Ex 7 Check the consistency of the following sets of clauses using lock resolution. Choose 2 different indexings for the literals.

7.2 $S_2 = \{p \vee \neg r, q \vee r, \neg p \vee \neg r, \neg q \vee \neg r\}$

THEORETICAL RESULTS

1) brief description and rules of lock resolution
2) brief description of level saturation strategy

1) $C_1 = \underset{(2)}{p} \vee \underset{(1)}{\neg r}$
$C_2 = \underset{(4)}{q} \vee \underset{(3)}{r}$
$C_3 = \underset{(6)}{\neg p} \vee \underset{(5)}{\neg r}$
$C_4 = \underset{(8)}{\neg q} \vee \underset{(7)}{\neg r}$

In order to prove consistency of a set S of clauses, lock resolution must be combined with the level saturation strategy.
if $S^k = \emptyset$ (last level of lock resolvents is empty), then the set S is consistent.

$S^0 = S = \{C_1, C_2, C_3, C_4\}$

$S^1 = \{Res^{lock}(C_i, C_j) \mid C_i \in S^0, C_j \in S^0\}$

$C_5 = Res^{lock}_r(C_1, C_2) = \underset{(2)}{p} \vee \underset{(4)}{q}$

$C_6 = Res^{lock}_r(C_2, C_3) = \underset{(4)}{q} \vee \underset{(6)}{\neg p}$

$C_7 = Res^{lock}_r(C_2, C_4) = \underset{(4)}{q} \vee \underset{(8)}{\neg q} \equiv T$

$S^2 = \{Res^{lock}(C_i, C_j) \mid C_i \in S^1, C_j \in S^1 \cup S^0\}$

$S^2 = \emptyset \Rightarrow$ last level of lock resolvents is empty $\Rightarrow$ S is consistent

2) $C_1 = \underset{(1)}{p} \vee \underset{(2)}{\neg r}$
$C_2 = \underset{(3)}{q} \vee \underset{(4)}{r}$
$C_3 = \underset{(5)}{\neg p} \vee \underset{(6)}{\neg r}$
$C_4 = \underset{(3)}{\neg q} \vee \underset{(6)}{\neg r}$

$S^0 = S = \{C_1, C_2, C_3, C_4\}$

$S^1 = \{Res^{lock}(C_i, C_j) \mid C_i \in S^0, C_j \in S^0\}$

$C_5 = Res^{lock}_p(C_1, C_3) = \underset{(2)}{\neg r} \vee \underset{(6)}{\neg r} = \underset{(2)}{\neg r}$

$C_6' = Res_2^{lock}(C_2, C_4) = \underset{(4)}{\neg r} \vee \underset{(6)}{\neg r} \equiv \top$ (tautology)

$S^1 = \{C_5', C_6'\}$

$S^2 = \{ Res^{lock}(C_i, C_j) \mid C_i \in S^1, C_j \in S^0 \cup S^1 \}$

$S^2 = \emptyset \Rightarrow$ last level of lock resolution is empty $\Rightarrow$ S consistent

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For pre-exercises with resolution in propositional logic, follow models from predicate:

- write main theorem used to prove what you need
- briefly cover the important aspects of the method you are using
- explain your results

  e.g. "Obtained ... therefore, ~~con~~ according to Theorem ... we can conclude ..."

  "Cannot resolve with C ... and C because ..."

idea here is to describe the process according to which you solve your problems, supporting it with the theoretical aspects you learned