UPSCALING PERMEABILITY
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1. Abstract

This study aims to find a representative value of permeability both in x and y direction among several small grids by employing upscaling rule. Consider a sand body where the fluid flows is split into smaller blocks, if the flow is parallel to the blocks, so harmonic averaging must be employed to obtain one representative permeability. Otherwise, if the flow is normal to the blocks, arithmetic averaging must be employed.

2. Objective

Objective of this assignment is as follows :

1. To find the uncertainties as result of upscaling represented by lower and upper bound permeability.
2. To understand the relation between aperture and number of fractures.

3. Result

A. Part 1 :

1. Matlab script to compute upper and lower bound of effective permeability in x and y direction :

```
disp('Part 1 Number 1 :')
disp(' ')
n = ceil(str2double(input('enter number of blocks in vertical arrangement
= ','s')));
m = ceil(str2double(input('enter number of blocks in horizontal
arrangement = ','s')));

for i = 1 : n
    for j = 1 : m
       fprintf('block %s,%s \n',num2str(i),num2str(j))
       kg(i,j) = str2double(input('enter permeability in mD = ','s'));
    end
end

kgx = kg;
kgy = kg;
kg1x = zeros(n,1);
kg2x = zeros(1,n);
kg1y = zeros(1,n);
kg2y = zeros(n,1);

for i = 1 : n
    kg1x(i) = m*prod(kgx(i,1:m))/sum(kgx(i,:)); % harmonic average
permeability in x direction
    kg2y(i) = sum(kgy(i,1:m))/(m); % aritmathic average permeability in y
direction
end

for i = 1 : m
    kg2x(i) = sum(kgx(1:n,i))/(n); % aritmathic average permeability in x
direction
```

```
    kg1y(i) = n*prod(kgy(1:n,i))/sum(kgy(:,i)); % harmonic average
permeability in y direction
end
kg_lox = sum(kg1x)/(n); % lower bound permeability in x direction
kg_upx = n*prod(kg2x)/sum(kg2x); % upper bound permeability in x direction
kg_loy = sum(kg1y)/(m); % lower bound permeability in y direction
kg_upy = m*prod(kg2y)/sum(kg2y); % upper bound permeability in y direction
disp(' ')
fprintf('with configuration %u x %u blocks \n',n,m)
fprintf('permeability value in x-direction lies between %f and %f mD.
\n',kg_lox,kg_upx)
fprintf('permeability value in y-direction lies between %f and %f mD.
\n',kg_loy,kg_upy)
```

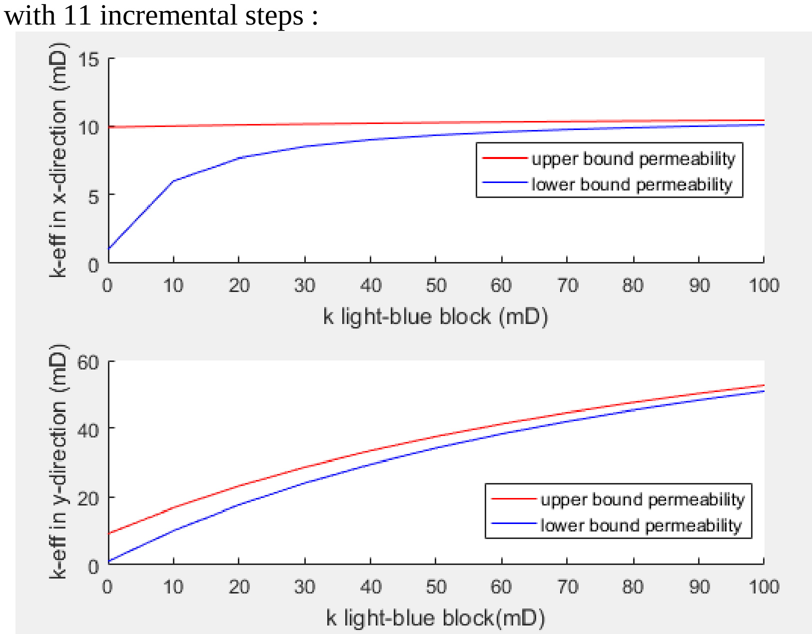
2. Based on configuration in figure 1, p effective ermeability in x-direction lies between 0.991099 and 9.909920 mD, while effective permeability in y-direction lies between 0.910091 and 9.099927 mD
   Matlab script to compute effective permeability of block configuration in figure 1 :

```
disp(' ')
disp('Part 1 Number 2 :')
disp(' ')
q = 2;
p = 2;
k = [1 100 ; 10 0.001 ];
kx = k;
ky = k;
k_4 = linspace(k(end,end),100,11);
k_1 = linspace(0,100,26);
k1x = zeros(q,1);
k2x = zeros(1,q);
k1y = zeros(1,q);
k2y = zeros(q,1);

for j = 1 : length(k_4)
    kx(end,end) = k_4(j); % sensitivity of light blue block in x-direction
    ky(end,end) = k_4(j); % sensitivity of light blue block in x-direction
    for i = 1 : q
        k1x(i) = p*prod(kx(i,1:p))/sum(kx(i,:)); % harmonic average
permeability in x direction
        k2y(i) = sum(ky(i,1:p))/(p); % aritmathic average permeability in
y direction
    end

    for i = 1 : p
        k2x(i) = sum(kx(1:q,i))/(q); % aritmathic average permeability in
x direction
        k1y(i) = q*prod(ky(1:q,i))/sum(ky(:,i)); % harmonic average
permeability in y direction
    end
    k_lox(j) = sum(k1x)/(q); % lower bound permeability in x direction
    k_upx(j) = q*prod(k2x)/sum(k2x); % upper bound permeability in x
direction
    k_loy(j) = sum(k1y)/(p); % lower bound permeability in y direction
    k_upy(j) = p*prod(k2y)/sum(k2y); % upper bound permeability in y
direction
```

```
end

fprintf('permeability in x-direction lies between %f and %f mD
\n',k_lox(1),k_upx(1))
fprintf('permeability in y-direction lies between %f and %f mD
\n',k_loy(1),k_upy(1))

disp(' ')
disp('Part 1 Number 4 is shown in figure(1)')
figure(1)
subplot (2,1,1)
hold on
plot(k_4,k_upx,'r')
plot(k_4,k_lox,'b')
ylabel('k-eff in x-direction (mD)')
xlabel('k light-blue block (mD)')
legend('upper bound permeability','lower bound permeability')
hold off

subplot (2,1,2)
hold on
plot(k_4,k_upy,'r')
plot(k_4,k_loy,'b')
ylabel('k-eff in y-direction (mD)')
xlabel('k light-blue block(mD)')
legend('upper bound permeability','lower bound permeability')
hold off
```

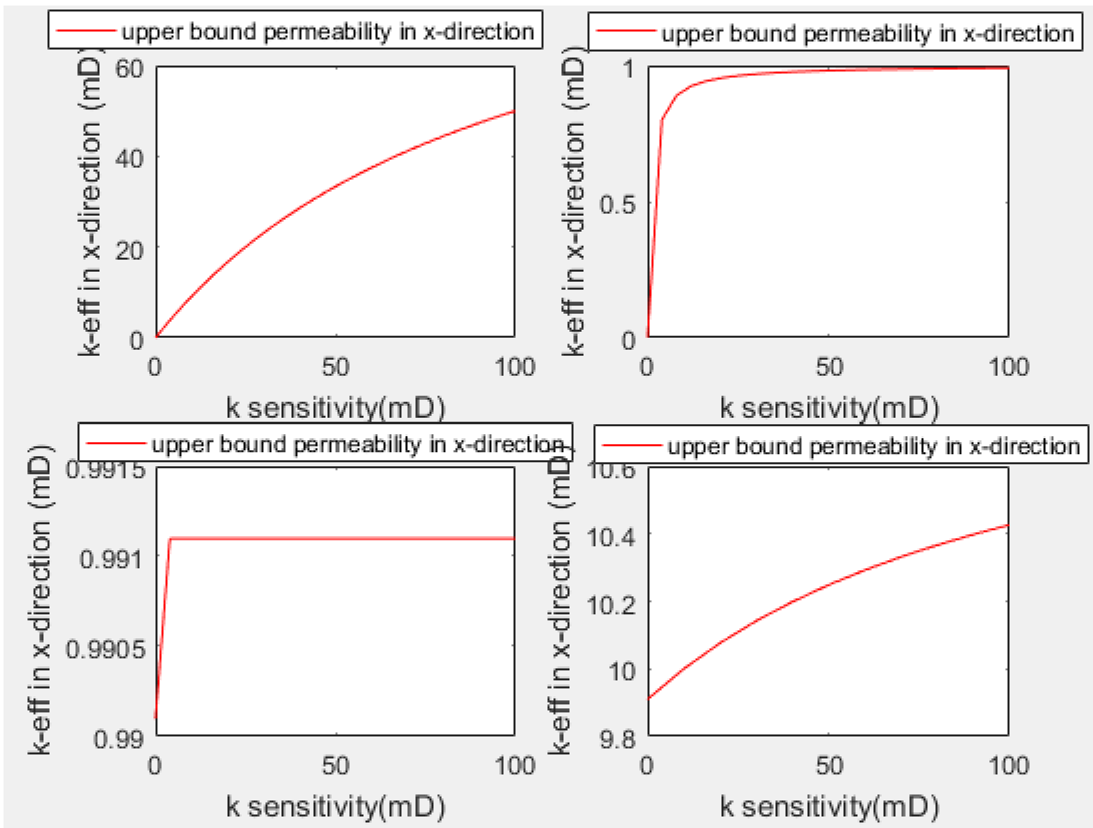
3. All fine cells have only one exact value of permeability both in xx and yy direction. However, after upscaling the value of permeability becomes uncertain. This uncertainty mainly depends on how we construct combination of blocks to average. Therefore, there will be a range which the value of permeability lies between. Those are upper and lower bound of permeability.

4. Sensitivity was undertaken by changing the value of light-blue block (block number 4 in figure 1) from 0.001 until 100 with 11 incremental steps :



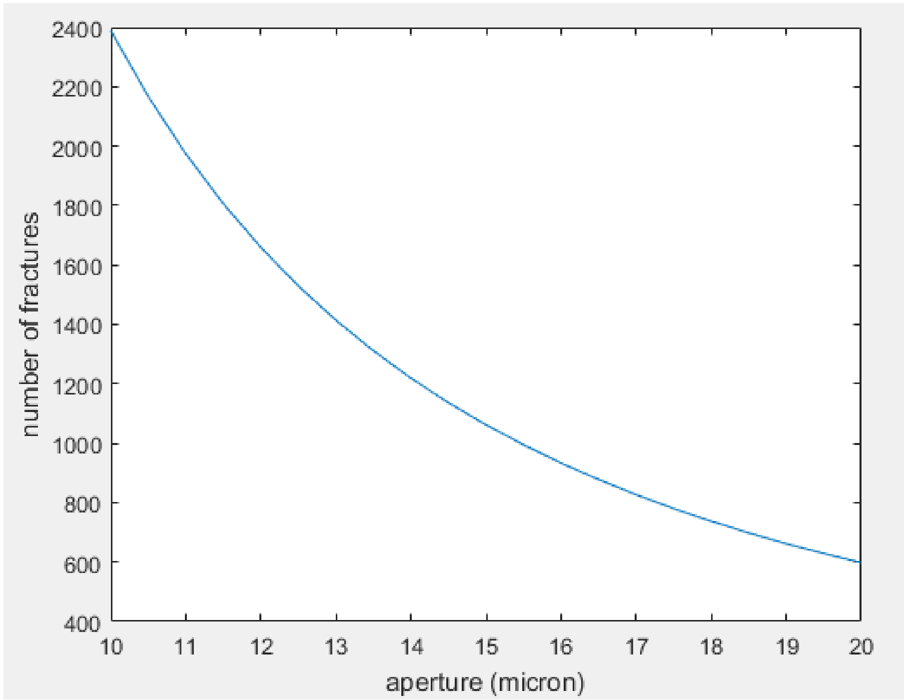
B. Part 2 :

1. Block number 1 should be changed to address the opportunity to obtain higher permeability rather than fracturing other blocks. Along x-axis, the flow will be split between upper and lower blocks. Since the block beside fractured block has 100 mD permeability, harmonic average between the first and second block is still high. Meanwhile in lower blocks, harmonic average between 10 mD and 0.001 mD is very low. Therefore, fluid tends to flow along upper blocks. This is also confirmed by sensitivity plot below :



2. Assuming the desired effective permeability in fractured block is 200 mD, 2389 fractures with aperture 10 micron must be generated.
3. Since the fracture lies along x-direction, it will alter significantly the value of effective permeability in x-direction. However, it will not much effective altering effective permeability in y-direction.

   Both aperture and number of fractures can give significant contribution to effective permeability. the more number of fractures, the smaller aperture. Sensitivity of aperture and number of fractures is shown in figure below :



Matlab script for fractured system in question 2 and question 3 is as follows :

```
disp('block 1,1 is chosen as fractured block due to its high value
of effective permeability in x-direction')
disp('as shown in figure(2)')
disp(' ')
k_efx = 200; % k-effective of fractured block in x-direction
k_efy = ky; % k-effective of fractured block in y-direction
b = 10; % fract aperture in micron
a = b*10^-6; % fract aperture in meter
a2 = 10^-6.*linspace(b,20,21);

k1xf = zeros(q,1);
k2xf = zeros(1,q);
k1yf = zeros(1,q);
k2yf = zeros(q,1);

kxf = kx;
kyf = ky;
kxf(1,1) = k_efx; % new value of kx in first block

for j = 1 : length(a2);
kf(j) = a2(j)^2/(12*10^-12)*1000;
nf(j) = ceil((k_efx-kx(1,1))*1/(a*(kf(j)-kx(1,1))));

for i = 1 : q
     k1xf(i) = p*prod(kxf(i,1:p))/sum(kxf(i,:)); % harmonic average
permeability in x direction
     k2yf(i) = sum(kyf(i,1:p))/(p); % aritmathic average
permeability in y direction
end
```

```
for i = 1 : p
    k2xf(i) = sum(kxf(1:q,i))/(q); % aritmathic average permeability
in x direction
    k1yf(i) = q*prod(kyf(1:q,i))/sum(kyf(:,i)); % harmonic average
permeability in y direction
end
end

k_loxf = sum(k1xf)/(q); % lower bound permeability in x direction
k_upxf = q*prod(k2xf)/sum(k2xf); % upper bound permeability in x
direction
k_loyf = sum(k1yf)/(p); % lower bound permeability in y direction
k_upyf = p*prod(k2yf)/sum(k2yf); % upper bound permeability in y
direction

figure(3)
plot(10^6.*a2,nf)
xlabel('aperture (micron)');
ylabel('number of fractures');

disp('Part 2 Number 2 :')
fprintf('if we have effective permeability along fractured block 1,1
= %1.2f mD. \n',k_efx)
fprintf('we need %g number of %g micron width uniform fracture.
\n',nf(1),b)
disp(' ')
disp('Part 2 Number 3 :')
fprintf('effective permeability in x-direction after fracturing
block 1,1 lies between %1.2f mD to %1.2f mD. \n',k_loxf,k_upxf)
fprintf('effective permeability in y-direction after fracturing
block 1,1 lies between %1.2f mD to %1.2f mD. \n',k_loyf,k_upyf)
```

4. There are 2 cases explained in Oda's paper. The first one is assuming the fluid flows only through fracture or ignoring matrix permeability. The second one is considering matrix permeability. Permeability in any direction can be approximated by this following equation :

$$k_{ij}^{(c)} = \gamma(1-\alpha_0)\left(P_{kk}\delta_{ij} - P_{ij}\right)$$

Where

$$P_{ij} = \frac{\pi\rho}{4}\int_0^{\infty}\int_0^{\infty}\int_{\omega} r^2 t^3 n_i n_j E(n,r,t)d\omega\, dr\, dt$$

$k_{ij}^{(c)}$ is then employed to construct a permeability tensor.

4. Conclusion

Several conclusion can be drawn from this assignment. Those are as follows :

1. Permeability upscaling can lead to uncertainties.
2. Approximate value of permeability lies between lower and upper bound of effective permeability.
3. Aperture of fractures even in micron scale can give significant contribution to the permeability of certain block.
4. Aperture fracture is inversely proportional to number of fractures.