(5) Formulele folosite:

a) $m = \int_D x \cdot f(x)\, dx$

b) $D = m_2[\xi] = E[(\xi - m)^2] = E\xi^2 - (E\xi)^2$

$D = m_2[X] = E[(X-m)^2] = EX^2 - (EX)^2$

$= \int_D x^2 \cdot f(x)\, dx - m^2.$

c) $A_s = E\left[\left(\frac{x-\mu}{\sigma}\right)^3\right] = E\left[\frac{(x-\mu)^3}{\sigma^3}\right] = \frac{1}{\sigma^3} \cdot E[(x-\mu)^3] =$

$= \frac{E[(x-\mu)^3]}{(E[(x-\mu)^2])^{3/2}} = \frac{m_3}{\sqrt{(E[(x-\mu)^2])^3}} = \frac{m_3}{\sqrt{D^3}}$ sau $\frac{m_3}{\sigma^3}$

$m_3 = E[(x-\mu)^3] = E[X^3] - 3\mu \cdot E[X^2] + 3\mu^2 E[X] - \mu^3 =$

$= E[X^3] - 3\mu\left(\underbrace{E[X^2] - \mu E[X]}_{D}\right) - \mu^3$

$= E[X^3] - 3\mu \cdot D - \mu^3.$

$\Rightarrow A_s = \frac{EX^3 - 3\mu D - \mu^3}{\sqrt{D^3}}$ sau $\frac{EX^3 - 3\mu D - \mu^3}{\sigma^3}$

5. a) Fie $f:\mathbb{R}\to\mathbb{R}$, $f(x)=\begin{cases} f_1(x), & x\in[0,1) \\ f_2(x), & x\in(1,2] \\ 0, & \text{în rest} \end{cases}$

$y = mx+n$

Se observă $f_1(x)=a$. $\begin{cases} P(0,a): & a = 0\cdot m+n \\ P(1,a): & a = 1m+n \end{cases} \Rightarrow n = a, m = 0.$

$f_2(x)$: $y = mx+n \Rightarrow \begin{cases} P(1,a): & a = 1\cdot m+n \\ P(2,0): & 0 = 2m+n \Rightarrow n = -2m \end{cases}$ $\Rightarrow m-2m = a \Rightarrow m = -a.$ $\Rightarrow n = 2a.$

$\Rightarrow f_2(x) = -ax+2a$

$f(x)=\begin{cases} a, & x\in(0,1) \\ -ax+2a, & x\in(1, \\ 0, & \text{rest.} \end{cases}$

Cond. $f(x)$ $\begin{cases} f(x)\geq 0. & \text{evident} \\ \int_0 f(x)dx = 1. \end{cases}$

$\int_0 f(x)dx = \int_0^1 a\,dx + \int_1^2 (-ax+2a)dx = a\frac{x^2}{2}\Big|_0^1 + 2a\,x\Big|_1^2 - \frac{a}{2}x^2\Big|_1^2 =$

$= \frac{a}{2} + 2a - \frac{3}{2}a \overset{\text{set}}{=} 1 \Rightarrow 2a+4a-3a = 2 \Leftrightarrow 3a = 2 \Rightarrow \boxed{a = \frac{2}{3}}$

Folosesc formulele menționate:

al m: $\int_0 x f(x)dx = \int_0^1 x\cdot\frac{2}{3}dx + \int_1^2 x\left(\frac{4}{3}-\frac{2}{3}x\right)dx =$

$= \frac{2}{3}\frac{x^2}{2}\Big|_0^1 + \frac{4}{3}\cdot\frac{x^2}{2}\Big|_1^2 - \frac{2}{3}\frac{x^3}{3}\Big|_1^2 = \frac{1}{3} + \frac{2}{3}\cdot 3 - \frac{2}{9}\cdot 7 = \frac{1}{3}+2-\frac{14}{9}$

$= \frac{7}{9}$

b) $D = \int_0 x^2 f(x)dx - m^2 = \int_0^1 x^2\cdot\frac{2}{3}dx + \int_1^2 x^2\left(\frac{4}{3}-\frac{2}{3}x\right)dx =$
$- m^2$

$$= \frac{2}{3}\frac{x^3}{3}\Big|_0^1 + \frac{4}{3}\frac{x^3}{3}\Big|_1^2 - \frac{2}{3}\frac{x^4}{4}\Big|_1^2 - \mu^2 = \frac{2}{9} + \frac{4}{9}\cdot 7 - \frac{2}{3}\cdot\frac{15}{4_2} - \mu^2 =$$

$$= \frac{30}{9} - \frac{15}{6} - \frac{49}{81} = 0{,}228 \approx \boxed{0{,}23 = D}$$

c) $$A_S = \frac{EX^3 - 3\mu D - \mu^3}{\sigma^3 \xrightarrow{} \sqrt{D^3}}$$

$$EX^3 = \int_0 x^3 f(x)\,dx = \int_0^1 x^3 \frac{2}{3}\,dx + \int_1^2 x^3\left(\frac{4}{3} - \frac{2}{3}x\right)dx =$$

$$= \frac{2}{3}\frac{x^4}{4}\Big|_0^1 + \frac{4}{3}\frac{x^4}{4}\Big|_1^2 - \frac{2}{3}\frac{x^5}{5}\Big|_1^2 = \frac{1}{6} + \frac{1}{3}\cdot 15 - \frac{2}{3}\cdot\frac{31}{5} =$$

$$= \frac{1}{6} + \frac{15}{3} - \frac{62}{15} = 1{,}0333 \approx 1{,}034.$$

$$\Rightarrow A_S = \frac{1{,}034 - 3\cdot\frac{7}{9}\cdot 0{,}23 - \frac{7\cdot 7\cdot 7}{9\cdot 9\cdot 9}}{\sqrt{(0{,}23)^3}} = \frac{0{,}0268}{0{,}0013} = 20{,}615$$

d) $$P(1 < x < 1.5) = \int_1^{1.5} f(x)\,dx = \int_1^{1.5}\left(\frac{4}{3} - \frac{2}{3}x\right)dx = \frac{4}{3}x\Big|_1^{1.5} - \frac{2}{3}\frac{x^2}{2}\Big|_1^{1.5} =$$

$$\cancel{= \frac{4}{3}\cdot\frac{1}{2} - \frac{2}{3}\cdot(2{,}25-1) = \frac{2}{3} - \frac{2}{3}\cdot 1{,}25 = \frac{2}{3}(1-}$$

$$P(1 < x < 1.5) = \int_1^{1.5} f(x)\,dx = \int_1^{1.5}\left(\frac{4}{3} - \frac{2}{3}x\right)dx = \frac{4}{3}x\Big|_1^{\frac{3}{2}} - \frac{2}{3}\cdot\frac{x^2}{2}\Big|_1^{3/2} =$$

$$= \frac{4}{3}\cdot\frac{1}{2} - \frac{1}{3}\cdot\left(\frac{9}{4} - 1\right) = \frac{2}{3} - \frac{1}{3}\cdot\frac{5}{4} = \frac{1}{4} = 25\%$$

e) $P(X>1{,}5 \mid X>1) = P_c$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P_c = \frac{P(X>1.5 \cap X>1)}{P(X>1)} =$$

$$= \frac{P(X>1.5)}{P(X>1)} = \frac{\int_{1.5}^{2} f(x)\,dx}{\int_{1}^{2} f(x)\,dx} = \frac{\int_{1.5}^{2}\left(\frac{4}{3}-\frac{2}{3}x\right)dx}{\int_{1}^{2}\left(\frac{4}{3}-\frac{2}{3}x\right)dx} =$$

$$= \frac{\frac{2}{3}\int_{1.5}^{2}(2-x)\,dx}{\frac{2}{3}\int_{1}^{2}(2-x)\,dx} = \frac{\cancel{\frac{2}{3}}\cdot\left(2x-\frac{x^2}{2}\right)\Big|_{1.5}^{2}}{\cancel{\frac{2}{3}}\left(2x-\frac{x^2}{2}\right)\Big|_{1}^{2}} = \frac{2\cdot 2-\frac{4}{2}-3+\frac{2{,}25}{2}}{4-\frac{4}{2}-2+\frac{1}{2}} =$$

$$= \frac{0{,}125}{0{,}5} = 0{,}25.$$

3. Fac echivalența cu o urnă cu 1 bilă albă și 2 negre.
Ev. scoaterii bilei albe reprezintă lovirea cu succes a țintei. (rep. Bern.)

$P(k: 1, k-1)$: probab. ca din k extrageri să obținem ~~minim~~ una albe și $k-1$.

Calc. prob. de a nu obține nicio albă.

Bernoulli: ~~media = Np~~ ~~dispersia = $\sqrt{}$~~ $X = \begin{pmatrix} 0 & 1 \\ 1-p & p \end{pmatrix}$

$$\Rightarrow X = \begin{pmatrix} 0 & 1 \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

$$\Rightarrow M(X) = 0 \cdot (1-p) + 1 \cdot p = p = \frac{1}{3}.$$

$$\sigma^2(x) = (0-p)^2 \cdot (1-p) + (1-p)^2 \cdot p = p(1-p) = \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9}.$$

Scanned with CamScanner

4. $\mu = 1500$ ore $\sigma = 200$

$n = 25$

$s = 1380$ ore.

$H_0: \mu = 1500$

$H_1: \mu < 1500$?

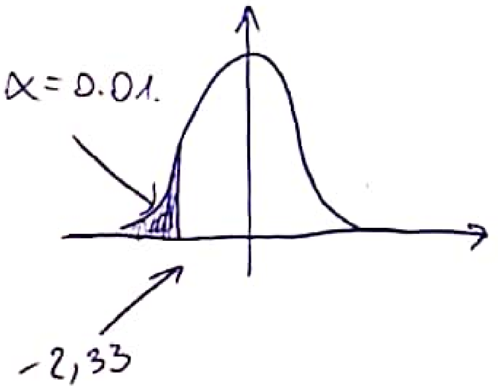
folosesc un test de medii one-tailed

$$\text{scor } z = \frac{s - \mu}{\sigma_s}; \quad \sigma_s = \frac{\sigma}{\sqrt{N}} = \frac{200}{5} = 40$$

$$\Rightarrow \text{scor } z = \frac{1380 - 1500}{40} = -3. < -2{,}33$$

→ am obținut valori mai extreme.

→ respingem ipoteza H_0 la un nivel de semnif. 0,01.



b. $\mu_1 = 1400$

$$\text{Scor } z = \frac{1400 - 1500}{40} = \frac{-100}{40} = \frac{-5}{2} = -2.5 \Rightarrow \Phi(2.5) = 0.006$$

$\Rightarrow 1 - 0{,}006$

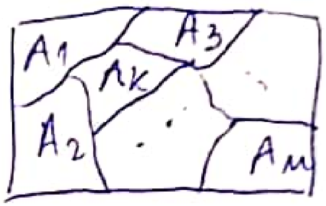
$\Rightarrow 0{,}994$

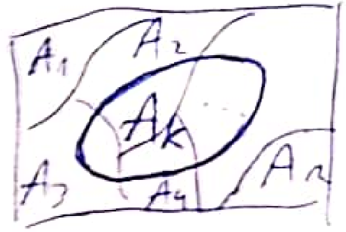
c. pentru ca $\alpha = 0.1 \Rightarrow$ scor $z = -1.28$ | val. extr. pentru semnif.

$$\Rightarrow \frac{X - 1500}{40} = -1{,}28 \Rightarrow X = -1{,}28 \cdot 40 + 1500 = 1448{,}8$$

1

$$P\left(\bigcap_{i=1}^{n} A_i\right) = \prod_{i=1}^{n} P(A_i)$$





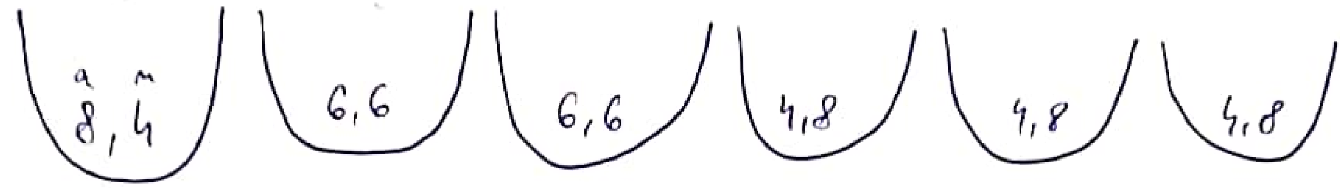
Formula valabilă în cazul în care evenimentele $A_i, i=\overline{1,n}$ sunt independente două câte două

Dacă nu,

$$P\left(\bigcap_{i=1}^{n} A_i\right) = P(A_1)\cdot P(A_2|A_1)\cdot P(A_3|A_1\cap A_2)\cdot \ldots \cdot P\left(A_n \,\middle|\, \bigcap_{i=1}^{n-1} A_i\right).$$

Justificare: Lipsa independenței semnifică o condiționare /influențare reciprocă a evenimentelor

2.



Probab căutată = 1 - Probab. de a extrage din urna 1.

$\frac{1}{6}$ = șansă alegere U_1 → $P(H_1)$

$\frac{2}{6}$ = șansă alegere U_2 → $\boxed{\text{de luat în considerare}}$ → $P(H_2)$

$\frac{3}{6}$ = șansă alegere U_3 → $P(H_3)$

Folosesc repart. hipergeometr.

~~U_1: $P(3$.~~

E - evidence că am coș 2a și 1n.

$$P(E|H_1) = \frac{C_8^2 \cdot C_4^1}{C_{12}^3} \quad P(E|H_2) = \frac{C_6^2 \cdot C_6^1}{C_{12}^3} \quad P(E|H_3) = \frac{C_4^2 \cdot C_8^1}{C_{12}^3}$$

Folosesc. $$P(H_1|E) = \frac{P(H_1) \cdot P(E|H_1)}{\sum_{i=1}^{n} P(H_i) \cdot p(E|H_i)}$$

$$\Rightarrow P(H_1|E) = \frac{C_8^2 \cdot C_4^1}{C_8^2 \cdot C_4^1 + 2 C_6^2 C_6^1 + 3 C_4^2 \cdot C_8^1} = \frac{28 \cdot 4}{28 \cdot 4 + 3 \cdot 6 \cdot 8 + 2 \cdot 15 \cdot 6} = 0,256$$

$\Rightarrow$ Probab. căutată este $1 - 0,256 = 0,744$

3. $\begin{pmatrix} 1 & 2 & 3 & \dots & n \\ \frac{1}{3} & \frac{2}{3^2} & \frac{2^2}{3^3} & \dots & \frac{2^{n-1}}{3^n} \end{pmatrix}$

$$m = \sum_{i=1}^{n} x_i p_i = \sum i \cdot \frac{2^{i-1}}{3^i} = \frac{1}{3} \sum i \left(\frac{2}{3}\right)^{i-1}$$

$$D = \sum_{i=1}^{n} (x_i - m^2) p_i$$

8. $$\sum_{i=1}^{\infty} p q^{i-1} = \frac{p}{1-q} = 1$$

$$EX = p \sum_{k=1}^{\infty} k \cdot q^{k-1} = \frac{1}{3} \sum_{1}^{\infty} k \cdot \left(\frac{2}{3}\right)^{k-1}$$

$$\frac{1}{3}\left(\frac{1}{\left(1-\frac{2}{3}\right)^2}\right) = \frac{\frac{1}{3}}{\left(\frac{1}{3}\right)^2} = 3. \quad m = 3$$

$$D = \frac{2}{p^2} = 6 \Rightarrow \sigma = \sqrt{6}.$$

Scanned with CamScanner