Listas e simulado P1

segunda-feira, 7 de fevereiro de 2022 13:55

Questionário 1

① $\sum_{i=1}^{50}(5+i) = 10K + \sum_{i=5}^{50} i \to 10K + 1265 = 1525 \to 260 = 10K$ $\boxed{K=26}$

$\sum_{i=1}^{50} 5 \to a.n = 5.50 = 250$

$\sum_{i=1}^{50} i \to 1+2+\dots+50 = \frac{50(n+1)}{2} = 25(51) = 1275$ } 1525

$\sum_{i=5}^{50} i \to 5+6+7+8+9+10+\dots+50$ (1 2 3 4 5 | 41 = 46 termos) $\frac{50\ 49}{55}$ ∴ $\frac{55.46}{2} = 1265$

② PG $S = \frac{a_1 (q^x - 1)}{q-1}$

$a_1 = 4$

$q = 2$

S_{10} $S = \frac{4.(2^{10}-1)}{2-1} = \frac{4(1024-1)}{1} = 4.1023 = 4092$

③ Determine o valor de n, sabendo que a = 14 e b = [14-2]

$\binom{a}{b} = \binom{n+1}{n} \to \binom{14}{12} = \binom{n+1}{n}$

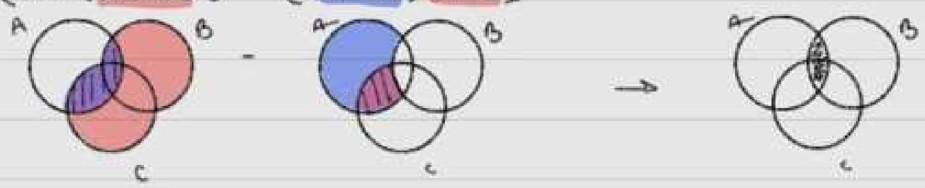
$\frac{a!}{b!(a-b)!} \to \frac{14!}{12!\,2!} = \frac{\overset{7}{\cancel{14}}.13.\cancel{12!}}{\cancel{12!}\,\cancel{2}} = 91$

$\frac{(n+1)!}{(n!(n+1-n)!} \to \frac{(n+1)!\cdot\cancel{n!}}{\cancel{n!}\ 1} \to n+1 = 91$ $\boxed{n = 90}$

$\binom{91}{\ } = \frac{91!}{\ } = \frac{91.90!}{\ } = 91$

Simulado 1

1 $[A\cap(B\cup C)] - [(A-B)\cap C]$



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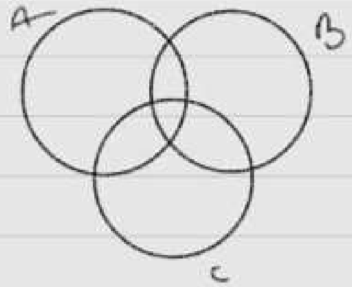


160 pessoas

a)

b)

3 $\sum_{i=0}^{51}(i+k) = (\sum_{i=1}^{51} i)\, 104$

$\sum_{i=0}^{51} i + \sum_{i=0}^{51} k \to \frac{52.\cancel{51}}{\cancel{2}} + 52.K = \frac{52.\cancel{51}}{\cancel{2}} . 104 = \boxed{2}$

4 78 C

106 E

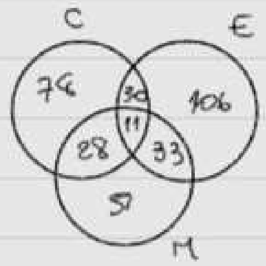
51 M

30 C e E

33 E e M

28 M e C

11 C e E e M



5 $B_2 = ?$

$\binom{1}{0} = \frac{1!}{0!\,1!} = 1$

$\binom{}{90} \quad 90!\,1 \quad \cancel{90!}\,1$

(4) Qual o menor valor de $\underline{n > 0}$ que satisfaz a seguinte equação:

$$\frac{n!}{2!(n-2)!} = 21$$

$$\frac{(n)(n-1)\cancel{(n-2)!}}{2!\cancel{(n-2)!}} = \frac{(n-1)n}{2 \cdot 1} = \frac{n^2-n}{2} = 21$$

$$n^2 - n = 42$$

$$n^2 - n - 42 = 0$$

$$\Delta = b^2 - 4 \cdot a \cdot c = 1 - 4 \cdot 1 \cdot (-42) = 169$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2 \cdot a} \rightarrow \boxed{x' = \frac{1+13}{2} = 7}$$

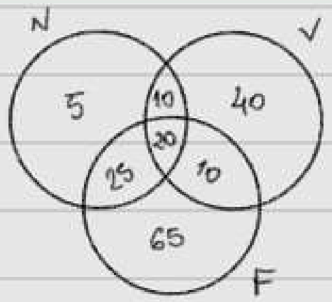
$$\rightarrow x'' = \frac{1-13}{2} = -6$$

Questionário 3

1 natação, volei, futebol (60, 80, 120)

Total 175

$120 - 55 = 65$

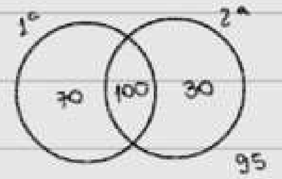


2 $A = \{0, 1, 2, 3, 4, 5\}$ $\quad (A-C) - (B-C)$

$B = \{4, 5, 6, 7\}$ $\quad \{0,1,2,3\} - \{7\}$

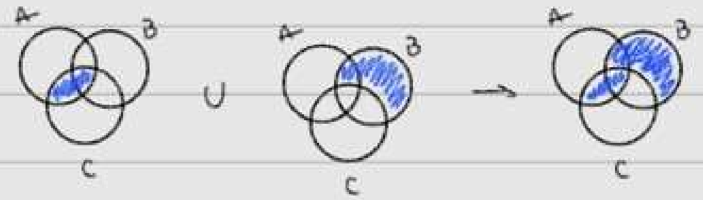
$C = \{4, 5, 6, 8\}$ $\quad \{0,1,2,3\}$

3



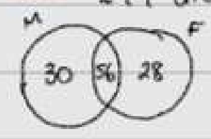
$200 - 95 = 295$

4



$(A \cap C) \cup (B - C)$

5 244 alunos; 86 matemática; 84 física; 56 m e f; ? nenhuma

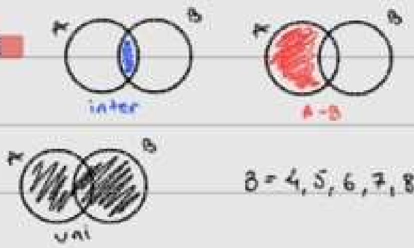


$30 + 56 + 28 = 114$

$244 - 114 = 130$

130

8



$B = 4, 5, 6, 7, 8$

$B_0 = 4$

$$B_n = \left[\sum_{i=0}^{n} (-3)^n \cdot \binom{n}{i}\right] + 2\underbrace{B_{n-1}}_{B_0} \qquad \binom{1}{1} = \frac{1!}{1!\,0!} = 1$$

$$B_1 = \left[\sum_{i=0}^{1} (-3)^1 \cdot \binom{1}{i}\right] + 2 \cdot 4 \qquad \binom{2}{0} = 1$$

$$B_1 = 8 + \left[(-3)^1 \cdot \binom{1}{0} + (-3)\binom{1}{1}\right] \qquad \binom{2}{1} = 2$$

$$B_1 = 8 + (-3) + (-3) \qquad \binom{2}{2} = 1$$

$$B_1 = 2$$

$$B_2 = 2 \cdot 2 + \left[\sum_{i=0}^{2} (-3)^2 \cdot \binom{2}{i}\right]$$

$$B_2 = 4 + \left[9 \cdot 1 + 9 \cdot 2 + 9 \cdot 1\right]$$

$$B_2 = 4 + 36 = \boxed{40}$$

6 $\binom{n+1}{4} = \binom{n}{3} \rightsquigarrow \binom{3+1}{4} = \binom{3}{3}$

$$\frac{n+1!}{4!(n+1-4)!} = \frac{n!}{3!(n-3)!} \qquad \binom{4}{4} = \binom{3}{3}$$

$$\frac{n+1!}{4!(n-3)!} = \frac{n!}{3!(n-3)!}$$

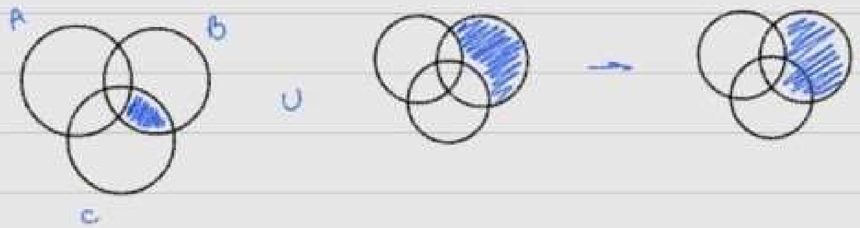
$$\frac{(n+1)(n)(n-1)(n-2)\cancel{(n-3)!}}{4!\cancel{(n-3)!}} = \frac{n \cdot (n-1)(n-2)\cancel{(n-3)!}}{3!\cancel{(n-3)!}}$$

$$\frac{(n+1)\cancel{(n)}\cancel{(n-1)}\cancel{(n-2)}}{4 \cdot \cancel{3!}} = \frac{\cancel{(n)}\cancel{(n-1)}\cancel{(n-2)}}{\cancel{3!}}$$

$$n + 1 = 4$$

$$\boxed{n = 3}$$

7 $[(C \cap B) - A] \cup [B - (A \cup C)]$



8 $a_1 = 5 \qquad a_n = 10 \cdot a_{n-1} + \ldots 2^{(1-1)}$

9 $U = \{0, 1, 2, 3, 4, 5, 6\}$

$A = \{1, 2\}$ $\qquad (U - A) \cap (B \cup C)$

$B = \{2, 3, 4\}$ $\qquad \{0, 3, 4, 5, 6\} \cap \{2, 3, 4, 5\}$

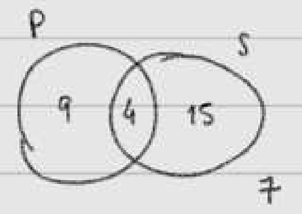
$C = \{4, 5\}$ $\qquad \{3, 4, 5\}$

10 35 alunos

19 salg.

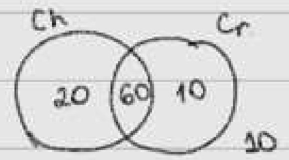
4 salg e pizza

7 ñ compraram

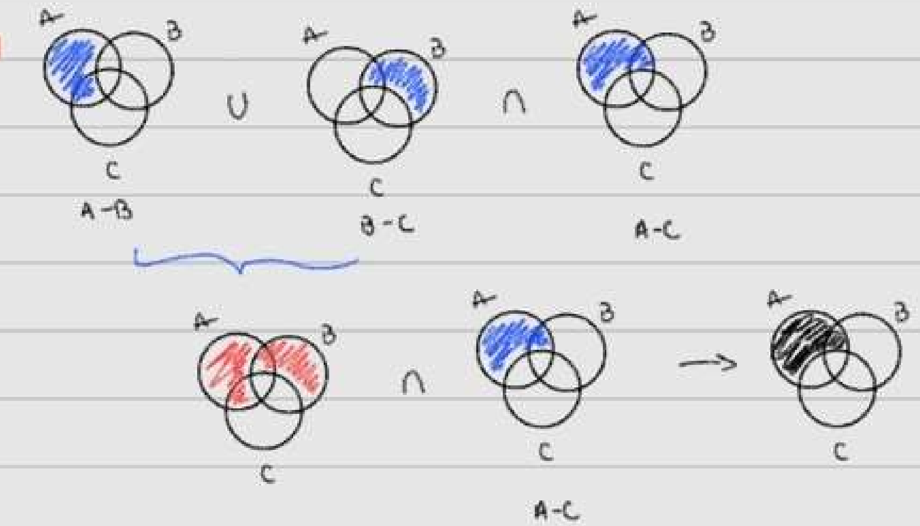


$\begin{array}{r} 19 \\ +7 \\ \hline 26 \end{array} \qquad \begin{array}{r} 35 \\ -26 \\ \hline 9 \end{array}$

11



12



6 $\binom{a}{b} = \binom{n+1}{n}$ $\quad a = 23$, $b = 21$

$\frac{a!}{b!(a-b)!} \to \frac{23!}{21!\,2!} \to \frac{23 \cdot \overset{11}{\cancel{22}} \cdot \cancel{21!}}{\cancel{21!}\,\cancel{2}} = 253$

$\begin{array}{r} 23 \\ \times 11 \\ \hline 23 \\ 23+ \\ \hline 253 \end{array}$

$\frac{n+1!}{n!\,(n-(n+1))!} = 253 \to \frac{n+1 \cdot \cancel{n!}}{\cancel{n!}\, 1}$ $\quad n+1 = 253$ $\quad \boxed{n = 252}$

Questionário 4

$a_0 = 5 \qquad a_1 = 18 \cdot a_0 + 2$

$a_n = 18\, a_{n-1} + 2^{(n-1)} \qquad = 18 \cdot 5 + 1 = 90$

$a_n = 18^{n-1} \cdot 90 + \sum_{i=2}^{n} 18^{n-i} \cdot 2^{(i-1)}$

$(2 \cdot 9)^{n-i} \cdot 2^{i-1}$

$2^{n-i}\, 9^{n-i} \cdot 2^{i-1}$

$9^{n-i} \cdot 2^{n-1}$

$2^{n-1} \sum_{i=2}^{n} 9^{n-i}$

$a_n = 18^{n-1} \cdot 90 + 2^{n-1} \cdot \frac{9^{-2}(9^{n-1} - 1)}{8}$

PG

$a_n = 18^{n-1} \cdot 90 + 2^{n-1} \cdot \frac{9^{n-3} - 9^{-2}}{8}$

$9^{0-2} + 9^{1-2} + 9^{2-2} + 9^{3-2}$

$\frac{1}{9^2} \overset{\times 9}{+} \frac{1}{9^1} + 1 + 9$

$S_n = \frac{a_1 \cdot (q^x - 1)}{q - 1}$

$S_n = \frac{9^{-2}(9^{n-1} - 1)}{9 - 1}$

$\therefore \boxed{2^{n-1} \cdot \frac{9^{-2}(9^{n-1} - 1)}{8}}$

Σ

9 $a_0 = 1 \qquad a_n = 11\, a_{n-1} - 30\, n-2$

$a_1 = 2$

$y^2 - 11y + 30 = 0$

$\begin{cases} x_1 + x_2 = a_0 \\ x_1 r_1 + x_2 r_2 = a_1 \end{cases}$

$\Delta = b^2 - 4 \cdot a \cdot c$

$= 121 - 4 \cdot 1 \cdot 30$

$\Delta = 1$

$\begin{cases} x_1 + \cancel{x_2} = 1 \quad (-5) \\ 6x_1 + \cancel{5x_2} = 2 \end{cases}$

$x_1 = -3$

$r = \frac{11 \pm 1}{2}$

$-3 + x_2 = 1$

$\boxed{x_2 = 4}$

$r_1 = \frac{11 + 1}{2} = 6$

$a_n = -3 \cdot 6^n + 4 \cdot 5^n$

2 $a_0 = 12 \;/\; a_n = -2a_{n-1} - 3$

$$a_n = -2^n \cdot 12 + (-3)\frac{(-2^n - 1)}{-2-1} \quad - \quad \frac{3 \cdot 2^n + 3}{3}$$

$$a_n = -2^n \cdot 12 - \left(\frac{3 \cdot 2^n + 3}{3}\right)$$

$$a_n = \frac{-36 \cdot 2^n - 3 \cdot 2^n - 3}{3}$$

$$\begin{array}{r} 12 \\ \times 3 \\ \hline 36 \end{array} \qquad a_n = \frac{-39 \cdot 2^n}{3} - 1 \qquad\qquad 39 \,\underline{|\,3} \quad 0 \;\; 13$$

$$a_n = -13 \cdot 2^n - 1$$

An = ((-13)*(2**n))-1

3) $a_0 = 5$

$a_n = 2a_{n-1} + 3$

$$a_n = 2^n \cdot 5 + \frac{3(2^n - 1)}{2-1}$$

$$a_n = 2^n \cdot 5 + 3.2^n - 3$$

$$a_n = 8 \cdot 2^n - 3$$

4) $a_0 = 1 \;/\; a_1 = 2 \;/\; a_n = 8a_{n-1} + 9a_{n-2}$

$y^2 - 8y - 9 = 0$

$\Delta = b^2 - 4.a.c$

$\Delta = 64 - 4.1.-9$

$\Delta = 100$

$r = \dfrac{8 \pm 10}{2}$ $\nearrow r_1 = 9$, $\searrow r_2 = -1$

$$\begin{cases} x_1 + x_2 = 1 \\ x_1.r_1 + x_2.r_2 = 2 \end{cases} \longrightarrow \frac{3}{10} + x_2 = 1$$

$$\begin{cases} x_1 + x_2 = 1 \\ 9.x_1 - 1.x_2 = 2 \end{cases}$$

$$10x_1 = 3 \Rightarrow x_1 = \frac{3}{10}$$

$$x_2 = \frac{10}{10} - \frac{3}{10}$$

$$x_2 = \frac{7}{10}$$

$$a_n = x_1.r_1^n + x_2.r_2^n \longrightarrow \boxed{a_n = \frac{3}{10} \cdot 9^n + \frac{7}{10} \cdot (-1)^n}$$

An = (3/10)*(9**n)+(7/10)*((-1)**n)

5 $a_0 = 3 \;/\; a_n = 7a_{n-1}$

$a_n = 3 \cdot 7^n$

$$r_2 = \frac{11-1}{2} = 5$$

10 $5 + 7 + 9 + \dots + (5 + 2n) = 5(n+1) + n^2 \quad n \geq 0$

Caso base

$n = 0$

$5 + 2.0 = 5(0+1) + 0^2$

$5 = 5$

Hipotese indutiva

$5 + 7 + 9 + \dots + (5 + 2n) = 5(n+1) + n^2$

Principio indutivo

$$\underbrace{5 + 7 + 9 + \dots + (5+2n)} + 5 + 2(n+1) = \overbrace{5((n+1)+1)}^{5(n+2)} + (n+1)^2$$

$$5n + 5 + n^2 + 5 + 2n + 2 = 5n + 10 + n^2 + 2n + 1$$

$$n^2 + 7n + 12 = n^2 + 7n + 11$$

6

$a_0 = 2$

$a_n = 3a_{n-1} + 3^{(n+2)}$

$a_1 = 3 \cdot 2 + 3^3$

$a_1 = 6 + 27 = 33$

$a_n = 3^{n-1} \cdot 33 + \sum_{i=2}^{n} 3^{n-i} \cdot 3^{i+2}$

$\sum_{i=2}^{n} \underbrace{3^{n-i+i+2}}_{3^{n+2}} \rightarrow 3^{n+2} \sum_{i=2}^{n} 1 \rightarrow 3^{n+2}(n-1)$

$a_n = 3^{n-1} \cdot 33 + 3^{n+2} \cdot (n-1)$

7 $a_0 = 4$

$a_n = a_{n-1} + 6n$

$a_1 = 1 \cdot a_0 + 6 \cdot 1$

$a_1 = 4 + 6 = \underline{10}$

$a_n = (1^{n-1})^{1} \cdot 10 + \sum_{i=2}^{n} (1^{n-i})^{1} \cdot 6i$

$a_n = 10 + \sum_{i=2}^{n} 6i \longrightarrow 6 \cdot 2 + 6 \cdot 3 + 6 \cdot 4 + 6 \cdot 5$

$12 + 18 + 24 + 30 + 6 \cdot n$

$a_n = 10 + \frac{(n-1)(6(n+2))}{2}$

$\frac{(n-1)(6(n+2))}{2} \rightarrow \frac{(n-1)}{2} \cdot 3(n+2)$

$a_n = \frac{20 + (n-1) \cdot 6(n+2)}{2}$

Exercícios aula presencial (4/2)

1. Calcule o valor de B2 sabendo que:

$B_0 = 2$

$\binom{1}{0} = \frac{1!}{0!\,1!} = 1$

$\binom{1}{1} = \frac{1!}{1!\,0!} = 1$

$B_n = \ldots \left[\sum^{n} (-1)^{n-i} \ldots \binom{n}{i} \right]$

$B_n = B_{n-1} + \left[\sum_{i=0} \ldots \right]$

$$B_1 = 2 + \sum_{i=0}^{1} (-1)^{1-i} \cdot \binom{1}{i}$$

$$B_1 = 2 + (-1)^{1-0} \cdot \binom{1}{0} + (-1)^{\overbrace{1-1}^{0}} \cdot \binom{1}{1}$$

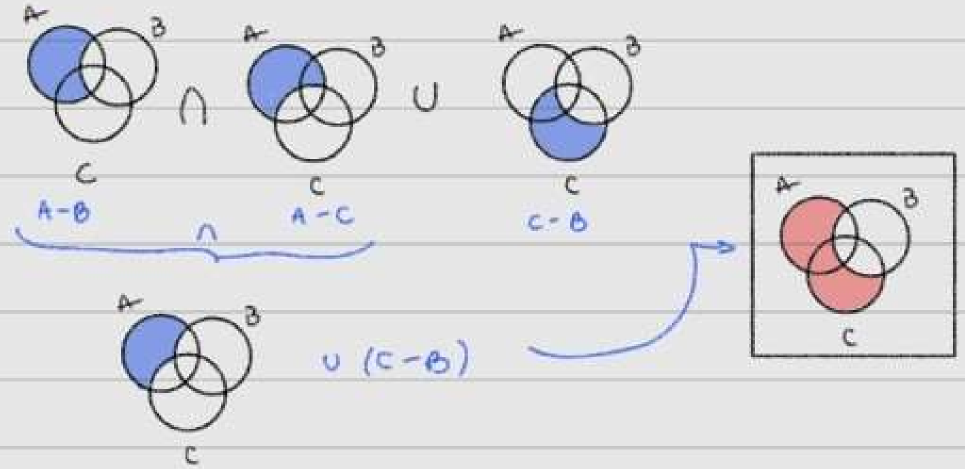
$$= 2 + (-1) \cdot 1 + (+1) \cdot 1$$

$$= 2 - 1 + 1$$

$$= 2$$

2. Como é a representação em diagrama de venn da seguinte exp:

$((A - B) \cap (A - C)) \cup (C - B)$



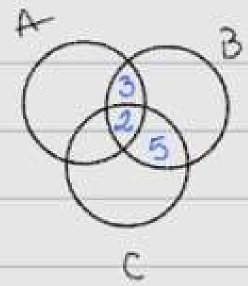
3. Considere os seguintes conjuntos:

$A = \{0, 1, 2, 3, 4\}$

$B = \{2, 3, 5, 6, 9\}$

$C = \{2, 4, 5, 7, 8\}$

Crie um expressão que resulte nos elementos $\{2, 3, 5\}$. Você terá que obrigat. colocar todos os conj. A, B e C na exp.



$(A \cap B) \cup (C \cap B)$