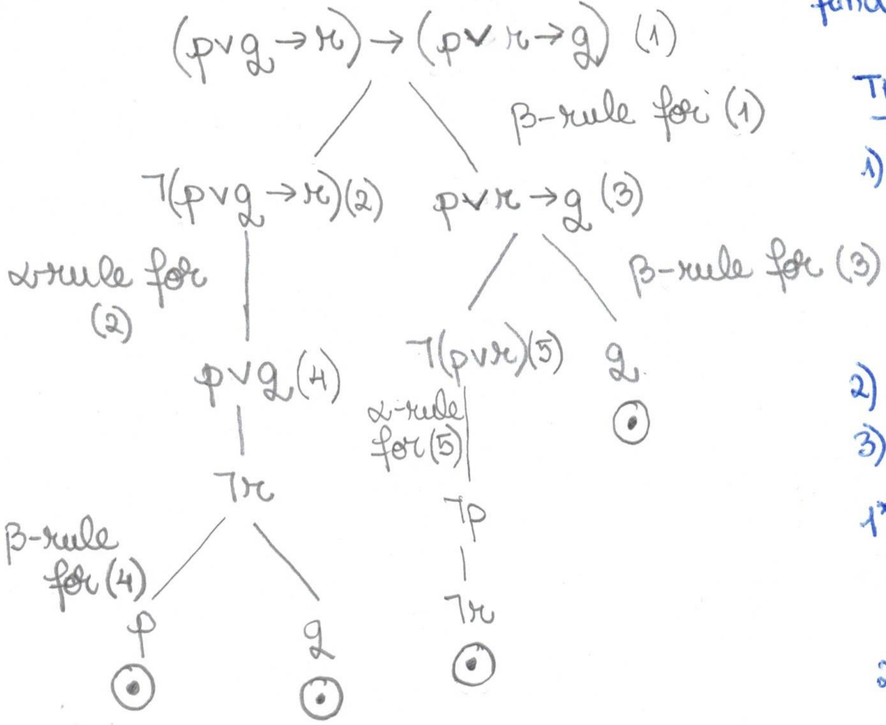
EX1-912

1.2 $U_2 = (p\vee q\to r)\to(p\vee r\to q)$

Use ST method to decide what kind of formula is U_2 (consistent, inconsistent, valid). If consistent, find all its models.



THEORETICAL RESULTS

1) consistent formula: has at least 1 model $\exists i\in\{p_1,\dots,p_m\}\to\{T,F\}$ s.t. $i(U)=T$

2) inconsistent:....

3) valid:....

1*) consistent formula has associated a complete and open ST

2*)... 3*)....

ABSORPTION

$U\vee(U\wedge V)\equiv U$

$U\wedge(U\vee V)\equiv U$

DNF

ST is disjunction of all its branches; graphical representation of Disjunctive Normal Form (DNF)

We obtain the models of the formula from the open branches.

$$DNF(U)=(\neg r\wedge p)\vee\underbrace{(\neg r\wedge q)\vee(\neg p\wedge\neg r)\vee q}_{\text{ABSORPTION}}$$

$$\equiv(\neg r\wedge p)\vee(\neg p\wedge\neg r)\vee q$$

Cube: $\neg r\wedge p = T$

$\downarrow \quad \downarrow$

$T \quad T$

$i_{1,2}:\{p,q,r\}\to\{T,F\}$

$i_1(p)=T$ $\quad$ $i_1(p)=T$

$i_1(q)=T$ $\quad$ $i_2(q)=F$

$i_1(r)=F$ $\quad$ $i_2(r)=F$

Cube: $\neg p\wedge\neg r=T$

$| \quad |$

$T \quad T$

$i_{3,4}:\{p,q,r\}\to\{T,F\}$

$i_3(p)=F$ $\quad$ $i_4(p)=F$

$i_3(q)=T$ $\quad$ $i_4(q)=F$

$i_3(r)=F$ $\quad$ $i_4(r)=F$

Cube: q

$i_5,i_6:\{p,q,r\}\to\{T,F\}$

$i_5(p)=T$ $\quad$ $i_6(p)=F$

$i_5(q)=T$ $\quad$ $i_6(q)=T$

$i_5(r)=T$ $\quad$ $i_6(r)=T$

The formula U_2 has 6 distinct models: $i_1, \ldots, i_6 \Rightarrow U_2$ consistent, but not a tautology (a tautology would have 8 models) $\Rightarrow U_2$ contingent.

4.3 — 9/3

$U_3 = p \to (q \wedge r) \vee q \wedge \neg p$

$\neg U3 = \neg(p \to (q \wedge r) \vee q \wedge \neg p)$

THEORETICAL RESULTS

α, β rules

We obtain the anti-models of U_3 by ~~&~~ assessing the open branches of the semantic table associated to $\neg U_3$. (T1)

⊙ = open branch

IDEMPOTENCE:

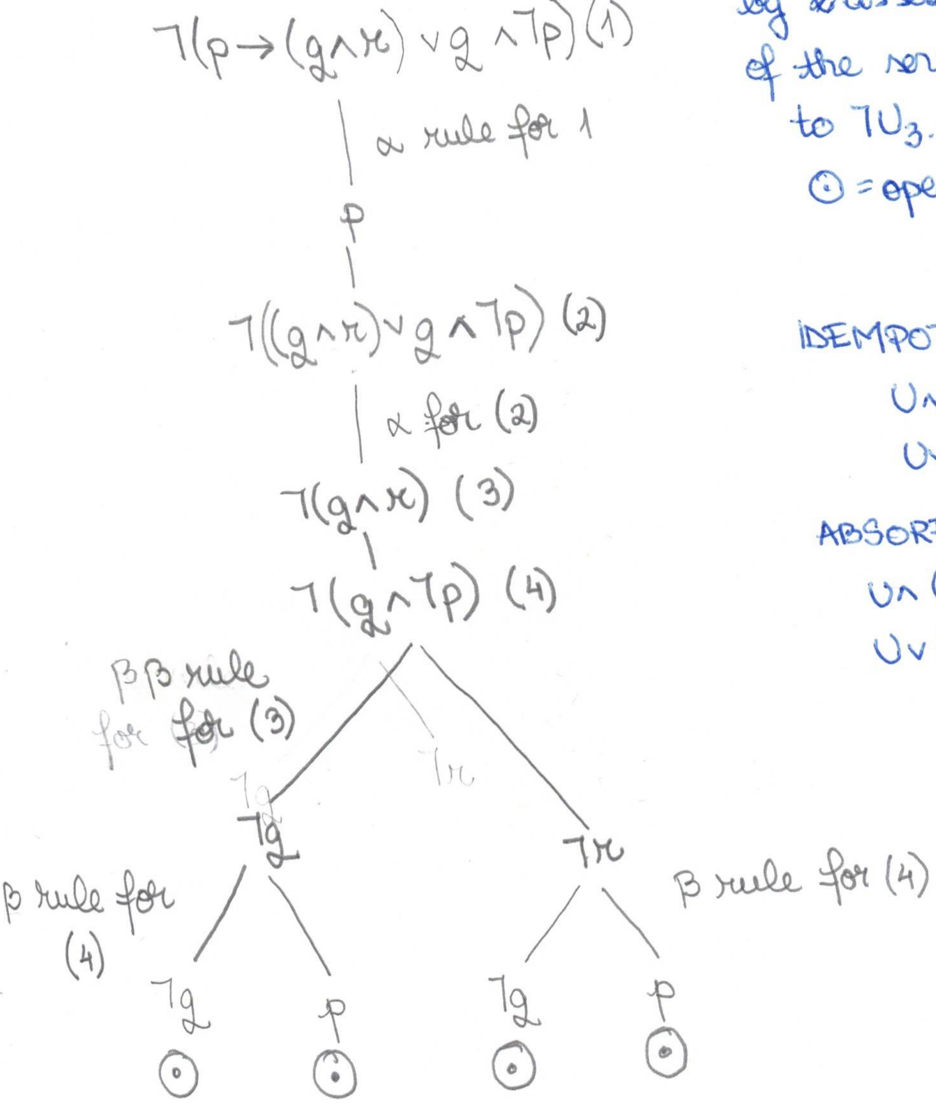
$U \wedge U \equiv U$

$U \vee U \equiv U$

ABSORPTION

$U \wedge (U \vee V) \equiv U$

$U \vee (U \wedge V) \equiv U$



$DNF(\neg U_3) = (p \wedge \overbrace{\neg q \wedge \neg q}^{\text{idempotence}}) \vee (p \wedge \neg q \wedge p) \vee (p \wedge \neg r \wedge \neg q) \vee (\overbrace{p \wedge \neg r \wedge p}^{\text{idempotence}})$

$\equiv \overbrace{(p \wedge \neg q) \vee (p \wedge \neg q)}^{\text{idempotence}} \vee (p \wedge \neg q \wedge \neg r) \vee (p \wedge \neg r)$

absorption

$\equiv (p \wedge \neg q) \vee (p \wedge \neg q \wedge \neg r) \vee (p \wedge \neg r)$

absorption

$\equiv (p \wedge \neg q) \vee (p \wedge \neg r)$

Cube 1: $\underset{T}{p} \wedge \underset{T}{\neg q} = T$

$i_1, i_2 : \{p, q, r\} \to \{T, F\}$

$i_1(p) = T$ $\quad$ $i_2(p) = T$

$i_1(q) = F$ $\quad$ $i_2(q) = F$

$i_1(r) = T$ ✓ $\quad$ $i_2(r) = F$ ✓

Cube 2: $\underset{\overset{|}{T}}{p} \land \underset{\overset{|}{T}}{\neg r} = T$

$i_3, i_4 : \{p, q, r\} \to \{T, F\}$

$i_3(p) = T$ $\quad$ $i_4(p) = T$

$i_3(q) = T$ $\quad$ $i_4(q) = F$

$i_3(r) = F$ ✓ $\quad$ $i_4(r) = F$

$i_4 = i_2$

The models of U_3 are $i_1, i_2, i_3 \overset{T1}{\Rightarrow}$ ~~the~~ i_1, i_2, i_3 anti-models for U_3.

ST5-

Check whether conclusion C is a logical consequence of the set of hypotheses using the ST. method.

H_1: All hummingbirds are richly colored.
H_2: No large birds live on honey.
H_3: Birds that do not live on honey are dull in color.
C: All hummingbirds are small.

$H_1, H_2, H_3 \vdash C$?

$H_1: (\forall x)(hb(x) \to rc(x))$

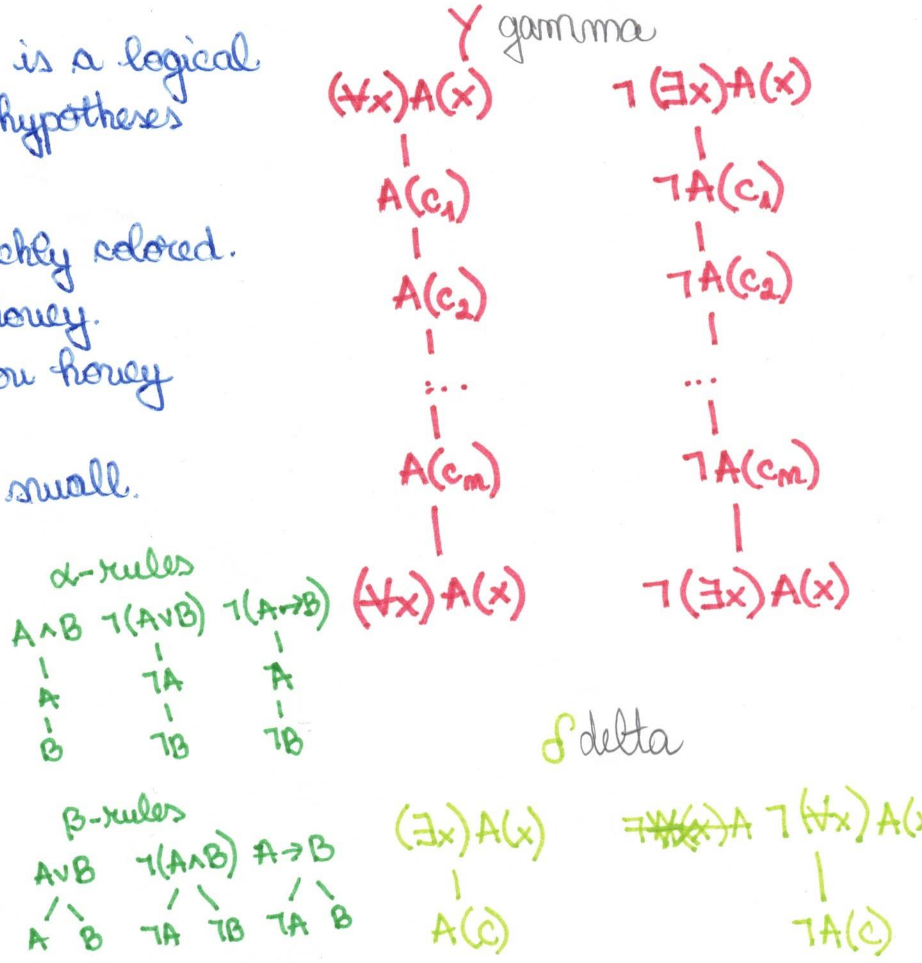
$H_2: \neg(\exists x)(\neg sb(x) \land lh(x))$
$\equiv (\forall x)(\neg sb(x) \to \neg lh(x))$

$H_3: (\forall x)(\neg lh(x) \to \neg rc(x))$

$C: (\forall x)(hb(x) \to sb(x))$

THEORETICAL RESULTS

Let $U_1, \dots, U_n, V$ be predicate formulas. $U_1, U_2, \dots, U_n \vDash V$ iff there is a closed semantic table associated with formula $U_1 \land U_2 \land \dots \land U_n \land \neg V$. (T1)



D = universe of birds

$hb: D \to \{T, F\}$, $hb(x)$ = "x hummingbird"

$rc: D \to \{T, F\}$, $rc(x)$ = "x richly colored"

$sb: D \to \{T, F\}$, $sb(x)$ = "x small bird"

$lh: D \to \{T, F\}$, $lh(x)$ = "x lives on hone

$$(\forall x)(hb(x) \to rc(x)) \land (\forall x)(\neg sb(x) \to \neg lh(x)) \land (\forall x)(\neg lh(x) \to \neg rc(x))$$
$$\land \neg(\forall x)(hb(x) \to sb(x)) \quad (1)$$
$$| \ \alpha \text{ for } (1)$$
$$(\forall x)(hb(x) \to rc(x)) \quad (2)$$
$$|$$
$$(\forall x)(\neg sb(x) \to \neg lh(x)) \quad (3)$$
$$|$$
$$(\forall x)(\neg lh(x) \to \neg rc(x)) \quad (4)$$
$$|$$
$$\neg((\forall x)(hb(x) \to sb(x)) \quad (5)$$

| δ for (5), a new constant

$\neg(hb(a) \to sb(a))$ (6)

| α rule for (6)

$hb(a)$

|

$\neg sb(a)$

| γ rule for (2), a used for instantiation

$hb(a) \to rc(a)$ (7)

|

$(\forall x)(hb(x) \to rc(x))$ copy of formula (2)

β rule for (7)

$\neg hb(a)$ ⊗ $\qquad$ $rc(a)$

| γ rule for (3) a used for instantiation

$\neg sb(a) \to \neg eh(a)$ (8)

|

$(\forall x)(\neg sb(x) \to \neg eh(x))$ copy of formula (3)

β rule for (8)

$sb(a)$ ⊗ $\qquad$ $\neg eh(a)$

| γ rule for (4) a used for instantiation

$\neg eh(a) \to \neg rc(a)$ (9)

|

$(\forall x)(\neg eh(x) \to \neg rc(x))$ copy of (4)

β rule for (9)

$eh(a)$ ⊗ $\qquad$ $\neg rc(a)$ ⊗

All 4 branches of the semantic table are closed (they contain pairs ($(hb(a), \neg hb(a))$, $(\neg sb(a), sb(a))$, $(\neg eh(a), eh(a))$ and $(rc(a), \neg rc(a))$) $\overset{T_4}{\Rightarrow}$ H_4 semantic table associated with formula $H_1 \wedge H_2 \wedge H_3 \wedge \neg C$ is closed $\overset{T_1}{\Rightarrow}$ $H_1, H_2, H_3 \vdash C$.

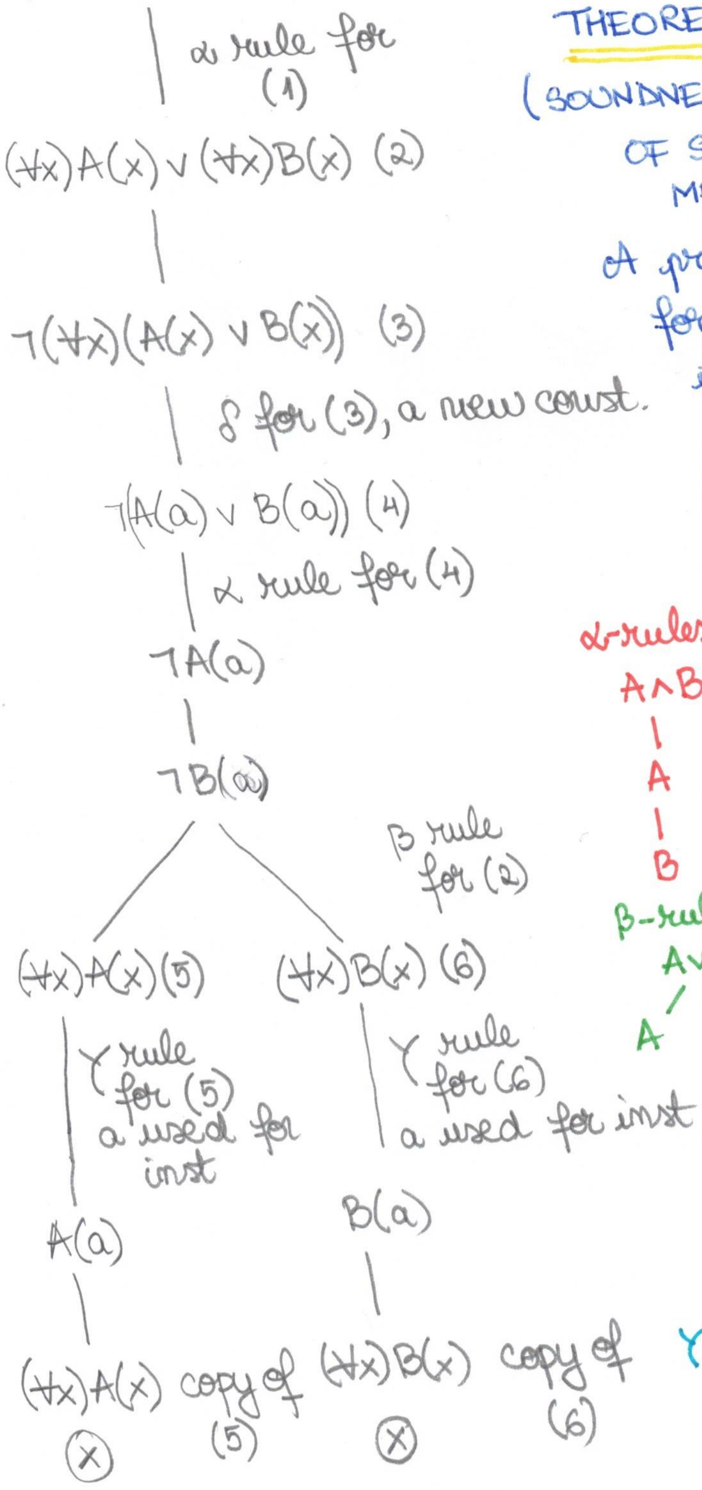
ST7.2 Using ST method, prove the following properties

$\models (\forall x)A(x) \vee (\forall x)B(x) \rightarrow (\forall x)(A(x) \vee B(x))$ U_1

$\not\models (\forall x)(A(x) \vee B(x)) \rightarrow ((\forall x)A(x) \vee (\forall x)B(x))$ U_2

„∀ is semi-distributive over ∨"

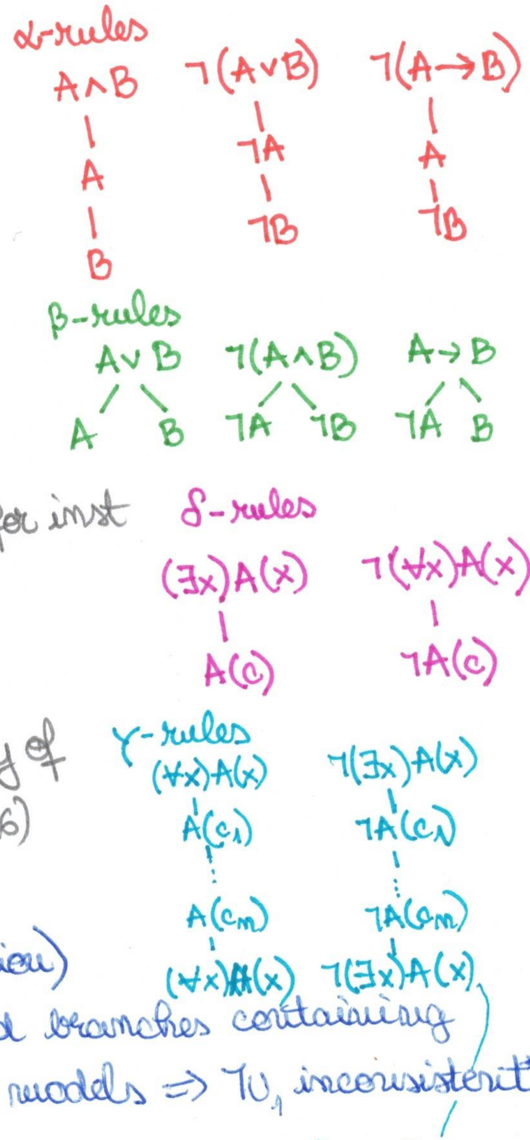
$$\neg U_1 = \neg((\forall x)A(x) \vee (\forall x)B(x) \rightarrow (\forall x)(A(x) \vee B(x))) \quad (1)$$



THEORETICAL RESULTS

(SOUNDNESS AND COMPLETENESS OF SEMANTIC TABLEAUX METHOD)

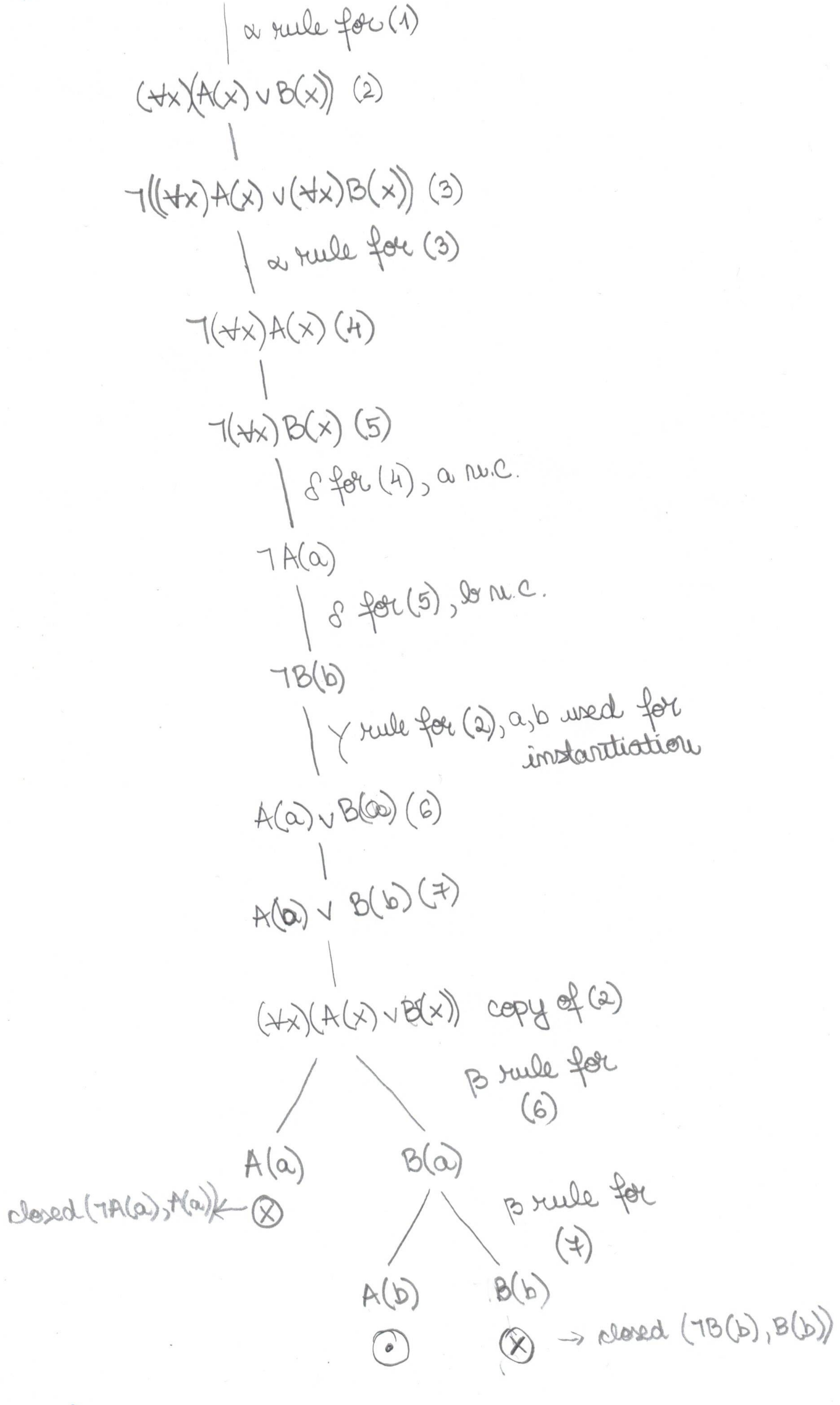
A propositional/predicate formula U is a tautology iff there is a closed semantic tableaux associated to ¬U. (T1)



⊗ → closed branch (contains formula and its negation)

→ the tableaux has only closed branches ⇔ (2 closed branches containing pairs $(\neg A(a), A(a))$ and $(\neg B(a), B(a))$ ⇒ $\neg U_1$ has no models ⇒ $\neg U_1$ inconsistent $\xRightarrow{T1} \models U_1$

$\neg U_2 = \neg((\forall x)(A(x) \vee B(x)) \to ((\forall x)A(x) \vee (\forall x)B(x)))$ (1)



⊙ = open branch

n.c. = new constant

The ~~p~~ open branch provides models for $\neg U_2$, which are anti-model of U_2.

$\neg A(a) \wedge \neg B(b) \wedge B(a) \wedge A(b)$

$I_1 = \langle D, m_1 \rangle$, $D = \{a, b\}$

$m_1(A) : D \to \{T, F\}$, $m_1(A)(a) = F$ $\quad m_1(A)(b) = T$

$m_1(B) : D \to \{T, F\}$, $m_1(B)(a) = T$ $\quad m_1(B)(b) = F$

↑

generic model

→ anti-model of $U_2 \Rightarrow U_2$ is not a tautology