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# METODE INTELIGENTE DE REZOLVARE A PROBLEMELOR REALE

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Graph-based learning

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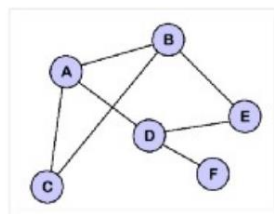
# Graph neural networks

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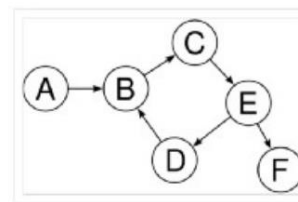
- Why?
- How?
- Applications

# Graphs

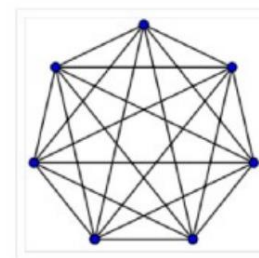
- a type of data structure having two components:
  - nodes (or vertices)
    - Homogenous
    - Heterogenous
  - edges, which connect two nodes
    - Unidirectional
    - Bidirectional
    - With or without weights
- a graph = a collection of loosely inter-connected nodes via edges



undirected graph



directed graph



complete graph

# Graph NNs

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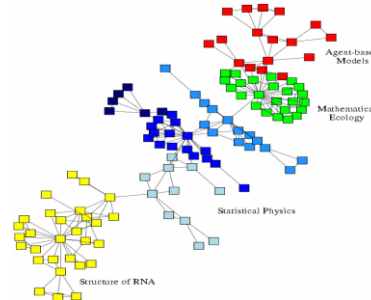
- neural network architectures that operate on a graph.
- Aim:
  - for each node in the graph to learn an embedding containing information about its neighborhood (nodes directly connected to the target node via edges).
  - This embedding can then be used for different problems like node labelling, node prediction, edge prediction, etc.
- Real-life applications
  - Social Network Analysis — Similar posts prediction, tags prediction, and recommending content to users.
  - Natural Sciences — GNNs have also gained popularity in dealing with molecular interactions like protein-protein interactions.
  - Recommender Systems — A heterogenous graph can be used to capture relationships between users and items to recommend relevant items to a buyer.

# Why graphs?

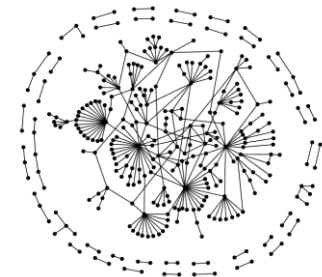
- Graphs (= networks, systems) are a general language for describing and modeling complex systems
- Many data are represented as graphs



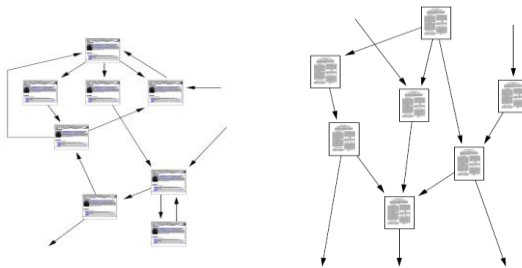
Social networks



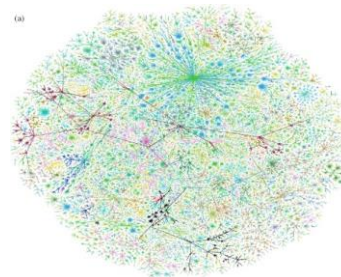
Economic networks



Biomedical networks

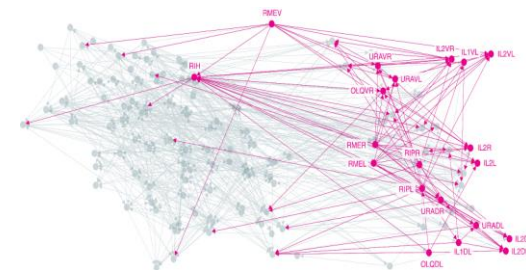


Information networks:  
Web & citations



Internet

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Networks of neurons

# Why graphs?

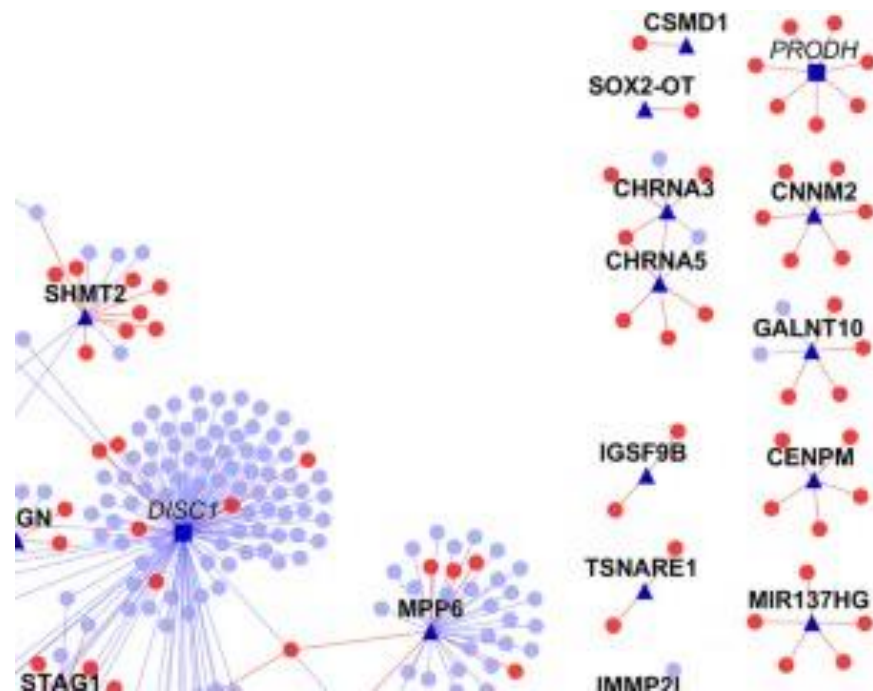
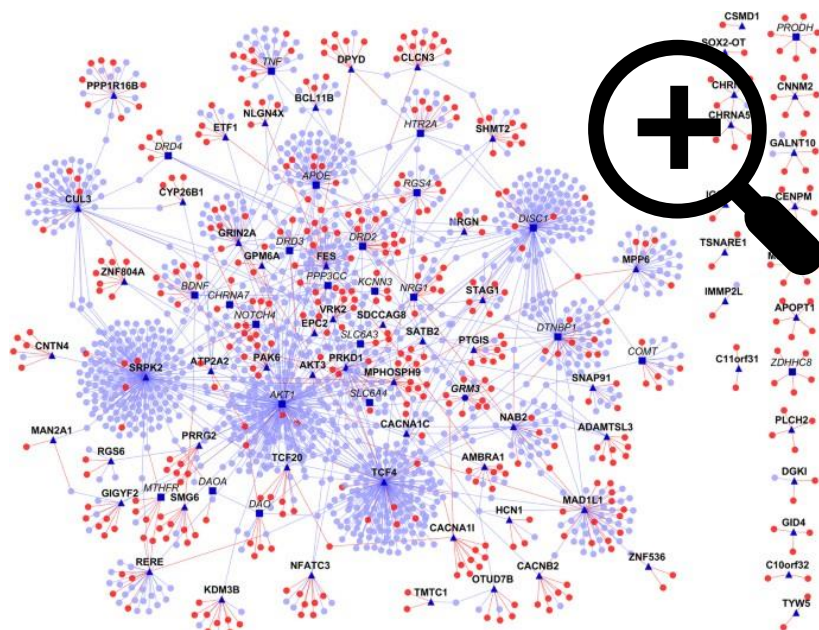
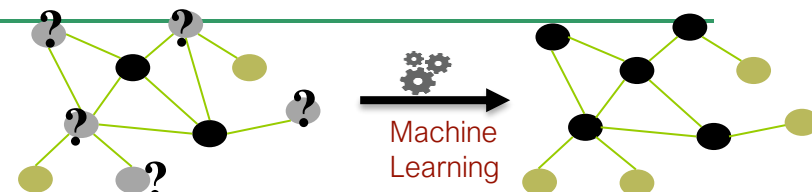
- ❑ Graphs (= networks, systems) are a general language for describing and modeling complex systems
- ❑ Many data are represented as graphs
- ❑ Universal language for describing complex data
  - Networks from science, nature, and technology are more similar than one would expect
- ❑ Shared vocabulary between fields
  - Computer Science, Social science, Physics, Economics, Statistics, Biology
- ❑ Data availability (+computational challenges)
  - Web/mobile, bio, health, and medical
- ❑ Impact!
  - Social networking, Social media, Drug design

# Machine learning with / in graphs

## □ Node classification

- Predict the type of a given node
- Classifying the function of proteins in the interactome!

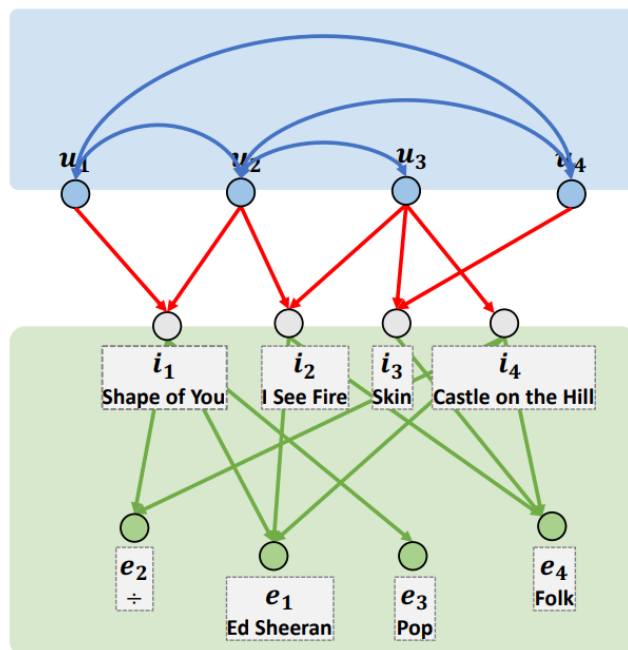
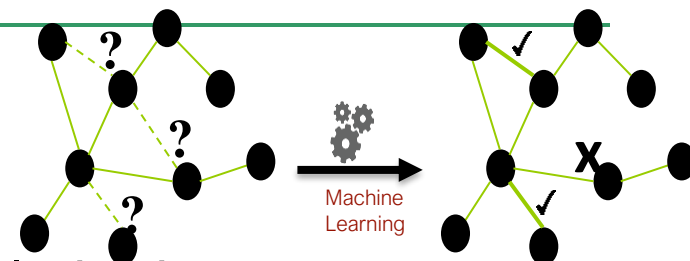
- See Ganapathiraju, M. K., Thahir, M., Handen, A., Sarkar, S. N., Sweet, R. A., Nimgaonkar, V. L., ... & Chaparala, S. (2016). Schizophrenia interactome with 504 novel protein–protein interactions. *NPJ schizophrenia*, 2(1), 1-10. <https://www.nature.com/articles/npjSchz201612>





# Machine learning with / in graphs

- Node classification
- Link prediction
  - Predict whether two nodes are linked
  - Recommender systems



## User-User Connections

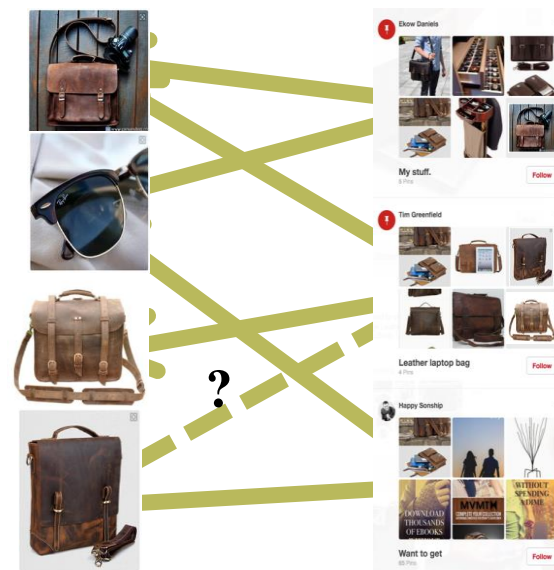
- Social Relations
- Same Profiles ...

## User-Item Interactions

- Implicit Feedback
- Explicit Feedback ...

## Item-Item Connections

- Same Attributes
- External Knowledge ...





# Machine learning with / in graphs

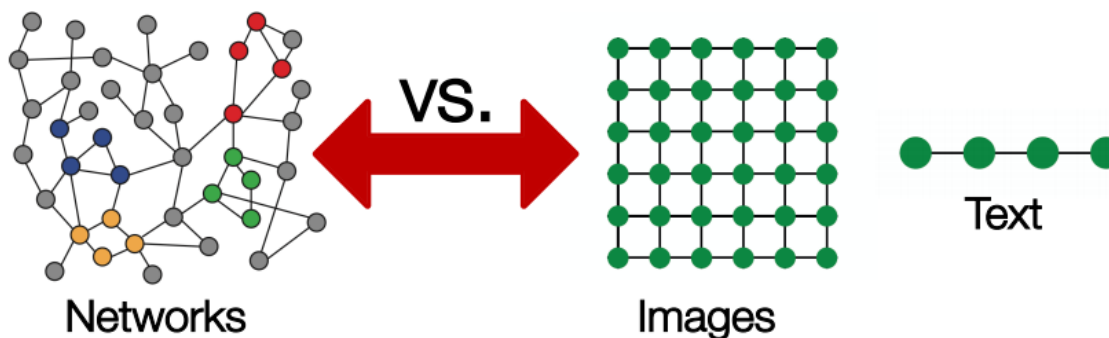
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- Node classification
  - Predict a type of a given node
- Link prediction
  - Predict whether two nodes are linked
- Community detection
  - Identify densely linked clusters of nodes
- Network similarity
  - How similar are two (sub)networks

# Graph neural networks

## □ Data structures

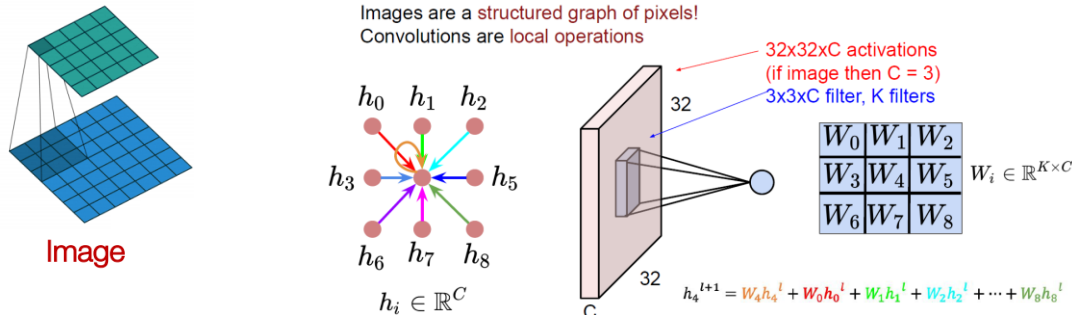
- Text/speech -> sequences -> RNN
- Images -> regular grids (matrix) -> CNN
- Graphs
  - Arbitrary size
  - Complex topological structure



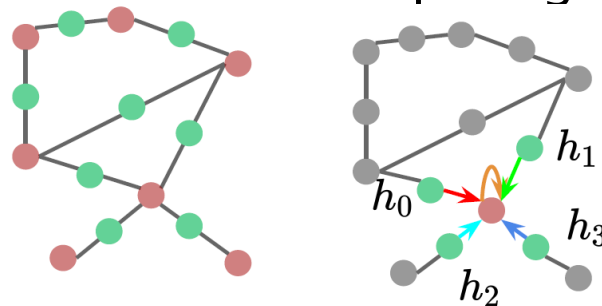
# Graph neural networks

## □ Convolutions over data

- Combine the information from the current element with that from neighbours  $\sum w_i h_i$



- How to deal with more complex graphs?



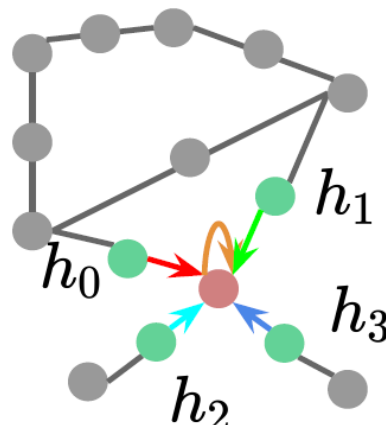
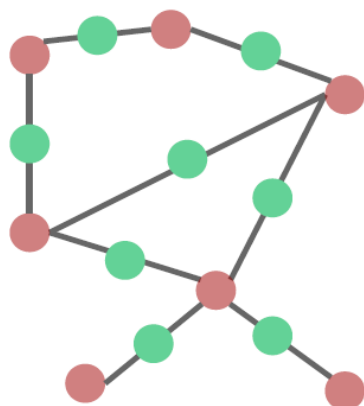
- Graph convolutions involve similar **local operations** on nodes.
- Nodes are now object representations and not activations
- The **ordering of neighbors** should not matter.
- The **number of neighbors** should not matter.
- $N(i)$  are the neighbors of node  $i$
- $c_{ij}$  is a normalization constant

$$h_4^{l+1} = W_4 h_4^l + W_0 h_0^l + W_1 h_1^l + W_2 h_2^l + W_3 h_3^l$$

$$h_i^{l+1} = W_i h_i^l + \sum_{j \in N(i)} \frac{1}{c_{ij}} W_j h_j^l$$

# Graph neural networks

## □ Convolutions over graphs



- Updates from some neighbors can be more important than others.
- Attention over neighbors allows graph convolutions to focus on specific neighbors
- $\sigma$  is a non-linearity, usually ReLU or LeakyReLU.

Without attention: 
$$h_i^{l+1} = W_i h_i^l + \sum_{j \in \mathcal{N}(i)} \frac{1}{c_{ij}} W_j h_j^l$$

With attention: 
$$h_i^{l+1} = W_i h_i^l + \sum_{j \in \mathcal{N}(i)} \frac{1}{c_{ij}} \alpha_{ij} W_j h_j^l$$

where 
$$\alpha_{ij} = \frac{e^{\sigma(a^T [Wh_i || Wh_j])}}{\sum_{k \in \mathcal{N}(i)} e^{\sigma(a^T [Wh_i || Wh_k])}}$$

# Graph neural networks

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## □ Naïve approach

- A fusion between the adjacency matrix and other node features -> input for an ANN
- + easy
- - a lot of parameters =  $O(\text{no of nodes})$
- - doesn't work for graphs of different sizes
- - not invariant to node ordering

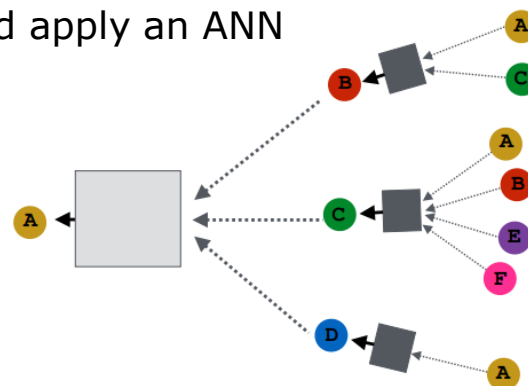
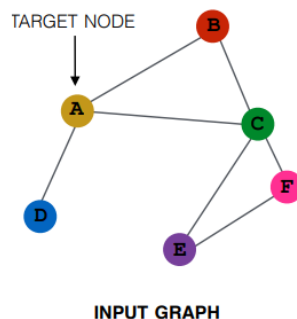
## □ Graph convolutional networks

- Node's neighbourhood defines a computational graph
- An ML algorithm is used to learn how to transform and transmit the information across the nodes

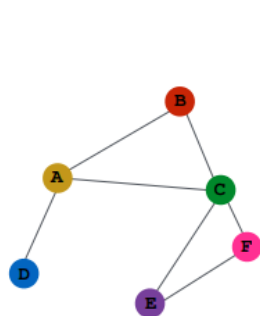
# Graph convolutional networks

- step1: generate node embeddings based on local graph neighbourhood

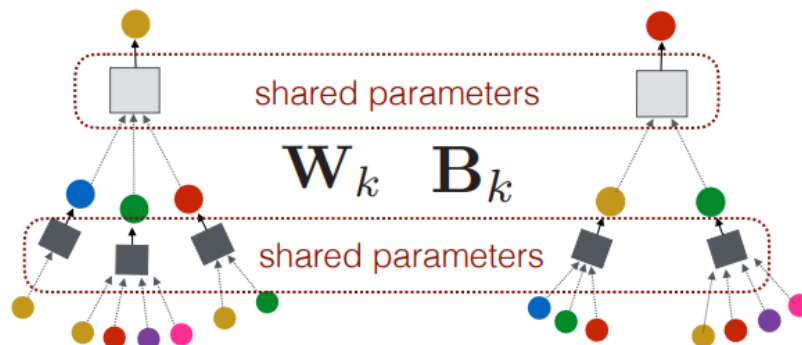
- Aggregate messages from neighbours and apply an ANN



- Every node defines a computation graph based on its neighbourhood



INPUT GRAPH

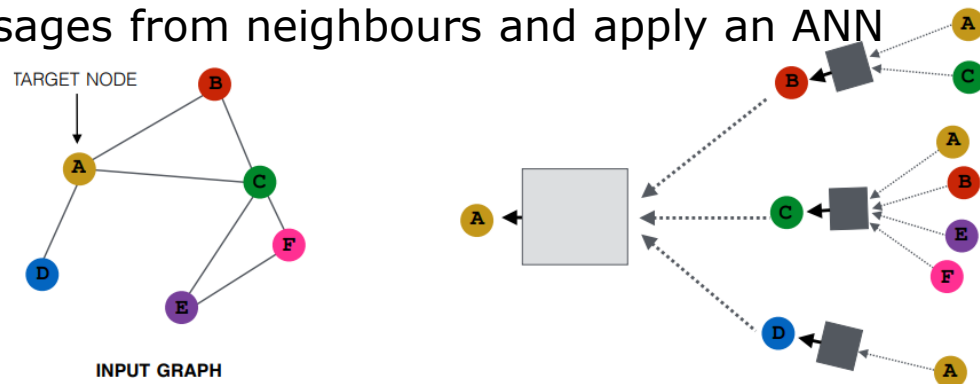


Compute graph for node A

Compute graph for node B

# Graph convolutional networks

- step1: setup for generating node embeddings based on local graph neighbourhood
  - Aggregate messages from neighbours and apply an ANN



- Aggregation by an order invariant operator
  - Average (mean) or Max or Sum
- ANN

Initial 0-th layer embeddings are equal to node features

$$\mathbf{h}_v^0 = \mathbf{x}_v$$

$$\mathbf{h}_v^k = \sigma \left( \mathbf{W}_k \sum_{u \in N(v)} \frac{\mathbf{h}_u^{k-1}}{|N(v)|} + \mathbf{B}_k \mathbf{h}_v^{k-1} \right), \quad \forall k \in \{1, \dots, K\}$$

Average of neighbor's previous layer embeddings

Non-linearity (e.g., ReLU)

Embedding after K layers of neighborhood aggregation

Previous layer embedding of v

$$\mathbf{z}_v = \mathbf{h}_v^K$$



# Graph convolutional networks

- step2: prepare the model training
  - Parameters of the graph model:  $W_k, B_k$
  - Classification weights  $\theta$
  - Loss function
    - E.g supervised binary classification task = safe or toxic drug

$$\mathcal{L} = \sum_{v \in V} y_v \log(\sigma(\mathbf{z}_v^T \boldsymbol{\theta})) + (1 - y_v) \log(1 - \sigma(\mathbf{z}_v^T \boldsymbol{\theta}))$$

Encoder output: node embedding

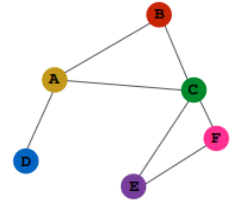
Classification weights

Node class label

Safe or toxic drug?

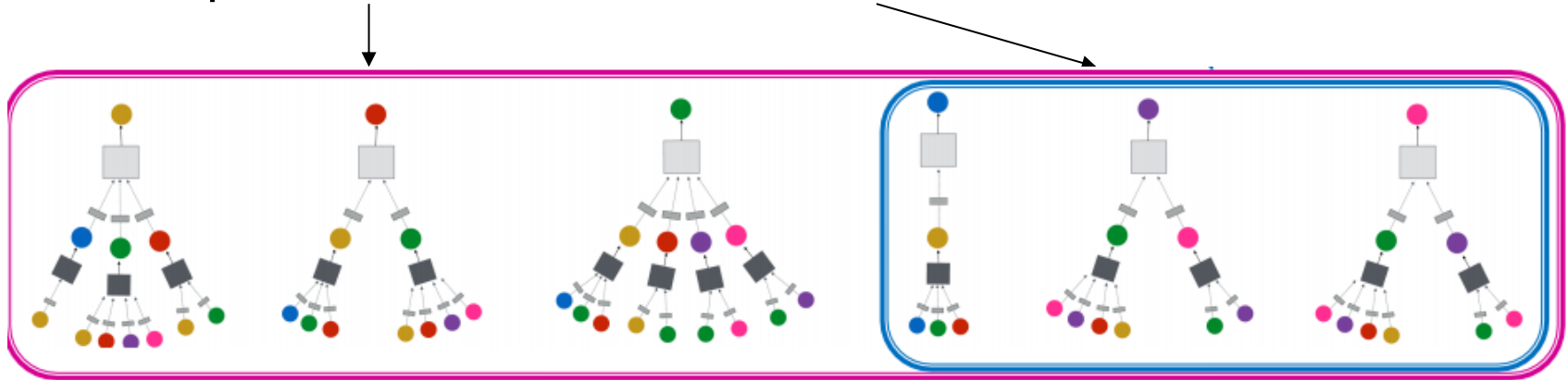
- E.g. unsupervised task – random walk optimization
  - See DeepWalk <https://arxiv.org/pdf/1403.6652.pdf>
  - See node2vec <https://cs.stanford.edu/~jure/pubs/node2vec-kdd16.pdf>

# Graph convolutional networks

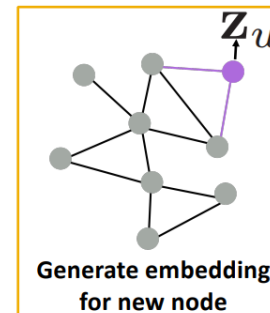
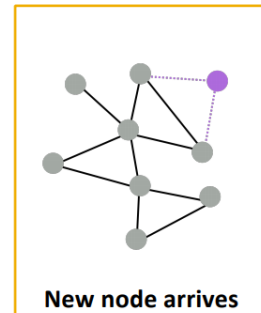
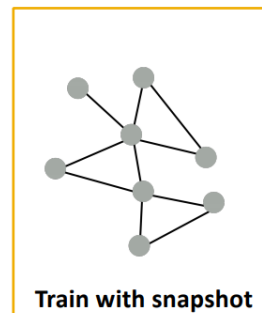


INPUT GRAPH

- step3: train the model and test



- Step 4: generalize for new nodes / graphs



# Graph CNNs - applications

- Action Genome: Understanding Action with Spatio-Temporal Scene Graphs
  - <http://actiongenome.org>
  - Ji, Krishna et al. Action Genome: Actions as Compositions of Spatio-Temporal Scene Graphs, CVPR 2020
- RecSys
  - Pinterest - PinSage
    - <https://medium.com/pinterest-engineering/pinsage-a-new-graph-convolutional-neural-network-for-web-scale-recommender-systems-88795a107f48>
    - <https://arxiv.org/pdf/1806.01973.pdf>
  - Model and predict side effects of drug pairs
    - <http://snap.stanford.edu/decagon/>
    - <https://arxiv.org/pdf/1802.00543.pdf>
- Data generation
  - Drug discovery = Molecule generation (with high value of a given chemical property)
    - <https://cs.stanford.edu/people/jure/pubs/gcpn-neurips18.pdf>
    - [https://github.com/bowenliu16/rl\\_graph\\_generation](https://github.com/bowenliu16/rl_graph_generation)
    - [https://www.cell.com/cell/pdf/S0092-8674\(20\)30102-1.pdf](https://www.cell.com/cell/pdf/S0092-8674(20)30102-1.pdf)

## □ Additional information

- Code example for GNN
  - <https://colab.research.google.com/drive/1DIQm9rOx2mT1bZETeeVUThxcrP1RKqAn>
- Data
  - SNAP project
    - <http://snap.stanford.edu/>
  - Open Graph Benchmark
    - <https://ogb.stanford.edu/>
- GNN and RecSys
  - <https://github.com/yazdotai/graph-networks#tensorflow-implementations>
  - <https://next-nus.github.io/slides/tuto-cikm2019-public.pdf>
- Graph-based Deep Learning
  - <https://github.com/naganandy/graph-based-deep-learning-literature>
  - [https://www.cs.mcgill.ca/~wlh/grl\\_book/](https://www.cs.mcgill.ca/~wlh/grl_book/)
  - <https://github.com/thunlp/GNNPapers>

## □ Materials are considered from various sources like:

- Fei-Fei Li's lecture about Graph Convolutions [http://vision.stanford.edu/teaching/cs231n/slides/2020/lecture\\_18.pdf](http://vision.stanford.edu/teaching/cs231n/slides/2020/lecture_18.pdf)
- Jure Leskovec's Lecture about Graph NNs <https://web.stanford.edu/class/cs224w/>
- <https://www.pyg.org/>
- Graph Representation Learning Book, by W. Hamilton [https://www.cs.mcgill.ca/~wlh/grl\\_book/](https://www.cs.mcgill.ca/~wlh/grl_book/)
- Network Science, by A. Barabasi <https://networksciencebook.com/>
- ...