MULTIPHASE TRANSPORT SIMULATOR

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Abstract

Numerical method using Matlab is employed to solve multiphase 1D and 2D transport equation in the absence of gravitational force and capillary pressure under incompressible condition. The simulation result is presented in water saturation profile. Ultimately, as a comparison, analytical solution are also provided to confirm the numerical simulation result.

Introduction

Transport equation is derived from mass conservation equation. Under incompressible condition, multiphase mass conservation is as following.

$$\frac{\partial(\phi S_\alpha)}{\partial t} - \nabla.(\lambda_\alpha \nabla P) = q_\alpha \quad (1)$$

$$\frac{\partial(\phi S_\alpha)}{\partial t} - \nabla.\left(\frac{\lambda_\alpha}{\lambda_T}\lambda_T \nabla P\right) = q_\alpha \quad (2)$$

Where terms $\lambda_T \nabla P$ is equal to total velocity along the domain, and terms $\frac{\lambda_\alpha}{\lambda_T}$ denotes fractional flow of a phase compared to total flow. Thus, equation 2 can be simplified to become as following.

$$\frac{\partial(\phi S_\alpha)}{\partial t} - \nabla.(f_\alpha U_T) = q_\alpha \quad (3)$$

Methodology

1. Numerical Solution

Equation 3 above can be solved numerically by splitting the domain into smaller grids in both space and time grids.. In order to find solution in form of saturation profile, two methods are employed based on time discretization.

a. Explicit Method :

Explicit method employs Euler forward method in time grids which enables saturation prediction in next time step based on known current saturation. In explicit method, equation 3 is discretized as following.

$$\phi \frac{{S_{ij}}^{n+1} - {S_{ij}}^{n}}{\Delta t} + \left(\frac{{f_{i,j}}^{n} U_{X\,i+1,j} - {f_{i-1,j}}^{n} U_{X\,i,j}}{\Delta x} \right) + \left(\frac{{f_{i,j}}^{n} U_{Y\,i,j+1} - {f_{i,j-1}}^{n} U_{Y\,i,j}}{\Delta y} \right) = q_{i,j} \quad (4)$$

Equation 4 can be rearranged as following.

$${S_{ij}}^{n+1} = {S_{ij}}^{n} + \frac{\Delta t}{\phi} \left[q_{ij} - \left(\frac{{f_{i,j}}^{n} U_{X\,i+1,j} - {f_{i-1,j}}^{n} U_{X\,i,j}}{\Delta x} \right) - \left(\frac{{f_{i,j}}^{n} U_{Y\,i,j+1} - {f_{i,j-1}}^{n} U_{Y\,i,j}}{\Delta y} \right) \right] \quad (5)$$

Where subscript i and j refer to space grid location, and superscript n refer to time step. In 1D domain, equation 5 is reduced as following.

$${S_i}^{n+1} = {S_i}^{n} + \frac{\Delta t}{\phi} \left[q_i - \left(\frac{{f_i}^{n} U_{i+1} - {f_i}^{n} U_{i,j}}{\Delta x} \right) \right] \quad (6)$$

b. Implicit Method

Implicit method employs Euler backward in time grids. Since current saturation is not known, this method can only be solved by iterative method. In implicit method, equation 3 is discretized as following.

$$\phi \frac{{S_{ij}}^{n+1} - {S_{ij}}^{n}}{\Delta t} + \left(\frac{f_{i,j}^{n+1} U_{X\,i+1,j} - f_{i-1,j}^{n+1} U_{X\,i,j}}{\Delta x} \right) + \left(\frac{f_{i,j}^{n+1} U_{Y\,i,j+1} - f_{i,j-1}^{n+1} U_{Y\,i,j}}{\Delta y} \right) = q_{i,j} \quad (7)$$

Introducing Taylor expansion to predict f in next time step.

$$f^{n+1}(S_{i,j}) = f^{v}(S_{i,j}) + \left(\frac{\partial f}{\partial S} \right)_{S_{i,j}}^{v} \delta S_{i,j}^{v+1} \quad (8)$$

$$\delta S_{i,j}^{v+1} = S_{i,j}^{v+1} - \delta S_{i,j}^{v} \quad (9)$$

Substitute equation 8 and equation 9 into equation 7 to become as following.

$$\frac{\phi}{\Delta t} \delta S_{i,j}^{v+1} + \frac{\left[\left(\frac{\partial f}{\partial S} \right)_{S_{i,j}}^{v} U_{X\,i+1,j}\, \delta S_{i,j}^{v+1} - \left(\frac{\partial f}{\partial S} \right)_{S_{i-1,j}}^{v} U_{X\,i,j}\, \delta S_{i-1,j}^{v+1} \right]}{\Delta x}$$
$$+ \frac{\left[\left(\frac{\partial f}{\partial S} \right)_{S_{i,j}}^{v} U_{Y\,i,j+1}\, \delta S_{i,j}^{v+1} - \left(\frac{\partial f}{\partial S} \right)_{S_{i,j-1}}^{v} U_{Y\,i,j}\, \delta S_{i,j-1}^{v+1} \right]}{\Delta x} = R(S^{v}) \quad (10)$$

Where $R(S^v)$ is defined as following.

$$R(S^{v}) = q_{i,j} - \left(S_{i,j}^{v} - S_{i,j}^{n} \right) - \frac{\left[f_{i,j}^{v} U_{X\,i+1,j} - f_{i-1,j}^{v} U_{X\,i,j} \right]}{\Delta x} - \frac{\left[f_{i,j}^{v} U_{Y\,i,j+1} - f_{i,j-1}^{v} U_{Y\,i,j} \right]}{\Delta y} \quad (11)$$

Similarly as explicit method, subscript i and j refer to space grid location, superscript n refer to time step, and superscript v refer to iteration. In 1D domain, equation 10 and 11 is reduced as following.

$$\frac{\phi}{\Delta t} \delta S_{i}^{v+1} + \frac{\left[\left(\frac{\partial f}{\partial S} \right)_{S_{i}}^{v} U_{i+1}\, \delta S_{i}^{v+1} - \left(\frac{\partial f}{\partial S} \right)_{S_{i-1}}^{v} U_{X\,i,j}\, \delta S_{i-1}^{v+1} \right]}{\Delta x} = R(S^{v}) \quad (12)$$

$$R(S^{v}) = q_i - (S_i^{v} - S_i^{n}) - \frac{[f_i^{v}\, U_{i+1} - f_{i-1}^{v}\, U_i]}{\Delta x} \quad (13)$$

2. **Analytical Solution**

   Analytical solution is employed to confirm the result from numerical solution. In this case, Buckley Leverett equation is being used to model two phase flow along the reservoir which then creates a shock front in between the phase's boundary. Saturation profile along the reservoir is obtained as multiplication product between time and characteristic velocity. The distinguished Buckley Leverett equation is provided in equation 14 below.

$$x_f = \frac{qt}{A\phi} \frac{\partial f_w}{\partial S_w} \quad (14)$$

Result and Discussion

Numerical and analytical solution is presented in terms of saturation profile along the reservoir. Some default input parameter are presented in table 1 below.

Table 1 : Default Input Parameter

Parameter	***1D***	***2D***	***Unit***	***Parameter***	***1D***	***2D***	***Unit***
LX	*100*	*100*	*m*	***krwe***	*0.7*	*0.7*	-
LY	-	*100*	*m*	***kroe***	*0.8*	*0.8*	-
NX	*500*	*50*	-	$\boldsymbol{S_{wc}}$	*0.2*	*0.2*	-
NY	-	*50*	-	$\boldsymbol{S_{or}}$	*0.1*	*0.1*	-
$\boldsymbol{\mu_w}$	10^{-3}	10^{-3}	*Pa.s*	$\boldsymbol{\phi}$	*0.3*	*0.3*	-
$\boldsymbol{\mu_o}$	10^{-2}	10^{-2}	*Pa.s*	***t***	*1000*	*30*	*s*

1. **1D Saturation Profile**

   One dimension reservoir with constant cross section area is injected by water so that the water flows from left part to the right part of the reservoir. Initially, the reservoir is saturated with oil and *Swc* fraction of water. The water begins to flow after time zero until determined time. Along with numerical simulation which consist of explicit and implicit method, analytical simulation is also provided to confirm saturation profile from numerical simulation.

   Both explicit and implicit method are using upwind in space grid which is derived from Taylor expansion. Since upwind approximation only takes two grids, it may result a first order error proportional to *dx* or distance between two grids. An error analysis had also been done to identify the error propagation towards the change of number of grids and presented in figure 1. From figure 1, it can be seen upwind approximation may result an average error in linear form.

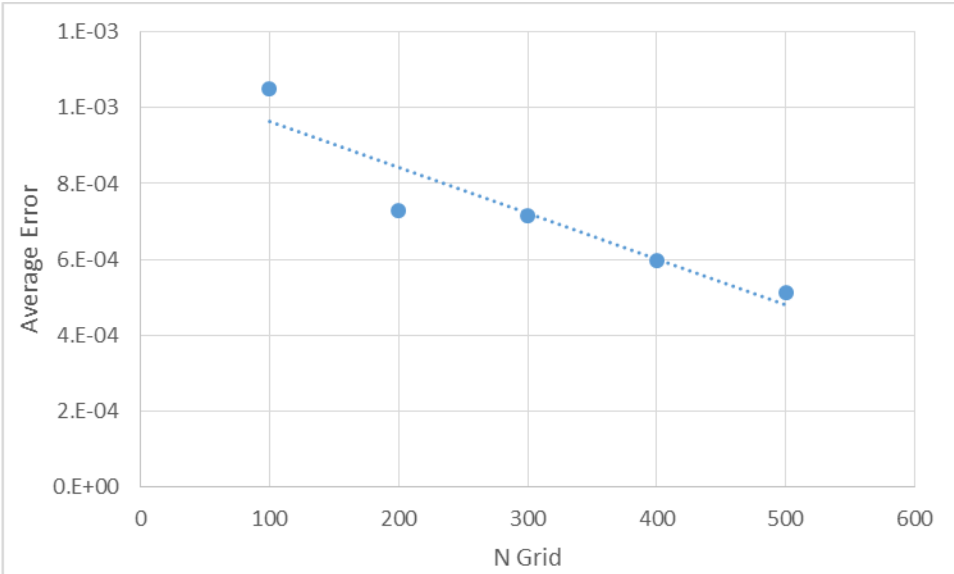

Figure 1 : Average Error towards the Change of Number of Grids

Those three methods perfectly match each other. They have rough similarity in shock front. In analytical method, shock front profile is caused by discontinuity in saturation function or unphysical situation where two saturation exist in one certain x position. To tackle this problem, Buckley Leverett solution is modified by balancing area under the graph. Thus, the saturation suddenly jumps from S_{wc} to $S_{w\,shock}$, and so does the velocity.

In numerical method, this shock front saturation does not actually happens. Unlike analytical method, shock front in numerical method possesses rapid change within narrow space, but continuous transition. This smooth transition leads to different saturation between numerical and analytical solution. From figure 1, numerical solution has slightly lower shock saturation and intersect analytical solution line between $S_{w\,shock}$ and S_{wc}. However, this difference is within allowable tolerance, and is still acceptable to be a correct solution. As scale of displacement grows larger, the front looks more likely to be a discontinuity and looks more and more similar to analytical solution. Comparison between numerical and analytical solution is presented in figure 2 below.

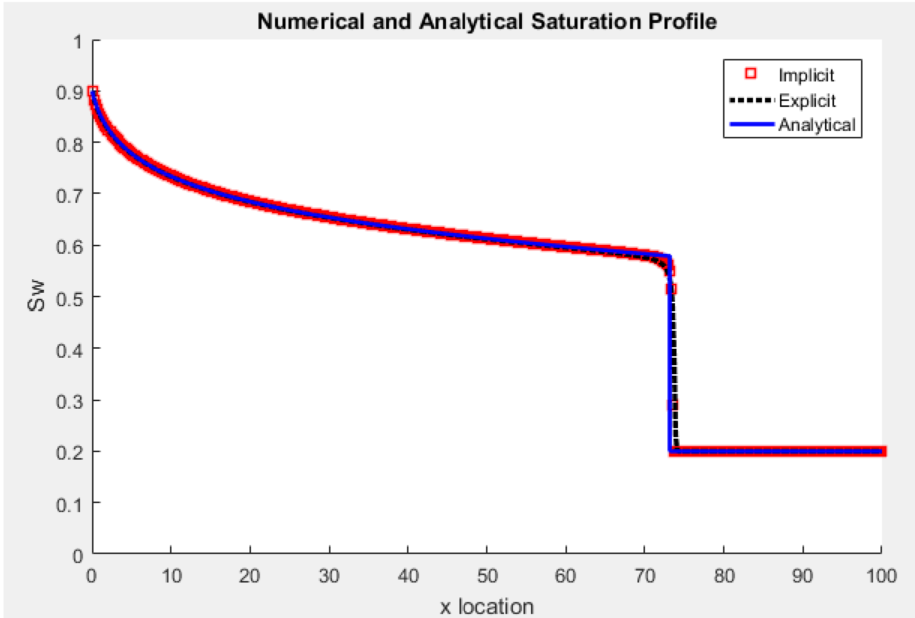

Figure 2 : Saturation Profile in 1D Reservoir with 500 Space Grid

In this study, CFL method is employed to ensure stability especially in explicit method which is not always stable. Implicit method, however, although it is unconditionally stable, stability condition is derived from linearity. Meanwhile transport equation is not a linear equation. Thus, sometime unstable solution may appear in both method. Since both method give similar result and have possibility of unstable solution, explicit method is more preferable to solve transport equation. It requires no iteration and less time consumed in calculation. Unstable solution in explicit and implicit method is presented in figure 3 below.

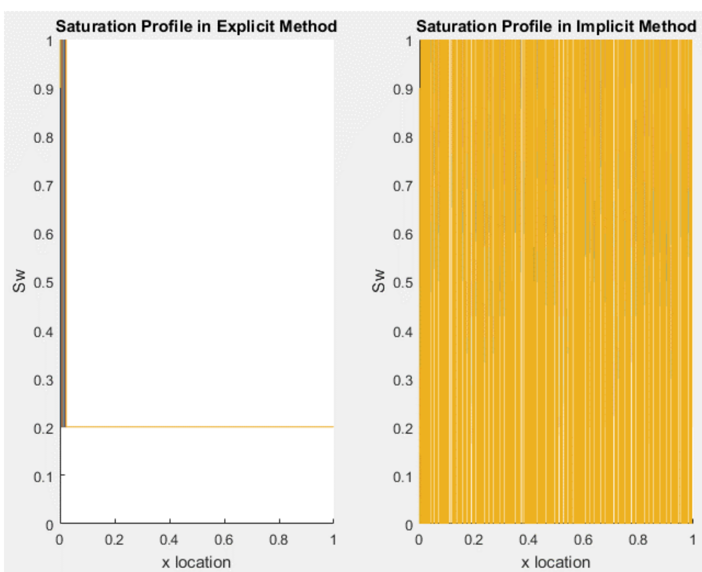

Figure 3 : Unstable Solution in Explicit and Implicit Method

Sensitivity is undertaken to identify the effect of fluid properties change into saturation profile. In this case, three sensitivity cases are built based on qualitative relation between water viscosity and oil viscosity.

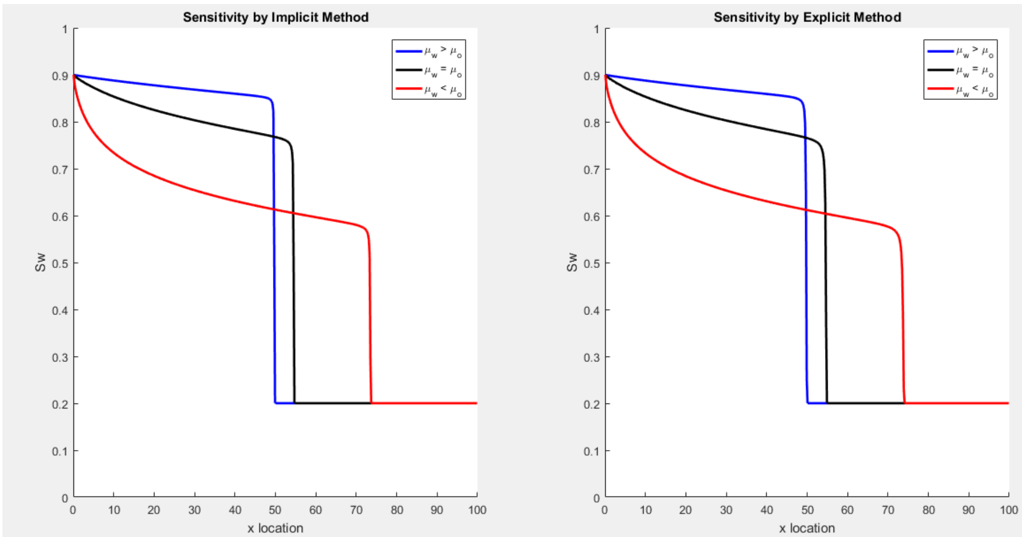

Figure 4 :Effect of Fluid Properties Change Towards Saturation Profile

From figure 4 above, water viscosity influences saturation profile by affecting mobility and fractional flow. As water viscosity becomes higher, water will be less mobile than oil. More oil will be displaced by water due to its high $S_{w\,shock}$ and later time to breakthrough. Inversely, if water viscosity is lower than oil viscosity, water will be more mobile and has higher velocity. Higher velocity leads to earlier breakthrough.

2. 2D Saturation Profile

Similarly as one dimension domain, in two dimension domain numerical simulation still shows shock profile where saturation change rapidly from $S_{w\,shock}$ to S_{wc}. However, the shock profile does not only happen in front, but also happen in left and right side of the water propagation. Again, this is not actually a discontinuity. Saturation change very rapidly within narrow space but still possesses continuous transition.

To ensure stability, the same CFL method is employed in 2D reservoir. Similar to what have been done in 1D reservoir, explicit method is more preferable to solve transport equation because easier and less calculation time consumed. Saturation profile in 2D reservoir is presented in figure 5 below

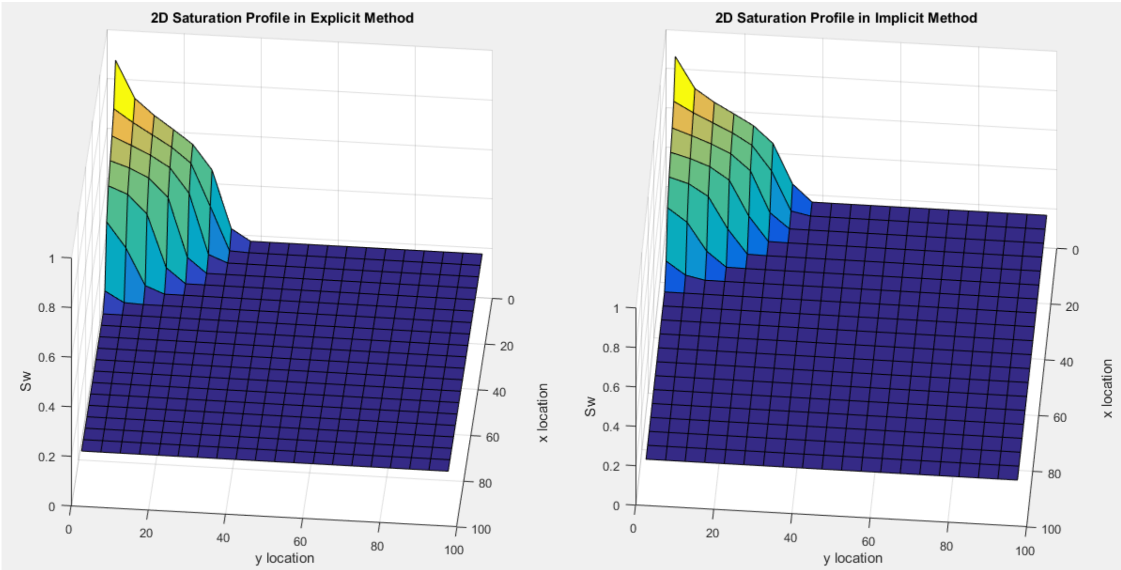

Figure 5 : Saturation Profile in 2D Reservoir Using Explicit method (left) and Implicit Method (right)

In 2D reservoir, water will flow in both x and y direction and sweeps the oil in entire reservoir area depending on the velocity in both x and y direction. Permeability anisotropy plays important role on this sweep efficiency. In this study, the reservoir model has homogenous and isotropic property. Thus the water movement grows forward and sideward in the same speed. Therefore, along with closer mobility ratio, isotropic reservoir is preferable to ensure better sweep efficiency.

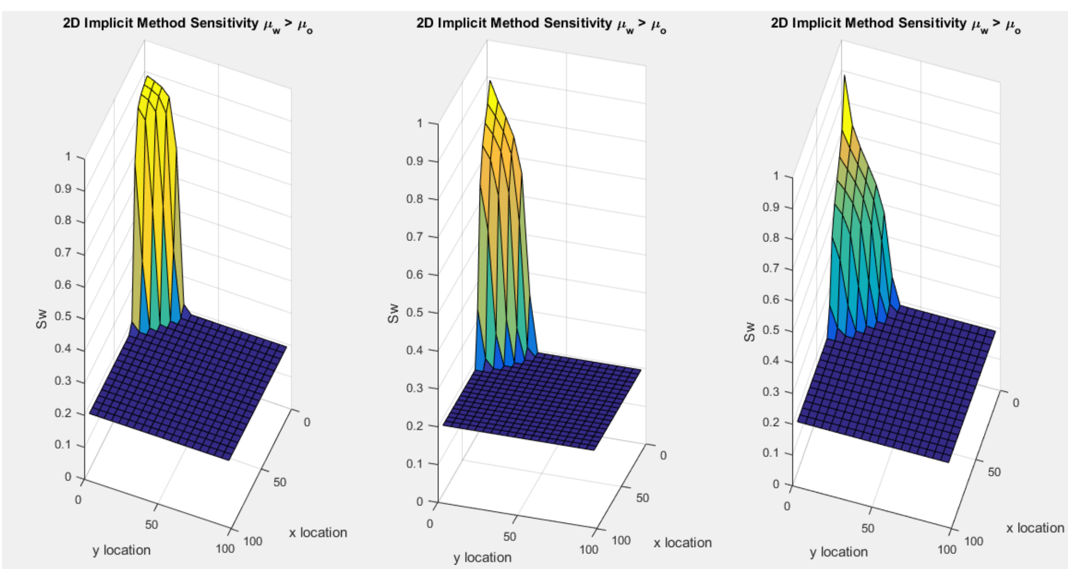

Figure 6 : Sensitivity Analysis in 2D Reservoir Using Implicit Method

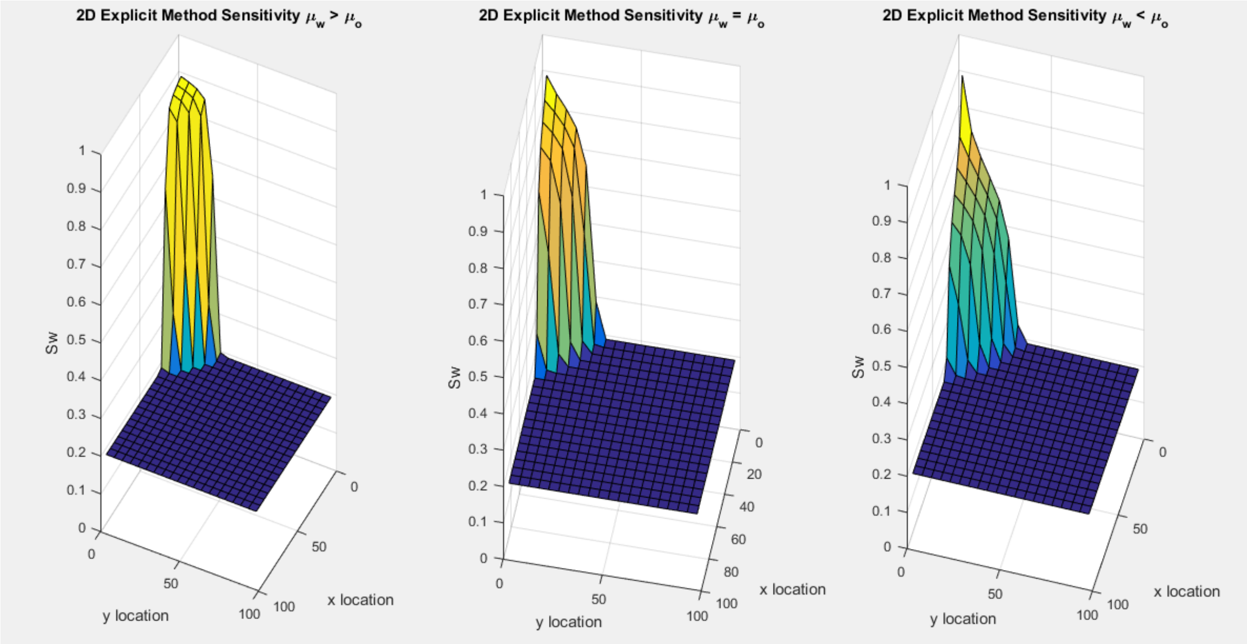

Figure 7 : Sensitivity Analysis in 2D Reservoir Using Explicit Method

Similar to sensitivity in 1D reservoir, water viscosity influences efficiency of water displacement. As water viscosity increases, $S_{w\,shock}$ also increases. Increasing water viscosity will make the water less mobile and leave less oil inside the reservoir. Once the breakthrough time is reached, water will flow freely into producer well.

Conclusions

Some conclusions can be withdrawn from this study as following.

1. Numerical solution using explicit and implicit method can be employed to solve transport equation.
2. Explicit method is more preferable due to less calculation time and requires no iteration.
3. Water mobility relative to oil mobility influences saturation profile. If water is less mobile than oil, than higher $S_{w\,shock}$ can be achieved, thus increasing efficiency.
4. In 2D reservoir, permeability anisotropy plays important role on determining sweep efficiency.

References

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