Midterm Test

1. Find $\inf, \sup, \min, \max$, the interior and the closure of the set $\{0.1, 0.11, 0.111, \ldots\}$.

2. Study the convergence of the following series:

   (a) $\displaystyle\sum_{n\geq 1} \frac{(n+1)^{n-1}}{n^{n+1}}$.

   (b) $\displaystyle\sum_{n\geq 1} \frac{a^n (n!)^2}{(2n)!}$, $a > 0$.

   (c) $\displaystyle\sum_{n\geq 1} \frac{\ln n}{n^2}$.

   (d) $\displaystyle\sum_{n\geq 1} n! \sin x \sin \frac{x}{2} \ldots \sin \frac{x}{n}$, $x \in (0, \pi)$.

3. Study the convergence and the absolute convergence of the series $\displaystyle\sum_{n\geq 1} (-1)^n (\sqrt{n} - \sqrt{n+1})$.

4. Using power series, find the sum of the following series:

   (a) $\displaystyle\sum_{n\geq 0} \frac{n+1}{4^n}$.

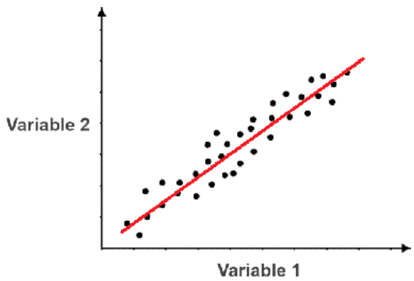
   (b) $\displaystyle\sum_{n\geq 2} \frac{n(n-1)}{2^n}$.

   (c) $\displaystyle\sum_{n\geq 0} \frac{(-1)^n}{2n+1}$.

5. Find the radius of convergence and the convergence set for the power series

$$\sum_{n\geq 1} \frac{x^n}{n^p},\ p \in \mathbb{R}.$$

6. Given the data points (x_i, y_i), $i \in \{1, \ldots, n\}$, the line of best fit f minimizes $\displaystyle\sum_{i=1}^{n} (y_i - f(x_i))^2$.
   Find the line of best fit that passes through the origin (and explain its uniqueness).



18th November 2022