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1 Code for simulation of Raman heterodyne like processes

1.1 Getting setup

1.1.1 Import required python libraries

```
In [1]: import sympy as sym
        sym.init_printing()

        import numpy as np
        from scipy import integrate,signal

        import math
        from math import pi
        #from sympy.functions import Heaviside, sin, cos, sqrt


        import matplotlib.pyplot as plt
        from sympy import I, Matrix, symbols
        from sympy.physics.quantum import TensorProduct,Dagger
        import scipy.optimize


        import scipy.constants as const

        import qutip

        from matplotlib.colors import Normalize as Norm

        %load_ext cython
```

1.1.2 Some things to help us make Liovillian superoperators

- Uses an approach very like qutip and the qotoolbox, reading the documention for those two packages might help make sense of spre, spost, etc
- s13 etc. are the atomic operators like |1><3| etc

```
In [2]: def spre(m):
            return TensorProduct(sym.eye(m.shape[0]),m)

        def spost(m):
            return TensorProduct(m.T,sym.eye(m.shape[0]))

        def collapse(c):
            temp = Dagger(c)*c/2
            return spre(c)*spost(Dagger(c))-spre(temp)-spost(temp)


        s13=Matrix([[0,0,1],[0,0,0],[0,0,0]])
        s23=Matrix([[0,0,0],[0,0,1],[0,0,0]])
        s12=Matrix([[0,1,0],[0,0,0],[0,0,0]])

        s31=s13.T
        s32=s23.T
        s21=s12.T

        s11 = s12*s21
        s22 = s21*s12
        s33 = s31*s13
```

1.1.3 Checking spre and spost agree with qutip

Note: This notebook uses Frotran (colum first) order when flattening matricies.

```
In [3]: sigmay = Matrix([[0, -I], [I, 0 ]])
        sigmay, qutip.sigmay()
```

```
Out[3]: (Matrix([
         [0, -I],
         [I,  0]]),
         Quantum object: dims = [[2], [2]], shape = (2, 2), type = oper, isherm = True
         Qobj data =
         [[0.+0.j 0.-1.j]
          [0.+1.j 0.+0.j]])
```

```
In [4]: qutip.sigmay()
```

Out[4]:
Quantum object: dims = [[2], [2]], shape = (2, 2), type = oper, isherm = True

$$\begin{pmatrix} 0.0 & -1.0j \\ 1.0j & 0.0 \end{pmatrix}$$

```
In [5]: spre(sigmay),qutip.spre(qutip.sigmay())
```

```
Out[5]: (Matrix([
         [0, -I, 0,  0],
         [I,  0, 0,  0],
         [0,  0, 0, -I],
         [0,  0, I,  0]]),
         Quantum object: dims = [[[2], [2]], [[2], [2]]], shape = (4, 4), type = super, isherm
         Qobj data =
         [[0.+0.j 0.-1.j 0.+0.j 0.+0.j]
          [0.+1.j 0.+0.j 0.+0.j 0.+0.j]
          [0.+0.j 0.+0.j 0.+0.j 0.-1.j]
          [0.+0.j 0.+0.j 0.+1.j 0.+0.j]])
```

```
In [6]: spost(sigmay),qutip.spost(qutip.sigmay())
```

```
Out[6]: (Matrix([
         [ 0,  0, I, 0],
         [ 0,  0, 0, I],
         [-I,  0, 0, 0],
         [ 0, -I, 0, 0]]),
         Quantum object: dims = [[[2], [2]], [[2], [2]]], shape = (4, 4), type = super, isherm
         Qobj data =
         [[0.+0.j 0.+0.j 0.+1.j 0.+0.j]
          [0.+0.j 0.+0.j 0.+0.j 0.+1.j]
          [0.-1.j 0.+0.j 0.+0.j 0.+0.j]
          [0.+0.j 0.-1.j 0.+0.j 0.+0.j]])
```

1.1.4 The symbolic variables we will use

- TODO: explain what they all are

```
In [7]: delta2,delta3 = sym.symbols('delta_2 delta_3',real=True)
        gamma13,gamma23,gamma2d,gamma3d,nbath,gammamu = sym.symbols('gamma_13 gamma_23 gamma_2
        omegao,omegam = sym.symbols('Omega_o Omega_mu', real=True, negative=False)
        rho11,rho12,rho13,rho21,rho22,rho23,rho31,rho32,rho33 = sym.symbols('rho_11 rho_12 rho

        a = sym.symbols('a')
        ar,ai = sym.symbols('a_r a_i',real=True)
        g = sym.symbols('g',real=True, negative=False)
```

1.2 Analytic calculations - Liovillian

```
In [8]: H =  omegam*s21 + omegao*s32 + g*a*s31
        H = H + Dagger(H)
        H = H + delta2*s22 + delta3*s33
        LH = -I*spre(H)+I*spost(H)

        L21 = gammamu*(nbath+1)*collapse(s12)
        L12 = gammamu*(nbath)*collapse(s21)
        L32 = gamma23*collapse(s23)
```

```
L31 = gamma13*collapse(s13)
L22 = gamma2d*collapse(s22)
L33 = gamma3d*collapse(s33)

L = LH + L21 + L12 + L32 + L31 + L22 + L33

#because Trace(rho) = 1 we can replace first row as below
#this gives us a matrix that can be inverted
L = L.row_insert(0,Matrix([[1,0,0,0,1,0,0,0,1]]))
L.row_del(1)

# The condition for the steady state is now L*rho=V
# where V = [1,0,0, 0,0,0, 0,0,0].T
```

1.2.1 Expressions for the various operators

```
In [9]: L
```

```
Out[9]:
```

$$\left[\begin{matrix}1 & 0 & 0 & 0 & 1 \\ -i\Omega_{\mu} & -i\delta_{2} - \frac{\gamma_{2d}}{2} - \frac{\gamma_{\mu} n_{b}}{2} - \frac{\gamma_{\mu} (n_{b}+1)}{2} & -i\Omega_{o} & 0 & i\Omega_{\mu} \\ -iag & -i\Omega_{o} & -i\delta_{3} - \frac{\gamma_{13}}{2} - \frac{\gamma_{23}}{2} - \frac{\gamma_{3d}}{2} - \frac{\gamma_{\mu} n_{b}}{2} & 0 & 0 \\ i\Omega_{\mu} & 0 & 0 & i\delta_{2} - \frac{\gamma_{2d}}{2} - \frac{\gamma_{\mu} n_{b}}{2} - \frac{\gamma_{\mu} (n_{b}+1)}{2} & -i\Omega_{\mu} \\ \gamma_{\mu} n_{b} & i\Omega_{\mu} & 0 & -i\Omega_{\mu} & -\gamma_{\mu}\left(n_{b}+1\right) \\ 0 & 0 & i\Omega_{\mu} & -iag & -i\Omega_{o} \\ ig\bar{a} & 0 & 0 & i\Omega_{o} & 0 \\ 0 & ig\bar{a} & 0 & 0 & i\Omega_{o} \\ 0 & 0 & ig\bar{a} & 0 & 0\end{matrix}\right.$$

```
In [10]: H
```

```
Out[10]:
```

$$\left[\begin{matrix}0 & \Omega_{\mu} & g\bar{a} \\ \Omega_{\mu} & \delta_{2} & \Omega_{o} \\ ag & \Omega_{o} & \delta_{3}\end{matrix}\right]$$

```
In [11]: #rho flattened out to a vector
         rho = Matrix([[rho11,rho21,rho31],[rho12,rho22,rho32],[rho13,rho23,rho33]])
         rho = 1*rho.T #because we are using "fortran" style matrix flatteneing
         rho[:]
         rhoflat = 1*rho.T
         rhoflat = rhoflat[:]
         rhoflat
```

```
Out[11]:
```

$$\left[\rho_{11}, \quad \rho_{21}, \quad \rho_{31}, \quad \rho_{12}, \quad \rho_{22}, \quad \rho_{32}, \quad \rho_{13}, \quad \rho_{23}, \quad \rho_{33}\right]$$

```
In [12]: L[:,0:3] #because the matrix gets clipped when exporting to PDF
```

```
Out[12]:
```

$$\left[\begin{matrix}1 & 0 & 0\\- i \Omega_{\mu} & - i \delta_{2} - \frac{\gamma_{2d}}{2} - \frac{\gamma_{\mu} n_{b}}{2} - \frac{\gamma_{\mu} \left(n_{b} + 1\right)}{2} & - i \Omega_{o}\\- i a g & - i \Omega_{o} & - i \delta_{3} - \frac{\gamma_{13}}{2} - \frac{\gamma_{23}}{2} - \frac{\gamma_{3d}}{2} - \frac{\gamma_{\mu} n_{b}}{2}\\i \Omega_{\mu} & 0 & 0\\\gamma_{\mu} n_{b} & i \Omega_{\mu} & 0\\0 & 0 & i \Omega_{\mu}\\i g \overline{a} & 0 & 0\\0 & i g \overline{a} & 0\\0 & 0 & i g \overline{a}\end{matrix}\right]$$

```
In [13]: L[:,3:6]
```

```
Out[13]:
```

$$\left[\begin{matrix}0 & 1 & 0\\0 & i \Omega_{\mu} & 0\\0 & 0 & i \Omega_{\mu}\\i \delta_{2} - \frac{\gamma_{2d}}{2} - \frac{\gamma_{\mu} n_{b}}{2} - \frac{\gamma_{\mu} \left(n_{b} + 1\right)}{2} & - i \Omega_{\mu} & - i g \overline{a}\\- i \Omega_{\mu} & - \gamma_{\mu} \left(n_{b} + 1\right) & - i \Omega_{o}\\- i a g & - i \Omega_{o} & i \delta_{2} - i \delta_{3} - \frac{\gamma_{13}}{2} - \frac{\gamma_{23}}{2} - \frac{\gamma_{2d}}{2} - \frac{\gamma_{3d}}{2} - \frac{\gamma_{\mu} \left(n_{b} + 1\right)}{2}\\i \Omega_{o} & 0 & 0\\0 & i \Omega_{o} & 0\\0 & 0 & i \Omega_{o}\end{matrix}\right]$$

```
In [14]: L[:,6:]
```

```
Out[14]:
```

$$\left[\begin{matrix}0 & 0 & 1\\0 & i a g & 0\\0 & 0 & i a g\\i \Omega_{o} & 0 & 0\\0 & i \Omega_{o} & \gamma_{23}\\0 & 0 & i \Omega_{o}\\i \delta_{3} - \frac{\gamma_{13}}{2} - \frac{\gamma_{23}}{2} - \frac{\gamma_{3d}}{2} - \frac{\gamma_{\mu} n_{b}}{2} & - i \Omega_{\mu} & - i g \overline{a}\\- i \Omega_{\mu} & - i \delta_{2} + i \delta_{3} - \frac{\gamma_{13}}{2} - \frac{\gamma_{23}}{2} - \frac{\gamma_{2d}}{2} - \frac{\gamma_{3d}}{2} - \frac{\gamma_{\mu} \left(n_{b} + 1\right)}{2} & - i \Omega_{o}\\- i a g & - i \Omega_{o} & - \gamma_{13} - \gamma_{23}\end{matrix}\right]$$

```
In [15]: rhoflat[6]
```

```
Out[15]:
```

$$\rho_{13}$$

1.2.2 We can transform the "basis" we use for our density matrix vector so that all the elements are real

In [16]:
```
#change of variables to make things real to make it a bit faster maybe
CtoR = Matrix([[2,0,0,0,0,0,0,0,0],
               [0,0,0,0,2,0,0,0,0],
               [0,0,0,0,0,0,0,0,2],
               [0,1,0,1,0,0,0,0,0],
               [0,I,0,-I,0,0,0,0,0],
               [0,0,1,0,0,0,1,0,0],
               [0,0,I,0,0,0,-I,0,0],
               [0,0,0,0,0,1,0,1,0],
               [0,0,0,0,0,I,0,-I,0]
              ])
CtoR=CtoR/2
```

1.2.3 The new organisation for our density matrix

In [17]:
```
#these is our new state vector
t = CtoR*Matrix(rhoflat)
t
```

Out[17]:

$$\left[\begin{matrix}\rho_{11}\\ \rho_{22}\\ \rho_{33}\\ \frac{\rho_{12}}{2}+\frac{\rho_{21}}{2}\\ -\frac{i\rho_{12}}{2}+\frac{i\rho_{21}}{2}\\ \frac{\rho_{13}}{2}+\frac{\rho_{31}}{2}\\ -\frac{i\rho_{13}}{2}+\frac{i\rho_{31}}{2}\\ \frac{\rho_{23}}{2}+\frac{\rho_{32}}{2}\\ -\frac{i\rho_{23}}{2}+\frac{i\rho_{32}}{2}\end{matrix}\right]$$

1.2.4 The Liovillian in this new basis

In [18]:
```
Lreal = sym.simplify(CtoR*L*CtoR.inv())
Lreal = Lreal.subs(a,ar+I*ai)
Lreal
```

Out[18]:

$$\left[\begin{array}{ccccccc}
1 & 1 & 1 & 0 & 0 & 0 & 0 \\
\gamma_\mu n_b & -\gamma_\mu (n_b+1) & \gamma_{23} & 0 & 2\Omega_\mu & 0 & 0 \\
0 & 0 & -\gamma_{13}-\gamma_{23} & 0 & 0 & 2a_i g & 2a \\
0 & 0 & 0 & -\frac{\gamma_{2d}}{2}-\gamma_\mu n_b-\frac{\gamma_\mu}{2} & -\delta_2 & 0 & - \\
\Omega_\mu & -\Omega_\mu & 0 & \delta_2 & -\frac{\gamma_{2d}}{2}-\gamma_\mu n_b-\frac{\gamma_\mu}{2} & \Omega_o & 0 \\
a_i g & 0 & -a_i g & 0 & -\Omega_o & -\frac{\gamma_{13}}{2}-\frac{\gamma_{23}}{2}-\frac{\gamma_{3d}}{2}-\frac{\gamma_\mu n_b}{2} & - \\
a_r g & 0 & -a_r g & \Omega_o & 0 & \delta_3 & -\frac{\gamma_{13}}{2}-\frac{\gamma_{23}}{2} \\
0 & 0 & 0 & a_i g & a_r g & 0 & \Omega \\
0 & \Omega_o & -\Omega_o & a_r g & -a_i g & -\Omega_\mu & 0
\end{array}\right.$$

```
In [19]: # A sanity check
         # sym.simplify(CtoR.inv()*Lreal*CtoR*Matrix(rhoflat)-(L*Matrix(rhoflat)).subs(a,ar+I*
```

1.2.5 C-code from our analytic expressions for the two different versions of the Liovillan

- this is copied and pasted into some of the functions below

```
In [20]: from sympy.printing import print_ccode
         Lflat = 1*L.T
         Lflat = Lflat[:]
         print_ccode(Lflat)

         print("-----------")
         #because this is destined for cython it's in row-major order
         Lflatreal = 1*Lreal.T
         Lflatreal = Lflatreal[:]
         print_ccode(Lflatreal)
         print("----------")
         for k in range(81):
             print("L[%d] = "%(k,),end="")
             print_ccode(Lflatreal[k])
```

```
// Not supported in C:
// conjugate
[1, -I*Omega_mu, -I*a*g, I*Omega_mu, gamma_mu*n_b, 0, I*g*conjugate(a), 0, 0, 0, -I*delta_2 - 
-----------
(1, gamma_mu*n_b, 0, 0, Omega_mu, a_i*g, a_r*g, 0, 0, 1, -gamma_mu*(n_b + 1), 0, 0, -Omega_mu,
----------
L[0] = 1
L[1] = gamma_mu*n_b
L[2] = 0
L[3] = 0
L[4] = Omega_mu
L[5] = a_i*g
L[6] = a_r*g
L[7] = 0
```

```
L[8] = 0
L[9] = 1
L[10] = -gamma_mu*(n_b + 1)
L[11] = 0
L[12] = 0
L[13] = -Omega_mu
L[14] = 0
L[15] = 0
L[16] = 0
L[17] = Omega_o
L[18] = 1
L[19] = gamma_23
L[20] = -gamma_13 - gamma_23
L[21] = 0
L[22] = 0
L[23] = -a_i*g
L[24] = -a_r*g
L[25] = 0
L[26] = -Omega_o
L[27] = 0
L[28] = 0
L[29] = 0
L[30] = -1.0/2.0*gamma_2d - gamma_mu*n_b - 1.0/2.0*gamma_mu
L[31] = delta_2
L[32] = 0
L[33] = Omega_o
L[34] = a_i*g
L[35] = a_r*g
L[36] = 0
L[37] = 2*Omega_mu
L[38] = 0
L[39] = -delta_2
L[40] = -1.0/2.0*gamma_2d - gamma_mu*n_b - 1.0/2.0*gamma_mu
L[41] = -Omega_o
L[42] = 0
L[43] = a_r*g
L[44] = -a_i*g
L[45] = 0
L[46] = 0
L[47] = 2*a_i*g
L[48] = 0
L[49] = Omega_o
L[50] = -1.0/2.0*gamma_13 - 1.0/2.0*gamma_23 - 1.0/2.0*gamma_3d - 1.0/2.0*gamma_mu*n_b
L[51] = delta_3
L[52] = 0
L[53] = -Omega_mu
L[54] = 0
L[55] = 0
```

```
L[56] = 2*a_r*g
L[57] = -Omega_o
L[58] = 0
L[59] = -delta_3
L[60] = -1.0/2.0*gamma_13 - 1.0/2.0*gamma_23 - 1.0/2.0*gamma_3d - 1.0/2.0*gamma_mu*n_b
L[61] = Omega_mu
L[62] = 0
L[63] = 0
L[64] = 0
L[65] = 0
L[66] = -a_i*g
L[67] = -a_r*g
L[68] = 0
L[69] = -Omega_mu
L[70] = -1.0/2.0*gamma_13 - 1.0/2.0*gamma_23 - 1.0/2.0*gamma_2d - 1.0/2.0*gamma_3d - 1.0/2.0*ga
L[71] = -delta_2 + delta_3
L[72] = 0
L[73] = -2*Omega_o
L[74] = 2*Omega_o
L[75] = -a_r*g
L[76] = a_i*g
L[77] = Omega_mu
L[78] = 0
L[79] = delta_2 - delta_3
L[80] = -1.0/2.0*gamma_13 - 1.0/2.0*gamma_23 - 1.0/2.0*gamma_2d - 1.0/2.0*gamma_3d - 1.0/2.0*ga
```

1.3 Numerical code for steady state

1.3.1 Python code

- Using analytic expressions above
- The code below is much slower than the cython version below, but useful for confirming correctness of cython version.

```
In [21]: Lfunc = sym.lambdify((a,delta2, delta3, gamma13, gamma23, gamma2d, gamma3d, nbath,gamm

         Lrealfunc = sym.lambdify((ar,ai,delta2, delta3, gamma13, gamma23, gamma2d, gamma3d, nb

         # finds the steady state density matrix
         # p is a python dictionary full of parameters

         def steady_rho(p):

             Lmatrix = Lfunc(p['a'],p['delta2'],p['delta3'],p['gamma13'],
                                         p['gamma23'],p['gamma2d'],p['gamma3d'],p['nbath'],
                                         p['gammamu'],p['omegao'],p['omegam'],p['g'])

             #return Lmatrix
```

```
    rho = np.linalg.solve(Lmatrix.astype('complex128'),np.matrix([[1,0,0,0,0,0,0,0,0]]
    return np.reshape(rho,(3,3),order='F')

# this code is commented out because it has been replaced by cython version
# will leave it for future sanity checks if

# Just the rho13 element of the density matrix:

# def steady_rho13_older(p):
#     Lmatrix = Lfunc(p['a'],p['delta2'],p['delta3'],p['gamma13'],
#                                    p['gamma23'],p['gamma2d'],p['gamma3d'],p['nbath'],
#                                    p['gammamu'],p['omegao'],p['omegam'],p['g'])
#     #return Lmatrix
#     rho = np.linalg.solve(Lmatrix.astype('complex128'),np.matrix([[1,0,0,0,0,0,0,0,
#     return rho[6]

# # another version
# def steady_rho13_old(p):
#     Lmatrix =  Lrealfunc(np.real(p['a']),np.real(p['a']),p['delta2'],p['delta3'],p[
#                                    p['gamma23'],p['gamma2d'],p['gamma3d'],p['nbath'],
#                                    p['gammamu'],p['omegao'],p['omegam'],p['g'])
#     rho = np.linalg.solve(Lmatrix.astype('double'),np.matrix([[1,0,0,0,0,0,0,0,0]])
#     return rho[5]+1j*rho[6]
```

1.3.2 Cython versions

Implemented in cython to make it (much) faster.

In [22]:
```
%%cython --compile-args=-fopenmp --link-args=-fopenmp --verbose --force
#### %%cython -a # this gives "highligted" output

#cython: boundscheck=False, wraparound=False, nonecheck=False
cimport cython
import numpy as np
from cython.parallel cimport prange
cimport scipy.linalg.cython_lapack as lapack
import numpy as np
from libc.math cimport exp


cdef int nextpow2(int x):
    cdef int n=1
    while(True):
        if n>=x:
            return n
        n=2*n

def steady_rho13(p):
    cdef double a_r,a_i,delta_2,delta_3,gamma_13,gamma_23,gamma_2d,gamma_3d,n_b,gamma
    cdef double L[81]
    cdef double V[9]
    cdef int n = 9
    cdef double complex a
    cdef int info
    cdef int lda = 9
    cdef int ldb = 9
    cdef int nrhs = 1
    cdef int workspace[9]

    V[:]= [1,0,0, 0,0,0, 0,0,0]
    a = p['a']
    a_r = a.real
    a_i = a.imag
    delta_2 = p['delta2']
    delta_3 = p['delta3']
    gamma_13 = p['gamma13']
    gamma_23 = p['gamma23']
    gamma_2d = p['gamma2d']
    gamma_3d = p['gamma3d']
    n_b = p['nbath']
    gamma_mu = p['gammamu']
    Omega_o = p['omegao']
    Omega_mu = p['omegam']
    g = p['g']
#   with nogil: # to make sure it's not doing any slow python calls
    L[:] = (1, gamma_mu*n_b, 0, 0, Omega_mu, a_i*g, a_r*g, 0, 0, 1, -gamma_mu*(n_b +

    lapack.dgesv(&n,&nrhs,L,&lda,workspace,V,&ldb,&info)
    return V[5]+1j*V[6]


cdef double complex steady_rho13_lots_of_args(double a_r,
                                double a_i,
                                double delta_2,
                                double delta_3,
                                double gamma_13,
                                double gamma_23,
                                double gamma_2d,
                                double gamma_3d,
                                double n_b,
                                double gamma_mu,
                                double Omega_o,
                                double Omega_mu,
                                double g) nogil:
    cdef double L[81]
    cdef double V[9]
cdef int n = 9
cdef double complex a
cdef int info
cdef int lda = 9
cdef int ldb = 9
cdef int nrhs = 1
cdef int workspace[9]
cdef int k

# Unfortunately cant use this syntax if we dont have dont have GIL
#V[:] = [1,0,0, 0,0,0, 0,0,0]
#L[:] = (1, gamma_mu*n_b, 0, 0, Omega_mu, a_i*g, a_r*g, 0, 0, 1, -gamma_mu*(n_b +

for k in range(9):
    V[k] = 0
V[0] = 1

L[0] = 1
L[1] = gamma_mu*n_b
L[2] = 0
L[3] = 0
L[4] = Omega_mu
L[5] = a_i*g
L[6] = a_r*g
L[7] = 0
L[8] = 0
L[9] = 1
L[10] = -gamma_mu*(n_b + 1)
L[11] = 0
L[12] = 0
L[13] = -Omega_mu
L[14] = 0
L[15] = 0
L[16] = 0
L[17] = Omega_o
L[18] = 1
L[19] = gamma_23
L[20] = -gamma_13 - gamma_23
L[21] = 0
L[22] = 0
L[23] = -a_i*g
L[24] = -a_r*g
L[25] = 0
L[26] = -Omega_o
L[27] = 0
L[28] = 0
L[29] = 0
L[30] = -1.0/2.0*gamma_2d - gamma_mu*n_b - 1.0/2.0*gamma_mu
```

```
L[31] = delta_2
L[32] = 0
L[33] = Omega_o
L[34] = a_i*g
L[35] = a_r*g
L[36] = 0
L[37] = 2*Omega_mu
L[38] = 0
L[39] = -delta_2
L[40] = -1.0/2.0*gamma_2d - gamma_mu*n_b - 1.0/2.0*gamma_mu
L[41] = -Omega_o
L[42] = 0
L[43] = a_r*g
L[44] = -a_i*g
L[45] = 0
L[46] = 0
L[47] = 2*a_i*g
L[48] = 0
L[49] = Omega_o
L[50] = -1.0/2.0*gamma_13 - 1.0/2.0*gamma_23 - 1.0/2.0*gamma_3d - 1.0/2.0*gamma_mu
L[51] = delta_3
L[52] = 0
L[53] = -Omega_mu
L[54] = 0
L[55] = 0
L[56] = 2*a_r*g
L[57] = -Omega_o
L[58] = 0
L[59] = -delta_3
L[60] = -1.0/2.0*gamma_13 - 1.0/2.0*gamma_23 - 1.0/2.0*gamma_3d - 1.0/2.0*gamma_mu
L[61] = Omega_mu
L[62] = 0
L[63] = 0
L[64] = 0
L[65] = 0
L[66] = -a_i*g
L[67] = -a_r*g
L[68] = 0
L[69] = -Omega_mu
L[70] = -1.0/2.0*gamma_13 - 1.0/2.0*gamma_23 - 1.0/2.0*gamma_2d - 1.0/2.0*gamma_3d
L[71] = -delta_2 + delta_3
L[72] = 0
L[73] = -2*Omega_o
L[74] = 2*Omega_o
L[75] = -a_r*g
L[76] = a_i*g
L[77] = Omega_mu
L[78] = 0
```

```
    L[79] = delta_2 - delta_3
    L[80] = -1.0/2.0*gamma_13 - 1.0/2.0*gamma_23 - 1.0/2.0*gamma_2d - 1.0/2.0*gamma_3
    lapack.dgesv(&n,&nrhs,L,&lda,workspace,V,&ldb,&info)
    return V[5]+1j*V[6]




# Steady state an ensemble of atoms, returns a matrix of rho13 values as well as the
# weights from the inhomogeneous broadening distribution.
# Convolve the two to get simulate single pass raman heterodyne signal as a function

def ensemble_steady_rho13(p,delta2vals,delta3vals):
    cdef double a_r,a_i,delta_2,delta_3,gamma_13,gamma_23,gamma_2d,gamma_3d,n_b,gamma
    a = p['a']
    a_r = a.real
    a_i = a.imag
    delta_2 = p['delta2']
    delta_3 = p['delta3']
    gamma_13 = p['gamma13']
    gamma_23 = p['gamma23']
    gamma_2d = p['gamma2d']
    gamma_3d = p['gamma3d']
    n_b = p['nbath']
    gamma_mu = p['gammamu']
    Omega_o = p['omegao']
    Omega_mu = p['omegam']
    g = p['g']

    cdef int ii
    cdef int jj

    cdef double m2 = p['mean_delta2']
    cdef double sd2 = p['sd_delta2']
    cdef double m3 = p['mean_delta3']
    cdef double sd3 = p['sd_delta3']
    #print(sd3)

    cdef double [:] delta2valsv = delta2vals
    cdef double [:] delta3valsv = delta3vals

    cdef int ndelta2 = len(delta2vals)
    cdef int ndelta3 = len(delta3vals)

    cdef double df2 = delta2valsv[1]-delta2valsv[0]
    cdef double df3 = delta3valsv[1]-delta3valsv[0]
    coh13 = np.zeros((ndelta2,ndelta3),dtype='complex128') #make numpy array
    cdef double complex [:,:] coh13v = coh13 #make cython  "view" of the data

    w = np.zeros((ndelta2,ndelta3),dtype='complex128') #make numpy array for gaussian
    cdef double complex [:,:] wv = w #make cython  "view" of the data

    for ii in prange(ndelta2, nogil=True):
    #for ii in range(ndelta2):
        delta_2 = delta2valsv[ii]
        for jj in range(ndelta3):
            delta_3 = delta3valsv[jj]
            coh13v[ii,jj] = steady_rho13_lots_of_args(a_r,
                                  a_i,
                                  delta_2,
                                  delta_3,
                                  gamma_13,
                                  gamma_23,
                                  gamma_2d,
                                  gamma_3d,
                                  n_b,
                                  gamma_mu,
                                  Omega_o,
                                  Omega_mu,
                                  g)
            wv[ii,jj] = 1.0/(2*3.14159265358979323846*sd3*sd2) * exp(-(delta_2-m2)**2/
    return (coh13,w)



# Steady state an ensemble of atoms, returns a matrix of rho values as well as the li
# weights from the inhomogeneous broadening distribution.
# Convolve the two to get simulate single pass raman heterodyne signal as a function

def ensemble_steady_rho(p,delta2vals,delta3vals):
    cdef double a_r,a_i,delta_2,delta_3,gamma_13,gamma_23,gamma_2d,gamma_3d,n_b,gamma_
    a = p['a']
    a_r = a.real
    a_i = a.imag
    delta_2 = p['delta2']
    delta_3 = p['delta3']
    gamma_13 = p['gamma13']
    gamma_23 = p['gamma23']
    gamma_2d = p['gamma2d']
    gamma_3d = p['gamma3d']
    n_b = p['nbath']
    gamma_mu = p['gammamu']
    Omega_o = p['omegao']
    Omega_mu = p['omegam']
    g = p['g']

    cdef int ii
    cdef int jj
    cdef int kk

    cdef double m2 = p['mean_delta2']
    cdef double sd2 = p['sd_delta2']
    cdef double m3 = p['mean_delta3']
    cdef double sd3 = p['sd_delta3']
    #print(sd3)

    cdef double [:] delta2valsv = delta2vals
    cdef double [:] delta3valsv = delta3vals

    cdef int ndelta2 = len(delta2vals)
    cdef int ndelta3 = len(delta3vals)

    cdef double df2 = delta2valsv[1]-delta2valsv[0]
    cdef double df3 = delta3valsv[1]-delta3valsv[0]

    rho = np.zeros((ndelta2,ndelta3,9),dtype='complex128') #make numpy array
    cdef double complex [:,:,:] rhov = rho #make cython  "view" of the data

    tmp = np.zeros((9,),dtype='complex128') #make numpy array
    cdef double complex [:] tmpv = tmp #make cython  "view" for temporary array

    w = np.zeros((ndelta2,ndelta3),dtype='complex128') #make numpy array for gaussian
    cdef double complex [:,:] wv = w #make cython  "view" of the data

    for ii in prange(ndelta2, nogil=True):
    #for ii in range(ndelta2):
        delta_2 = delta2valsv[ii]
        for jj in range(ndelta3):
            delta_3 = delta3valsv[jj]
#             tmp = steady_rho_lots_of_args(a_r,
#                                a_i,
#                                delta_2,
#                                delta_3,
#                                gamma_13,
#                                gamma_23,
#                                gamma_2d,
#                                gamma_3d,
#                                n_b,
#                                gamma_mu,
#                                Omega_o,
#                                Omega_mu,
#                                g)
```

```
                for kk in range(9):
                        rhov[ii,jj,kk] = tmpv[kk]
                wv[ii,jj] = 1.0/(2*3.14159265358979323846*sd3*sd2) * exp(-(delta_2-m2)**2/
        return (rho,w)




# Integrate over an ensemble using simpson's rule
cpdef ensemble_steady_rho13_integrated(p,double df):
    #cdef double df = 200e3 #frequency discretization
    cdef double a_r,a_i,delta_2,delta_3,gamma_13,gamma_23,gamma_2d,gamma_3d,n_b,gamma_
    a = p['a']
    a_r = a.real
    a_i = a.imag
    delta_2 = p['delta2']
    delta_3 = p['delta3']
    gamma_13 = p['gamma13']
    gamma_23 = p['gamma23']
    gamma_2d = p['gamma2d']
    gamma_3d = p['gamma3d']
    n_b = p['nbath']
    gamma_mu = p['gammamu']
    Omega_o = p['omegao']
    Omega_mu = p['omegam']
    g = p['g']

    cdef int ii
    cdef int jj

    cdef double m2 = p['mean_delta2']
    cdef double sd2 = p['sd_delta2']
    cdef double m3 = p['mean_delta3']
    cdef double sd3 = p['sd_delta3']

    cdef int w2,w3

    cdef int ndelta2 = 2*int(3*sd2/df)+1
    cdef int ndelta3 = 2*int(3*sd3/df)+1
    # print(ndelta2,ndelta3)

    delta2vals = np.linspace(-3,3,ndelta2)*sd2+m2
    delta3vals = np.linspace(-3,3,ndelta3)*sd3+m3

    cdef double [:] delta2valsv = delta2vals
    cdef double [:] delta3valsv = delta3vals
```

```
simpsonw2 = np.empty((ndelta2,),dtype='int32')
cdef int [:] simpsonw2v = simpsonw2
simpsonw3 = np.empty((ndelta3,),dtype='int32')
cdef int [:] simpsonw3v = simpsonw3

ii=0
while(ii<(ndelta2-1)):
    simpsonw2v[ii]=2
    ii+=1
    simpsonw2v[ii]=4
    ii+=1
simpsonw2v[0]=1
simpsonw2v[ndelta2-1]=1

ii=0
while(ii<(ndelta3-1)):
    simpsonw3v[ii]=2
    ii+=1
    simpsonw3v[ii]=4
    ii+=1
simpsonw3v[0]=1
simpsonw3v[ndelta3-1]=1



cdef double complex z = 0
cdef double sum_r = 0
cdef double sum_i = 0
cdef double gaussie
for ii in prange(ndelta2,nogil=True):
#for ii in range(ndelta2):
    delta_2 = delta2valsv[ii]
    w2 = simpsonw2v[ii]
    for jj in range(ndelta3):
        delta_3 = delta3valsv[jj]
        w3 = simpsonw3v[jj]
        gaussie = 1.0/(2*3.14159265358979323846*sd3*sd2) * exp(-(delta_2-m2)**2/(
        z = steady_rho13_lots_of_args(a_r,
                                      a_i,
                                      delta_2,
                                      delta_3,
                                      gamma_13,
                                      gamma_23,
                                      gamma_2d,
                                      gamma_3d,
                                      n_b,
                                      gamma_mu,
                                      Omega_o,
                        Omega_mu,
                        g)*gaussie*w2*w3
        #this is because cython doesn't know how to automatically do reductions f
        #complex variables but seems to for doubles
        sum_r += z.real
        sum_i += z.imag
return (sum_r+sum_i*1j)*df*df/9.0
```

```
[1/1] Cythonizing /home/jevon/.cache/ipython/cython/_cython_magic_82c47857b5e8c80de4403cb0ac8c
building '_cython_magic_82c47857b5e8c80de4403cb0ac8c1eab' extension
x86_64-linux-gnu-gcc -pthread -DNDEBUG -g -fwrapv -O2 -Wall -Wstrict-prototypes -g -fdebug-pre
x86_64-linux-gnu-gcc -pthread -shared -Wl,-O1 -Wl,-Bsymbolic-functions -Wl,-Bsymbolic-function
```

1.4 Calculations - just atoms

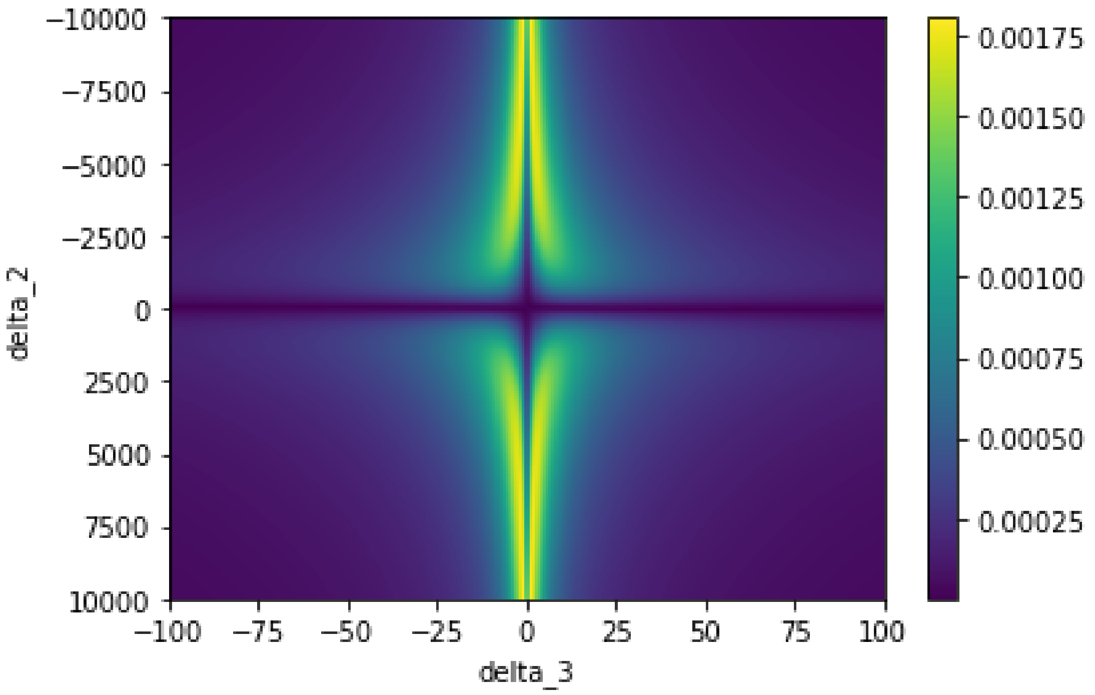
1.4.1 Single pass - no inhomogeneous broadening

Weird behavior -- driving both microwave and optical pump hard

- Signal persists at very high microwave detuning
- EIT like features

```
In [23]: #some default parameters
         p = {}
         p['a'] = 0
         p['delta2'] = 0.
         p['delta3'] = 0.
         p['gamma13'] = 1./22e-3
         p['gamma23'] = 1./22e-3
         p['gamma2d'] = 1./1e-6
         p['gamma3d'] = 1./1e-6
         p['nbath'] = 20
         p['gammamu'] = 1./((p['nbath']+1)*11e-3)
         p['omegao'] = 10e6
         p['omegam'] = 10e6
         p['g'] = 0.

         # Code currently requires these, even if we're not interested in
         # inhomogeneous brodening for the moment
         p['mean_delta2']=0
         p['sd_delta2']=20e6/2.355 #2.355 is to turn FWHM into standard deviation
         p['mean_delta3']=0
         p['sd_delta3']=300e6/2.355

         delta2vals = np.linspace(-10000e6,10000e6,501)
         delta3vals = np.linspace(-100e6,100e6,501)

         coh,_ = ensemble_steady_rho13(p,delta2vals,delta3vals)
```

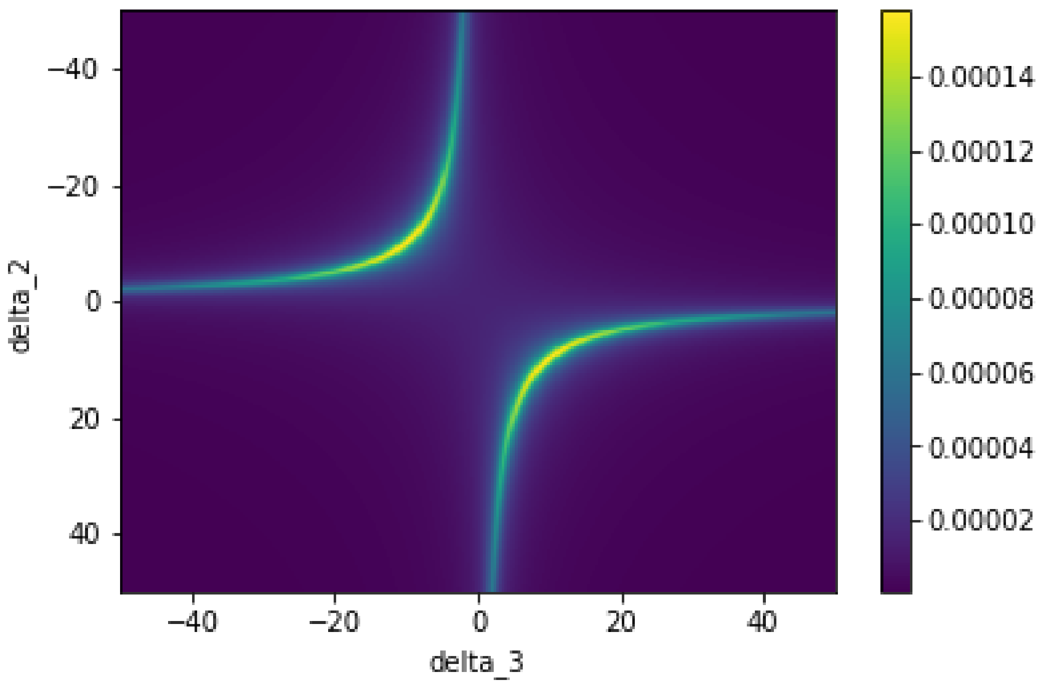
```
#plt.rcParams['figure.figsize'] = [10, 4]
plt.imshow(np.abs(coh),extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2va
plt.ylabel('delta_2')
plt.xlabel('delta_3')
plt.colorbar();
```



Strong optical pump, weak microwaves

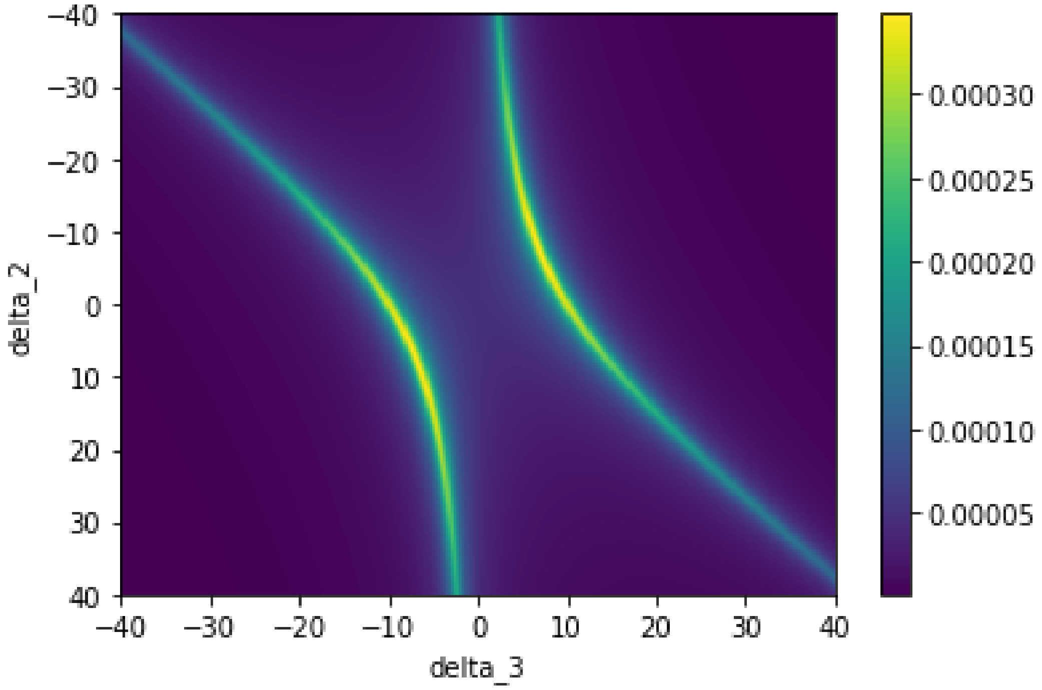
```
In [24]: #some default parameters
         p = {}
         p['a'] = 0.0
         p['delta2'] = 0.0
         p['delta3'] = 0.0
         p['gamma13'] = 1./22e-3
         p['gamma23'] = 1./22e-3
         p['gamma2d'] = 1./1e-6
         p['gamma3d'] = 1./1e-6
         p['nbath'] = 20
         p['gammamu'] = 1./((p['nbath']+1)*11e-3)
         p['omegao'] = 10e6
         p['omegam'] = 1e3
         p['g'] = 0.0
```

```
# Code currently requires these, even if we're not interested in
# inhomogeneous brodening for the moment
p['mean_delta2']=0
p['sd_delta2']=20e6/2.355 #2.355 is to turn FWHM into standard deviation
p['mean_delta3']=0
p['sd_delta3']=300e6/2.355


delta2vals = np.linspace(-50e6,50e6,505)
delta3vals = np.linspace(-50e6,50e6,505)


coh,_ = ensemble_steady_rho13(p,delta2vals,delta3vals)
coh = np.abs(coh)

#plt.rcParams['figure.figsize'] = [10, 4]
plt.imshow(coh,extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2vals)/1e6,
plt.ylabel('delta_2')
plt.xlabel('delta_3')
plt.colorbar();
```



Strong microwaves, weak optical pump

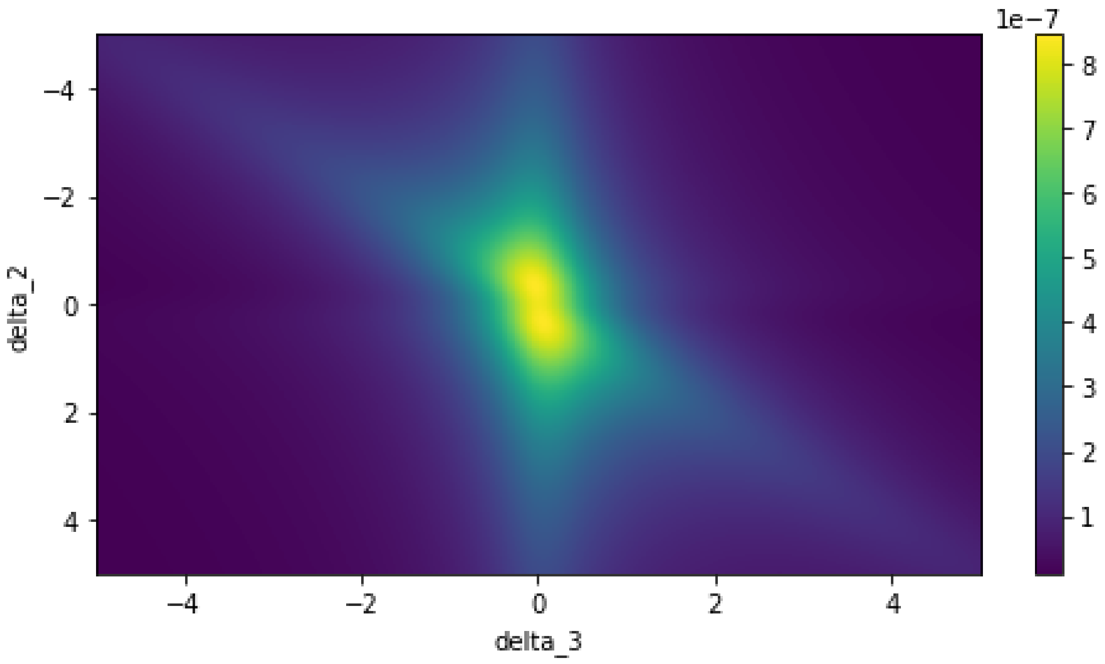
```
In [25]: #some default parameters
         p = {}
         p['a'] = 0
         p['delta2'] = 0.
         p['delta3'] = 0.
         p['gamma13'] = 1./22e-3
         p['gamma23'] = 1./22e-3
         p['gamma2d'] = 1./1e-6
         p['gamma3d'] = 1./1e-6
         p['nbath'] = 20
         p['gammamu'] = 1./((p['nbath']+1)*11e-3)
         p['omegao'] = 1e3
         p['omegam'] = 10e6
         p['g'] = 0.

         # Code currently requires these, even if we're not interested in
         # inhomogeneous brodening for the moment
         p['mean_delta2']=0
         p['sd_delta2']=20e6/2.355 #2.355 is to turn FWHM into standard deviation
         p['mean_delta3']=0
         p['sd_delta3']=300e6/2.355

         delta2vals = np.linspace(-40e6,40e6,501)
         delta3vals = np.linspace(-40e6,40e6,501)


         coh,_ = ensemble_steady_rho13(p,delta2vals,delta3vals)
         coh = np.abs(coh)


         #plt.rcParams['figure.figsize'] = [8, 4]
         plt.imshow(coh,extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2vals)/1e6,
         plt.ylabel('delta_2')
         plt.xlabel('delta_3')
         plt.colorbar();
```



Both microwaves and optical pump weak

```
In [26]: p = {}
         p['a'] = 0
         p['delta2'] = 0.
         p['delta3'] = 0.
         p['gamma13'] = 1./22e-3
         p['gamma23'] = 1./22e-3
         p['gamma2d'] = 1./1e-6
         p['gamma3d'] = 1./1e-6
         p['nbath'] = 20
         p['gammamu'] = 1./((p['nbath']+1)*11e-3)
         p['omegao'] = 1e3
         p['omegam'] = 1e3
         p['g'] = 0.

         # Code currently requires these, even if we're not interested in
         # inhomogeneous brodening for the moment
         p['mean_delta2']=0
         p['sd_delta2']=20e6/2.355 #2.355 is to turn FWHM into standard deviation
         p['mean_delta3']=0
         p['sd_delta3']=300e6/2.355

         delta2vals = np.linspace(-5e6,5e6,501)
```

```
delta3vals = np.linspace(-5e6,5e6,501)

coh,_ = ensemble_steady_rho13(p,delta2vals,delta3vals)
coh = np.abs(coh)

plt.rcParams['figure.figsize'] = [8, 4]
plt.imshow(coh,extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2vals)/1e6,m
plt.ylabel('delta_2')
plt.xlabel('delta_3')
plt.colorbar();
```



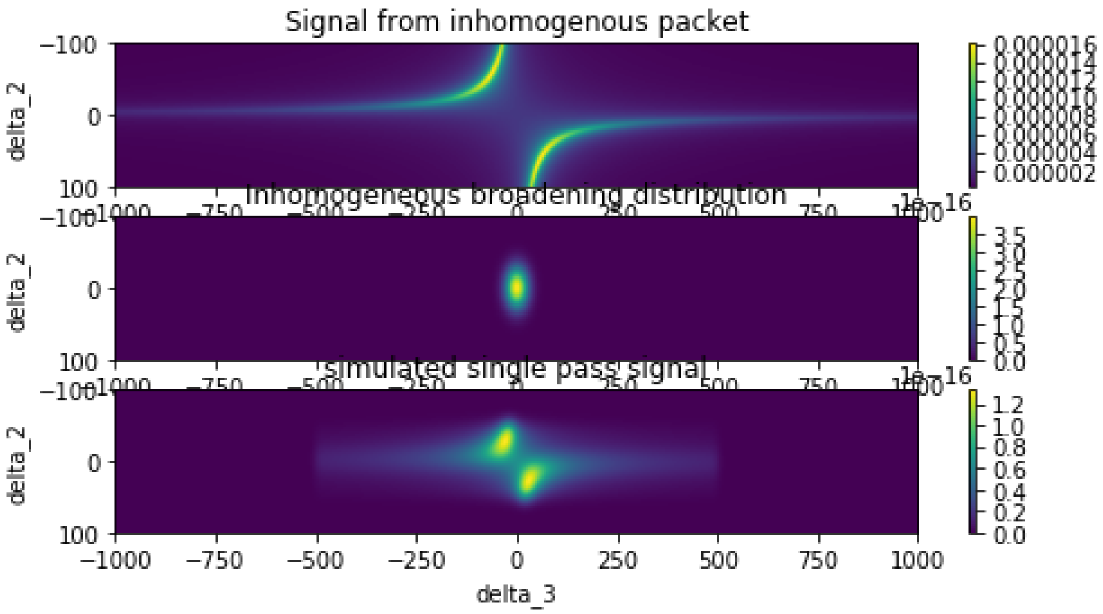
1.4.2 Single pass signal with inhomogneous broadening

```
In [27]: %%time
         p = {}
         p['a'] = 0
         p['delta2'] = 0.
         p['delta3'] = 0.
         p['gamma13'] = 1./22e-3
         p['gamma23'] = 1./22e-3
         p['gamma2d'] = 1./.1e-6 #unrealistically fast, but makes calculations easier
         p['gamma3d'] = 1./.1e-6
         p['nbath'] = 20
         p['gammamu'] = 1./((p['nbath']+1)*11e-3)
         p['omegao'] = 60e6
```

```
        p['omegam'] = 1e3
        p['g'] = 0.

        p['mean_delta2']=0
        p['sd_delta2']=20e6
        p['mean_delta3']=0
        p['sd_delta3']=20e6 #not realistic but makes calculations faster

        delta2vals = np.linspace(-100e6,100e6,1000)
        delta3vals = np.linspace(-1000e6,1000e6,10000)

        (coh13,w) = ensemble_steady_rho13(p,delta2vals,delta3vals)
        sig = signal.fftconvolve(coh13,w)
```

```
CPU times: user 18.4 s, sys: 1.3 s, total: 19.7 s
Wall time: 7.55 s
```

```
In [28]: #plt.rcParams['figure.figsize'] = [10, 20]
        plt.subplot(3,1,1)
        plt.imshow(np.abs(coh13),extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2v
        plt.ylabel('delta_2')
        plt.xlabel('delta_3')
        plt.title('Signal from inhomogenous packet')
        plt.colorbar();

        plt.subplot(3,1,2)
        plt.imshow(w.real,extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2vals)/1e
        plt.ylabel('delta_2')
        plt.xlabel('delta_3')
        plt.title('Inhomogeneous broadening distribution')
        plt.colorbar();

        plt.subplot(3,1,3)
        plt.imshow(np.abs(sig),extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2val
        plt.ylabel('delta_2')
        plt.xlabel('delta_3')
        plt.title('simulated single pass signal')
        plt.colorbar();
```



1.5 Introducing cavity dynamics

The (classical) equations of motion of the cavity mode amplitude are given by

$$\frac{d}{dt}a = -i\delta_c a - i\sum_k g_k\sigma_{13,k} - \frac{(\kappa_i+\kappa_c)}{2}a - \sqrt{\kappa_c}a_{in}$$

Here κ_i is the intrisic loss rate of our cavity and κ_c is the coupling induced loss.

Defining

$$S_{13}(a) = \sum_k g_k\sigma_{13,k}$$

We write $S_{13}(a)$ to remind us (explicitly) that the atoms states are a function of the cavity mode amplitude a.

Because we won't drive a this becomes

$$\frac{d}{dt}a = -i\delta_c a - iS_{13}(a) - \frac{(\kappa_i+\kappa_c)}{2}a$$

So to find the steady state of the cavity we need to find the roots of the function

$$f(a) = -i\delta_c a - iS_{13}(a) - \frac{(\kappa_i+\kappa_c)}{2}a$$

```
In [29]: def ffunc(p):
             df = p['df']
             a = p['a']
             N = p['N']
             g = p['g']
             deltac = p['deltac']
```

```
        kappa = p['kappaoi']+p['kappaoc']
        S13 = N*g*ensemble_steady_rho13_integrated(p,df)
        return -1j*deltac*a - 1j*S13 - kappa/2*a

    # the same function but in a form that maps a 2d vector to a 2d vector rather than co
    def ffunc_for_root(avec,p):
        a = avec[0]+1j*avec[1]
        # print(a)
        p['a']=a
        f = ffunc(p)
        return [f.real,f.imag]

    def steadya(p):
        # use value of a as steady state
        a = p['a']
        avec = [a.real,a.imag]
        df = p['df']
        result = scipy.optimize.root(ffunc_for_root,avec,args=(p))
        if not result.success:
            print(result)
            raise(AssertionError('The bloody thing didn\'t converge: '+result.message))
        #update p['a'] why not?
        a = result.x[0]+1j*result.x[1]
        p['a']=a
        return a
```

These aren't very realistic parameters, but lead to fast calcs

```
In [30]: # df = 0.1
         # p = {}
         # p['a'] = 0
         # p['delta2'] = 0.
         # p['delta3'] = 0.
         # p['gamma13'] = 1
         # p['gamma23'] = 1
         # p['gamma2d'] = 1
         # p['gamma3d'] = 1
         # p['nbath'] = 2
         # p['gammamu'] = 1
         # p['omegao'] = 1
         # p['omegam'] = .01
         # p['g'] = 1.
         # p['N'] = 1
         # p['deltac']=0
         # p['kappaoi']=1
         # p['kappaoc']=1
         # p['df']=0.1
```

```
        # p['mean_delta2']=0
        # p['sd_delta2']=2 #2.355 is to turn FWHM into standard deviation
        # p['mean_delta3']=0
        # p['sd_delta3']=3

        # omegamvals = np.logspace(-3,2,40)
        # avals = np.zeros(omegamvals.shape,dtype='complex128')
        # for i,omegam in enumerate(omegamvals):
        #     p['omegam']=omegam
        #     avals[i] = steadya(p)
```

```
In [31]: # plt.semilogx(omegamvals,(np.abs(avals)/(omegamvals))**2)
         # plt.ylabel('propto efficiency')
         # plt.xlabel('Omega_mu');
```

1.6 Time for some realistic simulations

```
In [65]: # calculate microwave rabi frequency from power in in dBm
         def omegam_from_Pin(Pin,p):
             mu0 = 4*pi*1e-7
             hbar=1.05457e-34; # in J*s
             Vsample=pi*((p['dsample']/2)**2) * p['Lsample'] # the volume of the Er:YSO sample
             V_microwave_cavity = Vsample/p['fillfactor']
             mwP = 1e-3 * 10**(Pin/10)  #convert from dBm to Watts
             Q=2*pi*p['freqm']/(p['kappami']+p['kappamc']*2) # Q factor
             S21=(4*p['kappamc']**2)/(p['kappami']+p['kappamc']*2)**2
             energy_in_cavity=math.sqrt(S21)*2/(2*pi*p['freqm']/Q)*mwP; # energy inside the mi
             Bmw=math.sqrt(mu0*(energy_in_cavity/V_microwave_cavity)/2); # Magnetic field of t
             p['omegam']=(p['mu12']*Bmw)/hbar*(-1); # in rad/sec???? in Hz
             return p['omegam']
```

```
In [66]: # calculate optical rabi frequency from power in in dBm
         def omegao_from_Pin(Pin,p):
             epsilon0=8.854187817e-12
             hbar=1.05457e-34; # in J*s

             optP = 1e-3 * 10**(Pin/10) #incident optical power in W
             pflux = optP/(2*pi*p['freqo']*hbar) #photon flux (photons/sec)
             n_in = pflux * p['kappaoc']*4/(p['kappaoc']+p['kappaoi'])**2 # num intracavity ph
                                                                          #is this right????
             Sspot = pi*p['Wcavity']**2 #cross sectional area of
                                        #optical mode
             V_cav = (Sspot*p['Lcavity_vac']+Sspot*p['Lsample']*p['nYSO']**3)/2;
             optEfield = math.sqrt(n_in*hbar*2*pi*p['freqo']/2/epsilon0/V_cav);
             p['omegao'] = p['d23']*optEfield/hbar*(-1);
             return p['omegao']
```

```
In [67]: # parameters from Stephen via slack
         # parameters commented out are not used in calculation
```

```
p = {}
p['a'] = 0
p['delta2'] = 0.
p['delta3'] = 0.


p['d13'] = 2e-32*math.sqrt(1/3)
p['d23'] = 2e-32*math.sqrt(2/3)
p['gamma13'] = p['d13']**2/(p['d13']**2+p['d23']**2)*1/11e-3
p['gamma23'] = p['d23']**2/(p['d13']**2+p['d23']**2)*1/11e-3
p['gamma2d'] = 1e6
p['gamma3d'] = 1e6
p['nbath'] = 20
p['gammamu'] = 1/(p['nbath']+1) * 1e3

p['g'] = 51.9  #optical coupling
p['N'] = 1.28e15 # number of atoms in the optical mode

p['deltac']=0 #detuning for
p['kappaoi']=2*pi*7.95e6 # intrinsic loss for optical resonator
p['kappaoc']=2*pi*1.7e6 # coupling loss for optical resonator
p['df']=0.1e6 # how small descretisation step to take when integrating over the
            # inhomogeneous lines

p['mean_delta2']=0
p['sd_delta2']=2*pi*25e6/2.355  #microwave inhomogeneous broadening
                                #2.355is to turn FWHM into standard deviation
p['mean_delta3']=0
p['sd_delta3']=2*pi*170e6/2.355 #optical inhomogeneous broadening

p['kappami'] = 650e3*2*pi # intrinsic loss for microwave cavity
p['kappamc'] = 70e3*2*pi  # coupling loss for optical cavity
                          # this is for one of the two output ports
p['Nmu'] = 2.22e16  #toal number of atoms
p['gmu'] = 1.04 #coupling between atoms and microwave field


muBohr=927.4009994e-26; # Bohr magneton in J/T in J* T^-1
p['mu12'] = 4.3803*muBohr # transition dipole moment for microwave cavity (J T^-1)

p['Lsample']=12e-3 # the length of the sample, in m
p['dsample']=5e-3 # the diameter of the sample, in m

p['fillfactor']=0.8 #microwave filling factor
p['freqm'] = 5.186e9
p['freqo'] = 195113.36e9

p['Lcavity_vac'] = 49.5e-3 # length of the vacuum part of the optical
```

```
                                   # Fabry Perot (m)
        p['Wcavity'] =  0.6e-3# width of optical resonator beam in sample (m)
        p['nYSO'] = 1.76 #refractive index of YSO

        # assume 1mW (=0dBm) input to both optical and to microwave cavities
        omegao_from_Pin(0,p)
        omegam_from_Pin(0,p)

        #see what rabi freqs we have
        p['omegao']/1e6,p['omegam']/1e6

        #%time ensemble_steady_rho13_integrated(p,p['df'])
```

```
Out[67]:
```

$$(-0.49209088755145475, \quad -4.7543084291639115)$$

In order to make things easier for the root finder we make the substiution

$$g \leftarrow \alpha g$$

$$N \leftarrow \frac{N}{\alpha^2}$$

This means that inorder to get the true intracavity field a we need to multiply the results of our calculations by α.

Trial and error suggests that using $\alpha = \sqrt{N}$ (the original N) is a good choice

```
In [68]: alpha = math.sqrt(p['N'])/40
         p['g'] = alpha *p['g']
         p['N'] = p['N']/alpha**2

         #this parameter shouldn't change
         math.sqrt(p['N'])*p['g']/1e9, " GHz"

Out[68]: (1.8568308485158254, ' GHz')
```

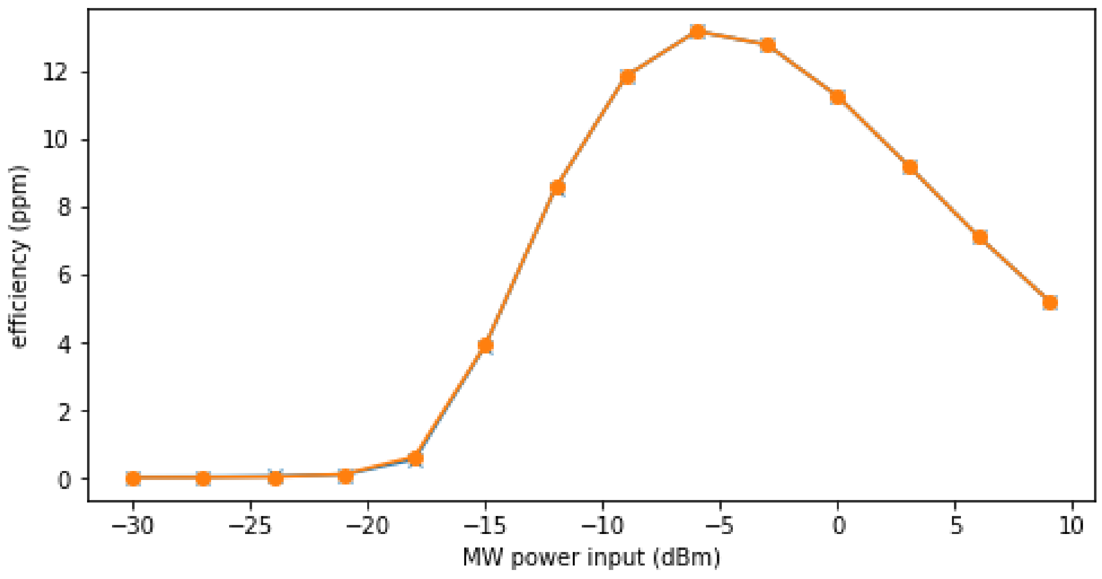
1.6.1 Efficiency vs microwave input power

```
In [69]: Pmu_vals_dBm = np.arange(-30,10,3) # in dBm
         Pmu_vals_W = 1e-3*np.exp(np.log(10)*Pmu_vals_dBm/10) #in W

         avals_approx = np.zeros(Pmu_vals_dBm.shape,dtype='complex128')
         avals = np.zeros(Pmu_vals_dBm.shape,dtype='complex128')

         for (i,P) in enumerate(Pmu_vals_dBm):
             #print("%d/%d"%(i+1,len(Pmu_vals_dBm)),end="")
             omegam_from_Pin(P,p)
             p['df'] = 2e6
             avals_approx[i] = steadya(p)*alpha
```

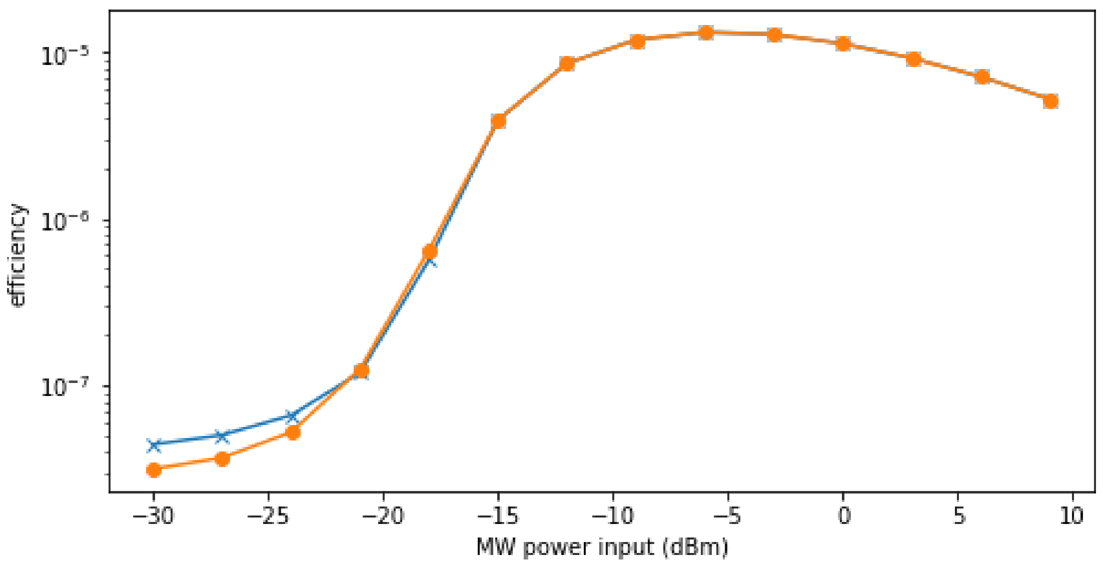
```
            p['df'] = 1e6
            avals[i] = steadya(p)*alpha
            #print(" -- completed")
```

```
In [70]: # this is the energy efficiency of the conversion
         effic = p['kappaoc']*np.abs(avals**2)*const.h*p['freqo'] /Pmu_vals_W
         effic_approx = p['kappaoc']*np.abs(avals_approx**2)*const.h*p['freqo'] /Pmu_vals_W

         # quantum efficiency
         effic  = effic*p['freqm']/p['freqo']
         effic_approx  = effic_approx*p['freqm']/p['freqo']
```

```
In [71]: plt.plot(Pmu_vals_dBm,effic*1e6,'x-',
                  Pmu_vals_dBm,effic_approx*1e6,'o-')
         plt.ylabel('efficiency (ppm)')
         plt.xlabel('MW power input (dBm)');
```



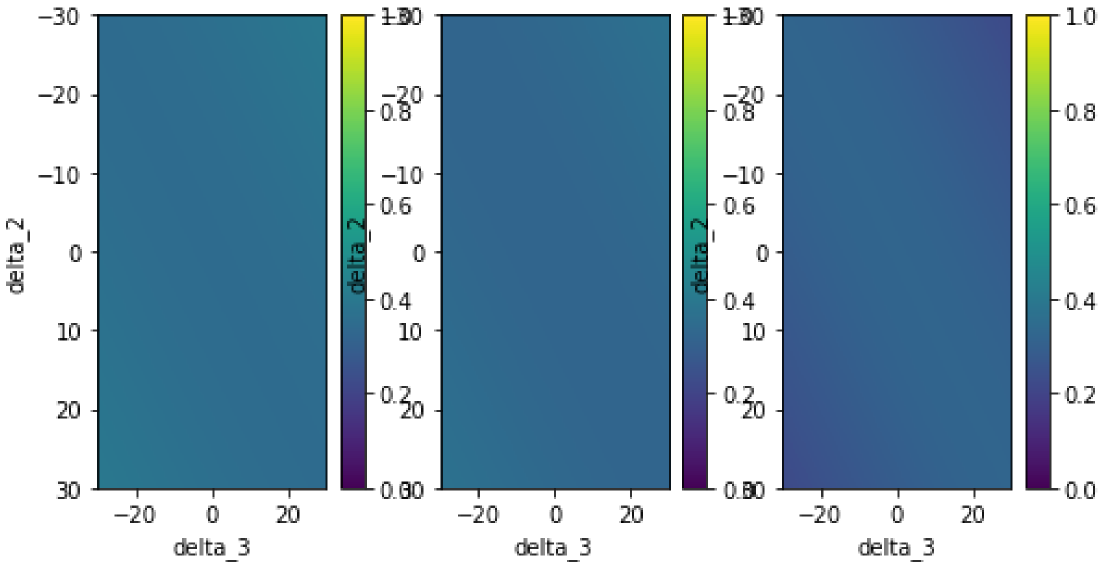
```
In [72]: plt.semilogy(Pmu_vals_dBm,effic,'x-',
                      Pmu_vals_dBm,effic_approx,'o-')#,Pmu_vals_dBm,np.abs(avals_approx)/Pmu_v
         plt.ylabel('efficiency')
         plt.xlabel('MW power input (dBm)');
```



1.6.2 Looking at how the population changes with microwave power

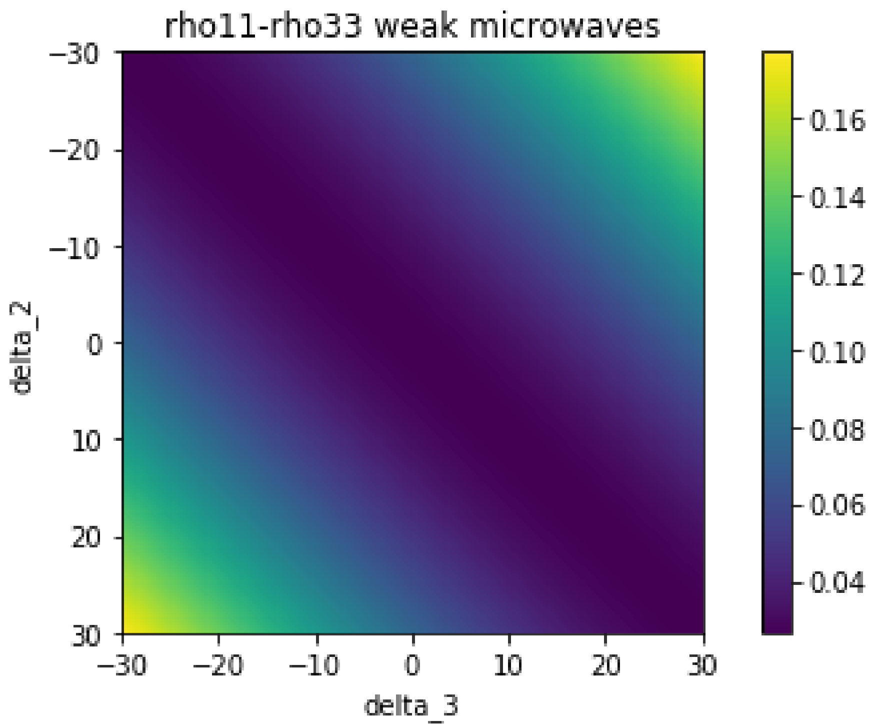
```
In [73]: #-30dBm in put power
         p['omegam']=0
         p['a']=0
         # omegam_from_Pin(-50,p)
         # p['a']=0
         # p['df']=2e6
         # steadya(p)
         # print(p['a'])
         # p['df']=1e6
         # steadya(p)
         # print(p['a'])

         delta2vals = np.linspace(-30e6,30e6,201)
         delta3vals = np.linspace(-30e6,30e6,201)

         pop = np.zeros((len(delta2vals),len(delta3vals),3))

         for ii in range(len(delta2vals)):
             for jj in range(len(delta3vals)):
                 p['delta2'] = delta2vals[ii]
                 p['delta3'] = delta3vals[jj]
                 rho = steady_rho(p)
                 for kk in range(3):
                     pop[ii,jj,kk] = rho[kk,kk].real

In [74]: plt.subplot(1,3,1)
```

```
        plt.rcParams['figure.figsize'] = [8, 4]
        plt.imshow(pop[:,:,0],
                   extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2vals)/1e6,min(d
                   norm=Norm(vmin=0, vmax=1),aspect='auto')
        plt.ylabel('delta_2')
        plt.xlabel('delta_3')
        plt.colorbar();

        plt.subplot(1,3,2)
        plt.rcParams['figure.figsize'] = [8, 4]
        plt.imshow(pop[:,:,1],extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2vals
                   norm=Norm(vmin=0, vmax=1),aspect='auto')
        plt.ylabel('delta_2')
        plt.xlabel('delta_3')
        plt.colorbar();

        plt.subplot(1,3,3)
        plt.rcParams['figure.figsize'] = [8, 4]
        plt.imshow(pop[:,:,2],extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2vals
                   norm=Norm(vmin=0, vmax=1),aspect='auto')
        plt.ylabel('delta_2')
        plt.xlabel('delta_3')
        plt.colorbar();
```



```
In [75]: plt.rcParams['figure.figsize'] = [8, 4]
         plt.imshow(pop[:,:,0]-pop[:,:,2],extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,ma
         plt.ylabel('delta_2')
         plt.xlabel('delta_3')
```

```
        plt.title('rho11-rho33 weak microwaves')
        plt.colorbar();
```

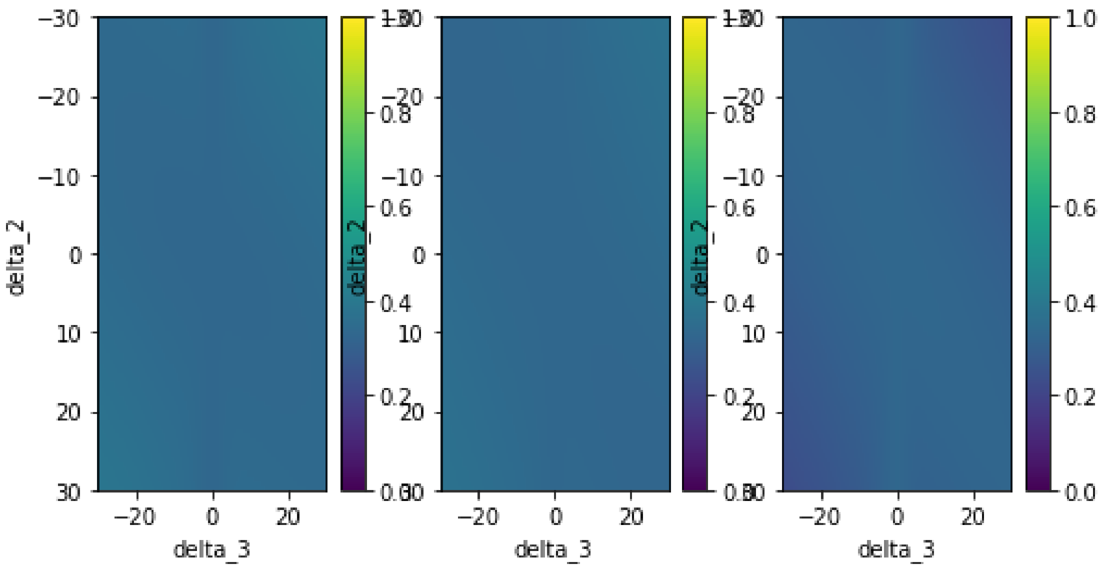


```
In [76]: #-5dBm in put power

         omegam_from_Pin(-15,p)
         p['a']=0
         p['df']=2e6
         steadya(p)
         print(p['a'])
         p['df']=1e6
         steadya(p)
         print(p['a'])

         delta2vals = np.linspace(-30e6,30e6,101)
         delta3vals = np.linspace(-30e6,30e6,101)

         pop = np.zeros((len(delta2vals),len(delta3vals),3))

         for ii in range(len(delta2vals)):
             for jj in range(len(delta3vals)):
                 p['delta2'] = delta2vals[ii]
                 p['delta3'] = delta3vals[jj]
                 rho = steady_rho(p)
```

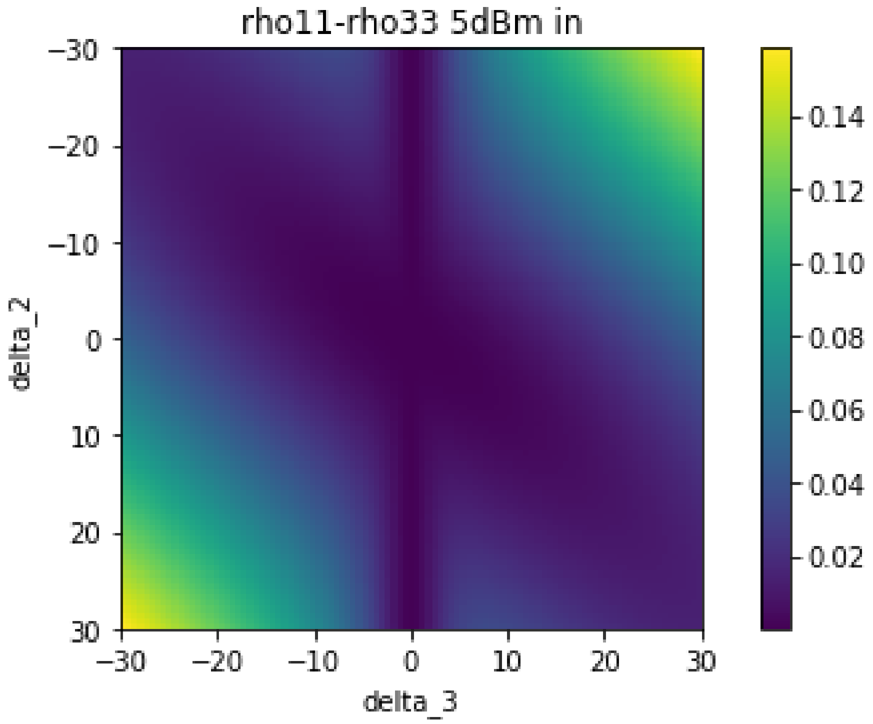
```
                for kk in range(3):
                    pop[ii,jj,kk] = rho[kk,kk].real
```

```
(-4.853697663066903e-14+0.0020574466778528473j)
(-2.8487804047696807e-14+0.0020528827087523266j)
```

```
In [77]: plt.subplot(1,3,1)
         plt.rcParams['figure.figsize'] = [8, 4]
         plt.imshow(pop[:,:,0],
                    extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2vals)/1e6,min(d
                    norm=Norm(vmin=0, vmax=1),aspect='auto')
         plt.ylabel('delta_2')
         plt.xlabel('delta_3')
         plt.colorbar();

         plt.subplot(1,3,2)
         plt.rcParams['figure.figsize'] = [8, 4]
         plt.imshow(pop[:,:,1],extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2vals
                    norm=Norm(vmin=0, vmax=1),aspect='auto')
         plt.ylabel('delta_2')
         plt.xlabel('delta_3')
         plt.colorbar();

         plt.subplot(1,3,3)
         plt.rcParams['figure.figsize'] = [8, 4]
         plt.imshow(pop[:,:,2],extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2vals
                    norm=Norm(vmin=0, vmax=1),aspect='auto')
         plt.ylabel('delta_2')
         plt.xlabel('delta_3')
         plt.colorbar();
```



```
In [78]: plt.rcParams['figure.figsize'] = [8, 4]
         plt.imshow(pop[:,:,0]-pop[:,:,2],extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,ma
         plt.ylabel('delta_2')
         plt.xlabel('delta_3')
         plt.title('rho11-rho33 5dBm in')
         plt.colorbar();
```



```
In [79]: p

Out[79]: {'a': (-2.8487804047696807e-14+0.0020528827087523266j),
          'delta2': 30000000.0,
          'delta3': 30000000.0,
          'd13': 1.1547005383792515e-32,
          'd23': 1.6329931618554522e-32,
          'gamma13': 30.303030303030297,
          'gamma23': 60.606060606060616,
          'gamma2d': 1000000.0,
          'gamma3d': 1000000.0,
          'nbath': 20,
          'gammamu': 47.61904761904761,
          'g': 46420771.21289563,
          'N': 1600.0000000000002,
          'deltac': 0,
          'kappaoi': 49951323.19207771,
          'kappaoc': 10681415.022205297,
          'df': 1000000.0,
          'mean_delta2': 0,
          'sd_delta2': 66700480.96793615,
          'mean_delta3': 0,
          'sd_delta3': 453563270.5819659,
          'kappami': 4084070.449666731,
          'kappamc': 439822.971502571,
          'Nmu': 2.22e+16,
          'gmu': 1.04,
          'mu12': 4.06229459767182e-23,
          'Lsample': 0.012,
          'dsample': 0.005,
          'fillfactor': 0.8,
          'freqm': 5186000000.0,
          'freqo': 195113360000000.0,
          'Lcavity_vac': 0.0495,
          'Wcavity': 0.0006,
          'nYSO': 1.76,
          'omegao': -492090.88755145477,
          'omegam': -845448.8788556679}
```

1.6.3 does lowering the temperature work? - this bit still broken

```
In [80]: #p['nbath']=20
```

```
In [47]: # Pmu_vals_dBm = np.arange(-30,11,3) # in dBm

         # Pmu_vals_W = 1e-3*np.exp(np.log(10)*Pmu_vals_dBm/10) #in W


         # avals_approx = np.zeros(Pmu_vals_dBm.shape,dtype='complex128')
         # avals_cold =   np.zeros(Pmu_vals_dBm.shape,dtype='complex128')

         # for (i,P) in enumerate(Pmu_vals_dBm):
         #     print("%d/%d"%(i+1,len(Pmu_vals_dBm)),end="")
         #     omegam_from_Pin(P,p)
         #     p['df'] = 2e6
         #     avals_approx[i] = steadya(p)*alpha
         #     p['df'] = 1e6
         #     avals_cold[i] = steadya(p)*alpha
         #     print(" -- completed")
```

```
In [48]: # # this is the energy efficiency of the conversion
         # effic_cold = p['kappaoc']*np.abs(avals_cold**2)*const.h*p['freqo'] /Pmu_vals_W
```

```
        # effic_approx = p['kappaoc']*np.abs(avals_approx**2)*const.h*p['freqo'] /Pmu_vals_W

        # #quantum efficiency
        # effic_cold  = effic*p['freqm']/p['freqo']
        # effic_approx  = effic_approx*p['freqm']/p['freqo']

In [49]: # plt.semilogy(Pmu_vals_dBm,effic*1e6,'x-',
         #          Pmu_vals_dBm,effic_cold*1e6,'o-')#,Pmu_vals_dBm,np.abs(avals_approx)/Pmu_v
         # plt.ylabel('efficiency (ppm)')
         # plt.xlabel('MW power input (dBm)');
         # plt.legend(('hotish','cold'))
```

1.6.4 scratchpad