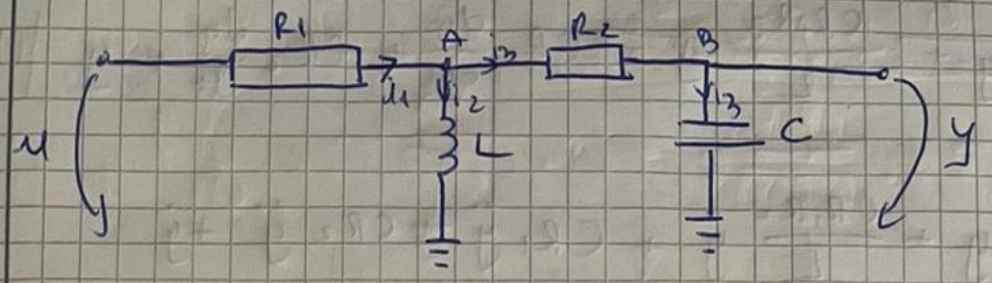


$$u_L = L \cdot \frac{di_L}{dt}$$

$$i_C = C \cdot \frac{du_C}{dt}$$

$$u_L = L \cdot \frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{u_L}{L}$$

$$i_L = \frac{1}{L}\int u_L$$

$$\left.\begin{array}{l} u = U_{R1} + U_{R2} + U_C \\ U_C = y \end{array}\right\} \Rightarrow u = U_{R1} + U_{R2} + y$$

$$\cancel{i_1 = i_2 + i_3 \Rightarrow \frac{U_{R1}}{R_1} = y + \frac{1}{L}\int u_L}$$

$$\left.\begin{array}{l} u_L = U_{R2} + U_C \Rightarrow u_L = U_{R2} + y \\ U_{R2} = i_3 \cdot R_2 \\ i_3 = i_C = C \cdot \frac{du_C}{dt} = C \cdot \frac{dy}{dt} \end{array}\right\} \Rightarrow U_{R2} = CR_2 \frac{dy}{dt} \Bigg\} \Rightarrow$$

$$u_L = y + CR_2 \frac{dy}{dt}$$

$$\cancel{U_{R1}} \quad i_1 = i_2 + i_3 \Rightarrow \frac{U_{R1}}{R_1} = i_L + i_C \Rightarrow \frac{U_{R1}}{R_1} = \frac{1}{L}\int u_L + C \cdot \frac{dy}{dt}$$

$$\frac{U_{R1}}{R_1} = \frac{1}{L}\int y + CR_2 \frac{dy}{dt} + C\frac{dy}{dt}$$

$$\frac{U_{R1}}{R_1} = \frac{1}{L}\int y + \frac{1}{L} CR_2 y + C\frac{dy}{dt} \quad \Big| \cdot R_1$$

$$U_{R1} = \frac{R_1}{L} \cdot \int y + \frac{CR_2R_1}{L} y + CR_1 \frac{dy}{dt}$$

$$u = \frac{R_1}{L}\int y + \frac{CR_1R_2}{L}\, y + CR_1 \frac{dy}{dt} + CR_2 \frac{dy}{dt} + y \quad \Big|\,()'$$

$$\dot{u} = \frac{R_1}{L}\, y + \frac{CR_1R_2}{L}\, \dot{y} + CR_1\, \ddot{y} + CR_2\, \ddot{y} + \dot{y}$$

$$\dot{u} = (CR_1 + CR_2)\ddot{y} + \dot{y}\left(\frac{CR_1R_2}{L} + 1\right) + \frac{R_1}{L}\, y \quad \Big|\, \frac{1}{CR_1+CR_2}$$

$$\frac{1}{C(R_1+R_2)} \cdot \dot{u} = \ddot{y} + \frac{\frac{CR_1R_2}{L} + 1}{C(R_1+R_2)} \cdot \dot{y} + \frac{\frac{R_1}{L}}{C(R_1+R_2)} \cdot y$$

$$\frac{1}{C(R_1+R_2)} \cdot \dot{u} = \ddot{y} + \frac{CR_1R_2 + L}{CL(R_1+R_2)} \cdot \dot{y} + \frac{R_1}{CL(R_1+R_2)} \cdot y$$

$$H(s) = \frac{\frac{1}{C(R_1+R_2)} \cdot s}{s^2 + \frac{CR_1R_2+L}{CL(R_1+R_2)} \cdot s + \frac{R_1}{CL(R_1+R_2)}}$$

$$\lim_{\omega \to 0} |H(j\omega)| = \lim_{\omega \to 0} \left| \frac{\frac{1}{C(R_1+R_2)} \cdot j\omega}{-\omega^2 + \frac{CR_1R_2+L}{CL(R_1+R_2)} \cdot j\omega + \frac{R_1}{CL(R_1+R_2)}} \right|$$

$$= 0 < \frac{\sqrt{2}}{2} \Rightarrow \text{tăiată}$$

$$\lim_{\omega\to\infty} |H(j\omega)| = \lim_{\omega\to\infty} \left| \frac{\frac{1}{C(R_1+R_2)} \cdot j\omega}{-\omega^2 + \frac{CR_1R_2+L}{CL(R_1+R_2)} j\omega + \frac{R_1}{CL(R_1+R_2)}} \right|$$

$$= 0 < \frac{\Gamma_2}{2} \Rightarrow \text{taiate}$$

$$\text{numitor} \Leftrightarrow s^2 + \frac{CR_1R_2+L}{CL(R_1+R_2)} s + \frac{R_1}{CL(R_1+R_2)}$$

$$s^2 + 2\xi\omega_n s + \omega_n^2$$

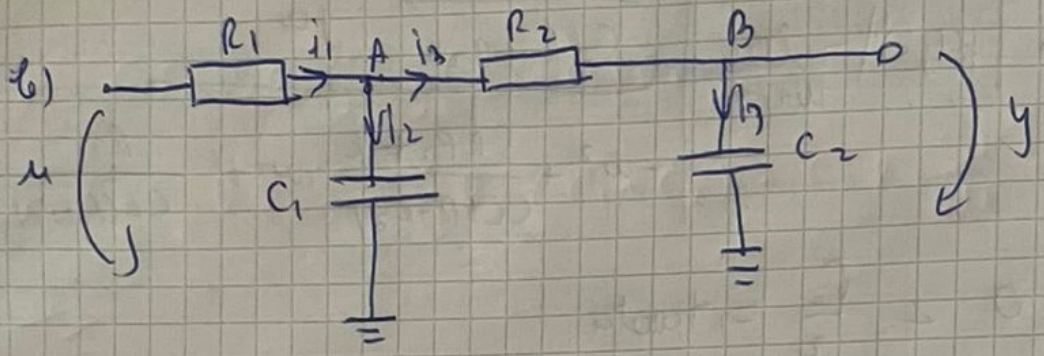
$$\omega_n = \sqrt{\frac{R_1}{CL(R_1+R_2)}} = \omega_0$$

$$\lim_{\omega\to\omega_0} |H(j\omega)| = \lim_{\omega\to\omega_0} \left| \frac{\frac{1}{\cancel{C(R_1+R_2)}} \cdot \cancel{j\omega}}{\frac{CR_1R_2+L}{\cancel{CL(R_1+R_2)}} \cdot \cancel{j\omega}} \right| = \lim_{\omega\to\omega_0}$$

$$\cancel{= \frac{L}{CR_1R_2+L}} \quad \frac{L}{CR_1R_2+L}$$

?

b)



$u_{C2} = y$

$u = U_{R1} + U_{R2} + y$

$u_{C1} = U_{R2} + y$

$i_1 = i_2 + i_3 \Rightarrow \frac{U_{R1}}{R_1} = i_{C1} + i_{C2}$

$\Rightarrow \frac{U_{R1}}{R_1} = C_1 \cdot \frac{du_{C1}}{dt} + C_2 \cdot \frac{du_{C2}}{dt}$

$\frac{U_{R1}}{R_1} = C_1 \cdot \frac{du_{C1}}{dt} + C_2 \cdot \frac{dy}{dt}$

$U_{R1} = u - V_A$

$u_{C1} = V_A - 0 \Rightarrow V_A = U_{C1}$

$\Rightarrow u = U_{R1} + U_{C1}$

$\frac{U_{R2}}{R_2} = C_2 \cdot \frac{du_{C2}}{dt} \Rightarrow U_{R2} = C_2 R_2 \cdot \frac{dy}{dt}$

$$U_{C1} = U_{R2} + U_{C2} \qquad u =$$

$$U_{C1} = C_2 R_2 \cdot \frac{dy}{dt} + y$$

$$\cancel{u = U_{R1} + U_{C1} \Rightarrow U_{R1} = u - U_{C1} = u - \left(C_2 R_2 \frac{dy}{dt} + y\right)}$$

$$\cancel{u = u - \left(C_2 R_2 \frac{dy}{dt} + y\right) + C_2 R_2 \frac{dy}{dt} + y}$$

$$i_1 = i_2 + i_3 \Rightarrow U_{R1} = i_{C1} + i_{C2}$$

$$\frac{U_{R1}}{R_1} = C_1 \cdot \frac{dU_{C1}}{dt} + C_2 \cdot \frac{dy}{dt}$$

$$\frac{U_{R1}}{R_1} = C_1 \cdot \frac{d\left(C_2 R_2 \cdot \frac{dy}{dt} + y\right)}{dt} + C_2 \frac{dy}{dt}$$

$$\frac{U_{R1}}{R_1} = C_1 \cdot \left(C_2 R_2 \ddot{y} + \dot{y}\right) + C_2 \dot{y}$$

$$\frac{U_{R1}}{R_1} = C_1 C_2 R_2 \ddot{y} + C_1 \dot{y} + C_2 \dot{y}$$

$$\frac{U_{R1}}{R_1} = C_1 C_2 R_2 \ddot{y} + \dot{y}(C_1 + C_2)$$

$$\cancel{u = C_1 C_2 R_2 \ddot{y} + \dot{y}(C_1 + C_2) + C_2 R_2 \dot{y} + y}$$

$$\cancel{u = C_1 C_2 R_2 \ddot{y} + \dot{y}(C_1 + C_2 + C_2 R_2) + y \quad \Big| \frac{1}{C_1 C_2 R_2}}$$

$$\cancel{\frac{1}{C_1 C_2 R_2} u = \ddot{y} + \frac{C_1 + C_2 + C_2 R_2}{C_1 C_2 R_2} \dot{y} + \frac{1}{C_1 C_2 R_2} y}$$

$$V_{R_1} = C_1 C_2 R_1 R_2 \ddot{y} + \dot{y}(C_1 + C_2) R_1$$

$$u = C_1 C_2 R_1 R_2 \ddot{y} + \dot{y} R_1 (C_1 + C_2) + C_2 R_2 \frac{dy}{dt} + y \quad \Big| \frac{1}{C_1 C_2 R_1 R_2}$$

$$\cancel{u =}$$

$$\frac{1}{C_1 C_2 R_1 R_2} u = \ddot{y} + \dot{y} \frac{\cancel{R_1}(C_1 + C_2)}{C_1 C_2 \cancel{R_1} R_2} + \dot{y} \frac{\cancel{C_2 R_2}}{C_1 \cancel{C_2} R_1 \cancel{R_2}} + \frac{1}{C_1 C_2 R_1 R_2}$$

$$\frac{1}{C_1 C_2 R_1 R_2} u = \ddot{y} + \dot{y} \frac{C_1 + C_2}{C_1 C_2 R_2} + \dot{y} \frac{1}{C_1 R_1} + \frac{1}{C_1 C_2 R_1 R_2}$$

$$\frac{1}{C_1 C_2 R_1 R_2} u = \ddot{y} + \dot{y} \left(\frac{C_1 + C_2}{C_1 C_2 R_2} + \frac{1}{C_1 R_1} \right) + y \cdot \frac{1}{C_1 C_2 R_1 R_2}$$

$$H(s) = \frac{\frac{1}{C_1 C_2 R_1 R_2}}{s^2 + \left(\frac{C_1 + C_2}{C_1 C_2 R_2} + \frac{1}{C_1 R_1} \right) s + \frac{1}{C_1 C_2 R_1 R_2}}$$

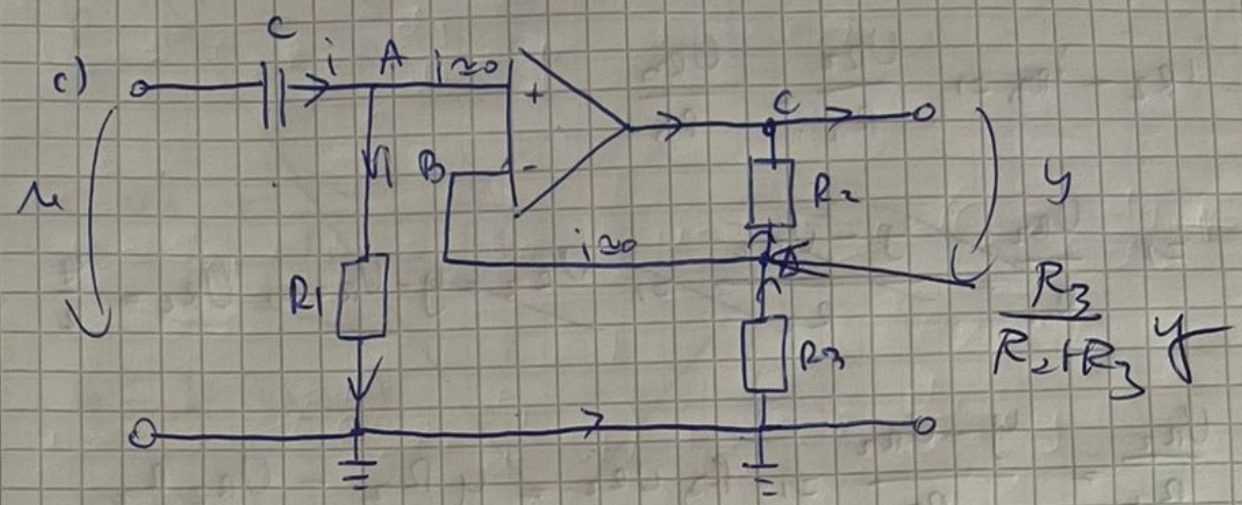
$$\lim_{\omega \to 0} |H(j\omega)| = 1 > \frac{\sqrt{2}}{2}$$

$$\Rightarrow \text{FTJ}$$

$$\lim_{\omega \to \infty} |H(j\omega)| = 0 < \frac{\sqrt{2}}{2}$$

$$\cancel{\omega_n^2 = \frac{1}{C_1 C_2 R_1 R_2}} \Rightarrow \cancel{\omega_n = \sqrt{\frac{1}{C_1 C_2 R_1 R_2}}}$$

c)



$V_C = K(V_A - V_B) \Rightarrow \cancel{y = K(V_A}$ $V_A - V_B = 0 \Rightarrow V_A = V_B$

$V_A - 0 = V_{R1} \Rightarrow V_{R1} = V_A$

$\left.\begin{array}{l} V_A = V_{R1} \\ V_B = V_{R3} \end{array}\right\} \Rightarrow V_{R1} = V_{R3}$

$u - V_A = V_C \Rightarrow V_{R1} = u - V_C$

$V_C = y$

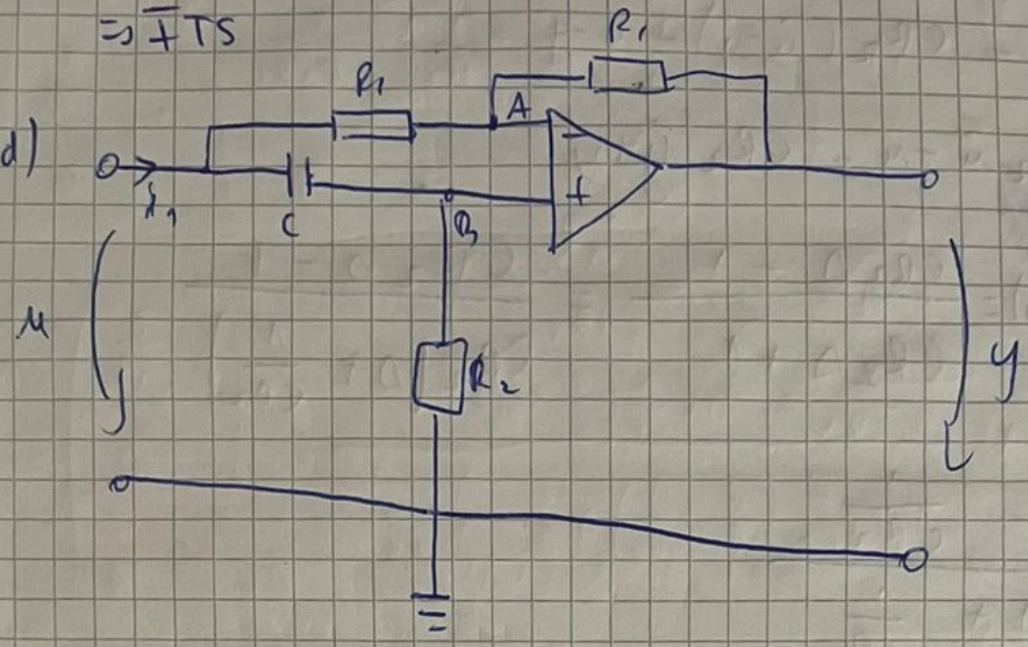
$\left.\begin{array}{l} y - V_B = V_{R2} \\ V_B = V_D \\ V_D - V_{R3} = 0 \Rightarrow V_{R3} = V_D \end{array}\right\} \Rightarrow y - V_{R3} = V_{R2}$

$y - V_{R1} = V_{R2} \Rightarrow V_{R1} = y - V_{R2}$

$\Rightarrow u - V_C = y - V_{R2}$

$\Rightarrow \bar{+}TS$

1d)



$u = u_C + U_{R_2}$

$V_A = V_B$

$\left.\begin{array}{l} u = u_C + U_{R_2} \\ u = U_{R_2} + U_{R_1} \end{array}\right\} \Rightarrow \boxed{u_C = U_{R_1}}$

$\left.\begin{array}{l} U_{R_1} = u - V_A \Rightarrow V_A = u - U_{R_1} \\ U_{R_2} = V_B - 0 \Rightarrow V_B = U_{R_2} \end{array}\right\} \Rightarrow$

$\Rightarrow U_{R_2} = u - U_{R_1}$

$U_{R_1} = V_A - y \Rightarrow V_A = U_{R_1} + y$

$V_A = u - U_{R_1}$

$\Rightarrow 2U_{R_1} = u - y$

$\left.\begin{array}{l} \Rightarrow \dfrac{u-y}{2} = U_{R_1} \\ U_{R_1} = u_C \end{array}\right\} \Rightarrow u_C = \dfrac{u-y}{2}$

$i_C = i_{R_2} \Rightarrow C \cdot \dfrac{du_C}{dt} = \dfrac{U_{R_2}}{R_2} \Rightarrow U_{R_2} = CR_2 \cdot \dot{u}_C$

U_{R_2}

$U_{R_2} = CR_2 \cdot \frac{1}{2}(\dot{u} - \dot{y})$

$$\Rightarrow u = CR_2 \frac{1}{2}(\dot{u}-\dot{y}) + \frac{u-y}{2}$$

$$2u = CR_2(\dot{u}-\dot{y}) + u - y$$

$$CR_2\dot{u} - u = CR_2\dot{y} + y$$

$$H(s) = \frac{CR_2 s - 1}{CR_2 s + 1} = \frac{CR_2 s - 1}{CR_2\left(s + \frac{1}{CR_2}\right)}$$

$$= \frac{s - \frac{1}{CR_2}}{s + \frac{1}{CR_2}}$$

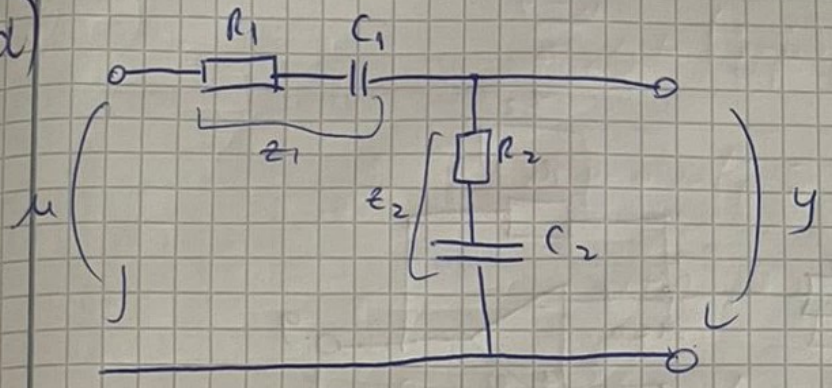
$$\lim_{\omega \to 0} |H(j\omega)| = 1 > \frac{\sqrt{2}}{2} \Rightarrow \text{true}$$

$$\cancel{\lim_{\omega \to 0} |H(j\omega)| = 1 > \frac{\sqrt{2}}{2} \Rightarrow \text{true}}$$

$$\cancel{\omega_n = \sqrt{\frac{1}{CR_2}}}$$

$$\lim_{\omega \to \infty} |H(j\omega)| = 1 > \frac{\sqrt{2}}{2} \Rightarrow \text{true}$$

2/a)



$$u = Z_1 + Z_2$$

$$y = Z_2$$

$$Z_1 = Z_{R_1} + Z_{C_1} = R_1 + \frac{1}{sC_1}$$

$$Z_2 = Z_{R_2} + Z_{C_2} = R_2 + \frac{1}{sC_2}$$

$$\frac{y}{u} = \frac{Z_2}{Z_1 + Z_2} = \frac{R_2 + \frac{1}{sC_2}}{R_1 + \frac{1}{sC_1} + R_2 + \frac{1}{sC_2}} =$$

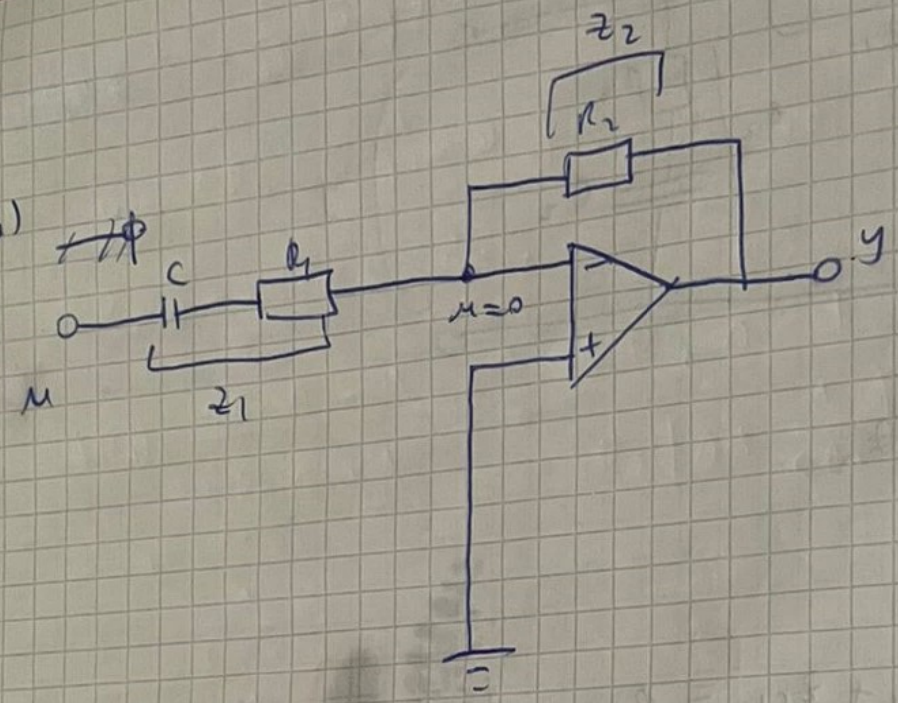
$$= \frac{R_2 + \frac{1}{sC_2}}{R_1 + R_2 + \frac{1}{sC_1} + \frac{1}{sC_2}} = \frac{R_2 + \frac{1}{sC_2}}{R_1 + R_2 + \frac{sC_2 + sC_1}{s^2 C_1 C_2}}$$

$$= \frac{\frac{sC_2R_2 + 1}{sC_2}}{\frac{s^2C_1C_2(R_1+R_2) + s(C_2+C_1)}{s^2C_1C_2}} = \frac{(sC_2R_2+1)\, s^{\cancel{2}} C_1 \cancel{C_2}}{(s^2C_1C_2(R_1+R_2) + s(C_2+C_1))\cancel{sC_2}}$$

$$= \frac{(sC_2R_2+1)\, \cancel{s} C_1}{s^{\cancel{2}}C_1C_2(R_1+R_2) + \cancel{s}(C_2+C_1)} = \frac{sC_1C_2R_2 + C_1}{sC_1C_2(R_1+R_1) + C_2 + C_1}$$

ordin 1

b)



$$I_{Z_1} = I_{Z_2} \Rightarrow \frac{u-0}{Z_1} = \frac{0-y}{Z_2} \Rightarrow \frac{u}{Z_1} = \frac{-y}{Z_2}$$

$$\Rightarrow \frac{y}{u} = -\frac{Z_2}{Z_1}$$

$$Z_1 = Z_C + Z_{R_1} = \frac{1}{sC} + R_1$$

$$Z_2 = R_2$$

$$\Rightarrow \frac{y}{u} = -\frac{R_2}{\frac{1}{sC} + R_1} = \frac{-R_2}{\frac{sCR_1+1}{sC}}$$

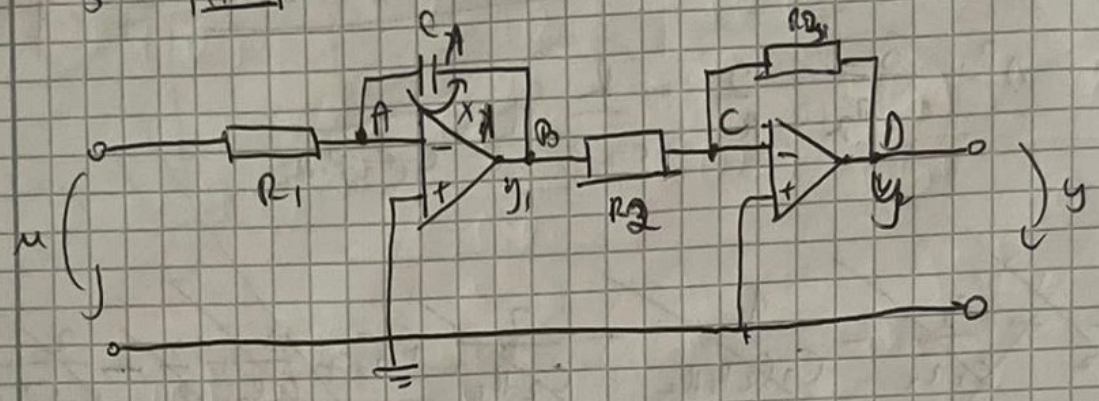
$$= \frac{-sCR_2}{sCR_1+1}$$

(ordin 1)

$$\lim_{\omega \to 0} |H(j\omega)| = 0 < \frac{\sqrt{2}}{2} \Rightarrow \text{tăietă (joase)}$$

$$\lim_{\omega \to \infty} |H(j\omega)| = \frac{R_2}{R_1} < \frac{\sqrt{2}}{2} \Rightarrow \text{tăietă (înalte)}$$

5.2. $\boxed{\text{Ex 2}}$



$$\left.\begin{array}{l} y_1 = k(V_+ - V_-) \\ V_+ = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} y_1 = k(0 - V_-) \\ V_- = V_A \end{array}\right\} \Rightarrow V_A = 0$$

$$V_A - y_1 = x_1 \Rightarrow 0 - y_1 = x_1 \Rightarrow y_1 = -x_1$$

$$y = k(V_+ - V_-)$$

$$\left.\begin{array}{l} V_- = V_C \\ V_+ = 0 \end{array}\right\} \Rightarrow V_C = 0$$

$$V_B - V_C = U_{R3} \Rightarrow -x_1 - 0 = U_{R3} \Rightarrow U_{R3} = -x_1$$

$$\cancel{V_C - V_D} \quad V_C - y = U_{R4} \Rightarrow y = -U_{R4}$$

$$\frac{U_{R3}}{R_3} = \frac{U_{R4}}{R_4} \Rightarrow U_{R4} = \frac{U_{R3} \cdot R_4}{R_3} = \frac{-x_1 \cdot R_4}{R_3}$$

$$\boxed{y = x_1 \cdot \frac{R_3}{R_2}}$$

$$u - V_A = U_{R1} \Rightarrow U_{R1} = u$$

$$\frac{U_{R1}}{R_1} = C_2 \cdot \dot{x}_1 \Rightarrow \dot{x}_1 = \frac{U_{R1}}{R_1 C_2} = \boxed{\frac{1}{C_2 R_1} \cdot u = \dot{x}_1}$$

$$\cancel{A = \frac{R_4}{R_3}} \qquad \cancel{B = 0}$$

$$\cancel{C = \frac{1}{C_2 R_1}} \qquad \cancel{D =}$$

$$A = 0 \qquad B = \frac{1}{C_2 R_1}$$

$$C = \frac{R_3}{R_2} \qquad D = 0$$

$$\cancel{H(s) = C \cdot (sI - A)^{-1} \cdot B + D}$$

$$\cancel{= \frac{R_4}{R_3} \cdot s \cdot \frac{1}{C_2 R_1} = \frac{s R_4}{C_2 R_3 R_1}}$$

$$y = \frac{R_3}{R_2} \cdot x_1 \Rightarrow \dot{y} = \frac{R_3}{R_2} \cdot \frac{u}{C_1 R_1}$$

$$\Rightarrow s \cdot Y(s) = \frac{R_3}{R_2 C_1 R_1} \cdot U(s) \Rightarrow \frac{Y(s)}{U(s)} = \frac{R_3}{s C_1 R_1 R_2}$$

$$\Rightarrow \boxed{H(s) = \frac{R_3}{R_1 R_2 C_1 s}}$$

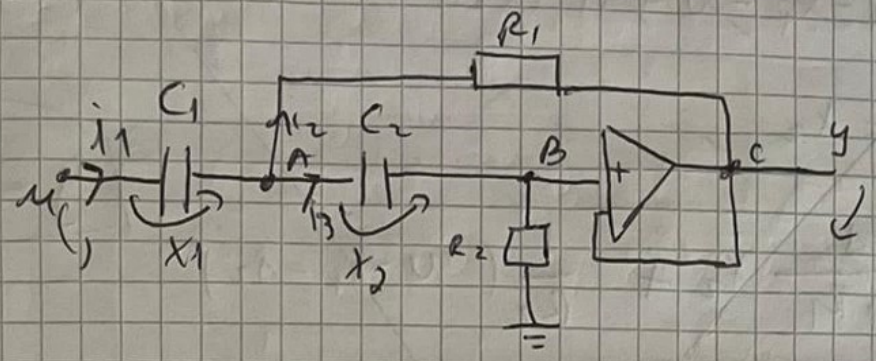
$$\lim_{\omega \to 0} |H(j\omega)| = \infty \Rightarrow \text{trec}$$

$$\Rightarrow \boxed{\text{FTJ}}$$

$$\lim_{\omega \to \infty} |H(j\omega)| = 0 \Rightarrow \text{nu trec}$$

Temă 2 (Laborator 4)

5.1)



$V_C = y$

$y = k(V_+ - V_-)$

$V_+ = V_B$

$V_- = y$

$u - V_A = x_1 \Rightarrow V_A = u - x_1$

$V_A - V_B = x_2 \Rightarrow u - x_1 - V_B = x_2 \Rightarrow V_B = u - x_1 - x_2$

$V_B - 0 = U_{R_2} \Rightarrow U_{R_2} = u - x_1 - x_2$

$\cancel{U_{R_1} = V_A - y = u - x_1 - y}$

$V_+ = V_- \Rightarrow y = V_B \Rightarrow \boxed{y = u - x_1 - x_2}$

$I_A: i_1 = i_2 + i_3 \Rightarrow C_1 \cdot \dot{x}_1 = \frac{U_{R_1}}{R_1} + C_2 \cdot \dot{x}_2$

$C_2 \dot{x}_2 = \frac{U_{R_2}}{R_2} \Rightarrow \boxed{\dot{x}_2 = \frac{1}{C_2 R_2} \cdot (u - x_1 - x_2)}$

$\cancel{\dot{x}_1 = \frac{U_{R_1}}{C_1 R_1} + \frac{1}{R_2} \cdot (u - x_1 - x_2)}$

Continuare Ex1

$$\dot{x}_1 = \frac{U_{R1}}{C_1R_1} + \frac{\cancel{C_2}}{C_1} \cdot \frac{1}{\cancel{C_2}R_2}(u - x_1 - x_2)$$

$$y = u - x_1 - x_2$$

$$\dot{x}_1 = \frac{U_{R1}}{C_1R_1} + \frac{1}{C_1R_2}(u - x_1 - x_2)$$

$$\dot{x}_2 = \frac{1}{C_2R_2}(u - x_1 - x_2)$$

$$U_A - y = U_{R1} \Rightarrow U_{R1} = u - x_1 - (u - x_1 - x_2) = x_2$$

$$\dot{x}_1 = \frac{1}{C_1R_1}x_2 + \frac{1}{C_1R_2}(u - x_1 - x_2)$$

$$= \frac{1}{C_1R_2}u - \frac{1}{C_1R_2}x_1 + x_2\left(\frac{1}{C_1R_1} - \frac{1}{C_1R_2}\right)$$

$$A = \begin{pmatrix} -\frac{1}{C_1R_2} & \frac{1}{C_1R_1} - \frac{1}{C_1R_2} \\ -\frac{1}{C_2R_2} & -\frac{1}{C_2R_2} \end{pmatrix} \qquad B = \begin{pmatrix} \frac{1}{C_1R_2} \\ \frac{1}{C_2R_2} \end{pmatrix}$$

$$C = \begin{pmatrix} -1 & -1 \end{pmatrix} \qquad D = [1]$$

$$H(s) = C(sI - A)^{-1} \cdot B + D$$

$$H(s) = (-1 \; -1) \begin{pmatrix} s + \frac{1}{C_1 R_2} & \frac{1}{C_1}\left(\frac{1}{R_2} - \frac{1}{R_1}\right) \\ \frac{1}{C_2 R_2} & s + \frac{1}{C_2 R_2} \end{pmatrix} \begin{pmatrix} \frac{1}{C_1 R_2} \\ \frac{1}{R_2 C_2} \end{pmatrix} + 1$$

$$= (-1 \; -1) \cdot \frac{1}{s^2 + \left(\frac{1}{C_1 R_2} + \frac{1}{C_2 R_2}\right) + \frac{1}{C_1 C_2 R_2^2} - \frac{1}{C_1 C_2 R_2}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)}$$

$$\begin{pmatrix} s + \frac{1}{C_2 R_2} & \frac{1}{C_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \\ -\frac{1}{C_2 R_2} & s + \frac{1}{C_1 R_2} \end{pmatrix}$$

$$\Rightarrow H(s) = \frac{1}{s^2 + \frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \cdot s + \frac{1}{C_1 C_2 R_1 R_2}} \begin{pmatrix} -s & -s - \frac{1}{C_1 R_1} \end{pmatrix} \begin{pmatrix} \frac{1}{C_1 R_2} \\ \frac{1}{R_2 C_2} \end{pmatrix} + 1$$

$$\Rightarrow H(s) = \frac{s^2}{s^2 + \frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \cdot s + \frac{1}{C_1 C_2 R_1 R_2}}$$

$\lim_{\omega \to 0} |H(j\omega)| = 0 \Rightarrow$ taiate

$\lim_{\omega \to \infty} |H(j\omega)| = 1 \Rightarrow$ trece

$\Rightarrow \boxed{FTS}$

5.3) $\boxed{a)}$ $u(t) = t + t^2$

$$y(t) = \frac{1}{2}t + \frac{1}{s}t^2 + \frac{1}{7}e^{-2t}$$

$$H(s) = \frac{\mathcal{L}\{y(t)\}(s)}{\mathcal{L}\{u(t)\}(s)}$$

$$\mathcal{L}\{y(t)\}(s) = \frac{1}{2}\cdot\frac{1}{s^2} + \frac{1}{s}\cdot\frac{2}{s^3} + \frac{1}{7}\cdot\frac{1}{s+2}$$

$$= \frac{1}{2s^2} + \frac{2}{s^4} + \frac{1}{7(s+2)}$$

$$\mathcal{L}\{u(t)\}(s) = \frac{1}{s^2} + \frac{2}{s^3}$$

$$H(s) = \frac{\frac{1}{2s^2} + \frac{2}{s^4} + \frac{1}{7(s+2)}}{\frac{1}{s^2} + \frac{2}{s^3}} =$$

$$= \frac{\frac{7(s+2)s^4 + 2\cdot 2s^2\cdot 7(s+2) + 2\cdot s^6}{14s^6(s+2)}}{\frac{s^3+2s^2}{s^5}}$$

$$= \frac{(7s^5 + 14s^4 + 28s^2(s+2) + 2s^6)\cdot \cancel{s^5}}{14s^6(s+2)(s^3+2s^2)}$$

$$= \frac{7s^5 + 14s^4 + 28s^3 + 54s^2 + 2s^6}{14s(s+2)(s^3+2s^2)} = \frac{2s^5 + 7s^4 + 14s^3 + 28s^2 + 54s}{14(s+2)\cdot s(s^2+2s)}$$

$$= \frac{2s^4+7s^3+14s^2+28s+54}{14s(s+2)(s+2)}$$

$$= \frac{2s^4+7s^3+14s^2+28s+54}{14s(s+2)^2}$$

$$\hat{s}_1 = 0$$

$$\hat{s}_{2,3} = -2$$

$$\overset{\circ}{s}_{1,2} = -2{,}1593 \pm 1{,}2118j$$

$$\overset{\circ}{z}_{3,4} = 0{,}4093 \pm 2{,}0583j$$

$\boxed{6)}$ $u(t) = e^{-t} + \sin t$

$$y(t) = \frac{1}{2}e^{-t}\sin t + 3e^{-t} + 5\sin t$$

$$\mathcal{L}\{y(t)\}(s) = \frac{1}{2}\cdot\frac{1}{(s+1)^2+1} + 3\cdot\frac{1}{s+1} + 5\cdot\frac{1}{s^2+1}$$

$$\mathcal{L}\{u(t)\}(s) = \frac{1}{s+1} + \frac{1}{s^2+1}$$

$$H(s) = \frac{\frac{1}{2(s+1)^2+2} + \frac{3}{s+1} + \frac{5}{s^2+1}}{\frac{1}{s+1} + \frac{1}{s^2+1}}$$

$$= \frac{(s+1)(s^2+1)+3(s+1)[2(s+1)^2+2]+5(s+1)[2(s+1)^2+2]}{\cancel{(s+1)}\cancel{(s+1)}[2(s+1)^2+2]}$$

$$\frac{s^2+1+s+1}{\cancel{(s+1)}\cancel{(s+1)}}$$

$$= \frac{\cancel{s^3+2}\, s^3+s+s^2+1+3(s^2+1)(2s^2+4s+4)+5(s+1)(2s^2+4s+4)}{(s^2+s+2)(2s^2+4s+4)}$$

$$= \frac{s^3+\boxed{s^2}+s+1+6s^4+12s^3+\boxed{12s^3}+\boxed{6s^2}+12s+12+10s^3+\boxed{20s^2}+20s+\boxed{10s^2}+20}{2(s^2+s+2)(s^2+2s+2)}$$

$$= \frac{6s^4+23s^3+49s^2+43s+33}{2(s^2+s+2)(s^2+2s+2)}$$

$$s^{\circ}_{1,2} = -1{,}5370 \pm 1{,}5124j$$

$$s^{\circ}_{3,4} = -0{,}3796 \pm 1{,}0192j$$

$$s^{\wedge}_{1,2} = -\frac{1}{2} \pm 1{,}3229j$$

$$s^{\wedge}_{3,4} = -1 \pm j$$

$\boxed{c)}$ $u(t) = 3t + 1$

$$y(t) = 7t^2 + 8t + 9 + 10e^{-5t}$$

$$\mathcal{L}\{y(t)\}(s) = 7 \cdot \frac{2}{s^3} + 8 \cdot \frac{1}{s^2} + \frac{9}{s} + 10 \cdot \frac{1}{s+5}$$

$$\mathcal{L}\{u(t)\}(s) = 3 \cdot \frac{1}{s^2} + \frac{1}{s}$$

$$H(s) = \frac{\frac{14}{s^3} + \frac{8}{s^2} + \frac{9}{s} + \frac{10}{s+5}}{\frac{3}{s^2} + \frac{1}{s}}$$

$$= \frac{\frac{14}{s^3} + \frac{\cancel{s}\,8}{s^2} + \frac{\cancel{s^2}\,9}{s}}{\frac{3}{s^2} + \frac{\cancel{s}\,1}{s}} + \frac{\frac{10}{s+5}}{\frac{3}{s^2} + \frac{1}{s}}$$

$$= \frac{\frac{14 + 8s + 9s^2}{s^3}}{\frac{3+s}{s^2}} + \frac{\frac{10}{s+5}}{\frac{3+s}{s^2}} =$$

$$= \frac{\cancel{s^2}(14 + 8s + 9s^2)}{s^{\cancel{3}}(3+s)} + \frac{10s^2}{(3+s)(s+5)}$$

$$= \frac{\cancel{s+5}\;14 + 8s + 9s^2}{s(3+s)} + \frac{\cancel{s}\;10s^2}{(3+s)(s+5)} =$$

$$= \frac{(s+5)(14+8s+9s^2)+10s^3}{s(3+s)(5+s)}$$

$$= \frac{14s+8s^2+9s^3+70+40s+45s^2+10s^3}{s(3+s)(5+s)}$$

$$= \frac{19s^3+53s^2+54s+70}{s(3+s)(5+s)}$$

$$s_{1,2}^{0} = -0{,}2679 \pm 1{,}25j$$

$$s_3^{0} = -2{,}2537+j$$

$$\hat{s}_1 = 0,\ \hat{s}_2 = -3,\ \hat{s}_3 = -5$$

$\boxed{d)}$ $u(t) = 100t + 5e^{-t}\sin t$

$$y(t) = 200t + 10e^{-t}\cos\left(t+\frac{\pi}{4}\right)$$

$$y(t) = 200t + 10e^{-t}\cos\left(t+\frac{\pi}{4}\right)$$

$$10e^{-t}\cos\left(t+\frac{\pi}{4}\right) = 10e^{-t}\cdot\left[\cos t\cdot\cos\frac{\pi}{4} - \sin t\sin\frac{\pi}{4}\right]$$

$$= 10e^{-t}\left(\cos t\cdot\frac{\sqrt{2}}{2} - \sin t\,\frac{\sqrt{2}}{2}\right)$$

$$= 5\sqrt{2}e^{-t}\cos t - 5\sqrt{2}e^{-t}\sin t$$

$$= 5\sqrt{2}\left(e^{-t}\cos t - e^{-t}\sin t\right) \Rightarrow y(t) = 200t + 5\sqrt{2}\left(e^{-t}\cos t - e^{-t}\sin t\right)$$

$$\mathcal{L}\{y(t)\}(s) = 200 \cdot \frac{1}{s^2} + 5\sqrt{2}\left(\frac{s+1}{(s+1)^2+1} + \frac{1}{(s+1)^2+1}\right)$$

$$= \frac{200}{s^2} + 5\sqrt{2} \cdot \frac{s+2}{(s+1)^2+1}$$

$$\mathcal{L}\{u(t)\}(s) = \frac{100}{s^2} + 5 \cdot \frac{1}{(s+1)^2+1}$$

$$H(s) = \frac{\frac{200}{s^2} + 5\sqrt{2} \cdot \frac{s+2}{(s+1)^2+1}}{\frac{100}{s^2} + \frac{5}{(s+1)^2+1}} = \frac{200[(s+1)^2+1] + 5\sqrt{2} \cdot s^2 (s+2)}{100[(s+1)^2+1] + 5s^2}$$

$$= \frac{200s^2 + 400s + 400 + 5\sqrt{2}s^3 + 10\sqrt{2}s^2}{100s^2 + 200s + 200 + 5s^2}$$

$$= \frac{5\sqrt{2}s^3 + s^2(10\sqrt{2} + 200) + 400s + 400}{105s^2 + 200s + 200}$$

$$s_{1,2}^{0} = -0{,}9622 \pm 1{,}0339j$$

$$s_3^{0} = -28{,}3599 + j$$

$$\theta_{1,2}^{\wedge} = -0{,}9524 \pm 0{,}9989j$$

$\boxed{e)}$ $u(t) = 1$

$$y(t) = 1 + e^{-5t} + e^{-7t} + e^{-9t}$$

$$\mathcal{L}\{y(t)\}(s) = \frac{1}{s} + \frac{1}{s+1} + \frac{1}{s+7} + \frac{1}{s+9}$$

$$\mathcal{L}\{u(t)\}(s) = \frac{1}{s}$$

$$H(s) = \frac{\frac{1}{s} + \frac{1}{s+5} + \frac{1}{s+7} + \frac{1}{s+9}}{\frac{1}{s}}$$

$$= 1 + \frac{s}{s+5} + \frac{s}{s+7} + \frac{s}{s+9} = \frac{(s+5)(s+7)(s+9) + (s+7)(s+9) + (s+5)(s+9) + (s+5)(s+7)}{(s+5)(s+7)(s+9)}$$

$$= \frac{(s+6)(s+7)(s+9) + (s+5)(2s+16)}{(s+5)(s+7)(s+9)}$$

$$= \frac{(s^2+13s+42)(s+9) + 2s^2+16s+10s+80}{(s+5)(s+7)(s+9)}$$

$$= \frac{s^3 + 5s^2 + 13s^2 + 117s + 42s + 378 + 2s^2 + 16s + 10s + 80}{(s+5)(s+7)(s+9)}$$

$$= \frac{s^3 + 24s^2 + 115s + 458}{(s+5)(s+7)(s+9)}$$

$s_1^0 = -10{,}7785$, $s_2^0 = -7{,}7108$, $s_3^0 = -5{,}5107$

$s_1^\infty = -5$, $s_2^\infty = -7$, $s_3^\infty = -9$

$\boxed{f)}$ $u(t) = e^{t} \sin t$

$y(t) = 0{,}5 e^{t} \sin t + 0{,}75 e^{-t} \sin t$

$$\mathcal{L}\{y(t)\}(s) = \cancel{\frac{1}{2} \cdot \frac{1}{(s+1)^2+1}} + \cancel{\frac{3}{4}} \cdot \frac{1}{2} \cdot \frac{1}{(s-1)^2+1} + \frac{3}{4} \cdot \frac{1}{(s+1)^2+1}$$

$$\mathcal{L}\{u(t)\}(s) = \frac{1}{(s-1)^2+1}$$

$$H(s) = \frac{\frac{1}{2(s-1)^2+2} + \frac{3}{4(s+1)^2+4}}{\frac{1}{(s-1)^2+1}}$$

$$= \frac{(s-1)^2+1}{2[(s-1)^2+1]} + \frac{3(s-1)^2+3}{4(s+1)^2+4}$$

$$= \frac{1}{2} + \frac{3s^2-6s+6}{\cancel{4(s+1)^2}\ 4s^2+8s+8}$$

$$= \frac{4s^2+8s+8+6s^2-12s+12}{8s^2+16s+16}$$

$$= \frac{10s^2-4s+20}{8s^2+16s+16} = \frac{5s^2-2s+10}{4s^2+8s+8}$$

$$\overset{\circ}{s}_{1,2} = 0{,}2 \pm 1{,}4j$$

$$\hat{s}_{1,2} = -1 \pm j$$

$\boxed{g)}$ ~~$u(t) = \delta(t) + 5$~~

~~$y(t)$~~ Exemplu Clasă (Laborator)

!

$\boxed{h}$ $u(t) = \sin t + \cos t$

$$y(t) = \frac{1}{2}t + 10\sin t + 7e^{-2t} - 5e^{-10t}$$

$$\mathcal{L}\{y(t)\}(s) = \frac{1}{2}\cdot\frac{1}{s^2} + 10\cdot\frac{1}{s^2+1} + 7\cdot\frac{1}{s+2} - 5\cdot\frac{1}{s+10}$$

$$= \frac{1}{2s^2} + \frac{10}{s^2+1} + \frac{7}{s+1} - \frac{5}{s+10}$$

$$\mathcal{L}\{u(t)\}(s) = \frac{1}{s^2+1} + \frac{s}{s^2+1} = \frac{s+1}{s^2+1}$$

$$H(s) = \frac{\frac{1}{2s^2} + \frac{10}{s^2+1} + \frac{7}{s+1} - \frac{5}{s+10}}{\cancel{\frac{1}{s^2+1}} + \frac{s+1}{s^2+1}}$$

$$\frac{1}{2s^2} + \frac{10}{s^2+1} + \frac{7}{s+1} - \frac{5}{s+10} =$$

$$= \frac{(s^2+1)(s+1)(s+10) + 20s^2(s+1)(s+10) + 14s^2(s^2+1)(s+10) + 10s^2(s^2+1)(s+1)}{2s^2(s^2+1)(s+1)(s+10)}$$

$$(s^2+1)(s+1)(s+10) + 20s^2(s+1)(s+10) + 14s^2(s^2+1)(s+10) + 10s^2(s^2+1)(s+1)$$

$$= (s^3+s^2+s+1)(s+10) + 20(s^3+s^2)(s+10) + 14(s^4+s^2)(s+10) + 10(s^4+s^2)(s+1)$$

$$= [s^4+10s^3+s^3+10s^2+s^2+10s+s+10] + [20(s^4+10s^3+s^3+10s^2)]$$

$$+ [14(s^6+10s^4+s^4+10s^2)] + [10(s^5+s^4+s^3+s^2)]$$

$$= [s^4+11s^3+11s^2+11s+10] + [20s^4+220s^3+200s^2]$$

$$+ [14s^6+140s^4+14s^4+140s^2] + [10s^5+10s^4+10s^3+10s^2]$$

$$= 14s^6+10s^5+171s^4+255s^3+361s^2+11s+10$$

$$H(s) = \frac{14s^6+10s^5+171s^4+255s^3+361s^2+11s+10}{2s^2(s+1)\cancel{(s^2+1)}(s+10)} \cdot \frac{\cancel{s^2+1}}{s+1}$$

$$= \frac{14s^6+10s^5+171s^4+255s^3+361s^2+11s+10}{2s^2(s+1)^2(s+10)}$$

$\hat{s}_{1,2} = 0$ $\qquad$ $\mathring{s}_{1,2} = 0{,}5074 \pm 3{,}394j$

$\hat{s}_{3,4} = -1$ $\qquad$ $\mathring{s}_{3,4} = -0{,}8591 \pm 1{,}1855j$

$\hat{s}_5 = -10$ $\qquad$ $\mathring{s}_{5,6} = -0{,}0054 \pm 0{,}1681j$

$\boxed{1}$ $u(t) = 3t$

$$y(t) = 12t + 7 \cdot e^{-3t} + 5 \cdot e^{-5t} - e^{-10t}$$

$$\mathcal{L}\{y(t)\}(s) = 12 \cdot \frac{1}{s^2} + 7 \cdot \frac{1}{s+3} + 5 \cdot \frac{1}{s+5} - \frac{1}{s+10} \qquad \mathcal{L}\{u(t)\}(s) = \frac{3}{s^2}$$

$$= \frac{12}{s^2} + \frac{7}{s+3} + \frac{5}{s+5} - \frac{1}{s+10}$$

$$= \frac{12(s+3)(s+5)(s+10) + 7s^2(s+5)(s+10) + 5s^2(s+3)(s+10) - s^2(s+3)(s+5)}{s^2(s+3)(s+5)(s+10)}$$

$$= \frac{12(s^2+8s+15)\cdot(s+10) + 7s^2(s^2+15s+50) + 5s^2(s^2+13s+30)}{s^2(s+3)(s+5)(s+10)}$$

$s^2(s^2+8s+15)$

$(s^4+8s^3+15s^2)$

$$= \frac{12(s^3+18s^2+95s+150) + 7(s^4+15s^3+50s^2) + 5(s^4+13s^3+30s^2)}{s^2(s+3)(s+5)(s+10)}$$

$s^4+8s^3+15s^2$

$$= \frac{12s^3 + 216s^2 + 1140s + 1800 + 7s^4 + 105s^3 + 350s^2 + 5s^4 + 65s^3 + 150s^2}{s^2(s+3)(s+5)(s+10)}$$

$$= \frac{11s^4 + 76s^3 + 701s^2 + 1140s + 1800}{s^2(s+3)(s+5)(s+10)}$$

$$H(s) = \frac{11s^4 + 76s^3 + 701s^2 + 1140s + 1800}{3(s+3)(s+5)(s+10)}$$

$\hat{s}_1 = -3, \ \hat{s}_2 = -5, \ \hat{s}_3 = -10 \qquad \mathring{s}_{1,2} = -2{,}6132 \pm 6{,}7040j$

$$s_{3,4}^{o} = -0{,}8414 \pm 1{,}5662j$$

5.4) a) $u(t) = 2 \cdot \delta(t)$

$$H(s) = \frac{5}{(s+1)(s+2)}$$

$$H(s) = 5 \cdot \frac{1}{(s+1)(s+2)}$$

$$\frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} = \frac{A(s+2) + B(s+1)}{(s+1)(s+2)}$$

$$A(s+2) + B(s+1) = 1$$

$$s(A+B) + 2A+B = 1$$

$$\begin{cases} A+B = 0 \\ 2A+B = 1 \end{cases} \ominus \Rightarrow -A = 1 \Rightarrow A = 1 \Rightarrow B = -1$$

$$H(s) = 5\left(\frac{1}{s+1} - \frac{1}{s+2}\right)$$

$$H(s) = \frac{\mathcal{L}\{y(t)\}(s)}{\mathcal{L}\{u(t)\}(s)}$$

$$\mathcal{L}\{u(t)\}(s) = 2$$

$$5\left(\frac{1}{s+1}-\frac{1}{s+2}\right)=\frac{Z\{y(t)\}(s)}{2}$$

$$Z\{y(t)\}(s)=10\left(\frac{1}{s+1}-\frac{1}{s+2}\right)$$

$$\Rightarrow \boxed{y(t)=10\cdot e^{-t}-10\cdot e^{-2t}}$$

b) $u(t)=3 \Rightarrow Z\{u(t)\}(s)=\frac{3}{s}$

$$H(s)=\frac{s}{(s+2)(s+4)}=s\cdot\frac{1}{(s+2)(s+4)}$$

$$\frac{s}{(s+2)(s+4)}=\frac{A}{s+2}+\frac{B}{s+4}=\frac{A(s+4)+B(s+2)}{(s+2)(s+4)}$$

$$A(s+4)+B(s+2)=s$$

$$s(A+B)+4A+2B=s$$

$$\begin{cases} A+B=1 \\ 4A+2B=0 \end{cases} \Rightarrow \begin{cases} A+B=1 \\ 2A+B=0 \end{cases} \quad -A=1 \Rightarrow A=-1$$

$$B=2$$

$$\frac{s}{(s+2)(s+4)}=\frac{-1}{s+2}+\frac{2}{s+4}$$

$$Z\{y(t)\}(s)=\frac{3}{s}\left(-\frac{1}{s+2}+\frac{2}{s+4}\right)=-3\cdot\frac{1}{s^2+2s}+6\cdot\frac{1}{s^2+4s}$$

$$\cancel{= -3 \cdot \frac{1}{(s+1)^2 - 1}}$$

$$\frac{1}{s^2+2s} = \frac{1}{s(s+2)} = \frac{1}{2}\left(\frac{1}{s} - \frac{1}{s+2}\right)$$

$$\frac{1}{s^2+4s} = \frac{1}{s(s+4)} = \frac{1}{4}\left(\frac{1}{s} - \frac{1}{s+4}\right)$$

$$\mathcal{L}\{y(t)\}(s) = -\frac{3}{2}\left(\frac{1}{s} - \frac{1}{s+2}\right) + \frac{6}{4}\left(\frac{1}{s} - \frac{1}{s+4}\right)$$

$$= -\frac{3}{2} \cdot \frac{1}{s} + \frac{3}{2} \cdot \frac{1}{s+2} + \frac{6}{4} \cdot \frac{1}{s} - \frac{6}{4} \cdot \frac{1}{s+4}$$

$$= \frac{1}{s}\left(\frac{6}{4} - \frac{3}{2}\right) + \frac{3}{2} \cdot \frac{1}{s+2} - \frac{6}{4} \cdot \frac{1}{s+4}$$

$$y(t) = \frac{3}{2} \cdot e^{-2t} - \frac{6}{4} \cdot e^{-4t}$$

c) $u(t) = 2 + \gamma(t) \Rightarrow U(s) = \frac{2}{s} + 1$

$$H(s) = \frac{1}{s^2+2s+5} \Rightarrow Y(s) = H(s) \cdot U(s)$$

$$\Rightarrow Y(s) = \frac{2+s}{s} \cdot \frac{1}{s^2+2s+5} = \frac{2}{s} \cdot \frac{1}{s^2+2s+5} + \frac{1}{(s+1)^2+2^2}$$

$$\frac{a}{s} + \frac{bs+c}{s^2+2s+5} = \frac{2}{s(s^2+2s+5)} \Rightarrow$$

$$\Rightarrow as^2 + 2as + 5a + bs^2 + cs = 2$$

$$\begin{cases} a+b=0 \\ 2a+c=0 \\ 5a=2 \end{cases} \Rightarrow a = \frac{2}{5},\ b = -\frac{2}{5},\ c = -\frac{4}{5}$$

$$\Rightarrow Y(s) = \frac{2}{5} \cdot \frac{1}{s} + \frac{2}{5} \frac{s+2}{(s+1)^2+2^2} + \frac{1}{(s+1)^2+2^2}$$

$$= \frac{2}{5} \cdot \frac{1}{s} - \frac{2}{5}\left[\frac{s+1}{(s+1)^2+2^2} + \frac{1}{(s+1)^2+2^2}\right] + \frac{1}{(s+1)^2+2^2}$$

$$\Rightarrow y(t) = \frac{2}{5} - \frac{2}{5} \cdot e^{-t} \cos(2t) - \frac{1}{5} \cdot e^{-t} \sin(2t)$$

$$+ \frac{1}{2} \cdot e^{-t} \sin(2t)$$

d) $u(t) = 1 + 2\,\delta(t) \Rightarrow U(s) = \frac{1}{s} + 2$

$$H(s) = \frac{s+8}{s^2+2s+5} \quad \text{si} \quad H(s) = \frac{Y(s)}{U(s)} \Rightarrow Y(s) = H(s)\cdot U(s)$$

$$\Rightarrow Y(s) = \frac{s+8}{s^2-2s+5}\cdot\frac{1}{s} + \frac{s+8}{s^2-2s+5}\cdot 2$$

$$\frac{a}{s} + \frac{bs+c}{s^2-2s+5} = \frac{s+8}{s^2-2s+5} \Rightarrow as^2 - 2as + 5a + bs^2 + cs = s+8$$

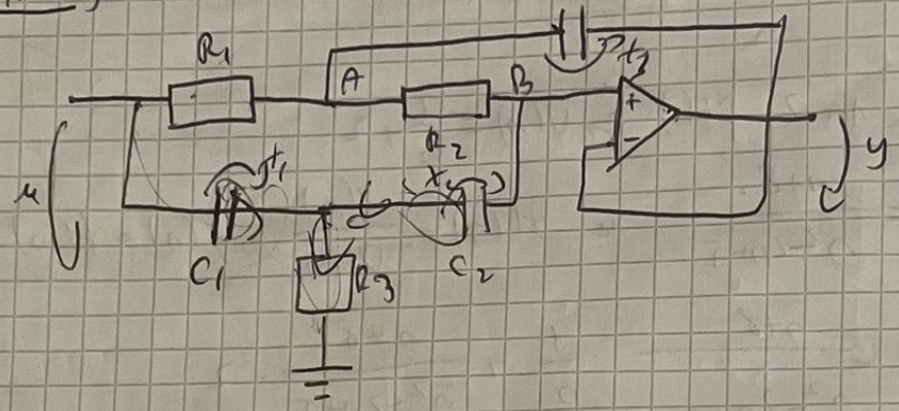
$$\Rightarrow \begin{cases} a+b=0 \\ c-2a=1 \\ 5a=8 \end{cases} \Rightarrow a = \frac{8}{5},\ b = -\frac{8}{5},\ c = \frac{21}{5}$$

$$Y(s) = \frac{8}{5s} + \frac{1}{5}\left(\frac{-8s}{s^2-2s+5} + \frac{21}{s^2-2s+5}\right) + 2\cdot\frac{s-1}{(s-1)^2+2^2}$$

$$+ 10\cdot\frac{1}{(s-1)^2+2^2}$$

$$\Rightarrow y(t) = \frac{8}{5} + \frac{2}{5}\cdot e^{t}\cos(2t) + \frac{103}{10}\cdot e^{t}\sin(2t)$$

Ex 5.5)



$y = V_B$

$V_C = U_{R3}$

$V_B = V_C + X_2 \Rightarrow V_B = X_2 + U_{R3}$

$u = X_1 + U_{R3} \Rightarrow U_{R3} = u - X_1 \Rightarrow V_B - X_2 = u - X_1$

$\Rightarrow V_B = u + X_2 - X_1$

$\Rightarrow \underline{y = u - X_1 + X_2}$

$V_A - y = X_3 \Rightarrow V_A = X_3 + y \Rightarrow V_A = X_2 + X_3 + u - X_1$

$U_{R2} = V_A - V_B = X_3 \Rightarrow \frac{U_{R2}}{R_2} = \dot{X}_2 \cdot C_2$

$\Rightarrow \boxed{\dot{X}_2 = \frac{X_3}{C_2 R_2}}$

$$I_{C_1} = I_{R_3} + I_{R_1} \Rightarrow I_{C_1} = \frac{u - x_1}{R_3} + \frac{x_3}{R_2}$$

$$\Rightarrow \boxed{\dot{x}_1 = \frac{u - x_1}{C_1 R_3} + \frac{x_3}{C_1 R_2}}$$

$$u - V_A = U_{R_1} \Rightarrow U_{R_1} = x_1 + x_2 - x_3$$

$$\Rightarrow I_{R_1} = \frac{x_1 + x_2 - x_3}{R_1}$$

$$I_{R_1} + I_{R_2} = I_{C_3} \Rightarrow I_{C_3} = \frac{x_1 + x_2 - x_3}{R_1} + \frac{x_3}{R_3}$$

$$\Rightarrow \boxed{\dot{x}_3 = \frac{x_1 + x_2 - x_3}{C_3 R_1} + \frac{x_3}{C_3 R_3}}$$

$$A = \begin{pmatrix} -\frac{2}{CR} & 0 & \frac{1}{CR} \\ 0 & 0 & \frac{1}{CR} \\ \frac{1}{2CR} & \frac{1}{2CR} & -\frac{1}{2CR} \end{pmatrix} \qquad B = \begin{pmatrix} \frac{2}{RC} \\ 0 \\ 0 \end{pmatrix}$$

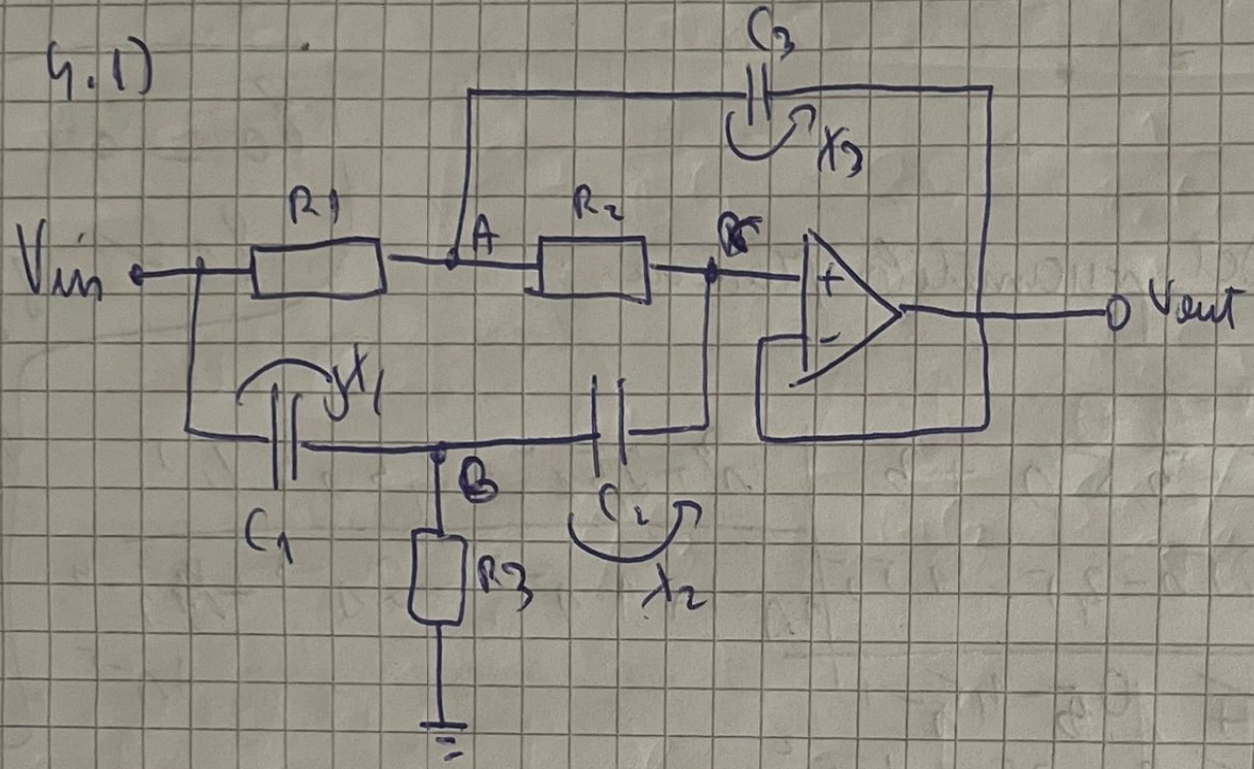
$$y = \begin{pmatrix} -1 & 1 & 0 \end{pmatrix} \qquad D = [1]$$

$$\cancel{sI - A = \begin{pmatrix} s + \frac{2}{} \end{pmatrix}}$$

$$sI - A = \begin{pmatrix} s + \frac{2}{RC} & 0 & -\frac{1}{RC} \\ 0 & s & -\frac{1}{RC} \\ -\frac{1}{2RC} & -\frac{1}{2RC} & s + \frac{1}{2RC} \end{pmatrix}$$

Laboratorio 5 Tema 1

4.1)



$AO \Rightarrow V_+ = V_- \Rightarrow \cancel{V_B = y}\ y = V_C$

$$\left.\begin{array}{l} V_B = U_{R3} \\ V_B - V_C = X_2 \end{array}\right\} \Rightarrow V_C = U_{R3} - X_2$$

$$\left.\begin{array}{l} \cancel{V_{in}}\ u - V_B = X_1 \Rightarrow u - U_{R3} = X_1 \Rightarrow U_{R3} = u - X_1 \end{array}\right\} \Rightarrow$$

$$V_C = u - x_1 - x_2$$

$$\Rightarrow \boxed{y = u - x_1 - x_2}$$

$$V_A - y = x_3 \Rightarrow V_A = y + x_3 = u - x_1 - x_2 + x_3$$

$$u - V_A = U_{R1} \Rightarrow U_{R1} = \cancel{u - u} + x_1 + x_2 - x_3 = x_1 + x_2 - x_3$$

$$U_{R2} = V_A - V_C = u - x_1 - x_2 + x_3 - u + x_1 + x_2 = x_3$$

$$I_{R1} = I_{C3} + I_{R2} \Rightarrow C_3 \cdot \frac{dx_3}{dt} = I_{R1} - I_{R2}$$

$$\Rightarrow \boxed{\dot{x}_3} = \frac{1}{C_3}\left(\frac{U_{R1}}{R_1} - \frac{U_{R2}}{R_2}\right) = \frac{1}{C_3}\left(\frac{x_1 + x_2 - x_3}{R_1} - \frac{x_3}{R_2}\right)$$

$$= \boxed{\frac{1}{C_3 R_1} x_1 + \frac{1}{C_3 R_1} x_2 - \frac{1}{C_3}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) x_3}$$

$$I_{R2} = I_{C2} \Rightarrow \boxed{\dot{x}_2} = \frac{1}{C_2} \cdot \frac{U_{R2}}{R_2} = \boxed{\frac{1}{C_2 R_2} x_3}$$

$$I_{C1} = I_{C2} + I_{R3} \Rightarrow \cancel{R_1 \dot{x}} \; C_1 \cdot \dot{x}_1 = C_2 \dot{x}_2 + \frac{U_{R3}}{R_3}$$

$$\dot{x}_1 = \frac{1}{C_1}\left(\frac{1}{R_2} x_3 + \frac{u - x_1}{R_3}\right) = \frac{1}{C_1 R_2} x_3 + \frac{1}{C_1 R_3} u - \frac{1}{C_1 R_3} x_1$$

$$\dot{X} = \begin{pmatrix} -\frac{1}{R_3 C_1} & 0 & -\frac{1}{R_2 C_1} \\ 0 & 0 & -\frac{1}{R_2 C_2} \\ \frac{1}{R_1 C_3} & \frac{1}{R_1 C_3} & -\frac{1}{C_3}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \end{pmatrix} X + \begin{pmatrix} \frac{1}{R_3 C_1} \\ 0 \\ 0 \end{pmatrix} u$$

$$y = \begin{bmatrix} -1 & -1 & 0 \end{bmatrix} X + \begin{bmatrix} 1 \end{bmatrix} u$$

$$H(s) = C\,(sI - A)^{-1} \cdot B + D$$

$$H(s) = [-1\ -1\ 0] \begin{pmatrix} s + \frac{1}{R_3 C_1} & 0 & \frac{1}{R_2 C_1} \\ 0 & s & \frac{1}{R_2 C_2} \\ -\frac{1}{R_1 C_3} & -\frac{1}{R_1 C_3} & s + \frac{1}{C_3}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \end{pmatrix} \begin{pmatrix} \frac{1}{R_3 C_1} \\ 0 \\ 0 \end{pmatrix} + 1$$

$$= (-1\ -1\ 0) \frac{\begin{pmatrix} s^2 + \frac{1}{C_3}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{1}{R_1 R_2 C_2 C_3} & x & x \\ -\frac{1}{R_1 R_2 C_2 C_3} & x & x \\ & s & x \end{pmatrix}}{\begin{vmatrix} s + \frac{1}{R_3 C_1} & 0 & \frac{1}{R_2 C_1} \\ 0 & s & \frac{1}{R_2 C_2} \\ -\frac{1}{R_1 C_3} & -\frac{1}{R_1 C_3} & s + \frac{1}{C_3}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \end{vmatrix}} \begin{pmatrix} \frac{1}{R_3 C_1} \\ 0 \\ 0 \end{pmatrix} + 1$$

Fie $R_1 = R_2 = R$, $R_3 = \frac{R}{2}$, $C_1 = C_2 = C$, $C_3 = 2C$

$$H(s) = \frac{(-1\ -1\ 0) \begin{pmatrix} s^2 + \frac{2}{RC} + \frac{1}{2R^2C^2} & x & x \\ -\frac{1}{2R^2C^2} & x & x \\ x & x & x \end{pmatrix} \begin{pmatrix} \frac{2}{RC} \\ 0 \\ 0 \end{pmatrix}}{s^3 + s^2 + \frac{3}{RC} + s\frac{3}{R^2C^2} + \frac{1}{R^3C^3}} + 1$$

$$H(s) = \frac{-\frac{2s}{RC}\left(0 + \frac{1}{RC}\right)}{\left(s + \frac{1}{RC}\right)^3} + 1 = \frac{-\frac{2s}{RC}}{s^2 + \frac{2s}{RC} + \left(\frac{1}{RC}\right)^2}$$

$$= \frac{s^2 + \frac{1}{R^2C^2}}{s^2 + \frac{2}{RC}s + \frac{1}{R^2C^2}}$$

$$\lim_{\omega \to 0} |H(j\omega)| = 1 \geq \frac{\sqrt{2}}{2} \Rightarrow \text{joase trec}$$

$$\lim_{\omega \to \infty} |H(j\omega)| = 1 > \frac{\sqrt{2}}{2} \Rightarrow \text{malte trec}$$

$$\omega_0 = \sqrt{\frac{1}{R^2C^2}} = \frac{1}{RC}$$

$$\lim_{\omega \to \omega_0} |H(j\omega)| = \frac{\frac{1}{R^2C^2} + \frac{1}{R^2C^2}}{\frac{1}{R^2C^2} + \frac{2}{RC}\cdot\frac{1}{RC} + \frac{1}{R^2C^2}} = \frac{\frac{2}{R^2C^2}}{\frac{4}{R^2C^2}} = \frac{1}{2} < \frac{\sqrt{2}}{2}$$

$\Rightarrow$ medie sunt tăiate

$$\Rightarrow \boxed{FOB}$$

Makow

$$\begin{array}{l|l}
s^2 + \frac{1}{R^2C^2} & s^2 + \frac{2}{RC}s + \frac{1}{R^2C^2} \\
\hline
-\left(s^2 + \frac{2}{RC}s + \frac{1}{R^2C^2}\right) & 1 - \frac{2}{RC}s^{-1} + \frac{4}{R^2C^2}s^{-2} - \frac{6}{R^3C^3}s^{-3} \\
\hline
-\frac{2}{RC}s & \\
\frac{2}{RC}s + \frac{4}{R^2C^2} + \frac{2}{R^3C^3}s^{-1} & \\
\hline
\frac{4}{R^2C^2} + \frac{2}{R^3C^3}s^{-1} & \\
-\frac{4}{R^2C^2} - \frac{8}{R^3C^3}s^{-1} + \frac{4}{R^4C^4}s^{-2} & \\
\hline
\end{array}$$

$\gamma_0 = 0$ $\qquad \gamma_3 = -\frac{6}{R^3C^3}$

$\gamma_1 = -\frac{2}{RC}$

$\gamma_2 = \frac{4}{R^2C^2}$

$$M_{2,2} = \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & \gamma_3 \end{pmatrix} = \begin{pmatrix} -\frac{2}{RC} & \frac{4}{R^2C^2} \\ \frac{4}{R^2C^2} & -\frac{6}{R^3C^3} \end{pmatrix}$$

$$|M_{2,2}| = \frac{12}{R^4C^4} - \frac{16}{R^4C^4} = -\frac{4}{R^4C^4} \neq 0 \Rightarrow$$

$\Rightarrow n' = 2 \ (= n) \Rightarrow f$ già in forma minimale

<u>FCC</u>

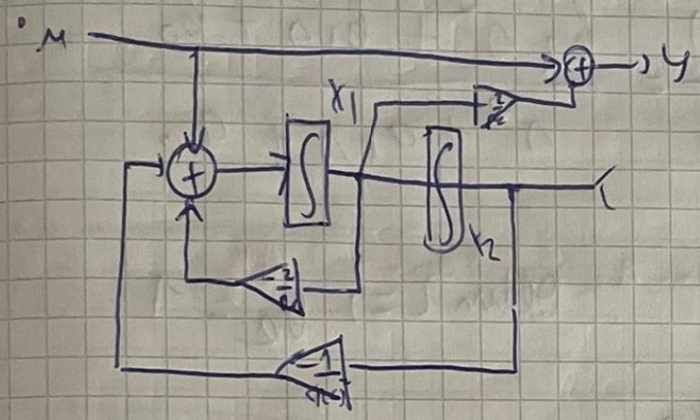
$$\left(\begin{array}{c|c} A_{FCC} & B_{FCC} \\ \hline C_{FCC} & D_{FCC} \end{array}\right)$$

$$H(s) = \frac{s^2 + \frac{1}{R^2C^2}}{s^2 + \frac{2}{RC}s + \frac{1}{R^4C^2}} = 1 - \frac{\frac{2}{RC}s}{s^2 + \underbrace{\frac{2}{RC}}_{a_{n1}}s + \underbrace{\frac{1}{R^2C^2}}_{a_{n2}}}$$

$d = 1$

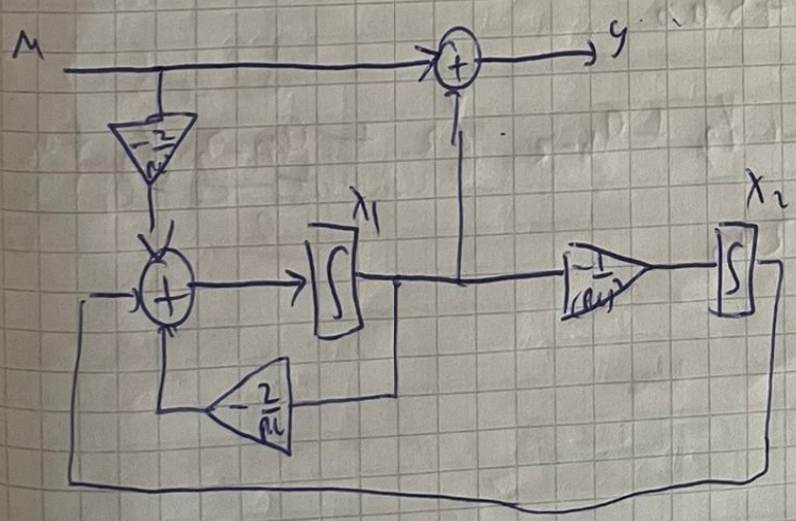
$$FCC = \left(\begin{array}{cc|c} -\frac{2}{RC} & -\frac{1}{R^4C^2} & 1 \\ 1 & 0 & 0 \\ \hline -\frac{2}{RC} & 0 & 1 \end{array}\right)$$

$$\Rightarrow \begin{cases} \dot{x}_1 = -\frac{2}{RC} \cdot x_1 - \frac{1}{(RC)^2} \cdot x_2 + u \\ \dot{x}_1 = x_1 \\ y = -\frac{2}{RC} x_1 + u \end{cases}$$



FCO

$$\left(\begin{array}{c|c} A_{FCO} & B_{FCO} \\ \hline C_{FCO} & D_{FCO} \end{array}\right) = \left(\begin{array}{cc|c} -\frac{2}{RC} & 1 & -\frac{2}{RC} \\ -\frac{1}{(RC)^2} & 0 & 0 \\ \hline 1 & 0 & 1 \end{array}\right) \Rightarrow \begin{cases} \dot{x}_1 = -\frac{2}{RC} x_1 + x_2 - \frac{2}{RC} \cdot u \\ \dot{x}_2 = -\frac{1}{(RC)^2} \cdot x_1 \\ y = x_1 + u \end{cases}$$



4.2

$\gamma_0 = 1,\ \gamma_1 = 5,\ \gamma_2 = -2,\ \gamma_3 = -4,\ \gamma_4 = 16$

$y_k = ?,\ k = \overline{5,10}$

$$\text{ordin } 2 \Rightarrow H(s) = \frac{\beta(s)}{\alpha(s)} = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$$

$b_n = \gamma_0 = 1$ $\qquad$ ordin $2 \Rightarrow b_2 = 1$

$b_{n-1} = \gamma_1 + a_{n-1} \cdot \gamma_0$

$b_1 = 5 + a_1 \cdot 1$

$b_0 = \gamma_2 + a_{n-1} \gamma_1 + a_{n-2} \cdot \gamma_0$

$b_0 = -2 + a_1 \cdot 5 + a_0 \cdot 1$

$b_{-1} = \gamma_3 + a_{n-1} \cdot \gamma_2 + a_{n-2} \cdot \gamma_1 + a_{n-3} \cdot \gamma_0$

$0 = -4 - 2a_1 + 5a_0$

$b_{-2} = \gamma_4 + a_{n-1} \gamma_3 + a_{n-2} \cdot \gamma_2 + \underbrace{a_{n-3}}_{0} \gamma_1 + \underbrace{a_{n-4}}_{0} \cdot \gamma_0$

$0 = 16 - 4a_1 - 2a_0$

$$\begin{cases} 5a_0 - 2a_1 = 4 & | \cdot 2 \\ 2a_0 + 4a_1 = 16 \end{cases} \Rightarrow \begin{array}{l} 10a_0 - 4a_1 = 8 \\ \underline{2a_0 + 4a_1 = 16} \quad +1 \end{array}$$

$$12a_0 = 24 \Rightarrow \underline{a_0 = 2} \Rightarrow \underline{a_1 = 3}$$

$$\underline{b_1 = 8}, \quad b_0 = \cancel{-2} + 15 + \cancel{2} = 15$$

$$\boxed{H(s) = \frac{s^2 + 8s + 15}{s^2 + 3s + 2}}$$

$$b_{n-5} = \gamma_5 + a_{n-1} \cdot \gamma_4 + a_{n-2} \cdot \gamma_3 + \underbrace{a_{n-3}}_{0} = 0$$

$$\Rightarrow 0 = \gamma_5 + a_{n-1} \cdot \gamma_4 + a_{n-2} \cdot \gamma_3$$

$$\Rightarrow \gamma_5 = 0 - 48 + 8 \Rightarrow \boxed{\gamma_5 = -40}$$

$$b_{n-6} = \gamma_6 + a_{n-1} \cdot \gamma_5 + a_{n-2} \cdot \gamma_4 + \underbrace{\quad}_{0}$$

$$\Rightarrow 0 = \gamma_6 + a_{n-1} \cdot \gamma_5 + a_{n-2} \cdot \gamma_4 \Rightarrow$$

$\cdot \gamma_0$

$$\Rightarrow \boxed{\gamma_6 = 88}$$

$\cancel{b_{n-7} = 0}$

$$b_{n-7} = \gamma_7 + a_{n-1} \cdot \gamma_6 + a_{n-2} \cdot \gamma_5$$

$$\Rightarrow \boxed{\gamma_7 = -184}$$

$$b_{n-8} = \gamma_8 + a_{n-1} \cdot \gamma_7 + a_{n-2} \cdot \gamma_6$$

$$\Rightarrow \boxed{\gamma_8 = 376}$$

$$b_{n-9} = \gamma_9 + a_{n-1} \cdot \gamma_8 + a_{n-2} \cdot \gamma_7$$

$$\Rightarrow \boxed{\gamma_9 = -760}$$

$$b_{n-10} = \gamma_{10} + a_{n-1} \cdot \gamma_9 + a_{n-2} \cdot \gamma_8$$

$$\Rightarrow \boxed{\gamma_{10} = 1528}$$

Lab6 Temă

2) $$\begin{cases} \dot{x} = \begin{pmatrix} -0,5 & 1 & 0 \\ 1 & 0 & 1 \\ 3 & 0 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 2,5 \\ -6 \end{pmatrix} u \\ y = (1\ 0\ 0)\, x \end{cases}$$

a) asimptotic stabil intern $\Rightarrow \operatorname{Re}\{\lambda_i\} < 0,\ \forall \lambda_i \in \Lambda(A)$

$\det(\lambda I_3 - A) = 0 \Rightarrow \det(\lambda I_3 - A) =$

$$= \begin{vmatrix} \lambda + 0,5 & -1 & 0 \\ -1 & \lambda & -1 \\ -3 & 0 & \lambda \end{vmatrix} = \lambda^3 + 0,5\lambda^2 - 3 - \lambda$$

$$= \lambda^3 + \lambda^2 \cdot 0,5 - \lambda - 3$$

$\lambda^3 + \lambda^2 \cdot 0,5 - \lambda - 3 = 0 \quad | \cdot 2$

$2\lambda^3 + \lambda^2 - 2\lambda - 6 = 0$

$\lambda_1 = 2 \quad \lambda_2 = -1+j \quad \lambda_3 = -1-j$

$Re\{\lambda_2, \lambda_3\} < 0$

$Re\{\lambda_1\} = \frac{3}{2} \Rightarrow$ sistem instabil intern

<u>Metoda II</u>

$$\begin{array}{c|ccc} \lambda^3 & 1 & -1 & 0 \\ \lambda^2 & \frac{1}{2} & -3 & 0 \\ \hline \lambda^1 & 5 & 0 & 0 \\ \lambda^0 & -3 & 0 & 0 \end{array}$$

$$b_1 = \frac{-\begin{vmatrix} 1 & -1 \\ \frac{1}{2} & -3 \end{vmatrix}}{\frac{1}{2}} = 5$$

$$b_2 = \frac{-\begin{vmatrix} 1 & 0 \\ \frac{1}{2} & 0 \end{vmatrix}}{\frac{1}{2}} = 0$$

avem o schimbare de semn

$$c_1 = \frac{-\begin{vmatrix} \frac{1}{2} & -3 \\ 5 & 0 \end{vmatrix}}{5} = -3$$

$\Rightarrow$ are o rad în C_+

$$c_2 = \frac{-\begin{vmatrix} 1 & 0 \\ \frac{1}{2} & 0 \end{vmatrix}}{5} = 0$$

$\Rightarrow$ sistem intern instabil

A) A, B, C sunt în forma FCO

$$\Rightarrow H(s) = \frac{s^2 + 2{,}5s - 6}{s^3 + 0{,}5s^2 - s - 3} + 0 \quad (d = 0)$$

$s^2 + 2{,}5s - 6 = 0 \Rightarrow s^2 + \frac{5}{2}s - 6 = 0 \mid \cdot 2$

$\Rightarrow 2s^2 + 5s - 12 = 0$

$\Delta = 25 - 4(-12) \cdot 2 = 121 \Rightarrow \sqrt{\Delta} = 11$

$s_{1,2} \Rightarrow s_1 = \frac{-5+11}{4} = \frac{3}{2}$ $\boxed{s_1 = \frac{3}{2}}$ $\boxed{s_2 = -4}$

$s_2 = \frac{-5-11}{4} = \frac{-16}{4} = -4$

$s^3 + 0 \cdot s^2 - s - 3 = 0$

$s_1^{\wedge} = \frac{3}{2}$, $s_{2,3}^{\wedge} = -1 \pm i$

$\frac{3}{2}$ - rad comune

$$H_{in}(s) = \frac{\cancel{\left(s - \frac{3}{2}\right)}(s+4)}{\cancel{\left(s - \frac{3}{2}\right)}(s^2+2s+2)} = \frac{s+4}{s^2+2s+2}$$

$s_2^{\circ} = -4$

$s_{1,2}^{\wedge} = -1 \pm i$

$Re\{s_{1,2}^{\wedge}\} < 0 \Rightarrow$ sistem stabil extern

Ex 1

b) $H(s) = \frac{5}{s+4}$

$\det(\lambda I - A) = 0 \Rightarrow \lambda + 4 = 0 \Rightarrow \lambda = -4 < 0 \Rightarrow$ s intern stabil

$s + 4 = 0 \Rightarrow s = -4 < 0 \Rightarrow$ s extern stabil

c) $H(s) = \frac{5}{s-4}$

$s - 4 \Rightarrow s = 4 > 0 \Rightarrow$ s extern instabil $\Rightarrow$ s intern instabil

a) $H(s) = \frac{10}{s^2+4s+4} = \frac{10}{(s+2)^2}$

$(s+2)^2 = 0 \Rightarrow s_{1,2} = -2 < 0 \Rightarrow$ extern stabil

e) $H(s) = \dfrac{s-2}{(s-2)(s^2+s+5)}$

$\det(\lambda I - A) = 0 \Rightarrow (\lambda - 2)(\lambda^2 + \lambda + 5) = 0 \Rightarrow \lambda_1 = 2 > 0$

$\Rightarrow$ s intern instabil

$H(s) = \dfrac{1}{s^2+s+5}$

$s^2 + s + 5 = 0 \Rightarrow s_{1,2} = -\frac{1}{2} \pm \frac{\sqrt{19}}{2} i$

$\mathrm{Re}\{s_{1,2}\} < 0 \Rightarrow$ s extern stabil

f) $H(s) = \dfrac{s+200}{s^2-2s+10}$

$\det(\lambda I - A) = 0 \Rightarrow \lambda^2 - 2\lambda + 10 = 0$

$\Delta = 4 - 40 = -36 \Rightarrow \lambda_{1,2} = 1 \pm 3i \Rightarrow \mathrm{Re}\{\lambda_i\} = 1 > 0$

$\Rightarrow$ sistem intern instabil

g) $H(s) = \frac{s}{(s+5)(s+10)}$

$(s+5)(s+10) = 0 \Rightarrow \left.\begin{array}{l} s_1 = -5 \\ s_2 = -10 \end{array}\right\} \Rightarrow Re\{s_i\} < 0 \Rightarrow$

s. extern stabil

h) $H(s) = \frac{s+2}{s(s+1)(s+100)}$

$\left.\begin{array}{l} s_1 = 0 \text{ (multiplicitate 1)} \\ s_2 = -1 \\ s_3 = -100 \end{array}\right\} \Rightarrow$ extern stabil

i) $H(s) = \frac{s+1}{s^2+2}$

$s^2+2 = 0 \Rightarrow s_{1,2} = \pm i\sqrt{2} \Rightarrow Re\{s_1, s_2\} = 0$ (multiplicitate 1)

$\Rightarrow$ extern instabil $\Rightarrow$ intern instabil

j) $H(s) = \frac{s+1}{(s^2+2)^2}$

$(s^2+2)^2 = 0 \Rightarrow s^2+2 = 0 \Rightarrow s_{1,2,3,4} = \pm i\sqrt{2}$

$\Rightarrow Re\{s_i\} = 0$ (mult. 4) $\Rightarrow$ extern instabil

$\Rightarrow$ intern instabil

k) $H(s) = -\frac{s+1}{s+2}$

$s+2=0 \Rightarrow s_1 = -2 \Rightarrow$ extern stabil

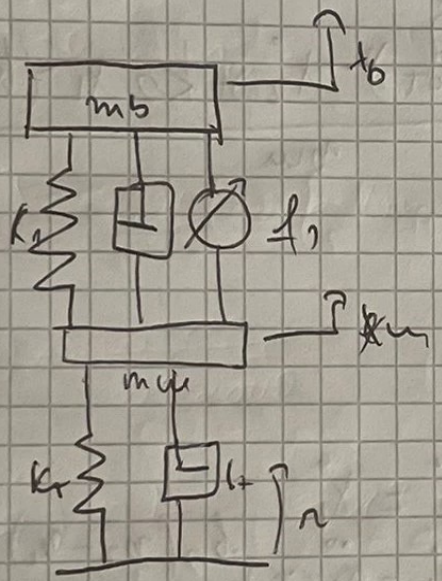
l) $H(s) = \frac{s-10}{(s-10.01)(s+5)}$

$s-10.01=0 \Rightarrow s_1 = 10.01$

$s+5 \Rightarrow s_2 = -5$

$\Rightarrow Re\{s_i\} > 0 \Rightarrow$ extern instabil

m)



(d)

$$m_b \frac{d^2x_b}{dt^2} + b_s \frac{dx_b}{dt} + k_s x_b = b_s \frac{dx_u}{dt} + k_s x_u + f_s$$

$$m_u \frac{d^2x_u}{dt^2} + (b_s + b_t)\frac{dx_u}{dt} + (k_s + k_t)x_u =$$

$$b_s \frac{dx_b}{dt} + k_s x_b + b_t \frac{dr}{dt} + k_t r - f_s$$

Fie $x_b = x_1$, $x_m = x_2$, $b_T = 0$

$$\Rightarrow \begin{cases} m_b \cdot \ddot{x}_1 + b_0 \cdot \dot{x}_1 + k_0 \cdot x_1 = b_0 \cdot \dot{x}_2 + k_0 \cdot x_2 + f_0 \\ m_m \cdot \ddot{x}_2 + (b_0 + b_T) \cdot \dot{x}_2 + (k_0 + k_T) \cdot x_2 = b_0 \cdot \dot{x}_1 \\ + k_0 \cdot x_1 + k_T \cdot r - f_0 \end{cases}$$

$$\Rightarrow \ddot{x}_1 = \frac{b_0(\dot{x}_2 - \dot{x}_1) + k_0(x_2 - x_1) + f_0}{m_b}$$

$$\ddot{x}_2 = \frac{b_0 \cdot \dot{x}_1 - (b_0 + b_T) \cdot \dot{x}_2 + k_0 \cdot x_1 - (k_0 + k_T) \cdot x_2}{m_m}$$

$$\underline{+ k_T \cdot r - f_0}$$

Fie

$$\begin{cases} x_1 = x_1 \\ v_1 = \dot{x}_1 \\ x_2 = x_2 \\ v_2 = \dot{x}_2 \end{cases} \Rightarrow \begin{cases} \dot{x}_1 = \dot{x}_1 \\ \dot{v}_1 = \ddot{x}_1 = \dfrac{b_0(v_2 - v_1) + k_0(x_2 - x_1) + f_0}{m_b} \\ \dot{x}_2 = \dot{x}_2 \\ \dot{v}_2 = \ddot{x}_2 = \dfrac{b_0 \cdot v_1 - (b_0 + b_T) \cdot v_2 + k_0 \cdot x_1}{m_m} \end{cases}$$

$$\underline{- (k_0 + k_T) \cdot x_2 + k_T \cdot r - f_0}$$

$$y = (x_1, \dot{x}_1, x_2, \dot{x}_2) = (x_1, v_1, x_2, v_2)$$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{k_2}{m_b} & \frac{-k_2}{m_b} & \frac{k_2}{m_b} & \frac{k_2}{m_b} \\ 0 & 0 & 1 & 0 \\ \frac{k_2}{m_u} & \frac{k_2}{m_u} & -\frac{k_2+k_T}{m_u} & -\frac{k_2+k_T}{m_u} \end{pmatrix} \quad [A_0 = 0]$$

$$B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{k_T}{m_u} \end{pmatrix} \qquad C = [1 \; 0 \; 0 \; 0] \qquad D = 0$$

6) $H(s) = \frac{\quad}{s(s+2)}$

$k = H(0) = \frac{1}{2}$

$s(s+2) = 0 \Rightarrow s_1 = 0,\ s_2 = -2 \Rightarrow T_1 = \frac{1}{2},\ T_2 = ?$

$s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + 2s$

$h(t) = \mathcal{L}^{-1}\{H(s)\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{2s} - \frac{\frac{1}{2}}{s+2}\right\}(t)$

$= \underbrace{\frac{1}{2}\cdot 1(t)}_{\text{zp}} - \underbrace{\frac{1}{2}\cdot e^{-2t}}_{\text{zl}}$

Modul răspunsului $= e^{-2t} \to 0 \Rightarrow$ sistem extern stabil

răspuns indicial: $y_i = \mathcal{L}^{-1}\left\{\frac{1}{s}\cdot H(s)\right\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\cdot\frac{1}{s(s+2)}\right\}(t)$

$= \mathcal{L}^{-1}\left\{\frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+2}\right\}(t)$

$\frac{1}{s^2(s+2)} = \frac{A(s+2) + Bs(s+2) + Cs^2}{s^2(s+2)}$

$As + 2A + Bs^2 + 2Bs + Cs^2 = 1$

$2A = 1 \Rightarrow A = \frac{1}{2}$

$As + 2Bs = 0 \Rightarrow Bs = -\frac{As}{2} = -\frac{1}{4}s \Rightarrow B = -\frac{1}{4}$

$$Bs^2 + Cs^2 = 0 \Rightarrow Cs^2 = \frac{1}{4}s^2 \Rightarrow C = \frac{1}{4}$$

$$\mathcal{L}^{-1}\left\{ \frac{\frac{1}{2}}{s^2} + \frac{-\frac{1}{4}}{s} + \frac{\frac{1}{4}}{s+2} \right\}(t)$$

$$= \underbrace{\frac{1}{2}t - \frac{1}{4}\,1(t)}_{T_1} + \underbrace{\frac{1}{4}e^{-2t}}_{T_2}$$

respuesta la compañ: $T_1 = \mathcal{L}^{-1}\left\{ \frac{1}{s^2} \cdot H(s) \right\}(t)$

$$= \mathcal{L}^{-1}\left\{ \frac{1}{s^2} \cdot \frac{1}{s(s+2)} \right\}(t) = \mathcal{L}^{-1}\left\{ \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s} + \frac{D}{s+2} \right\}(t)$$

$$\frac{1}{s^2(s+2)} = \frac{A(s+2) + Bs(s+2) + Cs^2(s+2) + Ds^3}{s^3(s+2)}$$

$$2A = 1 \Rightarrow A = \frac{1}{2}$$

$$As + 2Bs = 0 \Rightarrow Bs = \frac{-\frac{1}{2}}{2}s \Rightarrow B = -\frac{1}{4}$$

$$Bs^2 + 2Cs^2 = 0 \Rightarrow Cs^2 = -\frac{1}{2} \cdot \frac{1}{4}s^2 = \frac{1}{8} \Rightarrow C = \frac{1}{8}$$

$$Cs^3 + Ds^3 = 0 \Rightarrow Ds^3 = -\frac{1}{8}s^3 \Rightarrow D = -\frac{1}{8}$$

$$y_1 = \frac{1}{2} \cdot \frac{t^2}{2!} = \frac{1}{4} t + \frac{1}{8} 1(t) - \frac{1}{8} e^{-2t}$$

$$y_a = \frac{t^2}{4} - \frac{1}{4} t + \frac{1}{8} 1(t) - \frac{1}{8} e^{-2t}$$

c) $H(s) = \dfrac{40(s+1)}{s+10}$

$$k = H(0) = \frac{40}{10} = 4$$

$$s+10 = 0 \Rightarrow s_1 = -10 \Rightarrow T_1 = \frac{1}{10}$$

$$40(s+1) = 0 \Rightarrow T_i = 1$$

$$h(t) = \mathcal{L}^{-1}\{H(s)\}(t) = \mathcal{L}^{-1}\left\{\frac{40(s+1)}{s+10}\right\}(t) =$$

$$= 40\, \mathcal{L}^{-1}\left\{\frac{s+1}{s+10}\right\}(t) = 40\, \mathcal{L}^{-1}\left\{\frac{s+10}{s+10} - \frac{9}{s+10}\right\}(t)$$

$$= 40\, \mathcal{L}^{-1}\left\{1 - \frac{9}{s+10}\right\}(t) = 40\,\delta(t) - 360\, e^{-10t}$$

<u>risposta indicial</u>

$$\mathcal{L}^{-1}\left\{\frac{1}{s} \cdot H(s)\right\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s} \cdot \frac{40(s+1)}{s+10}\right\}(t)$$

$$= 40\, \mathcal{L}^{-1}\left\{\frac{s+1}{s(s+10)}\right\}(t) = 40\, \mathcal{L}^{-1}\left\{\frac{s+10}{s(s+10)} - \frac{9}{s(s+10)}\right\}(t)$$

$$= 40\, \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{\frac{9}{10}}{s} + \frac{\frac{9}{10}}{s+10}\right\}(t)$$

$$= 40 \cdot \frac{1}{10} \cdot 1(t) + 40 \cdot \frac{9}{10} \cdot e^{-10t} = 4\left(1(t) + 9 \cdot e^{-10t}\right)$$

rozpis do zlomků

$$y_k = \mathcal{L}^{-1}\left\{\frac{1}{s^2}\cdot H(s)\right\} = \mathcal{L}^{-1}\left\{\frac{40(s+1)}{s^2(s+10)}\right\}(t)$$

$$\cancel{= 40\,\mathcal{L}^{-1}\left\{\frac{s+10}{}\right\}} = 40\,\mathcal{L}^{-1}\left\{\frac{1}{s(s+10)} + \frac{1}{s^2(s+10)}\right\}(t)$$

$$\frac{1}{10}\cdot s(s+10) - \frac{1}{10}s^2 + \frac{1}{10}(s+10) + A(s+10)s + \frac{1}{100}s$$

$$\Rightarrow A = -\frac{1}{100}$$

$$\Rightarrow 40\cdot\frac{9}{100}\cdot 1(t) - 40\cdot\frac{9}{100}\cdot e^{-10t} + 40\cdot\frac{1}{10}t$$

$$= \frac{36}{10}\cdot 1(t) - \frac{36}{10}\cdot e^{-10t} + 4t$$

e) $H(s) = \frac{1}{(s+1)^2} = \frac{1}{s^2+2s+1} \Rightarrow s^2+2\zeta\omega_n s+\omega_n^2 = 1$

$\zeta = 1,\ \omega_n = 1$

$k = H(0) = 1$

$s+1 = 0 \Rightarrow s_1 = -1 \Rightarrow T_1 = 1$ (multiplicita 2)

<u>cazul lui impuls</u>

$$h(t) = \mathcal{L}^{-1}\{H(s)\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\}(t)$$

$$h(t) = e^{-t} \cdot \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}(t) = t \cdot e^{-t}$$

<u>răspuns indicial</u>

$$y_i = \mathcal{L}^{-1}\left\{\frac{1}{s} \cdot H(s)\right\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s} \cdot \frac{1}{(s+1)^2}\right\}(t)$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s} + \frac{A}{s+1} - \frac{1}{(s+1)^2}\right\}(t)$$

$$(s+1)^2 + As(s+1) - s = 1$$

$$2s + As - s = 0$$

$$As = -s \Rightarrow A = -1 \Rightarrow 1(t) - 1e^{-t} - te^{-t}$$

<u>răspuns rampă</u>

$$y_r = \mathcal{L}^{-1}\left\{\frac{1}{s^2} H(s)\right\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}(t)$$

$$= \mathcal{L}^{-1}\left\{\frac{A}{s} + \frac{1}{s^2} + \frac{s}{s+1} + \frac{1}{(s+1)^2}\right\}(t)$$

$$As(s+1)^2 + (s+1)^2 + As^2(s+1) + s^2 = 1$$

$$As^3 + 2As^2 + As + s^2 + 2s + 1 + Bs^3 + Bs^2 + s^2 = 1$$

$$As + 2s = 0 \Rightarrow A = -2$$

$$2As^2 + s^2 + Bs^2 + s^2 = 0 \Rightarrow Bs^2 = 2s^2 \Rightarrow B = 1$$

$$= \cancel{-2\mathbb{1}(t)} \; -2\,1(t) + t + 2e^{-t} + e^{-t} \cdot t$$

g) $H(s) = \frac{s+1}{s^2+1}$

$K = H(0) = 1$

$s^2+1=0 \Rightarrow s_{1,2} = \pm j \Rightarrow T_1 = \frac{1}{|j|} = \frac{1}{1} = 1$

$T_2 = 1$

$s+1=0 \Rightarrow s_1 = -1$

$\zeta = 0, \omega_n = 1$

<u>rsp lo impulso</u>

$h(t) = \mathcal{L}^{-1}\{H(s)\}(t) = \mathcal{L}^{-1}\left\{\frac{s+1}{s^2+1}\right\}(t)$

$= \mathcal{L}^{-1}\left\{\frac{s}{s^2+1} + \frac{1}{s^2+1}\right\}(t) = \cos t + \sin t$

<u>rsp lo trapto</u>

$\cancel{y(t) = \mathcal{L}^{-1}\{H(s)\}(t) = \mathcal{L}^{-1}}$

$y_i = \mathcal{L}^{-1}\left\{\frac{1}{s} \cdot H(s)\right\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s} \cdot \frac{s+1}{s^2+1}\right\}(t)$

$= \mathcal{L}^{-1}\left\{\frac{1}{s^2+1} + \frac{1}{s(s^2+1)}\right\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2+1} + \frac{A}{s} + \frac{Bs+C}{s^2+1}\right\}(t)$

$s + As^2 + A + Bs^2 + Cs = s+1$

$A = 1$

$$A p^2 + B p^2 = 0 \Rightarrow B = -1$$

$$p + C p = 1 \quad \Rightarrow C = 0$$

$$\mathcal{L}^{-1}\left\{\frac{1}{p^2+1} + \frac{1}{p} - \frac{p}{p^2+1}\right\}(t) = \sin(t) + 1(t) - \cos t$$

<u>Nap la sorpé</u>

$$y_1 = \mathcal{L}^{-1}\left\{\frac{1}{p^2} \cdot H(p)\right\}(t) = \mathcal{L}^{-1}\left\{\frac{p+1}{p^2(p^2+1)}\right\}(t)$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{p(p^2+1)} + \frac{1}{p^2(p^2+1)}\right\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{p} + \frac{Bp+C}{p^2+1} + \frac{E}{p} + \frac{1}{p^2}\right.$$

$$\left. + \frac{Fp+G}{p^2+1}\right\}$$

$$p^3 + p + Bp^2 + Cp^2 + Dp^3 + Dp + p^2 + 1 + Fp^3 + Gp^2 = p + 1$$

$$p + Dp = p \Rightarrow D = 0$$

$$Cp^2 + p^2 + Gp^2 = 0$$

$$C + G = -1$$

$$p^3 + Bp^3 + p^3 + Fp^3 = 0$$

$$B + D + F = -1$$

$$\mathcal{L}^{-1}\left\{\frac{1}{p} - \frac{p}{p^2+1} + \frac{1}{p^2} - \frac{1}{p^2+1}\right\}(t) = 1(t) - \cos t + t - \sin t$$

b) $H(s) = \frac{1+2s}{(s+1)^3}$

$K = H(0) = 1$

$s+1=0 \Rightarrow s_1 = -1 \Rightarrow \hat{m}_0 = 1$

$1-2s=0 \Rightarrow s_1 = \frac{1}{2} \Rightarrow \hat{m}_1 = 2$

<u>réponse impulsionnelle</u>

$$h(t) = \mathcal{L}^{-1}\{H(s)\}(t) = \mathcal{L}^{-1}\left\{\frac{1-2s}{(s+1)^3}\right\}(t) = \mathcal{L}^{-1}\left\{\frac{s-\frac{1}{2}}{(s+1)^3}\right\}(t)$$

$$= \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2} - \frac{\frac{3}{2}}{(s+1)^3}\right\}(t) = \mathcal{L}^{-1}\left\{\frac{-2}{(s+1)^2} + \frac{3}{(s+1)^2}\right\}$$

$$= -2te^{-t} + 3e^{-t} \cdot \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = -2te^{-t} + \frac{3e^{-t}t^2}{2}$$

<u>réponse transitoire</u>

$$y_t = \mathcal{L}^{-1}\left\{\frac{2}{s} \cdot \frac{1-2s}{(s+1)^2}\right\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s(s+1)^2} - \frac{2}{(s+1)^2}\right\}(t)$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+1} + \frac{s+1}{(s+1)^2} \cdot \frac{1}{(s+1)^2}\right\}(t)$$

$$= u(t) - e^{-t} - te^{-t} - 3\frac{t^2 \cdot e^{-t}}{2}$$

<u>cazul lui impuls</u>

$$h(t) = \mathcal{L}^{-1}\{H(s)\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\}(t)$$

$$h(t) = e^{-t} \cdot \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}(t) = t \cdot e^{-t}$$

<u>răspuns indicial</u>

$$y_i = \mathcal{L}^{-1}\left\{\frac{1}{s} \cdot H(s)\right\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s} \cdot \frac{1}{(s+1)^2}\right\}(t)$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s} + \frac{A}{s+1} - \frac{1}{(s+1)^2}\right\}(t)$$

$$(s+1)^2 + As(s+1) - s = 1$$

$$2s + As - s = 0$$

$$As = -s \Rightarrow A = -1 \Rightarrow 1(t) - 1e^{-t} - te^{-t}$$

<u>răspuns rampă</u>

$$y_r = \mathcal{L}^{-1}\left\{\frac{1}{s^2} H(s)\right\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}(t)$$

$$= \mathcal{L}^{-1}\left\{\frac{A}{s} + \frac{1}{s^2} + \frac{s}{s+1} + \frac{1}{(s+1)^2}\right\}(t)$$

$$As(s+1)^2 + (s+1)^2 + As^2(s+1) + s^2 = 1$$

$$As^3 + 2As^2 + As + s^2 + 2s + 1 + Bs^3 + Bs^2 + s^2 = 1$$

$$As + 2s = 0 \Rightarrow A = -2$$

$$2As^2 + s^2 + Bs^2 + s^2 = 0 \Rightarrow Bs^2 = 2s^2 \Rightarrow B = 1$$

$$= \cancel{-2\mathbb{1}(t)} \; -2\,1(t) + t + 2e^{-t} + e^{-t} \cdot t$$

g) $H(s) = \frac{s+1}{s^2+1}$

$K = H(0) = 1$

$s^2+1=0 \Rightarrow s_{1,2} = \pm j \Rightarrow T_1 = \frac{1}{|j|} = \frac{1}{1} = 1$

$T_2 = 1$

$s+1=0 \Rightarrow s_1 = -1$

$\zeta = 0, \omega_n = 1$

<u>rsp lo impulso</u>

$h(t) = \mathcal{L}^{-1}\{H(s)\}(t) = \mathcal{L}^{-1}\left\{\frac{s+1}{s^2+1}\right\}(t)$

$= \mathcal{L}^{-1}\left\{\frac{s}{s^2+1} + \frac{1}{s^2+1}\right\}(t) = \cos t + \sin t$

<u>rsp lo trapto</u>

$\cancel{y(t) = \mathcal{L}^{-1}\{H(s)\}(t) = \mathcal{L}^{-1}}$

$y_i = \mathcal{L}^{-1}\left\{\frac{1}{s} \cdot H(s)\right\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s} \cdot \frac{s+1}{s^2+1}\right\}(t)$

$= \mathcal{L}^{-1}\left\{\frac{1}{s^2+1} + \frac{1}{s(s^2+1)}\right\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2+1} + \frac{A}{s} + \frac{Bs+C}{s^2+1}\right\}(t)$

$s + As^2 + A + Bs^2 + Cs = s+1$

$A = 1$

$$A p^2 + B p^2 = 0 \Rightarrow B = -1$$

$$p + C p = 1 \quad \Rightarrow C = 0$$

$$\mathcal{L}^{-1}\left\{\frac{1}{p^2+1} + \frac{1}{p} - \frac{p}{p^2+1}\right\}(t) = \sin(t) + 1(t) - \cos t$$

<u>Nap la sorpé</u>

$$y_1 = \mathcal{L}^{-1}\left\{\frac{1}{p^2} \cdot H(p)\right\}(t) = \mathcal{L}^{-1}\left\{\frac{p+1}{p^2(p^2+1)}\right\}(t)$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{p(p^2+1)} + \frac{1}{p^2(p^2+1)}\right\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{p} + \frac{Bp+C}{p^2+1} + \frac{E}{p} + \frac{1}{p^2}\right.$$

$$\left. + \frac{Fp+G}{p^2+1}\right\}$$

$$p^3 + p + Bp^2 + Cp^2 + Dp^3 + Dp + p^2 + 1 + Fp^3 + Gp^2 = p + 1$$

$$p + Dp = p \Rightarrow D = 0$$

$$Cp^2 + p^2 + Gp^2 = 0$$

$$C + G = -1$$

$$p^3 + Bp^3 + p^3 + Fp^3 = 0$$

$$B + D + F = -1$$

$$\mathcal{L}^{-1}\left\{\frac{1}{p} - \frac{p}{p^2+1} + \frac{1}{p^2} - \frac{1}{p^2+1}\right\}(t) = 1(t) - \cos t + t - \sin t$$

b) $H(s) = \frac{1+2s}{(s+1)^3}$

$K = H(0) = 1$

$s+1=0 \Rightarrow s_1 = -1 \Rightarrow \hat{m}_0 = 1$

$1-2s=0 \Rightarrow s_1 = \frac{1}{2} \Rightarrow \hat{m}_1 = 2$

<u>réponse impulsionnelle</u>

$$h(t) = \mathcal{L}^{-1}\{H(s)\}(t) = \mathcal{L}^{-1}\left\{\frac{1-2s}{(s+1)^3}\right\}(t) = \mathcal{L}^{-1}\left\{\frac{s-\frac{1}{2}}{(s+1)^3}\right\}(t)$$

$$= \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2} - \frac{\frac{3}{2}}{(s+1)^3}\right\}(t) = \mathcal{L}^{-1}\left\{\frac{-2}{(s+1)^2} + \frac{3}{(s+1)^2}\right\}$$

$$= -2te^{-t} + 3e^{-t} \cdot \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = -2te^{-t} + \frac{3e^{-t}t^2}{2}$$

<u>réponse transitoire</u>

$$y_t = \mathcal{L}^{-1}\left\{\frac{2}{s} \cdot \frac{1-2s}{(s+1)^2}\right\}(t) = \mathcal{L}^{-1}\left\{\frac{1}{s(s+1)^2} - \frac{2}{(s+1)^2}\right\}(t)$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+1} + \frac{s+1}{(s+1)^2} \cdot \frac{1}{(s+1)^2}\right\}(t)$$

$$= u(t) - e^{-t} - te^{-t} - 3\frac{t^2 \cdot e^{-t}}{2}$$

<u>sólo la compª</u>

$$y_1 = 2^{-1}\left\{\frac{1}{s^2} - \frac{1-2s}{(s+1)^3}\right\}(t) = 2^{-1}\left\{\frac{1}{s^2(s+1)^3} - \frac{2}{s(s+1)^3}\right\}(t)$$

$$= 2^{-1}\left\{\frac{A}{s} + \frac{1}{s^2} + \frac{D}{s+1} + \frac{Bs+C}{(s+1)^2} + \frac{3}{(s+1)^3}\right\}(t)$$

$$A(s^3 + 2s^2 + 3s+1)\cdot s + s^3 + 3s^2 + 3s + 1 + Ds^2(s+1)^2 + Bs^4$$

$$+ Cs^2 + Bs^3 + Cs^2 + 3s^2 = 1 - 2s$$

$$As^4 + 3As^3 + 3As^2 + As + s^3 + 3s^2 + 3s + 1 + Ds^4 + 2Ds^3$$

$$+ Bs^4 + Cs^3 + Bs^2 + Cs^2 + 3s^2 = 1 - 2s$$

$$As + 3s = -2s \Rightarrow A = -5$$

$$D + C = -6 + 15 = 9$$

$B = 5 - D$ $\qquad$ $C - B = 4$

$C = 4 + B$

$D = 5 - B$ $\qquad$ $9A + 1 + 2D + 4 + B + B$

$A + D + B = 0$ $\qquad$ $D + C = 9$

$5 - B + C = 9$ $\qquad$ $2D + C + B = 14$

$D + B = 5$

$$= -5\cdot 1(t) + t + 5\cdot e^{-t} + 4te^{-t} + 3\frac{e^{-t}t^2}{2}$$

Lemnă

b) $H(s) = \dfrac{1}{s(s+2)}$

$\hat{s}_1 = 0, \hat{s}_2 = -2$

$\dfrac{1}{T_1} = \dfrac{1}{2}[s], \quad \dfrac{1}{T_2} = \dfrac{1}{2}[s]$

$k = H(0) = \dfrac{1}{2}$

Modurile asociate sunt e^{-0t}, e^{-2t}

Regimul: aperiodic amortizat (BIBO instabil)

<u>Performanțele asociate</u>

$t_r \simeq 1{,}5 \cdot 4 \cdot \hat{T} = 6 \cdot \frac{1}{2} \simeq 3s$

$\varepsilon_{sp} = 1 - H(0) = 1 - \frac{1}{2} = \frac{1}{2}$

$\varepsilon_{sv} = \infty$

c) $H(s) = \frac{40}{s+10}$

$K = H(0) = \frac{40}{10} = 4$

$s_1^{\wedge} = -10, s_1^{\circ} = -1$

$T_1^{\circ} = \frac{1}{1} = 1$

$T_1^{\wedge} = \frac{1}{10}$

Modurile naturale: e^{-10t}

Regimul: aperiodic amortizat

<u>Performanțele sistemului</u>

$t_1 \simeq 4 \cdot 1,5 \cdot \frac{1}{10} \simeq \frac{12}{2} \cdot \frac{1}{10} \simeq 0,6$ [s]

$\varepsilon_{rp} = 1 - H(0) = 1 - 4 = -3$

$\varepsilon_{rv} = \infty$

d) $H(s) = \frac{3}{(s+1)(s+2)}$

$s_1^{\wedge} = -1, s_2^{\wedge} = -2$

$T_1^{\wedge} = \frac{1}{1}, T_2^{\wedge} = \frac{1}{2}$

$K = H(0) = \frac{3}{2}$

Modurile naturale: e^{-t}, e^{-2t}

aperiodic amortizat

$$\Sigma_{opp} = 1 - H(0) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Sigma_{pro} = \lim_{p \to 0} \frac{1 - H(p)}{p} = \frac{1 - \frac{1}{2}}{0} = \infty$$

i) $H(p) = \frac{2p}{p^2 - 4} = \frac{2p}{(p-2)(p+2)}$

$p_1^0 = 0, \; p_1^1 = 2, \; p_2^1 = -2$

$T_1^1 = \frac{1}{2}, \; T_2^1 = \frac{1}{2}$

$K = H(0) = ?$

Modulul este: e^{-2t}, e^{2t}

sistem instabil extern

~~[illegible] este stabil~~

f) $H(p) = \frac{1}{p^2 + p + 1}$

$p^2 + p + 1 = 0$ $\qquad H(0) = 1$

$\Delta = 1 - 4 = -3 \Rightarrow \sqrt{-\Delta} = \sqrt{3}$

$p_{1,2} = -\frac{1}{2} \pm \sqrt{3} j$

$p^2 + 2\xi\omega_n p + 1$

$\boxed{\omega_n = 1} \Rightarrow \boxed{\xi = \frac{1}{2}}$

$\omega_{osc} = \omega_n\sqrt{1-\xi^2} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$

Mouvement : $e^{-\frac{1}{2}t} \cdot \sin(\sqrt{3}t + \phi)$

Oscillant amorti

$t_1 = \frac{4}{|Re|} = \frac{4}{|-\frac{1}{2}|} = 8\,(s)$

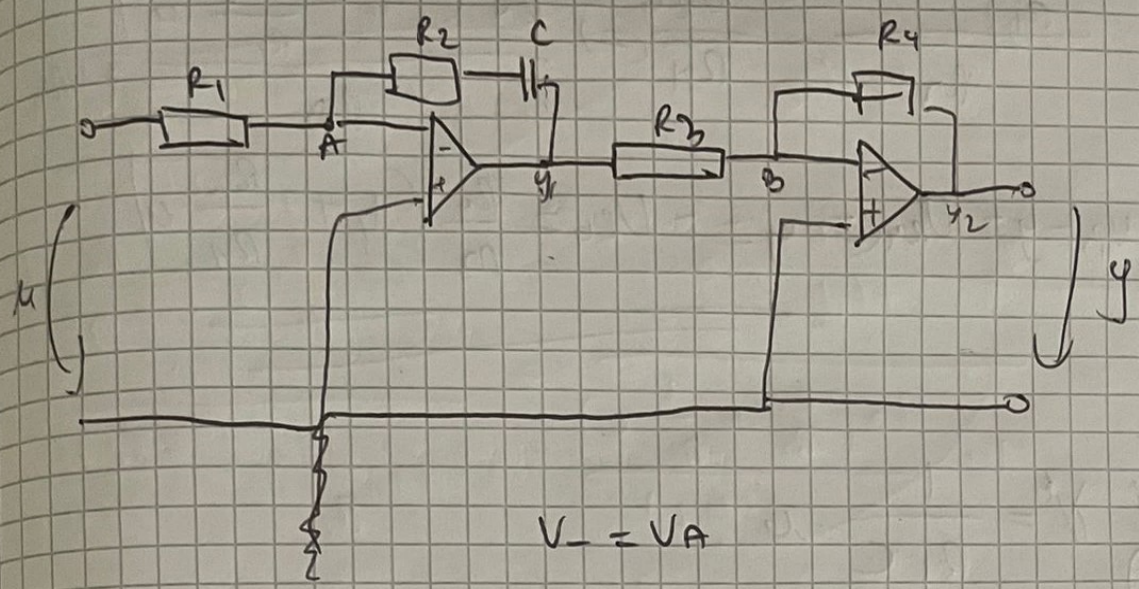
$\sigma = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = e^{\frac{-\pi\frac{1}{2}}{\frac{\sqrt{3}}{2}}} = e^{-\frac{\pi}{\sqrt{3}}} = 16,4\,\%$

$\varepsilon_{ssp} = 1 - H(0) = 0$

$\varepsilon_{ssv} = \lim_{p\to 0} \frac{1-H(p)}{p} = \lim_{p\to 0} \frac{1 - \frac{1}{p^2+p+1}}{p}$

$= \lim_{p\to 0} \frac{p^2+p}{p(p^2+p+1)} = \lim_{p\to 0} \frac{1+p}{p^2+p+1} = 1$

Tema 3



$V_- = V_A$

$$y_1 = k(V_+ - V_-) \Rightarrow \left.\begin{array}{l} \frac{y_1}{k} = V_+ - V_A \\ k \to \infty \end{array}\right\} \Rightarrow V_+ = V_A = 0$$

$$u - V_A = U_{R_1} \Rightarrow U_{R_1} = u$$

$$K_I : A : \frac{U_{R_1}}{R_1} = C \cdot \frac{dx_1}{dt} \Rightarrow \frac{dx_1}{dt} = \frac{u}{R_1 C}$$

$$U_{R_2} = i_{R_2} \cdot R_2 = \frac{u}{R_1 C} \cdot R_2$$

$$V_A - y_1 = U_{R_2} + x_1 \Leftrightarrow -y_1 = \frac{R_2}{R_1 C} \cdot u + x_1$$

$$\Rightarrow y_1 = -\frac{R_2}{R_1 C} \cdot u - x_1$$

$$V_B = 0 \Rightarrow y_1 - V_B = U_{R_3} \Rightarrow U_{R_3} = y_1 = \frac{-R_2}{R_1 C}$$

$$K_I : B : i_{R_3} = i_{R_4} \Leftrightarrow \frac{U_{R_3}}{R_3} = \frac{U_{R_4}}{R_4} \Rightarrow$$

$$\frac{-\frac{R_2}{R_1}\cdot u - x_1}{R_3} = \frac{U_{R4}}{R_4} \Leftrightarrow \frac{R_4\left(-x_1 - \frac{R_2}{R_1}\cdot u\right)}{R_3} = U_{R4}$$

$$U_B - y = U_{R4} \Leftrightarrow \underline{\underline{y = -U_{R4} = \frac{R_4}{R_3}\left(x_1 + \frac{R_2}{R_1}\cdot u\right)}}$$

$$\begin{cases} x_1' = \frac{1}{R_1 C}\cdot u \\ y = \frac{R_4}{R_3}\cdot x_1 + \frac{R_2 R_4}{R_3 R_1}\cdot u \end{cases}$$

$$\Rightarrow y' = x_1' \cdot \frac{R_4}{R_3} + u' \cdot \frac{R_2}{R_1 R_3}$$

$$\Rightarrow y' = \frac{R_4}{R_3}\cdot\frac{1}{R_1 C}\cdot u + \frac{R_2 R_4}{R_1 R_3}\cdot u' \quad \Big| \mathcal{L}\{\}(s)$$

$$\Rightarrow s\cdot Y(s) = \frac{R_4}{R_1 R_3 C}\cdot U(s) + \frac{R_2 R_4}{R_1 R_3}\cdot s\cdot U(s)$$

$$\Rightarrow s\cdot Y(s) = U(s)\left(\frac{R_4}{R_3 R_1 C} + \frac{R_2 R_4}{R_1 R_3}\cdot s\right)$$

$$\Rightarrow \frac{Y(s)}{U(s)} = \frac{1}{(R_1 R_3)s}\left(\frac{R_4}{C} + R_2 R_4 s\right)$$

$$\Rightarrow \frac{Y(s)}{U(s)} = \frac{R_2 R_4 C s + R_4}{(R_1 R_3 C) s} = \frac{\frac{R_2 R_4}{R_1 R_3} \cdot s + \frac{R_4}{R_3 R_1 C}}{s}$$

$$H_d(s) = H_{pi}(s) \cdot H_f(s) = \frac{10\left(\frac{R_2 R_4}{R_1 R_3} s + \frac{R_4}{R_1 R_3 C}\right)}{s(s+2)}$$

$$H_d(s) = H_{des}(s) \{ RN \; U \}$$

$$\Rightarrow H_{des} = \frac{10\left(\frac{R_2 R_4}{R_3 R_1} s + \frac{R_4}{R_3 R_1 C}\right)}{s(s+2)}$$

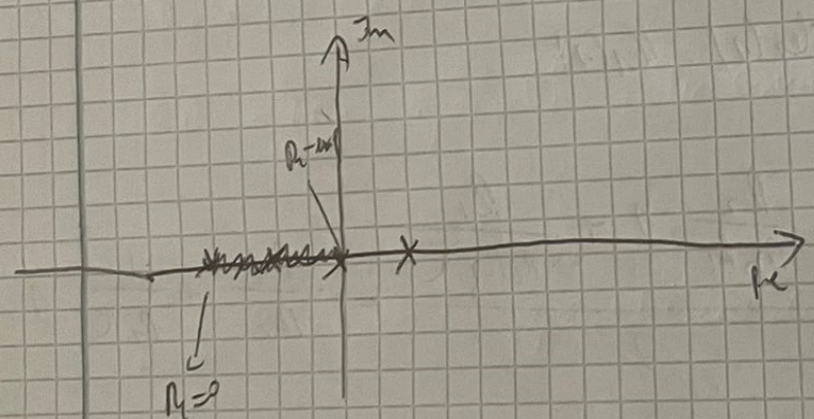
$$H_0(s) = \frac{H_d(s)}{1 + H_{des}(s) \cdot H_1(s)}$$

$$H_{pi}(s) = \frac{R_4 \cdot \frac{R_2}{R_3 R_1} \cdot s + \frac{R_4}{R_1 R_3 C}}{s} = \frac{R_2}{R_1} + \frac{1}{R_1 C} \cdot s$$

$$H_0(s) = \frac{H_d}{1+H_d(s)} = \frac{H_d}{1+\frac{10\left(\frac{R_2R_{10}}{R_1R_3}+\frac{R_2}{R_1R_3}C\right)}{s(s+2)}} =$$

$$= \frac{H_d}{1+\frac{10\left(\frac{R_2}{R_1}s+\frac{1}{R_1}C\right)}{s(s+2)}} = \frac{\frac{10\left(\frac{R_2}{R_1}s+\frac{R_2}{R_1R_2C}\right)}{s(s+2)}}{1+\frac{1}{R_1}\cdot\frac{10(R_2s+C)}{s(s+2)}}$$

$$\frac{1}{R_1}\cdot\frac{10(R_2s+C)}{s(s+2)} = 0 \Rightarrow \boxed{\begin{array}{l} s_1^0 = 0,\ s_2^0 = -2 \\ s_1^0 = -\frac{C}{R_2} \end{array}}$$



Cumutul este extern!

$\frac{1}{R_1} \in (0, 4{,}54 \cdot e^{-4})$: ap. amortizat: $e^{s_1^0 t}, e^{s_2^0 t}$

$\frac{1}{R_1} = 4{,}54 e^{-4}$: ap. amortizat: $e^{s_1^0 t}, e^{s_2^0 t} \cdot t$

$\frac{1}{R_1} \in (4{,}54 e^{-4}, +\infty)$: ap. amortizat: $e^{s_1^0 t}, e^{s_2^0 t}$

b) R_2 - variabil, $R_1 = 10k\Omega$, $C = 100nF$, $R_3 = R_2$

$$H_d = H_{pi}(s) \cdot H_f(s) = \left(\frac{R_2}{R_1} + \frac{1}{CR_1 s}\right) \cdot \frac{10}{s+2} =$$

$$= \frac{10(1+CR_2 s)}{R_1 C s(s+2)}$$

$$\boxed{H_{det} = H_d}$$

$$H_0 = \frac{H_d}{1+H_{det}} = \frac{\frac{10(1+R_2 C s)}{R_1 C \cdot s(s+2)}}{1+\frac{10(1+CR_2 s)}{R_1 C \cdot s(s+2)}}$$

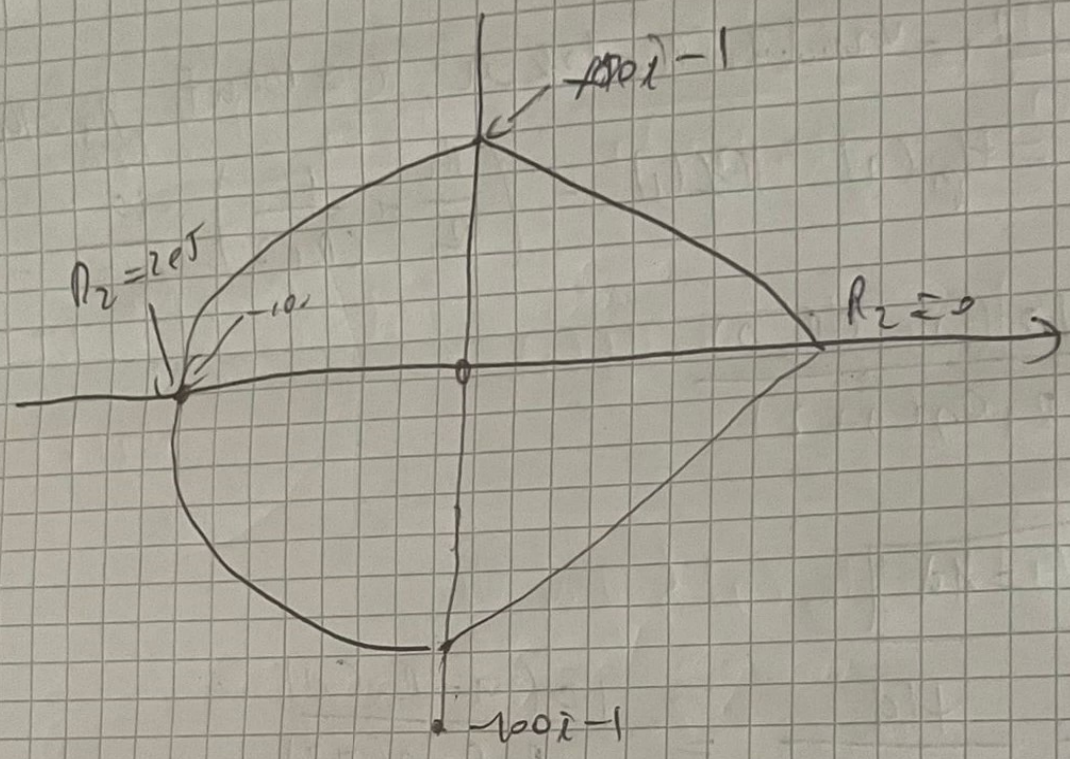
$$= \frac{10(1+R_2 C s)}{\cancel{R_1 C s(s+2)}} \cdot \frac{\cancel{R_1 C \cdot s(s+2)}}{R_1 C \cdot s(s+2) + 10(1+R_2 C s)}$$

$$= \frac{10(1+R_2 C s)}{R_1 C s^2 + 2R_1 C s + 10 R_2 C s + 10}$$

$$R_1 C \cdot s^2 + 2R_1 C s + 10 R_2 C \cdot s + 10 = 0$$

$$\Rightarrow 1 + \frac{10 R_2 C s}{R_1 \cdot C s^2 + 2R_1 C s + 10} = 0$$

$$\Rightarrow 1 + R_2 \cdot \frac{10 C s}{R_1 C s^2 + 2R_1 C s + 10} = 0$$



$R_2 = 0$: on amortit : $e^{-t} \cdot \sin(\sqrt{\omega_1^2 - \omega_2^2} + \phi)$

$R_2 \in (0, 2e5)$: on amortit : $e^{-t} \cdot \sin(\omega_1 \sqrt{1 - \xi^2} + \phi)$

$R_2 = 2e5$: périodic entre amortis : $\omega_1 = \omega_2$

$e^{\omega_1 t}, t \cdot e^{\omega_1 t}$

$R_2 \in (2e5, \infty)$: périodic amortis : $e^{\omega_1 t}, e^{\omega_2 t}$

$$H_1(p) = \frac{pT+1}{p^2}$$

$$H_d(p) = H_{pi}(p) \cdot H_1(p)$$

$$\Rightarrow \left(\frac{R_2}{R_1} + \frac{1}{CR_1 p}\right) \cdot \frac{pT+1}{p^2} = H_d(p)$$

$$\boxed{H_{des}(p) = H_d(p)}$$

$$\Rightarrow H_0(p) = \frac{H_d}{1+H_d} = \frac{\left(\frac{R_2}{R_1} + \frac{1}{CR_1 p}\right)\frac{pT+1}{p^2}}{1+\left(\frac{R_2}{R_1} + \frac{1}{CR_1 p}\right)\frac{pT+1}{p^2}}$$

$$= \frac{\left(\frac{R_2}{R_1} + \frac{1}{CR_1 p}\right)\frac{pT+1}{p^2}}{\left(\frac{1}{Cp} + R_2\right)\frac{pT+1}{p^2}}$$

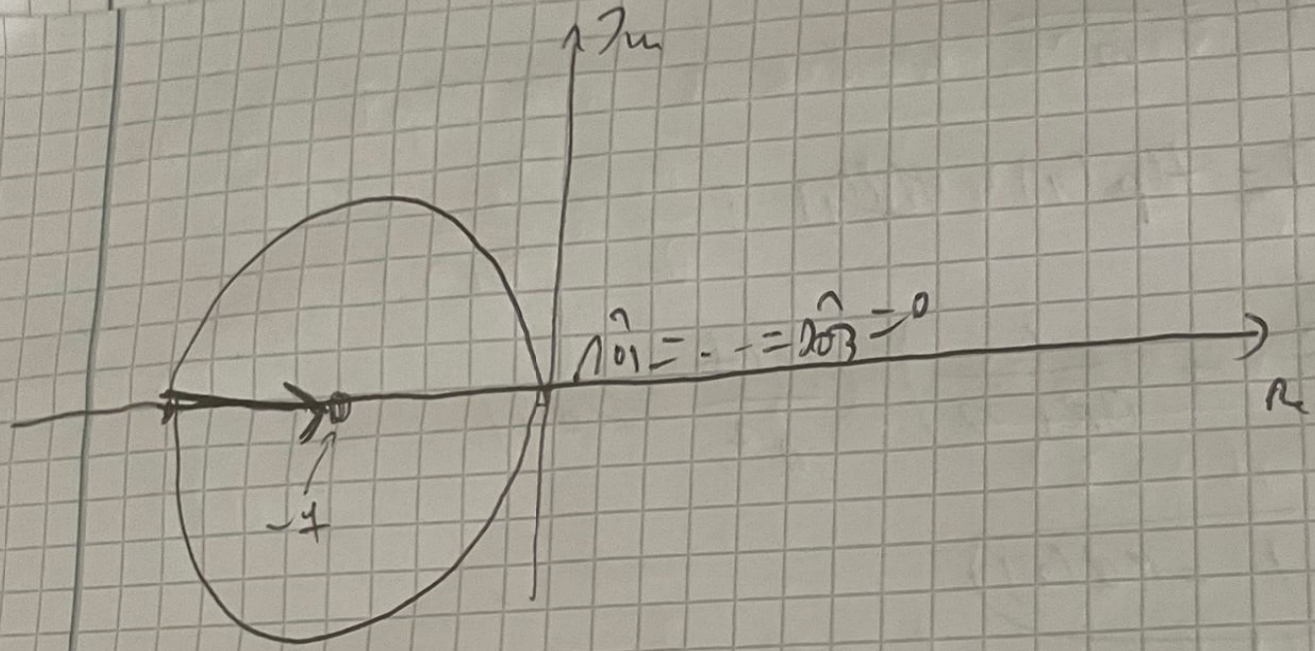
$$\left(\frac{1}{Cp} + R_2\right) \cdot \frac{pT+1}{p^2} = 0$$

$$\Rightarrow \left(\frac{R_2 Cp + 1}{Cp}\right)\frac{pT+1}{p^2} = 0$$

$$p_{01} = p_{02} = p_{03}$$

$$p_1^0 = -1, \quad p_2^0 = -\frac{1}{R_2 C}$$

$$H_{des}(p) = \frac{R_2 C p^2 + p(CR_2 T + 1) + T}{Cp^3}$$



$\frac{1}{R} = 0$ ($\hat{\omega}_1 = \hat{\omega}_2 = \hat{\omega}_3 = 0$), ap. am

$\frac{1}{R} \in (0, 8{,}27)$: on. am

$\frac{1}{R} = 8{,}27$: ap. am

$\frac{1}{R} \in (8{,}27, \infty)$: ap. am

Lab10

a) $H_{db}(s) = \frac{s+3}{s^2-4s+3}$

$s_1^0 = -3$

$s^2 - 4s + 3 = 0$

$\Delta = 16 - 12 = 4 \Rightarrow s_{1,2}^{\infty} = \frac{4 \pm 2}{2} \Rightarrow s_1^{\infty} = 3,\ s_2^{\infty} = 1$

$H_{db}'(s) = k' \cdot \frac{s+3}{(s-3)(s-1)}$

$k' = 1 > 0$

$\phi_{s_1^0} = \sum_{j=1}^{n} \angle s_j^0 - s_1^{\infty} - \sum_{\substack{i=1 \\ i \neq j}}^{m} \angle s_j^0 - s_i^{\infty} + (2l+1)\pi$

$\phi_{s_1^0} = \angle s_1^0 - s_1^{\infty} + \angle s_1^0 - s_2^{\infty} + (2l+1)\pi$

$= \angle -3-3 + \angle -3-1 + (2l+1)\pi$

$= \pi + \pi + (2l+1)\pi = 2\pi - \pi = \pi \quad (l = -1)$

$\phi_{s_1^{\infty}} = \sum_{i=1}^{m} \angle s_j^{\infty} - s_1^0 - \sum_{\substack{i=1 \\ i \neq j}}^{n} \angle s_j^{\infty} - s_i^{\infty} - (2l+1)\pi$

$= \lfloor 3+3 \rfloor - \lfloor 3-1 \rfloor - (2\ell+1)\pi$

$= 0 - 0 - (2\ell+1)\pi = x$

$\underline{\ell = -1} \Rightarrow x = \pi$

$\phi_{0,1} = 2\pi - \phi_{0,i} = 2\pi - \pi = \pi$, $n_0 = n - m = 1(2-1)$

$\sigma_0 = \dfrac{\sum \mathrm{Re}\{s_{0i}\} - \sum \mathrm{Re}\{s_{0i}\}}{n-m} = \dfrac{3+1-(-3)}{2-1} = 7$

$\phi_{n,m} = \dfrac{(2\ell+1)\pi}{a} = \pi$

<u>Punctele de ieșire</u>

$$\begin{cases} 1 + K \cdot H(s) = 0 \\ \dfrac{d}{ds} \cdot H(s) = 0 \end{cases}$$

$$\frac{d}{ds} \cdot \frac{s+3}{s^2-4s+3} = \frac{(s^2-4s+3) - (s+3)(s^2-4s+3)'}{(s^2-4s+3)^2}$$

$\Rightarrow -s^2 - 6s + 15 = 0$

$\Delta = 36 + 60 = 96$

$\Rightarrow s_{1,2} = \dfrac{6 \pm 9{,}79}{2} \Rightarrow s_1 = -7{,}89$, $s_2 = 1{,}89$

$$K = \frac{-1}{H'_{des}(s)}$$

$$k_{dem} = -\frac{1}{H'_{des}(-7{,}84)} = \frac{-\frac{1}{4{,}84}}{3{,}07-7{,}8673} \simeq 0{,}2$$

$$k_{gm} = -\frac{1}{H'_{des}(-7{,}84)} \simeq 17{,}8$$

5.a) $H'_{des}(s) = \dfrac{s+2}{(s^2+s+1)(s+5)}$

$s_1^0 = -2$, 稳定

$s_1^1 = -5$

$s^2+s+1=0 \Rightarrow s_2^1 = \dfrac{-1-j\sqrt{3}}{2}$, $s_3^1 = \dfrac{-1+j\sqrt{3}}{2}$

$$H'_{des}(s) = k' \cdot \frac{s+2}{(s+5)\left(s+\frac{1+j\sqrt{3}}{2}\right)\left(s-\frac{-1+j\sqrt{3}}{2}\right)}$$

$$(s^2+s+1)(s+5) = s^3+s^2+s+5s^2+5s+5 = s^3+6s^2+6s+5$$

$$\phi_{s_1} = \angle -2+5 + \angle -2+\frac{1+j\sqrt{3}}{2} + \angle -2-\frac{-1+j\sqrt{3}}{2} - (2l+1)\pi$$

$$\phi_{s_2} = \angle -1-\frac{j\sqrt{3}}{2}+2 - \angle -\frac{1+j\sqrt{3}}{2}+5 - \angle -\frac{1+j\sqrt{3}}{2} - -\frac{1+j\sqrt{3}}{2}$$

$$-(2l+1)\pi$$

$$\phi_{p_3} = \left\lfloor -\frac{1+j\sqrt{3}}{2} + 2 \right\rfloor - \left\lfloor \frac{-1+j\sqrt{3}}{2} + 5 \right\rfloor - \left\lfloor -\frac{1+j\sqrt{3}}{2} + \frac{1+j\sqrt{3}}{2} \right\rfloor$$

$$-(2\ell+1)\pi$$

$n_a = n - m = 2 \Rightarrow \phi_{a1}$ si ϕ_{a2}

$$\sigma_a = \frac{-5-1+2}{2} = -2$$

$$\phi_{a1} = \frac{(2-1)\pi}{2} = \frac{\pi}{2} \; ; \; \phi_{a2} = \frac{3\pi}{2} \; ; \; \phi_{a3} = \frac{9\pi}{2}$$

<u>Punctele de cotire</u>

$$\begin{cases} 1 + k \cdot H_{des}(s) = 0 \\ \frac{d}{ds} \cdot H_{des}(s) = 0 \end{cases}$$

$$\Rightarrow s^3 + 6s^2 + 6s + 5 - (3s^2 - 12s - 6)(s+2) = 0$$

$$\Rightarrow s^3 - 9s^2 - 30s - 7 = 0$$

$$s_1 = -2{,}97 \; ; \; s_2 = -0{,}25 \; , \; s_3 = 11{,}63 \notin \mathbb{R}$$

$$k = -\frac{1}{H'_{des}(-2{,}37)} = \frac{\frac{1}{-0{,}37}}{11{,}16} \simeq 30$$

$H_s(s)$ este extern stabil pt $(\forall)\alpha > 0$

Regim: op. amortizat.

Modul: $e^{s_1 t}$, $e^{\sigma_{12} t}$ [illegible] $\sin(\omega_{12} t)$ [illegible]

6) $H'(p) = \dfrac{p^2-2p+2}{p^2+4p+3}$

$p^2-2p+2=0$

$\Delta = -4 \Rightarrow p_1^0 = 1-j,\ p_2^0 = 1+j$

$p^2+4p+3=0$

$\Delta = 16-12=4 \Rightarrow p_1^\infty = -3,\ p_2^\infty = -1$

$\phi_{p_1^0} = \underline{/1-j+3} + \underline{/1-j+1} - \underline{/1-j-1-j} + (2\ell+1)\pi$

$= \underline{/4-j} + \underline{/2-j} - \underline{/-2j} + (2\ell+1)\pi$

$= -\frac{\pi}{2} - \frac{\pi}{2} - \frac{\pi}{2} + (2\ell+1)\pi$

$\underline{\underline{\ell=-1}}\quad -\frac{3\pi}{2} - \pi = -\frac{5\pi}{2}$

$\phi_{p_2^0} = \underline{/1+j+3} + \underline{/1+j+1} - \underline{/-1+j-1-j} + (2\ell+1)\pi$

$\phi_{\delta_1} = \angle -3-4j + \angle -3-1-j - \angle -3+1 - (2\ell+1)\pi$

$= \angle 4+j + \angle -4-j - \angle -2 - (2\ell+1)\pi =$

$= \frac{\pi}{2} - \frac{\pi}{2} - \pi + \pi = 0$

$\phi_{\delta_2} = \angle -1-4+j + \angle -1-4-j - \angle -1+3 - (2\ell+1)\pi$

$= -\pi$

$\omega = 0$

Pt. de croisi

$$\begin{cases} \lambda + k \cdot H' \text{ de } (\lambda) = 0 \\ \frac{d}{ds} \cdot H' \text{ de } (\lambda) = 0 \end{cases}$$

$\Rightarrow (2\lambda-2)(\lambda^2+4\lambda+3) - (\lambda^2-2\lambda+2)(2\lambda+4) = 0$

$2\lambda^3 - 2\lambda^2 + 8\lambda^2 - 8\lambda + 6\lambda - 6 - 2\lambda^3 + 4\lambda^2 - 4\lambda - 4\lambda^2$

$+ 8\lambda - 8 = 0$

$6\lambda^2 + 2\lambda + 14 = 0$

$\Delta = 340 \Rightarrow \lambda_1 = -1{,}7 \in \mathbb{R}$

$\lambda_2 = 4{,}36 \notin \mathbb{R}$

$$K_{gr} = -\frac{1}{H_{di}(-1,j)} \simeq 0,1$$

Ncu axa Oy

$$\alpha(s) + k\beta(s) = 0 \Rightarrow s^2 + 4s + 3 + s^2 k - 2sk + 2k = 0$$

$$(1+k)s^2 + (4-2k)s + 3 + 2k = 0$$

$$\begin{array}{c|cc} s^2 & 1+k & 3+2k \\ s & 4-2k & 0 \end{array}$$

$$\rightarrow 4 - 2k = 0 \Rightarrow k = 2$$

$H_0(s)$ est stabil pt $k \in (0,2)$ $k_{critic} = 2$

Regim : ep. amortizat

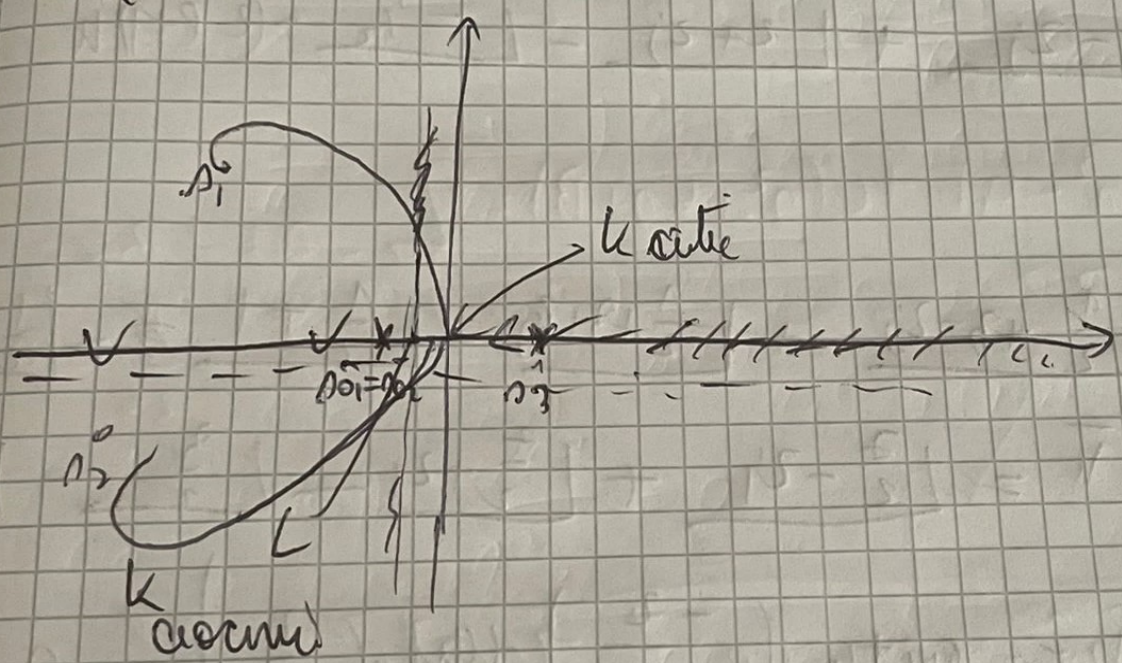
Temă 11

1) a) $H_{des}(s) = \dfrac{s^2+6s+13}{(s^2+2s+1)(s-\frac{1}{2})} = \dfrac{s^2+6s+13}{(s+1)^2(s-\frac{1}{2})}$

$\Delta = -16 \Rightarrow s_{1,2}^{0} = \dfrac{-6 \pm 4j}{2} = -3 \pm 2j$

$s_{01}^{0} = s_{02}^{0} = -1, \; s_3^{0} = \frac{1}{2}$

$k' = 1 > 0$



$\delta s_1^0 = \angle s_1^0 - s_1^0 + \angle s_1^0 - s_2^0 - \angle s_1^0 - s_2^0$

$- \angle s_1^0 - s_3^0 - (2l+1)\pi$

$= \angle -1+3-2j + \angle -1+3+2j - \angle -1-\frac{1}{2}$

$- (2l+1)\pi$

$$= \angle 2-2j + \angle 2+2j - \angle -1/2 - (2l+1)\pi$$

$$= \operatorname{arctg}\left(-\frac{2}{2}\right) + \operatorname{arctg}(1) - \pi - (2l+1)\pi$$

$$= \operatorname{arctg}(-1) + \operatorname{arctg} 1 - \pi - (2l+1)\pi$$

$$= -\frac{\pi}{4} + \frac{\pi}{4} - \pi - (2l+1)\pi = 0 \quad (l=-1)$$

$$\phi_{02} = \angle -1+3-2j + \angle -1+3+2j - \angle -1-\frac{1}{2} - (2l+1)\pi$$

$$= \angle 2-2j + \angle 2+2j - \angle -3/2 - (2l+1)\pi$$

$$= 0$$

$$\phi_{13} = \angle \frac{1}{2}+3-2j + \angle \frac{1}{2}+3+2j - \angle \frac{1}{2}+1 - \angle \frac{1}{2}\pi$$

$$-(2l+1)\pi = \angle \frac{7}{2}-2j + \angle \frac{7}{2}+2j - \angle \frac{3}{2} - \angle \frac{3}{2}$$

$$-(2l+1)\pi = \operatorname{arctg}\left(-\frac{2}{15}\right) + \operatorname{arctg}\left(\frac{2}{15}\right) - 0 - 0$$

$$-(2l+1)\pi = \pi$$

$$\phi_{11} = \angle \mathring{n}_1 - \mathring{n}_1 + \angle \mathring{n}_1 - \mathring{n}_2 + \angle \mathring{n}_1 - \mathring{n}_3 - \angle \mathring{n}_1 - \mathring{n}_2$$

$$+(2l+1)\pi$$

$$= \angle -3+2j+1 + \angle -3+2j+1 + \angle -3+2j-\frac{1}{2}$$

$$- \angle -3+2j+3+2j + (2l+1)\pi$$

$= \underline{\angle(-2+2j)} + \underline{\angle(-2-2j)} + \underline{\angle(-5+2j)} - \underline{\angle 4} + (2k+1)\pi$

$= \operatorname{arctg}(-1) + \operatorname{arctg}(-1) + \operatorname{arctg}\left(-\frac{2}{5}\right) - \frac{\pi}{2} + (2l+1)\pi$

$= -\frac{\pi}{2} - 29^\circ - \frac{\pi}{2} + (2l+1)\pi$

$= 2\pi - 29^\circ = 331^\circ$

$\sigma_a = \frac{-1-1+\frac{1}{2}+3+3}{1} = \frac{9}{2} =$

$\phi_{11} = \frac{(2\cdot 1-1)\pi}{1} = \pi$

$r_a = n - m = 3 - 2 = 1$

$$\begin{cases} 1 + k \cdot H_{dy}(s) \\ \frac{d}{ds} H_{dy}(s) = 0 \end{cases}$$

$$\frac{s^2+6s+13}{s^3+1{,}5s^2-0{,}5} = 0$$

$$(2s+6)(s^3+1{,}5s^2-0{,}5) - (s^2+6s+13)(3s^2+3s) = 0$$

$$= 2s^4 + 4s^3 - s + 3s^2 - 3 - (3s^4 + 3s^3 + 18s^3 + 18s^2 +$$

$$+ 39) = 0$$

$\Rightarrow s^4 - 12s^3 - 48s^2 - 400s = 0$

$s_{1,2} = -5.48 \pm 2,51j$

$s_3 = -1 \in \mathbb{R} \qquad s_3 = -1$

$s_4 = -0,08$

$$k = \frac{-1}{H_{d_1}(0,08)} = 0,039$$

$\cap Oy$

$$\left.\begin{array}{l} s^3 + 15s^2 - 0,5 + k(s^2 + 6s + 13) = 0 \\ s = 0 \end{array}\right\} \Rightarrow -0,5 + 13k = 0$$

$$\Rightarrow k = \frac{1}{26} = 0,0$$

$k = 0 : s_{01} = s_{02} = -1 \; e^{-t}, \; t \cdot e^{-t}, \; e^{0,5t}$

aperiodic critic neamortizat

$k \in (0, 0{,}031)$ $e^{r_1 t}, e^{r_2 t}, e^{r_3 t}$ aperiodi neamortizat

$k = 0{,}031$ $e^{r_1 t}, e^{r_2 t}$ = aperiodic amortizat

$k \in (0{,}031, 0{,}033)$ $= e^{r_1 t}, e^{r_2 t}, e^{r_3 t}$: aperiodic amortizat

$k = 0{,}033$: $r_1 = r_2 = r_3 = -0{,}01$: $e^{r_1 t}, e^{t r_2 t}, t \cdot e^{-0{,}01 t}$

aperiodic critic amortizat

$k \in (0{,}033, +\infty)$: $e^{r_1 t}, e^{k \{ r_2 t + \varphi \} \cdot \cos(\ )} \sin\{r_2 t + \varphi\}$

oscilant amortizat

$$H(s) = \frac{\quad}{s^2+2s+2}$$

$$s^2+4s+13=0$$

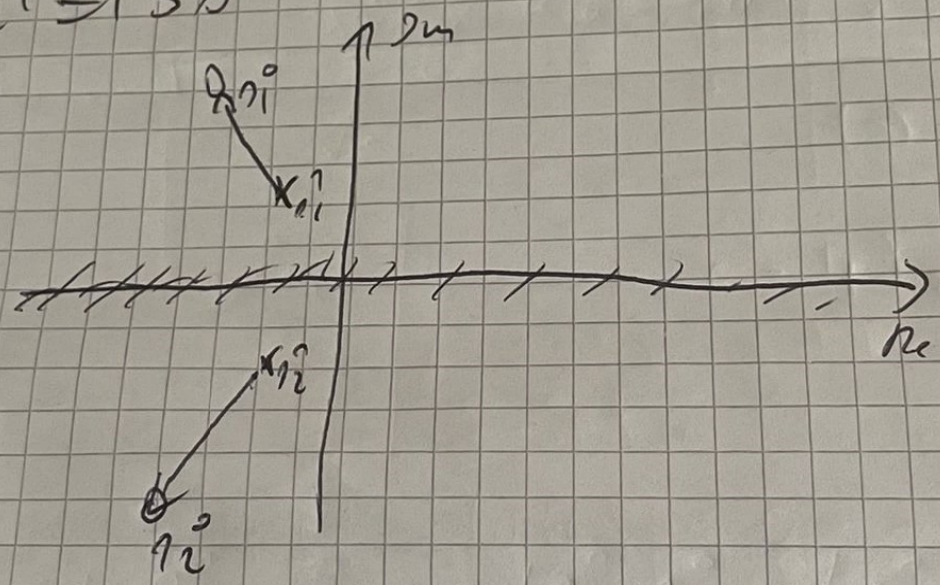
$$s_{1,2}^{\circ} = \frac{-4\pm 6i}{2} = -2\pm 3j \Rightarrow m=2$$

$$s^2+2s+2=0$$

$$s_{1,2}^{\hat{}} = -1\pm j \Rightarrow n=2$$

$$n_a = 0$$

$$K' = 1 > 0$$



$$\phi_{s_1^{\hat{}}} = \underline{\angle s_1^{\hat{}} - s_1^{\circ}} + \underline{\angle s_1^{\hat{}} - s_2^{\circ}} + \underline{\angle s_1^{\hat{}} - s_2^{\hat{}}}$$

$$-(l+1)\pi = \underline{\angle -1+j+2-3j} + \underline{\angle -1+j+2+3j}$$

$$-\underline{\angle -1+j+1+j} - (l+1)\pi$$

$= \angle 1-2j + \angle 1+4j - \frac{\pi}{2} - (2k+1)\pi$

$= -63^\circ + 76^\circ - \frac{\pi}{2} - (2k+1)\pi = -77^\circ - (2k+1)\pi$

$= -77^\circ + 180^\circ = 103^\circ \quad (k=-1)$

$\phi_{n_2^\circ} = 2\pi - 103^\circ = 257^\circ$

$\phi_{n_1^\circ} = \angle n_1^\circ - n_1^\circ + \angle n_1^\circ - n_2^\circ - \angle n_1^\circ - n_2^\circ + (2k+1)\pi$

$= \angle -2+3j+1-j + \angle -2+3j+1+4j - \angle -2+3j+2+3j + (2k+1)$

$= \angle -1+2j + \angle -1+4j - \angle 6j + (2k+1)\pi$

$= \operatorname{arctg}\left(\frac{2}{-1}\right) + \operatorname{arctg}\left(\frac{4}{-1}\right) - \frac{\pi}{2} + (2k+1)\pi$

$= 3\pi$

$\phi_{n_2^\circ} = 49^\circ$

Pct de echilibru: NU EXISTĂ!

Ciclu lim: NU EXISTĂ!

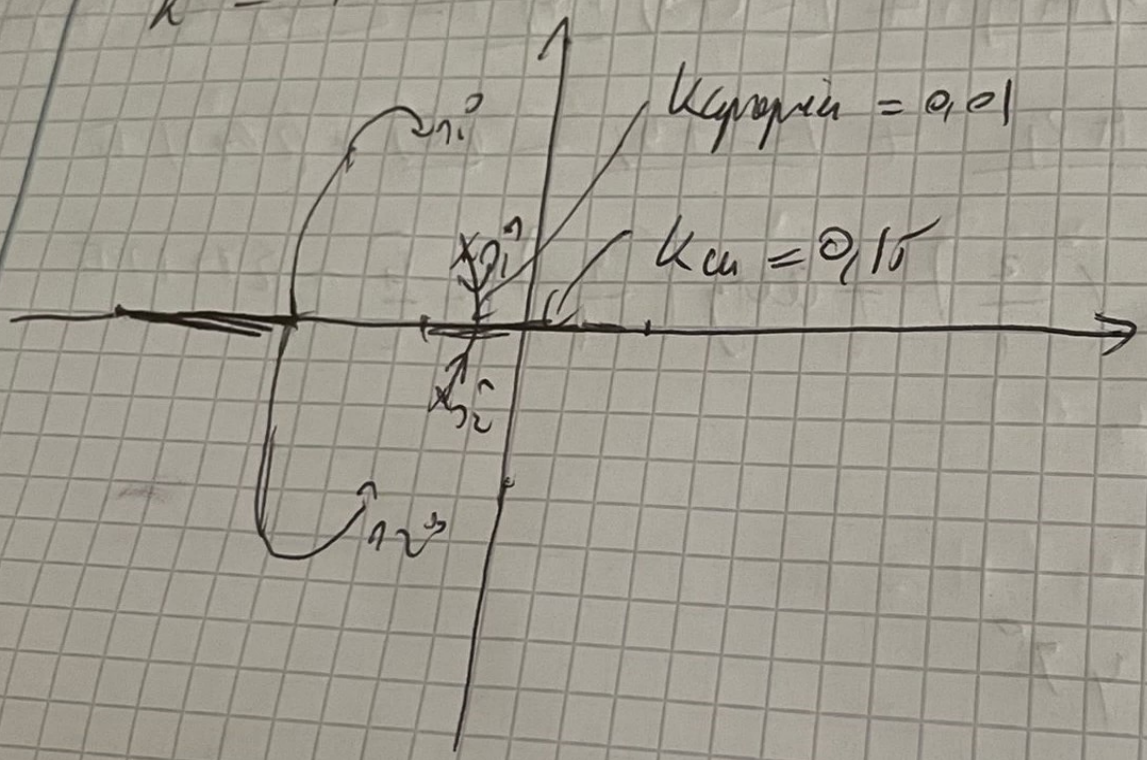
$m=2$

$p_1^0=-2+3j \quad p_2^0=-2-3j$

$n=2$

$p_1^1=-1+j \quad p_2^1=-1-j$

$k'=-1<0$



$$\Phi_{p_1^1}=\angle p_0^1-p_1^0+\angle p_0^1-p_2^0-\angle p_0^1-p_2^1-2l\pi$$

$$=\angle -1+j+2-3j+\angle -1+j+2+3j-\angle -1+j+1+j$$

$$-l\pi$$

$= \angle 1-2j + \angle 1+4j - \angle 2j - 2e\pi$

$= \operatorname{arctg}(-2) + \operatorname{arctg} 4 - \frac{\pi}{2} - 2e\pi$

$= -77^\circ + 20\pi = 283^\circ$

$\varphi_{12} = 2\pi - 203^\circ = 77^\circ$

$\varphi_{1}^{\circ} = \angle 1^\circ - 1^\circ + \angle 1^\circ - 1_2^\circ - \angle 1_1^\circ - 1_2^\circ + 2e\pi$

$= \operatorname{arctg}(-2) + \operatorname{arctg}(-4) - \frac{\pi}{2} + 2e\pi$

$= 2\pi - 229^\circ = 131^\circ$

$\varphi_{12}^{\circ} = 360^\circ - 131^\circ = 229^\circ$

$1_2 = 1 - 4 = 0\ 4^\circ$

Рст ск дифрсий

$$\int 1 + k \cdot H(d_1) = 0$$

$$\int \frac{d}{d_1} \cdot H' d_1(1) = 0$$

$$\frac{(-1^2 - 4_1 - 13)'(1^2 + 2_1 + 2) + (1^2 + 4_1 + 13)(1^2 + 2_1 + 2)'}{(1^2 + 2_1 + 2)^2}$$

$= 0$

$\Rightarrow (2_1 - 4)(1^2 + 2_1 + 2) + (1^2 + 4_1 + 13)(2_1 + 2) =$