

Dussapio (Ajudante 2)

P1 - 300

Calcule o seguinte limite:

$$\lim_{x \rightarrow 5} \frac{x^{200} - 25^{100}}{x^{100} - 5x^{99} + x - 5}$$

Tentando substituir o x por 5

$$\frac{5^{200} - 2 \cdot 5^{100}}{5^{100} - 5 \cdot 5^{99} + 5 - 5} =$$

$$\frac{5^{200} - (5^2)^{100}}{5^{100} - 5^{100} + 0} =$$

$$\frac{5^{200} - 5^{200}}{5^{100} - 5^{100}} = \frac{0}{0}$$

$\frac{0}{0}$ = Indeterminação

$$\begin{aligned} ax + bx + ay + by \\ x(a+b) + y(a+b) \\ (a+b) \cdot (x+y) \end{aligned}$$

$$\Rightarrow \lim_{x \rightarrow 5} \frac{x^{200} - 25^{100}}{x^{100} - 5x^{99} + x - 5} = \lim_{x \rightarrow 5} \frac{x^{200} - 5^{200}}{(x-5)(x^{99} + 1)} = \lim_{x \rightarrow 5} \frac{(x^{100})^2 - (5^{100})^2}{(x-5)(x^{99} + 1)}$$

$$= \lim_{x \rightarrow 5} \frac{(x^{100} - 5^{100})(x^{100} + 5^{100})}{(x-5)(x^{99} + 1)}$$

$$\begin{aligned} (x^{100} - 5x^{99}) + (x-5) \\ x^{99}(x-5) + (x-5) \\ (x-5)(x^{99} + 1) \end{aligned} = \begin{aligned} ax + a \\ \downarrow \\ a(x+1) \end{aligned}$$

$$= \lim_{x \rightarrow 5} \frac{(\cancel{x-5})(x^{99} \cdot 5^0 + x^{98} \cdot 5^1 + x^{97} \cdot 5^2 \dots x^0 \cdot 5^{99})(x^{100} + 5^{100})}{(\cancel{x-5})(x^{99} + 1)}$$

$$= \lim_{x \rightarrow 5} \frac{(x^{99} \cdot 5^0 + x^{98} \cdot 5^1 + x^{97} \cdot 5^2 \dots x^0 \cdot 5^{99})(x^{100} + 5^{100})}{x^{99} + 1}$$

$$= \lim_{x \rightarrow 5} \frac{\sum_{n=0}^{99} (x^{99-n} \cdot 5^n) \cdot (x^{100} + 5^{100})}{x^{99} + 1}$$

$$x^{99-n} = \frac{x^{99}}{x^n}$$

$$= \lim_{x \rightarrow 5} \frac{\sum_{n=0}^{99} \left(x^{99} \cdot \left(\frac{5}{x} \right)^n \right) \cdot (x^{100} + 5^{100})}{x^{99} + 1}$$

$$\frac{x^{99}}{x^n} \cdot 5^n = x^{99} \cdot \frac{5^n}{x^n} = \left(x^{99} \cdot \left(\frac{5}{x} \right)^n \right)$$

$$= \lim_{x \rightarrow 5} x^{99} \sum_{n=0}^{99} \left(\frac{5}{x}\right)^n \cdot \frac{(x^{100} + 5^{100})}{(x^{99} + 1)}$$

$$= 5^{99} \cdot \sum_{n=0}^{99} \left(\frac{5}{5}\right)^n \cdot \frac{(5^{100} + 5^{100})}{5^{99} + 1}$$

$$= 5^{99} \cdot \sum_{n=0}^{99} 1^n \cdot \frac{2 \cdot 5^{100}}{5^{99} + 1}$$

$$5^{100} + 5^{100} = 2(5^{100})$$

$$= \frac{5^{99} \cdot 100 \cdot 2 \cdot 5^{100}}{5^{99} + 1}$$

$$\sum_{n=0}^{99} 1^n = 0 \text{ a } 99 = 100 \text{ a } 1 \cdot 100 = 100$$

$$= \frac{5^{99} \cdot 200 \cdot 5^{100}}{5^{99} + 1} = \frac{5^{199} \cdot 200}{5^{99} + 1}$$

$$= \frac{200 \cdot 5^{199}}{5^{99} \left(1 + \frac{1}{5^{99}}\right)}$$

$$5^{99} + 1 = 5^{99} \left(1 + \frac{1}{5^{99}}\right)$$

$$5^{99} + \frac{1}{5^{99}} = 5^{99} + 1$$

$$= \frac{200 \cdot 5^{100}}{\frac{1}{5} + \frac{1}{5^{99}}} = \frac{200 \cdot 5^{100}}{1 + \approx 0}$$

$$\frac{1}{5^{99}} = 0,0000 \dots 1 = \approx 0$$

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$$= \frac{200 \cdot 5^{100}}{1} = \approx 200 \cdot 5^{100}$$

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Link de acesso:

<https://create.kahoot.it/share/calculo-1/30aca040-91d5-48e2-b7a1-87b2d25d48a8>