Optimal Transmission Range for Cluster-Based Wireless Sensor Networks With Mixed Communication Modes

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***Abstract*— Prolonging the network lifetime is one of the most important designing objectives in wireless sensor networks (WSN). We consider a heterogeneous cluster-based WSN, which consists of two types of nodes: powerful cluster-heads and basic sensor nodes. All the nodes are randomly deployed in a specific area. To better balance the energy dissipation, we use a simple mixed communication modes where the sensor nodes can communicate with cluster-heads in either single-hop or multi-hop mode. Given the initial energy of the basic sensor nodes, we derive the optimal communication range and identify the optimal mixed communication mode to maximize the WSN's lifetime through optimizations. Moreover, we also extend our model from 2-D space to 3-D space.**

***Index Terms*— Wireless sensor networks, network lifetime, clustering, Voronoi cell, optimization.**

I. Introduction

A WIRELESS sensor network consists of a large amount of sensor nodes, which have wireless communication capability and some level of ability for signal processing. Distributed wireless sensor networks enable a variety of applications for sensing and controlling the physical world [1], [2]. One of the most important applications is the monitor of a specific geographical area (e.g., to detect and monitor the environmental changes in forests) by spreading a great number of wireless sensor nodes across the area [3]–[6].

Because of the sensor nodes' self constraints (generally tiny size, low-energy supply, weak computation ability, etc.), it is challenging to develop a scalable, robust, and long-lived wireless sensor network. Much research effort has focused on this area which result in many new technologies and methods to address these problems in recent years. The combination of clustering and data-fusion is one of the most effective approaches to construct the large-scale and energy-efficient data gathering sensor networks [7]–[9].

In particular, the authors of [9] develop a distributed algorithm called Low-Energy Adaptive Clustering Hierarchy (LEACH) for homogeneous sensor networks where each sensor elects itself as a cluster-head with some probability and the cluster reconfiguration scheme is used to balance the energy load. The LEACH allows only *single-hop* clusters to be constructed. On the other hand, in [10] we proposed the similar clustering algorithms where sensors communicate with their cluster-heads in *multi-hop mode*. However, in these homogeneous sensor networks, the requirement that every node is capable of aggregating data leads to the extra hardware cost for all the nodes. Instead of using homogeneous sensor nodes and the cluster reconfiguration scheme, the authors of [11] focus on the heterogeneous sensor networks in which there are two types of nodes: supernodes and basic sensor nodes. The supernodes act as the cluster-heads. The basic sensor nodes communicate with their closest cluster-heads via multi-hop mode. The authors of [11] formulate an optimization problem to minimize the network cost which depends on the nodes' densities and initial energies.

In addition, The authors of [12] obtain the upper bounds on the lifetime of a sensor network through all possible routes/ communication modes. However, it is complicated to implement a distributed scheme to achieve the upper bound of the WSN lifetime because it is required to know the distance between every two sensor nodes in their scheme. The authors of [13] develop a simpler, but sub-optimal, scheme where the nodes employ the *mixed communication modes*: single-hop mode and multi-hop mode periodically. This mixed communication modes can better balance the energy load efficiently over WSNs. However, the authors of [13] do not obtain the optimal communication range for the multi-hop mode which is a critically important parameter for the mixed communication modes scheme. Also, their analytical model can only deal with the case of grid deployment, where the nodes are placed along the grids, without considering the random deployment.

In order to further increase the network lifetime of *heterogeneous* WSNs by remedying the deficiencies in the aforementioned pervious work, we develop the analytical models to determine the optimal transmission range of the sensor nodes and identify the optimal communication modes in this paper. In our models, the basic sensor nodes are allowed to communicate with their cluster-heads with mixed communication

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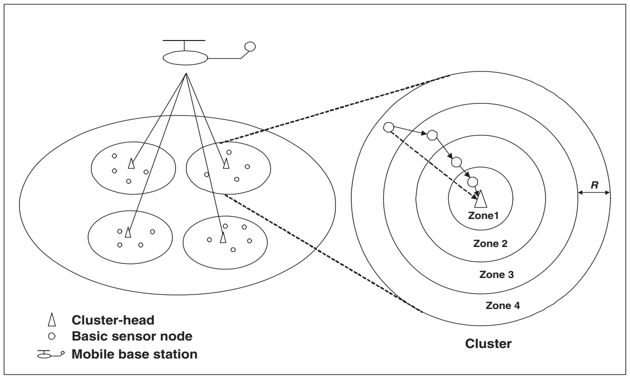

Fig. 1. An example of the wireless sensor data-gathering networks. In each round, the aircraft hovers above the cluster-heads in the monitored area to collect the aggregated data. Within each cluster, the basic sensor nodes can communicate with its cluster-head in either single-hop or multi-hop.

modes instead of only multi-hop mode or only single-hop mode. Applying the derived optimal transmission range and communication modes, we also study how the other WSN parameters (e.g., the density and initial energy of the cluster-heads, etc.) affect the network lifetime through simulation experiments. Moreover, simulation results verify our analytical models. We also extend our model from 2-D space to 3-D space.

The rest of the paper is organized as follows. Section II develops our proposed models and formulates the design procedure as an optimization problem. Section III solves the formulated optimization problem. Section IV presents the numerical and experimental results. Section V addresses the extended 3-D model. The paper concludes with Section VI.

II. System Model

We study the following WSN scenario in this paper. A heterogeneous sensor network consisting of two types of sensor nodes, i) the more powerful but more expensive cluster-head nodes with density of λ_1 and ii) the inexpensive basic sensor nodes with density of λ_2, is deployed in a specific area. The density of the basic sensor nodes is determined by the application requirements. A basic sensor node joins the cluster whose cluster-head is the closest in hops or distance to this basic sensor node. In each round, the cluster-heads send the aggregated data to the mobile base station (e.g., an aircraft or a satellite) after the cluster-heads receive and process the raw data from the basic sensor node. Fig. 1 shows an example of this type of sensor networks.

The definition of network lifetime used in this paper is the period in rounds from the time when the network starts working to the time when the first node dies [15]. Notice that energy dissipation is not uniform over the cluster-based WSNs, implying that some nodes in specific zones (e.g., the nodes which are close to the cluster-heads need to consume more energy for relaying traffic of other nodes in multi-hop mode) drain out their energy faster than others. Thus, the network lifetime of these critical nodes decides how long the WSN can survive. Hence, maximizing the network lifetime is equivalent to minimizing the energy consumption of the critical sensor nodes if the initial energy of sensor nodes is given. In this paper, our optimization objective is not to minimize the total energy consumption by all the sensor nodes, but to minimize the energy consumption of the critical nodes to prolong the network lifetime.

A. Node Architectures and Energy Models

A wireless sensor node typically consists of the following three parts: 1) the sensor component, 2) the transceiver component, and 3) the signal processing component. In this paper, we have the following assumptions for each component.

1) **For the sensor component:**
Assume that the sensor nodes sense constant amount of information every round. The energy consumed in sensing is simply $E_{sense}(l) = \gamma l$, where γ is the power consumption for sensing a bit of data and l is the length in bits of the information which a sensor node should sense in every data-gathering round. In general the value of l is a constant.

2) **For the transceiver component:**
We use a simple model for the radio hardware energy consumption [16]. The energy $E_{tx}(r,l)$ and the energy $E_{rx}(l)$ required for a node to transmit and receive a packet of l bits over r distance, respectively, can be expressed as follows.

$$\begin{cases} E_{tx}(r,l) &= (\alpha r^n + \beta)l \\ E_{rx}(l) &= \beta l \end{cases} \tag{1}$$

where αr^n accounts for the radiated power necessary to transmit over a distance of r between source and destination and β is the energy dissipated in the transmitter circuit (PLLs, VCOs, etc) which depends on the digital coding, modulation, etc. The value of path loss exponent n depends on the surrounding environment [16]. In general, $\alpha = 10pJ/bit/m^2$ when $n = 2$, $\alpha = 0.0013pJ/bit/m^4$ when $n = 4$, $\beta = 50nJ/bit$ [9].

3) **For the signal processing component:**
This component conducts data-fusion. Because the signal processing component usually consists of complicated and expensive gear such as Digital Signal Processors (DSP), Field Programmable Gate Arrays (FPGA), etc., the basic sensor nodes do not contain the signal processing component. Only the cluster-heads have these components and the ability for data-fusion. The energy spent in aggregating k streams of l bits raw information into a single stream is determined by

$$E_{aggr}(k,l) = k\delta l \tag{2}$$

where the typical value of δ is $5nJ/bit/stream$ [17].

B. Mixed Communication Modes

Notice that in the multi-hop mode, the closer the distance between the sensor node and its cluster-head, the more energy the sensor node consumes since the inner nodes are required to relay the more traffics than that for the outer nodes. On the other hand, for the case of pure single-hop mode, the basic sensor nodes which are closer to their cluster-heads dissipate less energy than those farther from the cluster-heads because the energy consumption increases as the n-th power of distance, where n is the path loss exponent.

In order to balance the energy load of the basic sensor nodes, our model employs the mixed communication modes which consist of the mixed single-hop and multi-hop mode. The basic sensor nodes use single-hop communication mode in some rounds but multi-hop communication mode in the other rounds. This kind of mixed communication modes is easy to implement. For example, the cluster-head can broadcast a notifying message periodically to all member nodes to inform of which communication mode should be used for next data-gathering round. We use parameter ξ to measure how often the single-hop mode is used.

Suppose T is the total rounds that the network can perform, T_s is the number of rounds that single-hop communication mode is used and T_m is the number of rounds that multi-hop communication mode is employed. Let $\xi = T_s/T = 1-T_m/T$ be the frequency with which the single-hop communication mode is used. Note that $\xi = 0$ means that the pure multi-hop communication mode is employed, while $\xi = 1$ represents that only the single-hop communication mode is used.

C. Deployment Models

The sensor nodes and the cluster-heads are randomly distributed on a 2-D circular area, whose radius is A unit. We can model such random deployment (e.g., deployed by the aircraft in a large-scale mode) as a spatial Poisson point process. Specifically, the cluster-heads and basic sensor nodes in the wireless sensor network are distributed according to two independent spatial Poisson processes PP1 and PP2 with densities equal to λ_1 and λ_2, respectively.

The basic sensor nodes will join the clusters in which the cluster-heads are the closest to the sensor nodes to form Voronoi cells [14]. The 2-D plane is thus partitioned into a number of Voronoi cells which correspond to a PP1 process point. The authors of [14] have studied the moments and tail of the distributions of the number of PP2 nodes (i.e., the basic sensor nodes) connected to a particular PP1 node (i.e, a cluster-head). Because PP1 and PP2 are homogeneous Poisson point processes, we can shift the origin to one of the PP1 nodes. Let $\mathcal{V}$ be the set of nodes which belong to the Voronoi cell corresponding to a PP1 node located at the origin and $S_{(r,\theta)}$ be a PP2 node whose polar coordinate is (r, θ). By using the results of [14], we can get the probability that $S_{(r,\theta)}$ belongs to $\mathcal{V}$ as follows:

$$Pr\left\{S_{(r,\theta)} \in \mathcal{V}\right\} = e^{-\lambda_1 \pi r^2} \tag{3}$$

Let $\mathcal{N}_\mathcal{V}$ be the number of PP2 nodes belonging to the $\mathcal{V}$. Then, the average $\mathcal{N}_\mathcal{V}$ can be given by

$$\begin{aligned} E[\mathcal{N}_\mathcal{V}] &= \int_0^{2\pi}\int_0^{\infty} Pr\left\{S_{(r,\theta)} \in \mathcal{V}\right\} \lambda_2 r dr d\theta \\ &= \int_0^{2\pi}\int_0^{\infty} e^{-\lambda_1 \pi r^2} \lambda_2 r dr d\theta \\ &= \frac{\lambda_2}{\lambda_1} \end{aligned} \tag{4}$$

where $\lambda_2 r dr d\theta$ denotes the number of PP2 nodes in a small area of $r dr d\theta$.

If we confine the cluster size to X hops (i.e., maximum number of X hops is allowed from the basic sensor node to its cluster-head), the average number of sensors which do not belong to any cluster-head, denoted by $E[\mathcal{N}_\mathcal{O}]$, can be expressed as follows:

$$\begin{aligned} E[\mathcal{N}_\mathcal{O}] &= \lambda_1 \pi A^2 \int_0^{2\pi}\int_{XR}^{\infty} Pr\left\{S_{(r,\theta)} \in \mathcal{V}\right\} \lambda_2 r dr d\theta \\ &= \lambda_2 \pi A^2 e^{-\lambda_1 \pi (XR)^2} \end{aligned} \tag{5}$$

Clearly, we want a small $E[\mathcal{N}_\mathcal{O}]$ with an appropriate cluster size X.

$$E\left[\mathcal{N}_\mathcal{O}\right] \le \epsilon \lambda_2 \pi A^2 \tag{6}$$

where $\epsilon\,(\epsilon > 0)$ is the percentage of sensor nodes that will not join any cluster-head.

Solving Eq. (5) and Eq. (6) together, we obtain the following inequality.

$$X \ge \sqrt{\frac{-\log \epsilon}{\pi \lambda_1 R^2}} \tag{7}$$

The average number of sensor nodes which do not join any cluster-head is less or equal than $\epsilon \lambda_2 \pi A^2$ if X is given by

$$X = \left\lceil \sqrt{\frac{-\log \epsilon}{\pi \lambda_1 R^2}} \right\rceil \tag{8}$$

Thus, we know that every cluster can be divided into X ring zones with the ring width equal to R when multi-hop communication mode is used.

The average number of sensor nodes in the i-th zone of the cluster can be written as follows:

$$\begin{aligned} E[\mathcal{N}_i] &= \int_0^{2\pi}\int_{(i-1)R}^{iR} Pr\left\{S_{(r,\theta)} \in \mathcal{V}\right\} \lambda_2 r dr d\theta \\ &= \frac{\lambda_2}{\lambda_1}\left(e^{-\lambda_1 \pi [(i-1)R]^2} - e^{-\lambda_1 \pi (iR)^2}\right) \end{aligned} \tag{9}$$

where i is an positive integer ranging from 1 to X.

D. The Wireless Sensor Network Lifetime

In our model, the nodes at the same zone will die at almost the same time. Therefore, the network lifetime is equivalent to the period from the time when the WSN begins working to the time when all nodes of a zone die.

i) The basic sensor nodes

The sensor nodes have the responsibility to relay the traffic from the peers laid in the outer zone in the multi-hop communication mode. We define the average number of packets by $\overline{\mathcal{Y}}(i)$, which a sensor node placed in the i-th zone need to relay. Because each basic sensor node sends out a packet of sensed information per round, $\overline{\mathcal{Y}}(i)$ is determined by the average number of nodes for which node i needs to forward messages and can be written as follows:

$$\overline{\mathcal{Y}}(i) = \frac{\sum_{j=i+1}^{X} E[\mathcal{N}_j]}{E[\mathcal{N}_i]}. \tag{10}$$

The energy consumption of the basic sensor nodes in the i-th zone can be written as the summation of the following 3 terms:

$$\begin{aligned} E_{sensor}(i,\xi) \;=\; & (1-\xi)\big\{E_{tx}(R,l) + \overline{\mathcal{Y}}(i)\big(E_{tx}(R,l) \\ & +E_{rx}(l)\big)\big\} + \xi E_{tx}(iR,l) + E_{sense}(l) \end{aligned} \tag{11}$$

where $\xi\,(0 \le \xi \le 1)$ is a parameter measuring the frequency with which the single-hop communication mode will be employed. The first term of Eq. (11) represents the energy spent in the relaying the traffic for the sensor nodes in the outer zone and transmitting its own traffic when the multi-hop communication mode is employed. The second term of Eq. (11) is energy consumption for transmitting a packet by single-hop. The third term of Eq. (11) is the energy dissipation for sensing.

In our model, because the basic sensor nodes in the same zone will consume almost the same energy in each round (i.e., sharing the same relaying traffic load when multi-hop communication mode is used), their lifetimes are equal if their initial energies are the same. Therefore, if the initial energy is the same for each basic sensor node, the sensor nodes which belong to the $(\arg\max_{1\le i\le X}\{E_{sensor}(i,\xi)\})$-th zone will cost the most energy and die first which decides the network lifetime. Given the initial energy, denoted by E_{init2}, which is carried by the basic sensor node, the network lifetime in rounds (T) can be written as follows:

$$T = \frac{E_{init2}}{\max_{1\le i\le X} E_{sensor}(i,\xi)} = \frac{E_{init2}}{E_{max}(\xi)} \tag{12}$$

where $E_{max}(\xi) \triangleq \max_{1\le i\le X} E_{sensor}(i,\xi)$. One of our objectives is to find the optimal ξ^* which is determined by

$$\xi^* = \arg\min_{0\le\xi\le 1}\{E_{max}(\xi)\} \tag{13}$$

ii) The cluster-heads

Because the main functions of cluster-heads include (1) sensing, (2) collecting data from the basic sensor nodes, (3) aggregating the raw data, and (4) transferring the processed data to the base station, the energy consumption of cluster-heads is the sum of the energy dissipation of these four parts for the above four parts. Therefore, the energy consumption for a cluster-head, denoted by E_{CH}, in each round can be expressed as summation of the following 4 terms:

$$\begin{aligned} E_{CH} \;=\; & E[\mathcal{N}]\,E_{rx}(l) + E_{tx}(H,l') + E_{sense}(l) + \\ & E_{aggr}(E[\mathcal{N}]+1,l) \end{aligned} \tag{14}$$

where $E[\mathcal{N}]$ is the average number of basic sensor nodes in a cluster, and H is the distance between the cluster-head and the mobile base-station.

The first term of Eq. (14) represents the energy consumed for receiving the packets from the basic sensor nodes. The second term of Eq. (14) is the energy spent in transmitting the aggregated information to the mobile base station. The third term of Eq. (14) denotes the energy dissipation for sensing and the fourth term of Eq. (14) means the energy consumption for aggregating $(E[\mathcal{N}]+1)$ packets, each with l bits, into one packet of l' bits.

Because $E[\mathcal{N}] = \lambda_2/\lambda_1$, E_{CH} depends on the ratio between λ_2 and λ_1. The energy consumption of cluster-heads is reversely proportional to the density of cluster-heads. The larger the density of cluster-heads, the smaller the value of E_{CH}. Thus, in order to ensure T_0 rounds network lifetime, the initial energy for the cluster-heads can be written as follows:

$$E_{init1} \ge T_0 E_{CH} \tag{15}$$

E. Connectivity

Because the mixed communication modes contains multi-hop communication mode, the communication range (R) is required to be large enough to ensure the connectivity of the network. When the nodes are assumed to be distributed with Poisson density λ in a disc of a unit area, the authors of [18] derived a lower bound on the communication range (R) to ensure the network connectivity with probability $Pr\{conn\}$, which is determined by

$$Pr\{conn\} \ge 1 - \lambda e^{-\lambda\pi R^2} \tag{16}$$

Hence, the following inequality need to hold to ensure the connectivity.

$$\lambda e^{-\lambda\pi R^2} \le \zeta \tag{17}$$

where $\zeta > 0$.

By resolving Eq. (17), we obtain the minimum transmission range, denoted by R_{min}, to ensure that the probability of connectivity is greater than $(1-\zeta)$.

$$\begin{aligned} R_{min} \;&=\; \sqrt{-\frac{1}{\lambda\pi}\log\frac{\zeta}{\lambda}} \\ &=\; \sqrt{\frac{1}{(\lambda_1+\lambda_2)\pi}\log\frac{(\lambda_1+\lambda_2)}{\zeta}} \end{aligned} \tag{18}$$

F. The Optimization Problem Formulation

Our objective is to find the optimal transmission range (R) and the parameter for the mixed communication modes (ξ) to maximize the network lifetime.

$$\textbf{Objective:}\quad \max\{T\} \tag{19}$$

$$\textbf{Subject to}\quad R \ge R_{min} \tag{20}$$

$$0 \le \xi \le 1 \tag{21}$$

$$0 \le E_{init2} \le E_0 \tag{22}$$

where the first constraint given by Eq. (20) is to ensure the connectivity of the network. The expression of R_{min} depends on the types of deployment. The second constraint given by Eq. (21) indicates that we can use mixed communication modes. The third constraint given by Eq. (22) is due to the very limited energy carried by the basic sensor nodes.

By observing Eq. (12), we can simplify the objective function as "$\min\{E_{max}\}$" and remove the constraint given by Eq. (22) by letting $E_{init2} = E_0$. Then, the simplified optimization problem can be written as follows:

$$\begin{array}{ll} \textbf{Objective:} & \min\{E_{max}\} \\ \textbf{Subject to} & R \geq R_{min} \\ & 0 \leq \xi \leq 1 \end{array} \tag{23}$$

III. Solutions for the Optimization Problem

First, we show that given a specified transmission range (R), the function of $E_{max}(\xi)$ is convex in ξ and the optimal $\xi^* = \arg\min_{0\leq\xi\leq1}\{E_{max}(\xi)\}$ is a function of R.

Because the energy consumption $E_{sensor}(i,\xi)$ of the sensor nodes in the i-th zone is a convex function of i (the proof is omitted for lack of space), the value of $E_{max}(\xi)$ will be achieved by the sensor nodes laid in either the 1st zone or the X-th zone, i.e.,

$$E_{max}(\xi) = \max\{E_{sensor}(1,\xi), E_{sensor}(X,\xi)\} \tag{24}$$

Note that $E_{sensor}(1,\xi)$ is a monotonically decreasing linear function of ξ while $E_{sensor}(X,\xi)$ is a monotonically increasing linear function of ξ, and $E_{sensor}(1,1) = E_{sensor}(X,0)$. Hence, we can find that the two lines corresponding to these two linear functions will intersect at the point where ξ is within the range between 0 to 1. Clearly, the intersecting point, denoted by ξ^*, yields the minimum value of $E_{max}(\xi)$. Therefore, $\xi = \xi^*$ is the optimal value when the following equation is satisfied.

$$E_{sensor}(1,\xi^*) = E_{sensor}(X,\xi^*) \tag{25}$$

By resolving Eq. (25), we obtain the solution for Eq. (13).

$$\begin{aligned} \xi^* &= \arg\min_{0\leq\xi\leq1}\{E_{max}(\xi)\} \\ &= \frac{\overline{\mathcal{Y}}(1)[E_{tx}(R)+E_{rx}]}{\overline{\mathcal{Y}}(1)[E_{tx}(R)+E_{rx}] + E_{tx}(XR) - E_{tx}(R)} \\ &= \frac{\overline{\mathcal{Y}}(1)(\alpha R^n + 2\beta)}{\overline{\mathcal{Y}}(1)(\alpha R^n + 2\beta) + \alpha R^n(X^n-1)} \\ &= \frac{\overline{\mathcal{Y}}(1)(R^n + 2\kappa)}{\overline{\mathcal{Y}}(1)(R^n + 2\kappa) + R^n(X^n-1)} \end{aligned} \tag{26}$$

where $\kappa = \beta/\alpha$, and we use $E_{tx}(R)$, $E_{tx}(XR)$, and E_{rx} instead of $E_{tx}(R,l)$, $E_{tx}(XR,l)$, and $E_{rx}(l)$ since the value of l is a constant for a specific application.

The factor κ measures to what extent R has the impact on the transmission energy consumption. For example, The transmission energy consumption is more sensitive to R when κ is larger. The factor κ also determines the cost of relaying traffic. The cost of relaying traffic increases with the increment of κ because receiving packets consumes more energy with a greater κ.

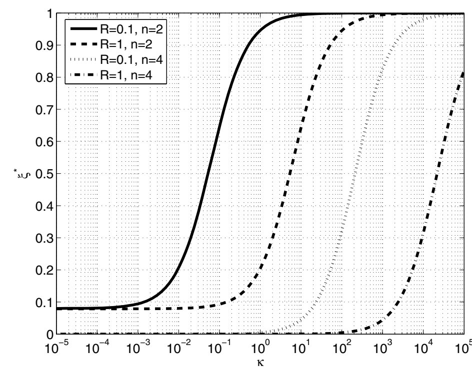

Fig. 2. The optimal ξ^* against κ.

If $\kappa \ll R^n$, Eq. (26) reduces to its approximated expression as follows:

$$\xi^* \approx \frac{\overline{\mathcal{Y}}(1)}{\overline{\mathcal{Y}}(1) + (X^n - 1)} \tag{27}$$

By substituting Eq. (9) and (10) into Eq. (26), we have

$$\xi^* = \frac{\frac{1-e^{-\lambda_1\pi(X^2-1)R^2}}{e^{\lambda_1\pi R^2}-1}(R^n+2\kappa)}{\frac{1-e^{-\lambda_1\pi(X^2-1)R^2}}{e^{\lambda_1\pi R^2}-1}(R^n+2\kappa) + R^n(X^n-1)} \tag{28}$$

Notice that ξ^* is a function of R if we substitute Eq. (8) into Eq. (28).

Let $\lambda_1 = 0.001$, $\lambda_2 = 3$, and $\alpha = 10^{-12}$. By using Eq. (28), we plot the optimal ξ^* against κ as shown in Fig. 2. We observe from Fig. 2 that the optimal ξ^* is almost 0 (i.e., pure multi-hop communication mode) if $n = 4$ and κ is small. The reason is because the energy consumption for transmission is proportional to R^4 and the term of αR^4 in the first part of Eq. (1) dominates the transmission energy consumption if κ is small. Thus, the energy consumption in single-hop mode is much more than that in multi-hop mode. In contrast, if κ is large, the multi-hop mode loses its advantage over the single-hop mode because the transmission energy consumption is dominated by the constant term of β in the first part of Eq. (1) and it is not sensitive to the transmission range.

So far, given the communication range (R), we obtain the minimum $E_{max}(\xi^*)$ for the basic sensor nodes in order to derive the solutions for objective function of Eq. (24) as follows:

$$E_{max}(\xi^*) = [\alpha R^n(1 + \xi^* X^n - \xi^*) + \beta + \gamma]l \tag{29}$$

Next, we want to identify the optimal R^* to minimize $E_{max}(\xi^*)$ when constraint given by Eq. (20) applies. Because it is difficult to find the closed form for the optimal R^*, we use a numerical solutions to determine the optimal R^* which is detailed for some scenarios in Section IV.

IV. The Numerical and Simulation Evaluations

For the following discussions, we set $\alpha = 10pJ/bit/m^2$, $\gamma = 50nJ/bit$, $\delta = 5nJ/bit/stream$, the packet length

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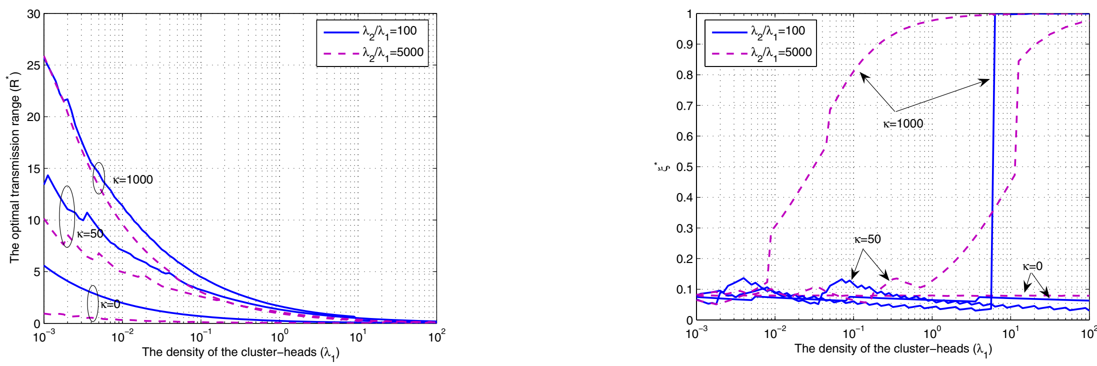

(a) (b)

Fig. 3. The optimal parameters versus the density of cluster-heads under different κ. (a) The optimal communication range R^*. (b) The optimal ξ^*.

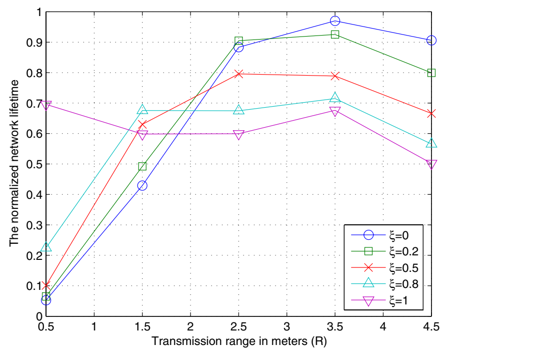

Fig. 4. The normalized network lifetime versus the transmission range (R) with $\kappa = 50$, $\lambda_1 = 0.1$, $\lambda_2/\lambda_1 = 5000$, and $\xi = 0, 0.2, 0.5, 0.8, 1$.

$l = 120\,bits$ and the path loss exponent $n = 2$. We consider various scenarios with three different values of κ, two average numbers of basic sensor nodes in a cluster (λ_2/λ_1) and various densities of cluster-heads (λ_1). According to the discussion in Section III, we get the numerical solutions for the optimal R^*, and then the value of optimal ξ^* can be calculated by using Eq. (28).

The numerical results of optimal R^* and ξ^* with different κ and (λ_2/λ_1) are shown in Fig. 3(a) and Fig. 3(b), respectively. The optimal R^* decreases as λ_1 increases. With the same λ_1, the larger κ, the larger the value of R^*. When κ is small (e.g., $\kappa = 0$ and $\kappa = 50$), the value of ξ^* is small (e.g., $\xi^* < 0.1$). This implies that the multi-hop communication mode dominates the single-hop mode. The reason for these observations includes the following two. First, the cost of relaying traffics is small since the receiving energy is small. Second, the transmission energy is sensitive to the transmission range.

We also conduct the simulation experiments to verify our analytical results. In our simulations, Minimum Transmission Energy (MTE) routing algorithm [19], which minimizes the total energy consumption for sending a packet, is used as the relaying scheme for the multi-hop communication mode. Let $\kappa = 50$, $\lambda_1 = 0.1$, and $\lambda_2/\lambda_1 = 5000$. We set the initial energy of cluster-heads (E_{init2}) high enough to guarantee that the cluster-heads can have longer lifetime than the basic sensor nodes. Fig. 4 shows the simulation results of network lifetime. The plots in Fig. 4 is the average results of 1000 experiments. It shows that in most cases (i.e., $\xi = 0, 0.2, 0.5$, and 0.8) the network lifetime is maximized when $R^* = 3.5$, which agrees with the numerical results shown in Fig. 3.

In the following simulations, the parameters are set as follows: the initial energy of basic sensor nodes $E_{init2} = 0.01J$, the distance between the cluster-heads and the mobile base station $H = 100m$, $l' = l = 120bits$ and $\lambda_2 = 1000$. Fig. 5 shows the network lifetime changes with the average number of the basic sensor nodes in a cluster by using the optimal R^* and ξ^* when κ is equal to 0, 100 and 1000. We observe that increasing the density of cluster-heads (i.e., λ_2/λ_1 decreases given constant λ_2) does not always help to extend the network lifetime. For example, the network lifetime can be increased by 51% when the density of cluster-heads changes from $\lambda_1 = 0.01$ or $\lambda_2/\lambda_1 = 10^6$ to $\lambda_1 = 0.1$ or $\lambda_2/\lambda_1 = 10^5$, while the network lifetime is almost the same when the density of cluster-heads is greater than $\lambda_1 = 1$ (i.e., the average of basic sensor nodes $\lambda_2/\lambda_1 \leq 10^4$). On the other hand, Fig. 6 shows the energy consumption of a cluster-head against the average number of the basic sensor nodes. We find that the average energy consumption of a cluster-head is proportional to (λ_2/λ_1) from Fig. 6. The required initial energy of cluster-heads increases with the decrease of the density of cluster-heads (λ_1). Thus, there is a trade-off between the network lifetime and the initial energy of the cluster-heads.

V. 3-D WSN Extension

Our work can be easily extended to a 3-D space model. The differences between the 3-D space model and the 2-D space model lie in the deployment models and the connectivity models.

The probability that a basic sensor node with spherical coordinate (r, θ, ϕ) belongs to a cluster-head located in the origin is

$$Pr\left\{S_{(r,\theta,\phi)} \in \mathcal{V}\right\} = e^{-\lambda_1 \frac{4}{3}\pi r^3} \tag{30}$$

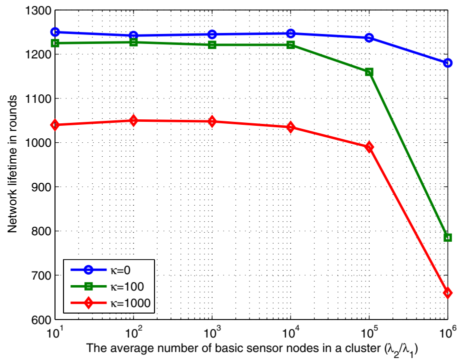

Fig. 5. Optimal network lifetime under various situations where $\lambda_2 = 1000$.

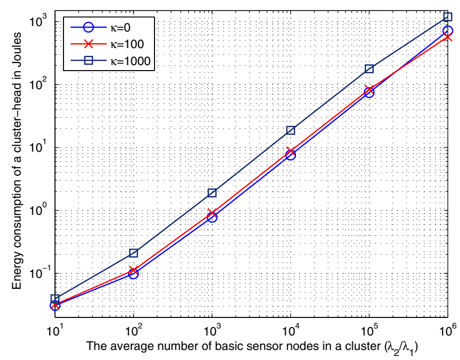

Fig. 6. The energy consumption of cluster-heads against the number of the basic sensor nodes.

The average number of basic sensor nodes in a cluster is determined by

$$
\begin{aligned}
&E[\mathcal{N}_\mathcal{V}] \\
&= \int_0^{\pi}\int_0^{2\pi}\int_0^{\infty} Pr\left\{S_{(r,\theta,\phi)} \in \mathcal{V}\right\}\lambda_2 r^2 \sin\phi\, dr d\theta d\phi \\
&= \int_0^{\pi}\int_0^{2\pi}\int_0^{\infty} e^{-\lambda_1 \frac{4}{3}\pi r^3}\lambda_2 r^2 \sin\phi\, dr d\theta d\phi \\
&= \frac{\lambda_2}{\lambda_1}
\end{aligned} \tag{31}
$$

In the similar way, we can obtain the number of basic sensor nodes in the i-th zone as follows:

$$
\begin{aligned}
&E\left[\mathcal{N}_\mathcal{V}\right] \\
&= \int_0^{\pi}\int_0^{2\pi}\int_{(i-1)R}^{iR} Pr\left\{S_{(r,\theta,\phi)} \in \mathcal{V}\right\}\lambda_2 r^2 \sin\phi\, dr d\theta d\phi \\
&= \frac{\lambda_2}{\lambda_1}\left(e^{-\lambda_1 \frac{4}{3}\pi[(i-1)R]^3} - e^{-\lambda_1 \frac{4}{3}\pi(iR)^3}\right)
\end{aligned} \tag{32}
$$

To satisfy the requirement of connectivity, the minimum communication range R_{min} can be written as follows:

$$
R_{min} = \sqrt[3]{\frac{3}{4(\lambda_1+\lambda_2)\pi}\log\frac{(\lambda_1+\lambda_2)}{\zeta}} \tag{33}
$$

Again, we can obtain the optimal ξ^* and R^* for the 3-D space model along the same manner as a class of the case of 2-D space WSN model.

VI. Conclusion

We investigated the optimal transmission range for a heterogeneous cluster-based sensor network which consists of two types of nodes, the super cluster-heads and the basic sensor nodes. To balance the energy load of the basic sensor nodes, the mixed communication modes are employed. By developing the analytical models, we numerically derive the optimal transmission range R^* and the frequency of single-hop mode ξ^* to achieve the longest network lifetime. Our analyses also showed that our proposed model can be easily extended from 2-D to 3-D. The simulation results validated our proposed analytical models. Our simulation results with the optimal R^* and ξ^* indicated that the high density of cluster-heads is not very helpful for prolonging the network lifetime.

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