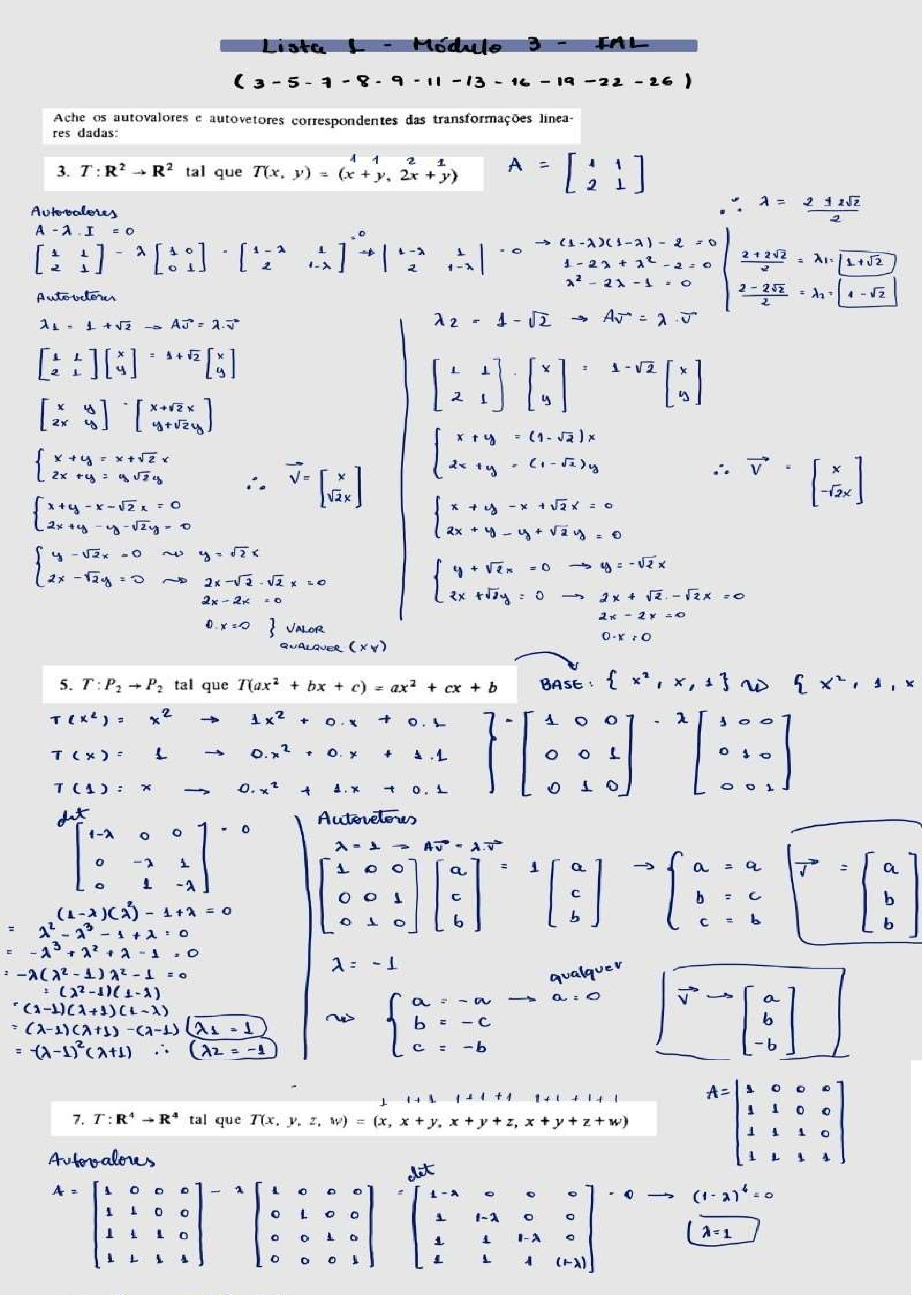
M3 Lista1, 2, 3

quinta-feira, 14 de outubro de 2021 14:17



Lista 1 - Módulo 3 - FAL

(3 - 5 - 7 - 8 - 9 - 11 - 13 - 16 - 19 - 22 - 26)

Ache os autovalores e autovetores correspondentes das transformações lineares dadas:

3. $T:\mathbf{R}^2 \to \mathbf{R}^2$ tal que $T(x, y) = (\overset{1}{x} + \overset{1}{y}, \overset{2}{2x} + \overset{1}{y})$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

Autovalores

$A - \lambda . I = 0$

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-\lambda & 1 \\ 2 & 1-\lambda \end{bmatrix} \xrightarrow{:0} \begin{vmatrix} 1-\lambda & 1 \\ 2 & 1-\lambda \end{vmatrix} = 0 \to (1-\lambda)(1-\lambda) - 2 = 0$$

$1 - 2\lambda + \lambda^2 - 2 = 0$

$\lambda^2 - 2\lambda - 1 = 0$

$\therefore \lambda = \frac{2 \pm 2\sqrt{2}}{2}$

$\frac{2+2\sqrt{2}}{2} = \lambda_1 = \boxed{1+\sqrt{2}}$

$\frac{2-2\sqrt{2}}{2} = \lambda_2 = \boxed{1-\sqrt{2}}$

Autovetores

$\lambda_1 = 1 + \sqrt{2} \to A\vec{v} = \lambda . \vec{v}$

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 1+\sqrt{2} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x & y \\ 2x & y \end{bmatrix} \cdot \begin{bmatrix} x+\sqrt{2}x \\ y+\sqrt{2}y \end{bmatrix}$$

$$\begin{cases} x + y = x + \sqrt{2}x \\ 2x + y = y\sqrt{2}y \end{cases} \qquad \therefore \vec{V} = \begin{bmatrix} x \\ \sqrt{2}x \end{bmatrix}$$

$$\begin{cases} x + y - x - \sqrt{2}x = 0 \\ 2x + y - y - \sqrt{2}y = 0 \end{cases}$$

$$\begin{cases} y - \sqrt{2}x = 0 \leadsto y = \sqrt{2}x \\ 2x - \sqrt{2}y = 0 \leadsto 2x - \sqrt{2} . \sqrt{2}x = 0 \end{cases}$$

$2x - 2x = 0$

$0.x = 0$ } VALOR QUALQUER (X Y)

$\lambda_2 = 1 - \sqrt{2} \to A\vec{v} = \lambda . \vec{v}$

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \end{bmatrix} = 1-\sqrt{2} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{cases} x + y = (1-\sqrt{2})x \\ 2x + y = (1-\sqrt{2})y \end{cases} \qquad \therefore \vec{V} = \begin{bmatrix} x \\ -\sqrt{2}x \end{bmatrix}$$

$$\begin{cases} x + y - x + \sqrt{2}x = 0 \\ 2x + y - y + \sqrt{2}y = 0 \end{cases}$$

$$\begin{cases} y + \sqrt{2}x = 0 \to y = -\sqrt{2}x \\ 2x + \sqrt{2}y = 0 \to 2x + \sqrt{2} . -\sqrt{2}x = 0 \end{cases}$$

$2x - 2x = 0$

$0.x = 0$

5. $T:P_2 \to P_2$ tal que $T(ax^2 + bx + c) = ax^2 + cx + b$

BASE: $\{x^2, x, 1\} \leadsto \{x^2, 1, x$

$T(x^2) = x^2 \to 1x^2 + 0.x + 0.1$

$T(x) = 1 \to 0.x^2 + 0.x + 1.1$

$T(1) = x \to 0.x^2 + 1.x + 0.1$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\det \begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & -\lambda & 1 \\ 0 & 1 & -\lambda \end{bmatrix} = 0$$

$(1-\lambda)(\lambda^2) - 1 + \lambda = 0$

$= \lambda^2 - \lambda^3 - 1 + \lambda = 0$

$= -\lambda^3 + \lambda^2 + \lambda - 1 = 0$

$= -\lambda(\lambda^2 - 1)\lambda^2 - 1 = 0$

$= (\lambda^2-1)(1-\lambda)$

$= (\lambda-1)(\lambda+1)(1-\lambda)$

$= (\lambda-1)(\lambda+1) - (\lambda-1)$ $\boxed{\lambda_1 = 1}$

$= -(\lambda-1)^2(\lambda+1) \quad \therefore$ $\boxed{\lambda_2 = -1}$

Autovetores

$\lambda = 1 \to A\vec{v} = \lambda . \vec{v}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ c \\ b \end{bmatrix} = 1 \begin{bmatrix} a \\ c \\ b \end{bmatrix} \to \begin{cases} a = a \\ b = c \\ c = b \end{cases} \qquad \boxed{\vec{v} = \begin{bmatrix} a \\ b \\ b \end{bmatrix}}$$

$\lambda = -1$

$$\leadsto \begin{cases} a = -a \to \overset{\text{qualquer}}{a = 0} \\ b = -c \\ c = -b \end{cases} \qquad \boxed{\vec{v} \to \begin{bmatrix} a \\ b \\ -b \end{bmatrix}}$$

1 1+1 1+1+1 1+1+1+1

7. $T:\mathbf{R}^4 \to \mathbf{R}^4$ tal que $T(x, y, z, w) = (x, x + y, x + y + z, x + y + z + w)$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Autovalores

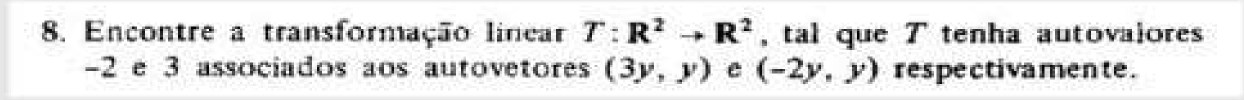
$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \det \begin{bmatrix} 1-\lambda & 0 & 0 & 0 \\ 1 & 1-\lambda & 0 & 0 \\ 1 & 1 & 1-\lambda & 0 \\ 1 & 1 & 1 & (1-\lambda) \end{bmatrix} = 0 \to (1-\lambda)^4 = 0$$

$\boxed{\lambda = 1}$

Autovetor $\rightarrow A\vec{v} = \lambda . \vec{v}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = 1 \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \rightarrow \begin{cases} x - x = 0 \longrightarrow 0x = 0 \\ x + y - y = 0 \rightarrow y = x \rightarrow y = 0 \\ x + y + z - z = 0 \rightarrow x + z = 0 \rightarrow z = 0 \\ x + y + z + w - \underbrace{w}_{\text{qualquer}} = 0 \rightarrow x + y + z = 0 \end{cases}$$

$$\boxed{\vec{v} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ w \end{bmatrix}}$$



8. Encontre a transformação linear $T: \mathbf{R}^2 \rightarrow \mathbf{R}^2$, tal que T tenha autovalores -2 e 3 associados aos autovetores $(3y, y)$ e $(-2y, y)$ respectivamente.

$\lambda_1 = -2$

$$\vec{v} = \begin{bmatrix} 3y \\ y \end{bmatrix} \qquad A.\vec{v} = \lambda . \vec{v}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3y \\ y \end{bmatrix} = -2 \begin{bmatrix} 3y \\ y \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3y \\ y \end{bmatrix} = \begin{bmatrix} -6y \\ -2y \end{bmatrix}$$

$$\begin{cases} 3\cancel{y}.a + \cancel{y}b = -6\cancel{y} \\ 3\cancel{y}c + \cancel{y}d = -2\cancel{y} \end{cases} \longrightarrow \begin{cases} 3a + b = -6 \\ 3c + d = -2 \end{cases}$$

$d = -2 - 3c$

$d = -2 - 3(-1)$

$d = -2 + 3$

$\boxed{d = 1}$

$b = -6 - 3a$

$b = -6 - 3.0$

$\boxed{b = -6}$

$\lambda_2 = 3$

$$\vec{v} = \begin{bmatrix} -2y \\ y \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} -2y \\ y \end{bmatrix} = 3 \begin{bmatrix} -2y \\ y \end{bmatrix}$$

$$\begin{cases} -2\cancel{y}.a + \cancel{y}.b = -6\cancel{y} \\ -2\cancel{y}.c + \cancel{y}.d = 3\cancel{y} \end{cases}$$

$$\begin{cases} -2a + b = -6 \longrightarrow -2a - 6 - 3a = -6 \\ -2c + d = 3 \end{cases}$$

$-5a = 0$

$\boxed{a = 0}$

$-2c - 2 - 3c = 3$

$-5c = 5$

$\boxed{c = -1}$

$$\boxed{A = \begin{bmatrix} 0 & -6 \\ -1 & 1 \end{bmatrix}}$$

Ache os autovalores e autovetores correspondentes das matrizes:

9. $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\det \begin{bmatrix} 1-\lambda & 2 \\ 0 & -1-\lambda \end{bmatrix} = 0 \longrightarrow (1-\lambda).(-1-\lambda) = 0$$

$1 - \lambda = 0 \rightarrow \boxed{\lambda = 1}$

$-1 - \lambda = 0 \rightarrow \boxed{\lambda = -1}$

$\lambda_1 = 1$

$$\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 1 \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{cases} x + 2y = x \rightarrow 2y = 0x \rightarrow y = 0 \\ -y = y \rightarrow -2y = 0 \rightarrow y = 0 \end{cases}$$

$x \rightsquigarrow$ qualquer

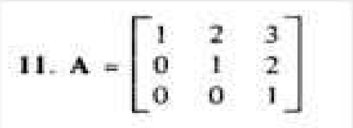
$$\boxed{\vec{v} = \begin{bmatrix} x \\ 0 \end{bmatrix}}$$

$$\lambda_2 = -1 \quad \therefore \quad \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = -1 \begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{cases} x + 2y = -x \longrightarrow 2y = -2x \rightarrow \boxed{y = -x} \\ -y = -y \longrightarrow y = y \end{cases}$$

deixa em função de x.

(ñ diz nada)

$$\boxed{\vec{v} = \begin{bmatrix} x \\ -x \end{bmatrix}}$$



11. $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$$\det \begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & 1-\lambda & 2 \\ 0 & 0 & 1-\lambda \end{bmatrix} = (1-\lambda)(1-\lambda)(1-\lambda) = 0$$

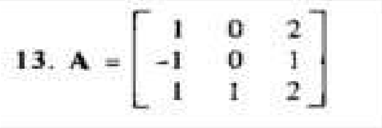
$\boxed{\lambda = 1}$

$\lambda_1 = 1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1 \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{cases} x + 2y + 3z = x \longrightarrow 2y + 3z = 0 \rightsquigarrow 2y = 0 \rightarrow y = 0 \\ y + 2z = y \longrightarrow 2z = 0 \\ z = z \end{cases}$$

$$\boxed{\vec{v} = \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix}}$$

$\det([A] - \lambda I) = 0$



13. $\mathbf{A} = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

$$\begin{vmatrix} 1-\lambda & 0 & 2 \\ -1 & -\lambda & 1 \\ 1 & 1 & 2-\lambda \end{vmatrix} = 0$$

$$\rightarrow (1-\lambda)(-\lambda)(2-\lambda)-2 - ((-2\lambda)+(1-\lambda)) = 0$$

$$(-\lambda+\lambda^2)(2-\lambda)-2 = -2\lambda+1-\lambda$$

$$-2\lambda+\lambda^2+2\lambda^2-\lambda^3-2 = -2\lambda+1-\lambda$$

$$-\lambda^3+3\lambda^2-2\lambda-2 = -3\lambda+1$$

$$-\lambda^3+3\lambda^2+x-3=0 \quad \therefore \quad \boxed{\lambda_1 = 3} \quad \boxed{\lambda_2 = -1} \quad \boxed{\lambda_3 = 1}$$

Autovalores

$\lambda_1 = 1$

$$\begin{bmatrix} 1 & 0 & 2 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{cases} x+2z = x \rightarrow z=0 \\ -x+z = y \rightarrow x=-y \\ x+y+2z = z \rightarrow x+y=0 \\ \qquad\qquad x=-y \end{cases}$$

$$\boxed{\vec{v} = \begin{bmatrix} -y \\ y \\ 0 \end{bmatrix}}$$

$\lambda_2 = -1$

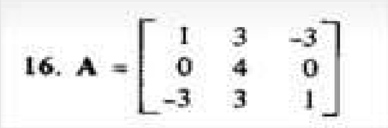
$$\begin{cases} x+2z = -x \rightarrow 2x+2z=0 \rightarrow \boxed{x=-z} \\ -x+z = -y \rightarrow y = x-z \rightarrow y = x+x \rightarrow \boxed{y=2x} \\ x+y+2z = -z \rightarrow x+2x-3x=0 \\ \qquad 0x=0 \end{cases}$$

$$\boxed{\vec{v} = \begin{bmatrix} x \\ 2x \\ -x \end{bmatrix}}$$

$\lambda_3 = = 3$

$$\begin{cases} x+2z = 3x \rightarrow 2z = 2x \rightarrow \boxed{z=x} \\ -x+z = 3y \rightarrow 3y=0 \rightarrow \boxed{y=0} \\ x+y+2z = 3z \rightarrow x+2x = 3x = 0 \end{cases}$$

$$\boxed{\vec{v} = \begin{bmatrix} x \\ 0 \\ x \end{bmatrix}}$$



16. $\mathbf{A} = \begin{bmatrix} 1 & 3 & -3 \\ 0 & 4 & 0 \\ -3 & 3 & 1 \end{bmatrix}$

$$\det\begin{bmatrix} 1-\lambda & 3 & -3 \\ 0 & 4-\lambda & 0 \\ -3 & 3 & 1-\lambda \end{bmatrix} = (1-\lambda)\cancel{(4-\lambda)}(1-\lambda) - \overset{-9}{(-3)\cancel{(4-\lambda)}(-3)}$$

$$1-2\lambda+\lambda^2-9=0$$

$$\lambda^2-2\lambda-8=0 \quad \boxed{\lambda_1 = -2 \mid \lambda_2 = 4}$$

Autovalores

$\lambda_1 = -2$

$$\begin{bmatrix} 1 & 3 & -3 \\ 0 & 4 & 0 \\ -3 & 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 3 \end{bmatrix} = -2\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{cases} x+3y-3z = -2x \\ 4y = -2y \\ -3x+3y+z = -2z \end{cases} \rightarrow \begin{cases} 3x+3y-3z=0 \rightarrow \boxed{x=z} \\ 6y=0 \rightarrow \boxed{y=0} \\ -3x+3y+3y=0 \rightarrow \boxed{x=z} \end{cases}$$

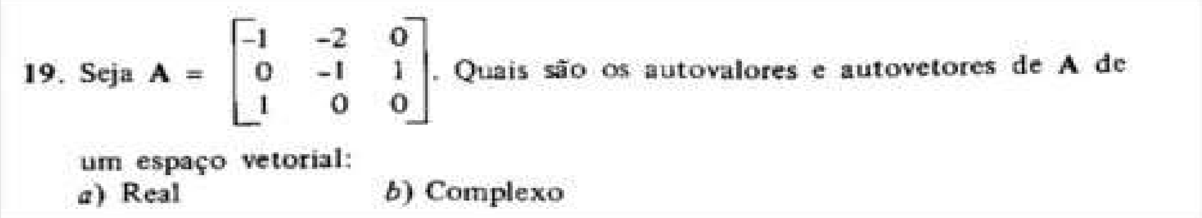
$$\boxed{\vec{v} = \begin{bmatrix} x \\ 0 \\ x \end{bmatrix}}$$

$\lambda_2 = 4$

$$\begin{cases} x+3y-3z = 4x \\ 4y = 4y \\ -3x+3y+z = 4z \end{cases} \rightarrow \begin{cases} -3x+3y-3z=0 \rightarrow -x+y-z=0 \rightarrow z=y-x \\ 4y-4y=0 \rightarrow \boxed{0=0}\ y = \text{qualquer} \\ -3x+3y-3z=0 \rightarrow -x+y-z=0 \end{cases}$$

$$\boxed{\vec{v} = \begin{bmatrix} x \\ y \\ y-x \end{bmatrix}}$$

Autovetores



19. Seja $\mathbf{A} = \begin{bmatrix} -1 & -2 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Quais são os autovalores e autovetores de **A** de um espaço vetorial:

a) Real *b*) Complexo

$$\det\begin{bmatrix} -1-\lambda & -2 & 0 \\ 0 & -1-\lambda & 1 \\ 1 & 0 & -\lambda \end{bmatrix} = (-1-\lambda)^2(-\lambda)-2 = 0$$

$$1+2\lambda+\lambda^2(-\lambda)-2=0$$

$$-\lambda-2\lambda^2-\lambda^3-2=0$$

$$\lambda^3+2\lambda^2+\lambda+2=0$$

$$\boxed{\lambda_1 = -2} \quad \boxed{\lambda_2 = i} \quad \boxed{\lambda_3 = -i}$$

a) $\boxed{\lambda_1 = -2}$

$$\begin{bmatrix} -1 & -2 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -2\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{cases} -x-2y = -2x \\ -y+z = -2y \\ x = -2z \end{cases}$$

$$\begin{cases} -2y = -x \rightarrow x = 2y \rightarrow y = \frac{x}{2} \\ z = -y \\ x = -2z \rightarrow z = -\frac{x}{2} \end{cases} \quad \vec{v} = \begin{bmatrix} x \\ x/2 \\ -x/2 \end{bmatrix}$$

b) $\boxed{\lambda_2 = i}$

$$\begin{cases} -x-2y = i.x \\ -y+z = iy \\ x = iz \end{cases} \rightarrow \begin{cases} -x-2y = i.x \\ z = iy+y \rightarrow z = (1+i)y \\ x = iz \rightarrow x = i(1+i)y \rightarrow x = (i-1)y \end{cases}$$

$$\vec{v} = \begin{bmatrix} (i-1)y \\ y \\ (1+i)y \end{bmatrix}$$

$\boxed{\lambda_3 = -i}$

$$\begin{cases} -x-2y = -i.x \\ -y+z = -i.y \rightarrow z = -i.y+y \rightarrow z = (1-i)y \\ x = -i.z \rightarrow x = -i.(1-i)y \rightarrow x = (-i-1)y \end{cases}$$

$$\boxed{\begin{matrix} i^2 = -1 \\ (-i).(-i) = -1 \end{matrix}}$$

$$\vec{V} = \begin{bmatrix} (-i-1)y \\ y \\ (1-i)y \end{bmatrix}$$

22. Seja $\mathbf{A} = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$.

a) Ache os autovalores de **A** e de $\mathbf{A}^{-1}$.
b) Quais são os autovetores correspondentes?

Ⓐ $\det \begin{bmatrix} -\lambda & 2 \\ 1 & 1-\lambda \end{bmatrix} = 0 \rightarrow (-\lambda)(1-\lambda) - 2 = 0$

$-\lambda + \lambda^2 - 2 = 0$

$\lambda^2 - \lambda - 2 = 0$

$\boxed{\lambda_1 = 2}\ \boxed{\lambda_2 = -1}$

$\lambda_1 = 2$

$$\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 2\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{cases} 2y = 2x \longrightarrow \boxed{y = x} \\ x + y = 2y \longrightarrow x = y \end{cases}$$

$$\boxed{\vec{V} = \begin{bmatrix} x \\ x \end{bmatrix}}$$

$\lambda_2 = -1$

$$\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = -1\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{cases} 2y = -x \longrightarrow \boxed{x = -2y} \\ x + y = -y \rightarrow 2y = -x \end{cases}$$

$y = -\frac{x}{2}$

$$\boxed{\vec{V} = \begin{bmatrix} x \\ -\frac{x}{2} \end{bmatrix}}$$

$$\begin{bmatrix} -0{,}5 & 1 \\ 0{,}5 & 0 \end{bmatrix}$$

Ⓐ$^{-1}$ $\det \begin{bmatrix} -0{,}5-\lambda & 1 \\ 0{,}5 & -\lambda \end{bmatrix} = 0 \rightarrow (-\lambda)(-0{,}5-\lambda) - 0{,}5 = 0$

$0{,}5\lambda + \lambda^2 - 0{,}5 = 0$

$\lambda^2 + 0{,}5\lambda - 0{,}5 = 0$

$\boxed{\lambda_1 = \frac{1}{2}}\ \boxed{\lambda_2 = -1}$

$\boxed{\lambda_1 = 1/2}$

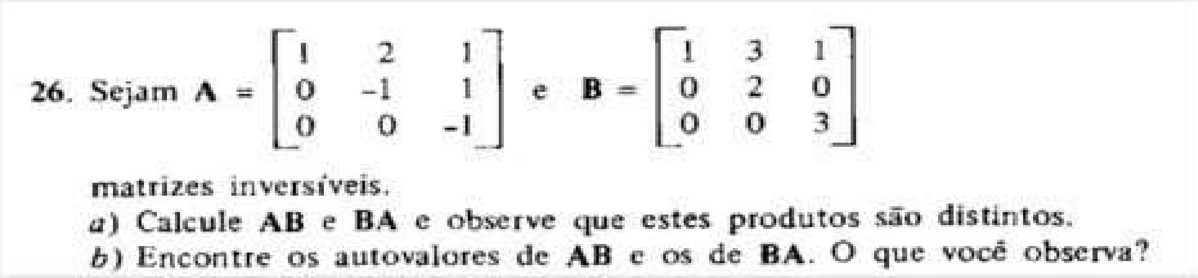
$$\begin{bmatrix} -1/2 & 1 \\ 1/2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 1/2\begin{bmatrix} x \\ y \end{bmatrix} \longrightarrow \begin{cases} -1/2x + y = 1/2\,x \\ 1/2x = 1/2y \end{cases}$$

$$\begin{cases} y = x \\ x = y \end{cases} \quad \boxed{\vec{V} = \begin{bmatrix} x \\ x \end{bmatrix}}$$

$\boxed{\lambda_2 = -1}$

$$\begin{bmatrix} -1/2 & 1 \\ 1/2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = -1\begin{bmatrix} x \\ y \end{bmatrix} \longrightarrow \begin{cases} -1/2x + y = -x \\ 1/2x = -y \end{cases} \rightarrow \begin{cases} y = -x + 1/2x \rightarrow \boxed{y = -\frac{x}{2}} \\ x = \frac{-y}{1/2} \rightarrow \boxed{x = -2y} \end{cases}$$

$$\vec{V} = \begin{bmatrix} x \\ -\frac{x}{2} \end{bmatrix} \text{ ou } \begin{bmatrix} -2y \\ y \end{bmatrix}$$



26. Sejam $\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$ e $\mathbf{B} = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

matrizes inversíveis.
a) Calcule **AB** e **BA** e observe que estes produtos são distintos.
b) Encontre os autovalores de **AB** e os de **BA**. O que você observa?

a)

$$A \cdot B = \begin{bmatrix} 1 & 7 & 4 \\ 0 & -2 & 3 \\ 0 & 0 & -3 \end{bmatrix} \quad B \cdot A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & -2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

b) $A \cdot B = \begin{bmatrix} 1 & 7 & 4 \\ 0 & -2 & 3 \\ 0 & 0 & -3 \end{bmatrix} \rightarrow \det \begin{bmatrix} 1-\lambda & 7 & 4 \\ 0 & -2-\lambda & 3 \\ 0 & 0 & -3-\lambda \end{bmatrix} = 0 \rightarrow \boxed{\lambda_1 = 1}\ \boxed{\lambda_2 = -2}\ \boxed{\lambda_3 = -3}$

$\lambda_1 = 1$

$$\begin{bmatrix} 1 & 7 & 4 \\ 0 & -2 & 3 \\ 0 & 0 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{cases} x + 7y + 4z = x \longrightarrow x = x \\ -2y + 3z = y \longrightarrow -3y = 0 \rightarrow \boxed{y = 0} \\ -3z = z \longrightarrow -4z = 0 \rightarrow \boxed{z = 0} \end{cases} \quad \vec{V} = \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix}$$

$\lambda_2 = -2$

$$\begin{cases} x + 7y + 4z = -2x \longrightarrow 7y = -3x \rightarrow \boxed{x = -\frac{7}{3}y} \\ -2y + 3z = -2y \longrightarrow \boxed{y = y} \\ -3z = -2z \longrightarrow -z = 0 \rightarrow \boxed{z = 0} \end{cases} \quad \vec{V} = \begin{bmatrix} -\frac{7}{3}y \\ y \\ 0 \end{bmatrix}$$

$\lambda_3 = -3$

$$\begin{cases} x + 7y + 4z = -3x \\ -2y + 3z = -3y \\ -3z = -3z \end{cases}$$

$$\begin{cases} -21z + 4z = -4x \\ 3z = -y \rightarrow \boxed{y = -3z} \\ \boxed{z = z} \end{cases}$$

$-4x = -17z$

$x = \frac{17}{4}z$

$$\vec{V} = \begin{bmatrix} \frac{17}{4}z \\ -3 \cdot z \\ z \end{bmatrix}$$

$B \cdot A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & -2 & 2 \\ 0 & 0 & -3 \end{bmatrix} \rightarrow \det \begin{bmatrix} 1-\lambda & -1 & 3 \\ 0 & -2-\lambda & 2 \\ 0 & 0 & -3-\lambda \end{bmatrix} = 0 \rightarrow \boxed{\lambda_1 = 1}\ \boxed{\lambda_2 = -2}\ \boxed{\lambda_3 = -3}$

$\lambda_1 \cdot 1$

$$\begin{bmatrix} 1 & -1 & 3 \\ 0 & -2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 1\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{cases} x - y + 3z = x \rightarrow \boxed{x = 0} \\ -2y + 2z = y \rightarrow -3y = 0 \rightarrow \boxed{y = 0} \end{cases} \quad \vec{V} = \begin{bmatrix} x \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & -3 \end{bmatrix}\begin{bmatrix} z \end{bmatrix} \quad \begin{bmatrix} z \end{bmatrix} \qquad -3z = z \rightarrow -4z = 0 \rightarrow \boxed{z=0}$$

$\lambda_2 = -2$

$$\begin{cases} x - y + 3z = -2x \longrightarrow 3x = y \rightarrow \boxed{x = \frac{y}{3}} \\ -2y + 2z = -2y \rightarrow \boxed{0y = 0} \text{ qualquer} \\ -3z = -2z \longrightarrow -z = 0 \rightarrow \boxed{z = 0} \end{cases} \qquad \vec{v} = \begin{bmatrix} y/3 \\ y \\ 0 \end{bmatrix} \text{ ou } \begin{bmatrix} y \\ 3y \\ 0 \end{bmatrix}$$

$\lambda_3 = -3$

$$\begin{cases} x - y + 3z = -3x \longrightarrow -4x = 2z + 3z \rightarrow \boxed{x = -\frac{5}{4}z} \\ -2y + 2z = -3y \rightarrow \boxed{y = -2z} \\ -3z = -3z \rightarrow \boxed{z = z} \end{cases} \qquad \vec{v} = \begin{bmatrix} -\frac{5}{4}z \\ -2z \\ z \end{bmatrix}$$

Autovalores são iguais

Autovetores de $\lambda = 1$ iguais, demais autovetores são diferentes

---

Módulo 3 - Lista 2 (3, 4, 5, 6)

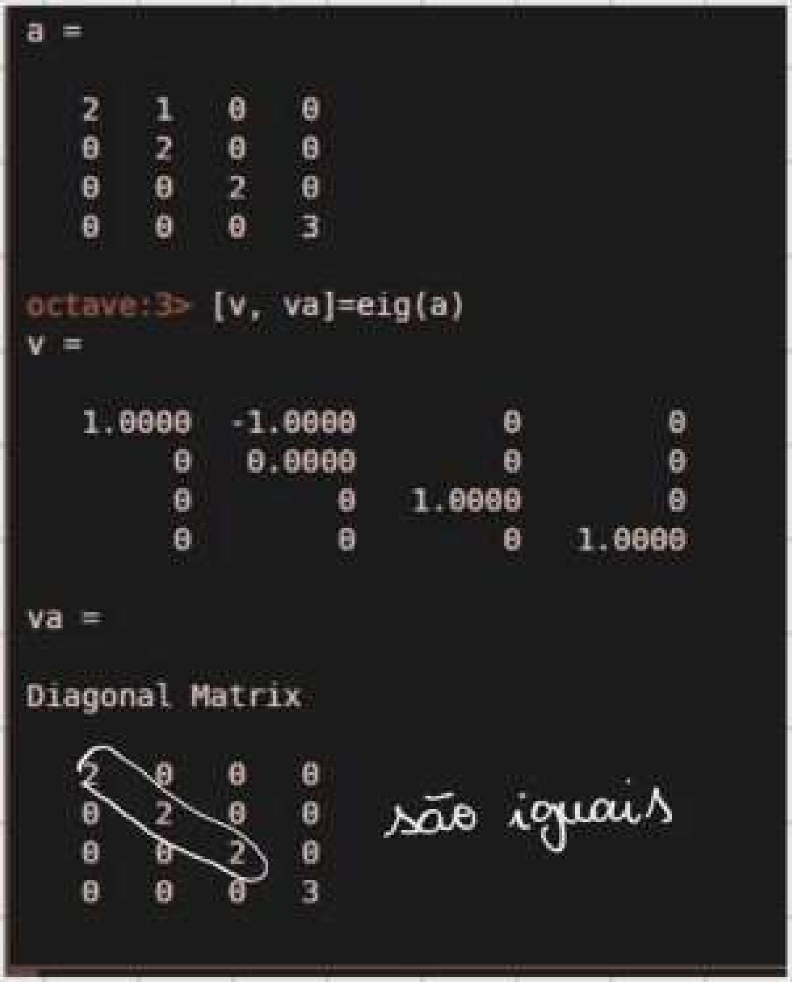
2. Dizemos que uma matriz $\mathbf{A}_{n \times n}$ é *diagonalizável* se seu operador associado $T_{\mathbf{A}} : \mathbf{R}^n \rightarrow \mathbf{R}^n$ for diagonalizável, ou seja, **A** é diagonalizável se, e somente se **A** admitir n autovetores LI. Baseado nisto, verifique quais das matrizes dos Exercícios 9 a 18 da secção 6.3 são diagonalizáveis.

3. Dada a matriz

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

a) **A** é diagonalizável (use a definição do exercício anterior).

b) Encontre seu polinômio minimal.



```
a =

   2   1   0   0
   0   2   0   0
   0   0   2   0
   0   0   0   3

octave:3> [v, va]=eig(a)
v =

   1.0000  -1.0000        0        0
        0   0.0000        0        0
        0        0   1.0000        0
        0        0        0   1.0000

va =

Diagonal Matrix

   2   0   0   0
   0   2   0   0
   0   0   2   0
   0   0   0   3
```

são iguais

③ a) Não, pois os autovalores não são distintos.

b)
$$\left| \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} \right| \rightarrow \left| \begin{bmatrix} 2-\lambda & 1 & 0 & 0 \\ 0 & 2-\lambda & 0 & 0 \\ 0 & 0 & 2-\lambda & 0 \\ 0 & 0 & 0 & 3-\lambda \end{bmatrix} \right| \longrightarrow P(\lambda) = (2-\lambda)^3 \cdot (3-\lambda)$$

$\therefore$ Polinômio minimal é

$$\underline{P(\lambda) = (2-\lambda)^2 (3-\lambda)}$$

$$P(\lambda)_1 = (2-\lambda)\cdot(3-\lambda) \longrightarrow P_1([T]^\alpha_\alpha) = (2I - [T]^\alpha_\alpha)\cdot(3I - [T]^\alpha_\alpha)$$

$$= \left(\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \right) \cdot \left(\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ Não zerou}$$

$$P(\lambda)_2 = (2-\lambda)^2\cdot(3-\lambda)$$
$$P(\lambda)_3 = (2-\lambda)^3\cdot(3-\lambda)$$

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}^2 \cdot \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

✓ → zerou a matriz

a) A - λI

✓ → zerou a matriz

4. Seja **A** uma matriz 3 × 3 triangular superior, com todos os seus elementos acima da diagonal distintos e não nulos.

$$\mathbf{A} = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

a) Autovalores A - λI

$$\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} a-\lambda & b & c \end{bmatrix} = (a-\lambda)(d-\lambda)(f-\lambda) = 0$$

a) Quais são os autovalores e autovetores de **A**?
b) Qual é o polinômio minimal de **A**?

$$\begin{bmatrix} 0 & d-\lambda & e \\ 0 & 0 & f-\lambda \end{bmatrix}$$

$\lambda_1 = a$
$\lambda_2 = d$
$\lambda = f$

b Autovetores

$\lambda_1 = a$

$$\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} - a\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{cases} \cancel{a}x + by + cz - \cancel{ax} = 0 \rightarrow by + cz = 0 \rightarrow x = x \\ dy + ez - ay = 0 \rightarrow y(d-a) + \overset{0}{ez} = 0 \xrightarrow{2)} \boxed{y=0} \\ f.z - a.z = 0 \longrightarrow z(f-a) = 0 \end{cases}$$

$1)\ \boxed{z=0}\ \therefore$

$$\boxed{V_1 = (x,0,0)}$$

$\lambda_2 = d$

$$\begin{cases} ax + by + \overset{0}{cz} = dx \rightarrow x(a-d) + by = 0 \rightarrow x = \frac{-by}{a-d} \\ \cancel{dy} + ez = \cancel{dy} \rightarrow ez = 0 \ \therefore y \text{ qualquer} \\ f.z = dz \rightarrow z(f-d) = 0 \\ z = 0 \end{cases}$$

$$\boxed{V_2 = \left(\frac{-by}{a-d}, y, 0\right)}$$

$\lambda_3 = f$

$$\begin{cases} ax + by + cz - fx = 0 \\ dy + ez - fy = 0 \rightarrow y(d-f) = -ez \\ \cancel{fz} - \cancel{fz} = 0 \end{cases}$$

$y = \frac{-e}{d-f} z$

z = qualquer

$x(a-f) + by + cz = 0$

$x(a-f) + b\left(\frac{-e.z}{d-f}\right) + cz = 0$

$x(a-f) = -b\left(\frac{-e.z}{d-f}\right) - cz$

$x(a-f) = -b\left(\frac{-e.z}{d-f}\right) - cz$

$= \frac{-bez}{d-f} - cz$

$\leftarrow x(a-f) = \frac{-bez - czd + czf}{d-f}$

$x = \frac{-be - cd + cf}{(d-f)(a-f)} . z$

$x = \frac{-be - cd + cf}{-f(d+a)} . z$

$$\boxed{V_3 = \left(\frac{-be - cd + cf}{-f(d+a)} . z, \frac{-e}{d-f} . z, z\right)}$$

5. Para quais valores de *a* as matrizes abaixo são diagonalizáveis?

a) $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & a \end{bmatrix}$ $[T]^{\beta}_{\beta} = \begin{bmatrix} \overset{v_1}{\uparrow} & \overset{v_2}{\uparrow} \\ x & x \\ x & x \end{bmatrix}$ *b*) $\mathbf{B} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$

a) $\begin{bmatrix} 1 & 1 \\ 0 & a \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \left|\begin{bmatrix} 1-\lambda & 1 \\ 0 & a-\lambda \end{bmatrix}\right| = (1-\lambda)(a-\lambda) = 0$

$\lambda_1 = 1$
$\lambda_2 = a$

$\lambda_1 = 1$

$$\begin{bmatrix} 1 & 1 \\ 0 & a \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} - 1\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{cases} x + y - x = 0 \rightarrow y = 0 \\ ay - y = 0 \end{cases}$$

$0x = 0$

$v_1 = (x, 0)$ x qualquer

$\lambda_2 = a$

$$\begin{cases} x + y - ax = 0 \\ ay - ay = 0 \end{cases}$$

$$\begin{cases} y = ax - x \longrightarrow y = x(a-1) \\ 0ay = 0 \end{cases}$$

$\rightarrow y \neq 0$
$\rightarrow x = 1$

$0 \neq a - 1$

$\boxed{a \neq 1}$

$v_2 = (x, ax - x)$

autovetores

$$\beta = \begin{bmatrix} x & x \\ 0 & ax - x \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 \\ 0 & a-1 \end{bmatrix}$$

$x = 1$

b) $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \left|\begin{bmatrix} 1-\lambda & a \\ 0 & 1-\lambda \end{bmatrix}\right| = (1-\lambda)^2 = 0$

$\lambda = 1$

$\lambda = 1$

$$\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{cases} x + a = x \\ y = y \end{cases} \rightarrow \begin{cases} \boxed{a = 0} \\ y = 0 \end{cases}$$

x qualquer

6. Sejam $T: \mathbf{R}^3 \rightarrow \mathbf{R}^3$ linear, $\alpha = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$, a base canônica de $\mathbf{R}^3$, $\beta = \{(0, 1, 1), (0, -1, 1), (1, 0, 1)\}$ e

$$[T]^{\alpha}_{\alpha} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix}$$

a) Encontre ~~o polinômio característico~~ de *T*, os autovalores de *T* e os autovetores ~~correspondentes.~~

b) Ache $[T]^{\beta}_{\beta}$ e o polinômio característico. Que observação você faz a este respeito?

c) Encontre uma base γ de $\mathbf{R}^3$, se for possível, tal que $[T]^{\gamma}_{\gamma}$ seja diagonal.

a) $\begin{bmatrix} 2 & 0 & 1 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \left|\begin{bmatrix} 2-\lambda & 0 & 1 \\ 0 & -3-\lambda & 0 \\ 0 & 0 & -3-\lambda \end{bmatrix}\right| = (2-\lambda)(-3-\lambda)^2 = 0$

$\lambda = 2$
$\lambda = -3$

$\boxed{\lambda = 2}$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} - 2\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

x = qualquer

$$\begin{cases} 2x + z - 2x = 0 \rightarrow z = 0 \\ -3y + z - 2y = 0 \rightarrow -5y = 0 \rightarrow y = 0 \\ -3z - 2z = 0 \rightarrow -5z = 0 \end{cases}$$

$v_1 = (x, 0, 0)$

$\boxed{\lambda = -3}$

$$\begin{cases} 2x + z + 3x = 0 \rightarrow 5x = 0 \\ -3y + z + 3y = 0 \rightarrow z = 0 \\ -3z - 2z + 3z = 0 \rightarrow -2z = 0 \end{cases}$$

y qualquer

$\boxed{z = 0}$

$v_2 = (0, y, 0)$

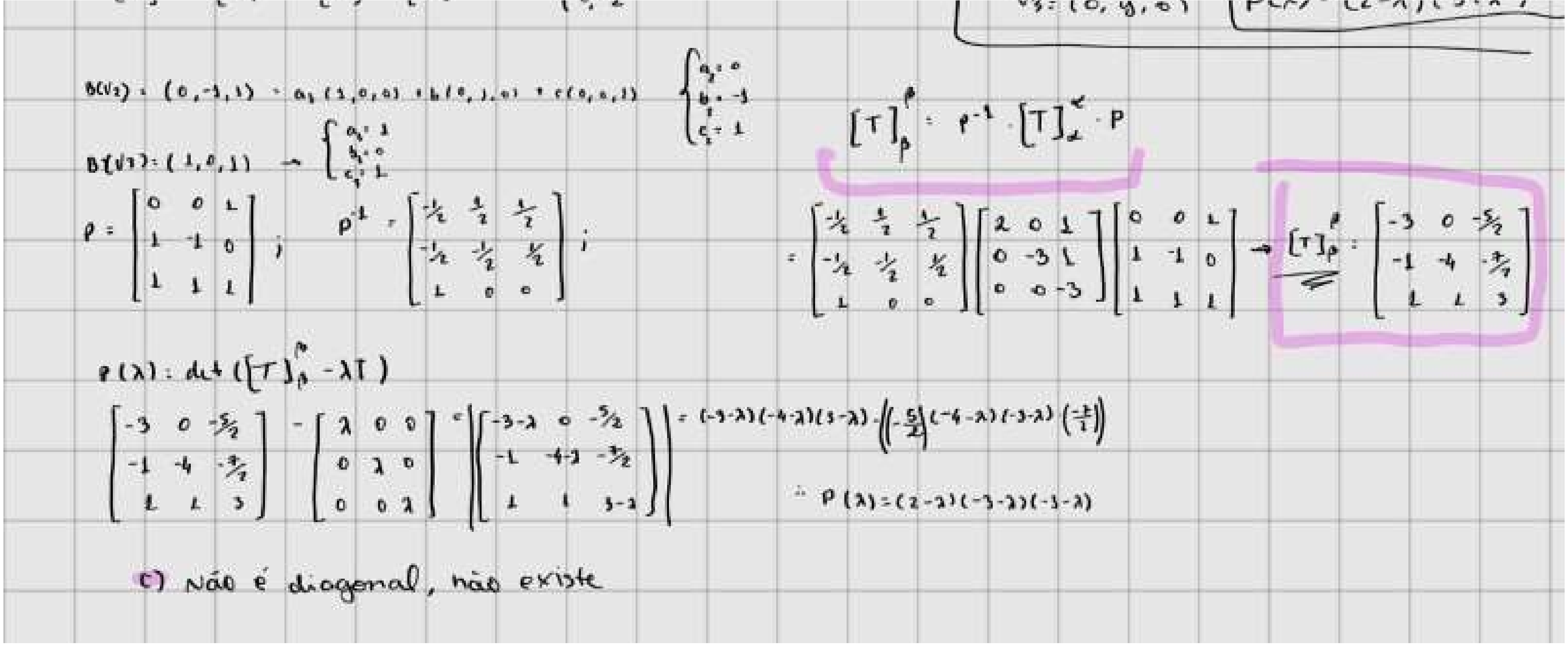
$P(x) = (2-\lambda)(3+\lambda)^2$

b)

$$\beta = \begin{bmatrix} v_1 & v_2 & v_3 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$\beta(v1) \rightarrow (0,1,1) = a_1(1,0,0) + b_1(0,1,0) + c_1(0,0,1)$

$$a_1\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \longrightarrow \begin{cases} a_1 = 0 \\ b_1 = 1 \\ c_1 = 1 \end{cases}$$



$B(v_2) = (0,-1,1) = a_1(1,0,0) + b(0,1,0) + c(0,0,1)$ $\begin{cases} a_1 = 0 \\ b_1 = -1 \\ c_1 = 1 \end{cases}$

$B(v_3) = (1,0,1) \rightarrow \begin{cases} a_1 = 1 \\ b_1 = 0 \\ c_1 = 1 \end{cases}$

$$\underbrace{[T]_\beta^\beta = P^{-1} \cdot [T]_\alpha^\alpha \cdot P}$$

$P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$; $P^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \end{bmatrix}$;

$$= \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \rightarrow \boxed{[T]_\beta^\beta = \begin{bmatrix} -3 & 0 & -\frac{5}{2} \\ -1 & -4 & -\frac{7}{2} \\ 1 & 1 & 3 \end{bmatrix}}$$

$P(\lambda) = \det([T]_\beta^\beta - \lambda I)$

$$\begin{bmatrix} -3 & 0 & -\frac{5}{2} \\ -1 & -4 & -\frac{7}{2} \\ 1 & 1 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \left|\begin{pmatrix} -3-\lambda & 0 & -\frac{5}{2} \\ -1 & -4-\lambda & -\frac{3}{2} \\ 1 & 1 & 3-\lambda \end{pmatrix}\right| = (-3-\lambda)(-4-\lambda)(3-\lambda) - \left(\left(-\frac{5}{2}\right)(-4-\lambda)(-3-\lambda)\left(-\frac{7}{1}\right)\right)$$

$\therefore P(\lambda) = (2-\lambda)(-3-\lambda)(-3-\lambda)$

c) Não é diagonal, não existe

Passo 1

Opaaa... vamos pra mais uma questãozinha???
Nessa aqui vamos usar a equação pra achar o polinômio característico:

$$P(\lambda) = \det(A - \lambda I)$$

E pra achar os autovalores:

$$P(\lambda) = 0$$

E os autovetores:

$$Ax = \lambda x$$

E também vamos precisar a relação pra mudança de base:

$$[T]_\alpha^\alpha = P[T]_\beta^\beta P^{-1}$$

Então:

$$[T]_\beta^\beta = P^{-1}[T]_\alpha^\alpha P$$

Passo 2

Pra encontrar o polinômio característico a gente resolve o determinante:

$$P(\lambda) = \det([T]_\alpha^\alpha - \lambda I)$$

Mas:

$$[T]_\alpha^\alpha - \lambda I = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[T]_\alpha^\alpha - \lambda I = \begin{bmatrix} 2-\lambda & 0 & 1 \\ 0 & -3-\lambda & 1 \\ 0 & 0 & -3-\lambda \end{bmatrix}$$

Daí a gente volta pro determinantes:

$$P(\lambda) = \det\begin{pmatrix} 2-\lambda & 0 & 1 \\ 0 & -3-\lambda & 1 \\ 0 & 0 & -3-\lambda \end{pmatrix}$$

E lembrando que o determinante de uma matriz triangular é só multiplicar os elementros da diagonal principal:

$$P(\lambda) = (2-\lambda)(-3-\lambda)(-3-\lambda)$$

Pra achar os autovalores fazemos:

$$P(\lambda) = 0$$

Pra achar os autovalores fazemos:

$$P(\lambda) = 0$$

$$(2-\lambda)(-3-\lambda)(-3-\lambda) = 0$$

Então ficamos com os seguintes autovalores:
$\lambda = 2$ ou $\lambda = -3$
E os autovetores:
- pra $\lambda = 2$:

$$\begin{bmatrix}2 & 0 & 1\\0 & -3 & 1\\0 & 0 & -3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = 2\begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\begin{cases}2x + 0y + 1z = 2x\\0x - 3y + 1z = 2y\\0x + 0y - 3z = 2z\end{cases}$$

$$v_1 = (x, 0, 0)$$

- pra $\lambda = -3$:

$$\begin{bmatrix}2 & 0 & 1\\0 & -3 & 1\\0 & 0 & -3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = -3\begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\begin{cases}2x + 0y + 1z = -3x\\0x - 3y + 1z = -3y\\0x + 0y - 3z = -3z\end{cases}$$

$$v_2 = (0, y, 0)$$

Passo 3

Vamos começar expressando a nossa base como combinação da base canônica:
- pro primeiro vetor, $(0,1,1)$:

$$(0,1,1) = a_1(1,0,0) + b_1(0,1,0) + c_1(0,0,1)$$

Fazendo o sisteminha:

$$\begin{cases}0 = a_1 + 0b_1 + 0c_1\\1 = 0a_1 + b_1 + 0c_1\\1 = 0a_1 + 0b_1 + c_1\end{cases}$$

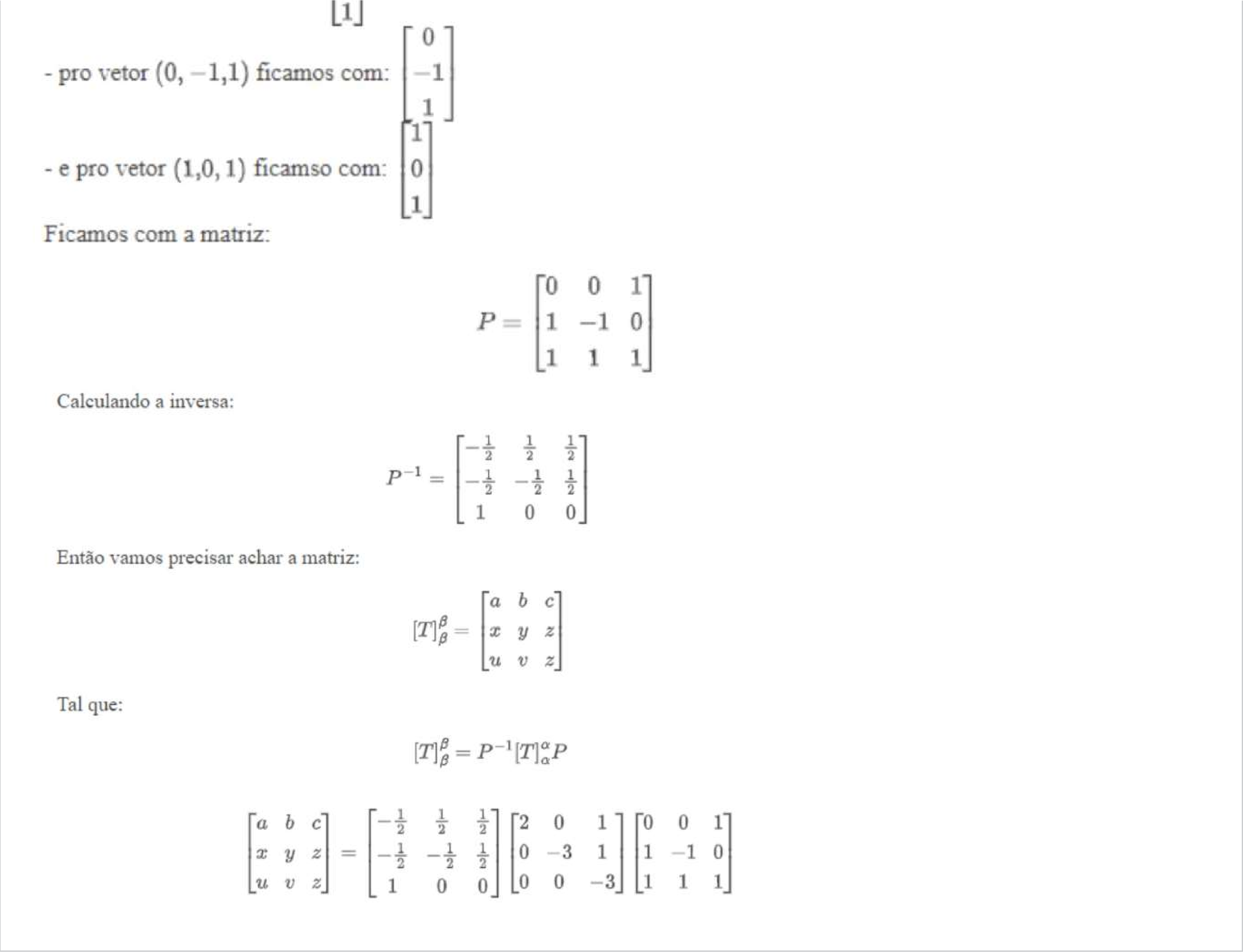
Resolvendo:

$$a_1 = 0$$

$$b_1 = 1$$

$$c_1 = 1$$

Então ficamos com o vetor: $\begin{bmatrix}0\\1\end{bmatrix}$



1

- pro vetor $(0,-1,1)$ ficamos com: $\begin{bmatrix}0\\-1\\1\end{bmatrix}$

- e pro vetor $(1,0,1)$ ficamso com: $\begin{bmatrix}1\\0\\1\end{bmatrix}$

Ficamos com a matriz:

$$P=\begin{bmatrix}0&0&1\\1&-1&0\\1&1&1\end{bmatrix}$$

Calculando a inversa:

$$P^{-1}=\begin{bmatrix}-\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\-\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\1&0&0\end{bmatrix}$$

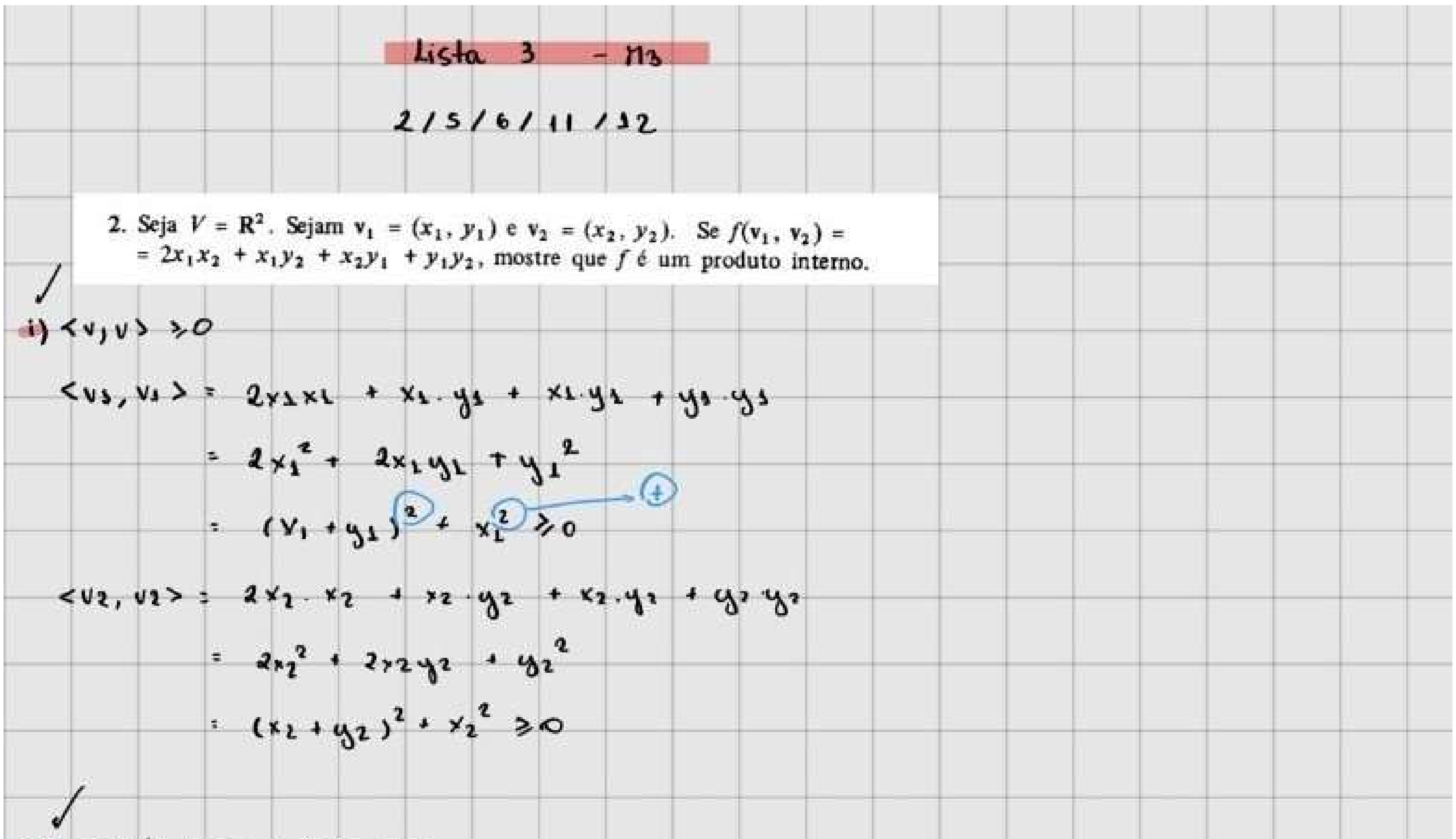
Então vamos precisar achar a matriz:

$$[T]^{\beta}_{\beta}=\begin{bmatrix}a&b&c\\x&y&z\\u&v&z\end{bmatrix}$$

Tal que:

$$[T]^{\beta}_{\beta}=P^{-1}[T]^{\alpha}_{\alpha}P$$

$$\begin{bmatrix}a&b&c\\x&y&z\\u&v&z\end{bmatrix}=\begin{bmatrix}-\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\\-\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\1&0&0\end{bmatrix}\begin{bmatrix}2&0&1\\0&-3&1\\0&0&-3\end{bmatrix}\begin{bmatrix}0&0&1\\1&-1&0\\1&1&1\end{bmatrix}$$



Lista 3 - N3

2 / 5 / 6 / 11 / 12

2. Seja $V=\mathbb{R}^2$. Sejam $v_1=(x_1,y_1)$ e $v_2=(x_2,y_2)$. Se $f(v_1,v_2)=$ $=2x_1x_2+x_1y_2+x_2y_1+y_1y_2$, mostre que f é um produto interno.

✓

i) $\langle v, v\rangle \geq 0$

$\langle v_1, v_1\rangle = 2x_1x_1 + x_1\cdot y_1 + x_1\cdot y_1 + y_1\cdot y_1$

$= 2x_1^2 + 2x_1y_1 + y_1^2$

$= (v_1+y_1)^{\textcircled{2}} + x_1^{\textcircled{2}} \geq 0 \longrightarrow \textcircled{+}$

$\langle v_2, v_2\rangle = 2x_2\cdot x_2 + x_2\cdot y_2 + x_2\cdot y_2 + y_2\cdot y_2$

$= 2x_2^2 + 2x_2y_2 + y_2^2$

$= (x_2+y_2)^2 + x_2^2 \geq 0$

✓

ii) $\langle \alpha v_1, v_2 \rangle = \alpha \langle v_1, v_2 \rangle$

$$\langle \alpha v_1, v_2 \rangle = \langle \alpha 2x_1 \cdot x_2 + \alpha x_1 \cdot y_2 + \alpha x_2 y_1 + \alpha y_1 \cdot y_2 \rangle$$

$$= \alpha (2x_1 \cdot x_2 + x_1 \cdot y_2 + x_2 \cdot y_1 + y_1 \cdot y_2 \rangle \quad \therefore \alpha \langle v_1, v_2 \rangle$$

$\langle v_1 + v_2, v_3 \rangle = \langle v_1, v_3 \rangle + \langle v_2, v_3 \rangle$

iii) $v_3 = (x_3, y_3)$

$$\langle v_1 + v_2, v_3 \rangle = 2(x_1 + x_2)x_3 + (x_1 + x_2)y_3 + x_3(y_1 + y_2) + (y_1 + y_2)y_3$$

$$= \underbrace{(2x_1 \cdot x_3) + (2x_2 \cdot x_3)}_{(v_1, v_3) + (v_2, v_3)} + \underbrace{(x_1 \cdot y_3) + (x_2 \cdot y_3)}_{v_1 v_3 + v_2 \cdot v_3} + \underbrace{(y_1 \cdot x_3) + (y_2 \cdot x_3)}_{v_1 \cdot v_3 + v_2 v_3} + \underbrace{(y_1 \cdot y_3) + (y_2 \cdot y_3)}_{\dots}$$

$$= \langle (v_1 \cdot v_3) \rangle + \langle (v_2, v_3) \rangle$$

iv) $\langle v_1, v_2 \rangle = \langle v_2, v_1 \rangle$

$$\langle v_2, v_1 \rangle = 2x_2 \cdot x_1 + x_2 y_1 + x_1 \cdot y_2 + y_2 \cdot y_1$$

$$= 2 \cdot x_1 \cdot x_2 + x_1 \cdot y_2 + x_2 \cdot y_1 + y_1 \cdot y_2 = \langle v_1, v_2 \rangle$$

∴ é um produto interno

5. Seja $\beta = \{(\overset{u_1}{1, 1, 0}), (\overset{u_2}{1, 0, 1}), (\overset{u_3}{0, 2, 0})\}$. Ache uma base ortonormal β' de $\mathbf{R}^3$, em relação ao produto interno usual.

PASSO 1 $\longrightarrow v_1 = u_1 = \boxed{(1, 1, 0)}$

PASSO 2 $\longrightarrow v_2 = u_2 - \dfrac{\langle u_2, v_1 \rangle}{\|v_1\|^2} \cdot v_1$

$$v_2 = (1,0,1) - \frac{\langle (1,0,1), (1,1,0) \rangle}{\sqrt{(1^2 + 1^2 + 0)}^2} \cdot (1,1,0)$$

$$= (1,0,1) - \frac{(1+0+0)}{2} \cdot (1,1,0)$$

$$= (1,0,1) - \frac{1}{2}(1,1,0) \longrightarrow \boxed{\left(\frac{1}{2}, \frac{-1}{2}, 1\right)}$$

PASSO 3

$$v_3 \to u_3 - \frac{\langle u_3, v_1 \rangle}{\|v_1\|^2} \cdot v_1 - \frac{\langle u_3, v_2 \rangle}{\|v_2\|^2} \cdot v_2$$

$$= (0,2,0) - \frac{\overbrace{\langle (0,2,0), (1,1,0) \rangle}^{(0+2+0)}}{2} \cdot (1,1,0) - \frac{\overbrace{\langle (0,2,0), (\frac{1}{2}, -\frac{1}{2}, 1) \rangle}^{(0-1+0)}}{\underbrace{\left(\sqrt{(\frac{1}{2})^2 + (-\frac{1}{2})^2 + 1^2}\right)^2}_{\frac{1}{4} + \frac{1}{4} + 1 = \frac{6}{4} = \frac{3}{2}}} \cdot \left(\frac{1}{2}, -\frac{1}{2}, 1\right) \quad -\frac{2}{3}$$

$$= (0,2,0) - \frac{2}{2} \cdot (1,1,0) + \frac{2}{3}\left(\frac{1}{2}, -\frac{1}{2}, 1\right)$$

$$= (0,2,0) - (1,1,0) + (1/3, -1/3, 2/3)$$

$$= (-1, 1, 0) + (1/3, -1/3, 2,3)$$

$$= \boxed{(-2/3, 2/3, 2/3)}$$

ORTO
Normalização

$$Q_1 = \frac{\vec{v_1}}{\|\vec{v_1}\|} = \frac{(1,1,0)}{\sqrt{1^2 + 1^2 + 0}} = \left(1/\sqrt{2}, \frac{1}{\sqrt{2}}, 0\right)$$

$$Q_2 = \frac{(1/2, -1/2, 1)}{\sqrt{3/2}} = \left(\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{\sqrt{6}}{3}\right)$$

$$\underbrace{2\sqrt{3/2} = \sqrt{4 \cdot \frac{3}{2}} = \sqrt{6}}$$

$$1 \cdot \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{2}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{2\sqrt{6}}{6}$$

$$\frac{1 \cdot \sqrt{6}}{3}$$

$$Q_3 = \frac{(-2/3, 2/3, 2/3)}{\sqrt{(\frac{2}{3})^2 + (\frac{2}{3})^2 + (\frac{2}{3})^2}}$$

$$\sqrt{\frac{4}{9} + \frac{4}{9} + \frac{4}{9}} = \frac{\overset{3}{\cancel{12}}}{9} = \sqrt{\frac{4}{3}}$$

$$Q_3 = \frac{(-2/3, 2/3, 2/3)}{2/\sqrt{3}}$$

$$\sqrt{4/3} = \frac{2}{\sqrt{3}}$$

$$Q_3 = \left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$$

$$\frac{2}{3} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{3} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$$

ou

$$Q_3 = \left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

6. Seja $\beta = \{(\overset{u_1}{-1, 1}), (\overset{u_2}{1, 1})\}$. Ache uma base ortonormal β' de $\mathbf{R}^2$, em relação ao produto interno definido no Exercício 2.

$\langle v_1, v_2\rangle = 2x_1 . x_2 + x_1 \cdot y_2 + x_2 \cdot y_1 + y_1 \cdot y_2$

PASSO 1 → $v_1 = \boxed{u_1 = (-1,1)}$

PASSO 2 → $v_2 = u_2 - \dfrac{\langle u_2, v_1\rangle}{\|v_1\|^2} . v_1 = (1,1) - \dfrac{\underbrace{\langle (1,1),(-1,1)\rangle}}{\sqrt{\langle -1,1\rangle}^2} \cdot (-1,1)$

$= (1,1) - \dfrac{(-1)}{1} . (-1,1)$

$= (1,1) + (-1,1)$

$\boxed{v_2 = (0,2)}$

$\langle (1,1),(-1,1)\rangle$

$= 2x_1 . x_2 + x_1 \cdot y_2 + x_2 \cdot y_1 + y_1 \cdot y_2$

$= 2 . 1 . (-1) + 1 . 1 + (-1) . 1 + 1 . 1$

$= -2 + 1 - 1 + 1 = -1$

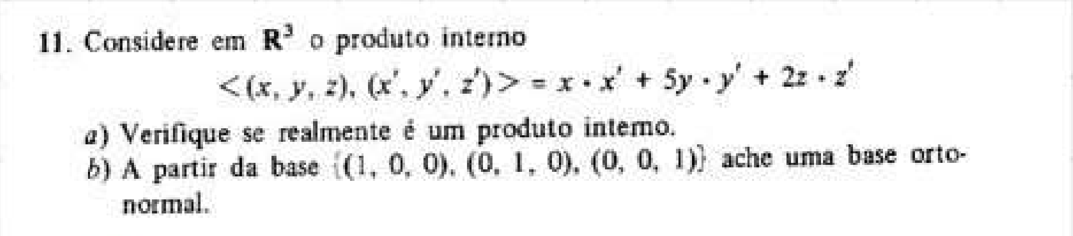
$\beta = \{(-1,1),(0,1)\}$

$\sqrt{\langle -1,1\rangle}^2$

$= 2x_1 . x_2 + x_1 \cdot y_2 + x_2 \cdot y_1 + y_1 \cdot y_2$

$= 2 . (-1) . (-1) + (-1) . 1 + (-1) . 1 + 1 . 1$

$= 2 - 2 + 1 = 1$



11. Considere em $\mathbf{R}^3$ o produto interno

$$<(x, y, z), (x', y', z')> = x \cdot x' + 5y \cdot y' + 2z \cdot z'$$

a) Verifique se realmente é um produto interno.

b) A partir da base $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ ache uma base ortonormal.

i)

a) $\langle (x,y,z),(x,y,z)\rangle = x^2 + 5y^2 + 2z^2 \geq 0$ (Raiz $\mathbb{R} = ⊕$)

$\langle (x',y',z'),(x',y',z')\rangle = x'^2 + 5y'^2 + 2z'^2 \geq$ ''

ii) $\langle \alpha v_1, v_2\rangle = \alpha x . x' + \alpha 5y \cdot y' + \alpha 2 . z . z' = \alpha (x . x' + 5y \cdot y' + 2 . z . z') = \alpha \langle v_1, v_2\rangle$

iii) $v_3 = (x_3, y_3, z_3)$

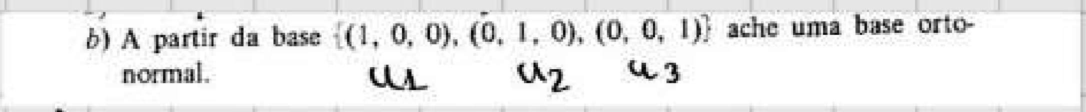
$\langle v_1 + v_2, v_3\rangle = (x+x') x_3 + 5(y+y') y_3 + 2(z+z') z_3 = \overset{\langle v_1,v_3\rangle + \langle v_2,v_3\rangle}{x . x_3 + x' . x_3 + 5y . y_3 + 5y' . y_3 +}$

$2z . z_3 + 2z' . z_3$

$= \langle v_1, v_3\rangle + \langle v_2, v_3\rangle$

iv) $\langle v_2, v_1\rangle = x' . x + 5y' . y + 2z' . z$

$= x . x' + 5y . y' + 2z . z' = \langle v_1, v_2\rangle$



b) A partir da base $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ ache uma base ortonormal.

$u_1 \quad u_2 \quad u_3$

PASSO 1 $v_1 = \boxed{u_1 = (1,0,0)}$

PASSO 2 $v_2 = u_2 - \dfrac{\langle u_2, v_1\rangle}{\|v_1\|^2} . v_1 = (0,1,0) - \dfrac{\langle (0,1,0),(1,0,0)\rangle}{\langle 1,0,0\rangle^2} . (1,0,0)$

$\nearrow 0.1 + 5.1.0 + 2.0.0 = 0$

$\rightarrow 1^2 + 5.0^2 + 2.0^2 = 1^2 = 1$

$= (0,1,0) - 0 (1,0,0)$

$\boxed{= (0,1,0)}$

PASSO 3

$V_3 = u_3 - \dfrac{\langle u_3, v_1\rangle}{\|v_1\|^2} . v_1 - \dfrac{\langle u_3, v_2\rangle}{\|v_2\|^2} . v_2$

$0.0 = 0$

$$V_3 = (0,0,1) - \frac{\overset{\to\, 0.0+5.0.0+2.0.0=0}{\langle (0,0,1),(1,0,0)\rangle}}{1}\cdot(1,0,0) - \frac{\overset{\to\, 0.0+5.0.}{\langle (0,0,1),(0,1,0)\rangle}}{\langle (0,1,0)\rangle^2}\cdot(0,1,0)$$

$\hookrightarrow 0^2+5.1^2+2.0^2=5$

$$\boxed{V_3=(0,0,1)}$$

ORTO NORMALIZAÇÃO

$$Q_1 = \frac{\vec{V_1}}{\|\vec{V_1}\|} = \frac{(1,0,0)}{\sqrt{1}} = (1,0,0)$$

$$Q_2 = \frac{\vec{V_2}}{\|V_2\|} = \frac{(0,1,0)}{\sqrt{5}} = \left(0,\frac{1}{\sqrt5},0\right)$$

$$Q_3 = \frac{\vec{V_3}}{\|V_3\|} = \frac{(0,0,1)}{\langle V_3,V_3\rangle} = \frac{(0,0,1)}{\sqrt2} = \left(0,0,\frac{1}{\sqrt2}\right)$$

$\hookrightarrow 0^2+5.0^2+2.1^2=2$

$$\beta = \left\{(1,0,0),\left(0,\frac{1}{\sqrt5},0\right),\left(0,0,\frac{1}{\sqrt2}\right)\right\}$$

$$\langle f,g\rangle = \int_{-1}^{1} f(t)\cdot g(t)\,dt$$

(12) a) Base $\{1, 1-t\}$ de $P2$

Vamos provar que $\langle f,g\rangle$ satisfaz

✓ i) $\langle v,v\rangle \geq 0$

$$\langle f,f\rangle = \int_{-1}^{1} 1.1\,dt = \overset{\text{integral de 1 é } t}{t\Big|_{-1}^{1}} = 1-(-1) = \boxed{2}\text{ ok}$$

$$\langle g,g\rangle = \int_{-1}^{1}(1-t)(1-t)\,dt = \int_{-1}^{1}(1-2t+t^2)\,dt = t - t^2 + \frac{t^3}{3}\Big|_{-1}^{1} = 1-1+\frac13-\left(-1-1-\frac13\right)$$

$$= \frac13 + 2^{\frac{6}{3}} = \boxed{\frac83}\text{ ok}$$

✓ ii) $\langle \alpha f,g\rangle = \int_{-1}^{1}\alpha\, 1\,.(1-t)\,dt \to \alpha\int_{-1}^{1}\overbrace{1(1-t)\,dt}^{\langle f,g\rangle} \to \alpha\langle f,g\rangle$

✓ iii) Seja $h = t^2 \in P2$ $\quad\because \langle v_1,v_3\rangle+\langle v_2,v_3\rangle$

$$\langle f+g,h\rangle = \langle 1+(1-t);\, t^2\rangle = \int_{-1}^{1}[1+(1-t)]\cdot t^2\,dt = \underbrace{\int_{-1}^{1}1.t^2\,dt}_{\langle 1,t^2\rangle} + \int_{-1}^{1}(1-t)t^2\,dt = \langle f,h\rangle+\langle g,h\rangle$$

$\langle 1,t^2\rangle + \langle (1-t),t^2\rangle$ ok

✓ iv) $\langle f,g\rangle = \int_{-1}^{1}1.(1-t)\,dt = \int_{-1}^{1}(1-t)\,1\,dt \therefore \langle g,f\rangle$

∴ é um produto interno, 4 propriedades OK

b) $\beta = \{1, 1-t\}$ (Base de polinomios)

$1-\frac12-\left(-1-\frac12\right) = \frac12+\frac32 = \frac42 = 2$

PASSO 1 → $v_1 = u_1 = \boxed{1}$

$\int_{-1}^{1}(1-t)\,1\,dt = t-\frac{t^2}{2}\Big|_{-1}^{1} = 2$

PASSO 2 → $v_2 = u_2 - \frac{\langle u_2,v_1\rangle}{\|v_1\|^2}\cdot v_1 = 1-t-\frac{\langle 1-t,1\rangle}{(\sqrt{\langle 1,1\rangle})^2}\cdot 1 = 1-t-\frac{2}{2}\cdot 1 = 1-t-1 = \boxed{-t}$

$\int_{-1}^{1}1.1\,dt = t\Big|_{-1}^{1} = 1-(-1) = 2$

∴ a base ortogonal é

$$\{1,-t\}$$

BASE

ORTO NORMAL :

$$Q_1 = \frac{\overrightarrow{V_1}}{\|\overrightarrow{V_1}\|} = \frac{1}{\sqrt{2}}$$

$$\therefore \left\{ \frac{1}{\sqrt{2}} , \frac{-t}{\sqrt{2/3}} \right\}$$

$$Q_2 = \frac{\overrightarrow{V_2}}{\|V_2\|} = \frac{-t}{\sqrt{\langle -t, -t \rangle}} = \frac{-t}{\sqrt{2/3}}$$

$$\hookrightarrow \int_{-1}^{1} -t \cdot -t \, dt = \int_{-1}^{1} t^2 \, dt = \frac{t^3}{3}\Big|_{-1}^{1} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$