Introdução à Álgebra Linear

8,5
muito bem

Lista 5
Turma 02 A
Grupo 22

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QUESTÃO 5)

Questão 5

a) diagonais $n \times n$

Seja $\begin{bmatrix} x_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & x_n \end{bmatrix}$ e $\begin{bmatrix} y_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & y_n \end{bmatrix} \in V$, $a \in \mathbb{R}$. $\begin{bmatrix} w_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & w_n \end{bmatrix} \in V$ e $b \in \mathbb{R}$

$$\begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{bmatrix} + \begin{bmatrix} y_1 & & 0 \\ & \ddots & \\ 0 & & y_n \end{bmatrix} = \begin{bmatrix} x_1+y_1 & & 0+0 \\ & \ddots & \\ 0+0 & & x_n+y_n \end{bmatrix} \in V$$

$$a \begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{bmatrix} = \begin{bmatrix} a \cdot x_1 & & a \cdot 0 \\ & \ddots & \\ a \cdot 0 & & a \cdot x_n \end{bmatrix} = \begin{bmatrix} a \cdot x_1 & & 0 \\ & \ddots & \\ 0 & & a \cdot x_n \end{bmatrix} \in V$$

1) $$\left(\begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{bmatrix} + \begin{bmatrix} y_1 & & 0 \\ & \ddots & \\ 0 & & y_n \end{bmatrix}\right) + \begin{bmatrix} w_1 & & 0 \\ & \ddots & \\ 0 & & w_n \end{bmatrix} = \begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{bmatrix} + \left(\begin{bmatrix} y_1 & & 0 \\ & \ddots & \\ 0 & & y_n \end{bmatrix} + \begin{bmatrix} w_1 & & 0 \\ & \ddots & \\ 0 & & w_n \end{bmatrix}\right)$$

$$\begin{bmatrix} x_1+y_1 & & 0 \\ & \ddots & \\ 0 & & x_n+y_n \end{bmatrix} + \begin{bmatrix} w_1 & & 0 \\ & \ddots & \\ 0 & & w_n \end{bmatrix} = \begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{bmatrix} + \begin{bmatrix} y_1+w_1 & & 0 \\ & \ddots & \\ 0 & & y_n+w_n \end{bmatrix}$$

$$\begin{bmatrix} x_1+y_1+w_1 & & 0 \\ & \ddots & \\ 0 & & x_n+y_n+w_n \end{bmatrix} = \begin{bmatrix} x_1+y_1+w_1 & & 0 \\ & \ddots & \\ 0 & & x_n+y_n+w_n \end{bmatrix}$$

2) $$\begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & y_1 \end{bmatrix} + \begin{bmatrix} y_1 & & 0 \\ & \ddots & \\ 0 & & y_n \end{bmatrix} = \begin{bmatrix} y_1 & & 0 \\ & \ddots & \\ 0 & & y_n \end{bmatrix} + \begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_1 \end{bmatrix}$$

$$\begin{bmatrix} x_1+y_1 & & 0 \\ & \ddots & \\ 0 & & x_n+y_n \end{bmatrix} = \begin{bmatrix} y_1+x_1 & & 0 \\ & \ddots & \\ 0 & & y_1+x_1 \end{bmatrix}$$

3) (∃) $$\begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{bmatrix} + \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} x_1+0 & \cdots & 0+0 \\ \vdots & \ddots & \vdots \\ 0+0 & \cdots & x_n+0 \end{bmatrix} = \begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{bmatrix}$$

4) $$\begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{bmatrix} + \begin{bmatrix} -x_1 & & 0 \\ & \ddots & \\ 0 & & -x_n \end{bmatrix} = \begin{bmatrix} x_1-x_1 & & 0+0 \\ & \ddots & \\ 0+0 & & x_n-x_n \end{bmatrix} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} = \vec{0}$$

5) $$(a \cdot b) \cdot \begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{bmatrix} = a \cdot \left(b \cdot \begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & y_n \end{bmatrix}\right)$$

$$\begin{bmatrix} a \cdot b \cdot x_1 & & a \cdot b \cdot 0 \\ & \ddots & \\ a \cdot b \cdot 0 & & a \cdot b \cdot x_n \end{bmatrix} = a \cdot \begin{bmatrix} b x_1 & & b \cdot 0 \\ & \ddots & \\ b \cdot 0 & & b \cdot x_n \end{bmatrix}$$

$$\begin{bmatrix} a \cdot b \cdot x_1 & & 0 \\ & \ddots & \\ 0 & & a \cdot b \cdot x_n \end{bmatrix} = \begin{bmatrix} a \cdot b \cdot x_1 & & \cancel{a \cdot 0} \\ & \ddots & \\ \cancel{a \cdot 0} & & a \cdot b \cdot x_n \end{bmatrix}$$

6) $$(a+b) \begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{bmatrix} = a \cdot \begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{bmatrix} + b \begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{bmatrix}$$

$$\begin{bmatrix} (a+b) \cdot x_1 & & (a+b) \cdot 0 \\ & \ddots & \\ (a+b) \cdot 0 & & (a+b) x_n \end{bmatrix} = \begin{bmatrix} a \cdot x_1 & & \cancel{a \cdot 0} \\ & \ddots & \\ \cancel{a \cdot 0} & & a \cdot x_n \end{bmatrix} + \begin{bmatrix} b x_1 & & \cancel{b \cdot 0} \\ & \ddots & \\ \cancel{b \cdot 0} & & b \cdot x_n \end{bmatrix}$$

$$\begin{bmatrix} a+b \cdot x_1 & & 0 \\ & \ddots & \\ 0 & & a+b \cdot x_n \end{bmatrix} = \begin{bmatrix} a \cdot x_1 + b \cdot x_1 & & 0+0 \\ & \ddots & \\ 0+0 & & a \cdot x_n + b \cdot x_n \end{bmatrix}$$

$$\begin{bmatrix} (a+b) x_1 & & 0 \\ & \ddots & \\ 0 & & (a+b) x_n \end{bmatrix} = \begin{bmatrix} x_1(a+b) & & 0 \\ & \ddots & \\ 0 & & x_n(a+b) \end{bmatrix}$$

7) $$a \left(\begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{bmatrix} + \begin{bmatrix} y_1 & & 0 \\ & \ddots & \\ 0 & & y_n \end{bmatrix}\right) = a \cdot \begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{bmatrix} + a \begin{bmatrix} y_1 & & 0 \\ & \ddots & \\ 0 & & y_n \end{bmatrix}$$

$$a \left(\begin{bmatrix} x_1+y_1 & & 0+0 \\ & \ddots & \\ 0+0 & & x_n+y_n \end{bmatrix}\right) = \begin{bmatrix} a \cdot x_1 & & \cancel{a \cdot 0} \\ & \ddots & \\ \cancel{a \cdot 0} & & a \cdot x_n \end{bmatrix} + \begin{bmatrix} a \cdot y_1 & & \cancel{a \cdot 0} \\ & \ddots & \\ \cancel{a \cdot 0} & & a \cdot y_n \end{bmatrix}$$

$$\begin{bmatrix} a \cdot (x_1+y_1) & & a \cdot 0 \\ & \ddots & \\ a \cdot 0 & & a \cdot (x_n \cdot y_n) \end{bmatrix} = \begin{bmatrix} (a \cdot x_1)+(a \cdot y_1) & & 0+0 \\ & \ddots & \\ 0+0 & & (a \cdot x_n)+(a \cdot x_n) \end{bmatrix}$$

$$\begin{bmatrix} a \cdot x_1 + a \cdot y_1 & & 0 \\ & \ddots & \\ 0 & & a x_n + a y_n \end{bmatrix} = \begin{bmatrix} a \cdot x_1 + a y_1 & & 0 \\ & \ddots & \\ 0 & & a \cdot x_n + a \cdot y_n \end{bmatrix}$$

8) $$1 \cdot \begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{bmatrix} = \begin{bmatrix} 1 x_1 & & 1 \cdot 0 \\ & \ddots & \\ 1 \cdot 0 & & 1 \cdot x_n \end{bmatrix} = \begin{bmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{bmatrix}$$

Sim

BASE

$$\left\{ \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ \vdots & \cdots & 0 & \cdots & \vdots \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}, \cdots, \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ \vdots & \cdots & 1 & \cdots & \vdots \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}, \cdots, \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ \vdots & \cdots & 0 & \cdots & \vdots \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & 0 & \cdots & 1 \end{bmatrix} \right\}$$

dimensão $= n$

b) Escalar $n \times n$

$$\begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}, \begin{bmatrix} y & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y \end{bmatrix}, \begin{bmatrix} w & 0 & 0 \\ 0 & w & 0 \\ 0 & 0 & w \end{bmatrix} \in V \text{ , } a, b \in \mathbb{R}$$

$$\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} x+y & 0+0 \\ 0+0 & x+y \end{bmatrix} \in V \qquad \left\{ a \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} a.x & a.0 \\ a.0 & a.x \end{bmatrix} = \begin{bmatrix} a.x & 0 \\ 0 & a.x \end{bmatrix} \in V \right.$$

1) $$\left(\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix}\right) + \begin{bmatrix} w & 0 \\ 0 & w \end{bmatrix} = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} + \left(\begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix} + \begin{bmatrix} w & 0 \\ 0 & w \end{bmatrix}\right) \checkmark$$

$$\begin{bmatrix} x+y & 0 \\ 0 & x+y \end{bmatrix} + \begin{bmatrix} w & 0 \\ 0 & w \end{bmatrix} = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y+w & 0 \\ 0 & y+w \end{bmatrix}$$

$$\begin{bmatrix} x+y+w & 0 \\ 0 & x+y+w \end{bmatrix} = \begin{bmatrix} x+y+w & 0 \\ 0 & x+y+w \end{bmatrix}$$

2) $$\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix} + \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}$$

$$\begin{bmatrix} x+y & 0 \\ 0 & x+y \end{bmatrix} = \begin{bmatrix} y+x & 0 \\ 0 & y+x \end{bmatrix} \checkmark$$

3) $$\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} + \underset{(\vec{0})}{\begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}} = \begin{bmatrix} x+0 & \cdots & 0+0 \\ \vdots & \ddots & \vdots \\ 0+0 & \cdots & x+0 \end{bmatrix} = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} \checkmark$$

4) $$\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} + \begin{bmatrix} -x & 0 \\ 0 & -x \end{bmatrix} = \begin{bmatrix} x-x & 0+0 \\ 0+0 & x-x \end{bmatrix} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} = \vec{0} \checkmark$$

5) $$(a.b).\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = a.\left(b.\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}\right) \checkmark$$

$$\begin{bmatrix} ab.x & ab.0 \\ ab.0 & ab.x \end{bmatrix} = a.\begin{bmatrix} bx & b.0 \\ b0 & b.x \end{bmatrix}$$

$$\begin{bmatrix} ab\,x & 0 \\ 0 & a.b.x \end{bmatrix} = \begin{bmatrix} a.b.x & \cancel{a0}\ 0 \\ \cancel{a0}\ 0 & a.b.x \end{bmatrix} \checkmark$$

6) $$(a+b)\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix} = a.\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix} + b\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix} \checkmark$$

$$\begin{bmatrix} (a+b).x & (a+b).0 \\ (a+b).0 & (a+b)x \end{bmatrix} = \begin{bmatrix} a.x & \cancel{a0}\ 0 \\ \cancel{a0}\ 0 & a.x \end{bmatrix} + \begin{bmatrix} bx & \cancel{b0}\ 0 \\ \cancel{b0}\ 0 & bx \end{bmatrix}$$

$$\begin{bmatrix} a+b.x & 0 \\ 0 & a+b.x \end{bmatrix} = \begin{bmatrix} a.x+b.x & 0+0 \\ 0+0 & a.x+b.x \end{bmatrix}$$

$$\begin{bmatrix} (a+b)x & 0 \\ 0 & (a+b)x \end{bmatrix} = \begin{bmatrix} x(a+b) & 0 \\ 0 & x(a+b) \end{bmatrix}$$

8) $$1.\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} 1x & 1.0 \\ 1.0 & 1.x \end{bmatrix} = \begin{bmatrix} x_1 & 0 \\ 0 & x \end{bmatrix} \checkmark$$

7) $$a\left(\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix}\right) = a.\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} + a\begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix}$$

$$a\left(\begin{bmatrix} x+y & 0+0 \\ 0+0 & x+y \end{bmatrix}\right) = \begin{bmatrix} a.x & \cancel{a0}\ 0 \\ \cancel{a0}\ 0 & a.x \end{bmatrix} + \begin{bmatrix} a.y & \cancel{a0}\ 0 \\ \cancel{a0}\ 0 & a.y \end{bmatrix}$$

$$\begin{bmatrix} a.(x+y) & a.0 \\ a.0 & a.(x+y) \end{bmatrix} = \begin{bmatrix} (a.x)+(a.y) & 0+0 \\ 0+0 & (a.x)+(a.x) \end{bmatrix}$$

$$\begin{bmatrix} a.x+a.y & 0 \\ 0 & ax+ay \end{bmatrix} = \begin{bmatrix} a.x+ag & 0 \\ 0 & a.x+a.y \end{bmatrix} \checkmark$$

$\boxed{\text{Sim}}$

BASE $= \left\{ \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & & & \vdots \\ 0 & \cdots & 1 & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 1 \end{bmatrix} \right\}$

dimensão = 1

✓

c) $\left\{\begin{bmatrix} a & a+b \\ a & b \end{bmatrix} : a, b \in \mathbb{R}\right\}$ $\quad \gamma$ e $\beta \in \mathbb{R}$ (constantes)

$a = 0$ e $b = 0 \rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in V$ ✓ $\in \mathbb{R}$

$$\begin{bmatrix} a & a+b \\ a & b \end{bmatrix} + \begin{bmatrix} c & c+d \\ c & d \end{bmatrix} = \begin{bmatrix} a+c & a+b+c+d \\ a+c & b+d \end{bmatrix} \in \mathbb{R}$$ ✓

$$\gamma \cdot \begin{bmatrix} a & a+b \\ a & b \end{bmatrix} = \begin{bmatrix} \gamma a & \gamma(a+b) \\ \gamma a & \gamma \cdot b \end{bmatrix} \in \mathbb{R}$$

1) $$\left(\begin{bmatrix} a & a+b \\ a & b \end{bmatrix} + \begin{bmatrix} c & c+d \\ c & d \end{bmatrix}\right) + \begin{bmatrix} e & e+f \\ e & f \end{bmatrix} = \begin{bmatrix} a & a+b \\ a & b \end{bmatrix}\left(\begin{bmatrix} c & c+d \\ c & d \end{bmatrix} + \begin{bmatrix} e & e+f \\ e & f \end{bmatrix}\right)$$

$$\begin{bmatrix} a+c & a+b+c+d \\ a+c & b+d \end{bmatrix} + \begin{bmatrix} e & e+f \\ e & f \end{bmatrix} = \begin{bmatrix} a & a+b \\ a & b \end{bmatrix} + \begin{bmatrix} c+e & c+d+e+f \\ c+e & d+f \end{bmatrix}$$

$$\begin{bmatrix} a+c+e & a+b+c+d \\ a+c+e & b+d+f \end{bmatrix} = \begin{bmatrix} a+c+e & a+b+c+d \\ a+c+e & b+d+f \end{bmatrix} \in \mathbb{R}$$

2) $$\begin{bmatrix} a & a+b \\ a & b \end{bmatrix} + \begin{bmatrix} c & c+d \\ c & d \end{bmatrix} = \begin{bmatrix} c & c+d \\ c & d \end{bmatrix} + \begin{bmatrix} a & a+b \\ a & b \end{bmatrix}$$ ✓

$$\begin{bmatrix} a+c & a+b+c+d \\ a+c & b+d \end{bmatrix} = \begin{bmatrix} c+a & c+d+a+b \\ c+a & d+b \end{bmatrix} \in \mathbb{R}$$

3) $$\begin{bmatrix} a & a+b \\ a & b \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a+0 & a+b+0 \\ a+0 & b+0 \end{bmatrix} = \begin{bmatrix} a & a+b \\ a & b \end{bmatrix}$$ ✓

4) $$\begin{bmatrix} a & a+b \\ a & b \end{bmatrix} + \begin{bmatrix} -a & -(a+b) \\ -a & -b \end{bmatrix} = \begin{bmatrix} a-a & a+b-(a+b) \\ a-a & b-b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \vec{0}$$

5) $$(\alpha \cdot \beta) \cdot \begin{bmatrix} a & a+b \\ a & b \end{bmatrix} = \alpha\left(\beta \cdot \begin{bmatrix} a & a+b \\ a & b \end{bmatrix}\right)$$

$$\begin{bmatrix} \alpha \cdot \beta a & \alpha \cdot \beta(a+b) \\ \alpha \cdot \beta a & \alpha \cdot \beta b \end{bmatrix} = \alpha \cdot \begin{bmatrix} \beta a & \beta(a+b) \\ \beta a & \beta b \end{bmatrix}$$

$$\begin{bmatrix} \alpha \cdot \beta a & \alpha \cdot \beta(a+b) \\ \alpha \cdot \beta a & \alpha \cdot \beta b \end{bmatrix} = \begin{bmatrix} \alpha \cdot \beta a & \alpha \cdot \beta(a+b) \\ \beta a & \beta b \end{bmatrix}$$

6) $$(\alpha + \beta)\begin{bmatrix} a & a+b \\ a & b \end{bmatrix} = \alpha\begin{bmatrix} a & a+b \\ a & b \end{bmatrix} + \beta\begin{bmatrix} a & a+b \\ a & b \end{bmatrix}$$ ✓

$$\begin{bmatrix} \alpha a + \beta a & \alpha a + \alpha b + \beta a + \beta b \\ \alpha a + \beta a & \alpha a + \beta b \end{bmatrix} \quad \begin{bmatrix} \alpha a & \alpha(a+b) \\ \alpha a & \alpha b \end{bmatrix} + \begin{bmatrix} \beta a & \beta(a+b) \\ \beta a & \beta b \end{bmatrix}$$

$$= \begin{bmatrix} \alpha a + \beta a & \alpha(a+b) + \beta(a+b) \\ \alpha a + \beta a & \alpha a + \beta b \end{bmatrix}$$

$$\begin{bmatrix} \alpha a + \beta a & \alpha a + \alpha b + \beta a + \beta b \\ \alpha a + \beta a & \alpha a + \beta b \end{bmatrix} = \begin{bmatrix} \alpha a + \beta a & \alpha a + \alpha b + \beta a + \beta b \\ \alpha a + \beta a & \alpha a + \beta b \end{bmatrix}$$

7) $$\alpha\left(\begin{bmatrix} a & a+b \\ a & b \end{bmatrix} + \begin{bmatrix} c & c+d \\ c & d \end{bmatrix}\right) = \alpha\begin{bmatrix} a & a+b \\ a & b \end{bmatrix} + \alpha\begin{bmatrix} c & c+d \\ c & d \end{bmatrix}$$

$$\alpha\begin{bmatrix} a+c & a+b+c+d \\ a+c & a+b \end{bmatrix} = \begin{bmatrix} \alpha a & \alpha a + \alpha b \\ \alpha a & \alpha b \end{bmatrix} + \begin{bmatrix} \alpha c & \alpha c + \alpha d \\ \alpha c & \alpha d \end{bmatrix}$$

$$\begin{bmatrix} \alpha a + \alpha c & \alpha a + \alpha b + \alpha c + \alpha d \\ \alpha a + \alpha c & \alpha b + \alpha d \end{bmatrix} = \begin{bmatrix} \alpha a + \alpha c & \alpha a + \alpha b + \alpha c + \alpha d \\ \alpha a + \alpha c & \alpha b + \alpha d \end{bmatrix}$$ ✓

8) $$1 \cdot \begin{bmatrix} a & a+b \\ a & b \end{bmatrix} = \begin{bmatrix} 1 \cdot a & 1 \cdot a + 1 \cdot b \\ 1a & 1 \cdot b \end{bmatrix} = \begin{bmatrix} a & a+b \\ a & b \end{bmatrix}$$ ✓

$\boxed{\text{SIM}}$

Base

$$\left[\begin{array}{cc|c} a & a+b & 0 \\ a & a & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & \frac{a+b}{a} & 0 \\ a & a & 0 \end{array}\right] \rightarrow a\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$\frac{a}{a} + b$

$L_1 = L_1 \cdot \frac{1}{a}$ $\quad L_2 = \frac{1}{a} \cdot L_2$

$$\left\{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}\right\}$$

dimensão = 2

✓

d) $V = \{(a, a, \dots, a) \in \mathbb{R}^n : a \in \mathbb{R}\}$ $\quad \alpha, \beta \in \mathbb{R}$ (constantes)

$a = 0 \Rightarrow \vec{0}$ ✓ | $(a, \dots, a) + (b, \dots, b) = (a+b, \dots, a+b)$ ✓ $\in \mathbb{R}$ | $\alpha(a, \dots, a) = (\alpha a, \dots, \alpha a)$ ✓ $\in \mathbb{R}$

① $\big((a, \dots, a) + (b, \dots, b)\big) + (c, \dots, c) = (a, \dots, a) + \big((b, \dots, b) + (c, \dots, c)\big)$

$(a+b, \dots, a+b) + (c, \dots, c) = (a, \dots, a) + (b+c, \dots, b+c)$

$(a+b+c, \dots, a+b+c) = (a+b+c, \dots, a+b+c)$ ✓

② $(a, \dots, a) + (b, \dots, b) = (b, \dots, b) + (a, \dots, a)$

$(a+b, \dots, a+b) = (b+a, \dots, b+a)$ ✓

③ $(0, \dots, 0) + (a, \dots, a) = (a+0, \dots, a+0) = (a, \dots, a)$ ✓

④ $(a, \dots, a) + (-a, \dots, -a) = (a-a, \dots, a-a) = (0, \dots, 0) = \vec{0}$ ✓

⑤ $(\alpha \cdot \beta) \cdot (a, \dots, a) = \alpha(\beta(a, \dots, a)$

$(\alpha \cdot \beta \cdot a, \dots, \alpha \cdot \beta \cdot a) = \alpha(\beta a, \dots, \beta a)$

$(\alpha \cdot \beta \cdot a, \dots, \alpha \cdot \beta \cdot a) = (\alpha \cdot \beta a, \dots, \alpha \cdot \beta \cdot a)$ ✓

⑥ $(\alpha + \beta) \cdot (a, \dots, a) = \alpha(a, \dots, a) + \beta(a, \dots, a)$

$(\alpha a + \alpha\beta, \dots, \alpha a + \alpha\beta) = (\alpha a, \dots, \alpha a) + (\beta a, \dots, \beta a)$

$(\alpha a + \alpha\beta, \dots, \alpha a + \alpha\beta) = (\alpha a + \alpha\beta, \dots, \alpha a + \alpha\beta)$ ✓

⑦ $\alpha\big((a, \dots, a) + (b, \dots, b)\big) = \alpha(a, \dots, a) + \alpha(b, \dots, b)$

$\alpha(a+b, \dots, a+b) = (\alpha a, \dots, \alpha a) + (\alpha b, \dots, \alpha b)$

$(\alpha a + \alpha b, \dots, \alpha a + \alpha b) = (\alpha a + \alpha b, \dots, \alpha a + \alpha b)$ ✓

⑧ $1 \cdot (a, \dots, a) = (1 \cdot a, \dots, 1 \cdot a) = (a, \dots, a)$ ✓

SIM

✓

BASE $\quad (a, \dots, a)$

$a(1, \dots, 1)$

$\therefore \{1, 1, \dots, 1\}$

dimensão = 1

e) $\{(1, a, b) : a, b \in \mathbb{R}\}$ NÃO

$a = 0,\ b = 0 \quad \therefore \quad (1, 0, 0) \notin \vec{0}$

✓

f) A reta $\{(x, x+3) \quad x \in \mathbb{R}\}$

$x = 0 \quad \therefore \quad (0, 3) \notin \vec{0}$

✓

NÃO

g) $\{(a, 2a, 3a) : a \in \mathbb{R}\}$

$a = 0 \therefore (0, 0, 0) = \vec{0}$

$(a, 2a, 3a) + (b, 2b, 3b) = (a+b, 2a+2b, 3a+3b) \in \mathbb{R}$ | $\alpha(a, 2a, 3a) = (\alpha a, \alpha 2a, \alpha 3a) \in \mathbb{R}$

1) $\big((a, 2a, 3a) + (b, 2b, 3b)\big) + (c, 2c, 3c) = (a, 2a, 3a) + \big((b, 2b, 3b) + (c, 2c, 3c)\big)$

$(a+b, 2a+2b, 3a+3b) + (c, 2c, 3c) = (a, 2a, 3a) + (b+c, 2b+2c, 3b+3c)$

$(a+b+c, 2a+2b+2c, 3a+3b+3c) = (a+b+c, 2a+2b+2c, 3a+3b+3c)$ ✓

2) $(a, 2a, 3a) + (b, 2b, 3b) = (b, 2b, 3b) + (a, 2a, 3a)$

$(a+b, 2a+2b, 3a+2b) = (b+a, 2b+2a + 3b+3a)$

$\| \quad = (a+b, 2a+2b + 3a+3b)$ ✓

3) $(a, 2a, 3a) + (0, 0, 0) =$

$(a+0, 2a+0, 3a+0) =$

$(a, 2a, 3a)$

4) $(a, 2a, 3a) + (-a, -2a, -3a) = (a-a, 2a-2a, 3a-3a) = (0, 0, 0) = \vec{0}$ ✓

5) $(\alpha \cdot \beta)(a, 2a, 4a) = \alpha(\beta(a, 2a, 3a)$

$(\alpha \cdot \beta a, \alpha \cdot \beta \cdot 2a, \alpha \cdot \beta \cdot 3a) = \alpha(\beta a, \beta 2a, \beta 3a)$

$\| \quad = (\alpha \beta a, \alpha \cdot \beta 2a, \alpha \cdot \beta \cdot 3a)$ ✓

6) $(\alpha + \beta)(a, 2a, 3a) = \alpha(a, 2a, 3a) + \beta(a, 2a, 3a)$

$(\alpha a + \beta a, \alpha 2a + \beta 2a, \alpha 3a + \beta 3a) = (\alpha a, \alpha 2a, \alpha 3a) + (\beta a, \beta 2a, \beta 3a)$

$\| \quad = (\alpha a + \beta a, \alpha 2a + \beta 2a, \alpha 3a + \beta 3a)$ ✓

7) $\alpha\big((a, 2a, 3a) + (b, 2b, 3b)\big) = \alpha(a, 2a, 3a) + \alpha(b, 2b, 3b)$

$\alpha(a+b, 2a+2b, 3a+3b) = (\alpha a, \alpha 2a, \alpha 3a) + (\alpha b, \alpha 2b, \alpha 3b)$

$(\alpha a + \alpha b, \alpha 2a + \alpha 2b, \alpha 3a + \alpha 3b) = (\alpha a + \alpha b, \alpha 2a + \alpha 2b, \alpha 3a + \alpha 3b)$ ✓

8) $1 \cdot (a, 2a, 3a) =$

$(1 \cdot a, 1 \cdot 2a, 1 \cdot 3a) =$

$(a, 2a, 3a)$ ✓

SIM

✓

Base $\quad (a, 2a, 3a) \rightarrow a(1, 2, 3) \therefore$

$\hookrightarrow \{(1, 2, 3)\}$ dimensão = 1

QUESTÃO 9)

9-> $\vec{V_1} = \begin{bmatrix}1&0\\0&0\end{bmatrix}$ $\vec{V_2} = \begin{bmatrix}0&1\\0&0\end{bmatrix}$ $\vec{V_3} = \begin{bmatrix}0&0\\1&0\end{bmatrix}$

$V_4 = \begin{bmatrix}0&0\\0&1\end{bmatrix}$

i) $a\begin{bmatrix}1&0\\0&0\end{bmatrix} + b\begin{bmatrix}0&1\\0&0\end{bmatrix} + c\begin{bmatrix}0&0\\1&0\end{bmatrix} + d\begin{bmatrix}0&0\\0&1\end{bmatrix} =$

$\begin{bmatrix}0&0\\0&0\end{bmatrix}$ então $\begin{bmatrix}a&0\\0&0\end{bmatrix} + \begin{bmatrix}0&b\\0&0\end{bmatrix} + \begin{bmatrix}0&0\\c&0\end{bmatrix} + \begin{bmatrix}0&0\\0&c\end{bmatrix} = \begin{bmatrix}0&0\\0&0\end{bmatrix}$

então $\begin{bmatrix}a&b\\c&d\end{bmatrix} = \begin{bmatrix}0&0\\0&0\end{bmatrix}$ então

$a=0$ $b=0$
$c=0$ $d=0$

$\{V_1, V_2, V_3, V_4\}$ é LI

~~(ii)~~

ii) $a\begin{bmatrix}1&0\\0&0\end{bmatrix} + b\begin{bmatrix}0&1\\0&0\end{bmatrix} + c\begin{bmatrix}0&0\\1&0\end{bmatrix} + d\begin{bmatrix}0&0\\0&1\end{bmatrix} = \begin{bmatrix}x&y\\z&w\end{bmatrix}$

$\begin{bmatrix}a&0\\0&0\end{bmatrix} + \begin{bmatrix}0&b\\0&0\end{bmatrix} + \begin{bmatrix}0&0\\c&0\end{bmatrix} + \begin{bmatrix}0&0\\0&d\end{bmatrix} = \begin{bmatrix}x&y\\z&w\end{bmatrix}$

$\begin{bmatrix}a&b\\c&d\end{bmatrix} = \begin{bmatrix}x&y\\z&w\end{bmatrix}$ $\{V_1, V_2, V_3, V_4\}$ gera R^4

$\{V_1, V_2, V_3, V_4\}$ são Base para R^4

QUESTÃO 11)

11-)

$B = \{(1,1,1), (-1,1,0), (1,0,-1)\}$

$x_A = (1,0,0)$ x_B ?

$x_B = m\, x_A \qquad m = B^{-1} A$

$$B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \qquad B^{-1} = \begin{bmatrix} 1/3 & 1/3 & -7/3 \\ -1/3 & 2/3 & -5/3 \\ 1/3 & 1/3 & 2/3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & -1 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$L_2 = L_2 - L_3 \qquad L_1 = L_1 + L_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 1 & -1 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 1 & -1 \\ 0 & 0 & -3 & -1 & -1 & -2 \end{array}\right] \sim$$

$L_3 = L_3 - L_1 \qquad L_3 = L_3 \cdot \left(-\frac{1}{3}\right)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1/3 & 1/3 & 2/3 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & -1 \\ 0 & 1 & 0 & -1/3 & 2/3 & -5/3 \\ 0 & 0 & 1 & 1/3 & 1/3 & 2/3 \end{array}\right]$$

$L_2 = L_2 - L_3 \qquad L_1 = L_1 - 2 \cdot L_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & 1/3 & -7/3 \\ 0 & 1 & 0 & -1/3 & 2/3 & -5/3 \\ 0 & 0 & 1 & 1/3 & 1/3 & 2/3 \end{array}\right]$$

$x_B = B^{-1} x_A$

$$x_B = \begin{bmatrix} 1/3 & 1/3 & -7/3 \\ -1/3 & 2/3 & -5/3 \\ 1/3 & 1/3 & 2/3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$x_B = \begin{bmatrix} 1/3 \\ -1/3 \\ 1/3 \end{bmatrix}$$

QUESTÃO 15)

15 - $\begin{pmatrix} 1 & -5 \\ -4 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ -1 & 5 \end{pmatrix} = \begin{pmatrix} 2 & -4 \\ -5 & 7 \end{pmatrix}$

$$x\begin{pmatrix} 1 & -5 \\ -4 & 2 \end{pmatrix} + y\begin{pmatrix} 1 & 1 \\ -1 & 5 \end{pmatrix} + z\begin{pmatrix} 1 & -7 \\ -5 & 1 \end{pmatrix} = 0$$

$$\begin{cases} x + y + z = 0 \\ -5x + y - 7z = 0 \\ -4x - y - 5z = 0 \\ 2x + 5y + z = 0 \end{cases}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ -5 & 1 & -7 & 0 \\ -4 & -1 & -5 & 0 \\ 2 & 5 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 6 & -2 & 0 \\ 0 & 3 & -1 & 0 \\ 0 & 3 & -1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 6 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} =$$

$y = 2z$

$x = -y - z = -2z - z = -3z$

$$-3z\begin{pmatrix} 1 & -5 \\ -4 & 2 \end{pmatrix} + 2z\begin{pmatrix} 1 & 1 \\ -1 & 5 \end{pmatrix} + z\begin{pmatrix} 1 & -7 \\ -5 & 1 \end{pmatrix} = 0$$

$$\begin{pmatrix} 1 & -7 \\ -5 & 1 \end{pmatrix} = -3\begin{pmatrix} 1 & -5 \\ -4 & 2 \end{pmatrix} + 2\begin{pmatrix} 1 & 1 \\ -1 & 5 \end{pmatrix}, \text{ se } z \neq 0$$

$$\left\{\begin{pmatrix} 1 & -5 \\ -4 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ -1 & 5 \end{pmatrix}\right\} = \text{base } W$$

$\dim W = 2$

QUESTÃO 19)

19 - $x(1,1,0) + y(0,-1,1) + z(1,1,1) = 0$

$$\left.\begin{aligned} x + z &= 0 \\ x - y + z &= 0 \\ y + z &= 0 \end{aligned}\right\} = \begin{aligned} x &= -z \\ y &= 0 \\ z &= 0 \end{aligned}$$

$x = y = z = 0 = LI$

$\{V_1, V_2, V_3\} = 3$, então $\{V_1, V_2, V_3\} = \mathbb{R}^3$

QUESTÃO 32)

32) $\beta_1 = \{(1,0),(0,2)\}$, $\beta_2 = \{(-1,0),(1,1)\}$, $\beta_3 = \{(-1,-1),(0,-1)\}$

a) $[I]^{\beta_2}_{\beta_1} = ?$ seja $V_1 = (-1,0) = a_{11}(1,0) + a_{21}(0,2)$

$-1 = a_{11} \Rightarrow \boxed{a_{11} = -1}$

$0 = 2a_{21} \quad \boxed{a_{21} = 0}$

$V_2 = (1,1) = a_{12}(1,0) + a_{22}(0,2)$

$1 = a_{12} \Rightarrow \boxed{a_{12} = 1}$

$1 = 2a_{22} \quad \boxed{a_{22} = 1/2}$

$\therefore [I]^{\beta_2}_{\beta_1} = \begin{bmatrix} -1 & 1 \\ 0 & 1/2 \end{bmatrix}$

b) $[I]^{\beta_3}_{\beta_2} = ?$

$z_1 = (-1,-1) = a_{11}(-1,0) + a_{21}(1,1)$

$(-1,-1) = (-a_{11}, 0) + (a_{21}, a_{21})$

$-1 = -a_{11} + a_{21} \quad \boxed{a_{11} = 0}$

$\Rightarrow -1 = 0 + a_{21} \Rightarrow \boxed{a_{21} = -1}$

$-1 = -a_{11} + (-1)$

$-1 + 1 = -a_{11}$

$0 = -a_{11}$

$\boxed{0 = a_{11}}$

$z_2 = (0,-1) = a_{12}(-1,0) + a_{22}(1,1)$

$0 = a_{12}(-1) + a_{22}(1)$

$-1 = a_{12} \cdot 0 + a_{22}(1)$

$\boxed{-1 = a_{22}}$

$0 = -a_{12} + (-1)$

$\boxed{a_{12} = 1}$

$\boxed{\therefore [I]^{\beta_3}_{\beta_2} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}}$

c) $[I]^{\beta_3}_{\beta_1} = ?$ $w_1 = (-1,-1) = a_{11}(1,0) + a_{21}(0,2)$

$-1 = a_{11} \Rightarrow \boxed{a_{11} = -1}$

$-1 = 2a_{21} \Rightarrow \boxed{-1/2 = a_{21}}$

$w_2 = (0,-1) = a_{12}(1,0) + a_{22}(0,2) \Rightarrow \boxed{0 = a_{12}}$; $-1 = 2a_{22}$

$\boxed{a_{22} = -1/2}$

$\boxed{\therefore [I]^{\beta_3}_{\beta_1} = \begin{bmatrix} -1 & 0 \\ -1/2 & -1/2 \end{bmatrix}}$

d) $[I]^{\beta_2}_{\beta_1} \cdot [I]^{\beta_3}_{\beta_2} = \begin{bmatrix} -1 & 1 \\ 0 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 - 1 & -1 - 1 \\ 0 - 1/2 & 0 - 1/2 \end{bmatrix} \Rightarrow$

$\boxed{\therefore [I]^{\beta_2}_{\beta_1} \cdot [I]^{\beta_3}_{\beta_2} = \begin{bmatrix} -1 & -2 \\ -1/2 & -1/2 \end{bmatrix}}$

b) A 1ª Linha de todas as matrizes é não nula

QUESTÃO 33)

33) $\beta_1 = \left\{ \begin{bmatrix}1&0\\0&0\end{bmatrix}, \begin{bmatrix}0&1\\0&0\end{bmatrix}, \begin{bmatrix}0&0\\0&1\end{bmatrix} \right\}$

$\beta = \left\{ \begin{bmatrix}1&0\\0&0\end{bmatrix}, \begin{bmatrix}0&1\\0&0\end{bmatrix}, \begin{bmatrix}0&0\\0&1\end{bmatrix} \right\}$

→ 1ª Linha

→ 3ª Linha

$$\begin{bmatrix}1&0\\0&0\end{bmatrix} = 1\begin{bmatrix}1&0\\0&0\end{bmatrix} + 0\begin{bmatrix}0&1\\0&0\end{bmatrix} + 0\begin{bmatrix}0&0\\0&1\end{bmatrix}$$

$$\begin{bmatrix}1&1\\0&0\end{bmatrix} = 1\begin{bmatrix}1&0\\0&0\end{bmatrix} + 1\begin{bmatrix}0&1\\0&0\end{bmatrix} + 0\begin{bmatrix}0&0\\0&1\end{bmatrix}$$

$$\begin{bmatrix}1&1\\0&1\end{bmatrix} = 1\begin{bmatrix}1&0\\0&0\end{bmatrix} + 1\begin{bmatrix}0&1\\0&0\end{bmatrix} + 1\begin{bmatrix}0&0\\0&1\end{bmatrix}$$

→ 2ª Linha

$$\boxed{\therefore [I]^{\beta_1}_{\beta} = \begin{bmatrix}1&1&1\\0&1&1\\0&0&1\end{bmatrix}}$$