Assignment 2

Time Series Analysis 02417
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Note that all code and figures are in our GitHub repository.

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1 Stability

1.1 Determine if the process is stationary for $\phi_1 = -0.7$ and $\phi_2 = -0.2$ by analyzing the roots of the characteristic equation

The given AR(2) process can be rewritten as:

$$X_t = -\phi_1 X_{t-1} - \phi_2 X_{t-2} + \epsilon_t$$

The corresponding characteristic equation is:

$$\lambda^2 + 0.7\lambda + 0.2 = 0$$

Solving for the roots:

$$|\lambda_1| = 0.45, \quad |\lambda_2| = 0.45$$

Since the absolute values of the roots are within the unit circle ($|\lambda| < 1$), the process is **stationary**.

1.2 Is the process invertible?

An AR(p) process is always invertible because it is defined in terms of past values of the time series, rather than past noise terms. Therefore, there are no additional invertibility constraints for an AR(2) process.

1.3 Autocorrelation function $\rho(k)$ for the AR(2) process as function of ϕ_1 and ϕ_2.

Using the **Yule-Walker equations**, we obtain:

$$\rho(1) + \phi_1 + \phi_2\rho(1) = 0$$

$$\rho(2) + \phi_1\rho(1) + \phi_2 = 0$$

Solving for $\rho(1)$:

$$\rho(1) = \frac{-\phi_1}{1 + \phi_2}$$

Next, we calculate $\rho(2)$:

$$\rho(2) = -\phi_1\rho(1) - \phi_2$$

Substituting the given values $\phi_1 = -0.7$, $\phi_2 = -0.2$:

$$\rho(1) = -\frac{-0.7}{1 + (-0.2)} = \frac{0.7}{0.8} \approx 0.875$$

$$\rho(2) = -(-0.7) \times (-0.875) - (-0.2) = 0.4083 - 0.2 = -0.4125$$

1.4 Plot of the autocorrelation function $\rho(k)$ up to $n_{\mathbf{lag}} = 30$

The figure below illustrates the behavior of the autocorrelation function $\rho(k)$ within the lag range. As expected, the autocorrelation values gradually decay and approach zero as the lag increases, consistent with the theoretical properties of an AR(2) process. The values become close to zero after lag 2.

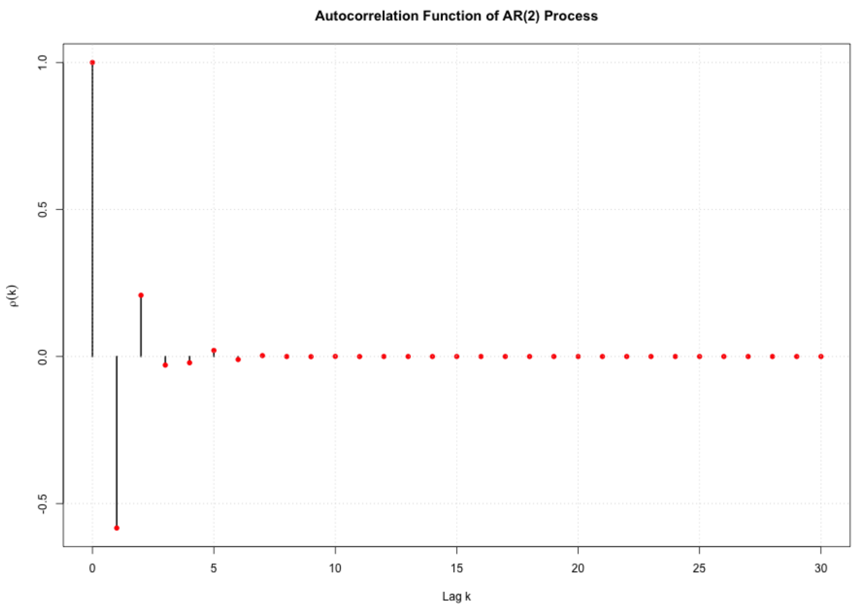

Figure 1: Autocorrelation function values for the AR(2) process

2 Simulating seasonal processes

A process $\{Y_t\}$ is said to follow a multiplicative $(p, d, q) \times (P, D, Q)_s$ seasonal model if

$$\phi(B)\Phi(B^s)\nabla^d\nabla_s^D Y_t = \theta(B)\Theta(B^s)\epsilon_t$$

where $\{\epsilon_t\}$ is a white noise process, and $\phi(B)$ and $\theta(B)$ are polynomials of order p and q, respectively. Furthermore, $\Phi(B^s)$ and $\Theta(B^s)$ are polynomials in B^s. All according to Definition 5.22 in the textbook.

Note: `arima.sim` does not have a seasonal module, so model formulations as standard ARIMA processes have to be made when using that function.

Simulate the following models. Plot the simulations and the associated autocorrelation functions (ACF and PACF). Comment on each result:

2.1 A $(1, 0, 0) \times (0, 0, 0)_{12}$ model with the parameter $\phi_1 = 0.6$

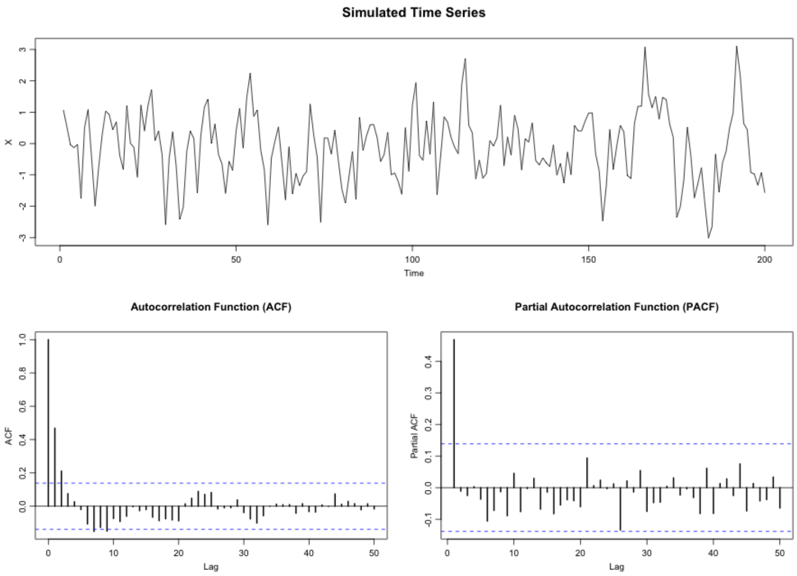

Figure 2: SARIMA Model (1,0,0) $\times$ (0,0,0)$_{12}$

The time series shows a smooth and persistent pattern, since each value seems to be influenced by the previous one. This happens because the short-term dependence is positive, so values tend to stay high or low for longer periods before changing. The autocorrelation function gradually decreases over time, which is typical for a process where each value depends on the one before it. The process mainly depends on the most recent value rather than on seasonal effects.

2.2 A $(0, 0, 0) \times (1, 0, 0)_{12}$ model with the parameter $\Phi_1 = -0.9$

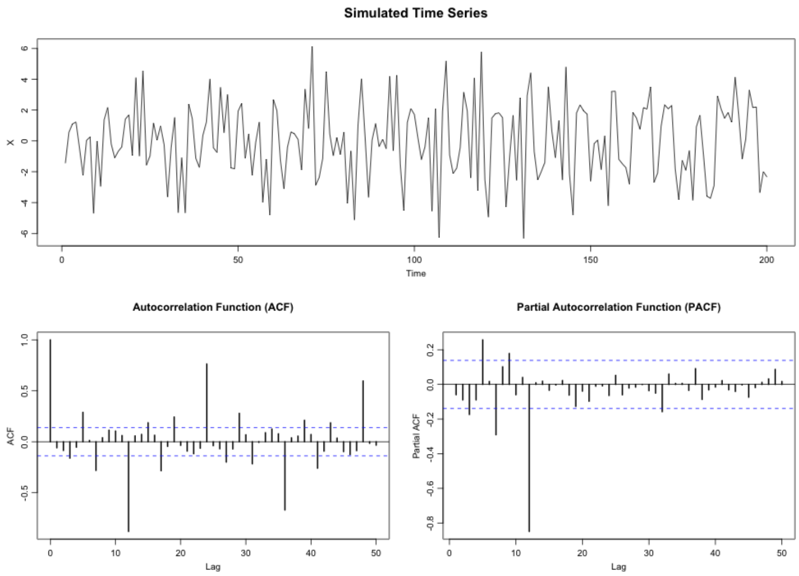

Figure 3: SARIMA Model (0,0,0) $\times$ $(1,0,0)_{12}$

The time series has a clear seasonal pattern because of the seasonal dependence at lag 12. Since the seasonal coefficient is negative, the values alternate sharply between high and low. The autocorrelation function shows a strong spike at lag 12, followed by a gradual decrease, which confirms the seasonal nature of our trend.

2.3 A $(1, 0, 0) \times (0, 0, 1)_{12}$ model with the parameters $\phi_1 = 0.9$ and $\Theta_1 = -0.7$

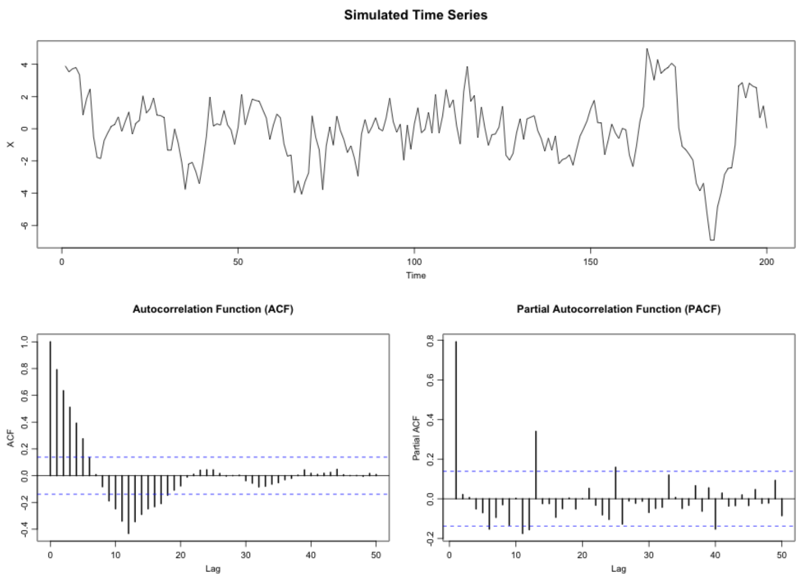

Figure 4: SARIMA Model (1,0,0) X $(0,0,1)_{12}$

This process has both short-term dependence and seasonal effects, leading to a time series with less dramatic spikes. The partial autocorrelation function has a strong spike at lag 1 and some weaker seasonal effects, which makes sense for this type of model.

2.4 A $(1,0,0) \times (1,0,0)_{12}$ model with the parameters $\phi_1 = -0.6$ and $\Phi_1 = -0.8$

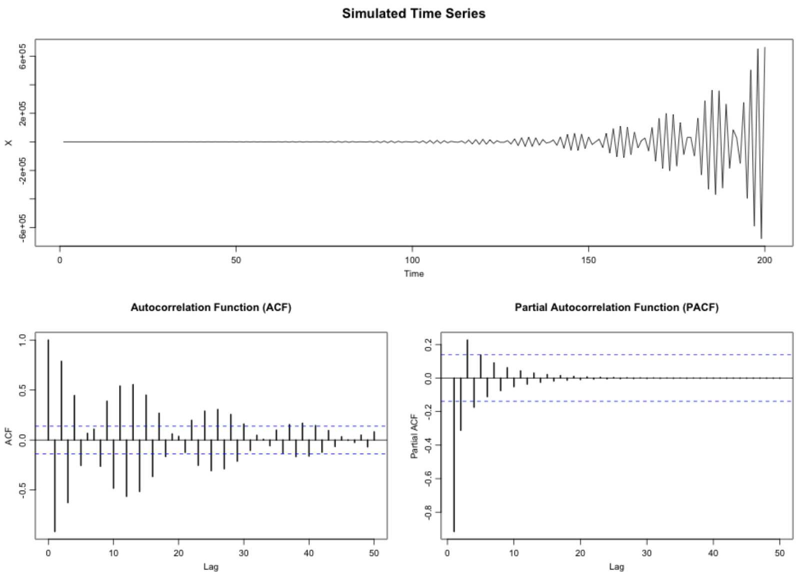

Figure 5: SARIMA Model (1,0,0) X $(1,0,0)_{12}$

The negative values indicate a strong reverse-effect in both short-term and seasonal trends, therefore making values shift directions very frequently.

2.5 A $(0,0,1) \times (0,0,1)_{12}$ model with the parameters $\theta_1 = 0.4$ and $\Theta_1 = -0.8$

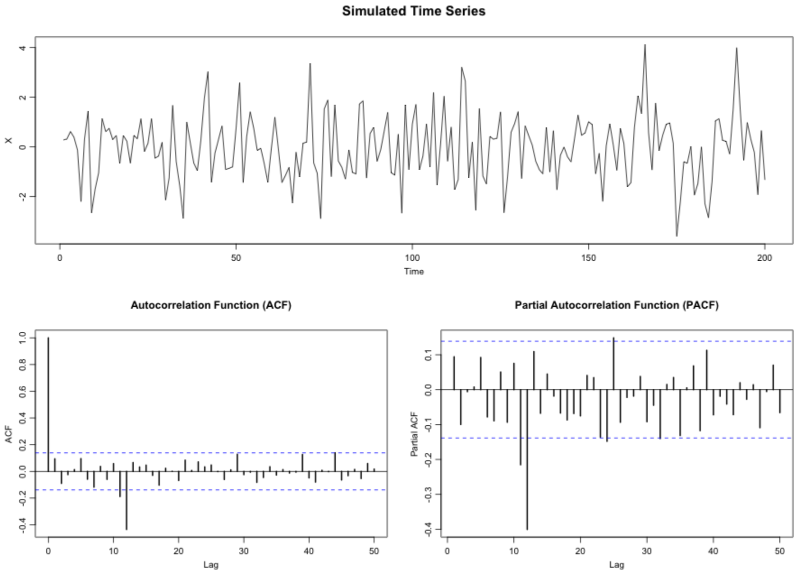

Figure 6: SARIMA Model (0,0,1) X $(0,0,1)_{12}$

The process has both short-term and seasonal components, as it has smooth fluctuations with seasonality.

2.6 A $(0,0,1) \times (1,0,0)_{12}$ model with the parameters $\theta_1 = -0.4$ and $\Phi_1 = 0.7$

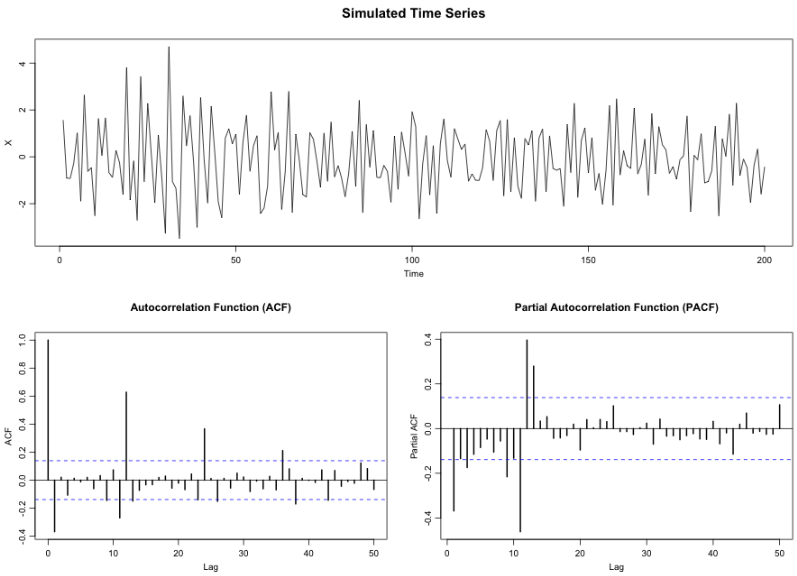

Figure 7: SARIMA Model (0,0,1) X $(1,0,0)_{12}$

This model blends a short-term average with a seasonal pattern, helping to reduce random noise while also picking up regular, repeating trends.

2.7 Summarize your observations on the processes and the autocorrelation functions. Which conclusions can you draw on the general behavior of the autocorrelation function for seasonal processes?

Sometimes it is difficult to spot clear patterns just by looking at the time series plots alone, especially if the data has noise or is complex. However, ACF and PACF plots can help identify those patterns much easier by showing the relationships between observations at different time lags. These plots highlight patterns that aren't obvious in the time series itself. When the process only includes either AR or MA lags, it is easier to identify the pattern of the time series, rather than when they are both included. This is because the behavior of the process is simpler: either it depends on past values or on past errors.

3 Identifying arma models

Below are plots of three simulated ARMA processes: time series plot, ACF and PACF. Guess the ARMA model structure for each of them, and give a short reasoning of your guess.

3.1 Process 1:

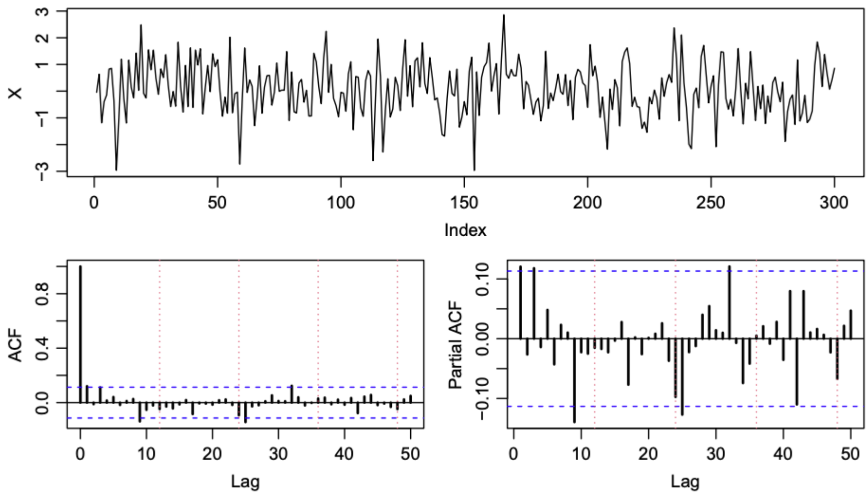

Figure 8: Time series plot, ACF (left) and PACF (right)

The time series seems to have no obvious trend or seasonality and so it is stationary. As long as I could see there is no significant ACF or PACF values, which, alongside the randomness of the time series subplot of Figure 8, could point towards the fact that observation is independent and identically distributed (iid). Therefore my best guess of ARMA model structure is **ARMA(0,0)**, that is, white noise.

3.2 Process 2:

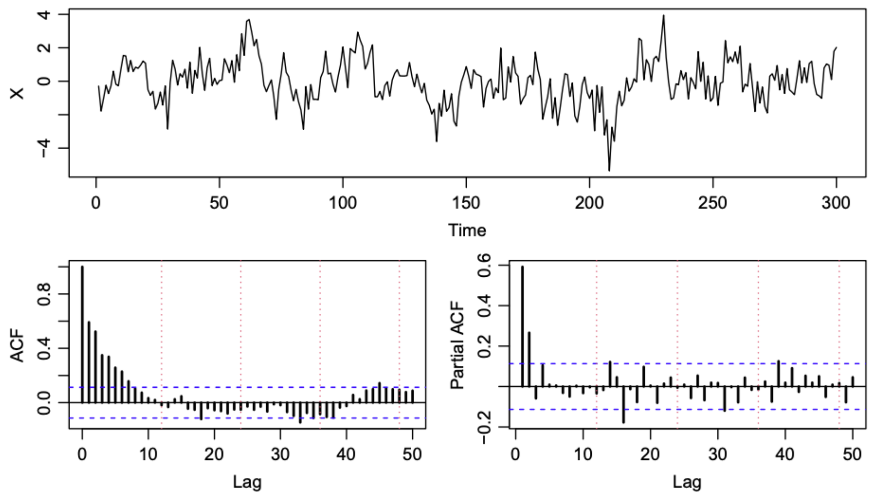

Figure 9: Time series plot, ACF (left) and PACF (right)

The above process in Figure 9 matches with the model structure of an **AR(2)** process. The reasoning behind this is that there is no clear upward or downward trend in the time series, ACF decays gradually and PACF cuts off after lag 2 - more specifically it can be seen strong spikes at lag 1 and 2, then they drop to close to zero.

3.3 Process 3:

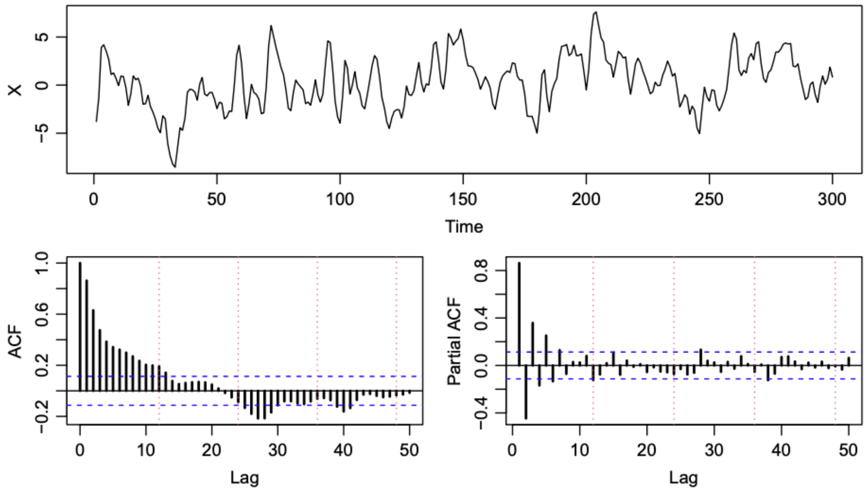

Figure 10: Time series plot, ACF (left) and PACF (right)

Figure 10 resembles an **ARMA(1,1)** process. This is due to ACF's exponential decay after lag 1 as well as the very sharp drop of PACF values after lag 1. Could also possibly be an **ARMA(2,1)** process due to the fact that in PACF there is a sudden drop with a negative value at lag 2, however its description for ACF does not match as closely in this case.