EDO 2 – s3 e 4

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Lista de fixação Semana 4

1) Encontre a solução geral:

(a) $y''(t) + 3y'(t) - 4y(t) = 0$ $\to R^2 + 3R - 4 = 0$

$\Delta = 9 - 4 \cdot 1 \cdot (-4) = 25$ $\rightsquigarrow$ $\Delta > 0$: $y_1(t) = e^{R_1 t}$, $y_2(t) = e^{R_2 t}$

$R = \frac{-3 \pm 5}{2}$ $\to R_1 = 1$, $\to R_2 = -4$

$\therefore \boxed{y_1(t) = e^{t}, \; y_2(t) = e^{-4t}}$

(b) $y''(t) - 2y'(t) + y(t) = 0$ $\to R^2 - 2R + 1 = 0$

$\Delta = 4 - 4 \cdot 1 \cdot 1 = 0$ $\rightsquigarrow$ $\Delta = 0 \to y_1(t) = e^{Rt}$, $y_2(t) = t\,e^{Rt}$

$R = \frac{2 \pm 0}{2} = 1$

$\boxed{\therefore y_1(t) = e^{t}, \; y_2(t) = e^{t}}$

(c) $y''(t) - 4y'(t) + 4y(t) = 0$ $\to R^2 - 4R + 4 = 0$

$\Delta = 16 - 4 \cdot 1 \cdot 4 = 0$

$\boxed{\therefore y_1(t) = e^{2t}, \; y_2(t) = 2e^{2t}}$

$R = \frac{4 \pm 0}{2} = 2$

(d) $y''(t) + 4y'(t) + 13y(t) = 0$ $\to R^2 + 4R + 13 = 0$

$\Delta = 16 - 4 \cdot 1 \cdot 13 = -36$ $\rightsquigarrow$ $\Delta < 0 \to y_1(t) = e^{at} \cdot \cos(bt)$

$y_2(t) = e^{at} \cdot \operatorname{sen}(bt)$

$R = \frac{-b}{2a} \pm \frac{\sqrt{|\Delta|} \cdot i}{2 \cdot a}$

$R = -\frac{2}{2} \pm \frac{\sqrt{36} \cdot i}{2} \to \boxed{R = \underbrace{-2}_{a} \pm \underbrace{6}_{b} i}$

$\therefore \boxed{\begin{aligned} y_1(t) &= e^{-2t} \cdot \cos(3t) \\ y_2(t) &= e^{-2t} \cdot \operatorname{sen}(3t) \end{aligned}}$

2) Encontre todos os valores de k para os quais a equação diferencial $y''(t)+ky'(t)+ky(t)=0$ tenha uma solução geral da forma dada: $\rightarrow R^2 + KR + K = 0$

(a) $y(t) = c_1e^{at} + c_2e^{bt}$ $\Delta > 0$

(b) $y(t) = c_1e^{at} + c_2te^{at}$ $\Delta = 0$

$a \pm bi$ $\Delta < 0$

(c) $y(t) = c_1e^{at}cos(bt) + c_2e^{at}sen(bt)$

$\Delta = K^2 - 4K > 0$

$K(K-4) > 0$

$\boxed{K > 0 \text{ ou } K > 0}$

$\Delta = K^2 - 4K = 0$

$K(K-4) = 0$

$\boxed{K = 0 \text{ ou } K = 4}$

$\Delta = K^2 - 4K < 0$

?

$0 < K < 4$

3) Resolva os problemas de valores iniciais (PVIs):

(a) $y'' + 2y'(t) - 3y(t) = 0, \quad y(0) = 1, y'(0) = 9$

$R^2 + 2R - 3 = 0$

$\Delta = 4 - 4(-3) = 16 \rightarrow \Delta > 0 : y_1 = e^{R_1t}$

$R = \frac{-2 \pm 4}{2}$ $\rightarrow R_1 = 1$, $\rightarrow R_2 = -3$

$y_2 = e^{R_2t}$

$\therefore \underline{y(t) = C_1 e^t + C_2 . e^{-3t}}$

$y'(t) = C_1 e^t - 3C_2 . e^{-3t}$

$y(0) = C_1 e^0 + C_2 \cdot e^{-3.0} = C_1 + C_2$

$y'(0) = C_1 . e^0 - 3C_2 \cdot e^{-3.0} = C_1 - 3C_2$

$3C_1 \quad 3C_2 \quad 3$

$C_1 + C_2 = 1 \quad (\times 3) \rightarrow C_2 = 1 - 3$

$\underline{C_2 = -2}$

$C_1 - 3C_2 = 9$

$4C_1 = 12$

$\underline{C_1 = 3}$

$$\boxed{y(t) = 3.e^t - 2e^{-3t}}$$

(b) $y'' + 6y'(t) + 9y(t) = 0, \quad y(0) = 2, y'(0) = -5$

$R^2 + 6R + 9 = 0$

$\Delta = 36 - 4.9 = 0 \rightarrow \Delta = 0 \rightarrow y_1(t) = e^{Rt}$

$R = \frac{-6 \pm 0}{2} = -3$ $\qquad y_2(t) = t \cdot e^{Rt}$

$y(t) = C_1 \cdot e^{-3t} + C_2 \cdot t \cdot e^{-3t}$

$y(0) = C_1 \cdot e^0 + C_2 \cdot 0 \cdot e^0 \rightarrow \underline{C_1 = 2}$

$y'(t) = -3 \cdot C_1 \cdot e^{-3t} + \overbrace{C_2 \left(e^{-3t} - 3t \cdot e^{-3t} \right)}^{f' \cdot g + f \cdot g'}$

$y'(0) = -3C_1 \cdot e^0 + C_2 \left(e^0 - 3 \cdot 0 \cdot e^0 \right) \rightarrow -3C_1 + C_2 = -5$

$C_2 = -5 + 6$

$\underline{C_2 = 1}$

$$\boxed{y(t) = 2 \cdot e^{-3t} + t \cdot e^{-3t}}$$

$\rightarrow R^2 + 4R + 5 = 0$

(c) $y'' + 4y'(t) + 5y(t) = 0, \qquad y(0) = -3, y'(0) = 0$

$\Delta = 16 - 4 \cdot 5 = -4 \quad \overset{\Delta < 0}{\leadsto} \quad y_1 = e^{at} \cdot \cos(bt)$

$R = -4/2 \pm 2i/2 \qquad y_2 = e^{at} \cdot \text{sen}(bt)$

$R = -2 \pm i$

$y(t) = C_1 \cdot e^{-2t} \cdot \cos(t) + C_2 \cdot e^{-2t} \cdot \text{sen}(t)$

$y(0) = C_1 \cdot e^0 \cdot \cancel{\cos(0)}^{1} + C_2 \cdot e^0 \cdot \cancel{\text{sen}(0)}^{0} \rightarrow \underline{C_1 = -3}$

$y'(t) = C_1(-2e^{-2t} \cdot \cos(t) - e^{-2t} \text{sen}(t)) + C_2(-2 e^{-2t} \cdot \text{sen}(t) + e^{-2t} \cdot \cos(t)$

$y'(t) = C_1(-2 \cdot e^0 \cdot \cancel{\cos(0)}^{1} - e^0 \cdot \cancel{\text{sen}(0)}^{0}) + C_2(-2 \cdot e^0 \cdot \cancel{\text{sen}(0)}^{0} + e^0 \cdot \cancel{\cos(0)}^{1})$

$C_1(-2 - 0) + C_2(0 + 1)$

$-2C_1 + C_2 = 0 \rightarrow \underline{C_2 = -6}$

$y(t) = -3 \cdot e^{-2t} \cdot \cos(t) - 6 \cdot e^{-2t} \text{sen}(t)$

$$\boxed{y(t) = -e^{-2t} \left(3\cos(t) + 6\,\text{sen}(t)\right)}$$

(d) $y'' - 6y'(t) + 13y(t) = 0, \qquad y(0) = -2, y'(0) = 0$

$R^2 - 6R + 13 = 0$

$\Delta = 36 - 4.13 = -16 \quad \therefore \quad R = \dfrac{6 \pm 4i}{2} = 3 \pm 2i$

$y(t) = C_1 \cdot e^{3t} \cos(2t) + C_2 \cdot e^{3t} \cdot \text{sen}(2t)$

$y(0) = C_1 \cdot e^0 \cdot \cos(0) + C_2\, e^0 \cdot \text{sen}(0) \rightarrow \underline{C_1 = -2}$

$y'(t) = C_1\left(3e^{3t} \cdot \cos(2t) - e^{3t} \cdot 2\,\text{sen}(2t)\right) + C_2\left(3e^{3t} \cdot \text{sen}(2t) + e^{3t} \cdot 2\cos(2t)\right)$

$y'(0) = C_1(3.1.1 - 1.2.0) + C_2(3.1.0 + 1.2.1)$

$y'(0) = C_1(3-0) + C_2(0+2) \rightarrow 3C_1 + 2C_2 = 0 \rightarrow \underline{C_2 = 3}$

$y(t) = -2 \cdot e^{3t} \cdot \cos(2t) + 3 \cdot e^{-3t} \cdot \text{sen}(2t)$

$$\boxed{y(t) = e^{3t}\left(-2\cos(2t) + 3\,\text{sen}(2t)\right)}$$

4) Encontre a solução geral pelo método da variação dos parâmetros (MVP):

(a) $y''(t) + y(t) = \tan(t)$

$R^2 + 0R + 1 = 0 \qquad \underline{g(t) = tg(t)}$

$\Delta = -4 \qquad y(t) = C_1\, e^{0.t} \cdot \cos(t) + C_2 \cdot e^{0.t}\, \text{sen}(t)$

$R = 0 \pm i \qquad y(t) = \underbrace{C_1 \cdot \cos(t)}_{y_1} + \underbrace{C_2 \cdot \text{sen}(t)}_{y_2}$

$$w(y_1(t), y_2(t)) = \det\begin{pmatrix} \cos(t) & \text{sen}(t) \\ -\text{sen}(t) & \cos(t) \end{pmatrix} = \cos^2(t) + \text{sen}^2(t) = 1$$

$$C_1 = \int \frac{-y_2 \cdot g(t)}{w(t)} \cdot dt = \int \frac{-\text{sen}(t) \cdot tg(t)\, dt}{1} = -1 \int \frac{\text{sen}^2(t)}{\cos(t)}$$

$\frac{1}{\cos} - \cos$

$$= (-1) \int \frac{1-\cos^2(t)}{\cos(t)} \cdot dt = (-1) \int \sec(t) - \cos(t)$$

$$= -\ln|tg(t) + \sec(t)| + \text{sen}(t) + C1$$

$$C_2 = \int \frac{y_1 \cdot g(t)}{w(t)} \cdot dt = \int \frac{\cos(t) \cdot tg(t)}{1}\, dt = \int \text{sen}(t) \cdot dt$$

$$= -\cos(t) + C2$$

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$\text{sen}(t) \cdot \cos(t) = 0$

$$y_g(t) = (-\ln|tg(t) + \sec(t)| + \text{sen}(t) + C1)(\cos(t)) + (-\cos(t) + C2)\text{sen}(t)$$

$$y_g(t) = (-\ln|tg(t) + \sec(t)|) \cdot (\cos(t)) + 0 + C1 \cdot \cos(t) + 0 + C2 \cdot \text{sen}(t)$$

$$y_g(t) = \underbrace{-\cos(t) \cdot \ln|tg(x) + \sec(x)|}_{p} + \underbrace{C1 \cos(t) + C2\, \text{sen}(t)}_{h}$$

(b) $y''(t) - y(t) = e^{2t}$

$R^2 + 0R - 1 = 0$

$\Delta : 4$

$R = \frac{0 \pm 2}{2}$ $\to -1$, $\to 1$

$$y(t) = \underbrace{C_1 \cdot e^{-t}}_{y_1} + \underbrace{C_2 \cdot e^{t}}_{y_2}$$

$$w(t) \overset{\det}{=} \begin{vmatrix} e^{-t} & e^{t} \\ -e^{-t} & e^{t} \end{vmatrix} = e^0 + e^0 = 2$$

$$g(t) = e^{2t}$$

$$C_1 = \int \frac{-y_2 \cdot g(t)}{w(t)} \cdot dt = \int \frac{-e^{t} \cdot e^{2t}}{2} \cdot dt = \frac{1}{2} \int -e^{3t}\, dt = -\frac{1}{6} e^{3t} + C1$$

$$C_2 = \int \frac{y_1 \cdot g(t)}{w(t)}\, dt = \int \frac{e^{-1} \cdot e^{2t}}{2} = \frac{1}{2} e^{t} + C_2$$

$$y_g(t) = \left(-1/6 \cdot e^{3t} + C_1\right) e^{-t} + \left(1/2\, e^{t} + C_2\right) e^{t}$$

$$y_g(t) = -1/6\, e^{2t} + C_1 \cdot e^{-t} + 1/2\, e^{2t} + C_2 \cdot e^{t}$$

$$y_g(t) = \underbrace{1/3\, e^{2t}}_{y_p} + \underbrace{C_1 \cdot e^{-t} + C_2 \cdot e^{t}}_{y_h}$$

(c) $y''(t) + y'(t) - 2y(t) = e^{-2t}$

$$y^2 + y - 2 = 0 \qquad y(t) = C_1 \cdot e^{t} + C_2 \cdot e^{-2t}$$

$$\Delta = 1 + 8 = 9$$

$$R = \frac{-1 \pm 3}{2} \begin{cases} \to 1 \\ \to -2 \end{cases} \qquad w(t) = \begin{vmatrix} e^{t} & e^{-2t} \\ e^{t} & -2e^{-2t} \end{vmatrix} = -2e^{-t} - e^{-t} = \underline{-3e^{-t}}$$

$$g(t) = e^{-2t}$$

$$C_1 = \int \frac{-y_2 \cdot g(t)}{w(t)}\, dt = \int \frac{-e^{-2t} \cdot e^{-2t}}{-3e^{-t}}\, dt = \frac{1}{3} \int e^{-3t}\, dt = \frac{1}{3} \int -\frac{1}{3} \cdot e^{u}\, du$$

$$= -\frac{1}{9} \cdot e^{-3t} + C_1$$

$$C_2 = \int \frac{y_1 \cdot g(t)}{w(t)}\, dt = \int \frac{e^{t} \cdot e^{-2t}}{-3e^{-t}}\, dt = -\frac{1}{3} \int \cancel{e^{0}}^{1}\, dt = -\frac{1t}{3} + C_2$$

$$y_g(t) = \left(-1/9 \cdot e^{-t} + C_1\right) e^{t} + \left(-1t/3 + C_2\right) e^{-2t}$$

$$= -\frac{1}{9} \cdot 1 + C_1 \cdot e^{t} - \frac{1}{3} \cdot t \cdot e^{-2t} + C_2 \cdot e^{-2t}$$

$$= -\frac{1}{9} - \frac{t}{3} \cdot e^{-2t} + C_1 \cdot e^{t} + C_2 \cdot e^{-2t}$$

gabarito : $-\frac{t}{3}\cdot e^{-2t} + C_1 \cdot e^{t} + C_2 \cdot e^{-2t}$

X

(d) $y''(t) + 4y(t) = t\cos(2t)$

$y^2 + 4y + 0 = 0$ $\quad y(t) = C_1 \cdot e^{0t} + C_2 \cdot e^{-4t}$

$\Delta = 16$ $\quad y(t) = C_1 + C_2 \cdot e^{-4t}$

$R = \frac{-4 \pm 4}{2} \begin{cases} \to 0 \\ \to -4 \end{cases}$

$W(t) = \begin{vmatrix} 1 & e^{-4t} \\ 0 & -4 \cdot e^{-4t} \end{vmatrix} = -4e^{-4t}$ $\quad g(t) = t \cdot \cos(2t)$

$C_1 = \int \frac{-e^{-4t} \cdot t \cdot \cos(t)}{-4e^{-4t}} dt = \frac{1}{4} \int t \cdot \cos(t) \cdot dt = \frac{t \cdot \text{sen}(t) - \cos(t)}{4} + C1$

$\int t \cdot \cos(t) \cdot dt \to \int u \cdot dv = u \cdot v - \int v \cdot du$

$= t \cdot \text{sen}(t) - \int \text{sen}(t) \cdot 1$

$= t \cdot \text{sen}(t) - \cos(t)$

$u = t \quad du = 1$

$dv = \cos(t)dt \quad v = \text{sen}(t)$

$C_2 = \int \frac{1 \cdot t \cdot \cos(t) \cdot dt}{-4 \cdot e^{-4t}} = \frac{1}{4} \int$

Oi?