Universidade de Brasília – UnB **Faculdade UnB Gama – FGA**
Cálculo 1 - Prof. Ricardo Fragelli

Faça um esboço dos gráficos seguintes com base nas informações dadas:

GRÁFICO 1

Domínio: $\mathbb{R}$

		x=-5	x=-2	x=0	x=1	x=3	
y'	+	+	+	-	-	+	
y''	-	+	-	-	+	+	

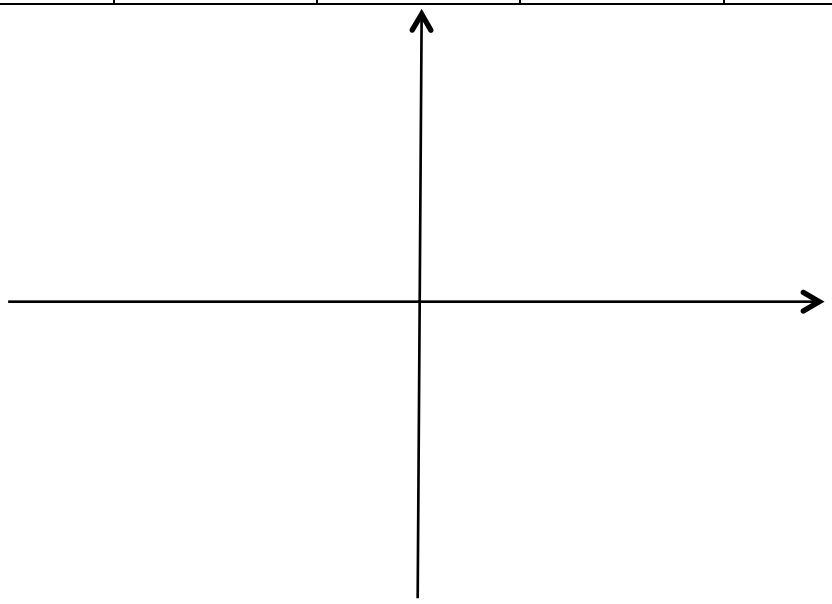


GRÁFICO 2

Domínio: $\mathbb{R}$

Pontos: (-5, -3), (-2, 0), (0, 2), (1, 0), (3, -2)

		x=-5	x=-2	x=0	x=1	x=3	
y'	+	+	+	-	-	+	
y''	-	+	-	-	+	+	

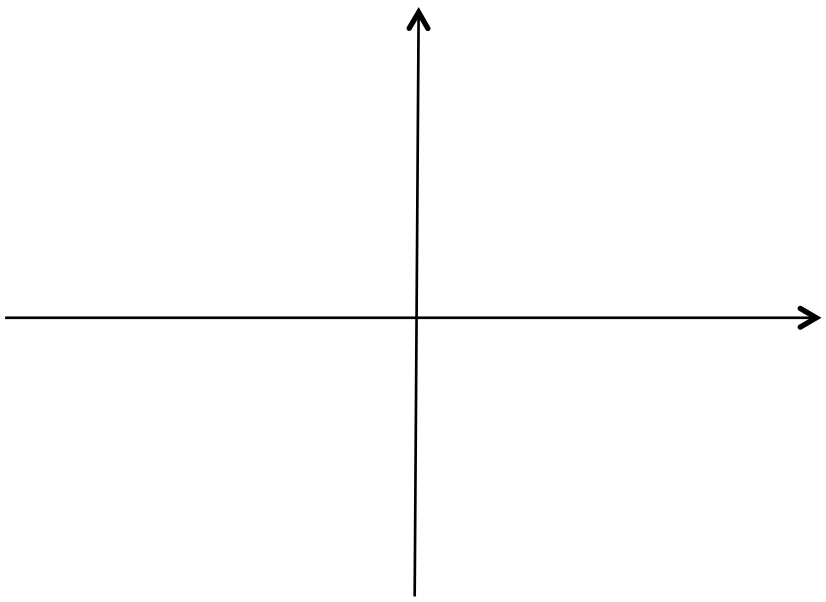


GRÁFICO 3

Domínio: $\mathbb{R}$

Pontos: (-3, 1), (-1, 2), (0, 0), (1, 2), (3, 1)

		x=-3		x=-1		x=0		x=1		x=3	
y'	+		+		-		+		-		-
y''	+		-		-		-		-		+

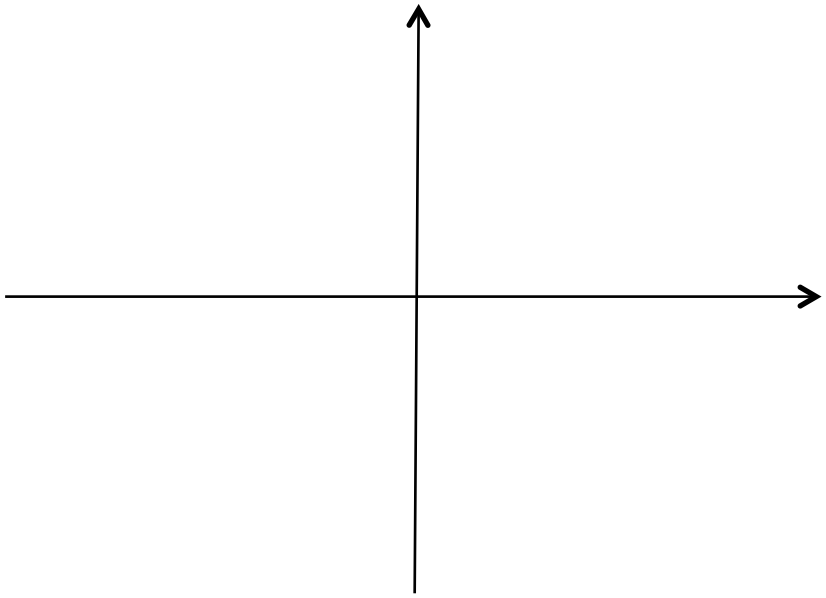


GRÁFICO 4

Domínio: $\mathbb{R}$

$\lim_{x \to -\infty} f(x) = 0$

$\lim_{x \to +\infty} f(x) = -3$

Pontos: (-3, 1), (-1, 2), (0, 0), (1, 2), (3, 1)

		x=-3		x=-1		x=0		x=1		x=3	
y'	+		+		-		+		-		-
y''	+		-		-		-		-		+

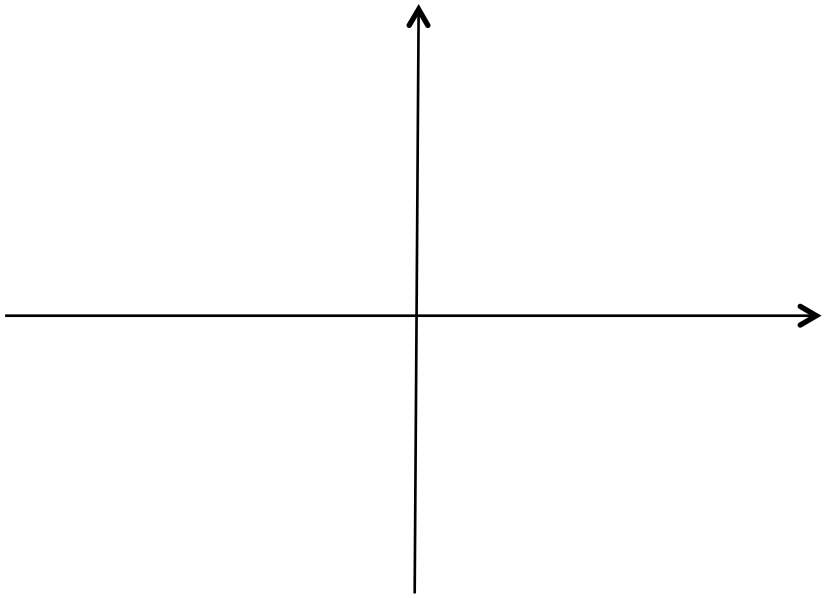


GRÁFICO 5

Domínio: $\mathbb{R}$ - {0}

$\lim_{x\to-\infty} f(x) = -\infty$ $\quad$ $\lim_{x\to+\infty} f(x) = 0$

$\lim_{x\to0-} f(x) = +\infty$ $\quad$ $\lim_{x\to0+} f(x) = 1$

Pontos: (-5, 2), (-2, 0), (1, 2), (3, 1)

	x=-5	x=-2	x=0	x=1	x=3	
y'	+ ↗	- ↘	+ ↗	+ ↗	- ↘	- ↘
y''	+ ∪	+ ∪	+ ∪	- ∩	- ∩	+ ∪

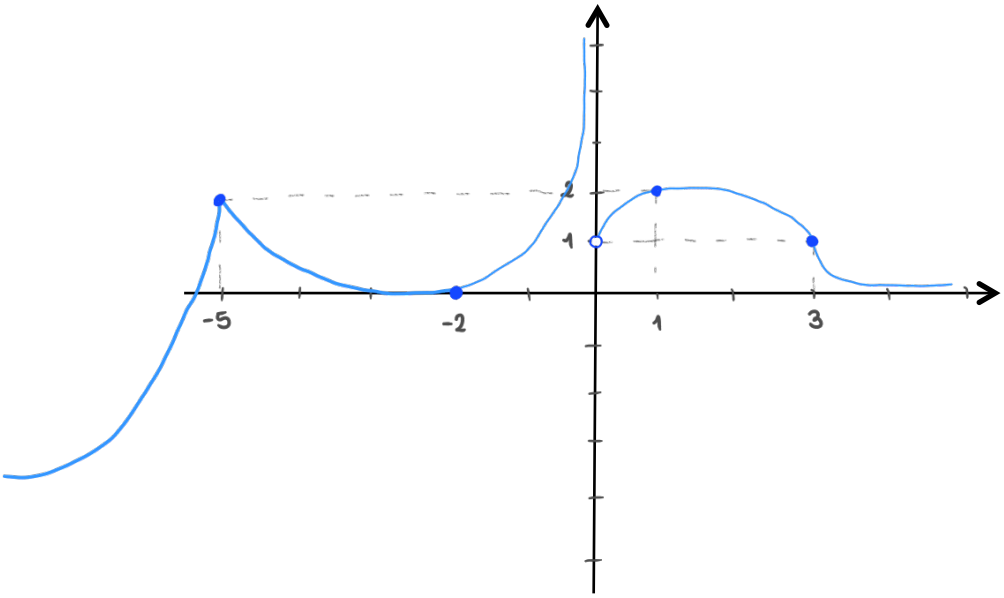


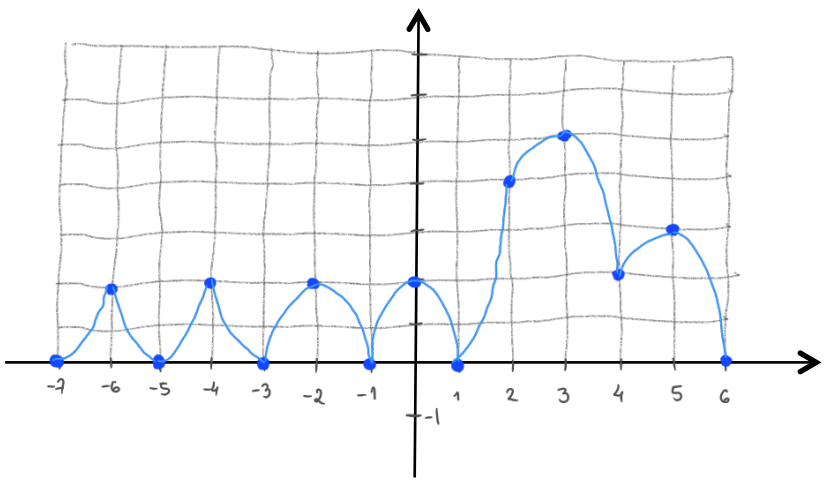
GRÁFICO 6

Domínio: [-7, 6]

$$\lim_{x\to-7+} f(x) = \lim_{x\to6-} f(x) = f(-7) = f(-5) = f(-3) = f(-1) = f(1) = f(6) = 0$$

$f(-6) = f(-4) = f(-2) = f(0) = f(4) = 2$, $f(2) = 4$, $f(3)=5$, $f(5)=3$

	x=-6	x=-5	x=-4	x=-3	x=-2	x=-1	x=0	x=1	x=2	x=3	x=4	x=5
y'	↗ +	↘ -	↗ +	↘ -	↗ +	↘ -	↗ +	↘ -	↗ +	↗ +	↘ -	↗ +	↘ -
y''	∪ +	∪ +	∪ +	∪ +	∩ -	∩ -	∩ -	∩ -	∪ +	∩ -	∩ -	∩ -	∩ -



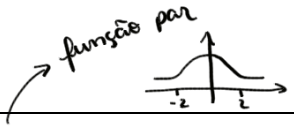


GRÁFICO 7

$y = e^{-x^2}$ Domínio = $\mathbb{R}$ $\lim_{x \to \pm\infty} e^{-x} = 0$

$e^{-x} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

$y' = e^{-x^2} \cdot (-2x)$

$y' = 0 \rightarrow e^{-x^2}(-2x) = 0$

$\therefore -2x = 0$

$x = 0$ → Ponto crítico (1ª ordem)

exp não pode ser 0.

$y'' = e^{-x^2}(-2x)(-2x) + e^{-x^2} \cdot (-2)$

$y'' = e^{-x^2}(4x^2 - 2)$

$y'' = 0 \rightarrow e^{-x^2}(4x^2 - 2) = 0$

$\therefore 4x^2 - 2 = 0$

$4x^2 = 2$

$x^2 = \frac{1}{2}$

$x = \pm\sqrt{1/2}$ → Pontos críticos de 2ª ordem (muda a concavidade)

Pontos do gráfico

$y(0) = e^0 = 1$

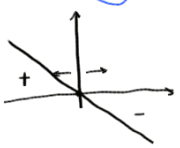
$y(-\sqrt{\frac{1}{2}}) = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$

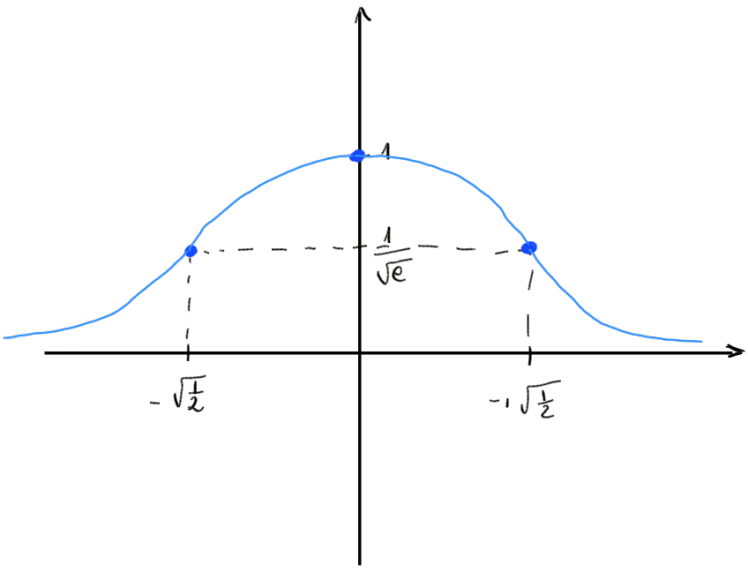
$y(+\sqrt{\frac{1}{2}}) = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$

	$x = -\sqrt{\frac{1}{2}}$	$x = 0$	$x = +\sqrt{\frac{1}{2}}$
y'	+	+	−
y''	∪ +	∩ −	∩ − ∪ +

$y' = 2x$ influencia o sinal

$y'' = +4x^2 - 2$





$y = xe^x$ Domínio : $\mathbb{R}$

1°) Domínios
2°) Limites
3°) Pontos críticos
4°) ESTUDO do sinal
5°) Pontos no gráfico

$\lim_{x\to-\infty} x \cdot e^x = \lim_{x\to-\infty} \frac{x}{e^{-x}} \overset{L'H}{=} \lim_{x\to-\infty} \frac{1}{-e^{-x}} = 0$

$\lim_{x\to+\infty} x \cdot e^x = +\infty$

$y = x \cdot e^x$

$y' = 1 \cdot e^x + x \cdot e^x$

$y' = e^x(1+x)$

$0 = e^x(1+x)$

$0 = 1+x$

$x = -1$

$y'' = e^x + e^x + x \cdot e^x$

$y'' = 2e^x + x e^x$

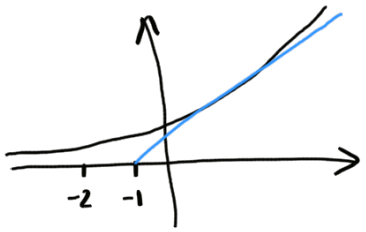
$y'' = e^x(2+x)$

$0 = e^x(2+x)$

$0 = 2+x$

$x = -2$

ESBOÇO:



		-2		-1	
y'	−		−		+
y''	−		+		+

$y' = 1 \cdot e^x + x \cdot e^x$

$y'(-10) = 1 \cdot e^{-10} - 10 \cdot e^{-10}$ ⊖ | $y'(1) = e^1 + 1 \cdot e^1 = 2e =$ ⊕

$y''(-10) = 2 \cdot e^{-10} - 10 \cdot e^{-10}$ ⊖ | se a exp vsbe vai ficar ⊕

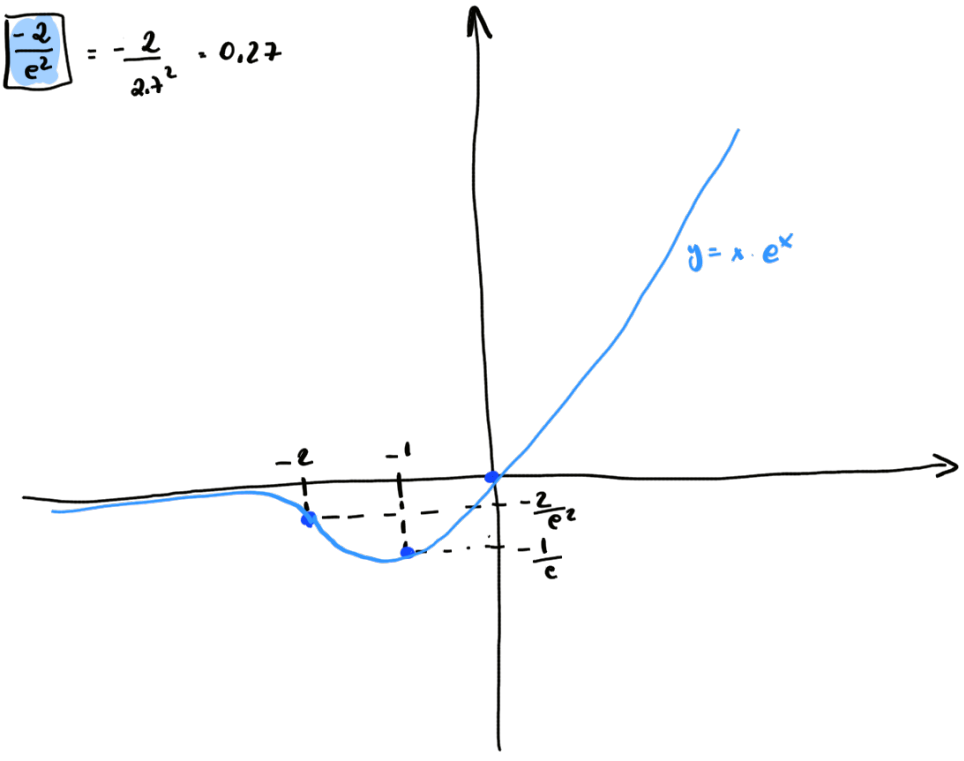
$y'' = 2e^x + x e^x$

Pontos : $y = x \cdot e^x$

$y(-1) = -e^{-1} = \frac{-1}{e} = -\frac{1}{2{,}7} = -0{,}3$

$y(-2) = -2e^{-2} = \frac{-2}{e^2} = -\frac{2}{2{,}7^2} \approx 0{,}27$

$y(0) = 0$



$y = x + \frac{1}{x}$ Domínio : $\mathbb{R} - \{0\}$

1) Limites

$\lim_{x \to \pm\infty} \left(x + \frac{1}{x}\right) = \pm\infty$ ($\frac{1}{x} \to 0$)

$\lim_{x \to 0\pm} \left(x + \frac{1}{x}\right) = \pm\infty$

$\frac{1}{0} = +\infty / -\infty / \nexists$

$\lim_{x \to 0+} x + \frac{1}{x} = +\infty$

$\lim_{x \to 0-} x + \frac{1}{x} = -\infty$

2) derivadas

$y = x + \frac{1}{x}$

$y' = 1 - \frac{1}{x^2}$

$y' = 1 - x^{-2}$

$y'' = 2x^{-3}$

$y'' = \frac{2}{x^3}$

$0 = 1 - \frac{1}{x^2}$

$\frac{1}{x^2} = 1$

$x^2 = 1 \rightarrow x = \pm 1$

3) Pontos críticos

$x = \pm 1$

Ponto crítico de 1ª ordem

$0 = \frac{2}{x^3} = \nexists$

não tem ponto crítico de 2ª ordem

função impar

4) Sinais

	-1	0	1	
y'	+	−	−	+
y''	−	−	+	+

$y' = 1 - \frac{1}{x^2}$ | $(-2)\ y = 1 - \frac{1}{(-2)^2} = \frac{3}{4} = (+)$ | $(-\frac{1}{2})\ y = 1 - 1(-2)^2\ y = -4\ (-)$

$y'' = \frac{2}{x^3}$

$x > 0 = (+)$

$x < 0 = (-)$

5) Pontos

$y = x + \frac{1}{x}$ $(-1) = -2$ $(1) = 2$