EX 1 BOOLEAN FUNCTIONS

For the following Boolean fns of 3 vars, given by their truth tables, write the corresponding DCF (DISJUNCTIVE CANONICAL FORM) and CCF (CONJUNCTIVE CANONICAL FORM).
Using Karnaugh diagrams simplify both DCF and CCF.

	x	y	z	f_8
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	0
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

THEORETICAL RESULTS

1) CCF is conjunction of maxterms corresponding to the values 0 of the fn
2) DCF = disjunction of minterms corresponding to values 1 of fn

minterm = monom (conjunction of variables) that contains all vars of fn

maxterm = disjunction containing all variables of fn

minterm form: $x_1^{\alpha_1} \wedge \dots \wedge x_n^{\alpha}$, $\alpha_i \in B_2$
maxterm form: $x_1^{\alpha_1} \vee x_2^{\alpha_2} \vee \dots \vee x_n^{\alpha_n}$

$f : (B_2)^n \to B_2$

SOLUTION

$DCF(f_8) = m_0 \vee m_2 \vee m_5 \vee m_6$
$= (\bar{x} \wedge \bar{y} \wedge \bar{z}) \vee (\bar{x} \wedge y \wedge \bar{z}) \vee (x \wedge \bar{y} \wedge z) \vee (x \wedge y \wedge \bar{z})$

x	y	z	f_8	m_0	m_2	m_5	m_6
0	0	0	1	1	0	0	0
0	0	1	0	0	0	0	0
0	1	0	1	0	1	0	0
0	1	1	0	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	0	0	1	0
1	1	0	1	0	0	0	1
1	1	1	0	0	0	0	0

minterm → boolean fn which is 1 only for 1 argument

index of minterm = conversion to decimal of binary numbers composed of the digits which represent the powers of the n (here = 3) variables that form the minterm

$m_0 = x^0 \wedge y^0 \wedge z^0 = \bar{x} \wedge \bar{y} \wedge \bar{z}$ $\quad 000_{(2)} \to 0_{(10)}$
$m_2 = x^0 \wedge y^1 \wedge z^0 = \bar{x} \wedge y \wedge \bar{z}$ $\quad 010_{(2)} \to 2_{(10)}$
$m_5 = x^1 \wedge y^0 \wedge z^1 = x \wedge \bar{y} \wedge z$ $\quad 101_{(2)} \to 5_{(10)}$
$m_6 = x^1 \wedge y^1 \wedge z^0 = x \wedge y \wedge \bar{z}$ $\quad 110_{(2)} \to 6_{(10)}$

$$CCF(f_8) = M_1 \wedge M_3 \wedge M_4 \wedge M_7$$
$$= (x \vee y \vee \bar{z}) \wedge (x \vee \bar{y} \vee \bar{z}) \wedge$$
$$\wedge (\bar{x} \vee y \vee z) \wedge (\bar{x} \vee \bar{y} \vee \bar{z})$$

$$M_1 = M_{001_{(2)}} = x^{\bar{0}} \vee y^{\bar{0}} \vee z^{\bar{1}}$$
$$= x^1 \vee y^1 \vee z^0$$
$$= x \vee y \vee \bar{z}$$

$$M_3 = M_{011_{(2)}} = x^{\bar{0}} \vee y^{\bar{1}} \vee z^{\bar{1}}$$
$$= x^1 \vee y^0 \vee z^0$$
$$= x \vee \bar{y} \vee \bar{z}$$

$$M_4 = M_{100_{(2)}} = x^{\bar{1}} \vee y^{\bar{0}} \vee z^{\bar{0}}$$
$$= x^0 \vee y^1 \vee z^1$$
$$= \bar{x} \vee y \vee z$$

$$M_7 = M_{111_{(2)}} = \bar{x} \vee \bar{y} \vee \bar{z}$$

x	y	z	f_8	M_1	M_3	M_4	M_7
0	0	0	1	1	1	1	1
0	0	1	0	0	1	1	1
0	1	0	1	1	1	1	1
0	1	1	0	1	0	1	1
1	0	0	0	1	1	0	1
1	0	1	1	1	1	1	1
1	1	0	1	1	1	1	1
1	1	1	0	1	1	1	0

maxterm = Boolean fn which is 0 for only 1 argument

index obtained by conversion in decimal of the binary no. composed of the dual of the digits, digits representing the powers of all n variables from the expression of maxterm.

SIMPLIFICATION

$$DCF(f_8) = m_0 \vee m_2 \vee m_5 \vee m_6$$
$$= \bar{x}\bar{y}\bar{z} \vee \bar{x}y\bar{z} \vee x\bar{y}z \vee xy\bar{z}$$

Factorization process (Karnaugh diagram)

x_1 \ x_2x_3	00	01	11	10
0	m_0			m_2
1		m_5		m_6

max_1, max_2

$max_1 = \bar{x}\bar{y}\bar{z} \vee \bar{x}y\bar{z} = \bar{x}\bar{z} = m_0 \vee m_2$

$max_2 = \bar{x}y\bar{z} \vee xy\bar{z} = y\bar{z} = m_2 \vee m_6$

$max_3 = m_5$ (isolated minterm)

We have to obtain $M(f_8)$ = set of maximal monoms

$C(f_8)$ = set of central monoms.

→ apply factorization by first trying n-factorization, n-1 factorization, ... simple factorization and 0-factorization (isolated minterms)

$M(f_8) = \{max_1, max_2, max_3\}$ maximal monoms

$C(f_8) = \{max_1, max_2, max_3\}$ central monoms

has m_0 circled once

has m_6 circled once

A max monom is a central monom if the corresponding group of minterms contains at least 1 minterm circled exactly once.

$M(f_8) = C(f_8) \Rightarrow$ Case 1 of simplification algorithm

$$\Rightarrow 1 \text{ solution} \Rightarrow f_8^{DS} = \bigvee_{m \in M(f_8)} m = max_1 \vee max_2 \vee max_3$$
$$= \bar{x}\bar{z} \vee y\bar{z} \vee x\bar{y}z$$

$$CCF(f_8) = M_1 \wedge M_3 \wedge M_4 \wedge M_7$$
$$= (x \vee y \vee \bar{z}) \wedge (x \vee \bar{y} \vee \bar{z}) \wedge (\bar{x} \vee y \vee z) \wedge (\bar{x} \vee \bar{y} \vee \bar{z})$$

Dual factorization process (Karnaugh diagram): to simplify CCF

x_1 \ x_2x_3	00	01	11	10
0		M_1	M_3	
1	M_4		M_7	

$$maxd_1 = M_1 \wedge M_3 = (\overset{1}{x} \vee \overset{1}{y} \vee \overset{0}{\bar{z}}) \wedge (\overset{1}{x} \vee \overset{0}{\bar{y}} \vee \overset{0}{\bar{z}})$$

001 / 110   011 / 100

$$= (y \wedge \bar{y}) \vee x \vee \bar{z} = x \vee \bar{z}$$

$$maxd_2 = M_3 \wedge M_7 = (\overset{1}{x} \vee \overset{0}{\bar{y}} \vee \overset{0}{\bar{z}}) \wedge (\overset{0}{\bar{x}} \vee \overset{0}{\bar{y}} \vee \overset{0}{\bar{z}})$$

011 / 100   111 / 000

$$= (x \wedge \bar{x}) \vee \bar{y} \vee \bar{z}$$
$$= \bar{y} \vee \bar{z}$$

$maxd_3 = M_4$ (isolated maxterm)

→ in the diagram, headers of lines and columns are used to express indices of maxterms, represent duals of the powers of the vars from the maxterm expression

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$Md(f_8) = \{maxd_1, maxd_2, maxd_3\}$

$Cd(f_8) = \{maxd_1, maxd_2, maxd_3\}$

$Md(f_8) = Cd(f_8) \Rightarrow$ Case 1 of dual simplification algorithm

$\Rightarrow$ unique conjunctive simplified form

$$\Rightarrow f_8^{CS}(x,y,z) = (x \vee \bar{z}) \wedge (\bar{y} \vee \bar{z}) \wedge (\bar{x} \vee y \vee z)$$
$$= maxd_1 \wedge maxd_2 \wedge maxd_3$$

EX 2 BOOLEAN FNS | Simplify using Karnaugh diagrams.

2.3. $f_3(x_1,x_2,x_3,x_4) = \underset{m_{15}}{x_1x_2x_3x_4} \vee \underset{m_{14}}{x_1x_2x_3\bar{x}_4} \vee \underset{m_6}{\bar{x}_1x_2x_3\bar{x}_4} \vee \underset{m_7}{\bar{x}_1x_2x_3x_4} \vee \underset{m_0}{\bar{x}_1\bar{x}_2\bar{x}_3\bar{x}_4}$

$\vee \underset{m_2}{\bar{x}_1\bar{x}_2x_3\bar{x}_4} \vee \underset{m_9}{x_1\bar{x}_2\bar{x}_3x_4} \vee \underset{m_1}{\bar{x}_1\bar{x}_2\bar{x}_3x_4} \vee \underset{m_3}{\bar{x}_1\bar{x}_2x_3x_4}$

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00	m_0	m_1	m_3	m_2
01			m_7	m_6
11			m_{15}	m_{14}
10		m_9		

max_1, max_2, max_4

We start with double factorization, then do simple factorization. (no triple factorization possible, no 0-factorization etc)

$max_1 = m_0 \vee m_1 \vee m_3 \vee m_2 = \bar{x}_1\bar{x}_2$ (what is the common part? they are all on the row with $x_1x_2 \to 00$)

$max_2 = m_7 \vee m_6 \vee m_{15} \vee m_{14} = x_2x_3$

$\cancel{max_4 = m_3 \vee m_2 \vee m_7 \vee m_6 = \bar{x}_1x_3}$

$max_4 = m_1 \vee m_9 = \bar{x}_1\bar{x}_2\bar{x}_3x_4 \vee x_1\bar{x}_2\bar{x}_3x_4 = \bar{x}_2\bar{x}_3x_4$

$M(f_3) = \{max_1, max_2, \cancel{max_3}, max_4\}$ maximal monoms

$C(f_3) = \{max_1, max_2, max_4\}$ central monoms $\Rightarrow$

max_1: has m_0 circled once

max_2: has m_{14}, m_{15} circled once

max_4: has m_9 circled once

$\Rightarrow M(f_3) = C(f_3) \rightarrow$ first case of simplification algorithm

$\Rightarrow$ unique simplified form

$$f_3^S(x_1,x_2,x_3,x_4) = \bigvee_{m \in M(f_3)} m = max_1 \vee max_2 \vee max_4 = \bar{x}_1\bar{x}_2 \vee x_2x_3 \vee \bar{x}_2\bar{x}_3x_4$$

EX3 BOOLEAN FNS

Using Veitch diagrams, simplify the function:

3.3. $f_3(x_1,x_2,x_3) = \overline{x}_1(x_2 \downarrow x_3) \vee x_1\overline{x}_2x_3 \vee \overline{(x_1 \vee (x_2 \uparrow x_3))} \vee x_1x_2x_3$

THEORETICAL RESULTS USED

$x \downarrow y = \overline{x \vee y} = \overline{x} \wedge \overline{y}$ (NOR)

$x \uparrow y = \overline{x \wedge y} = \overline{x} \vee \overline{y}$ (NAND)

$\overline{x}_1(x_2 \downarrow x_3) = \overline{x}_1(\overline{x_2 \vee x_3})$
$= \overline{x}_1\overline{x}_2\overline{x}_3$

$\overline{x_1 \vee (x_2 \uparrow x_3)} = \overline{x_1 \vee \overline{x_2 \wedge x_3}}$

$= \overline{x_1 \vee \overline{x}_2 \vee \overline{x}_3} = \overline{x}_1x_2x_3$

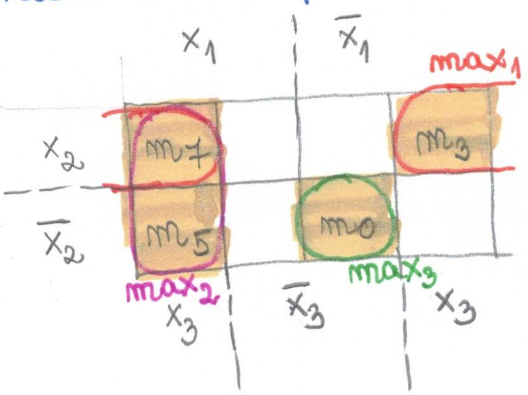
→ we bring fn to DCF

$\overline{x}_1(x_2 \downarrow x_3) = \overline{x}_1\overline{x}_2\overline{x}_3$

$\overline{(x_1 \vee (x_2 \uparrow x_3))} = \overline{x_1 \vee \overline{x}_2 \vee \overline{x}_3} = \overline{x}_1x_2x_3$

$$\Rightarrow f_3(x_1,x_2,x_3) = \underbrace{\overset{000}{\overline{x}_1\overline{x}_2\overline{x}_3}}_{m_0} \vee \underbrace{\overset{101}{x_1\overline{x}_2x_3}}_{m_5} \vee \underbrace{\overset{011}{\overline{x}_1x_2x_3}}_{m_3} \vee \underbrace{\overset{111}{x_1x_2x_3}}_{m_7}$$

Factorization process (Veitch diagram)



$max_1 = m_7 \vee m_3 = x_1x_2x_3 \vee \overline{x}_1x_2x_3$
$= x_2x_3$

$max_2 = m_7 \vee m_5 = x_1x_2x_3 \vee x_1\overline{x}_2x_3$
$= x_1x_3$

$max_3 = m_0$ (isolated minterm)

$M(f_3) = \{max_1, max_2, max_3\}$ set of maximal monoms

m_3 circled once m_5 circled once $\Rightarrow$

$\Rightarrow C(f_3) = \{max_1, max_2, max_3\}$

$M(f_3) = C(f_3) \Rightarrow$ Case 1 of simplification algorithm

$\Rightarrow$ unique simplified form:

$$f_3^S(x_1, x_2, x_3) = max_1 \vee max_2 \vee max_3$$
$$= x_2 x_3 \vee x_1 x_3 \vee \overline{x}_1 \overline{x}_2 \overline{x}_3$$

EX4 BOOLEAN FNS

Simplify fn using Veitch diagram.

$$f_3(x_1,x_2,x_3,x_4) = x_1x_4 \vee \overset{1}{x_1}\overset{0}{\overline{x_2}}\overset{0}{\overline{x_3}}\overset{0}{\overline{x_4}} \vee \overline{x_1}\,\overline{x_2}\,\overline{x_4} \vee \overline{x_1}\,\overline{x_3} \vee x_3x_4$$

minterm m_8

what are the minterms that cover them?

x_1x_4: $\overset{1}{x_1}\overset{1}{x_2}\overset{1}{x_3}\overset{1}{x_4}$ m_{15}; $\overset{1}{x_1}\overset{1}{x_2}\overset{0}{\overline{x_3}}\overset{1}{x_4}$ m_{13}; $\overset{1}{x_1}\overset{0}{\overline{x_2}}\overset{1}{x_3}\overset{1}{x_4}$ m_{11}; $\overset{1}{x_1}\overset{0}{\overline{x_2}}\overset{0}{\overline{x_3}}\overset{1}{x_4}$ m_9

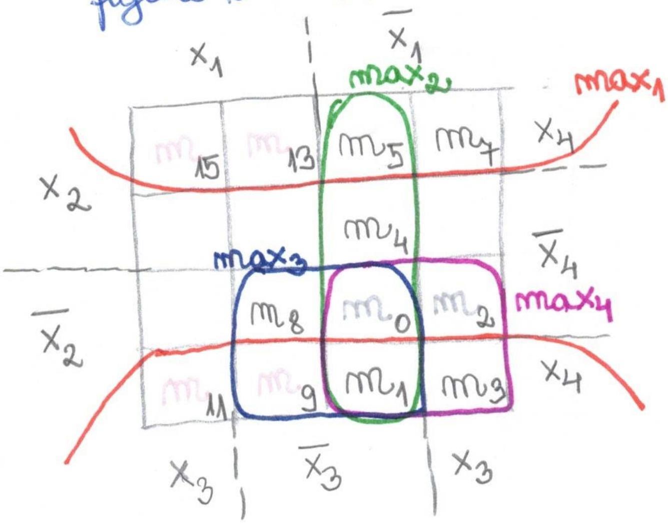
$\overline{x_1}\,\overline{x_2}\,\overline{x_4}$: $\overset{0}{\overline{x_1}}\overset{0}{\overline{x_2}}\overset{0}{\overline{x_3}}\overset{0}{\overline{x_4}}$ m_0; $\overset{0}{\overline{x_1}}\overset{0}{\overline{x_2}}\overset{1}{x_3}\overset{0}{\overline{x_4}}$ m_2

$\overline{x_1}\,\overline{x_3}$: $\overset{0}{\overline{x_1}}\overset{1}{x_2}\overset{0}{\overline{x_3}}\overset{1}{x_4}$ m_5; $\overset{0}{\overline{x_1}}\overset{1}{x_2}\overset{0}{\overline{x_3}}\overset{0}{\overline{x_4}}$ m_4; $\overset{0}{\overline{x_1}}\overset{0}{\overline{x_2}}\overset{0}{\overline{x_3}}\overset{1}{x_4}$ m_1; $\overset{0}{\overline{x_1}}\overset{0}{\overline{x_2}}\overset{0}{\overline{x_3}}\overset{0}{\overline{x_4}}$ m_0

x_3x_4: $\overset{1}{x_1}\overset{1}{x_2}\overset{1}{x_3}\overset{1}{x_4}$ m_{15}; $\overset{1}{x_1}\overset{0}{\overline{x_2}}\overset{1}{x_3}\overset{1}{x_4}$ m_{11}; m_7 $\overset{0}{\overline{x_1}}\overset{1}{x_2}\overset{1}{x_3}\overset{1}{x_4}$; m_3 $\overset{0}{\overline{x_1}}\overset{0}{\overline{x_2}}\overset{1}{x_3}\overset{1}{x_4}$

We have the minterms: $m_0, m_1, m_2, m_3, m_4, m_5, m_7, m_8, m_9, m_{11}, m_{13}, m_{15}$

OR We can draw the Veitch diagram directly and use it to figure out what minterms we have:



→ triple factorization

$max_1 = m_1 \vee m_3 \vee m_5 \vee m_7 \vee m_9 \vee m_{11} \vee m_{13} \vee m_{15} = x_4$

$max_2 = m_5 \vee m_4 \vee m_0 \vee m_1 = \overline{x_1}\,\overline{x_3}$

$max_3 = m_8 \vee m_0 \vee m_9 \vee m_1 = \overline{x_2}\,\overline{x_3}$

$max_4 = m_0 \vee m_2 \vee m_1 \vee m_3 = \overline{x_1}\,\overline{x_2}$

— : minterms in the area of diagram in which we have x_1, and x_4

— : —||— —||— —||— —||— —||— $\overline{x_1}, \overline{x_2}, \overline{x_4}$

... similar for $\overline{x_1}\,\overline{x_3}$ and x_3x_4

↓ fill in indices after

$M(f_3) = \{max_1, max_2, max_3, max_4\}$ set of maximal monoms

has m_2 circled once

has m_8 circled once

has m_{13}, m_{15}, m_7, m_{11} circled once

has m_4 circled once

$\Rightarrow C(f_3) = \{max_1, max_2, max_3, max_4\}$ set of central monoms

$M(f_3) = C(f_3) \Rightarrow$ Case 1 of simplification algorithm

$\Rightarrow$ 1 simplified form:

$$f_3^S(x_1, x_2, x_3, x_4) = max_1 \vee max_2 \vee max_3 \vee max_4$$

$$= x_4 \vee \overline{x_1}\,\overline{x_3} \vee \overline{x_2}\,\overline{x_3} \vee \overline{x_1}\,\overline{x_2}$$