Assignment
#1: Due May 31

1. For each of the following statements, state whether it is true or false; if true give a proof; if false give a counter example: [You may assume that all functions are nonnegative, increasing and tend to infinity.]

   (a) $[f(n) = O(g(n))] \Leftrightarrow [(f(n))^2 = O((g(n))^2)]$

   (b) $[f(n) = O(g(n))] \Rightarrow [2^{f(n)} = O(2^{g(n)})]$

   (c) $[f(n) = O(g(n))] \Leftarrow [2^{f(n)} = O(2^{g(n)})]$

2. Suppose $f(n) = O(g(n))$ and $f(n) \neq o(g(n))$, is the statement that $f(n) = \Theta(g(n))$ true? If your answer is yes, prove it; if it is no, provide a counter example.

3. Exercises 3.2-4

4. Problem 3.3 (a)

5. Show that (i)$\sum_{i=1}^{i=n} i^2 = \Theta(n^3)$; (ii) Show that $\sum_{i=1}^{i=n} \frac{1}{i} = \Theta(\lg n)$

6. Challenge Problems: Do not turn it in. No answers will be provided.

   (a) Show that $\sum_{i=1}^{i=n} i^k = \Theta(n^{k+1})$ for any positive integer k.

   (b) 3.2-5