EMTopicModel

December 20, 2020

[19]:
```
%matplotlib inline
%load_ext autoreload
%autoreload 2

import matplotlib.pyplot as plt
import numpy as np
import seaborn as sns
import pandas as pd

from scipy.special import logsumexp

from utils import test_case_checker, perform_computation
```

```
The autoreload extension is already loaded. To reload it, use:
  %reload_ext autoreload
```

1 *Assignment Summary

EM Topic models The UCI Machine Learning dataset repository hosts several datasets recording word counts for documents at https://archive.ics.uci.edu/ml/datasets/Bag+of+Words. You will use the NIPS dataset. You will find (a) a table of word counts per document and (b) a vocabulary list for this dataset at the link. You must implement the multinomial mixture of topics model, lectured in class. For this problem, you should write the clustering code yourself (i.e. not use a package for clustering). * Cluster this to 30 topics, using a simple mixture of multinomial topic model, as lectured in class. * Produce a graph showing, for each topic, the probability with which the topic is selected. * Produce a table showing, for each topic, the 10 words with the highest probability for that topic.

2 *EM for Topic model in Matrix Form

For you convenience, we bring the reading assignment file here so that you can use it.

Caution Depending on your browser, you might need to right click on this pdf document to see the display options.

[20]:
```
from IPython.display import IFrame
IFrame("./EMTopicModel.pdf", width=1000, height=800)
```

[20]: <IPython.lib.display.IFrame at 0x7fac67dc8790>

3 0. Data

3.1 0.1 Description

There are multiple collection of word-count datasets available at https://archive.ics.uci.edu/ml/datasets/Bag+of+Words . We will be using the NIPS collection of words in this exercise. This dataset is composed of papers presented at the Conference of Neural Information Processing Systems (formerly NIPS, which is now knows as NeurIPS).

3.2 0.2 Information Summary

- **Input/Output**: There are a total of 12419 words counted, and 1500 documents were surveyed. Therefore, the data can be stored in a count array with a shape of `(1500, 12419)`.
- **Missing Data**: There is no missing data.
- **Final Goal**: We want to fit an EM topic model for clustering the documents.

[21]:
```
data_file = f'words/docword.nips.txt'

with open(data_file) as fh:
    for line_num, line in enumerate(fh):
        if line_num == 0:
            N = int(line) # Number of documents
        elif line_num == 1:
            d = int(line) # Number of words
            X = np.zeros((N, d))
        elif line_num == 2:
            NNZ = int(line)
        else:
            doc_id, word_id, count = tuple([int(a) for a in line.split(' ')])
            X[doc_id-1, word_id-1] = count

assert X[X>0].size == NNZ
```

[22]:
```
with open('words/vocab.nips.txt') as fh2:
    words = [line.rstrip() for line in fh2]

assert len(words) == d
```

4 1. Implementing the EM Topic Model

5 Task 1

In this task, we want to implement the E-step.

Write a function `find_logW` that calculates the $\log W_{i,j}$ matrix, and takes the following arguments as input:

1. `X`: A numpy array of the shape `(N,d)` where `N` is the number of documents and `d` is the number of words. Do not assume anything about `N` or `d` other than being a positive integer. This variable is equivalent to the data matrix X in the review document above.
2. `log_P`: A numpy array of the shape `(t,d)` where `t` is the number of topics for clustering and `d` is the number of words. Again, do not assume anything about `t` or `d` other than being a positive integer. This variable is equivalent to the element-wise natural log of the topic probability matrix P in the review document above, which we also showed by $\tilde{P}$.
3. `log_pi`: A numpy array of the shape `(t,1)` where `t` is the number of topics for clustering. This variable is equivalent to the element-wise natural log of the prior probabilities vector π in the review document above, which we also showed by $\tilde{\pi}$.

Your model should return the numpy array `log_W` with the shape of `(N, t)` whose i^{th} row and j^{th} column should be

$$\log W_{i,j} = \log\left(\frac{\pi_j \prod_{k=1}^{d} P_{j,k}^{x_{i,k}}}{\sum_{l=1}^{t} \pi_l \prod_{k=1}^{d} P_{l,k}^{x_{i,k}}}\right).$$

Important Note: You **should** use the `logsumexp` function imported above from scipy's library to make sure that numerical stability would not be a problem.

[37]:
```python
def find_logW(X, log_P, log_pi):
    N, d = X.shape
    t = log_pi.shape[0]
    # your code here

    R_hat = np.dot( np.ones((N,1)), log_pi.T ) + np.dot( X, log_P.T )
    S_hat = logsumexp(R_hat, axis=1, keepdims = True)
    log_W = R_hat - S_hat

    assert log_W.shape == (N, t)
    return log_W
```

[38]:
```python
some_X = 1 + (np.arange(35).reshape(7,5) ** 13) % 20
some_log_P = np.log(some_X[:3, :]/np.sum(some_X[:3, :], axis=1).reshape(-1,1))
some_log_pi = np.log(some_X[:3, 0]/np.sum(some_X[:3, 0])).reshape(-1,1)
some_log_W = find_logW(some_X, some_log_P, some_log_pi)

assert np.array_equal(some_log_W.round(2), np.array([[ -0.   ,  -9.07,  -6.1 ],
                                                     [-24.61,  -0.   , -12.27],
                                                     [-12.59,  -6.01,  -0.   ],
```

```
                                                     [-23.81,  -0.   , -29.1 ],
                                                     [ -0.   ,  -9.07,  -6.1 ],
                                                     [-24.61,  -0.   , -14.62],
                                                     [-29.96,  -0.   , -10.82]]))

# Checking against the pre-computed test database
test_results = test_case_checker(find_logW, task_id=1)
assert test_results['passed'], test_results['message']
```

6 Task 2

In this task, we want to implement the first part of the M-step.

Write a function `update_logP` that does the maximization step for the $\log P_{i,j}$ matrix, and takes the following arguments as input:

1. `X`: A numpy array of the shape `(N,d)` where `N` is the number of documents and `d` is the number of words. Do not assume anything about `N` or `d` other than being a positive integer. This variable is equivalent to the data matrix X in the review document above.
2. `log_W`: A numpy array of the shape `(N,t)` where `N` is the number of documents and `t` is the number of topics for clustering. Again, do not assume anything about `t` other than being a positive integer. This variable is equivalent to the element-wise natural log of the W matrix referenced in the document above and in the textbook. We also used the notation $\tilde{W}$ for this matrix in the document above. `log_W` is the same as the output from the previous function you wrote.
3. `eps`: A very small ϵ scalar added to make sure the log operation has enough numerical stability. The document above suggests computing the matrix E using the following relation

$$E_{t\times d} = [W^T]_{t\times N} \cdot X_{N\times d}.$$

However, we will make a small modification to this calculation by incorporating an insurance epsilon.

$$E_{t\times d} = [W^T]_{t\times N} \cdot X_{N\times d} + \epsilon.$$

You should implement the $E = W^T \cdot X + \epsilon$ in your code.

Your model should return the numpy array `log_P` with the shape of `(t, d)` whose j^{th} row should be

$$\log \mathbf{p}_j = \log \left(\frac{\sum_{i=1}^{N} \mathbf{x}_i W_{i,j}}{\sum_{i=1}^{N} (\mathbf{x}_i^T \mathbf{1}) W_{i,j}} \right).$$

Here, log is the element-wise logarithm in the natural basis.

Important Note: You **should** use the `logsumexp` function imported above from scipy's library to make sure that numerical stability would not be a problem.

[49]:
```
def update_logP(X, log_W, eps=1e-100):
    N, d = X.shape
    t = log_W.shape[1]
    assert log_W.shape[0] == N
```

```
    # your code here

    E = np.dot( np.exp(log_W).T, X ) + eps
    log_E = np.log(E)
    F = logsumexp( log_E, axis = 1, keepdims = True )
    log_P = log_E - F

    assert log_P.shape == (t, d)
    return log_P
```

```
[50]: some_X = 1 + (np.arange(35).reshape(7,5) ** 13) % 20
some_log_P = np.log(some_X[:3, :]/np.sum(some_X[:3, :], axis=1).reshape(-1,1))
some_log_pi = np.log(some_X[:3, 0]/np.sum(some_X[:3, 0])).reshape(-1,1)
some_log_W = find_logW(some_X, some_log_P, some_log_pi)

assert np.array_equal(update_logP(some_X, some_log_W, eps=1e-100).round(2), np.
 ↪array([[-3.22, -2.53, -0.65, -1.83, -1.61],
                                                                                ␣
 ↪      [-1.87, -1.16, -1.81, -1.86, -1.55],
                                                                                ␣
 ↪      [-3.8 , -1.32, -1.24, -1.17, -2.2 ]]))

# Checking against the pre-computed test database
test_results = test_case_checker(update_logP, task_id=2)
assert test_results['passed'], test_results['message']
```

7 Task 3

In this task, we want to implement the second part of the M-step.

Write a function `update_log_pi` that does the maximization step for the $\log \pi$ vector, and takes the following arguments as input:

1. `log_W`: A numpy array of the shape `(N,t)` where `N` is the number of documents and `t` is the number of topics for clustering. Again, do not assume anything about `t` other than being a positive integer. This variable is equivalent to the element-wise natural log of the W matrix referenced in the document above and in the textbook. We also used the notation $\tilde{W}$ for this matrix in the document above. `log_W` is the same as the output from the previous functions you wrote.

The output of the function should be the `log_pi` numpy array with a shape of `(t,1)` whose j^{th} element should be

$$\log \pi_j = \log \left(\frac{\sum_{i=1}^{N} W_{i,j}}{N} \right).$$

Important Note: You **should** use the `logsumexp` function imported above from scipy's library to make sure that numerical stability would not be a problem.

[67]:
```python
def update_log_pi(log_W):
    N, t = log_W.shape

    # your code here
    log_pi = ( logsumexp(log_W, axis = 0, keepdims = True) - np.log(N) ).
 ↪reshape(-1,1)

    assert log_pi.shape == (t,1)
    return log_pi
```

[68]:
```python
some_X = 1 + (np.arange(35).reshape(7,5) ** 13) % 20
some_log_P = np.log(some_X[:3, :]/np.sum(some_X[:3, :], axis=1).reshape(-1,1))
some_log_pi = np.log(some_X[:3, 0]/np.sum(some_X[:3, 0])).reshape(-1,1)
some_log_W = find_logW(some_X, some_log_P, some_log_pi)

assert np.array_equal(update_log_pi(some_log_W).round(2), np.array([[-1.26],
                                                                    [-0.56],
                                                                    [-1.94]]))

# Checking against the pre-computed test database
test_results = test_case_checker(update_log_pi, task_id=3)
assert test_results['passed'], test_results['message']
```

8 2. Running the Topic Model EM Algorithm

[69]:
```python
def TopicModel(X, t, iterations=100, seed=12345):
    N, d = X.shape

    np_random = np.random.RandomState(seed=seed)
    pi_init = np.ones((t,1))/float(t)

    if True:
        P_init = np_random.uniform(0, 1, (t, d))
    else:
        X_copy = X.copy()
        np_random.shuffle(X_copy)

        c = N//t
        P_init = np.zeros((t, d))
        for k in range(t):
            P_init[k, :] = (X_copy[(c*k):(c*(k+1)), :]).sum(axis=0) + 1e-1

    P_init = P_init/P_init.sum(axis=1).reshape(-1, 1)
```

```
    log_pi = np.log(pi_init) # log_pi.shape == (t,1)
    log_P = np.log(P_init) # log_P.shape == (t,d)
    assert log_pi.shape == (t,1)

    log_W = None
    for iteration in range(iterations):
        print('.', end='')
        #The E-Step
        log_W = find_logW(X, log_P, log_pi)

        #The M-Step
        log_P = update_logP(X, log_W)
        log_pi = update_log_pi(log_W)

    return log_pi, log_P, log_W
```

Let's use 30 topics (as instructed in the assignment summary) and 100 iterations for a start.

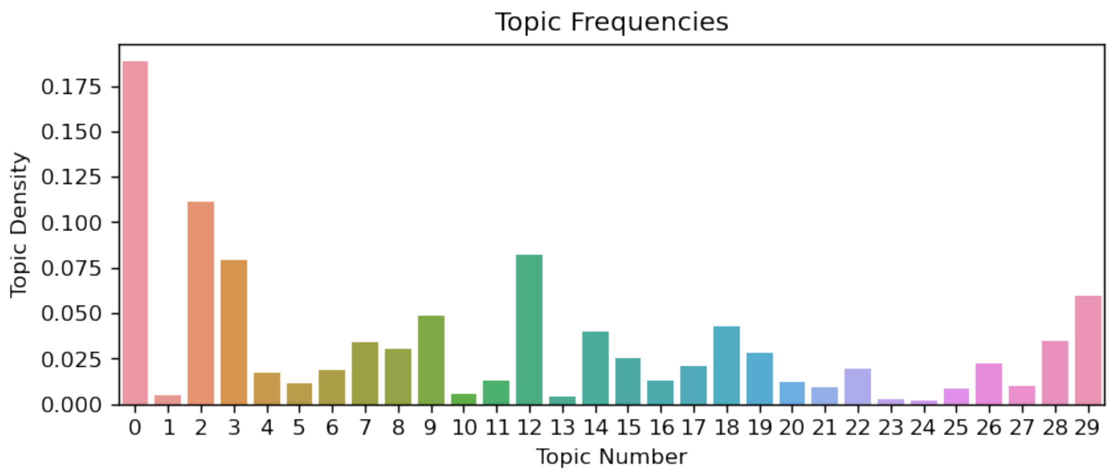
It is a wonderful thought exercise to play with the number of iterations, and see where the results seem to be unaffected by the more iterations, which is possibly a sign of the algorithm converging.

[75]:
```
if perform_computation:
    log_pi, log_P, log_W = TopicModel(X, t=30, iterations=10, seed=12345)
```

…

8.1 2.1 Visualizing Topic Frequencies

[76]:
```
if perform_computation:
    fig, ax=plt.subplots(figsize=(8,3), dpi=120)
    sns.barplot(x=np.arange(30), y=np.exp(log_pi).reshape(-1), ax=ax)
    ax.set_title(f'Topic Frequencies')
    ax.set_xlabel(f'Topic Number')
    _ = ax.set_ylabel(f'Topic Density')
```



8.2 2.2 Printing The Most Frequent Words in Each Topic

[72]:
```
if perform_computation:
    top_indices = np.argsort(log_P, axis=1)[:,::-1][:, :10]
    top_words = [[words[x] for x in top_indices_row] for top_indices_row in␣
↪top_indices]

    fig, ax = plt.subplots(figsize=(8,3), dpi=120)
    col_labels = ['1st Word', '2nd Word', '3rd Word'] + [f'{i}th Word' for i in␣
↪range(4,11)]
    row_labels = [f'Topic {t_idx}' for t_idx in range(log_P.shape[0])]
    table_ = ax.table(top_words, colLabels=col_labels, rowLabels=row_labels)

    table_.auto_set_font_size(False)
    table_.set_fontsize(32)
    table_.scale(4, 4)

    # Removing ticks and spines enables you to get the figure only with table
    plt.tick_params(axis='x', which='both', bottom=False, top=False,␣
↪labelbottom=False)
    plt.tick_params(axis='y', which='both', right=False, left=False,␣
↪labelleft=False)
    for pos in ['right','top','bottom','left']:
        plt.gca().spines[pos].set_visible(False)

    fig.tight_layout()
```

	1st Word	2nd Word	3rd Word	4th Word	5th Word	6th Word	7th Word	8th Word	9th Word	10th Word
Topic 0	network	function	learning	model	algorithm	neural	set	weight	error	input
Topic 1	motion	output	stress	point	learning	function	chip	velocity	input	circuit
Topic 2	network	learning	model	unit	function	input	neural	algorithm	system	output
Topic 3	network	input	neural	training	set	function	model	system	output	learning
Topic 4	model	cell	neuron	system	frequency	input	network	visual	output	field
Topic 5	model	motion	system	data	network	set	learning	object	unit	training
Topic 6	algorithm	learning	model	function	data	set	vector	number	method	problem
Topic 7	network	cell	model	input	unit	training	data	error	problem	set
Topic 8	network	neural	system	function	net	data	training	neuron	learning	set
Topic 9	network	input	model	neuron	system	neural	learning	cell	function	output
Topic 10	neuron	model	network	neural	learning	item	problem	star	weight	system
Topic 11	network	algorithm	set	function	model	learning	data	distribution	neural	weight
Topic 12	network	learning	system	neural	training	set	input	model	unit	algorithm
Topic 13	problem	solution	method	action	algorithm	monte	carlo	system	belief	point
Topic 14	network	model	input	cell	neuron	unit	motion	pattern	function	neural
Topic 15	network	algorithm	function	input	model	unit	learning	output	data	weight
Topic 16	model	data	network	algorithm	point	distribution	learning	gaussian	matrix	component
Topic 17	model	input	function	neuron	current	algorithm	set	point	number	network
Topic 18	set	model	data	network	training	algorithm	function	learning	error	parameter
Topic 19	learning	model	algorithm	function	input	unit	task	problem	set	system
Topic 20	network	neuron	neural	system	learning	input	object	chip	weight	task
Topic 21	model	system	cell	function	word	speaker	visual	context	recognition	data
Topic 22	model	system	network	cell	neural	neuron	input	distance	function	field
Topic 23	function	decision	probability	bound	term	tree	result	distribution	leaves	exponent
Topic 24	instruction	classifier	learning	block	segment	features	set	labeling	images	schedule
Topic 25	network	output	net	neural	function	input	recognition	set	system	hand
Topic 26	model	data	algorithm	learning	function	set	vector	space	point	error
Topic 27	network	model	neuron	unit	input	learning	cell	noise	pattern	function
Topic 28	network	learning	unit	input	function	output	neural	weight	model	cell
Topic 29	network	model	input	neural	function	neuron	data	system	learning	output

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