EX5 BOOLEAN FNS

Using Quine's method, simplify the following Boolean fns given by their values 0.

5.2. $f_2(0,0,0) = f_2(0,0,1) = f_2(1,1,1) = 0$

THEORETICAL RESULTS

We need the support set of f_2,

$$S_f = \{(x_1,x_2,x_3) \mid f(x_1,x_2,x_3) = 1\}$$

and sort it in ascending/descending order, with respect to the no. of values 1 in each tuple.

SOLUTION

$$S_{f_2} = \{(0,1,0),(0,1,1),(1,0,0),(1,0,1),(1,1,0)\}$$

$$S_{f_2} = \{(0,1,0),(1,0,0),(0,1,1),(1,0,1),(1,1,0)\}$$

$$f_2(x_1,x_2,x_3) = \overline{x}_1 x_2 \overline{x}_3 \vee x_1 \overline{x}_2 \overline{x}_3 \vee \overline{x}_1 x_2 x_3 \vee x_1 \overline{x}_2 x_3 \vee x_1 x_2 \overline{x}_3$$

$$= m_2 \vee m_4 \vee m_3 \vee m_5 \vee m_6$$

→ next we represent these minterms in a table

REPRESENTATION

		x_1	x_2	x_3	
I	✓	0	1	0	m_2
	✓	1	0	0	m_4
	✓	0	1	1	m_3
II	✓	1	0	1	m_5
	✓	1	1	0	m_6
III		0	1	–	$m_2 \vee m_3 = \overline{x}_1 x_2 = max_1$
		–	1	0	$m_2 \vee m_6 = x_2 \overline{x}_3 = max_2$
		1	0	–	$m_4 \vee m_5 = x_1 \overline{x}_2 = max_3$
		1	–	0	$m_4 \vee m_6 = x_1 \overline{x}_3 = max_4$

SIMPLE FACTORIZATIONS

The set of maximal monoms contains the monom corresponding all the unmarked rows from the table.

$M(f_2) = \{max_1, max_2, max_3, max_4\}$

To obtain the central monoms, a new table - table of correspondence is used.

MAX MONOMS / MINTERMS	max_1	max_2	max_3	max_4
m_2	*	*		
m_4			*	*
m_3	*			
m_5			*	
m_6		*		*

* in cell [i,j] $\Rightarrow$ minterm on row i was used in ~~monom~~ maximal monom on column j

A maximal monom is a central monom if there is a symbol * on its column which is unique on its row.

$C(f) = \{max_1, max_3\}$

$\left.\begin{array}{l} M(f) \neq C(f) \\ C(f) \neq \emptyset \end{array}\right\} \Rightarrow$ Case 2 of simplification algorithm

We denote by $g(x_1, x_2, x_3) = \overline{x_1} x_2 \vee x_1 \overline{x_2} = max_1 \vee max_3$ the disjunction of central monoms that belongs in all simplified forms of the fn

$\Rightarrow m_6$ is uncovered

$\Rightarrow$ can use either max_2 or $max_4 \Rightarrow$ 2 simplified forms with same no. of overlaps

$f^S_{2_1}(x_1, x_2, x_3) = \overline{x_1} x_2 \vee x_1 \overline{x_2} \vee x_2 \overline{x_3} = max_1 \vee max_2 \vee max_3$

$f^S_{2_2}(x_1, x_2, x_3) = \overline{x_1} x_2 \vee x_1 \overline{x_2} \vee x_1 \overline{x_3} = max_1 \vee max_3 \vee max_4$

EXERCISE 8 - BOOLEAN FNS | Moisil method to simplify fn

8.2. $f_2(x_1,x_2,x_3) = m_0 \vee m_1 \vee m_3 \vee m_4 \vee m_7 \to$ DCF

$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$, $m_1 = \bar{x}_1\bar{x}_2 x_3$, $m_3 = \bar{x}_1 x_2 x_3$, $m_4 = x_1\bar{x}_2\bar{x}_3$, $m_7 = x_1 x_2 x_3$

In Moisil's method → initial fn in DCF (Disjunctive Canonical Form). The set of maximal monoms calculated using either Veitch-Karnaugh or Quine, and we can use propositional logic to obtain simplified forms from the set of maximal monoms.

STEP 1. We use Quine's method to find $M(f_2)$ = set of maximal monoms

$S_{f_2} = \{(0,0,0), (0,0,1), (1,0,0), (0,1,1), (1,1,1)\}$

	x_1	x_2	x_3	
I ✓	0	0	0	m_0
✓	0	0	1	m_1
II ✓	1	0	0	m_4
III ✓	0	1	1	m_3
IV ✓	1	1	1	m_7
V = I+II	0	0	—	$m_0 \vee m_1 = \bar{x}_1\bar{x}_2 = max_1$
	—	0	0	$m_0 \vee m_4 = \bar{x}_2\bar{x}_3 = max_2$
VI = II+III	0	—	1	$m_1 \vee m_3 = \bar{x}_1 x_3 = max_3$
VII = III+IV	—	1	1	$m_3 \vee m_7 = x_2 x_3 = max_4$

STEP 2: p_i: "max_i belongs to the simplified form of f"

$i = 1, 2, \dots, 4$ (because we have 4 max monoms)

We have the following true sentences (because each minterm must be covered by a max monom in the simplified form)

"m_0 is covered by max_1 or by max_2": $p_1 \vee p_2$

"m_1 covered by max_1 or max_3": $p_1 \vee p_3$

"m_4 covered by max_2": p_2

"m_3 covered by max_3 or max_4": $p_3 \vee p_4$

"m_7 covered by max_4": p_4

$\Rightarrow$ all minterms must be covered by a minimum number of max monoms, with minimum number of overlaps $\Rightarrow$

$$(p_1 \vee p_2) \wedge (p_1 \vee p_3) \wedge p_2 \wedge (p_3 \vee p_4) \wedge p_4 \equiv T \quad \text{(CNF with 5 clauses)}$$

$$\xrightarrow{\text{absorption}} p_2 \wedge p_4 \wedge (p_1 \vee p_3)$$

OR

$$(p_1 \vee p_2) \wedge (p_1 \vee p_3) \wedge p_2 \wedge (p_3 \vee p_4) \wedge p_4 \equiv T$$

$$(p_1 \vee (p_2 \wedge p_3)) \wedge p_2 \wedge ((p_3 \wedge p_4) \vee (p_4 \wedge p_4)) \equiv T$$

$$(p_1 \vee (p_2 \wedge p_3)) \wedge p_2 \wedge ((p_3 \wedge p_4) \vee p_4) \equiv T$$

$\xrightarrow{\text{DISTRIBUTIVITY}}$

$$(p_1 \wedge p_2 \wedge p_3 \wedge p_4) \vee (p_1 \wedge p_2 \wedge p_4) \vee (p_2 \wedge p_3 \wedge p_2 \wedge p_3 \wedge p_4) \vee (p_2 \wedge p_3 \wedge p_2 \wedge p_4) \equiv T$$

$\xrightarrow{\text{IDEMPOTENCY}}$

$$(p_1 \wedge p_2 \wedge p_3 \wedge p_4) \vee (p_1 \wedge p_2 \wedge p_4) \vee (p_2 \wedge p_3 \wedge p_4) \vee (p_2 \wedge p_3 \wedge p_4) \equiv T$$

$\xrightarrow{\text{ABSORPTION}}$

$$T \equiv (p_1 \wedge p_2 \wedge p_4) \vee (p_2 \wedge p_3 \wedge p_4) \Rightarrow \text{DNF with 2 cubes}$$

A DNF is true if one of the cubes is true.
We consider the cubes with the min no. of propositional variables from the DNF to obtain f_2^S (the simplified forms of f_2)

$\Rightarrow$ for cube $p_1 \wedge p_2 \wedge p_4 \Rightarrow f^S_{2_1}(x_1, x_2, x_3) = max_1 \vee max_2 \vee max_4$

$$= \overline{x_1}\overline{x_2} \vee \overline{x_2}\overline{x_3} \vee \overline{x_2}x_3$$

for cube $p_2 \wedge p_3 \wedge p_4 \Rightarrow f^S_{2_2}(x_1, x_2, x_3) = max_2 \vee max_3 \vee max_4$

$$= \overline{x_2}x_3 \vee \overline{x_1}x_3 \vee x_2x_3$$