# M2 Lista 2

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**Introdução à Álgebra Linear**

Lista 5
Turma 02 A
Grupo 22
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# Questão 5

a) diagonais $n \times n$

Seja $\begin{bmatrix} x_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & x_n \end{bmatrix}$ e $\begin{bmatrix} y_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & y_n \end{bmatrix} \in V$ e $a \in \mathbb{R}$. $\begin{bmatrix} w_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & w_n \end{bmatrix} \in V$ e $b \in \mathbb{R}$

$$\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix} + \begin{bmatrix} y_1 & 0 \\ 0 & y_n \end{bmatrix} = \begin{bmatrix} x_1+y_1 & 0+0 \\ 0+0 & x_n+y_n \end{bmatrix} \in V \quad \left\{ a\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix} = \begin{bmatrix} a\cdot x_1 & a\cdot 0 \\ a\cdot 0 & a\cdot x_n \end{bmatrix} = \begin{bmatrix} a\cdot x_1 & 0 \\ 0 & a\cdot x_n \end{bmatrix} \in V \right.$$

1) $\left(\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix} + \begin{bmatrix} y_1 & 0 \\ 0 & y_n \end{bmatrix}\right) + \begin{bmatrix} w_1 & 0 \\ 0 & w_n \end{bmatrix} = \begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix} + \left(\begin{bmatrix} y_1 & 0 \\ 0 & y_n \end{bmatrix} + \begin{bmatrix} w_1 & 0 \\ 0 & w_n \end{bmatrix}\right)$ ✓

$$\begin{bmatrix} x_1+y_1 & 0 \\ 0 & x_n+y_n \end{bmatrix} + \begin{bmatrix} w_1 & 0 \\ 0 & w_n \end{bmatrix} = \begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix} + \begin{bmatrix} y_1+w_1 & 0 \\ 0 & y_n+w_n \end{bmatrix}$$

$$\begin{bmatrix} x_1+y_1+w_1 & 0 \\ 0 & x_n+y_n+w_n \end{bmatrix} = \begin{bmatrix} x_1+y_1+w_1 & 0 \\ 0 & x_n+y_n+w_n \end{bmatrix}$$

2) $\begin{bmatrix} x_1 & 0 \\ 0 & y_1 \end{bmatrix} + \begin{bmatrix} y_1 & 0 \\ 0 & y_n \end{bmatrix} = \begin{bmatrix} y_1 & 0 \\ 0 & y_n \end{bmatrix} + \begin{bmatrix} x_1 & 0 \\ 0 & x_1 \end{bmatrix}$

$$\begin{bmatrix} x_1+y_1 & 0 \\ 0 & x_n+y_n \end{bmatrix} = \begin{bmatrix} y_1+x_1 & 0 \\ 0 & y_1+x_1 \end{bmatrix}$$ ✓

3) $\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix} + \underset{(\vec{0})}{\begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}} = \begin{bmatrix} x_1+0 & \cdots & 0+0 \\ \vdots & \ddots & \vdots \\ 0+0 & \cdots & x_n+0 \end{bmatrix} = \begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix}$ ✓

4) $\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix} + \begin{bmatrix} -x_1 & 0 \\ 0 & -x_n \end{bmatrix} = \begin{bmatrix} x_1-x_1 & 0+0 \\ 0+0 & x_n-x_n \end{bmatrix} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} = \vec{0}$ ✓

5) $(a.b).\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix} = a.\left(b.\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix}\right)$ ✓

$$\begin{bmatrix} ab.x_1 & ab.0 \\ ab.0 & ab.x_n \end{bmatrix} = a.\begin{bmatrix} bx_1 & b.0 \\ b.0 & b.x_n \end{bmatrix}$$

$$\begin{bmatrix} a.b\,x_1 & 0 \\ 0 & a.b.x_n \end{bmatrix} = \begin{bmatrix} a.b.x_1 & \cancel{a.0}^{\,0} \\ \cancel{a.0}^{\,0} & a.b.x_n \end{bmatrix}$$ ✓

6) $(a+b)\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix} = a.\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix} + b\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix}$ ✓

$$\begin{bmatrix} (a+b).x_1 & (a+b).0 \\ (a+b).0 & (a+b)x_n \end{bmatrix} = \begin{bmatrix} a.x_1 & \cancel{a.0}^{\,0} \\ \cancel{a.0}^{\,0} & a.x_n \end{bmatrix} + \begin{bmatrix} bx_1 & \cancel{b.0}^{\,0} \\ \cancel{b.0}^{\,0} & b.x_n \end{bmatrix}$$

$$\begin{bmatrix} a+b.\,x_1 & 0 \\ 0 & a+b.\,x_n \end{bmatrix} = \begin{bmatrix} a.x_1+b.x_1 & 0+0 \\ 0+0 & a.x_n+b.x_n \end{bmatrix}$$

$$\begin{bmatrix} (a+b)\,x_1 & 0 \\ 0 & (a+b)\,x_n \end{bmatrix} = \begin{bmatrix} x_1(a+b) & 0 \\ 0 & x_n(a+b) \end{bmatrix}$$

7) $a\left(\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix} + \begin{bmatrix} y_1 & 0 \\ 0 & y_n \end{bmatrix}\right) = a.\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix} + a\begin{bmatrix} y_1 & 0 \\ 0 & y_n \end{bmatrix}$

$$a\left(\begin{bmatrix} x_1+y_1 & 0+0 \\ 0+0 & x_n+y_n \end{bmatrix}\right) = \begin{bmatrix} a.x_1 & \cancel{a.0}^{\,0} \\ \cancel{a.0}^{\,0} & a.x_n \end{bmatrix} + \begin{bmatrix} a.y_1 & \cancel{a.0}^{\,0} \\ \cancel{a.0}^{\,0} & a.y_n \end{bmatrix}$$

$$\begin{bmatrix} a.(x_1+y_1) & a.0 \\ a.0 & a.(x_n.y_n) \end{bmatrix} = \begin{bmatrix} (a.x_1)+(a.y_1) & 0+0 \\ 0+0 & (a.x_n)+(a.x_n) \end{bmatrix}$$

$$\begin{bmatrix} a.x_1+a.y_1 & 0 \\ 0 & a x_n + a y_n \end{bmatrix} = \begin{bmatrix} a.x_1+a y_1 & 0 \\ 0 & a.x_n+a.y_n \end{bmatrix}$$ ✓

8) $1.\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix}$

$$= \begin{bmatrix} 1\,x_1 & 1\,0 \\ 1.0 & 1.x_n \end{bmatrix}$$

$$= \begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix}$$ ✓

$\boxed{\text{Sim}}$

BASE

$$\left\{\begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}, \ldots, \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \vdots & & 1 & & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}, \ldots, \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \vdots & & 0 & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 1 \end{bmatrix}\right\}$$

dimensão $= n$

b) Escalar $n \times n$

$\begin{bmatrix} x & \cdots & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & x & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & x \end{bmatrix}, \begin{bmatrix} y & \cdots & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & y & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & y \end{bmatrix}, \begin{bmatrix} w & \cdots & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & w & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & w \end{bmatrix} \in V$ e $a, b \in \mathbb{R}$

$$\begin{bmatrix} x & & 0 \\ & \ddots & \\ 0 & & x \end{bmatrix} + \begin{bmatrix} y & & 0 \\ & \ddots & \\ 0 & & y \end{bmatrix} = \begin{bmatrix} x+y & & 0+0 \\ & \ddots & \\ 0+0 & & x+y \end{bmatrix} \in V \qquad \left\{ a \begin{bmatrix} x & & 0 \\ & \ddots & \\ 0 & & x \end{bmatrix} = \begin{bmatrix} a.x & & a0 \\ & \ddots & \\ a.0 & & a.x \end{bmatrix} = \begin{bmatrix} a.x & & 0 \\ & \ddots & \\ 0 & & a.x \end{bmatrix} \in V \right.$$

1) $$\left(\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix}\right) + \begin{bmatrix} w & 0 \\ 0 & w \end{bmatrix} = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} + \left(\begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix} + \begin{bmatrix} w & 0 \\ 0 & w \end{bmatrix}\right) \checkmark$$

$$\begin{bmatrix} x+y & 0 \\ 0 & x+y \end{bmatrix} + \begin{bmatrix} w & 0 \\ 0 & w \end{bmatrix} = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y+w & 0 \\ 0 & y+w \end{bmatrix}$$

$$\begin{bmatrix} x+y+w & 0 \\ 0 & x+y+w \end{bmatrix} = \begin{bmatrix} x+y+w & 0 \\ 0 & x+y+w \end{bmatrix}$$

2) $$\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix} + \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}$$

$$\begin{bmatrix} x+y & 0 \\ 0 & x+y \end{bmatrix} = \begin{bmatrix} y+x & 0 \\ 0 & y+x \end{bmatrix}$$

$\checkmark$

(=)

3) $$\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} + \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} = \begin{bmatrix} x+0 & \cdots & 0+0 \\ \vdots & & \vdots \\ 0+0 & \cdots & x+0 \end{bmatrix} = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} \checkmark$$

4) $$\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} + \begin{bmatrix} -x & 0 \\ 0 & -x \end{bmatrix} = \begin{bmatrix} x-x & 0+0 \\ 0+0 & x-x \end{bmatrix} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix} = \vec{0} \checkmark$$

$\checkmark$ $\checkmark$

5) $$(a.b).\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = a.\left(b.\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}\right)$$

$$\begin{bmatrix} ab.x & ab.0 \\ ab.0 & ab.x \end{bmatrix} = a.\begin{bmatrix} bx & b.0 \\ b0 & b.x \end{bmatrix}$$

$$\begin{bmatrix} abx & 0 \\ 0 & ab.x \end{bmatrix} = \begin{bmatrix} a.b.x & \cancel{a0}^{\,0} \\ \cancel{a0}^{\,0} & a.b.x \end{bmatrix} \checkmark$$

6) $$(a+b)\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix} = a.\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix} + b\begin{bmatrix} x_1 & 0 \\ 0 & x_n \end{bmatrix}$$

$$\begin{bmatrix} (a+b).x & (a+b).0 \\ (a+b).0 & (a+b)x \end{bmatrix} = \begin{bmatrix} a.x & \cancel{a0}^{\,0} \\ \cancel{a0}^{\,0} & a.x \end{bmatrix} + \begin{bmatrix} bx & \cancel{b0}^{\,0} \\ \cancel{b0}^{\,0} & bx \end{bmatrix}$$

$$\begin{bmatrix} a+b.x & 0 \\ 0 & a+b.x \end{bmatrix} = \begin{bmatrix} a.x+b.x & 0+0 \\ 0+0 & a.x+b.x \end{bmatrix}$$

$$\begin{bmatrix} (a+b)x & 0 \\ 0 & (a+b)x \end{bmatrix} = \begin{bmatrix} x(a+b) & 0 \\ 0 & x(a+b) \end{bmatrix}$$

8) $$1.\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} 1x & 1.0 \\ 1.0 & 1.x \end{bmatrix} = \begin{bmatrix} x_1 & 0 \\ 0 & x \end{bmatrix} \checkmark$$

7) $$a\left(\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix}\right) = a.\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} + a\begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix}$$

$$a\left(\begin{bmatrix} x+y & 0+0 \\ 0+0 & x+y \end{bmatrix}\right) = \begin{bmatrix} a.x & \cancel{a0}^{\,0} \\ \cancel{a0}^{\,0} & a.x \end{bmatrix} + \begin{bmatrix} a.y & \cancel{a0}^{\,0} \\ \cancel{a0}^{\,0} & a.y \end{bmatrix}$$

$$\begin{bmatrix} a.(x+y) & a.0 \\ a.0 & a.(x.y) \end{bmatrix} = \begin{bmatrix} (a.x)+(a.y) & 0+0 \\ 0+0 & (a.x)+(a.x) \end{bmatrix}$$

$$\begin{bmatrix} a.x+a.y & 0 \\ 0 & ax+ay \end{bmatrix} = \begin{bmatrix} a.x+ag & 0 \\ 0 & a.x+a.y \end{bmatrix}$$

$\checkmark$

BASE $$= \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$$

dimensão = 1

$\boxed{\text{Sim}}$

c) $\left\{\begin{bmatrix} a & a+b \\ a & b \end{bmatrix} : a, b \in \mathbb{R}\right\}$ $\quad \gamma$ e $\beta \in \mathbb{R}$ (constantes)

$a=0$ e $b=0 \to \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in V$ $\checkmark$ $\in \mathbb{R}$

$$\begin{bmatrix} a & a+b \\ a & b \end{bmatrix} + \begin{bmatrix} c & c+d \\ c & d \end{bmatrix} \checkmark$$

$$= \begin{bmatrix} a+c & a+b+c+d \end{bmatrix}$$

$$\gamma.\begin{bmatrix} a & a+b \\ a & b \end{bmatrix}$$

$$= \begin{bmatrix} \gamma a & \gamma(a+b) \end{bmatrix}$$

$$\begin{bmatrix} a+c & b+d \end{bmatrix} \quad \begin{bmatrix} \gamma a & \gamma . b \end{bmatrix}$$
$\in R \qquad \in R$

1) $$\left(\begin{bmatrix} a & a+b \\ a & b \end{bmatrix} + \begin{bmatrix} c & c+d \\ c & d \end{bmatrix}\right) + \begin{bmatrix} e & e+f \\ e & f \end{bmatrix} = \begin{bmatrix} a & a+b \\ a & b \end{bmatrix} \left(\begin{bmatrix} c & c+d \\ c & d \end{bmatrix} + \begin{bmatrix} e & e+f \\ e & f \end{bmatrix}\right)$$
$$\begin{bmatrix} a+c & a+b+c+d \\ a+c & b+d \end{bmatrix} + \begin{bmatrix} e & e+f \\ e & f \end{bmatrix} = \begin{bmatrix} a & a+b \\ a & b \end{bmatrix} + \begin{bmatrix} c+e & c+d+e+f \\ c+e & d+f \end{bmatrix}$$
$$\begin{bmatrix} a+c+e & a+b+c+d \\ a+c+e & b+d+f \end{bmatrix} = \begin{bmatrix} a+c+e & a+b+c+d \\ a+c+e & b+d+f \end{bmatrix} \in R$$

2) $$\begin{bmatrix} a & a+b \\ a & b \end{bmatrix} + \begin{bmatrix} c & c+d \\ c & d \end{bmatrix} = \begin{bmatrix} c & c+d \\ c & d \end{bmatrix} + \begin{bmatrix} a & a+b \\ a & b \end{bmatrix} ✓$$
$$\begin{bmatrix} a+c & a+b+c+d \\ a+c & b+d \end{bmatrix} = \begin{bmatrix} c+a & c+d+a+b \\ c+a & d+b \end{bmatrix} \in R$$

3) $$\begin{bmatrix} a & a+b \\ a & b \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a+0 & a+b+0 \\ a+0 & b+0 \end{bmatrix} = \begin{bmatrix} a & a+b \\ a & b \end{bmatrix} ✓$$

4) $$\begin{bmatrix} a & a+b \\ a & b \end{bmatrix} + \begin{bmatrix} -a & -(a+b) \\ -a & -b \end{bmatrix} = \begin{bmatrix} a-a & a+b-(a+b) \\ a-a & b-b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \vec{0}$$

5) $$(\alpha . \beta) \begin{bmatrix} a & a+b \\ a & b \end{bmatrix} = \alpha \left(\beta . \begin{bmatrix} a & a+b \\ a & b \end{bmatrix}\right)$$
$$\begin{bmatrix} \alpha . \beta a & \alpha . \beta(a+b) \\ \alpha . \beta a & \alpha . \beta b \end{bmatrix} = \alpha . \begin{bmatrix} \beta a & \beta(a+b) \\ \beta a & \beta b \end{bmatrix}$$
$$\begin{bmatrix} \alpha . \beta a & \alpha . \beta(a+b) \\ \alpha . \beta a & \alpha . \beta b \end{bmatrix} = \begin{bmatrix} \alpha . \beta a & \alpha . \beta(a+b) \\ \beta a & \beta b \end{bmatrix}$$

6) $$(\alpha+\beta) \begin{bmatrix} a & a+b \\ a & b \end{bmatrix} = \alpha \begin{bmatrix} a & a+b \\ a & b \end{bmatrix} + \beta \begin{bmatrix} a & a+b \\ a & b \end{bmatrix} ✓$$
$$\begin{bmatrix} \alpha a+\beta a & \alpha a+\alpha b+\beta a+\beta b \\ \alpha a+\beta a & \alpha a+\beta b \end{bmatrix} \quad \begin{bmatrix} \alpha a & \alpha(a+b) \\ \alpha a & \alpha b \end{bmatrix} + \begin{bmatrix} \beta a & \beta(a+b) \\ \beta a & \beta b \end{bmatrix}$$
$$= \begin{bmatrix} \alpha a+\beta a & \alpha(a+b)+\beta(a+b) \\ \alpha a+\beta a & \alpha a+\beta b \end{bmatrix}$$
$$\begin{bmatrix} \alpha a+\beta a & \alpha a+\alpha b+\beta a+\beta b \\ \alpha a+\beta a & \alpha a+\beta b \end{bmatrix} = \begin{bmatrix} \alpha a+\beta a & \alpha a+\alpha b+\beta a+\beta b \\ \alpha a+\beta a & \alpha a+\beta b \end{bmatrix}$$

7) $$\alpha\left(\begin{bmatrix} a & a+b \\ a & b \end{bmatrix} + \begin{bmatrix} c & c+d \\ c & d \end{bmatrix}\right) = \alpha \begin{bmatrix} a & a+b \\ a & b \end{bmatrix} + \alpha \begin{bmatrix} c & c+d \\ c & d \end{bmatrix}$$
$$\alpha \begin{bmatrix} a+c & a+b+c+d \\ a+c & a+b \end{bmatrix} = \begin{bmatrix} \alpha a & \alpha a+\alpha b \\ \alpha a & \alpha b \end{bmatrix} + \begin{bmatrix} \alpha c & \alpha c+\alpha d \\ \alpha c & \alpha d \end{bmatrix}$$
$$\begin{bmatrix} \alpha a+\alpha c & \alpha a+\alpha b+\alpha c+\alpha d \\ \alpha a+\alpha c & \alpha b+\alpha d \end{bmatrix} = \begin{bmatrix} \alpha a+\alpha c & \alpha a+\alpha b+\alpha c+\alpha d \\ \alpha a+\alpha c & \alpha b+\alpha d \end{bmatrix}$$
✓

8) $$1 . \begin{bmatrix} a & a+b \\ a & b \end{bmatrix} = \begin{bmatrix} 1.a & 1.a+1.b \\ 1a & 1.b \end{bmatrix}$$
$$= \begin{bmatrix} a & a+b \\ a & b \end{bmatrix} ✓$$

$\boxed{\text{SIM}}$

Base

$$\left[\begin{array}{cc|c} a & a+b & 0 \\ a & a & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & \frac{a+b}{a} & 0 \\ a & a & 0 \end{array}\right] \to a \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + b \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$\frac{a}{a} + b$

$L_1 = L_1 \cdot \frac{1}{a} \qquad L_2 = \frac{1}{a} \cdot L_2$

$$\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \right\}$$

dimensão = 2

d) $V = \{(a, a, \dots, a) \in R^n : a \in R\}$ $\quad \alpha, \beta \in R$ (constantes)

$a = 0 \to \vec{0}$ ✓ $\quad (a, \dots, a) + (b, \dots, b) = (a+b, \dots, a+b)$ ✓ $\in R$ $\quad \alpha(a, \dots, a) = (\alpha a, \dots, \alpha a)$ ✓ $\in R$

① $((a, \dots, a) + (b, \dots, b)) + (c, \dots, c) = (a, \dots, a) + ((b, \dots, b) + (c, \dots, c))$

$(a+b, \dots, a+b) + (c, \dots, c) = (a, \dots, a) + (b+c, \dots, b+c)$

$(a+b+c, \dots, a+b+c) = (a+b+c, \dots, a+b+c)$ ✓

SIM

② $(a, \dots, a) + (b, \dots, b) = (b, \dots, b) + (a, \dots, a)$

$(a+b, \cdots, a+b) = (b+a, \cdots, b+a)$ ✓

③ $(0, \cdots, 0) + (a, \cdots a) = (a+0, \cdots, a+0) = (a, \cdots, a)$ ✓

④ $(a, \cdots, a) + (-a, \cdots, -a) = (a-a, \cdots, a-a) = (0, \cdots, 0) = \vec{0}$ ✓

⑤ $(\alpha \cdot \beta) \cdot (a, \cdots, a) = \alpha(\beta(a, \cdots, a)$

$(\alpha \cdot \beta \cdot a, \cdots, \alpha \beta \cdot a) = \alpha(\beta a, \cdots, \beta a)$

$(\alpha \cdot \beta \cdot a, \cdots, \alpha \cdot \beta \cdot a) = (\alpha \cdot \beta a, \cdots, \alpha \cdot \beta \cdot a)$ ✓

⑧ $1 \cdot (a, \cdots, a) = (1 \cdot a, \cdots, 1 \cdot a) = (a, \cdots, a)$ ✓

⑥ $(\alpha + \beta) \cdot (a, \cdots, a) = \alpha(a, \cdots, a) + \beta(a, \cdots, a)$

$(\alpha a + \alpha\beta, \cdots, \alpha a + \alpha\beta) = (\alpha a, \cdots, \alpha a) + (\beta a, \cdots, \beta a)$

$(\alpha a + \alpha\beta, \cdots, \alpha a + \alpha\beta) = (\alpha a + \alpha\beta, \cdots, \alpha a + \alpha\beta)$ ✓

⑦ $\alpha((a, \cdots a) + (b, \cdots b)) = \alpha(a, \cdots, a) + \alpha(b, \cdots, b)$

$\alpha(a+b, \cdots, a+b) = (\alpha a, \cdots, \alpha a) + (\alpha b, \cdots, \alpha b)$

$(\alpha a + \alpha b, \cdots, \alpha a + \alpha b) = (\alpha a + \alpha b, \cdots, \alpha a + \alpha b)$ ✓

BASE

$(a, \cdots, a)$

$a(1, \cdots, 1)$

$\therefore \{1, 1, \cdots, 1\}$

dimensão = 1

e) $\{(1, a, b) : a, b \in \mathbb{R}\}$ NAO

$a = 0,\ b = 0 \ \therefore (1, 0, 0) \notin \vec{0}$

f) A reta $\{(x, x+3) : x \in \mathbb{R}\}$

$x = 0 \ \therefore (0, 3) \notin \vec{0}$

NAO

g) $\{(a, 2a, 3a) : a \in \mathbb{R}\}$

$a = 0 \ \therefore (0,0,0) = \vec{0}$

$(a, 2a, 3a) + (b, 2b, 3b) = (a+b, 2a+2b, 3a+3b) \in \mathbb{R}$

$\alpha(a, 2a, 3a) = (\alpha a, \alpha 2a, \alpha 3a) \in \mathbb{R}$

1) $((a, 2a, 3a) + (b, 2b, 3b)) + (c, 2c, 3c) = (a, 2a, 3a) + ((b, 2b, 3b) + (c, 2c, 3c))$

$(a+b, 2a+2b, 3a+3b) + (c, 2c, 3c) = (a, 2a, 3a) + (b+c, 2b+2c, 3b+3c)$

$(a+b+c, 2a+2b+2c, 3a+3b+3c) = (a+b+c, 2a+2b+2c, 3a+3b+3c)$ ✓

2) $(a, 2a, 3a) + (b, 2b, 3b) = (b, 2b, 3b) + (a, 2a, 3a)$

$(a+b, 2a+2b, 3a+2b) = (b+a, 2b+2a+3b+3a)$

" $= (a+b, 2a+2b+3a+3b)$ ✓

3) $(a, 2a, 3a) + (0, 0, 0) = (a+0, 2a+0, 3a+0) = (a, 2a, 3a)$

4) $(a, 2a, 3a) + (-a, -2a, -3a) = (a-a, 2a-2a, 3a-3a) = (0, 0, 0) = \vec{0}$ ✓

5) $(\alpha \cdot \beta)(a, 2a, 3a) = \alpha(\beta(a, 2a, 3a)$

$(\alpha \cdot \beta a, \alpha \cdot \beta \cdot 2a, \alpha \cdot \beta \cdot 3a) = \alpha(\beta a, \beta 2a, \beta 3a)$

" $= (\alpha \cdot \beta a, \alpha \cdot \beta 2a, \alpha \cdot \beta \cdot 3a)$ ✓

6) $(\alpha + \beta)(a, 2a, 3a) = \alpha(a, 2a, 3a) + \beta(a, 2a, 3a)$

$(\alpha a + \beta a, \alpha 2a + \beta 2a, \alpha 3a + \beta 3a) = (\alpha a, \alpha 2a, \alpha 3a) + (\beta a, \beta 2a, \beta 3a)$

" $= (\alpha a + \beta a, \alpha 2a + \beta 2a, \alpha 3a + \beta 3a)$ ✓

7) $\alpha((a, 2a, 3a) + (b, 2b, 3b)) = \alpha(a, 2a, 3a) + \alpha(b, 2b, 3b)$

$\alpha(a+b, 2a+2b, 3a+3b) = (\alpha a, \alpha 2a, \alpha 3a) + (\alpha b, \alpha 2b, \alpha 3b)$

$(\alpha a + \alpha b, \alpha 2a + \alpha 2b, \alpha 3a + \alpha 3b) = (\alpha a + \alpha b, \alpha 2a + \alpha 2b, \alpha 3a + \alpha 3b)$ ✓

8) $1 \cdot (a, 2a, 3a) = (1 \cdot a, 1 \cdot 2a, 1 \cdot 3a) = (a, 2a, 3a)$ ✓

SIM

Base $(a, 2a, 3a) \to a(1, 2, 3) \ \therefore$

$\{(1, 2, 3)\}$ dimensão = 1

**QUESTÃO 9)**

9 → $\vec{V_1} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $\vec{V_2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $\vec{V_3} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$V_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

i) $a\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + d\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} =$

$\begin{bmatrix}0&0\\0&0\end{bmatrix}$ então $\begin{bmatrix}a&0\\0&0\end{bmatrix}+\begin{bmatrix}0&b\\0&0\end{bmatrix}+\begin{bmatrix}0&0\\c&0\end{bmatrix}+\begin{bmatrix}0&0\\0&c\end{bmatrix}=$

então $\begin{bmatrix}a&b\\c&d\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$ então $\begin{bmatrix}0&0\\0&0\end{bmatrix}$

$\{v_1, v_2, v_3, v_4\}$ é LI $\quad a=0 \quad b=0 \quad c=0 \quad d=0$

(ii)

(ii) $a\begin{bmatrix}1&0\\0&0\end{bmatrix}+b\begin{bmatrix}0&1\\0&0\end{bmatrix}+c\begin{bmatrix}0&0\\1&0\end{bmatrix}+d\begin{bmatrix}0&0\\0&1\end{bmatrix}=\begin{bmatrix}x&y\\z&w\end{bmatrix}$

$\begin{bmatrix}a&0\\0&0\end{bmatrix}+\begin{bmatrix}0&b\\0&0\end{bmatrix}+\begin{bmatrix}0&0\\c&0\end{bmatrix}+\begin{bmatrix}0&0\\0&d\end{bmatrix}=\begin{bmatrix}x&y\\z&w\end{bmatrix}$

$\begin{bmatrix}a&b\\c&d\end{bmatrix}=\begin{bmatrix}x&y\\z&w\end{bmatrix}$ $\{v_1, v_2, v_3, v_4\}$ gera $R^4$

$\{v_1, v_2, v_3, v_4\}$ são Base para $R^4$

**QUESTÃO 11)**

11-)

$B = \{(1,1,1), (-1,1,0), (1,0,-1)\}$

$x_A = (1,0,0) \quad x_B$ ?

$x_B = m x_A \quad m = B^{-1} A$

$B = \begin{bmatrix}1&-1&1\\1&1&0\\1&0&-1\end{bmatrix} \quad B^{-1} = \begin{bmatrix}1/3&1/3&-7/3\\-1/3&2/3&-5/3\\1/3&1/3&2/3\end{bmatrix}$

$\left[\begin{array}{ccc|ccc}1&-1&1&1&0&0\\1&1&0&0&1&0\\1&0&-1&0&0&1\end{array}\right] \sim \left[\begin{array}{ccc|ccc}1&-1&1&1&0&0\\0&1&1&0&1&-1\\1&0&-1&0&0&1\end{array}\right]$

$L_2 = L_2 - L_3 \qquad L_1 = L_1 + L_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 1 & -1 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 1 & -1 \\ 0 & 0 & -3 & -1 & -1 & -2 \end{array}\right] \sim$$

$L_3 = L_3 - L_1$ $\qquad$ $L_3 = L_3 \cdot \left(-\frac{1}{3}\right)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & -1 \\ 0 & 1 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1/3 & 1/3 & 2/3 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & -1 \\ 0 & 1 & 0 & -1/3 & 2/3 & -5/3 \\ 0 & 0 & 1 & 1/3 & 1/3 & 2/3 \end{array}\right]$$

$L_2 = L_2 - L_3$ $\qquad$ $L_1 = L_1 - 2 \cdot L_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & 1/3 & -7/3 \\ 0 & 1 & 0 & -1/3 & 2/3 & -5/3 \\ 0 & 0 & 1 & 1/3 & 1/3 & 2/3 \end{array}\right]$$

$$X_B = B^{-1} X_A$$

$$x_B = \begin{bmatrix} 1/3 & 1/3 & -7/3 \\ -1/3 & 2/3 & -5/3 \\ 1/3 & 1/3 & 2/3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$x_B = \begin{bmatrix} 1/3 \\ -1/3 \\ 1/3 \end{bmatrix}$$

**QUESTÃO 15)**

15 - $\begin{pmatrix} 1 & -5 \\ -4 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ -1 & 5 \end{pmatrix} = \begin{pmatrix} 2 & -4 \\ -5 & 7 \end{pmatrix}$

$$x\begin{pmatrix} 1 & -5 \\ -4 & 2 \end{pmatrix} + y\begin{pmatrix} 1 & 1 \\ -1 & 5 \end{pmatrix} + z\begin{pmatrix} 1 & -7 \\ -5 & 1 \end{pmatrix} = 0$$

$$\begin{cases} x + y + z = 0 \\ -5x + y - 7z = 0 \\ -4x - y - 5z = 0 \\ 2x + 5y + z = 0 \end{cases}$$

$\downarrow$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ -5 & 1 & -7 & 0 \\ -4 & -1 & -5 & 0 \\ 2 & 5 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 6 & -2 & 0 \\ 0 & 3 & -1 & 0 \\ 0 & 3 & -1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 6 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \quad \begin{array}{l} y = 2z \\ x = -y - z = -2z - z = -3z \end{array}$$

$$-3z\begin{pmatrix} 1 & -5 \\ -4 & 2 \end{pmatrix} + 2z\begin{pmatrix} 1 & 1 \\ -1 & 5 \end{pmatrix} + z\begin{pmatrix} 1 & -7 \\ -5 & 1 \end{pmatrix} = 0$$

$$\begin{pmatrix}1&-7\\-5&1\end{pmatrix} = -3\begin{pmatrix}1&-5\\-4&2\end{pmatrix} + 2\begin{pmatrix}1&1\\-1&5\end{pmatrix}, \text{ se } z \neq 0$$

$$\left\{\begin{pmatrix}1&-5\\-4&2\end{pmatrix}, \begin{pmatrix}1&1\\-1&5\end{pmatrix}\right\} = \text{base } W$$

$\dim W = 2$

**QUESTÃO 19)**

19-

$$x(1,1,0) + y(0,-1,1) + z(1,1,1) = 0$$

$$\left.\begin{aligned} x + z &= 0 \\ x - y + z &= 0 \\ y + z &= 0 \end{aligned}\right\} = \begin{aligned} x &= -z \\ y &= 0 \\ z &= 0 \end{aligned}$$

$x = y = z = 0 = Li$

$\{V_1, V_2, V_3\} = 3$, então $\{V_1, V_2, V_3\} = \mathbb{R}^3$

**QUESTÃO 32)**

32) $\beta_1 = \{(1,0),(0,2)\}$, $\beta_2 = \{(-1,0),(1,1)\}$, $\beta_3 = \{(-1,-1),(0,-1)\}$

a) $[I]^{\beta_2}_{\beta_1} = ?$ seja $v_1 = (-1,0) = a_{11}(1,0) + a_{21}(0,2)$

$-1 = a_{11} \Rightarrow \boxed{a_{11} = -1}$

$0 = 2a_{21} \quad \boxed{a_{21} = 0}$

$v_2 = (1,1) = a_{12}(1,0) + a_{22}(0,2)$

$1 = a_{12} \Rightarrow \boxed{a_{12} = 1}$

$1 = 2a_{22} \quad \boxed{a_{22} = 1/2}$

$\therefore [I]^{\beta_2}_{\beta_1} = \begin{bmatrix} -1 & 1 \\ 0 & 1/2 \end{bmatrix}$

b) $[I]^{\beta_3}_{\beta_2} = ?$

$z_1 = (-1,-1) = a_{11}(-1,0) + a_{21}(1,1)$

$(-1,-1) = (-a_{11}, 0) + (a_{21}, a_{21})$

$-1 = -a_{11} + a_{21} \quad \boxed{a_{11} = 0}$

$\Rightarrow -1 = 0 + a_{21} \Rightarrow \boxed{a_{21} = -1}$

$-1 = -a_{11} + (-1)$

$-1 + 1 = -a_{11}$

$0 = -a_{11}$

$\boxed{0 = a_{11}}$

$z_2 = (0,-1) = a_{12}(-1,0) + a_{22}(1,1)$

$0 = a_{12}(-1) + a_{22}(1)$

$-1 = a_{12} \cdot 0 + a_{22}(1)$

$\boxed{-1 = a_{22}}$

$0 = -a_{12} + (-1)$

$\boxed{a_{12} = 1}$

$\boxed{\therefore [I]^{\beta_3}_{\beta_2} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}}$

c) $[I]^{\beta_3}_{\beta_1} = ?$ $w_1 = (-1,-1) = a_{11}(1,0) + a_{21}(0,2)$

$$\begin{cases} -1 = a_{11} \\ -1 = 2a_{21} \end{cases} \Rightarrow \boxed{a_{11} = -1} \quad \Rightarrow \boxed{-1/2 = a_{21}}$$

$w_2 = (0,-1) = a_{12}(1,0) + a_{22}(0,2) \Rightarrow \boxed{0 = a_{12}}$ ; $-1 = 2a_{22}$ ; $\boxed{a_{22} = -1/2}$

$$\therefore [I]^{\beta_3}_{\beta_1} = \begin{bmatrix} -1 & 0 \\ -1/2 & -1/2 \end{bmatrix}$$

/ /

d) $[I]^{\beta_2}_{\beta_1} \cdot [I]^{\beta_3}_{\beta_2} = \begin{bmatrix} -1 & 1 \\ 0 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 0-1 & -1-1 \\ 0-1/2 & 0-1/2 \end{bmatrix} \Rightarrow$

$$\therefore [I]^{\beta_2}_{\beta_1} \cdot [I]^{\beta_3}_{\beta_2} = \begin{bmatrix} -1 & -2 \\ -1/2 & -1/2 \end{bmatrix}$$

b) A 1ª Linha de todas as matrizes é não nula

**QUESTÃO 33)**

33) $\beta_1 = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

$\beta = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

→ 1ª Linha

→ 3ª Linha

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = 1\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 0\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 0\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = 1\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 1\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 0\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = 1\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 1\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 1\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

↳ 2ª Linha

$$\therefore [I]^{\beta_1}_{\beta} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$