Introdução à Álgebra Linear

Lista 6
Turma 02 A
Grupo 22
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7,5

Silas Neres

QUESTÃO 2

2 - a) $f: R^2 \to R^2$
$(x, y) \to (x+y, x-y)$

$u = (x_1, y_1)$
$v = (x_2, y_2)$

$f(u+v) = f(x_1 + x_2, y_1 + y_2)$
$(x_1 + x_2 + y_1 + y_2, x_1 + x_2 - y_1 - y_2)$
$= (x_1 + y_1, x_1 - y_1) + (x_2 + y_2, x_2 - y_2)$
$f(x_1, y_1) + f(x_2, y_2) = f(u) + f(v)$

$f(\lambda u) = f(\lambda x_1, \lambda y_1)$
$(\lambda x_1 + \lambda y_1, \lambda x_1 - \lambda y_1)$
$\lambda f(u)$

é uma aplicação linear

b) $g: R^2 \to R$
$(x, y) \to xy$

$g(u) = g(1,1) = 1.1 = 1$
$g(2.u) = g(2,2) = 2.2 = 4$
$2g(u) = 2\,g(1,1) = 2.1 = 2$
$g(2u) \neq 2g(u)$

não é uma aplicação

c) $h: M_2 \to R$
$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \to \det \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$h(u) = \det \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = 1$
$h(3u) = \det \begin{pmatrix} 3 & 3 \\ 3 & 6 \end{pmatrix} = 9$
$3h(u) = 3 . \det \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = 3$
$h(3u) \neq 3h(u)$

não é uma aplicação

d) $K: P_2 \to P_3$
$ax^2 + bx + c \to ax^3 + bx^2 + cx$

$u = a_1x^2 + a_2x + a_3$
$v = b_1x^2 + b_2x + b_3$

$K(u+v) = K(a_1x^2 + a_2x + a_3 + b_1x^2 + b_2x + b_3)$
$K((a_1 + b_1)x^2 + (a_2 + b_2)x + (a_3 + b_3))$
$(a_1 + b_1)x^3 + (a_2 + b_2)x^2 + (a_3 + b_3)x$
$a_1x^3 + a_2x^2 + a_3x + b_1x^3 + b_2x^2 + b_3x$
$K(u) + K(v)$

$K(\lambda u) = K(\lambda a_1x^2 + \lambda a_2x + \lambda a_3)$
$\lambda a_1x^3 + \lambda a_2x^2 + \lambda a_3x$
$\lambda(a_1x^3 + a_2x^2 + a_3x)$

é uma aplicação linear

e) $M: R^3 \to R^2$
$(x, y, z) \to (x, y, z) \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 1 & 1 \end{bmatrix}$

$u = (x_1, y_1, z_1)$
$v = (x_2, y_2, z_2)$

$A = \begin{pmatrix} 1 & 2 \\ 0 & -1 \\ 1 & 1 \end{pmatrix}$

$M(u+v) = (u+v)A = uA + uv = M(u) + M(v)$
$M(\lambda u) = (\lambda u)A = \lambda(u.A) = \lambda M(u)$

é uma aplicação linear

f) $N: R \to R$
$x \to |x|$

$u = 2 \quad \lambda = -1$

$N(u) = |2| = 2$
$N(\lambda u) = |(-1).2| = 2$
$\lambda N(u) = (-1).|2| = -2$
$N(\lambda u) \neq \lambda N(u)$

não é uma apicação linear

QUESTÃO 3

3 - a) $(x, y, z) = x(1,0,0) + y(0,1,0) + z(0,0,1)$

$T(x,y,z) = T(x(1,0,0) + y(0,1,0) + z(0,0,1))$

$xT(1,0,0) + yT(0,1,0) + zT(0,0,1)$

$x(2,0) + y(1,1) + z(0,-1)$

$(2x+y, y-z)$

b) $v = (x, y, z)$

$T(v) = (2x+y, y-z) = (3,2)$

$\begin{cases} 2x+y=3 \\ y-z=2 \end{cases} \rightarrow x = \frac{3-y}{2} \quad z = y-2$

$\left(\frac{3-y}{2}, y, y-2\right)$

QUESTÃO 4

4a-) $T(1,1) = (3,2,1)$

$T(0,-2) = (0,1,0)$

$(x,y,z) = a(1,1) + b(0,-2)$

$(x,y,z) = (a,a) + (0,-2b)$

$(x,y,z) = (a, a-2b)$

$x = a$ $\quad y = x - 2b \quad b = \frac{y-x}{2}$

$y = a - 2b$

$(x,y,z) = x(1,1) + \left(\frac{y-x}{2}\right)(0,-2)$

$T(x,y,z) = \cancel{x}\, T(1,1) - \left(\frac{y-x}{2}\right) T(0,-2)$

$T(x,y,z) = x(3,2,1) - \left(\frac{y-x}{2}\right)(0,1,0)$

$T(x,y,z) = (3x, 2x, 1x) - \left(0, \frac{y-x}{2}, 0\right)$

$T(x,y,z) = \left(3x, 2x - \left(\frac{y-x}{2}\right), x\right)$ ✓

b-> $T(1,0)$ em $(3x, 2x - (\frac{y-x}{2}), x)$

$(3, 2 - (\frac{0-1}{2}), 1)$

$(3, \frac{4-0-1}{2}, 1) = (3, \frac{3}{2}, 1)$

$T(0,1)$ em $(3x, 2x - (\frac{y-x}{2}), x) =$

$(0, 0 - (\frac{1-0}{2}), 0) = (0, -\frac{1}{2}, 0)$

c-> $T(3,2,1) = (1,1)$
$T(0,1,0) = (0,-2)$
$T(0,0,1) = (0,0)$

$(x,y,z) = a(3,2,1) + b(0,1,0) + c(0,0,1)$

$(x,y,z) = (3a, 2a, a) + (0, b, 0) + (0,0,c)$

$(x,y,z) = (3a, 2a+b, a+c)$

$x = 3a$ $\quad a = x/3$
$y = 2a+b$ $\quad y = 2(x/3) + b$ $\quad b = y - \frac{2x}{3}$
$z = a+c$ $\quad z = \frac{x}{3} + c$ $\quad c = z - \frac{x}{3}$

tilibra

$(x,y,z) = \frac{x}{3}(3,2,1) + \left(y - \frac{2x}{3}\right)(0,1,0) +$

$z - \frac{x}{3}(0,0,1)$

$T(x,y,z) = \frac{x}{3}T(3,2,1) + \left(y - \frac{2x}{3}\right)T(0,1,0) +$

$\left(z - \frac{x}{3}\right)T(0,0,1)$

$T(x,y,z) = \frac{x}{3}(1,1) + \left(y - \frac{2x}{3}\right)(0,-2) +$

$\left(z - \frac{x}{3}\right)(0,0)$

$T(x,y,z) = (x/3, x/3) + \left(0, y - \frac{2x}{3}\right) +$

$(0,0)$

$T(x,y,z) = \left(x/3, \frac{x}{3} + y - \frac{2x}{3}\right)$

$T(x,y,z) = \left(x/3, \frac{x+3y}{3} - 2x\right.$

$T(x,y,z) = \left(x/3, -\frac{x+3y}{3}\right)$

$(-6y+5x)/3$

d $\to$ $P = S \circ T$

$S = \left(\frac{x}{3}, -\frac{x+3y}{3}\right)$ $\quad T = \left(3x, 2x - \left(\frac{y-x}{2}\right), x\right)$

$\left(3x + \frac{y}{3}, -\frac{x+3y}{3} + 2x - \left(\frac{y-x}{2}\right), x\right)$

$\left(\frac{9x+x}{3}, \frac{-2x+6y+12x-3y+3x}{6}, x\right)$

×

$\left(\frac{10x}{3}, \frac{13x+3y}{6}, x\right) = P$

p(x,y)=(x,y)

QUESTÃO 11

$11 \to \alpha = \{(1,-1),(0,2)\}$ e $\beta = \{(1,0,-1),(0,1,2),(1,2,0)\}$.

$$[T]^{\alpha}_{\beta} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & -1 \end{bmatrix}$$

a) $T(\vec{v_1}) = T(1,-1) = 1(1,0,-1) + 1(0,1,2) + 0(1,2,0)$

$T(1,-1) = (1,0,-1) + (0,1,2)$
$T(1,-1) = (1,1,1)$
$T(\vec{v_2}) = T(0,2) = 0(1,0,-1) + 1(0,1,2) + (-1)\cdot(1,2,0)$
$T(0,2) = (0,1,2) + (-1,-2,0)$
$T(0,2) = (-1,-1,2)$

$(x,y) = a(1,-1) + b(0,2)$
$(x,y) = (a,-a) + (0,2b)$
$(x,y) = (a,-a+2b)$

$a = x$

$y = -a + 2b \quad b = \frac{y+x}{2}$

$(x,y) = x(1,-1) + \left(\frac{y+x}{2}\right)(0,2)$

$T(x,y) = x\,T(1,-1) + \left(\frac{y+x}{2}\right) T(0,2)$

$T(x,y) = x(1,1,1) + \left(\frac{y+x}{2}\right)(-1,-1,2)$

$T(x,y) = (x,x,x) + \left(-\left(\frac{y+x}{2}\right), -\left(\frac{y+x}{2}\right), (y+x)\right)$

$T(x,y) = \left(\frac{2x-y+x}{2}, \frac{2x-y+x}{2}, x+y+x\right)$

$T(x,y) = \left(\frac{3x-y}{2}, \frac{3x-y}{2}, 2x+y\right)$ ×

(x-y)/2

b) $S(x,y) = (2y, x-y, x)$

$[S]^{\alpha}_{\beta}$

$S(1,-1) = (-2, 2, 1)$

$T(\vec{v}_1) = a_{11}(1,0,-1) + a_{21}(0,1,2) + a_{31}(2,2,0)$

$T(\vec{v}) = (a_{11}, 0, -a_{11}) + (0, a_{21}, 2a_{21}) + (a_{31}, 2a_{31}, 0)$

$$(a_{11}+a_{31},\ a_{21}+2a_{31},\ -a_{11}+2a_{21}) = (-2, 2, 1)$$

$$a_{11}+a_{31}=2 \qquad \cancel{a_{11} = -2 - a_{31}}$$
$$a_{21}+2a_{31}=2 \qquad \cancel{2 + a_{31} + a_{21} = 1}$$
$$-a_{11}+2a_{21}=1$$

$$2a_{21} = 1 + a_{11} \qquad \frac{1+a_{11}}{2} + 2a_{31} = 2$$
$$\underline{a_{21} = \frac{1+a_{11}}{2}}$$

$$2a_{31} = 2 - \frac{1+a_{11}}{2}$$
$$2a_{31} = \frac{4-1+a_{11}}{2} \cdot \frac{1}{2}$$

$$\underline{a_{31} = \frac{4-1+a_{11}}{4}} \qquad a_{11} + \frac{4-1+a_{11}}{4} = -2$$

$$\frac{4a_{11}+4-1+a_{11}}{4} = -2 \qquad 4a_{11}+4-1+a_{11} = -2 \cdot 4$$

$$4a_{11}+3+a_{11} = -8 \qquad 5a_{11} = -8-3$$
$$5a_{11} = \frac{-11}{[illegible]} \qquad a_{11} = \frac{-11}{5}$$

$$a_{21} = \frac{1+(-11/5)}{2}$$

$$a_{21} = \frac{\frac{5-11}{5}}{2} \qquad a_{21} = \frac{-6}{5} \cdot \frac{1}{2} \qquad a_{21} = \frac{-6}{10} = \frac{3}{-5}$$

$$a_{31} = \frac{3+(11/5)}{4} = \frac{15-11}{5} \cdot \frac{1}{4} = \frac{4}{20} = \frac{2}{10} = \frac{1}{5}$$

$S(0,2)=(4,2,0)$

$T(\vec{v}_2) = a_{12}(1,0,-1) + a_{22}(0,1,2) + a_{32}(1,2,0)$

$T(\vec{v}_2) = (a_{12}, 0, -a_{12}) + (0, a_{22}, 2a_{22}) + (a_{32}, 2a_{32}, 0)$

$T(\vec{v}_2) = (a_{12} + a_{32}, a_{22} + 2a_{32}, -a_{12} + a_{22})$

$a_{12} + a_{32} = 4$

$a_{22} + 2a_{32} = -2$

$-a_{12} + 2a_{22} = 0$

~~$a_{22} + 2(4 - a_{12}) = -2$~~

$a_{22} + 2(4 - a_{12}) = -2$

$a_{22} + 8 - 2a_{12} = -2$

$a_{32} = 4 - a_{12}$

$a_{22} = -2 - 8 + 2a_{12}$

~~$-a_{12} + 2(-2-8+2a_{12})$~~

$-a_{12} + 2(-10 + 2a_{12}) = 0$

$-a_{12} - 20 + 4a_{12} = 0$

$3a_{12} = 20$

$a_{12} = \frac{20}{3}$

$a_{22} = -10 + 2 \cdot \frac{20}{3}$

$a_{22} = -10 + \frac{40}{3} = \frac{-30+40}{3} = \frac{10}{3}$

$a_{32} = -4 - \frac{20}{3} = \frac{-12-20}{3} = -\frac{32}{3}$

$$[S]_{\beta}^{\alpha} = \begin{bmatrix} -11/5 & 20/3 \\ -3/5 & 10/3 \\ 1/5 & -32/3 \end{bmatrix}$$

c-) $T(x,y) = \left(\frac{3x-y}{2}, \frac{3x-y}{2}, 2x+y\right)$

$T(1,-1) = (2,2,4)$

$T(0,2) = (-1,-1,2)$

$I(\vec{v_1}) = 1(x_1, y_1, z_1) + 0(x_2, y_2, z_2) + 0(x_3, y_3, z_3)$

$I(\vec{v_1}) = (x_1, y_1, z_1)$ $x_1 = 2$ $y_1 = 2$ $z_1 = 4$

$I(\vec{v_2}) = 0(x_1, y_1, z_1) + 0(x_2, y_2, z_2) + 1(x_3, y_3, z_3)$

$x_3 = -1$ $y_3 = -1$ $z_3 = 2$

$V = \{(2,2,4), (x_2, y_2, z_2), (-1,-1,2)\}$
sendo x_2, y_2 e z_2 quaisquer valores.

QUESTÃO 14

ITEM A)

14)

$$\beta = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+d, b+c)$$

a) $[T]^{\beta}_{\alpha} \Rightarrow T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+d, b+c)$

Seja:

$$T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = (1+0, 0+0) \Rightarrow \boxed{T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = (1, 0)}$$

$$T\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) = (0+0, 1+0) \Rightarrow \boxed{T\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) = (0, 1)}$$

$$T\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) = (0+0, 1+0) \Rightarrow \boxed{T\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) = (0, 1)}$$

$$T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = (0+1, 0+0) \Rightarrow \boxed{T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = (1, 0)}$$

$$\boxed{\therefore [T]^{\beta}_{\alpha} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}}$$

✓

ITEM B)

Se $S: \mathbb{R}^2 \to V$ e $[S]^{\alpha}_{\beta} = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -1 & 0 \\ 0 & 1 \end{bmatrix}$

b) Ache S e, se for possível (a,b) tal que $S(a,b) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ canônica

$(x,y)\alpha = x(1,0) + y(0,1)$

$S(x,y))\beta = [S]^{\alpha}_{\beta} \, [(x,y)]\alpha$

$= \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x & +y \\ x & -y \\ -x & 0 \\ 0 & -y \end{bmatrix}$

$S(x,y) = (2x+y)\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + (x-y)\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - x\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + y\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{pmatrix} 2x+y & x-y \\ -x & y \end{pmatrix}$

$\begin{bmatrix} 2x+y & x-y \\ -x & y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$2x+y = 1 \to 2 \cdot 0 + 1 = 1$

$x - y = 1 \to X$

$\left.\begin{matrix} \boxed{-x = 0} \\ \boxed{y = 1} \end{matrix}\right\} \; 0 - 1 = -1$

Não é possível

QUESTÃO 19

(19) $T: \mathbb{R}^3 \to \mathbb{R}^3$ dada por $T(x,y,z) = (z, x-y, -z)$

a) Determine uma base do núcleo de T.

$N(T) = Ker(T) = \{(x,y,z) \in \mathbb{R}^3$ tal que $T(x,y,z) = (0,0,0)\}$

ou seja:

$(z, x-y, -z) = (0,0,0)$

$\boxed{x = y}$ e $\boxed{z = 0}$

$N(T) = \{(x,x,0);\ x \in \mathbb{R}\}$

$\therefore$ BASE $= \{(1,1,0)\}$ $\quad$ Dim $\{N(T)\} = 1$

c) T é sobrejetora? Justifique

Não, pois a dimensão da imagem é diferente da dimensão do contradomínio.

b) Dê a dimensão da imagem de T

$Im\,T = \{(z, x-y, -z) \in \mathbb{R}^3,\ x, y, z \in \mathbb{R}^3\}$

$= \{x(0,1,0) + y(0,-1,0) + z(1,0,1)\}$

$= \{(0,1,0), (0,-1,0), (1,0,\overset{(-1)}{1})\}$

dimensão $= 2\ 1$

d) Faça um esboço de Ker T e Im T.

$T(1,1,1) \quad T(x,y,z) = (1,0,-1)$