AST 381: Planetary Astrophysics: Homework #3

Raquel Martinez

October 20, 2015

Code can be found at `https://github.com/raquizzi/ast381_hw3`.

Part 1: Write a computer code that takes stellar properties (temperature and radius) as input, and computes the flux density in Jansky at a user-specified orbital radius. Test it for Fomalhaut, at orbital radii of 10 AU and 130 AU, and plot the resulting spectra.

- Wrote code "part1.pro" that takes stellar temperature (K), stellar radius (solar radii), orbital radius (AU), and wavelength (microns) and returns the observed stellar spectrum in Jansky's at the indicated orbital radius.
- Figure 1 shows the resultant spectrum that would be observed from 10 AU and 130 AU for Fomalhaut, a 8590 K, 1.842 solar radius star.

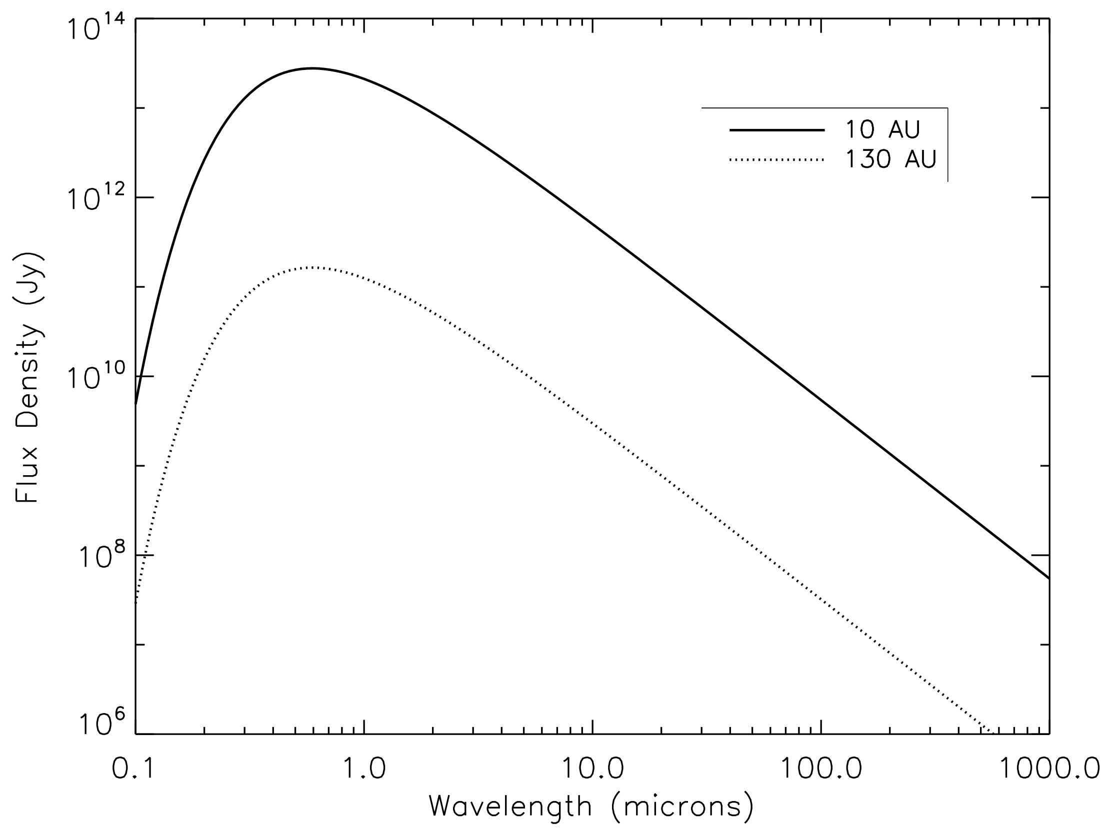

Figure 1: Flux density as would be observed 10 AU (solid) and 130 AU (dotted) from Fomalhaut.

Part 2: Write a computer code that takes the output of Part 1, and calculates the total energy absorbed by a dust grain of "astrosilicate" if it has a size of 0.1 microns, 1 micron, or 10 microns, or by a perfect absorber with a radius of 1 millimeter. Test it on each dust grain type at 10 AU and 130 AU away from Fomalhaut.

- Wrote code "part2.pro" that takes in a wavelength array (microns), stellar flux density array (Jy), Q_{abs}, and grain size (microns) and integrates via the trapezoid rule to calculate the total flux received by the dust grain. The output power absorbed is given in Watts.
- Table 1 lists the powers absorbed by different types of "astrosilicate"/perfect absorber and at different orbital radii.

Table 1: Power Absorbed by Different-sized Grains at Various Orbital Radii

Grain Size (μm)	Power (10 AU) (J s^{-1})	Power (130 AU) (J s^{-1})
0.1	1.53×10^{-12}	9.05×10^{-15}
1.0	4.01×10^{-10}	2.37×10^{-12}
10.0	4.70×10^{-8}	2.78×10^{-10}
1000	5.06×10^{-4}	2.99×10^{-6}

Part 3: Write a computer code that takes the output of Part 2, and computes the equilibrium temperature and emission spectrum for a dust grain of each size/type. Test it on each dust grain type and at 10 AU and 130 AU away from Fomalhaut, calculating the temperature and luminosity and making a plot of the spectrum in each case.

- Wrote code "part3.pro" that takes in power absorbed (Watts), Q_{abs}, a temperature array (K), a wavelength array (microns), and grain size, and finds the grain temperature that when its modified blackbody intensity is integrated over the specified wavelength array best matches the input power absorbed.
- Table 2 lists the equilibrium dust temperature and emitted luminosities for each grain type and distance from Fomalhaut.
- Figure 2 shows the spectral luminosities expected for each grain type at distances of 10 AU and 130 AU.

Table 2: Equilibrium Grain Temperature and Power Emitted by Different-sized Grains at Various Orbital Radii

Grain Size (μm)	10 AU T_g (K)	Power (10 AU) (J s^{-1})	130 AU T_g (K)	Power (130 AU) (J s^{-1})
0.1	263	1.52×10^{-12}	99	9.22×10^{-15}
1.0	195	4.01×10^{-10}	78	2.46×10^{-12}
10.0	146	4.71×10^{-8}	45	2.66×10^{-10}
1000	163	5.03×10^{-4}	45	2.92×10^{-6}

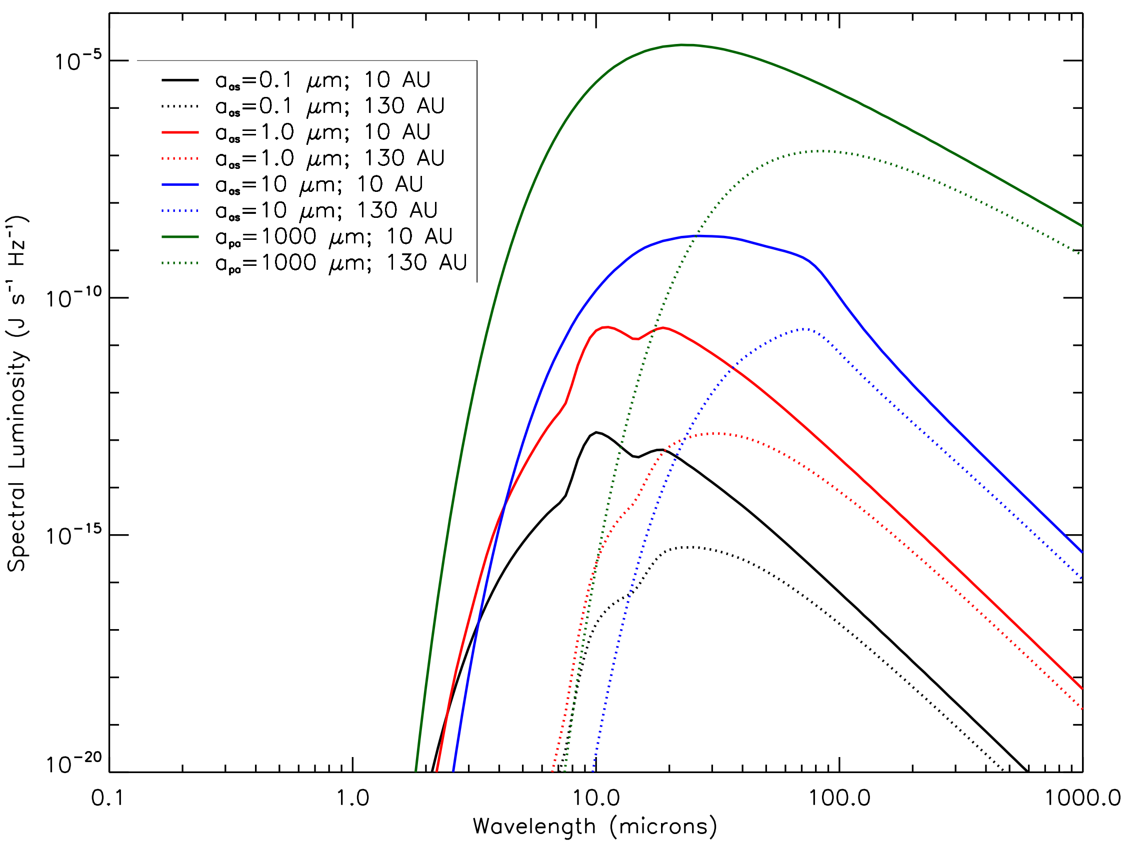

Figure 2: Spectral luminosity of different sized "astrosilicate" (black, red, blue) and 1 mm perfect absorber (dark green) dust grains at 10 AU (solid) and 130 AU (dotted) from Fomalhaut.

Part 4: Go dig up an infrared+millimeter SED of Fomalhaut's two (warm+cold) debris disk out of the literature, and compare it to the SEDs computed in Part 3. For each dust grain size/type, how many dust grains are needed to roughly match the observed SED of each component? What is the corresponding dust mass, if you assume the Fomalhaut debris ring is made out of grains of that size and that dust grains have a density somewhere around 2 g/cc? Which dust grain size produces the best fit to the SED shape?

- Calculations for number and mass of dust grains of each size/type were made in code "hw3.pro".
- From Su et al. 2013, I estimated the warm belt to have a maximum flux density of 700 mJy (Fig. 7) and the cold belt to have a maximum flux density of 10^4 mJy (Fig. 8). I divided these maxima by the maximum flux density we would observe the differing grains to have (take output spectral luminosity from Part 3, multiply by the cross-sectional area the Earth would "see" and then divide by the surface area of the sphere whose radius is the Fomalhaut-Earth distance).
- Grains were assumed to be spherical, so to calculate mass, I multiplied their volume (in m^3) by the given density of 2 g/cc (2000 kg/m^3) and the number of grains calculated in the previous bullet. Table 3 lists the final numbers.
- The dust grain size/type that best matches the observed SED shape is the 10 μm "astrosilicate." The perfect absorbing, 1 mm grain also peaks in the right place, but the observed SED falls off faster than the blackbody tail the 1 mm grain emits.
- The total masses I calculated seem too small, which I'm chalking up to a units error/misunderstanding. I've been reviewing my functions over and over and can't convince myself what the problem is...

Table 3: Comparison of Model Debris Disk to Observed Fomalhaut System

Grain Size (μm)	Number of Dust Grains at 10 AU	10 AU Dust Mass (kg)	Number of Dust Grains at 10 AU	130 AU Dust Mass (kg)
0.1	2.71×10^{35}	2.27×10^{18}	1.02×10^{39}	8.57×10^{21}
1.0	1.64×10^{31}	1.37×10^{17}	4.09×10^{34}	3.42×10^{20}
10.0	1.96×10^{27}	1.65×10^{16}	2.60×10^{30}	2.18×10^{19}
1000	1.85×10^{19}	1.55×10^{14}	4.56×10^{22}	3.82×10^{17}

Part 5: In class we talked about radiation pressure and Poynting-Robertson drag in terms of ideal blackbodies. However, for previous problems you've already calculated the actual energy absorption and emission of real particles, which should allow you to calculate better version of each force. For each particle size, cacluate the real radiation pressure and Poynting-Robertson drag at each radius. Estimate a characteristic timescale for dust grains of each size to be removed from the system, assuming they start at 10 AU or 130 AU.

- F_{rad} is given by $\frac{P_{in}}{c}$.
- F_{PR} is given by $\frac{P_{in}v}{c^2}$ with $v = \sqrt{\frac{GM_*}{r}}$.
- Timescale estimated using Equation 7 from Klacka & Kocifaj 2008 ($\tau = \frac{400 * a^2}{\beta}$; a is the semi-major axis in au, $\beta \equiv \frac{F_{rad}}{F_{grav}}$, τ is the timescale in years).

Table 4: Radiation Force Experienced by Each Grain Size/Type

Grain Size (μm)	$F_{rad,10AU}$ (N)	$F_{rad,130AU}$ (N)
0.1	5.10×10^{-21}	3.02×10^{-23}
1.0	1.34×10^{-18}	7.92×10^{-21}
10.0	1.57×10^{-16}	9.27×10^{-19}
1000	1.69×10^{-12}	9.99×10^{-15}

Table 5: Poynting-Robertson Drag Experienced by Each Grain Size/Type

Grain Size (μm)	$F_{PR,10AU}$ (N)	$F_{PR,130AU}$ (N)
0.1	2.22×10^{-25}	3.65×10^{-28}
1.0	5.83×10^{-23}	9.56×10^{-26}
10.0	6.82×10^{-21}	1.12×10^{-23}
1000	7.35×10^{-17}	1.21×10^{-19}

Table 6: Timescale for Infall for Each Grain Size/Type

Grain Size (μm)	τ_{10AU} (years)	τ_{130AU} (years)
0.1	7.48×10^{3}	1.26×10^{6}
1.0	2.85×10^{4}	4.82×10^{6}
10.0	2.44×10^{5}	4.12×10^{7}
1000	2.26×10^{7}	3.82×10^{9}