LISTA 1

Lista 1

1 / 2 / 3 / 4 / 5 / 6 / 7 / 9 / 12 / 14

Questão 1

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \end{bmatrix}_{2\times3}$, $B = \begin{bmatrix} -2 & 0 & 1 \\ 3 & 0 & 1 \end{bmatrix}_{2\times3}$, $C = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}_{3\times1}$ e $D = \begin{bmatrix} 2 & -1 \end{bmatrix}_{1\times2}$

a) $A + B \rightarrow \begin{bmatrix} 1+(-2) & 2+0 & 3+1 \\ 2+3 & 1+0 & -1+1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 4 \\ 5 & 1 & 0 \end{bmatrix}$

b) $A \cdot C \rightarrow \begin{bmatrix} 1.(-1) + 2.2 + 3.4 \\ 2(-1) + 1.2 + (-1).4 \end{bmatrix} = \begin{bmatrix} 15 \\ -4 \end{bmatrix}$

c) $B \cdot C \rightarrow \begin{bmatrix} -2.(-1) + 0.2 + 1.4 \\ 3.(-1) + 0.2 + 1.4 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$

d) $C \cdot D \rightarrow \begin{bmatrix} (-1).2 & (-1).(-1) \\ 2.2 & 2.(-1) \\ 4.2 & 4.(-1) \end{bmatrix} = \begin{bmatrix} - \\ 4 \\ 8 \end{bmatrix}$

e) $D \cdot A \rightarrow \begin{bmatrix} 2.1 + (-1).2 & 2.2 + (-1).1 & 2.3 + (-1).(-1) \end{bmatrix}$
$= \begin{bmatrix} 0 & 3 & 7 \end{bmatrix}$

f) $D \cdot B \rightarrow \begin{bmatrix} 2.(-2) + (-1).3 & 2.0 + (-1).0 & 2.1 + (-1).1 \end{bmatrix} = \begin{bmatrix} -7 & 0 & 1 \end{bmatrix}$

g) $-A \rightarrow -\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -2 & -3 \\ -2 & -1 & 1 \end{bmatrix}$

h) $-D \rightarrow \begin{bmatrix} -2 & 1 \end{bmatrix}$

Questão 2 Se $A = \begin{bmatrix} 2 & x^2 \\ 2x-1 & 0 \end{bmatrix}$. Se $A' = A$, então $x = 1$

$A' = \begin{bmatrix} 2 & 2x-1 \\ x^2 & 0 \end{bmatrix} \rightarrow$ $2x - 1 = x^2 \rightarrow x^2 - 2x + 1 = 0$

$\Delta = b^2 - 4.a.c \rightarrow \Delta = 4 - 4 = 0$

$x = \frac{-b \pm \sqrt{\Delta}}{2.a}$ → $x' = \frac{2+0}{2} = 1$; $x'' = \frac{2-0}{2} = 1$ $\therefore \boxed{x = 1}$

Questão 3 Se A é uma matriz simétrica, então $A - A' = 0$

$A = \begin{bmatrix} e & \pi & \sqrt{2} \\ \pi & 7 & 1 \\ \sqrt{2} & 1 & -1 \end{bmatrix} - A' = \begin{bmatrix} e & \pi & \sqrt{2} \\ \pi & 7 & 1 \\ \sqrt{2} & 1 & -1 \end{bmatrix} = \boxed{0}$

Questão 4

Questão 5 A = diagonal, A' = diag

$A = \begin{bmatrix} e & \pi & \sqrt{2} \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$, $A' = \begin{bmatrix} e & 0 & 0 \\ \pi & 2 & 0 \\ \sqrt{2} & 1 & 3 \end{bmatrix} \rightsquigarrow \boxed{\text{Triangular inferior}}$

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $A' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Questão 6

a) $(-A)' = -(A')$ ⇝ Verdadeiro

$A = \begin{bmatrix} 2 & 7 & 3 \\ 1 & 0 & -1 \end{bmatrix} \to -A \begin{bmatrix} -2 & -7 & -3 \\ -1 & 0 & 1 \end{bmatrix} \to (-A)' = \begin{bmatrix} -2 & -1 \\ -7 & 0 \\ -3 & 1 \end{bmatrix}$

$A' = \begin{bmatrix} 2 & 1 \\ 7 & 0 \\ 3 & -1 \end{bmatrix} \to -(A') = \begin{bmatrix} -2 & -1 \\ -7 & 0 \\ -3 & 1 \end{bmatrix}$ (=)

b) $(A+B)' = B' + A' \to$ Verdadeiro

$A = \begin{bmatrix} 2 & 7 & 3 \\ 1 & 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \end{bmatrix}$

$A+B = \begin{bmatrix} 3 & 7 & 5 \\ 4 & 1 & -1 \end{bmatrix} \to (A+B)' = \begin{bmatrix} 3 & 4 \\ 7 & 1 \\ 5 & -1 \end{bmatrix}$

$B' = \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 2 & 0 \end{bmatrix}$ $A' = \begin{bmatrix} 2 & 1 \\ 7 & 0 \\ 3 & -1 \end{bmatrix} \to B' + A' =$

c) $A.B = 0$, então $A = 0$ ou $B = 0$.

$A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \neq 0$

$A \cdot B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ Falso

d) $(K_1.A)(K_2.B) = (K_1.K_2)A.B$ Verdadeiro

$A = \begin{bmatrix} 2 & 7 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

$2.A \begin{bmatrix} 4 & 14 \\ 2 & 0 \end{bmatrix}$, $3.B = \begin{bmatrix} 3 & 0 \\ 9 & 3 \end{bmatrix}$

$(2A)(3B) = \begin{bmatrix} 12+126 & 0+42 \\ 6+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 138 & 42 \\ 6 & 0 \end{bmatrix}$

$A \cdot B = \begin{bmatrix} 2.1+7.3 & 2.0+7.1 \\ 1.1+0.3 & 1.0+0.0 \end{bmatrix} = \begin{bmatrix} 23 & 7 \\ 1 & 0 \end{bmatrix} \to$

$6(A \cdot B) = \begin{bmatrix} 138 & 42 \\ 6 & 0 \end{bmatrix}$

e) $(-A).(-B) = -(AB)$ Falso

$A = \begin{bmatrix} 2 & 7 & 3 \\ 1 & 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \end{bmatrix}$

$-A = \begin{bmatrix} -2 & -7 & -3 \\ -1 & 0 & 1 \end{bmatrix}$, $-B = \begin{bmatrix} -1 & 0 & 2 \\ -3 & -1 & 0 \end{bmatrix}$

$(-A).(-B) = \begin{bmatrix} 2 & 0 & -6 \\ 3 & 0 & 0 \end{bmatrix}$

$A.B = \begin{bmatrix} 2 & 0 & 6 \\ 3 & 0 & 0 \end{bmatrix} \to -(AB) = \begin{bmatrix} -2 & 0 & -6 \\ -3 & 0 & 0 \end{bmatrix}$

f) Se A e B são matrizes simétricas, então $AB = BA$ falso

$A = \begin{bmatrix} 7 & 2 \\ 2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ 1 & 8 \end{bmatrix}$

$A \cdot B = \begin{bmatrix} 0+2 & 7+16 \\ 0+4 & 2+32 \end{bmatrix} = \begin{bmatrix} 2 & 23 \\ 4 & 34 \end{bmatrix}$

$\neq$

$B.A = \begin{bmatrix} 0+2 & 0+4 \\ 7+16 & 2+32 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 23 & 34 \end{bmatrix}$

g) Se $A.B = 0$, então $B.A = 0$ Falso

$A \cdot B \neq B \cdot A$, por isso o não é necessariamente iguais

$A = \begin{bmatrix} 1 & 0 \end{bmatrix}$ $B \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

h) Se podemos efetuar o produto $A \cdot A$, então A é uma matriz quadrada verdadeiro

se $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$2 \times 1 \cdot 2 \times 1$

$A \cdot B = [0+0] = 0$

$B \cdot A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$A \cdot A = \cancel{\exists} \quad \neq$

se $A = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$

$2\times 2 \cdot 2 \times 2$

$A \cdot A = \begin{bmatrix} 0+2 & 0+1 \\ 0+2 & 2+1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$

Questão 14 Se $A = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$, ache B, de modo que $B^2 = A$

$\begin{bmatrix} x & y \\ z & w \end{bmatrix}^2 = \begin{bmatrix} x\cdot x + y\cdot z & x\cdot y + y\cdot w \\ z\cdot x + w\cdot z & z\cdot y + w w \end{bmatrix}$

$x\cdot x + y\cdot z = 3 \qquad x\cdot y + y\, w = -2$

$z\cdot x + w\cdot z = -4 \qquad zy + w\cdot w = 3$

$x^2 + y\cdot z = 3$

$(-)\ w^2 + y\cdot z = 3$

$x^2 - w^2 = 0$

$\underline{x = w}$

$xy + yw = -2$

$wy + wy = -2$

$2wy = -2$

$w = -1/y$

$z\cdot x + wz = -4$

$zw + zw = -4$

$2zw = -4$

$z = \frac{-2}{w}$

$z = \frac{-2}{-1/y}$

$\underline{z = 2.y}$

$z\cdot y + w^2 =$

$2\cdot y^2 + \left(\frac{-1}{y}\right)^2 =$

$\frac{2y^2}{1} + \frac{1}{y^2} = 3$

$\frac{2y^4+1}{y^2} = 3$

$2y^4 + 1 = 3y$

$3y^2 - 2y^4 =$

$y^2(3-2y^2) =$

$y^2 = 1$

$\boxed{y = 1}$

$3 - 2y^2 = 1$

$2y^2 = 2$

$\underline{y^2 = \sqrt{2}}$

$\times\ B = \begin{bmatrix} -1/\sqrt{2} & \sqrt{2} \\ 2\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

$\times\ A \begin{bmatrix} 4.5 & -2 \\ -4 & 4.5 \end{bmatrix}$

$B = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}$

$B^2 = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$

LISTA 3

Lista 3 - IAL

3 / 4 / 5 / 6 / 8 / 9 / 13 / 14 / 21 /

3

$$\det \begin{bmatrix} 2 & 0 & -1 \\ 3 & 0 & 2 \\ 4 & -3 & 7 \end{bmatrix}$$

a) Pela definição

$$\begin{vmatrix} 2 & 0 & -1 \\ 3 & 0 & 2 \\ 4 & -3 & 7 \end{vmatrix} \begin{matrix} 2 & 0 \\ 3 & 0 \\ 4 & -3 \end{matrix} \rightarrow 0 + 0 + 9 - (0 - 12 + 0)$$

$$9 + 12 = \boxed{21}$$

b) Laplace

$$\begin{vmatrix} 2 & 0 & -1 \\ 3 & 0 & 2 \\ 4 & -3 & 7 \end{vmatrix} \rightarrow 0 \cdot \Delta_{12} + 0 \cdot \Delta_{22} - 3 \cdot \Delta_{32} =$$

$$\cancel{0 \cdot (-1)^3 \cdot \begin{vmatrix} 3 & 2 \\ 4 & 7 \end{vmatrix}} + \cancel{0 \cdot (-1)^4 \cdot \begin{vmatrix} 2 & -1 \\ 4 & 7 \end{vmatrix}} - 3 \cdot (-1)^5 \cdot \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = 3(-4(-3)$$

$$= 3 \cdot 7 = \boxed{2$$

4 $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix}$

a) det A + det B

$A = \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} \rightarrow A = 0 - 2 = -2$ $\quad B = \begin{vmatrix} 3 & -1 \\ 0 & 1 \end{vmatrix} \rightarrow B = 3 - 0 = 3$ $\quad$ det(A) + det(B) =

b) det (A + B)

$A+B = \begin{vmatrix} 4 & 1 \\ 1 & 1 \end{vmatrix} \rightarrow 4 - 1 = 3$ $\quad \boxed{\det(a+b) = 3}$

5 **a) det (AB) = det (BA)** → Verdadeiro

$A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \cdot B \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4+2 & 1+2 \\ 4+0 & 1+0 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 4 & 1 \end{bmatrix} = \begin{vmatrix} 6 & 3 \\ 4 & 1 \end{vmatrix} = 6 - 12 \rightarrow$ det (AB) = -

$$B=\begin{bmatrix}4 & 1\\ 1 & 1\end{bmatrix}\cdot A=\begin{bmatrix}1 & 2\\ 1 & 0\end{bmatrix}=\begin{bmatrix}4+1 & 8+0\\ 1+2 & 2+0\end{bmatrix}=\begin{bmatrix}5 & 8\\ 3 & 2\end{bmatrix}=10-16=-6 \rightsquigarrow \underline{\det(B.A)=-6}$$

b) $\det(A^t)=\det A$ $\rightsquigarrow$ Verdadeiro

$$\det(A)=\begin{bmatrix}1 & 2\\ 1 & 0\end{bmatrix}\to\begin{vmatrix}1 & 2\\ 1 & 0\end{vmatrix}=1-2=\underline{-1}$$

$$\det(A)^t=\begin{bmatrix}1 & 1\\ 2 & 0\end{bmatrix}\to\begin{vmatrix}1 & 1\\ 2 & 0\end{vmatrix}=1-2=\underline{\underline{-1}}$$

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6 $A=\begin{bmatrix}2 & 3 & 1 & -2\\ \cancel{5} & \cancel{3} & 1 & \cancel{4}\\ 0 & 1 & 2 & 2\\ 3 & -1 & -2 & 4\end{bmatrix}$

a) A_{23} $=\begin{bmatrix}2 & 3 & -2\\ 0 & 1 & 2\\ 3 & -1 & 4\end{bmatrix}$

b) $|A_{23}|$

$$\left|\begin{matrix}2 & 3 & -2\\ 0 & 1 & 2\\ 3 & -1 & 4\end{matrix}\right|\begin{matrix}2 & 3\\ 0 & 1\\ 3 & -1\end{matrix}\to 26+10=$$

c) Δ_{23} $=(-1)^5\cdot\begin{vmatrix}2 & 3 & -2\\ 0 & 1 & 2\\ 3 & -1 & 4\end{vmatrix}\to(-1)\cdot 36$

$=\boxed{-36}$

d) $\det(A)$

$$\begin{vmatrix}2 & 3 & 1 & -2\\ 5 & 3 & 1 & 4\\ 0 & 1 & 2 & 2\\ 3 & -1 & -2 & 4\end{vmatrix}\to 1\cdot(-1)^4\cdot\underbrace{\begin{vmatrix}5 & 3 & 4\\ 0 & 1 & 2\\ 3 & -1 & 4\end{vmatrix}}_{38-2=36}+1\cdot(-1)^5\cdot\underbrace{\begin{vmatrix}2 & 3 & -2\\ 0 & 1 & 2\\ 3 & -1 & 4\end{vmatrix}}_{36}+2\cdot(-1)^6\underbrace{\begin{vmatrix}2 & 3 & -2\\ 5 & 3 & 4\\ 3 & -1 & 4\end{vmatrix}}_{70-34=36}+(-2)(-1)^7\underbrace{\begin{vmatrix}2 & 3 & \\ 5 & 3 & \\ 0 & 1 & \end{vmatrix}}_{2-38}$$

$$1.36+1.(-1).(36)+2.(36)+(-2).(-2)(-36)$$

$$36-36+2.36-2.36=\boxed{0}$$

8 a) $\det(A)=\begin{bmatrix}3 & -1 & 5 & 0\\ 0 & 2 & 0 & 1\\ 2 & 0 & -1 & 3\\ 1 & 1 & 2 & 0\end{bmatrix}\to 0+2.(-1)^4\underbrace{\begin{vmatrix}3 & 5 & 0\\ 2 & -1 & 3\\ 1 & 2 & 0\end{vmatrix}}_{15-18=-3}+0+1.(-1)^6.\underbrace{\begin{vmatrix}3 & -1 & 5\\ 2 & 0 & -1\\ 1 & 1 & 2\end{vmatrix}}_{11-(\ldots)=1\ldots}$

$$2.(-3)+1.(18)=\boxed{12}$$

b) $\det(A)=\begin{bmatrix}3 & 0 & 0 & 0 & 0\\ 19 & 18 & 0 & 0 & 0\\ -6 & \pi & -5 & 0 & 0\\ 4 & \sqrt{2} & \sqrt{3} & 0 & 0\\ 8 & 3 & 5 & 6 & -1\end{bmatrix}\to\begin{bmatrix}3 & 0 & 0 & \boxed{0} & 0\\ 19 & 18 & 0 & \boxed{0} & 0\\ -6 & \pi & -5 & \boxed{0} & 0\\ 4 & \sqrt{2} & \sqrt{3} & \boxed{0} & 0\\ 8 & 3 & 5 & \boxed{0} & -1\end{bmatrix}\rightsquigarrow 0$

$\times 6$

$+$

c) $\det(A) = \begin{bmatrix} i & 3 & \boxed{2} & -i \\ 3 & -i & \boxed{1} & i \\ 2 & 1 & \boxed{-1} & 0 \\ -i & i & \boxed{0} & 1 \end{bmatrix} \to 2\cdot(-1)^4 \cdot \begin{vmatrix} 3 & -i & i \\ 2 & 1 & 0 \\ -i & i & 1 \end{vmatrix} + 1\cdot(-1)^5 \cdot \begin{vmatrix} i & 3 & -i \\ 2 & 1 & 0 \\ -i & i & 1 \end{vmatrix} + (-1)\cdot(-1)^6 \cdot \begin{vmatrix} i & 3 & -i \\ 3 & -i & i \\ -i & i & 1 \end{vmatrix} + 0$

$2(3+3i^2+2i) + (-1)(i-2i^2-6) + (-1)(-7i^2-9) =$

$6+6i^2+4i - i + 3i^2 + 6 + 7i^2 + 9$

$\boxed{16i^2+3i+21}$

$\begin{vmatrix} 3 & -i & i \\ 2 & 1 & 0 \\ -i & i & 1 \end{vmatrix}\begin{matrix} 3 & -i \\ 2 & 1 \\ -i & i \end{matrix} \to 3+0+2i^2-(-i^2+0-2i)$

$3+2i^2+i^2+2i =$

$3+3i^2+2i$

$\begin{vmatrix} i & 3 & -i \\ 2 & 1 & 0 \\ -i & i & 1 \end{vmatrix}\begin{matrix} i & 3 \\ 2 & 1 \\ -i & i \end{matrix} \to i+0-2i^2-(i^2+0+6)$

$i-3i^2-6$

$\begin{vmatrix} i & 3 & -i \\ 3 & -i & i \\ -i & i & 1 \end{vmatrix}\begin{matrix} i & 3 \\ 3 & -i \\ -i & i \end{matrix} \to -i^2-3i^2-3i^2-(\cancel{-i^3}+\cancel{i^3}+9)$

$-7i^2-9$

c) $\det(2A) = 2\cdot\det(A)$ → Falso

$2\cdot\det(A) = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \to 2\cdot(2-6) =$

$2\cdot(-4) = \boxed{-8}$

$\det(2A) = \begin{bmatrix} 2 & 4 \\ 6 & 4 \end{bmatrix} = 6-24 =$

$\boxed{-16}$

d) $\det(A^2) = (\det A)^2$ → Verdadeiro

$\det(A^2) = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \to \det(A^2) = \begin{bmatrix} 1+6 & 2+4 \\ 3+6 & 6+4 \end{bmatrix} \to \det(A^2) = \begin{bmatrix} 7 & 6 \\ 9 & 10 \end{bmatrix} = 70-54$

$= \boxed{16}$

$(\det(A))^2 = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \to (\det(A))^2 = 2-6 \to \det(A) = (-4)^2 = \boxed{16}$

e) $\text{Det } A_{ij} < \det A$ → Falso

$\det(A) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \to \begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}\begin{matrix} 1 & 2 \\ 2 & 1 \\ 1 & 0 \end{matrix} \to 1+0+0-(3+0+4)$

$= 1-7$

$\boxed{-6}$

$\det(A_{12}) = \begin{bmatrix} \cancel{1} & \boxed{\cancel{2}} & \cancel{3} \\ 2 & \cancel{1} & 0 \\ 1 & \cancel{0} & 1 \end{bmatrix} \to \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 2-0$

$= \boxed{2}$

$\left.\right\} A_{ij} = 2 > A = -6$

f) Verdadeiro

$\begin{vmatrix} \boxed{1 \quad 0 \quad 1} \\ 2 \quad 2 \quad 1 \end{vmatrix} \to 1\cdot(-1)^2 \begin{vmatrix} 2 & 1 \\ 3 & 7 \end{vmatrix} + 0\cdot(-1)^3 \cancel{\begin{vmatrix} 2 & 1 \\ 3 & 7 \end{vmatrix}} + 1\cdot(-1)^4 \begin{vmatrix} 2 & 2 \\ 3 & 3 \end{vmatrix}$

$$\begin{vmatrix} 3 & 3 & 7 \end{vmatrix} \qquad 1.(14-3) + 1.(6-6) = \boxed{11}$$

$$\begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ 3 & 3 & 7 \end{vmatrix} \to 2.(-1)^2 . \begin{vmatrix} 0 & 1 \\ 3 & 7 \end{vmatrix} + 2.(-1)^3 . \begin{vmatrix} 1 & 1 \\ 3 & 7 \end{vmatrix} + 1.(-1)^4 . \begin{vmatrix} 1 & 0 \\ 3 & 3 \end{vmatrix}$$

$$2(0-3) + 2.(-1)(\overset{4}{\cancel{7}}-3) + 1(3-0)$$

$$-6 - 8 + 3 = \boxed{11}$$

9 Calcule A^{-1}

a)

$$A = \left[\begin{array}{cccc|cccc} 4 & -1 & 2 & -2 & 1 & 0 & 0 & 0 \\ 3 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 7 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cccc|cccc} 1 & -1/4 & 1/2 & -1/2 & 1/4 & 0 & 0 & 0 \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \end{array}\right]$$

$L_1 = \frac{L_1}{4}$

$L_2 = L_2 - 4.L_1$

$L_3 = L_3 - 2.L_1$

9) $A = \left[\begin{array}{cccc|cccc} 4 & -1 & 2 & -2 & 1 & 0 & 0 & 0 \\ 3 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 7 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$ $L1 \to \frac{1}{4} L1$ N

A^{-1}?

$$\left[\begin{array}{cccc|cccc} 1 & -\frac{1}{4} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 \\ 3 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 7 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad L2 \to 3L1 - L2 \text{ N}$$

$$\left[\begin{array}{cccc|cccc} 1 & -\frac{1}{4} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{3}{2} & -\frac{3}{2} & \frac{3}{4} & -1 & 0 & 0 \\ 2 & 3 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 7 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad L3 \to 2L1 - L3 \text{ N}$$

$$\left[\begin{array}{cccc|cccc} 1 & -\frac{1}{4} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{3}{2} & -\frac{3}{2} & \frac{3}{4} & -1 & 0 & 0 \\ 0 & -7/2 & 0 & -1 & \frac{1}{2} & 0 & -1 & 0 \\ 0 & 7 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad L_2 \to 4L_2 \; N$$

$$\left[\begin{array}{cccc|cccc} 1 & -\frac{1}{4} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 1 & 6 & -6 & 3 & -4 & 0 & 0 \\ 0 & -\frac{7}{2} & 0 & -1 & \frac{1}{2} & 0 & -1 & 0 \\ 0 & 7 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} L_1 \to L_1 + \left(\frac{1}{4}\right) \cdot L_2 \\ \\ L_3 \to \frac{7}{2} L_2 + L_3 \\ L_4 \to 7L_2 - L_4 \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & -2 & 1 & -1 & 0 & 0 \\ 0 & 1 & 6 & -6 & 3 & -4 & 0 & 0 \\ 0 & 0 & 21 & -22 & 11 & -14 & -1 & 0 \\ 0 & 0 & 41 & -43 & 21 & -28 & 0 & -1 \end{array}\right] \quad \begin{array}{l} L_3 \to L_3 \cdot \frac{1}{21} \\ \\ L_4 \to (-1) L_4 \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & -2 & 1 & -1 & 0 & 0 \\ 0 & 1 & 6 & -6 & 3 & -4 & 0 & 0 \\ 0 & 0 & 1 & -\frac{22}{21} & \frac{11}{21} & -\frac{2}{3} & -\frac{1}{21} & 0 \\ 0 & 0 & -41 & +43 & -21 & +28 & 0 & +1 \end{array}\right] \quad \begin{array}{l} L_1 \to L_1 - 2L_3 \; N \\ L_2 \to L_2 - 6L_3 \\ \\ L_4 \to L_3 \cdot 41 + L_4 \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac{2}{21} & -\frac{1}{21} & +1/3 & \frac{2}{21} & 0 \\ 0 & 1 & 0 & \frac{2}{7} & -\frac{1}{7} & 0 & \frac{2}{7} & 0 \\ 0 & 0 & 1 & -\frac{22}{21} & \frac{11}{21} & -\frac{2}{3} & -\frac{1}{21} & 0 \\ 0 & 0 & 0 & \frac{1}{21} & \frac{10}{21} & \frac{2}{3} & -\frac{41}{21} & 1 \end{array}\right] \quad \begin{array}{l} L_1 \to L_1 - 2L_4 \; N \\ L_2 \to L_2 - 6L_4 \\ L_3 \to L_3 + 22L_4 \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & -1 & 4 & -2 \\ 0 & 1 & 0 & 0 & -3 & -4 & 12 & -6 \\ 0 & 0 & 1 & 0 & 11 & 14 & -43 & 22 \\ 0 & 0 & 0 & \frac{1}{21} & \frac{10}{21} & \frac{2}{3} & -\frac{41}{21} & 1 \end{array}\right] \quad L_4 \to L_4 \cdot 21 \; N$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & -1 & 4 & -2 \\ 0 & 1 & 0 & 0 & -3 & -4 & 12 & -6 \\ 0 & 0 & 1 & 0 & 11 & 14 & -43 & 22 \\ 0 & 0 & 0 & 1 & 10 & 14 & -41 & 21 \end{array}\right] \Rightarrow \boxed{A^{-1} = \begin{bmatrix} -1 & -1 & 4 & -2 \\ -3 & -4 & 12 & -6 \\ 11 & 14 & -43 & 22 \\ 10 & 14 & -41 & 21 \end{bmatrix}}$$

2) b)

$$\left[\begin{array}{cccc|cccc} 1 & -1 & i & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & -i & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] L_1 \to L_2 \sim \left[\begin{array}{cccc|cccc} 0 & -i & -2 & i & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & -i & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$L_4 \to L_1 - L_4 \sim \left[\begin{array}{cccc|cccc} 1 & -1 & i & 1 & 0 & 1 & 0 & 0 \\ 0 & -i & -2 & i & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & -i & 0 & 0 & 1 & 0 \\ 0 & -2 & (i-1) & 1 & 0 & 1 & 0 & -1 \end{array}\right] L_2 \to \left(-\frac{1}{i}\right) L_2 \sim$$

$$\left[\begin{array}{cccc|cccc} 1 & -1 & i & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & (-2i) & -1 & i & 0 & 0 & 0 \\ 0 & -1 & 1 & -i & 0 & 0 & 1 & 0 \\ 0 & -2 & (i-1) & 1 & 0 & 1 & 0 & -1 \end{array}\right] \begin{array}{l} L_1 \to L_1 + L_2 \sim \\ \\ L_3 \to L_2 + L_3 \\ L_4 \to 2L_2 + L_4 \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & -i & 0 & i & 1 & 0 & 0 \\ 0 & 1 & -2i & -1 & i & 0 & 0 & 0 \\ 0 & 0 & (1-2i) & (-1-i) & i & 0 & 1 & 0 \\ 0 & 0 & (-1-3i) & -1 & 2i & -1 & 1 & -1 \end{array}\right]$$

$$L_3 \to [1/(1-2i)] \cdot L_3 \sim \left[\begin{array}{cccc|cccc} 1 & 0 & -i & 0 & i & 1 & 0 & 0 \\ 0 & 1 & -2i & -1 & i & 0 & 0 & 0 \\ 0 & 0 & 1 & \left(\frac{1}{5}-\frac{3}{5}i\right) & \left(-\frac{2}{5}+\frac{1}{5}i\right) & 0 & \left(\frac{1}{5}+\frac{2}{5}i\right) & 0 \\ 0 & 0 & (-1-3i) & -1 & 2i & -1 & 1 & -1 \end{array}\right] \begin{array}{l} L_1 \to \\ L_1 + iL_3 \\ L_2 \to \\ L_2 + 2iL_3 \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \left(\frac{3}{5}+\frac{1}{5}i\right) & \frac{1}{5}(3+i) & 1 & (-2+i)/5 & 0 \\ 0 & 1 & 0 & (1+2i)/5 & (i-2)/5 & 0 & (2i-4)/5 & 0 \\ 0 & 0 & 1 & \left(\frac{1}{5}-\frac{3}{5}i\right) & \left(-\frac{2}{5}+\frac{1}{5}i\right) & 0 & \left(\frac{1}{5}+\frac{2}{5}i\right) & 0 \\ 0 & 0 & (-1-3i) & -1 & 2i & -1 & 1 & -1 \end{array}\right] L_4 \to L_3(-1-3i) - L_4 \sim$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac{3+i}{5} & \frac{1}{5}(3+i) & 1 & (-2+i)/5 & 0 \\ 0 & 1 & 0 & \frac{1+2i}{5} & \frac{i-2}{5} & 0 & \frac{2i-4}{5} & 0 \\ 0 & 0 & 1 & \frac{1-3i}{5} & \frac{-2+i}{5} & 0 & \frac{1+2i}{5} & 0 \\ 0 & 0 & 0 & -1 & 1-i & -1 & 1-i & 1 \end{array}\right] L_4 \to (-1)L_4 \sim$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac{3+i}{5} & \frac{i}{5}(3+i) & 1 & \frac{-2+i}{5} & 0 \\ 0 & 1 & 0 & \frac{1+2i}{5} & \frac{-2+i}{5} & 0 & \frac{-1+2i}{5} & 0 \\ 0 & 0 & 1 & \frac{1-3i}{5} & \frac{-2+i}{5} & 0 & \frac{1+2i}{5} & 0 \\ 0 & 0 & 0 & 1 & -1+i & 1 & -1+i & -1 \end{array}\right] \begin{array}{l} L_1 \to L_1 + L_4\left(\frac{3+i}{5}\right)(\\ L_2 \to L_2 + L_4\left(\frac{1-2i}{5}\right)(- \\ L_3 \to \left(\frac{3i-1}{5}\right) L_4 + L \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{5}(3+i) & \frac{2-i}{5} & \frac{2-i}{5} & \frac{3+i}{5} \\ 0 & 1 & 0 & 0 & \frac{1+2i}{5} & \frac{-1-2i}{5} & \frac{-1+3i}{5} & \frac{1+2i}{5} \\ 0 & 0 & 1 & 0 & \frac{-4-3i}{5} & \frac{-1+3i}{5} & \frac{-1-2i}{5} & \frac{1-3i}{5} \\ 0 & 0 & 0 & 1 & -1+i & 1 & -1+i & -1 \end{array}\right]$$

$$\Rightarrow A^{-1} = \begin{bmatrix} (3i-1)/5 & (2-i)/5 & (2-i)/5 & (3+i)/5 \\ (1+2i)/5 & (-1-2i)/5 & (-1+3i)/5 & (1+2i)/5 \\ (-4-3i)/5 & (-1+3i)/5 & (-1-2i)/5 & (1-3i)/5 \\ -1+i & 1 & -1+i & -1 \end{bmatrix}$$

9) c) $\left[\begin{array}{ccc|ccc} 1 & 0 & x & 1 & 0 & 0 \\ 1 & 1 & x^2 & 0 & 1 & 0 \\ 2 & 2 & x^2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} L_2 \to L_2 - L_1 \\ L_3 \to L_3 - 2L_1 \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & x & 1 & 0 & 0 \\ 0 & 1 & x^2-x & -1 & 1 & 0 \\ 0 & 2 & x^2-2x & -2 & 0 & 1 \end{array}\right] L_3 \to L_3 - 2L_2$

$\left[\begin{array}{ccc|ccc} 1 & 0 & x & 1 & 0 & 0 \\ 0 & 1 & x^2-x & -1 & 1 & 0 \\ 0 & 0 & -x^2 & 0 & -2 & 1 \end{array}\right] L_3 \to \left(\frac{-1}{x^2}\right) L_3 \sim \left[\begin{array}{ccc|ccc} 1 & 0 & x & 1 & 0 & 0 \\ 0 & 1 & (x^2-x) & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & \frac{2}{x^2} & \left(\frac{-1}{x^2}\right) \end{array}\right] \begin{array}{l} L_1 \to L_1 - xL_3 \\ L_2 \to L_2 - L_3(x^2-x) \end{array}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -\frac{2}{x} & \frac{1}{x} \\ 0 & 1 & 0 & -1 & \left(-1+\frac{2}{x}\right) & 1-\frac{1}{x} \\ 0 & 0 & 1 & 0 & \frac{2}{x^2} & -\frac{1}{x^2} \end{array}\right] \Rightarrow \boxed{A^{-1} = \begin{bmatrix} 1 & -\frac{2}{x} & \frac{1}{x} \\ -1 & -1+\frac{2}{x} & 1-\frac{1}{x} \\ 0 & \frac{2}{x^2} & -\frac{1}{x^2} \end{bmatrix}}$

13) $\det \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix} = (a-b)(b-c)(c-a)$

$(a-b)(b-c)(c-a) = (ab - ac - b^2 + bc)(c-a) = abc - ac^2 - b^2c + bc^2 - a^2b + a^2c + ab^2 = \ldots$

$= \boxed{-ac^2 - b^2c + bc^2 - a^2b + a^2c + ab^2}$

$\left[\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ a & b & c & a & b \\ a^2 & b^2 & c^2 & a^2 & b^2 \end{array}\right]$ $\det = bc^2 + ca^2 + ab^2 - a^2b - b^2c - c^2a$

$\boxed{\det = -ac^2 - b^2c + bc^2 - a^2b + a^2c + ab^2}$

$\boxed{\text{Logo: } \det \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix} = (a-b)(b-c)(c-a)}$

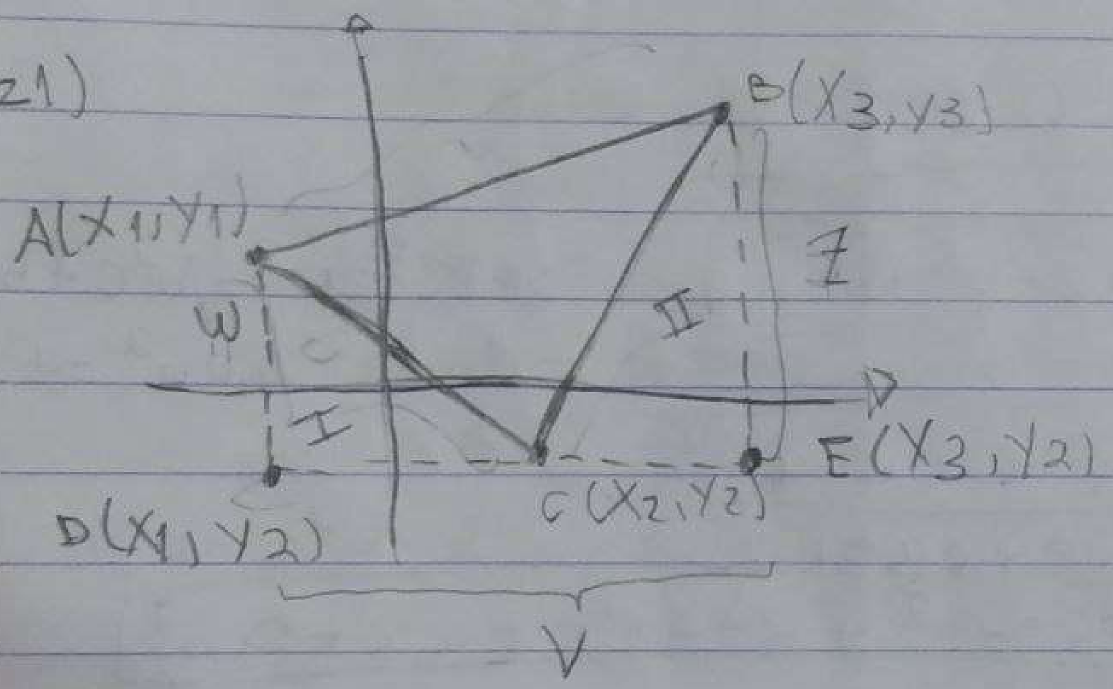
Dado que: $B = P^{-1}AP$, então:

$\det(B) = \det(P^{-1} \cdot A \cdot P) \Rightarrow$

$\Rightarrow \det(B) = \det(P^{-1}) \cdot \det(P) \cdot \det(A) \Rightarrow$

$\Rightarrow \boxed{\det(B) = \det(A)}$

21)



$$A_T = (Z + W) \cdot V \cdot \frac{1}{2} - A_I - A_{II}$$

$$A_T = [(Y_3 - Y_2) + (Y_1 - Y_2)][X_3 - X_1] \cdot \frac{1}{2}$$

$$- [X_2 - X_1][Y_1 - Y_2] \cdot \frac{1}{2} -$$

$$- [Y_3 - Y_2][X_3 - X_2] \frac{1}{2}$$

$$\frac{1}{2}\left|\begin{matrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{matrix}\;\begin{matrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{matrix}\right| = \frac{1}{2}\left[x_1y_2 + y_1x_3 + x_2y_3 - x_3y_2 - y_3x_1 - x_2y_1\right]$$

Determinante $= \boxed{\frac{x_1y_2}{2} + \frac{y_1x_3}{2} + \frac{x_2y_3}{2} - \frac{x_3y_2}{2} - \frac{y_3x_1}{2} - \frac{x_2y_1}{2}}$

$$A_T = [(Y_3 - Y_2 + Y_1 - Y_2)(X_3 - X_1)] \cdot \frac{1}{2} - [X_2 - X_1][Y_1 - Y_2]$$

$$- [Y_3 - Y_2][X_3 - X_2] \frac{1}{2}$$

$$A_T = [Y_3 - 2Y_2 + Y_1](X_3 - X_1) \cdot \frac{1}{2} - (X_2Y_1 - X_2Y_2 - X_1Y_1 + X_1Y_2) \frac{1}{2}$$

$$- (Y_3X_3 - Y_3X_2 - Y_2X_3 + Y_2X_2] \frac{1}{2}$$

$$A_T = \left[\cancel{\frac{Y_3X_3}{2}} - \frac{2Y_2X_3}{2} + \frac{Y_1X_3}{2} - \frac{Y_3X_1}{2} + \frac{2Y_2X_1}{2} - \cancel{\frac{Y_1X_1}{2}}\right] +$$

$$- \cancel{\frac{X_2Y_1}{2}} + \cancel{\frac{X_2Y_2}{2}} + \cancel{\frac{X_1Y_1}{2}} - \frac{X_1Y_2}{2} - \cancel{\frac{Y_3X_3}{2}} + \frac{Y_3X_2}{2} + \frac{Y_2X_3}{2} - \cancel{\frac{Y_2X_2}{2}}$$

$$A_T = - \frac{2Y_2X_3}{2} + \frac{Y_2X_3}{2} + \frac{Y_1X_3}{2} - \frac{Y_3X_1}{2} + \frac{Y_2X_1}{2} - \frac{Y_1X_2}{2} + \frac{Y_3X_2}{2}$$

$$A_T = - \frac{Y_2X_3}{2} + \frac{Y_1X_3}{2} - \frac{Y_3X_1}{2} + \frac{Y_2X_1}{2} - \frac{Y_1X_2}{2} + \frac{Y_3X_2}{2}$$

$$\boxed{A_T = \frac{X_1Y_2}{2} + \frac{Y_1X_3}{2} + \frac{X_2Y_3}{2} - \frac{X_3Y_2}{2} - \frac{Y_3X_1}{2} - \frac{X_2Y_1}{2}}$$

$\therefore$ A área é igual o Determinante