Lab 6 PMP

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November 2025

Exercise 1

Let's suppose we are investigating a disease (B), which affects only 1(P(B) = 0.01). We have a diagnostic test for this disease with the following characteristics:

- Sensitivity (the probability that the test is positive if the person has the disease): P($Test = Positive \mid B$) = 0.95.
- Specificity (the probability that the test is negative if the person does not have the disease): P($Test = Negativ \mid \neg B$) = 0.90.

a) If a person is tested and the result is positive, what is the probability that they actually have the disease?
Explain the result.
b) What should be the minimum specificity for the above probability to reach 50%?

a) P($Test = Negativ \mid \neg B$) = 0.90 $\Rightarrow$ $P(Test = Positive \mid \neg B) = 0.10$

$$\text{P}(B \mid Test = Positive) = \frac{P(Test = Positive \mid B) * P(B)}{P(Test = Positive)}$$
$$= \frac{P(Test = Positive \mid B) * P(B)}{P(Test = Positive \mid B) * P(B) + P(Test = Positive \mid \neg B) * P(\neg B)}$$
$$= \frac{0.95 * 0.01}{0.95 * 0.01 + 0.10 * 0.99}$$
$$= \frac{0.0095}{0.1085} \approx 0.088$$

b) P($B \mid Test = Positive$) = 0.5

$$\text{P}(B \mid Test = Positive) = \frac{P(Test = Positive \mid B) * P(B)}{P(Test = Positive \mid B) * P(B) + P(Test = Positive \mid \neg B) * P(\neg B)}$$
$$0.5 = \frac{0.95 * 0.01}{0.95 * 0.01 + (1 - P(Test = Negativ \mid \neg B)) * 0.99}$$

$$0.0095 + (1 - P(Test = Negativ \mid \neg B)) * 0.99 = \frac{0.0095}{0.5}$$

$$(1 - P(Test = Negativ \mid \neg B)) * 0.99 = 0.019 - 0.0095$$

$$1 - P(Test = Negativ \mid \neg B) = \frac{0.0095}{0.99}$$

$$\mathrm{P}(Test = Negativ \mid \neg B) = 1 - \frac{0.0095}{0.99}$$

$$\mathrm{P}(Test = Negativ \mid \neg B) = \frac{0.9805}{0.99}$$

$$\mathrm{P}(Test = Negativ \mid \neg B) \approx 0.99$$