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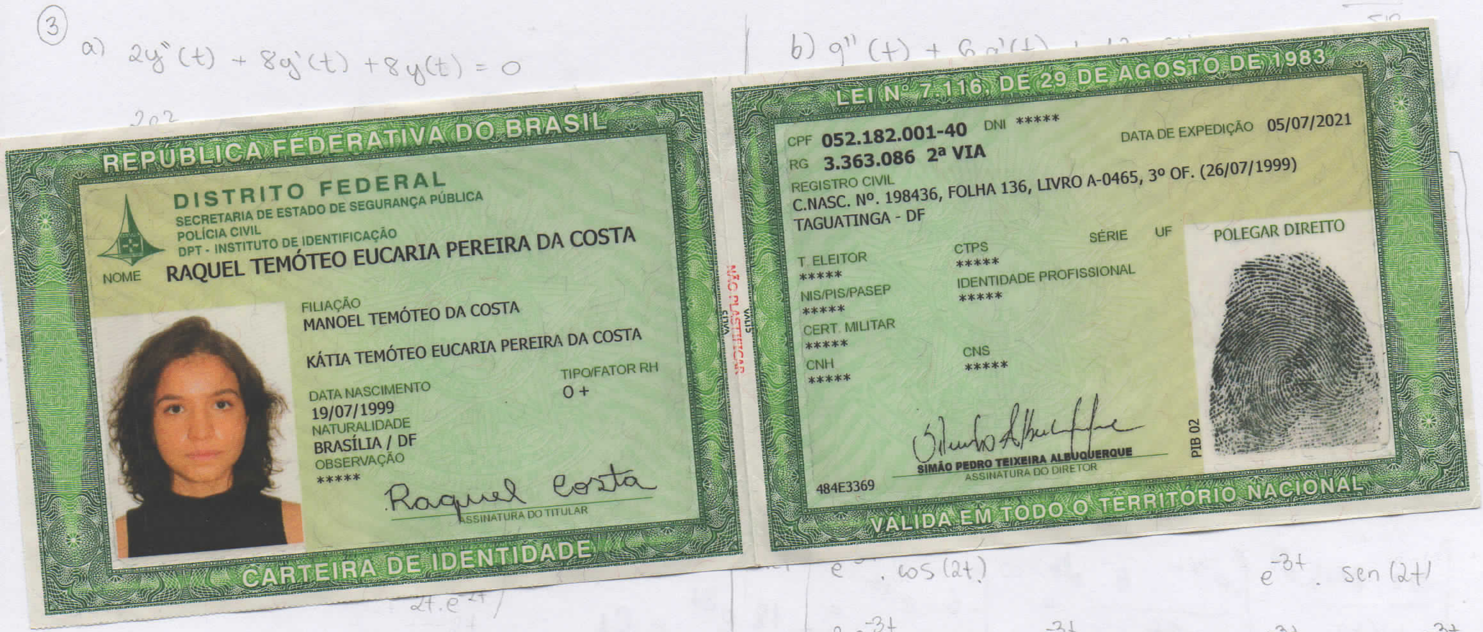
③ a) $2y''(t) + 8y'(t) + 8y(t) = 0$



$2t.e^{-2t})$

$C_1 = \int \frac{-y_2 \cdot g(t)}{W(t)} \, dt = \int \frac{t \cdot e^{-2t} \cdot 8}{e^{-4t}} \, dt = 8 \int t \cdot e^{2t} \, dt$

$u = t \quad du = 1$

$dv = e^{2t} \quad v = \frac{e^{2t}}{2}$

$= 8\left(t \cdot \frac{e^{2t}}{2} - \int \frac{e^{2t}}{2} \cdot 1 \, dt\right) = 8\left(t \cdot \frac{e^{2t}}{2} - \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{} e^{2t} + C_1\right)$

$= 8\left(t \cdot \frac{e^{2t}}{2} - e^{2t} + C_1\right) = 8 \cdot e^{2t}\left(\frac{t}{2} - 1\right) + 8C_1$

$C_2 = \int \frac{y_1 \cdot g(t)}{W(t)} \, dt = \int \frac{e^{-2t} \cdot 8}{e^{-4t}} \, dt = 8 \int e^{2t} \, dt = 16\, e^{2t} + 8C_2$

$y_g = \left(8\, e^{2t}\left(\frac{t}{2} - 1\right) + 8C_1\right) \cdot \left(e^{-2t}\right) + \left(16 \cdot e^{2t} + 8C_2\right)\left(t \cdot e^{-2t}\right)$

$y_g = 8 \cdot \frac{t \cdot e^{-2t}}{2} - e^{-2t} + 8 \cdot C_1 \cdot e^{-2t} + 16 + 8C_2 \cdot t \cdot e^{-2t}$

$(\div 8)$

$y_g = \frac{t \cdot e^{-2t}}{2} - \frac{e^{-2t}}{8} + C_1 \cdot e^{-2t} + \frac{8}{1} + C_2 \cdot t \cdot e^{-2t}$

$\boxed{y_g = \frac{8t \cdot e^{-2t} - 2e^{-2t}}{16} + 8 + C_1 \cdot e^{-2t} + C_2 \cdot t \cdot e^{-2t}}$

b) $y''(t) + 6y'(t) + 13 \ldots$

$\begin{vmatrix} e^{-3t} \cdot \cos(2t) & e^{-3t} \cdot sen(2t) \\ -3e^{-3t} \cdot \cos(2t) - e^{-3t} \cdot sen(2t) & -3e^{-3t} \cdot sen(2t) + e^{-3t} \cdot \cos(2t) \end{vmatrix}$

$\cancel{-3e^{-6t} \cdot tg(2t)} + e^{-6t} \cdot \cos^2(2t) -$

$(\cancel{-3e^{-6t} \cdot tg(2t)} - e^{-6t} \cdot sen^2(2t)) =$

$e^{-6t} \cdot \cos^2(2t) + e^{-6t} \cdot sen^2(2t)$

$e^{-6t}\left(\overbrace{\cos^2(2t) + sen^2(2t)}^{1}\right) =$

$W(t) = e^{-6t}$

$C_1 = \int \frac{-y_2 \cdot g(t)}{e^{-6t}} \, dt = 13 \int \frac{-e^{-3t} \cdot sen(2t)}{e^{-6t}} = 13 \int -e^{3t} \cdot sen(2t)$

$u = t \rightarrow du = 1$

$= 13 \int 0 \cdot du = 13C_1$

$C_2 = \int \frac{y_1 \cdot g(t)}{W(t)} \, dt = \int \frac{e^{-3t} \cdot \cos(2t) \cdot 13}{e^{-6t}} \, dt$

$= 13 \int e^{3t} \cdot \cos(2t) \, dt = t + C_2$

$y_g = C_1 \cdot e^{-3t} \cdot \cos(2t) + (t + C_2)(e^{-3t} \cdot sen(2t))$

$\boxed{y_g = t \cdot e^{-3t} \cdot sen(2t) + C_1 \cdot e^{-3t} \cdot \cos(2t) + C_2 \cdot e^{-3t} \cdot sen(2t)}$

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a) $2y''(t) + 8y'(t) + 8y(t) = 0$

$2R^2 + 8R + 8 = 0$

$\Delta = b^2 - 4.a.c \rightsquigarrow y_1(t) = e^{Rt} \; ; \; y_2(t) = t.\, e^{Rt}$

$8^2 - 4.2.8$

$64 - 64 = 0$

$y_1(t) = e^{-2t} \; , \; y_2(t) = t.\, e^{-2t}$

$R = \frac{-8}{2.2} = -2$

$y(t) = C_1 . e^{-2t} + C_2 . t . e^{-2t}$

$$w(t) = \det \begin{pmatrix} e^{-2t} & t.e^{-2t} \\ -2.e^{-2t} & e^{-2t} - 2t.e^{-2t} \end{pmatrix} = e^{-4t}$$

$$C_1 = \int \frac{-y_2 . g(t)}{w(t)} . dt = \int \frac{t.e^{-2t} . 8}{e^{-4t}} dt = 8 \int t . e^{2t} . dt$$

$u = t \quad du = 1$

$dv = e^{2t} \quad v = \frac{e^{2t}}{2}$

$$= 8\left(t.\frac{e^{2t}}{2} - \int \frac{e^{2t}}{2} . 1\, dt\right) = 8\left(t.\frac{e^{2t}}{2} - \frac{1}{2} . 2e^{2t} + C_1\right)$$

$$= 8\left(t.\frac{e^{2t}}{2} - e^{2t} + C_1\right) = 8.e^{2t}\left(\frac{t}{2} - 1\right) + 8C_1$$

$$C_2 = \int \frac{y_1 . g(t)}{w(t)} dt = \int \frac{e^{-2t} . 8}{e^{-4t}} dt = 8 \int e^{2t} . dt = 16\, e^{2t} + 8C_2$$

$$y_g = \left(8\, e^{2t}\left(\frac{t}{2} - 1\right) + 8C_1\right) . \left(e^{-2t}\right) + \left(16 . e^{2t} + 8C_2\right)\left(t . e^{-2t}\right)$$

$$y_g = 8 . \frac{t.e^{-2t}}{2} - e^{-2t} + 8.C_1.e^{-2t} + 16 + 8C_2.t.e^{-2t}$$

$(\div 8)$

$$y_g = \frac{t.e^{-2t}}{2} - \frac{e^{-2t}}{8} + C_1.e^{-2t} + \frac{8}{1} + C_2.t.e^{-2t}$$

$$y_g = \frac{8t.e^{-2t} - 2e^{-2t}}{16} + 8 + C_1.e^{-2t} + C_2.t.e^{-2t}$$

b) $q''(t) + 6q'(t) + 13q(t) = 0$

$R^2 + 6R + 13 = 0$

$\Delta = 36 - 4.1.13$

$36 - 52 = -16$

$R = \frac{-6 \pm 4i}{2}$

$R = -3 \pm 2i$

$y_1(t) = e^{-3t} . \cos(2t) . C_1 +$

$y_2(t) = e^{-3t} . \text{sen}(2t) . C_2$

$w(t) =$

$$\det \begin{pmatrix} e^{-3t} . \cos(2t) & e^{-3t} . \text{sen}(2t) \\ -3e^{-3t} . \cos(2t) - e^{-3t} . \text{sen}(2t) & -3e^{-3t} . \text{sen}(2t) + e^{-3t} . \cos(2t) \end{pmatrix}$$

$$\cancel{-3\, e^{-6t} . tg(2t)} + e^{-6t} . \cos^2(2t) -$$

$$\left(\cancel{-3\, e^{-6t} . tg(2t)} - e^{-6t} . \text{sen}^2(2t)\right) =$$

$$e^{-6t} . \cos^2(2t) + e^{-6t} . \text{sen}^2(2t)$$

$$e^{-6t}\left(\underbrace{\cos^2(2t) + \text{sen}^2(2t)}_{1}\right) =$$

$$w(t) = e^{-6t}$$

$$C_1 = \int \frac{-y_2 . g(t)}{e^{-6t}} . dt = 13 \int \frac{-e^{-3t} . \text{sen}(2t)}{e^{-6t}} = 13 \int -e^{3t} . \text{sen}(2t)$$

$u = t \rightarrow du = 1$

$$= 13 \int 0 . du = 13C_1$$

$$C_2 = \int \frac{y_1 . g(t)}{w(t)} . dt = \int \frac{e^{-3t} . \cos(2t) . 13}{e^{-6t}} . dt$$

$$= 13 \int e^{3t} . \cos(2t) . dt = t + C_2$$

$$y_g = C_1 . e^{-3t} . \cos(2t) + (t + C_2)\left(e^{-3t} . \text{sen}(2t)\right)$$

$$y_g = t . e^{-3t} . \text{sen}(2t) + C_1 . e^{-3t} . \cos(2t) + C_2 . e^{-3t} . \text{sen}(2t)$$

$13 \times 4 = 52$

$52 - 36 = 16$

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1 c) $y''(t) + 5y'(t) + 6y(t) = 0$

$R^2 + 5R + 6 = 0$

$y_1(t) = e^{-3t}$, $y_2(2) = e^{-2t}$

$\Delta = 25 - 4 \cdot 1 \cdot 6 = 1$

$R = \dfrac{-5 \pm 1}{2}$ → $R_1 = -3$; → $R_2 = -2$

$W(t) = \det \begin{pmatrix} e^{-3t} & e^{-2t} \\ -3e^{-3t} & -2e^{-2t} \end{pmatrix} = -2e^{-5t} + 3e^{-5t} = e^{-5t}$

$C_1 = \int \dfrac{-y_2 \cdot g(t)}{w(t)} \cdot dt = \int \dfrac{-e^{-2t} \cdot 6}{e^{-5t}} \cdot dt = \int -6 \cdot e^{3t} = -18 e^{3t} + C_1$

$C_2 = \int \dfrac{y_1 \cdot g(t)}{W(t)} \, dt = \int \dfrac{e^{-3t} \cdot 6}{e^{-5t}} = \int 6 \cdot e^{2t} = 12 e^{2t} + C2$

$y_g = (-18 e^{3t} + C_1) e^{-3t} + (12 e^{2t} + C2) e^{-2t}$

$y_g = -18 + C_1 \cdot e^{-3t} + 12 + C_2 \cdot e^{-2t}$

$\boxed{y_g = -6 + C_1 \cdot e^{-3t} + C_2 \cdot e^{-2t}}$

2) $(1-x^2)\,y''(x) - x y'(x) + 16\,y(x) = 0$

a) $16\,y(x) = \sum_{n=0}^{\infty} 16\, C_n . x^n$

$-x\,y'(x) = \sum_{n=0}^{\infty} -C_n.(n).x^n$

$(1-x^2)\,y''(x) = \sum_{n=0}^{\infty}\left(C_{n+2}(n+2)(n+1)\,x^n - C_n\,(n).(n-1).\,x^n\right)$

b) $\left(C_{n+2}(n+2)(n+1) - C_n(n)(n-1) - C_n(n) + 16\,C_n\right)x^n = 0$

$C_{n+2}\,(n+2)(n+1) = C_n(n)(n-1) + C_n(n) - 16\,C_n$

$$C_{n+2} = \frac{C_n\left(\overbrace{n(n-1)}^{n^2-n} + \cancel{n} - 16\right)}{(n+2)(n+1)}$$

$$\boxed{C_{n+2} = \frac{(n^2-16)}{(n+2)(n+1)} . C_n}$$

$C_2 = C_{0+2} = \frac{0^2-16}{(2)(1)} . 1 = -8$

$C_4 = C_{2+2} = \frac{2^2-16}{(4)(3)} = \frac{-12}{12} = -1$

$C_6 = C_{4+2} = \frac{4^2-16}{(6)(5)} = 0$

d)

$y_1(x) = 1 + 0 + C_2 x^2 + C_3 x^3 + \dots$

$C_0 = 1 \to C_2 = -8 \to C_4 = -1 \to C_6 = 0 \dots = 0$

$C_1 = 0 \to 0 \dots$

$\boxed{y_1(x) = 1 - 8x^2 - x^4}$

e) função par

c) $y_2(x) = 0 + 1 + C_2 x^2 + C_3 x^3 + \dots$

$C_0 = 0 \to 0 \dots$

$C_1 = 1 \to \neq 4 \to$ Não zera $\therefore$ $y_2(x)$ não é um polinômio

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$\int -e^{3t} \cdot sen(2t) \rightarrow \int -e^{1} \cdot \cancel{sen(0)}^{0} = \int 0 = C_1$

$u = t \rightarrow du = 1$

$\int e^{3t} \cdot cos(2t) \rightarrow \int \overbrace{e^{3.1} \cdot cos(2.1)}^{const.} du = t + C_2$

$u = t \Rightarrow du = 1$

$C_1 \cdot e$