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1 Code for simulation of Raman heterodyne like processes

1.1 Getting setup

1.1.1 Import required python libraries

```
In [1]: import sympy as sym
        sym.init_printing()

        import numpy as np
        from scipy import integrate,signal

        import math
        from math import pi
        #from sympy.functions import Heaviside, sin, cos, sqrt


        import matplotlib.pyplot as plt
        from sympy import I, Matrix, symbols
        from sympy.physics.quantum import TensorProduct,Dagger
        import scipy.optimize

        import qutip

        %load_ext cython
```

1.1.2 Some things to help us make Liovillian superoperators

- Uses an approach very like qutip and the qotoolbox, reading the documention for those two packages might help make sense of spre, spost, etc
- s13 etc. are the atomic operators like |1><3| etc

```
In [2]: def spre(m):
            return TensorProduct(sym.eye(m.shape[0]),m)

        def spost(m):
            return TensorProduct(m.T,sym.eye(m.shape[0]))
```

```
def collapse(c):
    temp = Dagger(c)*c/2
    return spre(c)*spost(Dagger(c))-spre(temp)-spost(temp)


s13=Matrix([[0,0,1],[0,0,0],[0,0,0]])
s23=Matrix([[0,0,0],[0,0,1],[0,0,0]])
s12=Matrix([[0,1,0],[0,0,0],[0,0,0]])

s31=s13.T
s32=s23.T
s21=s12.T

s11 = s12*s21
s22 = s21*s12
s33 = s31*s13
```

1.1.3 Checking spre and spost agree with qutip

Note: This notebook uses Frotran (colum first) order when flattening matricies.

```
In [3]: sigmay = Matrix([[0, -I], [I, 0 ]])
        sigmay, qutip.sigmay()
```

```
Out[3]: (Matrix([
         [0, -I],
         [I,  0]]),
         Quantum object: dims = [[2], [2]], shape = (2, 2), type = oper, isherm = True
         Qobj data =
         [[0.+0.j 0.-1.j]
          [0.+1.j 0.+0.j]])
```

```
In [4]: qutip.sigmay()
```

Out[4]:
Quantum object: dims = [[2], [2]], shape = (2, 2), type = oper, isherm = True

$$\begin{pmatrix} 0.0 & -1.0j \\ 1.0j & 0.0 \end{pmatrix}$$

```
In [5]: spre(sigmay),qutip.spre(qutip.sigmay())
```

```
Out[5]: (Matrix([
         [0, -I, 0,  0],
         [I,  0, 0,  0],
         [0,  0, 0, -I],
         [0,  0, I,  0]]),
         Quantum object: dims = [[[2], [2]], [[2], [2]]], shape = (4, 4), type = super, isherm
         Qobj data =
```

```
 [[0.+0.j 0.-1.j 0.+0.j 0.+0.j]
  [0.+1.j 0.+0.j 0.+0.j 0.+0.j]
  [0.+0.j 0.+0.j 0.+0.j 0.-1.j]
  [0.+0.j 0.+0.j 0.+1.j 0.+0.j]])
```

In [6]:
```
spost(sigmay),qutip.spost(qutip.sigmay())
```

Out[6]:
```
(Matrix([
 [ 0,  0, I, 0],
 [ 0,  0, 0, I],
 [-I,  0, 0, 0],
 [ 0, -I, 0, 0]]),
 Quantum object: dims = [[[2], [2]], [[2], [2]]], shape = (4, 4), type = super, isherm
 Qobj data =
 [[0.+0.j 0.+0.j 0.+1.j 0.+0.j]
  [0.+0.j 0.+0.j 0.+0.j 0.+1.j]
  [0.-1.j 0.+0.j 0.+0.j 0.+0.j]
  [0.+0.j 0.-1.j 0.+0.j 0.+0.j]])
```

1.1.4 The symbolic variables we will use

- TODO: explain what they all are

In [7]:
```
delta2,delta3 = sym.symbols('delta_2 delta_3',real=True)
gamma13,gamma23,gamma2d,gamma3d,nbath,gammamu = sym.symbols('gamma_13 gamma_23 gamma_2
omegao,omegam = sym.symbols('Omega_o Omega_mu', real=True, negative=False)
rho11,rho12,rho13,rho21,rho22,rho23,rho31,rho32,rho33 = sym.symbols('rho_11 rho_12 rho_

a = sym.symbols('a')
ar,ai = sym.symbols('a_r a_i',real=True)
g = sym.symbols('g',real=True, negative=False)
```

1.2 Analytic calculations - Liovillian

In [8]:
```
H =  omegam*s21 + omegao*s32 + g*a*s31
H = H + Dagger(H)
H = H + delta2*s22 + delta3*s33
LH = -I*spre(H)+I*spost(H)

L21 = gammamu*(nbath+1)*collapse(s12)
L12 = gammamu*(nbath)*collapse(s21)
L32 = gamma23*collapse(s23)
L31 = gamma13*collapse(s13)
L22 = gamma2d*collapse(s22)
L33 = gamma3d*collapse(s33)

L = LH + L21 + L12 + L32 + L31 + L22 + L33

#because Trace(rho) = 1 we can replace first row as below
```

```
#this gives us a matrix that can be inverted
L = L.row_insert(0,Matrix([[1,0,0,0,1,0,0,0,1]]))
L.row_del(1)

# The condition for the steady state is now L*rho=V
# where V = [1,0,0, 0,0,0, 0,0,0].T
```

1.2.1 Expressions for the various operators

In [9]: L

Out[9]:

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 1 \\
-i\Omega_\mu & -i\delta_2 - \frac{\gamma_{2d}}{2} - \frac{\gamma_\mu n_b}{2} - \frac{\gamma_\mu(n_b+1)}{2} & -i\Omega_o & 0 & i\Omega_\mu \\
-iag & -i\Omega_o & -i\delta_3 - \frac{\gamma_{13}}{2} - \frac{\gamma_{23}}{2} - \frac{\gamma_{3d}}{2} - \frac{\gamma_\mu n_b}{2} & 0 & 0 \\
i\Omega_\mu & 0 & 0 & i\delta_2 - \frac{\gamma_{2d}}{2} - \frac{\gamma_\mu n_b}{2} - \frac{\gamma_\mu(n_b+1)}{2} & -i\Omega_\mu \\
\gamma_\mu n_b & i\Omega_\mu & 0 & -i\Omega_\mu & -\gamma_\mu\left(n_b+1\right) \\
0 & 0 & i\Omega_\mu & -iag & -i\Omega_o \\
ig\bar{a} & 0 & 0 & i\Omega_o & 0 \\
0 & ig\bar{a} & 0 & 0 & i\Omega_o \\
0 & 0 & ig\bar{a} & 0 & 0
\end{bmatrix}$$

In [10]: H

Out[10]:

$$\begin{bmatrix}
0 & \Omega_\mu & g\bar{a} \\
\Omega_\mu & \delta_2 & \Omega_o \\
ag & \Omega_o & \delta_3
\end{bmatrix}$$

In [11]:
```
#rho flattened out to a vector
rho = Matrix([[rho11,rho21,rho31],[rho12,rho22,rho32],[rho13,rho23,rho33]])
rho = 1*rho.T #because we are using "fortran" style matrix flatteneing
rho[:]
rhoflat = 1*rho.T
rhoflat = rhoflat[:]
rhoflat
```

Out[11]:

$$[\rho_{11},\ \rho_{21},\ \rho_{31},\ \rho_{12},\ \rho_{22},\ \rho_{32},\ \rho_{13},\ \rho_{23},\ \rho_{33}]$$

In [12]:
```
L[:,0:3] #because the matrix gets clipped when exporting to PDF
```

Out[12]:

$$\begin{bmatrix} 1 & 0 & 0 \\ -i\Omega_\mu & -i\delta_2 - \frac{\gamma_{2d}}{2} - \frac{\gamma_\mu n_b}{2} - \frac{\gamma_\mu(n_b+1)}{2} & -i\Omega_o \\ -iag & -i\Omega_o & -i\delta_3 - \frac{\gamma_{13}}{2} - \frac{\gamma_{23}}{2} - \frac{\gamma_{3d}}{2} - \frac{\gamma_\mu n_b}{2} \\ i\Omega_\mu & 0 & 0 \\ \gamma_\mu n_b & i\Omega_\mu & 0 \\ 0 & 0 & i\Omega_\mu \\ ig\bar{a} & 0 & 0 \\ 0 & ig\bar{a} & 0 \\ 0 & 0 & ig\bar{a} \end{bmatrix}$$

In [13]: `L[:,3:6]`

Out[13]:

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & i\Omega_\mu & 0 \\ 0 & 0 & i\Omega_\mu \\ i\delta_2 - \frac{\gamma_{2d}}{2} - \frac{\gamma_\mu n_b}{2} - \frac{\gamma_\mu(n_b+1)}{2} & -i\Omega_\mu & -ig\bar{a} \\ -i\Omega_\mu & -\gamma_\mu\,(n_b+1) & -i\Omega_o \\ -iag & -i\Omega_o & i\delta_2 - i\delta_3 - \frac{\gamma_{13}}{2} - \frac{\gamma_{23}}{2} - \frac{\gamma_{2d}}{2} - \frac{\gamma_{3d}}{2} - \frac{\gamma_\mu(n_b+1)}{2} \\ i\Omega_o & 0 & 0 \\ 0 & i\Omega_o & 0 \\ 0 & 0 & i\Omega_o \end{bmatrix}$$

In [14]: `L[:,6:]`

Out[14]:

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & iag & 0 \\ 0 & 0 & iag \\ i\Omega_o & 0 & 0 \\ 0 & i\Omega_o & \gamma_{23} \\ 0 & 0 & i\Omega_o \\ i\delta_3 - \frac{\gamma_{13}}{2} - \frac{\gamma_{23}}{2} - \frac{\gamma_{3d}}{2} - \frac{\gamma_\mu n_b}{2} & -i\Omega_\mu & -ig\bar{a} \\ -i\Omega_\mu & -i\delta_2 + i\delta_3 - \frac{\gamma_{13}}{2} - \frac{\gamma_{23}}{2} - \frac{\gamma_{2d}}{2} - \frac{\gamma_{3d}}{2} - \frac{\gamma_\mu(n_b+1)}{2} & -i\Omega_o \\ -iag & -i\Omega_o & -\gamma_{13} - \gamma_{23} \end{bmatrix}$$

In [15]: `rhoflat[6]`

Out[15]:

$$\rho_{13}$$

1.2.2 We can transform the "basis" we use for our density matrix vector so that all the elements are real

```
In [16]: #change of variables to make things real to make it a bit faster maybe
         CtoR = Matrix([[2,0,0,0,0,0,0,0,0],
                        [0,0,0,0,2,0,0,0,0],
                        [0,0,0,0,0,0,0,0,2],
                        [0,1,0,1,0,0,0,0,0],
                        [0,I,0,-I,0,0,0,0,0],
                        [0,0,1,0,0,0,1,0,0],
                        [0,0,I,0,0,0,-I,0,0],
                        [0,0,0,0,0,1,0,1,0],
                        [0,0,0,0,0,I,0,-I,0]
                       ])
         CtoR=CtoR/2
```

1.2.3 The new organisation for our density matrix

```
In [17]: #these is our new state vector
         t = CtoR*Matrix(rhoflat)
         t
```

Out[17]:

$$\begin{bmatrix} \rho_{11} \\ \rho_{22} \\ \rho_{33} \\ \frac{\rho_{12}}{2} + \frac{\rho_{21}}{2} \\ -\frac{i\rho_{12}}{2} + \frac{i\rho_{21}}{2} \\ \frac{\rho_{13}}{2} + \frac{\rho_{31}}{2} \\ -\frac{i\rho_{13}}{2} + \frac{i\rho_{31}}{2} \\ \frac{\rho_{23}}{2} + \frac{\rho_{32}}{2} \\ -\frac{i\rho_{23}}{2} + \frac{i\rho_{32}}{2} \end{bmatrix}$$

1.2.4 The Liovillian in this new basis

```
In [18]: Lreal = sym.simplify(CtoR*L*CtoR.inv())
         Lreal = Lreal.subs(a,ar+I*ai)
         Lreal
```

Out[18]:

$$\begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 \\
\gamma_\mu n_b & -\gamma_\mu (n_b+1) & \gamma_{23} & 0 & 2\Omega_\mu & 0 & 0 \\
0 & 0 & -\gamma_{13}-\gamma_{23} & 0 & 0 & 2a_i g & 2a_i \\
0 & 0 & 0 & -\frac{\gamma_{2d}}{2}-\gamma_\mu n_b-\frac{\gamma_\mu}{2} & -\delta_2 & 0 & -\Omega \\
\Omega_\mu & -\Omega_\mu & 0 & \delta_2 & -\frac{\gamma_{2d}}{2}-\gamma_\mu n_b-\frac{\gamma_\mu}{2} & \Omega_o & 0 \\
a_i g & 0 & -a_i g & 0 & -\Omega_o & -\frac{\gamma_{13}}{2}-\frac{\gamma_{23}}{2}-\frac{\gamma_{3d}}{2}-\frac{\gamma_\mu n_b}{2} & -a \\
a_r g & 0 & -a_r g & \Omega_o & 0 & \delta_3 & -\frac{\gamma_{13}}{2}-\frac{\gamma_{23}}{2}- \\
0 & 0 & 0 & a_i g & a_r g & 0 & \Omega \\
0 & \Omega_o & -\Omega_o & a_r g & -a_i g & -\Omega_\mu & 0
\end{bmatrix}$$

```
In [19]: # A sanity check
         # sym.simplify(CtoR.inv()*Lreal*CtoR*Matrix(rhoflat)-(L*Matrix(rhoflat)).subs(a,ar+I*
```

1.2.5 C-code from our analytic expressions for the two different versions of the Liovillan

- this is copied and pasted into some of the functions below

```
In [20]: from sympy.printing import print_ccode
         Lflat = 1*L.T
         Lflat = Lflat[:]
         print_ccode(Lflat)

         print("-----------")
         #because this is destined for cython it's in row-major order
         Lflatreal = 1*Lreal.T
         Lflatreal = Lflatreal[:]
         print_ccode(Lflatreal)
         print("----------")
         for k in range(81):
             print("L[%d] = "%(k,),end="")
             print_ccode(Lflatreal[k])
```

```
// Not supported in C:
// conjugate
[1, -I*Omega_mu, -I*a*g, I*Omega_mu, gamma_mu*n_b, 0, I*g*conjugate(a), 0, 0, 0, -I*delta_2 - 
-----------
(1, gamma_mu*n_b, 0, 0, Omega_mu, a_i*g, a_r*g, 0, 0, 1, -gamma_mu*(n_b + 1), 0, 0, -Omega_mu,
----------
L[0] = 1
L[1] = gamma_mu*n_b
L[2] = 0
L[3] = 0
L[4] = Omega_mu
L[5] = a_i*g
L[6] = a_r*g
L[7] = 0
```

```
L[8] = 0
L[9] = 1
L[10] = -gamma_mu*(n_b + 1)
L[11] = 0
L[12] = 0
L[13] = -Omega_mu
L[14] = 0
L[15] = 0
L[16] = 0
L[17] = Omega_o
L[18] = 1
L[19] = gamma_23
L[20] = -gamma_13 - gamma_23
L[21] = 0
L[22] = 0
L[23] = -a_i*g
L[24] = -a_r*g
L[25] = 0
L[26] = -Omega_o
L[27] = 0
L[28] = 0
L[29] = 0
L[30] = -1.0/2.0*gamma_2d - gamma_mu*n_b - 1.0/2.0*gamma_mu
L[31] = delta_2
L[32] = 0
L[33] = Omega_o
L[34] = a_i*g
L[35] = a_r*g
L[36] = 0
L[37] = 2*Omega_mu
L[38] = 0
L[39] = -delta_2
L[40] = -1.0/2.0*gamma_2d - gamma_mu*n_b - 1.0/2.0*gamma_mu
L[41] = -Omega_o
L[42] = 0
L[43] = a_r*g
L[44] = -a_i*g
L[45] = 0
L[46] = 0
L[47] = 2*a_i*g
L[48] = 0
L[49] = Omega_o
L[50] = -1.0/2.0*gamma_13 - 1.0/2.0*gamma_23 - 1.0/2.0*gamma_3d - 1.0/2.0*gamma_mu*n_b
L[51] = delta_3
L[52] = 0
L[53] = -Omega_mu
L[54] = 0
L[55] = 0
```

```
L[56] = 2*a_r*g
L[57] = -Omega_o
L[58] = 0
L[59] = -delta_3
L[60] = -1.0/2.0*gamma_13 - 1.0/2.0*gamma_23 - 1.0/2.0*gamma_3d - 1.0/2.0*gamma_mu*n_b
L[61] = Omega_mu
L[62] = 0
L[63] = 0
L[64] = 0
L[65] = 0
L[66] = -a_i*g
L[67] = -a_r*g
L[68] = 0
L[69] = -Omega_mu
L[70] = -1.0/2.0*gamma_13 - 1.0/2.0*gamma_23 - 1.0/2.0*gamma_2d - 1.0/2.0*gamma_3d - 1.0/2.0*g
L[71] = -delta_2 + delta_3
L[72] = 0
L[73] = -2*Omega_o
L[74] = 2*Omega_o
L[75] = -a_r*g
L[76] = a_i*g
L[77] = Omega_mu
L[78] = 0
L[79] = delta_2 - delta_3
L[80] = -1.0/2.0*gamma_13 - 1.0/2.0*gamma_23 - 1.0/2.0*gamma_2d - 1.0/2.0*gamma_3d - 1.0/2.0*g
```

1.3 Numerical code for steady state

1.3.1 Python code

- Using analytic expressions above
- The code below is much slower than the cython version below, but useful for confirming correctness of cython version.

```
In [21]: Lfunc = sym.lambdify((a,delta2, delta3, gamma13, gamma23, gamma2d, gamma3d, nbath,gam

         Lrealfunc = sym.lambdify((ar,ai,delta2, delta3, gamma13, gamma23, gamma2d, gamma3d, n

         # finds the steady state density matrix
         # p is a python dictionary full of parameters

         def steady_rho(p):

             Lmatrix = Lfunc(p['a'],p['delta2'],p['delta3'],p['gamma13'],
                                      p['gamma23'],p['gamma2d'],p['gamma3d'],p['nbath'],
                                      p['gammamu'],p['omegao'],p['omegam'],p['g'])

             #return Lmatrix
```

```
    rho = np.linalg.solve(Lmatrix.astype('complex128'),np.matrix([[1,0,0,0,0,0,0,0,0]
    return np.reshape(rho,(3,3),order='F')

# this code is commented out because it has been replaced by cython version
# will leave it for future sanity checks if

# Just the rho13 element of the density matrix:

# def steady_rho13_older(p):
#     Lmatrix = Lfunc(p['a'],p['delta2'],p['delta3'],p['gamma13'],
#                                 p['gamma23'],p['gamma2d'],p['gamma3d'],p['nbath'],
#                                 p['gammamu'],p['omegao'],p['omegam'],p['g'])
#     #return Lmatrix
#     rho = np.linalg.solve(Lmatrix.astype('complex128'),np.matrix([[1,0,0,0,0,0,0,0,0,
#     return rho[6]

# # another version
# def steady_rho13_old(p):
#     Lmatrix =  Lrealfunc(np.real(p['a']),np.real(p['a']),p['delta2'],p['delta3'],p[
#                                 p['gamma23'],p['gamma2d'],p['gamma3d'],p['nbath'],
#                                 p['gammamu'],p['omegao'],p['omegam'],p['g'])
#     rho = np.linalg.solve(Lmatrix.astype('double'),np.matrix([[1,0,0,0,0,0,0,0,0]])
#     return rho[5]+1j*rho[6]
```

1.3.2 Cython versions

Implemented in cython to make it (much) faster.

```
In [22]: %%cython --compile-args=-fopenmp --link-args=-fopenmp --verbose --force
         #### %%cython -a # this gives "highligted" output

         #cython: boundscheck=False, wraparound=False, nonecheck=False
         cimport cython
         import numpy as np
         from cython.parallel cimport prange
         cimport scipy.linalg.cython_lapack as lapack
         import numpy as np
         from libc.math cimport exp


         cdef int nextpow2(int x):
             cdef int n=1
             while(True):
                 if n>=x:
                     return n
                 n=2*n

         def steady_rho13(p):
```

```
    cdef double a_r,a_i,delta_2,delta_3,gamma_13,gamma_23,gamma_2d,gamma_3d,n_b,gamma_
    cdef double L[81]
    cdef double V[9]
    cdef int n = 9
    cdef double complex a
    cdef int info
    cdef int lda = 9
    cdef int ldb = 9
    cdef int nrhs = 1
    cdef int workspace[9]

    V[:]= [1,0,0, 0,0,0, 0,0,0]
    a = p['a']
    a_r = a.real
    a_i = a.imag
    delta_2 = p['delta2']
    delta_3 = p['delta3']
    gamma_13 = p['gamma13']
    gamma_23 = p['gamma23']
    gamma_2d = p['gamma2d']
    gamma_3d = p['gamma3d']
    n_b = p['nbath']
    gamma_mu = p['gammamu']
    Omega_o = p['omegao']
    Omega_mu = p['omegam']
    g = p['g']
#   with nogil: # to make sure it's not doing any slow python calls
    L[:] = (1, gamma_mu*n_b, 0, 0, Omega_mu, a_i*g, a_r*g, 0, 0, 1, -gamma_mu*(n_b + 1

    lapack.dgesv(&n,&nrhs,L,&lda,workspace,V,&ldb,&info)
    return V[5]+1j*V[6]


cdef double complex steady_rho13_lots_of_args(double a_r,
                                double a_i,
                                double delta_2,
                                double delta_3,
                                double gamma_13,
                                double gamma_23,
                                double gamma_2d,
                                double gamma_3d,
                                double n_b,
                                double gamma_mu,
                                double Omega_o,
                                double Omega_mu,
                                double g) nogil:
    cdef double L[81]
    cdef double V[9]
```

```
cdef int n = 9
cdef double complex a
cdef int info
cdef int lda = 9
cdef int ldb = 9
cdef int nrhs = 1
cdef int workspace[9]
cdef int k

# Unfortunately cant use this syntax if we dont have dont have GIL
#V[:] = [1,0,0, 0,0,0, 0,0,0]
#L[:] = (1, gamma_mu*n_b, 0, 0, Omega_mu, a_i*g, a_r*g, 0, 0, 1, -gamma_mu*(n_b +

for k in range(9):
    V[k] = 0
V[0] = 1

L[0] = 1
L[1] = gamma_mu*n_b
L[2] = 0
L[3] = 0
L[4] = Omega_mu
L[5] = a_i*g
L[6] = a_r*g
L[7] = 0
L[8] = 0
L[9] = 1
L[10] = -gamma_mu*(n_b + 1)
L[11] = 0
L[12] = 0
L[13] = -Omega_mu
L[14] = 0
L[15] = 0
L[16] = 0
L[17] = Omega_o
L[18] = 1
L[19] = gamma_23
L[20] = -gamma_13 - gamma_23
L[21] = 0
L[22] = 0
L[23] = -a_i*g
L[24] = -a_r*g
L[25] = 0
L[26] = -Omega_o
L[27] = 0
L[28] = 0
L[29] = 0
L[30] = -1.0/2.0*gamma_2d - gamma_mu*n_b - 1.0/2.0*gamma_mu
```

```
L[31] = delta_2
L[32] = 0
L[33] = Omega_o
L[34] = a_i*g
L[35] = a_r*g
L[36] = 0
L[37] = 2*Omega_mu
L[38] = 0
L[39] = -delta_2
L[40] = -1.0/2.0*gamma_2d - gamma_mu*n_b - 1.0/2.0*gamma_mu
L[41] = -Omega_o
L[42] = 0
L[43] = a_r*g
L[44] = -a_i*g
L[45] = 0
L[46] = 0
L[47] = 2*a_i*g
L[48] = 0
L[49] = Omega_o
L[50] = -1.0/2.0*gamma_13 - 1.0/2.0*gamma_23 - 1.0/2.0*gamma_3d - 1.0/2.0*gamma_mu
L[51] = delta_3
L[52] = 0
L[53] = -Omega_mu
L[54] = 0
L[55] = 0
L[56] = 2*a_r*g
L[57] = -Omega_o
L[58] = 0
L[59] = -delta_3
L[60] = -1.0/2.0*gamma_13 - 1.0/2.0*gamma_23 - 1.0/2.0*gamma_3d - 1.0/2.0*gamma_mu
L[61] = Omega_mu
L[62] = 0
L[63] = 0
L[64] = 0
L[65] = 0
L[66] = -a_i*g
L[67] = -a_r*g
L[68] = 0
L[69] = -Omega_mu
L[70] = -1.0/2.0*gamma_13 - 1.0/2.0*gamma_23 - 1.0/2.0*gamma_2d - 1.0/2.0*gamma_3d
L[71] = -delta_2 + delta_3
L[72] = 0
L[73] = -2*Omega_o
L[74] = 2*Omega_o
L[75] = -a_r*g
L[76] = a_i*g
L[77] = Omega_mu
L[78] = 0
```

```
    L[79] = delta_2 - delta_3
    L[80] = -1.0/2.0*gamma_13 - 1.0/2.0*gamma_23 - 1.0/2.0*gamma_2d - 1.0/2.0*gamma_3
    lapack.dgesv(&n,&nrhs,L,&lda,workspace,V,&ldb,&info)
    return V[5]+1j*V[6]



# Steady state an ensemble of atoms, returns a matrix of rho13 values as well as the
# weights from the inhomogeneous broadening distribution.
# Convolve the two to get simulate single pass raman heterodyne signal as a function

def ensemble_steady_rho13(p,delta2vals,delta3vals):
    cdef double a_r,a_i,delta_2,delta_3,gamma_13,gamma_23,gamma_2d,gamma_3d,n_b,gamma_
    a = p['a']
    a_r = a.real
    a_i = a.imag
    delta_2 = p['delta2']
    delta_3 = p['delta3']
    gamma_13 = p['gamma13']
    gamma_23 = p['gamma23']
    gamma_2d = p['gamma2d']
    gamma_3d = p['gamma3d']
    n_b = p['nbath']
    gamma_mu = p['gammamu']
    Omega_o = p['omegao']
    Omega_mu = p['omegam']
    g = p['g']

    cdef int ii
    cdef int jj

    cdef double m2 = p['mean_delta2']
    cdef double sd2 = p['sd_delta2']
    cdef double m3 = p['mean_delta3']
    cdef double sd3 = p['sd_delta3']
    #print(sd3)

    cdef double [:] delta2valsv = delta2vals
    cdef double [:] delta3valsv = delta3vals

    cdef int ndelta2 = len(delta2vals)
    cdef int ndelta3 = len(delta3vals)

    cdef double df2 = delta2valsv[1]-delta2valsv[0]
    cdef double df3 = delta3valsv[1]-delta3valsv[0]
```

```
    coh13 = np.zeros((ndelta2,ndelta3),dtype='complex128') #make numpy array
    cdef double complex [:,:] coh13v = coh13 #make cython  "view" of the data

    w = np.zeros((ndelta2,ndelta3),dtype='complex128') #make numpy array for gaussian
    cdef double complex [:,:] wv = w #make cython  "view" of the data

    for ii in prange(ndelta2, nogil=True):
    #for ii in range(ndelta2):
        delta_2 = delta2valsv[ii]
        for jj in range(ndelta3):
            delta_3 = delta3valsv[jj]
            coh13v[ii,jj] = steady_rho13_lots_of_args(a_r,
                                a_i,
                                delta_2,
                                delta_3,
                                gamma_13,
                                gamma_23,
                                gamma_2d,
                                gamma_3d,
                                n_b,
                                gamma_mu,
                                Omega_o,
                                Omega_mu,
                                g)
            wv[ii,jj] = 1.0/(2*3.14159265358979323846*sd3*sd2) * exp(-(delta_2-m2)**2/
    return (coh13,w)

# Integrate over an ensemble using simpson's rule
cpdef ensemble_steady_rho13_integrated(p,double df):
    #cdef double df = 200e3 #frequency discretization
    cdef double a_r,a_i,delta_2,delta_3,gamma_13,gamma_23,gamma_2d,gamma_3d,n_b,gamma_
    a = p['a']
    a_r = a.real
    a_i = a.imag
    delta_2 = p['delta2']
    delta_3 = p['delta3']
    gamma_13 = p['gamma13']
    gamma_23 = p['gamma23']
    gamma_2d = p['gamma2d']
    gamma_3d = p['gamma3d']
    n_b = p['nbath']
    gamma_mu = p['gammamu']
    Omega_o = p['omegao']
    Omega_mu = p['omegam']
    g = p['g']

    cdef int ii
    cdef int jj
```

```
cdef double m2 = p['mean_delta2']
cdef double sd2 = p['sd_delta2']
cdef double m3 = p['mean_delta3']
cdef double sd3 = p['sd_delta3']

cdef int w2,w3

cdef int ndelta2 = 2*int(3*sd2/df)+1
cdef int ndelta3 = 2*int(3*sd3/df)+1
# print(ndelta2,ndelta3)

delta2vals = np.linspace(-3,3,ndelta2)*sd2+m2
delta3vals = np.linspace(-3,3,ndelta3)*sd3+m3

cdef double [:] delta2valsv = delta2vals
cdef double [:] delta3valsv = delta3vals


simpsonw2 = np.empty((ndelta2,),dtype='int32')
cdef int [:] simpsonw2v = simpsonw2
simpsonw3 = np.empty((ndelta3,),dtype='int32')
cdef int [:] simpsonw3v = simpsonw3

ii=0
while(ii<(ndelta2-1)):
    simpsonw2v[ii]=2
    ii+=1
    simpsonw2v[ii]=4
    ii+=1
simpsonw2v[0]=1
simpsonw2v[ndelta2-1]=1

ii=0
while(ii<(ndelta3-1)):
    simpsonw3v[ii]=2
    ii+=1
    simpsonw3v[ii]=4
    ii+=1
simpsonw3v[0]=1
simpsonw3v[ndelta3-1]=1



cdef double complex z = 0
cdef double sum_r = 0
cdef double sum_i = 0
cdef double gaussie
```

```
            for ii in prange(ndelta2,nogil=True):
            #for ii in range(ndelta2):
                delta_2 = delta2valsv[ii]
                w2 = simpsonw2v[ii]
                for jj in range(ndelta3):
                    delta_3 = delta3valsv[jj]
                    w3 = simpsonw3v[jj]
                    gaussie = 1.0/(2*3.14159265358979323846*sd3*sd2) * exp(-(delta_2-m2)**2/(
                    z = steady_rho13_lots_of_args(a_r,
                                                  a_i,
                                                  delta_2,
                                                  delta_3,
                                                  gamma_13,
                                                  gamma_23,
                                                  gamma_2d,
                                                  gamma_3d,
                                                  n_b,
                                                  gamma_mu,
                                                  Omega_o,
                                                  Omega_mu,
                                                  g)*gaussie*w2*w3
                    #this is because cython doesn't know how to automatically do reductions f
                    #complex variables but seems to for doubles
                    sum_r += z.real
                    sum_i += z.imag
            return (sum_r+sum_i*1j)*df*df/9.0
```

```
[1/1] Cythonizing /home/jevon/.cache/ipython/cython/_cython_magic_b6305d45ddd9fca58a04cb1f0d958
building '_cython_magic_b6305d45ddd9fca58a04cb1f0d95874d' extension
x86_64-linux-gnu-gcc -pthread -DNDEBUG -g -fwrapv -O2 -Wall -Wstrict-prototypes -g -fdebug-prei
x86_64-linux-gnu-gcc -pthread -shared -Wl,-O1 -Wl,-Bsymbolic-functions -Wl,-Bsymbolic-functions
```

1.4 Calculations - just atoms

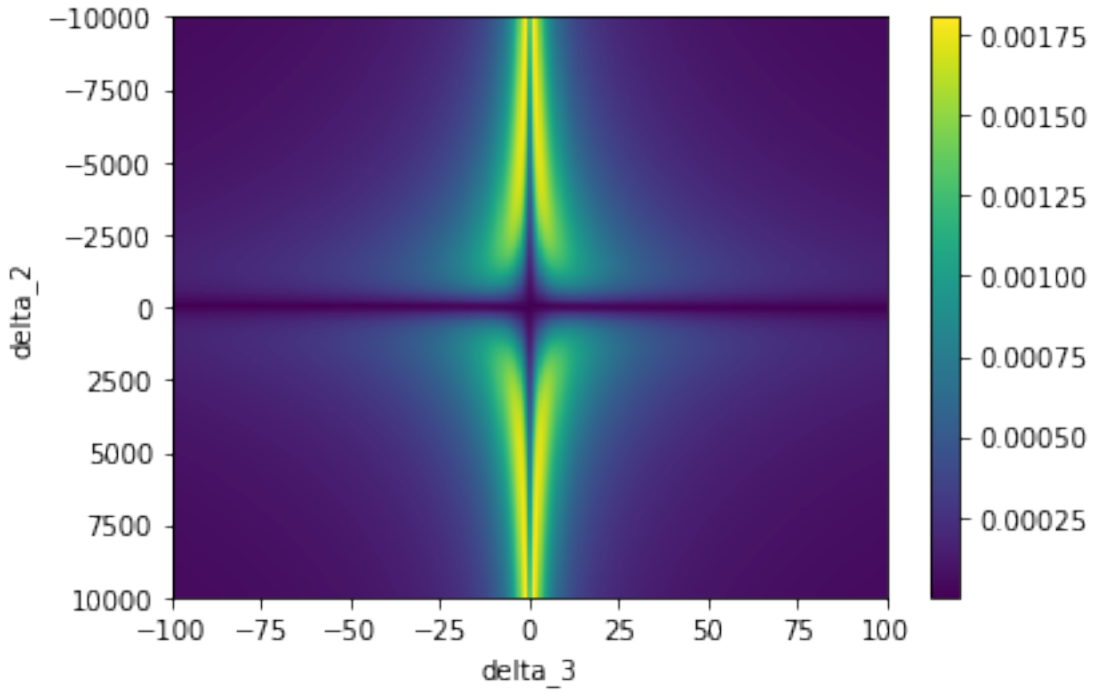
1.4.1 Single pass - no inhomogeneous broadening

Weird behavior -- driving both microwave and optical pump hard

- Signal persists at very high microwave detuning
- EIT like features

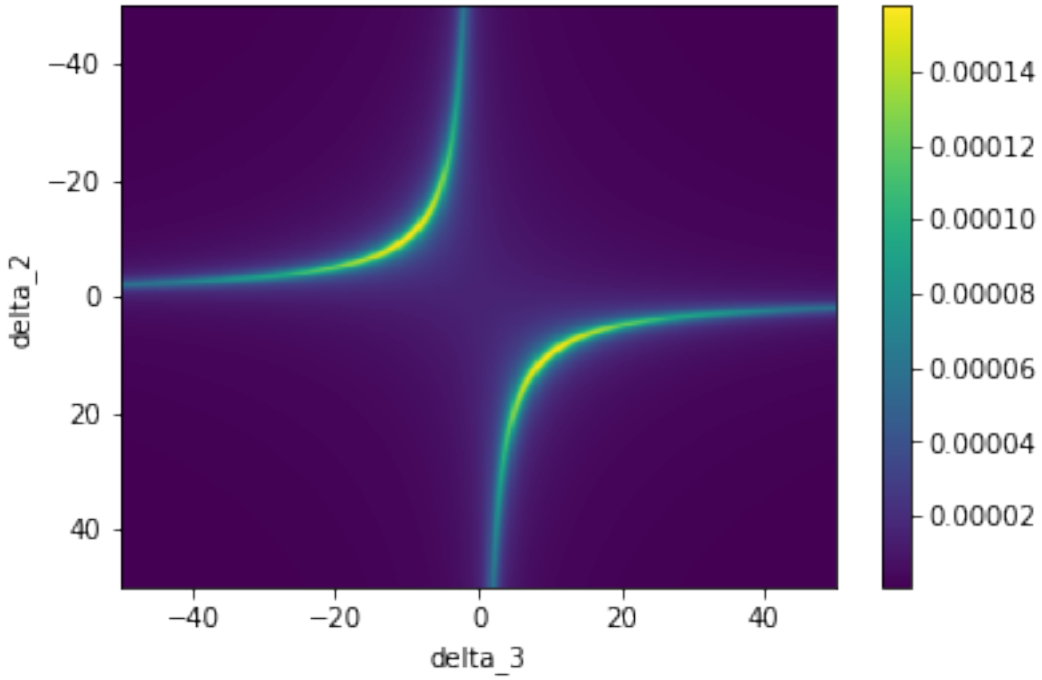
```
In [23]: #some default parameters
         p = {}
         p['a'] = 0
         p['delta2'] = 0.
         p['delta3'] = 0.
         p['gamma13'] = 1./22e-3
         p['gamma23'] = 1./22e-3
```

```
p['gamma2d'] = 1./1e-6
p['gamma3d'] = 1./1e-6
p['nbath'] = 20
p['gammamu'] = 1./((p['nbath']+1)*11e-3)
p['omegao'] = 10e6
p['omegam'] = 10e6
p['g'] = 0.

# Code currently requires these, even if we're not interested in
# inhomogeneous brodening for the moment
p['mean_delta2']=0
p['sd_delta2']=20e6/2.355 #2.355 is to turn FWHM into standard deviation
p['mean_delta3']=0
p['sd_delta3']=300e6/2.355

delta2vals = np.linspace(-10000e6,10000e6,501)
delta3vals = np.linspace(-100e6,100e6,501)

coh,_ = ensemble_steady_rho13(p,delta2vals,delta3vals)

#plt.rcParams['figure.figsize'] = [10, 4]
plt.imshow(np.abs(coh),extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2va
plt.ylabel('delta_2')
plt.xlabel('delta_3')
plt.colorbar();
```



Strong optical pump, weak microwaves

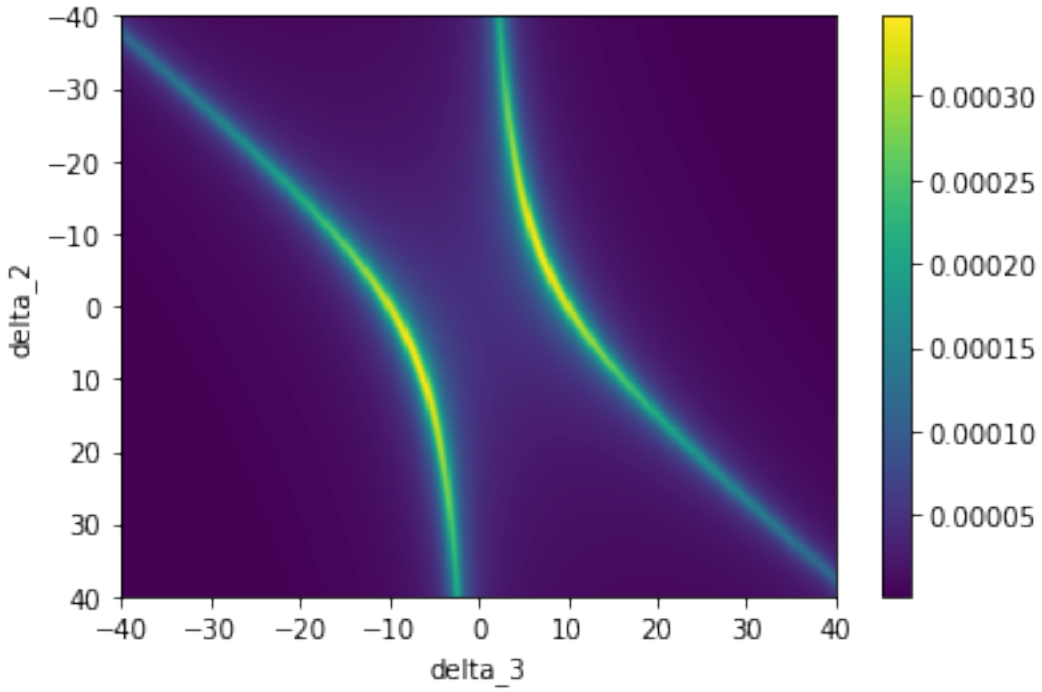
In [24]:
```
#some default parameters
p = {}
p['a'] = 0.0
p['delta2'] = 0.0
p['delta3'] = 0.0
p['gamma13'] = 1./22e-3
p['gamma23'] = 1./22e-3
p['gamma2d'] = 1./1e-6
p['gamma3d'] = 1./1e-6
p['nbath'] = 20
p['gammamu'] = 1./((p['nbath']+1)*11e-3)
p['omegao'] = 10e6
p['omegam'] = 1e3
p['g'] = 0.0


# Code currently requires these, even if we're not interested in
# inhomogeneous brodening for the moment
p['mean_delta2']=0
p['sd_delta2']=20e6/2.355 #2.355 is to turn FWHM into standard deviation
p['mean_delta3']=0
p['sd_delta3']=300e6/2.355


delta2vals = np.linspace(-50e6,50e6,505)
delta3vals = np.linspace(-50e6,50e6,505)


coh,_ = ensemble_steady_rho13(p,delta2vals,delta3vals)
coh = np.abs(coh)

#plt.rcParams['figure.figsize'] = [10, 4]
plt.imshow(coh,extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2vals)/1e6,m
plt.ylabel('delta_2')
plt.xlabel('delta_3')
plt.colorbar();
```



Strong microwaves, weak optical pump

In [25]:
```
#some default parameters
p = {}
p['a'] = 0
p['delta2'] = 0.
p['delta3'] = 0.
p['gamma13'] = 1./22e-3
p['gamma23'] = 1./22e-3
p['gamma2d'] = 1./1e-6
p['gamma3d'] = 1./1e-6
p['nbath'] = 20
p['gammamu'] = 1./((p['nbath']+1)*11e-3)
p['omegao'] = 1e3
p['omegam'] = 10e6
p['g'] = 0.

# Code currently requires these, even if we're not interested in
# inhomogeneous brodening for the moment
p['mean_delta2']=0
p['sd_delta2']=20e6/2.355 #2.355 is to turn FWHM into standard deviation
p['mean_delta3']=0
p['sd_delta3']=300e6/2.355
```

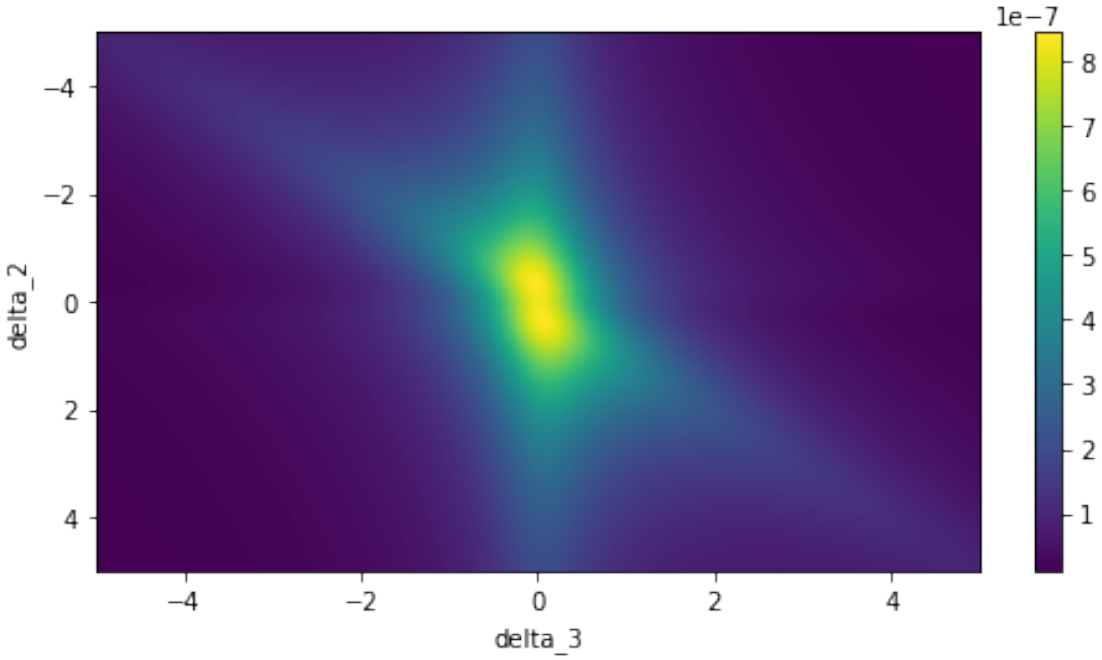
```
delta2vals = np.linspace(-40e6,40e6,501)
delta3vals = np.linspace(-40e6,40e6,501)


coh,_ = ensemble_steady_rho13(p,delta2vals,delta3vals)
coh = np.abs(coh)


#plt.rcParams['figure.figsize'] = [8, 4]
plt.imshow(coh,extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2vals)/1e6,
plt.ylabel('delta_2')
plt.xlabel('delta_3')
plt.colorbar();
```



Both microwaves and optical pump weak

In [26]:
```
p = {}
p['a'] = 0
p['delta2'] = 0.
p['delta3'] = 0.
p['gamma13'] = 1./22e-3
p['gamma23'] = 1./22e-3
p['gamma2d'] = 1./1e-6
p['gamma3d'] = 1./1e-6
```

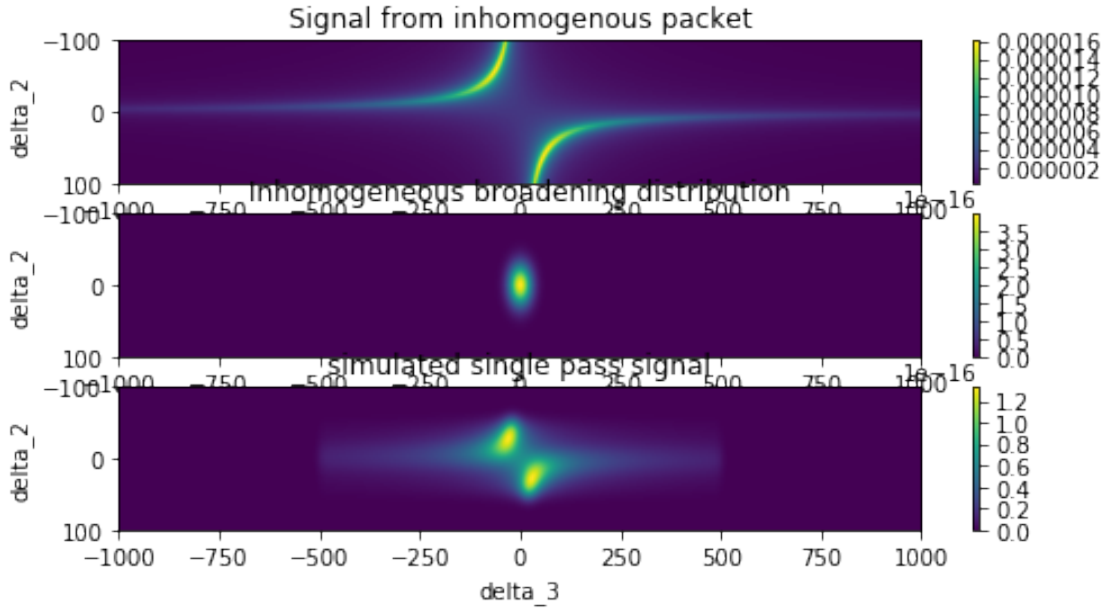
```
p['nbath'] = 20
p['gammamu'] = 1./((p['nbath']+1)*11e-3)
p['omegao'] = 1e3
p['omegam'] = 1e3
p['g'] = 0.

# Code currently requires these, even if we're not interested in
# inhomogeneous brodening for the moment
p['mean_delta2']=0
p['sd_delta2']=20e6/2.355 #2.355 is to turn FWHM into standard deviation
p['mean_delta3']=0
p['sd_delta3']=300e6/2.355

delta2vals = np.linspace(-5e6,5e6,501)
delta3vals = np.linspace(-5e6,5e6,501)

coh,_ = ensemble_steady_rho13(p,delta2vals,delta3vals)
coh = np.abs(coh)

plt.rcParams['figure.figsize'] = [8, 4]
plt.imshow(coh,extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2vals)/1e6,m
plt.ylabel('delta_2')
plt.xlabel('delta_3')
plt.colorbar();
```



1.4.2 Single pass signal with inhomogneous broadening

```
In [27]: %%time
         p = {}
         p['a'] = 0
         p['delta2'] = 0.
         p['delta3'] = 0.
         p['gamma13'] = 1./22e-3
         p['gamma23'] = 1./22e-3
         p['gamma2d'] = 1./.1e-6 #unrealistically fast, but makes calculations easier
         p['gamma3d'] = 1./.1e-6
         p['nbath'] = 20
         p['gammamu'] = 1./((p['nbath']+1)*11e-3)
         p['omegao'] = 60e6
         p['omegam'] = 1e3
         p['g'] = 0.

         p['mean_delta2']=0
         p['sd_delta2']=20e6
         p['mean_delta3']=0
         p['sd_delta3']=20e6 #not realistic but makes calculations faster

         delta2vals = np.linspace(-100e6,100e6,1000)
         delta3vals = np.linspace(-1000e6,1000e6,10000)

         (coh13,w) = ensemble_steady_rho13(p,delta2vals,delta3vals)
         sig = signal.fftconvolve(coh13,w)

CPU times: user 18.2 s, sys: 1.41 s, total: 19.6 s
Wall time: 7.54 s


In [28]: #plt.rcParams['figure.figsize'] = [10, 20]
         plt.subplot(3,1,1)
         plt.imshow(np.abs(coh13),extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2
         plt.ylabel('delta_2')
         plt.xlabel('delta_3')
         plt.title('Signal from inhomogenous packet')
         plt.colorbar();

         plt.subplot(3,1,2)
         plt.imshow(w.real,extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2vals)/1
         plt.ylabel('delta_2')
         plt.xlabel('delta_3')
         plt.title('Inhomogeneous broadening distribution')
         plt.colorbar();

         plt.subplot(3,1,3)
         plt.imshow(np.abs(sig),extent = [min(delta3vals)/1e6,max(delta3vals)/1e6,max(delta2val
```

```
plt.ylabel('delta_2')
plt.xlabel('delta_3')
plt.title('simulated single pass signal')
plt.colorbar();
```



1.5 Introducing cavity dynamics

The (classical) equations of motion of the cavity mode amplitude are given by

$$\frac{d}{dt}a = -i\delta_c a - i\sum_k g_k \sigma_{13,k} - \frac{(\kappa_i + \kappa_c)}{2}a - \sqrt{\kappa_c}a_{in}$$

Here κ_i is the intrisic loss rate of our cavity and κ_c is the coupling induced loss.

Defining

$$S_{13}(a) = \sum_k g_k \sigma_{13,k}$$

We write $S_{13}(a)$ to remind us (explicitly) that the atoms states are a function of the cavity mode amplitude a.

Because we won't drive a this becomes

$$\frac{d}{dt}a = -i\delta_c a - iS_{13}(a) - \frac{(\kappa_i + \kappa_c)}{2}a$$

So to find the steady state of the cavity we need to find the roots of the function

$$f(a) = -i\delta_c a - iS_{13}(a) - \frac{(\kappa_i + \kappa_c)}{2}a$$

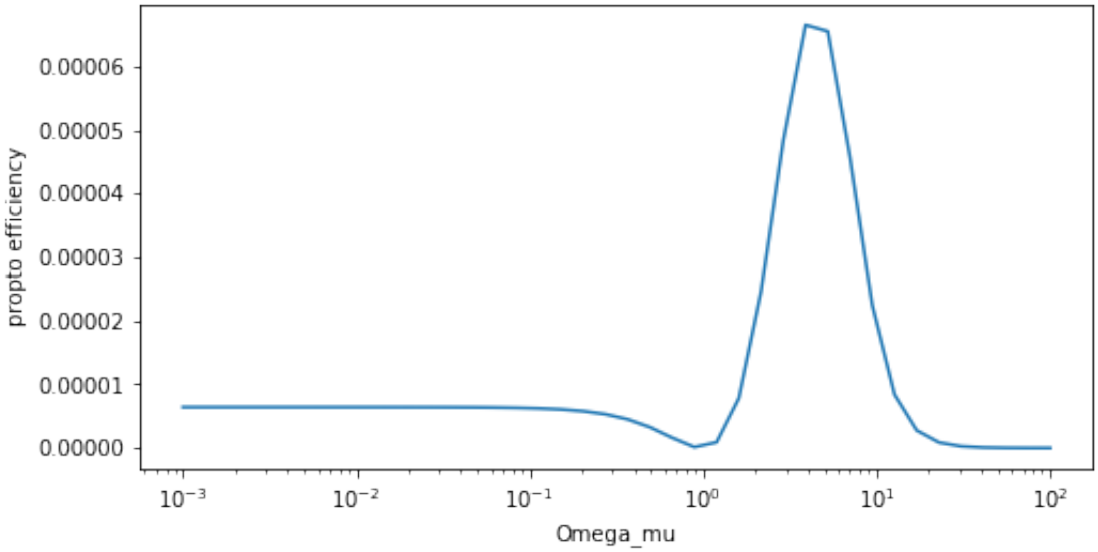
```
In [29]: def ffunc(p):
             df = p['df']
             a = p['a']
             N = p['N']
             g = p['g']
             deltac = p['deltac']
             kappa = p['kappaoi']+p['kappaoc']
             S13 = N*g*ensemble_steady_rho13_integrated(p,df)
             return -1j*deltac*a - 1j*S13 - kappa/2*a

         # the same function but in a form that maps a 2d vector to a 2d vector rather than co
         def ffunc_for_root(avec,p):
             a = avec[0]+1j*avec[1]
             # print(a)
             p['a']=a
             f = ffunc(p)
             return [f.real,f.imag]

         def steadya(p):
             # use value of a as steady state
             a = p['a']
             avec = [a.real,a.imag]
             df = p['df']
             result = scipy.optimize.root(ffunc_for_root,avec,args=(p))
             if not result.success:
                 print(result)
                 raise(AssertionError('The bloody thing didn\'t converge: '+result.message))
             #update p['a'] why not?
             a = result.x[0]+1j*result.x[1]
             p['a']=a
             return a
```

These aren't very realistic parameters, but lead to fast calcs

```
In [30]: df = 0.1
         p = {}
         p['a'] = 0
         p['delta2'] = 0.
         p['delta3'] = 0.
         p['gamma13'] = 1
         p['gamma23'] = 1
         p['gamma2d'] = 1
         p['gamma3d'] = 1
         p['nbath'] = 2
         p['gammamu'] = 1
         p['omegao'] = 1
         p['omegam'] = .01
         p['g'] = 1.
```

```
p['N'] = 1
p['deltac']=0
p['kappaoi']=1
p['kappaoc']=1
p['df']=0.1

p['mean_delta2']=0
p['sd_delta2']=2 #2.355 is to turn FWHM into standard deviation
p['mean_delta3']=0
p['sd_delta3']=3

omegamvals = np.logspace(-3,2,40)
avals = np.zeros(omegamvals.shape,dtype='complex128')
for i,omegam in enumerate(omegamvals):
    p['omegam']=omegam
    avals[i] = steadya(p)
```

In [31]:
```
plt.semilogx(omegamvals,(np.abs(avals)/(omegamvals))**2)
plt.ylabel('propto efficiency')
plt.xlabel('Omega_mu');
```



1.6 Time for some realistic simulations

In [32]:
```
# calculate microwave rabi frequency from power in in dBm
def omegam_from_Pin(Pin,p):
    mu0 = 4*pi*1e-7
    hbar=1.05457e-34; # in J*s
    Vsample=pi*((p['dsample']/2)**2) * p['Lsample'] # the volume of the Er:YSO sample
```

```
    V_microwave_cavity = Vsample/p['fillfactor']
    mwP = 1e-3 * 10**(Pin/10)  #convert from dBm to Watts
    Q=2*pi*p['freqm']/(p['kappami']+p['kappamc']*2) # Q factor
    S21=(4*p['kappamc']**2)/(p['kappami']+p['kappamc']*2)**2
    energy_in_cavity=math.sqrt(S21)*2/(2*pi*p['freqm']/Q)*mwP; # energy inside the mi
    Bmw=math.sqrt(mu0*(energy_in_cavity/V_microwave_cavity)/2); # Magnetic field of t
    p['omegam']=(p['mu12']*Bmw)/hbar*(-1); # in rad/sec???? in Hz
    return p['omegam']
```

TODO: in above ask stephen why -1? why Hz?
TODO in below check V_cav?
TODO check n_in

In [33]:
```
# calculate optical rabi frequency from power in in dBm
def omegao_from_Pin(Pin,p):
    epsilon0=8.854187817e-12
    hbar=1.05457e-34; # in J*s

    optP = 1e-3 * 10**(Pin/10) #incident optical power in W
    pflux = optP/(2*pi*p['freqo']*hbar) #photon flux (photons/sec)
    n_in = pflux * p['kappaoc']*4/(p['kappaoc']+p['kappaoi'])**2 # num intracavity ph
                                                    #is this right????
    Sspot = pi*p['Wcavity']**2 #cross sectional area of
                                #optical mode
    V_cav = (Sspot*p['Lcavity_vac']+Sspot*p['Lsample']*p['nYSO']**3)/2;
    optEfield = math.sqrt(n_in*hbar*2*pi*p['freqo']/2/epsilon0/V_cav);
    p['omegao'] = p['d23']*optEfield/hbar*(-1);
    return p['omegao']
```

In [34]:
```
# parameters from Stephen via slack
# parameters commented out are not used in calculation
p = {}
p['a'] = 0
p['delta2'] = 0.
p['delta3'] = 0.


p['d13'] = 2e-32*math.sqrt(1/3)
p['d23'] = 2e-32*math.sqrt(2/3)
p['gamma13'] = p['d13']**2/(p['d13']**2+p['d23']**2)*1/11e-3
p['gamma23'] = p['d23']**2/(p['d13']**2+p['d23']**2)*1/11e-3
p['gamma2d'] = 1e6
p['gamma3d'] = 1e6
p['nbath'] = 20
p['gammamu'] = 1/(p['nbath']+1) * 1/1e3

p['g'] = 51.9  #optical coupling
p['N'] = 1.28e15 # number of atoms in the optical mode
```

```
p['deltac']=0 #detuning for
p['kappaoi']=2*pi*7.95e6 # intrinsic loss for optical resonator
p['kappaoc']=2*pi*1.7e6 # coupling loss for optical resonator
p['df']=4e6 # how small descretisation step to take when integrating over the
            # inhomogeneous lines

p['mean_delta2']=0
p['sd_delta2']=2*pi*25e6/2.355  #microwave inhomogeneous broadening
                                #2.355is to turn FWHM into standard deviation
p['mean_delta3']=0
p['sd_delta3']=2*pi*170e6/2.355 #optical inhomogeneous broadening

p['kappami'] = 650e3*2*pi # intrinsic loss for microwave cavity
p['kappamc'] = 70e3*2*pi  # coupling loss for optical cavity
                          # this is for one of the two output ports
p['Nmu'] = 2.22e16  #toal number of atoms
p['gmu'] = 1.04 #coupling between atoms and microwave field


muBohr=927.4009994e-26; # Bohr magneton in J/T in J* T^-1
p['mu12'] = 4.3803*muBohr # transition dipole moment for microwave cavity (J T^-1)

p['Lsample']=12e-3 # the length of the sample, in m
p['dsample']=5e-3 # the diameter of the sample, in m

p['fillfactor']=0.8 #microwave filling factor
p['freqm'] = 5.186e9
p['freqo'] = 195113.36e9

p['Lcavity_vac'] = 49.5e-3 # length of the vacuum part of the optical
                           # Fabry Perot (m)
p['Wcavity'] =  0.6e-3# width of optical resonator beam in sample (m)
p['nYSO'] = 1.76 #refractive index of YSO


# assume 1mW (=0dBm) input to both optical and to microwave cavities
omegao_from_Pin(0,p)
omegam_from_Pin(0,p)

#see what rabi freqs we have
p['omegao']/1e6,p['omegam']/1e6

#%time ensemble_steady_rho13_integrated(p,p['df'])
```

Out[34]:

$$(-0.49209088755145475,\quad -4.7543084291639115)$$

```
In [35]: %time steadya(p)
```

```
    fjac: array([[-0.99999844,  0.00176688],
       [-0.00176688, -0.99999844]])
     fun: array([1.81175332e+02, 1.30350380e+12])
 message: 'The iteration is not making good progress, as measured by the \n  improvement from t
    nfev: 13
     qtf: array([ 2.30313159e+09, -1.30350182e+12])
       r: array([ 9.27176643e+06, -4.92847517e+05,  2.78314650e+08])
  status: 5
 success: False
       x: array([-1.94970919e-05,  3.67525265e-04])
```

```
---------------------------------------------------------------------------
AssertionError                            Traceback (most recent call last)

<timed eval> in <module>()

<ipython-input-29-fd215916e166> in steadya(p)
     25     if not result.success:
     26         print(result)
---> 27         raise(AssertionError('The bloody thing didn\'t converge: '+result.message))
     28     #update p['a'] why not?
     29     a = result.x[0]+1j*result.x[1]

AssertionError: The bloody thing didn't converge: The iteration is not making good prog
improvement from the last ten iterations.
```

In [36]:
```
#commented out because it takes and age to run

# omegamvals = np.logspace(-3,2,8)
# avals = np.zeros(omegamvals.shape,dtype='complex128')
# for i,omegam in enumerate(omegamvals):
#     p['omegam']=omegam
#     print("%d/%d"%(i+1,len(omegamvals)),end="")
#     avals[i] = steadya(p)
#     print(" -- completed")
```

In [37]:
```
# plt.semilogx(omegamvals,(np.abs(avals)/(omegamvals))**2,'x-')
# plt.ylabel('propto efficiency')
# plt.xlabel('Omega_mu')
```

In [38]:
```
!hostname
```

bb8

```
In [39]: p['N']*p['g']*p['g']
```

Out[39]:

$$3.4478208e + 18$$