Seminar 13

1. Compute the following integrals:

   (a) $\displaystyle\iint_R \cos x \sin y \,\mathrm{d}x\,\mathrm{d}y$, where $R = [0, \pi/2] \times [0, \pi/2]$.

   (b) $\displaystyle\iint_R \frac{1}{(x+y)^2}\,\mathrm{d}x\,\mathrm{d}y$ and $\displaystyle\iint_R ye^{xy}\,\mathrm{d}x\,\mathrm{d}y$, where $R = [1, 2] \times [0, 1]$.

   (c) ★$\displaystyle\iint_R \min\{x, y\}\,\mathrm{d}x\,\mathrm{d}y$, where $R = [0, 1] \times [0, 1]$.

2. Let $R = [0, 1] \times [0, 1]$. Sketch the solid and find its volume for the following:

   (a) $\displaystyle\iint_R (2 - x - y)\,\mathrm{d}x\,\mathrm{d}y$. (b) $\displaystyle\iint_R (2 - x^2 - y^2)\,\mathrm{d}x\,\mathrm{d}y$. (c) ★$\displaystyle\iint_R xy\,\mathrm{d}x\,\mathrm{d}y$.

3. Let $D \subseteq \mathbb{R}^2$ be the subset bounded by the parabola $y = x^2$ and the lines $x = 2$ and $y = 0$.

   (a) Express D as a simple set first w.r.t. the y-axis and then w.r.t. the x-axis.

   (b) Compute $\displaystyle\iint_D xy\,dx\,dy$ in two ways.

4. (a) Prove that the volume of the unit ball is $\displaystyle\iiint_{B(0,1)} 1\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \frac{4}{3}\pi$.

   (b) ★Prove that the volume of the ball of radius r is $\displaystyle\iiint_{B(0,r)} 1\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \frac{4}{3}\pi r^3$.

5. By changing the order of integration, evaluate the following:

   (a) $\displaystyle\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2 - y^2}\,\mathrm{d}x\,\mathrm{d}y$.

   (b) $\displaystyle\int_0^1 \int_y^1 \sin(x^2)\,\mathrm{d}x\,\mathrm{d}y$.

   (c) ★$\displaystyle\int_0^{\pi/2} \int_x^{\pi/2} \frac{\sin(y)}{y}\,\mathrm{d}y\,\mathrm{d}x$.

   (d) ★$\displaystyle\int_0^1 \int_x^1 e^{y^2}\,\mathrm{d}y\,\mathrm{d}x$.

Homework questions are marked with ★.
Solutions should be handed in at the beginning of next week's lecture.