SINGLE PHASE FLOW SIMULATOR

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Abstract

Numerical method using Matlab is employed to solve single phase 1D and 2D flow under three different flow systems : incompressible flow, slightly compressible flow, and compressible flow. The simulation result is presented in pressure and velocity solution. Ultimately, as a comparison, analytical solution are also provided to confirm the numerical simulation result.

Introduction

In petroleum engineering world, mass conservation equation describes the behavior of fluid flow through a medium. To solve mass conservation equation, several methods can be employed. Starting from general mass conservation equation, pressure and velocity solution along the reservoir can be obtained under three different flow types.

a. Incompressible Flow Equation :

   In incompressible flow, density is considered to be constant along the space and time. Mass conservation equation in single phase incompressible flow is a linear equation as follows :

$$-\nabla.\left[\lambda\,\nabla P\right] = q$$

b. Slightly compressible Flow Equation :

   In slightly compressible flow, density is considered to be constant along the space, but changed along the time. Mass conservation equation in single phase slightly compressible flow is as follows :

$$(\phi c_e)\frac{\partial P}{\partial t} - \nabla.\left[\lambda\,\nabla P\right] = q$$

c. Compressible Flow Equation :

   In compressible flow equation, density is always changing by time and space. Thus, mass conservation equation in single phase compressible flow is following general mass conservation equation :

$$\frac{\partial(\rho\phi)}{\partial t} - \nabla.\left[\rho\lambda\,\nabla P\right] = \rho q$$

Methodology

1. Discretization

Numerical simulation splits the domain into several smaller grids, in which certain parameter value are assigned into each grids. In this case, N grid are constructed to split pressure into N number of pressure which are evaluated in grid center, while N+1 number of velocity are evaluated in grid interface. Velocity is function of mobility. Thus, mobility is also evaluated in grid interface. Moreover, to simplify the matrix system later, transmissibility is introduced which is division product between mobility and square distance between two grids. Schematic of discretization system in one dimension is presented in figure 1 below :

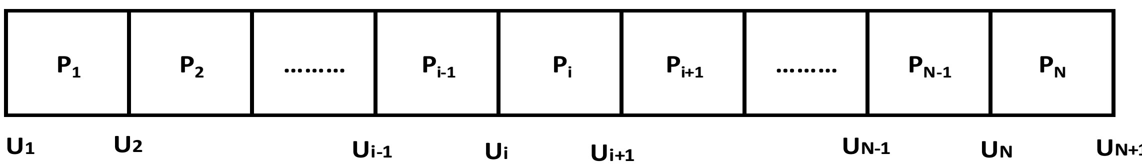
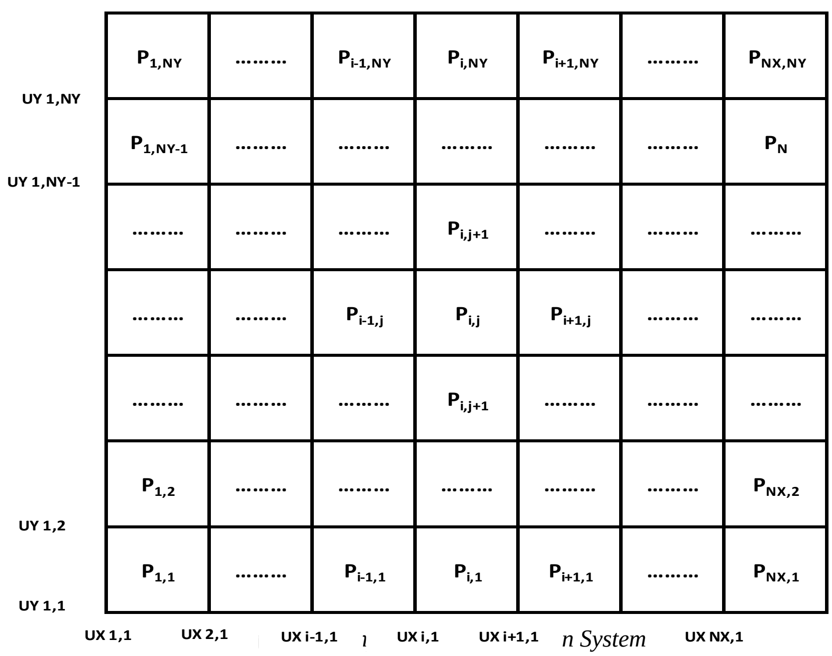


Figure 1 : Simple 1D Discretization System

In two dimensions domain with domain length LX and width LY, NX times NY grids are constructed which pressure are assigned into each grid. Velocity has two direction and are assigned at grid boundary. Thus, NX+1 times NY velocity in x-direction and NX times NY+1 velocity in y-direction are constructed. Schematic of discretization system in two dimensions is presented in figure 2 below.



In incompressible system, since density is constant by time and space, the discretization results a matrix system in linear form as follows :

$$AP = q$$

Where A is a matrix consists of a set of transmissibility, P is vector of unknown pressure, and q is source term.

Secondly, in slightly compressible system, since density is only changing by time and porosity is constant, the discretization results two types of matrix system which can be solved explicitly and implicitly :

$$P^{n+1} = C\backslash[q - A\ P^n + C\ P^n]$$

$$P^{n+1} = [C + A]\backslash[q + C\ P^n]$$

Where C is a matrix consists of $\phi c_{eff}/\Delta t$, and superscript *n* and *n+1* denote specific time step and next time step.

However, in fully compressible flow system, density and porosity is function of pressure. And since pressure is function of time, the solution is obtained using iterative method which give residual term approaching zero. The final matrix system can be presented in implicit and explicit form. In this case, implicit solution is preferable due to its stability advantage.

2. Boundary Condition

This simulator enables one to apply two types of boundary condition. Firstly, Dirichlet boundary condition is applied. This will give constant pressure at domain boundaries. Secondly, well constrained boundary condition which keeps wellbore pressure constant.

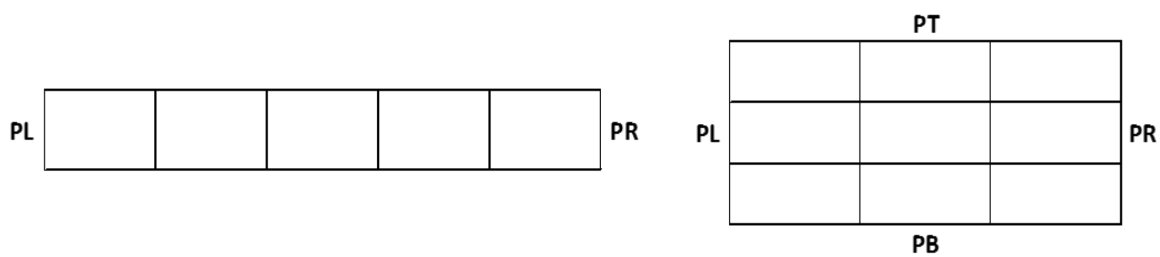


Figure 3 : Dirichlet Boundary Condition in 1D Domain (left), and 2D Domain (right)

3. Analytical Solution

Analytical solution in incompressible flow system can be obtained by twice integration of space. By applying Dirichlet boundary condition, 1D incompressible flow can be solved analytically as follows.

$$-\nabla.[\lambda\ \nabla P] = q$$

$$P = \frac{PR - PL}{L}x + PL$$

While in 1D slightly compressible flow, analytical solution can be obtained as follows :

$$\sigma = \sqrt{2Dt}$$

$$P(x,t) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{1}{2}\frac{x^2}{\sigma^2})$$

Result and Discussion

Numerical simulation is presented in pressure and velocity solution. Using default input data as presented in table 1 below and well constrained boundary condition, pressure and velocity profile for both 1D and 2D cases can be obtained, note that in 1D problem in x direction, LY and NY can be neglected.

Table 1 : Default Input Data for Numerical Simulation

Parameter	***Incompressible***	***Slightly Compressible***	***Fully Compressible***
LX	*10*	*10*	*10*
LY	*10*	*10*	*10*
NX	*20*	*20*	*20*
NY	*20*	*20*	*20*
Nwell	*2*	*2*	*2*
PI	*[1000 1000]*	*[1000 1000]*	*[1000 1000]*
P_W	*[1 0]*	*[1 0]*	*[1 0]*
Grid	*[1 N]*	*[1 N]*	*[1 N]*
Type	*Homogenous*	*Homogenous*	*Homogenous*
μ	10^{-3}	10^{-3}	10^{-3}
k	10^{-14}	10^{-14}	10^{-14}
t	-	*1*	*1*
nt	-	*10*	*10*
C_{eff}	-	*1*	-
C_r	-	-	*1*
C_f	-	-	*1*
ϕ_i	-	*0.1*	*0.1*
ρ_1	-	*0.1*	*0.1*

1. Pressure Solution

Numerical simulation is undertaken to find pressure solution in 1D and 2D reservoir under constant wellbore pressure (*Pw*) boundary condition. Pressure solution along 1D and 2D reservoir is respectively presented in line plot in figure 4 and in surface plot in figure 5.

In incompressible system, pressure is only function of space and has no time dependency as presented in figure 4 and figure 5 left. Pressure in any specific space is constant throughout time.

In slightly compressible system, pressure is not only changing by space, but also changing by time. However, since density is only changing by time and constant by space, pressure solution at late time or when the system reaches steady state condition acts like incompressible system. At steady state condition, slightly compressible equation becomes linear equation, thus the solution is acting like incompressible solution. Pressure solution in slightly compressible system is presented in figure 4 and figure 5 middle. Once steady state is reached, pressure solution becomes linear line and exactly the same profile as incompressible system.

However, in fully compressible flow system, density and porosity is function of pressure. Since pressure depends on time, density and porosity will also has time dependency. When the flow is reaching steady state condition, the equation does not become a linear equation due to density. Divergence of density times pressure gradient will always be constant. Thus, both density and pressure gradient will adjust each other to make the multiplication constant.

Physically, density is proportional to pressure. When pressure is high, density is also high, it needs lower pressure gradient to push the same mass of fluid. Reversely, when pressure is low, density is also low, it needs higher pressure gradient to push the same mass of fluid. Pressure solution in fully compressible system is presented in figure 4 and figure 5 right.

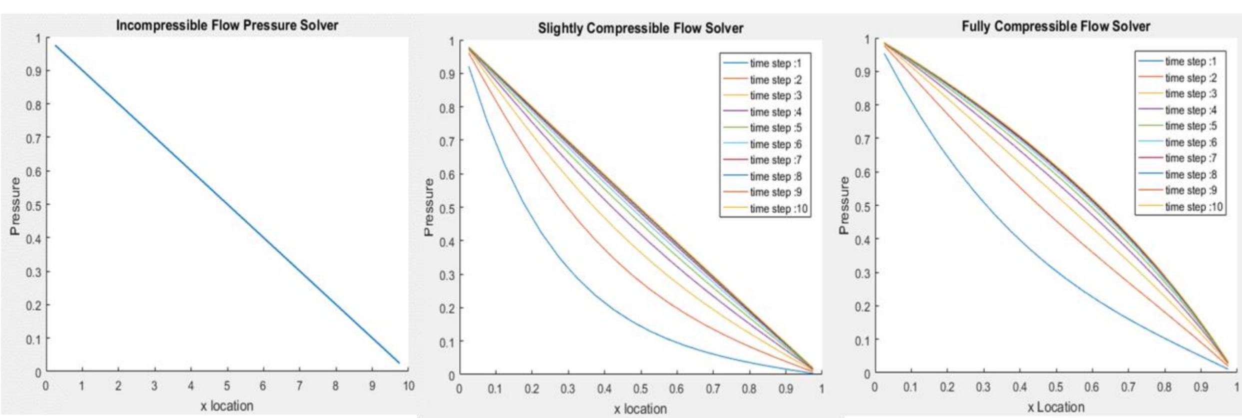


Figure 4 : Pressure Profile in 1D Reservoir Under Three Different System

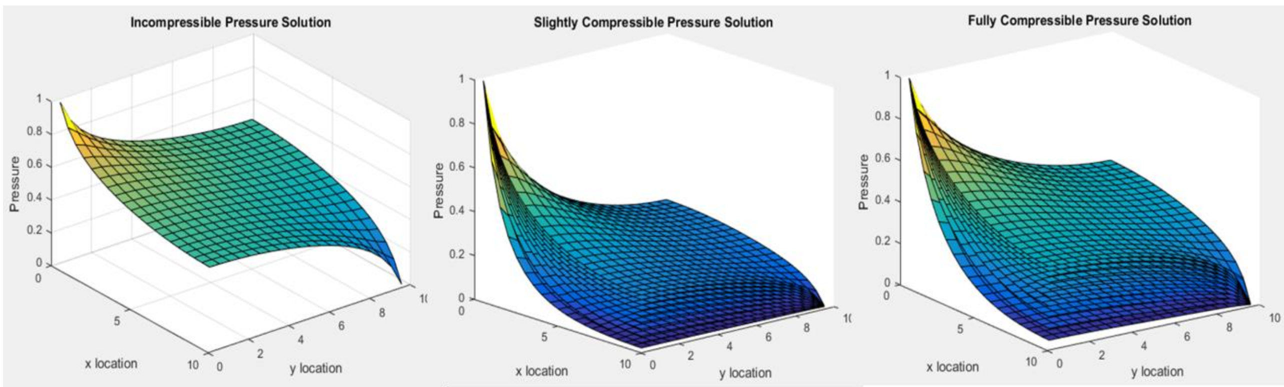


Figure 5 : Pressure Profile in 1D Reservoir Under Three Different System with Constant Wellbore Pressure

2. Velocity Solution

According to Darcy's law, velocity is function of pressure gradient or change in pressure divided by distance between two points. Similarly as pressure solution, velocity solution along 1D and 2D reservoir is respectively presented in line plot in figure 6 and in surface plot in figure 7. In multi dimension domain, permeability becomes tensor. Hence enables fluid to flow in multiple direction. In 2D case, only velocity in x and y direction are evaluated.

In incompressible flow system, velocity is always constant throughout time both in 1D and 2D domain. Pressure along the reservoir follows linear equation. Hence pressure gradient is always the same. This uniform pressure gradient leads to constant velocity except in location where well is put. Source term added by wells influences the velocity. Velocity profile in 1D and 2D incompressible reservoir can be seen in figure 6 and figure 7 left.

Velocity profile in slightly compressible system tends to approach constant as the flow entering steady state condition which no more pressure changes by time. Physically, at earlier time high pressure gradient needs to be applied in order to move static fluid. As time increases, pressure becomes more stable as well as velocity get stabilized. Velocity profile in 1D and 2D slightly compressible reservoir can be seen in figure 6 and figure 7 middle.

In compressible system, since density and porosity is always changing by pressure, steady state condition does not simply linearized the equation. In order to hold mass conservation principle, density and velocity must adjust each other so that the mass amount flowing into certain grid is constant. If density is higher, then velocity is lower. In other words, density is inversely proportional to velocity. Velocity profile in 1D and 2D fully compressible reservoir can be seen in figure 6 and figure 7 right.

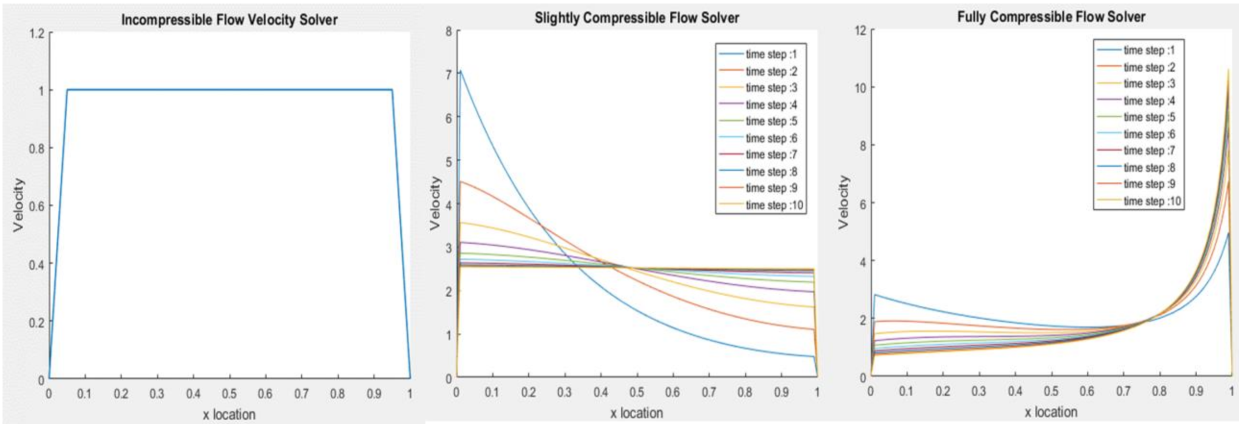


Figure 6 : Velocity Profile in 1D Reservoir Under Three Different System with Constant Wellbore Pressure

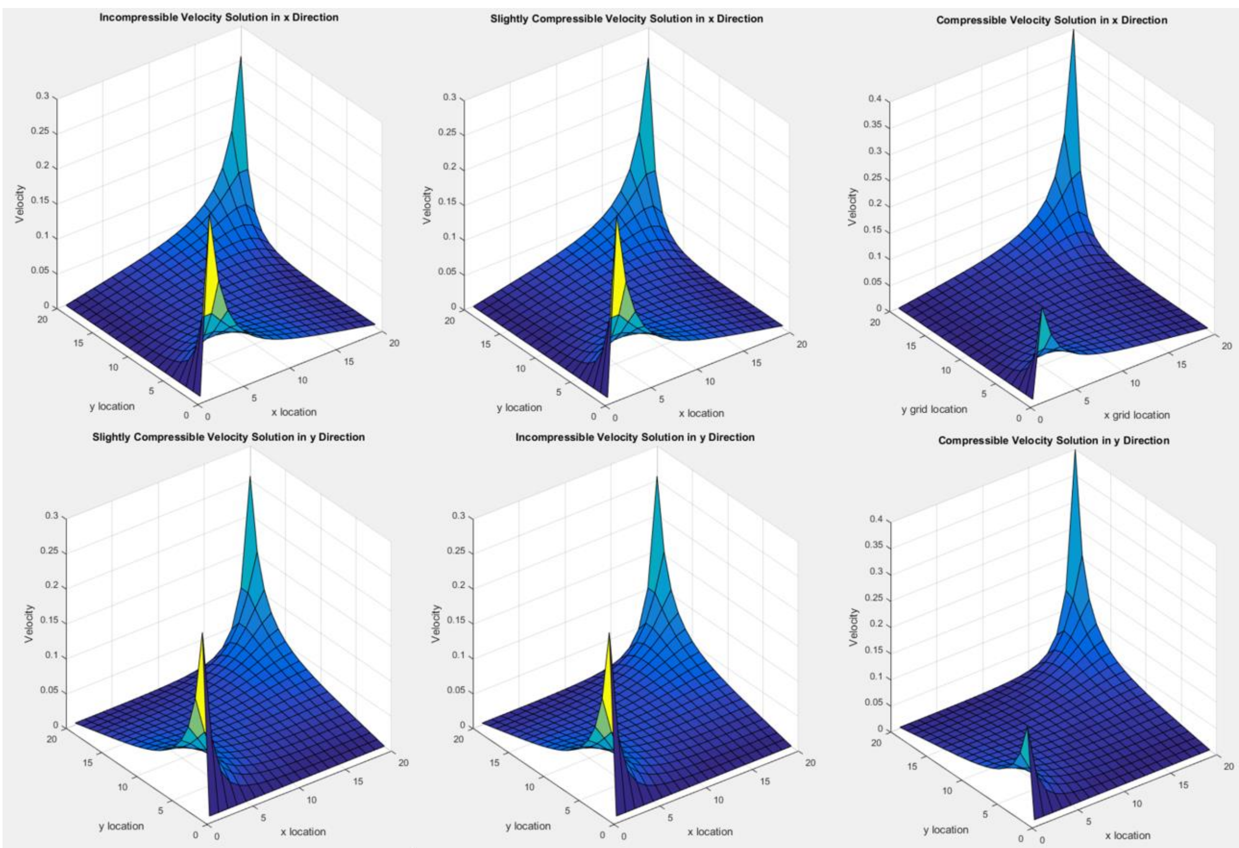


Figure 7 : Velocity Profile in 2D Reservoir Under Three Different System with Constant Wellbore Pressure in x-direction (upper), and y-direction (lower)

Validation

This simulator is validated by analytical solution. In this case, only incompressible and slightly compressible in 1D reservoir are presented. Dirichlet and Dirac pulse boundary condition are employed to validate 1D incompressible flow and slightly compressible flow equation respectively.

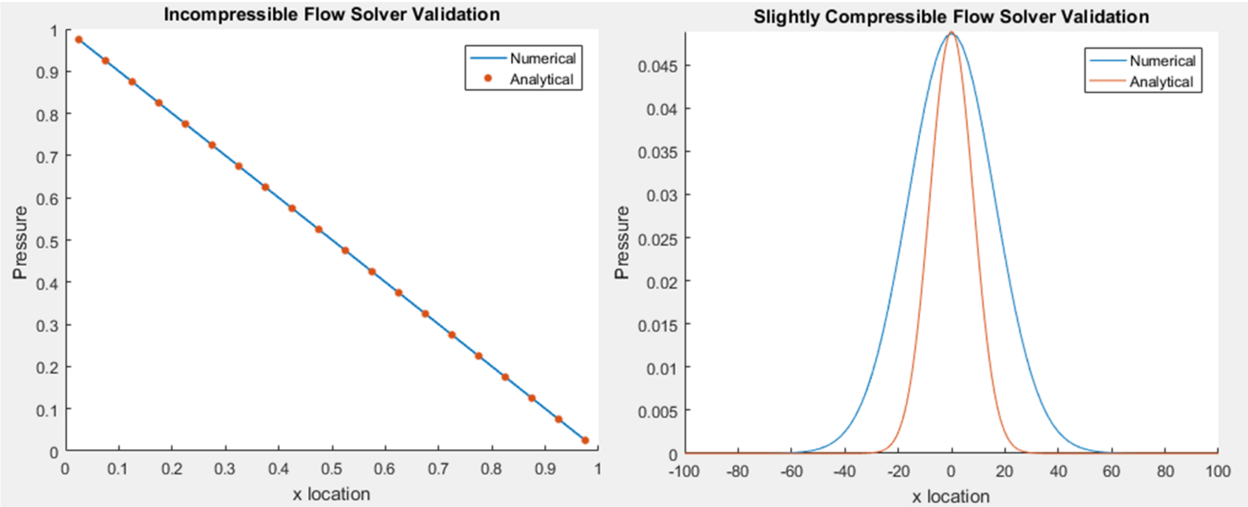


Figure 8 : 1D Reservoir Validation Using Incompressible (left) and Slightly Compressible Flow System (right)

This simulator works perfectly in 1D incompressible flow system. Both numerical and analytical solution coincides each other and gives the same pressure solution. In slightly compressible flow system numerical and analytical give small difference. This error can be reduced by fining the time and space grid.

Conclusions

Some conclusions can be taken from this numerical simulation study :

1. Pressure and velocity is always constant by time in incompressible flow system.
2. In slightly compressible system, pressure and velocity is function of time and space until reaches steady state condition. After steady state, pressure and velocity are acting like pressure and velocity in incompressible system.
3. In fully compressible system, density and porosity are function of pressure. Thus pressure and velocity solution are always changing by time.
4. This numerical simulator gives the correct solution in incompressible flow system.

References

Hajibeygi, Hadi. Lecture Notes 2017 AES1350 Reservoir Simulation.

Cusini, Matteo. Assignment 1 2017 Notes.