# Logical Aspects of Multi-Agent Systems
# Homework set F

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## 2.22

We have a new logic $Epist_{(m)}$ for m agents. It comprises of the axiom and rules of $S5_{(m)}$ for the knowledge operators $K_i$, the axiom and rules $KD45_{(m)}$ for the belief operators $B_i$ and a new mixed axiom $K_i\varphi \rightarrow B_i\varphi$. The Kripke models are furthermore defined as $M = \langle S, \pi, R_1^K, ..., R_m^K, R_1^B, ..., R_k^B \rangle$, with S being the set of states, $\pi$ the valuation and K and B representing knowledge and belief respectively.

### b.

We have the following structure: $\langle S, R_1^K, ..., R_m^K, R_1^B, ..., R_m^B \rangle$ for which for some $i <= m$ and some $v, w \in S$ one has $vR_i^Bw$ and $\neg vR_i^Kw$.

In order to find the valuation $\pi'$ such that for $M' = \langle S, \pi', R_1^K, ..., R_m^K, R_1^B, ..., R_k^B \rangle$ we have $(M', v) \nvDash K_ip \rightarrow B_ip$, we evaluate $K_ip \rightarrow B_ip$ in M'.

Because $vR_i^Bw$ holds, then $(M', v) \models R_i^Bw$ is true. Assuming that $(M', v) \models K_ip \rightarrow B_ip$ is true, $(M', w) \models K_ip$ also implies that $(M', w) \models B_ip$.

However, we also know that $\neg vR_i^Kw$, and this means that we in fact have $(M', w) \nvDash K_ip$. This means that $K_ip \rightarrow B_ip$ is false in $(M', v)$. In conclusion, $(M', v) \nvDash K_ip \rightarrow B_ip$.

### c.

For this exercise, I have chosen the following mixed theorem of $Epist_{(m)}$: $(K_i\varphi \wedge K_i\psi) \rightarrow (B_i\varphi \wedge B_i\psi)$.

1. $K_i\varphi \wedge K_i\psi$
2. $K_i\varphi$ (From: 1)
3. $K_i\psi$ (From: 1)
4. $K_i\varphi \rightarrow B_i\varphi$ (Mixed theory of $Epist_{(m)}$, $K_i\varphi \rightarrow B_i\varphi$)
5. $K_i\psi \rightarrow B_i\psi$ (Mixed theory of $Epist_{(m)}$, $K_i\varphi \rightarrow B_i\varphi$)
6. $B_i\varphi$ (From: 2, 4)

7. $B_i\psi$ (From: 3, 5)
8. $B_i\varphi \wedge B_i\psi$ (From: 6, 7)
9. $(K_i\varphi \wedge K_i\psi) \rightarrow (B_i\varphi \wedge B_i\psi)$ (From: 1, 8)

We can therefore conclude that $(K_i\varphi \wedge K_i\psi) \rightarrow (B_i\varphi \wedge B_i\psi)$ is a valid mixed theorem in $Epist_{(m)}$.