 5: Eigenvectors for Truss

```
Eigenvector (Mode Shape) for Mesh 4:  [ 1.12012040e-03  8.96362828e-02  5.11020874e-01  2.18788522e-03
  1.69887564e-01  1.90668775e-02  3.15338601e-03  8.50753884e-02
 -6.41435223e-01  3.97149427e-03 -9.38580729e-02 -6.50555655e-01
  2.48371532e-02 -7.56263528e-02  3.47833956e-01 -9.39062655e-02
 -5.38597786e-02  4.49890946e-01 -1.42060481e-01 -2.95757425e-02
 -1.22532125e-01 -4.01894790e-02 -3.90930559e-03 -6.10631664e-01
 -3.19106442e-02  2.89396706e-01 -1.26087544e+00 -2.21402729e-02
  4.47845481e-01  1.71639414e-01 -1.13350423e-02  2.28245700e-01
  1.31648259e+00 -2.00960451e-02 -5.14192163e-02 -1.47551016e-01
 -2.40853829e-03 -6.21493601e-02  1.77033665e-01  5.64884100e-02
 -2.61285362e-02  8.31047052e-02  1.53259873e-02 -3.44467111e-01
 -9.08830480e-01  2.59641279e-02 -4.02277090e-01  5.41087503e-01
  3.53886782e-02 -1.33894930e-01  1.29821748e+00  4.31591250e-02
  4.02461259e-02 -3.61107822e-01  1.26330510e-01  6.79630868e-02
 -1.88765708e-01  1.09142172e-01  9.25033825e-02  3.15226385e-01
  5.47570215e-03  1.12719974e-01  3.93717113e-01 -1.99901545e-02
  1.27667918e-01 -3.21684401e-01 -2.72073234e-02  2.33712591e-01
 -2.85197012e-01 -3.31527938e-02  2.05673485e-01  5.13241789e-01
 -3.75486682e-02  3.34453556e-02  6.60940786e-01 -1.30133530e-01
 -1.00427656e-01  7.32618327e-02  1.02629729e-03  2.14023704e-02
  7.31624484e-01  1.18649919e-01  1.27806118e-01 -2.65986696e-04
  3.95507481e-02 -2.22911447e-01 -1.28780196e+00  3.40937283e-02
 -4.17555387e-01 -4.39419593e-02  2.70431323e-02 -2.35438541e-01
  1.31120104e+00  1.87285122e-02  6.69881341e-02  6.77995852e-01
 -2.43443526e-02  6.80359969e-02 -2.17849729e-01  6.33391133e-02
  6.59037868e-02 -3.51525772e-01  9.69206104e-02  6.06911653e-02
  1.51786906e-01 -2.19873759e-02  5.26417759e-02  7.76629352e-01
 -2.30961087e-02 -1.54650693e-01  6.75611501e-01 -2.31253055e-02
 -2.11963705e-01 -2.70207229e-01 -2.20736015e-02 -4.85163294e-02
 -8.81843332e-01 -1.79146236e-02  1.22234248e-01  8.63651408e-02
 -2.35653620e-02  9.60741744e-02 -2.24021592e-01 -6.39287353e-02
  2.41037675e-02 -9.71007298e-02]
```

Figure 6: Eigenvectors for Truss

4.3 Question 3

4.3.1 Full-order Dynamic Stiffness Matrix (Direct)

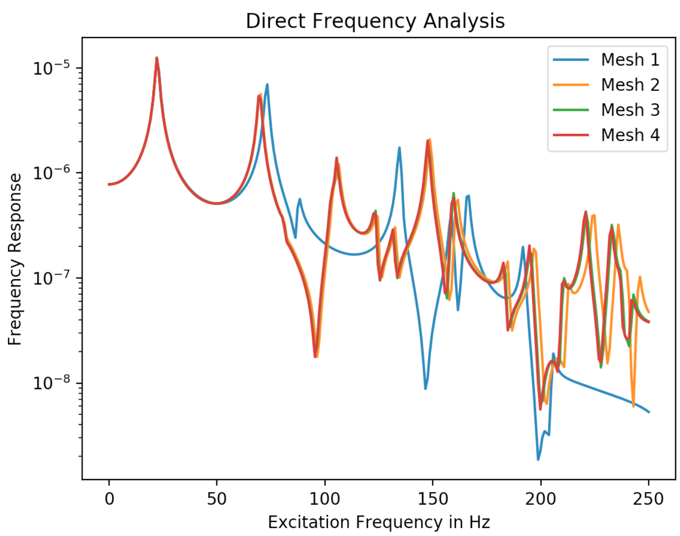


Figure 7: Frequency Response for Meshes 1 to 4. Note X axis is excitation frequency

Like the analysis for natural frequency, it can be seen from Figure 7 that for Mesh 2 and after, extra refinements do not substantially change the frequency response. That is, it rapidly converges from mesh 1 to mesh 2, while the convergence for additional elements per beam (mesh 3 and 4) is slow. In addition, from 0 to 50Hz all meshes result in the same frequency response, after that the differences in their frequency response for each mesh varies.

4.3.2 Modal Analysis

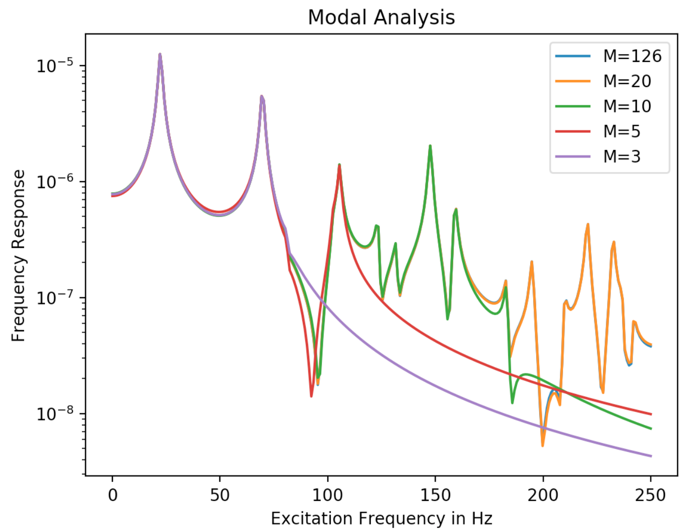


Figure 8: Frequency Response for Mesh 1

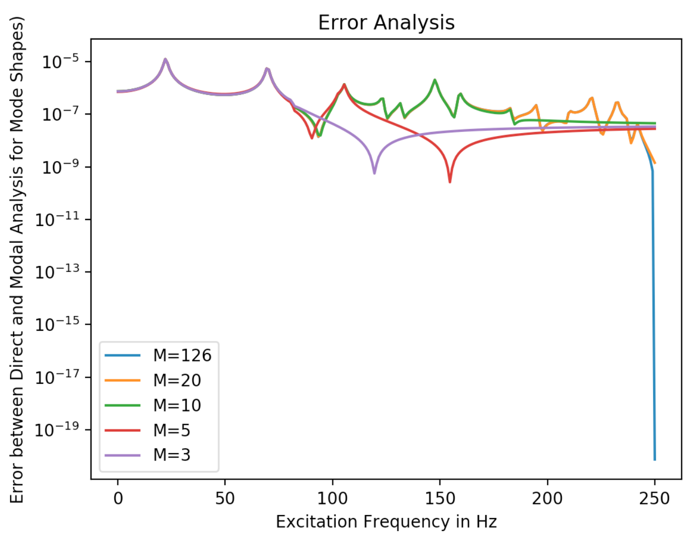


Figure 9: Difference (error) between using Full-order dynamic stiffness vs modal analysis for Mesh 4

Figure 7 shows the frequency response using 5 mode numbers, one of which is the full eigenvector and eigenvalue matrix (full mode – M=126). As can be seen, the frequency response looks very similar to Mesh 4 in Figure 7, especially when using the full mode. In fact, the full mode should be the same as Mesh 4 in 7.

A study of Mesh 4 was conducted using various values of M (mode numbers) and comparing the error of using modal analysis against the values gotten for the Frequency Response for the full-order-dynamic-stiffness-method in Section 4.3.1, the results of which are shown in Figure 8. The error increases as fewer number of modes are used, but is nonetheless a good approximation. To ensure that the frequency response is approximated with modal analysis agrees with the full-order approach for the region of 0 to 250Hz, use 20 modes.

5 Bonus: Quasi-static Correction Scheme

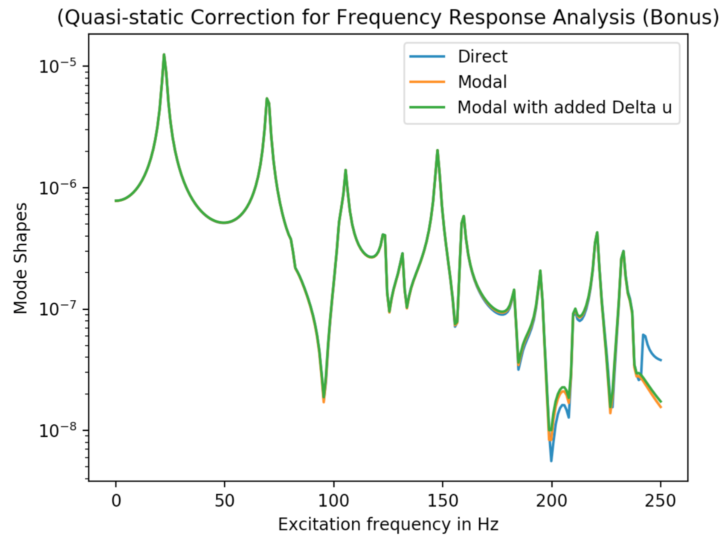


Figure 10: Error from Quasi-static Correction Scheme for Frequency Response Analysis

Figure 10 shows the Frequency response using a direct frequency analysis (full-order) vs modal analysis for m=15 vs modal analysis with the correction scheme. The correction is a constant that is added to the frequency response values. The differences is small, however, it can be seen that the modal analysis with the correction scheme is slightly closer to the direct frequency analysis.