PROVA 3

4 $\int \sec^2 x \cdot tg^3 x \cdot dx$

$\begin{cases} u = tg\,x \\ du = \sec^2 x \cdot dx \end{cases}$

$\int u^3 \cdot du =$

$\frac{u^4}{4} + C = \boxed{\frac{tg^4 x}{4} + C}$

5 $\int \frac{2x+55}{x^2+3x-40} \cdot dx =$

$\frac{2x+55}{(x+8)(x-5)} = \frac{A}{(x+8)} + \frac{B}{(x-5)} = \frac{A(x-5)+B(x+8)}{(x+8)(x-5)} =$

$x = 5 \rightarrow B(5+8) = 2.5 + 55 \rightarrow B = 65/13 \rightarrow B = 5$

$x = -8 \rightarrow A(-13) = 2.8 + 55 \rightarrow A = -71/13$

$A + B = \frac{65}{13} - \frac{71}{13} = \boxed{-\frac{6}{13}}$

6 $\int \frac{2}{x-3} \cdot dx + \int \frac{-5}{x+1} \cdot dx$

$2\int \frac{1}{x-3} \cdot dx + -5\int \frac{1}{x+1} \cdot dx$

$\boxed{2 \cdot \ln|x-3| - 5 \cdot \ln|x+1| + C}$

8 $\int_1^2 \frac{9x - 3x^2}{x} \cdot dx$

$\int_1^2 \left(\frac{9x}{x} - \frac{3x^2}{x}\right) dx$

$\int_1^2 (9 - 3x) \cdot dx$

$\left[9x - \frac{3x^2}{2}\right]_1^2$

$\left(9.2 - \frac{3.2^2}{2}\right) - \left(9.1 - \frac{3.1^2}{2}\right)$

$18 - 6 - 9 + \frac{3}{2}$

$\frac{6}{2} + \frac{3}{2} = \boxed{\frac{9}{2}}$

9 $\int_1^e 10 \cdot \frac{\ln(x)}{x} \cdot dx \rightarrow 10 \int_1^e u \cdot dx \rightarrow 10 \frac{u^2}{2}\Big|_1^e \rightarrow \frac{10(\ln(x))^2}{2}\Big|_1^e \rightarrow 5 \cdot (\ln e)^2 - 5(\ln 1)^2 = \boxed{5}$

$u = \ln x$

$du = \frac{1}{x} \cdot dx$

$x^{-3} = \frac{x^{-2}}{-2}$

10 (i) $\int_0^1 \frac{1}{x^3} dx = \lim_{A \to 0^+} \int_A^1 \frac{1}{x^3} \cdot dx = \lim_{A \to 0^+} \frac{x^{-2}}{-2}\Big|_0^1 = \lim_{A \to 0^+} -\frac{1}{2 \cdot x^2}\Big|_0^1 = -\frac{1}{2.1^2} - \frac{1}{\cancel{2.0^2}} = -\infty \rightarrow$ divergente

$(+\infty)$ $0{,}0001$

(ii) $\int_1^{+\infty} \frac{1}{x^3} dx = \lim_{B \to +\infty} \int_1^B x^{-3} \cdot dx = \lim_{B \to +\infty} \frac{x^{-2}}{-2}\Big|_1^B = \lim_{B \to +\infty} \frac{1}{2x^2}\Big|_B^1 = \frac{1}{2.1} - \frac{1}{\cancel{2.\infty^2}} = \frac{1}{2} \rightarrow$ convergente

0

(iii) $\int_{-\infty}^{-1} \frac{1}{x^3} dx = \lim_{C \to -\infty} \int_C^{-1} x^{-3} \cdot dx = \lim_{C \to -\infty} \frac{x^{-2}}{-2}\Big|_C^{-1} = \lim_{C \to -\infty} \frac{1}{2x^2}\Big|_{-1}^{C} = \frac{1}{\cancel{2.-\infty}} - \frac{1}{2.(-1)^2} = +\infty \rightarrow$ divergente

$+\infty$

11 $\int \frac{1}{x^2\sqrt{9-x^2}} \cdot dx = \int \frac{3 \cdot \cos\theta \cdot d\theta}{(3 \cdot \text{sen}\,\theta)^2 \sqrt{9 - (3\,\text{sen}\,\theta)^2}}$

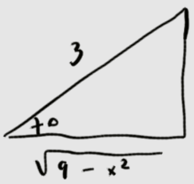
$x = 3\,\text{sen}\,\theta$

$dx = 3\cos\theta \cdot d\theta$

$= \int \frac{3\cos\theta \cdot d\theta}{9 \cdot \text{sen}^2\theta \sqrt{9 - 9 \cdot \text{sen}^2\theta}}$

$x = 3\,\text{sen}\,\theta \therefore \text{sen}\,\theta = \frac{x}{3}$

$= \int \frac{3\cos\theta \cdot d\theta}{9\,\text{sen}^2\theta \sqrt{9(1 - \text{sen}^2\theta)}}$



$= \int \frac{3\cos\theta \cdot d\theta}{9\,\text{sen}^2\theta \sqrt{9 \cdot \cos^2\theta}}$

$= \int \frac{\cancel{3\cos\theta} \cdot d\theta}{9 \cdot \text{sen}^2\theta \cdot \cancel{3 \cdot \cos\theta}}$

$= \frac{1}{9}\int \frac{1}{\text{sen}^2\theta} \cdot d\theta$

$= \frac{1}{9}\int \csc^2\theta \cdot d\theta$

$\boxed{-\frac{1}{9} \cdot \frac{\sqrt{9-x^2}}{x} + C} \leftarrow = \boxed{-\frac{1}{9} \cdot \cot g\,\theta + C}$

$\cot g = \frac{\cos}{\text{sen}}$

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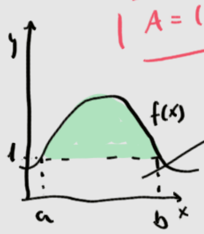
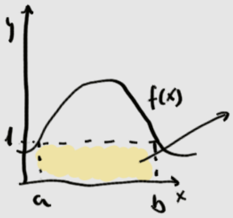
(1) $\int_a^b f(x) \cdot dx$



$(b-a) \cdot 1$

$\boxed{A = (1) - (2)}$



(2) $(b-a) \cdot 1$

$A = \int_a^b f(x) \cdot dx - (b-a)$

$A = \boxed{\int_a^b f(x)\,dx - b + a}$

14 $A = -\int_{-1}^{0} x^3 dx + \int_0^2 x^3 dx = \int_0^{-1} x^3 dx + \int_0^2 x^3 dx = \frac{x^4}{4}\Big|_0^{-1} + \frac{x^4}{4}\Big|_0^2 = \frac{(-1)^4}{4} + \frac{(2)^4}{4} = \frac{17}{4}$

$4A = \frac{17}{4}$

$\therefore \boxed{A = 17}$