AA
lab2

1. a)

```
i = 0;
while (v[i] != 0) {
    if (v[i] % 2 == 0)
        printf("%d", v[i]);
    i++
}
```

COST	Nr. repetări
c_1	1
c_2	t
c_3	$t-1$
c_4	p
c_5	$t-1$

b) $T(n) = c_1 \cdot 1 + c_2 \cdot t + c_3(t-1) + c_4 p + c_5(t-1)$

1. cazul cel mai favorabil: $t=1$ și $p=0$

$\left.\begin{array}{l} t=1 \\ p=0 \end{array}\right\} \Rightarrow T(n) = c_1 + c_2 + c_3 \cdot 0 + c_4 \cdot 0 + c_5 \cdot 0 \Rightarrow$

$\Rightarrow T(n) = c_1 + c_2 = \Theta(1)$

2. cazul cel mai defavorabil: $t = n$ și $p = n-1$

$T(n) = c_1 + c_2 n + c_3(n-1) + c_4(n-1) + c_5(n-1) =$

$= n(c_2 + c_3 + c_4 + c_5) + (c_1 - c_3 - c_4 - c_5) = \Theta(n)$

3. cazul mediu : $t = (n+1)/2$, $p = (n-1)/2$

$$T(n) = c_1 + \frac{n+1}{2} c_2 + \frac{n-1}{2} c_3 + \frac{n-1}{2} c_4 + \frac{n-1}{2} c_5$$

$$= \frac{1}{2}(2c_1 + c_2 - c_3 - c_4 - c_5) + \frac{n}{2}(c_2 + c_3 + c_4 + c_5)$$

$$= \Theta\left(\frac{n}{2}\right) = \frac{1}{2}\Theta(n) = \Theta(n)$$

Operații critice:

1. cazul cel mai favorabil: 1, 2

$T(n) = c_1 + c_2 = \Theta(n)$

2. cazul cel mai defavorabil: 3, 4, 5

$$T(n) = c_3(n-1) + c_4(n-1) + c_5(n-1)$$

$$= n(c_3 + c_4 + c_5) - (c_3 + c_4 + c_5) = \Theta(n)$$

3. cazul mediu : 3, 4, 5

$t = h_1 \cdot 1 - (1-h_1)n$

$p = h_2 \cdot 0 + (1-h_2)(n-1)$, h_1 și $h_2 \in (0,1)$

$$T(n) = c_3[h_1 - (1-h_1)n - 1] + c_4[(1-h_2)(n-1)] +$$
$$+ c_5[h_1 - (1-h_1)n - 1]$$

$$T(n) = c_3(h_1 - n + h_1 n - 1) + c_4(n - h_2 n - 1 + h_2) +$$
$$+ c_5(h_1 - n + h_1 n - 1)$$

$$= n(-c_3 + c_3 h_1 + c_4 - c_4 h_2 - c_5 + c_5 h_1) +$$
$$+ (h_1 c_3 - c_3 - c_4 + c_4 h_2 + c_5 h_1 - c_5) =$$

$$= n[c_3(1-h_1) + c_4(1-h_2) + c_5(1-h_1)] \cdot$$

$+ c_3(h_1 - 1) - c_4(1 - h_2) + c_5(h_1 - 1) = \Theta(n)$

2. a) $n^2 + n = O(n^2)$

$O(g(n)) = \{ f : \mathbb{N}^* \to \mathbb{R}_+^* \mid \exists c \in \mathbb{R}_+^*, \exists n_0 \in \mathbb{N}^*$ a.î. $f(n) \le c \cdot g(n), \forall n \ge n_0 \}$.

$\exists c \in \mathbb{R}_+^*, \exists n_0 \in \mathbb{N}^*$ a.î. $n^2 + n \le c n^2$?

$c = 2 \Rightarrow n^2 + n \le n^2 \cdot 2 \Rightarrow n \le n^2 \Rightarrow n \ge 1 \Rightarrow$

$\Rightarrow n_0 = 1 \in \mathbb{N}^* \Rightarrow n^2 + n = O(n^2)$

b) $n^2 + n \ne o(n^2)$

Proprietate: $f(n) = o(g(n)) \Leftrightarrow \lim\limits_{n \to \infty} \frac{f(n)}{g(n)} = 0$

$$\lim_{n \to \infty} \frac{n^2 + n}{n^2} = \underbrace{\lim_{n \to \infty} \frac{n^2}{n^2}}_{1} + \underbrace{\lim_{n \to \infty} \frac{n}{n^2}}_{0} = 1 + 0 = 1 \ne 0 \Rightarrow$$

$\Rightarrow n^2 + n \ne o(n^2)$

c) $n^3 + n^2 = \Theta(n^3)$

~~$n^2 = \Theta(n^3 + n^2)$~~

~~$n^2 = \Theta(n^3 + n^2) \Leftrightarrow \exists c_1, c_2 \in \mathbb{R}_+^*, \exists n_0 \in \mathbb{N}^*$ a.î.~~

~~$c_1(n^3 + n^2) \le n^2 \le c_2(n^3 + n^2), \forall n \ge n_0$~~

$\exists c_1, c_2 \in \mathbb{R}_+^*, \exists n_0 \in \mathbb{N}^*$ a.î. $c_1 n^3 \le n^3 + n^2 \le c_2 n^3, \forall n \ge n_0$

$c_1 = \frac{1}{2}$ și $c_2 = 2 \Rightarrow \frac{1}{2} n^3 \le n^3 + n^2 \le 2n^3 \mid \cdot 2 \Leftrightarrow n^3 \le 2n^3 + 2n^2 \le 4n^3$

$n^3 \le 2n^3 + 2n^2 \Leftrightarrow 0 \le n^3 + 2n^2 \Rightarrow n \ge 0 \Rightarrow n_0 = 0$

$2n^3 + 2n^2 \le 4n^3 \Leftrightarrow 2n^2 \le 2n^3 \Leftrightarrow n^2 \le n^3 \Leftrightarrow 1 \le n \Rightarrow n_0 = 1$ $\Big\} \Rightarrow$

$\Rightarrow n_0 = 1 \in \mathbb{N}^* \Rightarrow \quad n^3 + n^2 = \Theta(n^3)$

d) $f(n) = O(g(n)) \Leftrightarrow g(n) = \Omega(f(n))$.

$f(n) = O(g(n)) \Leftrightarrow \exists\, c \in \mathbb{R}_+^*,\ \exists\, n_0 \in \mathbb{N}^*$ a. î. $f(n) \leq c\,g(n)$, $\forall n \geq n_0$

$$\left.\begin{array}{l} f(n) \leq c\,g(n) \mid \cdot \frac{1}{c} \Rightarrow \frac{1}{c} f(n) \leq g(n) \\ \left.\begin{array}{l} c' = \frac{1}{c} \\ c \in \mathbb{R}_+^* \end{array}\right\} \Rightarrow c' \in \mathbb{R}_+^* \end{array}\right\} \Rightarrow \underbrace{c' f(n) \leq g(n)}_{c' \in \mathbb{R}_+^*} \Big\} \Rightarrow$$

$\Rightarrow g(n) = \Omega(f(n)) \quad (1.)$

$g(n) = \Omega(f(n)) \Rightarrow \exists\, c \in \mathbb{R}_+^*,\ \exists\, n_0 \in \mathbb{N}^*$ a. î. $g(n) \geq c\,f(n)$

$$g(n) \geq c\,f(n) \mid \cdot \frac{1}{c} \Rightarrow \left.\underbrace{\frac{1}{c}}_{c' \in \mathbb{R}_+^*} g(n) \geq f(n) \right\} \Rightarrow f(n) = O(g(n)) \ (2).$$

$1 \wedge 2 \Rightarrow f(n) = O(g(n)) \Leftrightarrow g(n) = \Omega(f(n))$

e) $f(n) = \Theta(g(n)) \Leftrightarrow \begin{cases} f(n) = O(g(n)) \\ \text{și} \\ f(n) = \Omega(g(n)) \end{cases}$

"=>" $f(n) = \Theta(g(n))$

$\exists\, c_1, c_2 \in \mathbb{R}_+^*, \exists\, n_0 \in \mathbb{N}^*$ a.î. $c_1 g(n) \leq f(n) \leq c_2 g(n), \forall n \geq n_0$

$c_1 g(n) \leq f(n) \Rightarrow f(n) = \Omega(g(n))$ (1.)

$f(n) \leq c_2 g(n) \Rightarrow f(n) = O(g(n))$

"<=" $f(n) = O(g(n))$ și $f(n) = \Omega(g(n)) \Rightarrow$

$\Rightarrow \exists\, c_1 \in \mathbb{R}_+^*, \exists\, n_0 \in \mathbb{N}^*$ a.î. $f(n) \leq c_1 g(n)$

$\exists\, c_2 \in \mathbb{R}_+^*, \exists\, n_0' \in \mathbb{N}^*$ a.î. $c_2 g(n) \leq f(n)$

I. $n_0 \geq n_0' \Rightarrow \forall n \geq n_0$ atunci $\exists\, c_1, c_2 \in \mathbb{R}_+^*$ a.î.

$c_2 g(n) \leq f(n) \leq c_1 g(n) \Rightarrow$

$\Rightarrow f(n) = \Theta(n)$

II. $n_0' > n_0 \Rightarrow \forall n \geq n_0'$ atunci $\exists\, c_1, c_2 \in \mathbb{R}_+^*$ a.î.

$c_2 g(n) \leq f(n) \leq c_1 g(n) \Rightarrow$

$\Rightarrow f(n) = \Theta(n)$

$\mathrm{I} \wedge \mathrm{II} \Rightarrow f(n) = \Theta(n)$

$$f(n) = \Theta(g(n)) \Leftrightarrow \begin{cases} f(n) = \Omega(g(n)) \\ f(n) = O(g(n)) \end{cases}$$