CONTROL AND HIGHER ORDER FUNCTIONS 1

COMPUTER SCIENCE 61A

January 29, 2015

1 Control

Control structures direct the flow of logic in a program. For example, conditionals allow a program to skip sections of code, while iteration allows a program to repeat a section.

1.1 Conditional Statements

Conditional statements let programs execute different lines of code depending on certain conditions. In Python, we can use the `if`- `elif`-`else` block:

```
if <conditional expression>:
    <suite of statements>
elif <conditional expression>:
    <suite of statements>
else:
    <suite of statements>
```

Some notes:

- The `else` and `elif` statements are optional.
- You can have any number of `elif` statements.
- A **conditional expression** is a Python expression. All that matters for control is whether its value is a true value or a false value.
- The code that is executed is the **suite** that is indented under the first `if`/`elif` that has a true **conditional expression**. If none are true, then the `else` suite is executed.
- Once one suite is executed, the rest are skipped.

Note: in Python, there are a few things that are treated as false values:

- The boolean `False`
- The integer `0`
- The value `None`
- And more...

Python also includes **boolean operators** `and`, `or`, and `not`. These operators are used to combine and manipulate boolean values.

- `not True` evaluates to `False`, and `not False` evaluates to `True`.
- `True and True` evaluates to `True`, but a false value on either side makes it `False`.
- `False or False` evaluates to `False`, but a true value on either side makes it `True`.

1.2 Iteration

Iteration lets a program repeat statements multiple times. A common iterative block of code is the **`while` loop**:

```
while <conditional clause>:
    <body of statements>
```

This block of code states: "while the conditional clause is still `True`, continue executing the indented body of statements." Here is an example:

```
>>> def countdown(x):
...     while x > 0:
...         print(x)
...         x = x - 1
...     print("Blastoff!")
...
>>> countdown(3)
3
2
1
Blastoff!
```

1.3 Questions

1. Fill in the `is_prime` function, which returns `True` if `n` is a prime number and `False` otherwise.

   Hint: use the `%` operator: `x % y` returns the remainder of `x` when divided by `y`.

```
def is_prime(n):
```

```
def is_prime(n):
 if n == 1:
  return False
 k = 2
 while n > k:
  if n % k == 0:
   return False
  k += 1
 return True
```

1.4 Extra Questions

1. Fill in the `choose` function, which returns the number of ways to choose `k` items from `n` items. Mathematically, `choose(n, k)` is defined as:

$$\frac{n \times (n-1) \times (n-2) \times \cdots \times (n-k+1)}{k \times (k-1) \times (k-2) \times \cdots \times 2 \times 1}$$

```
def choose(n, k):
    """Returns the number of ways to choose K items from
       N items.

    >>> choose(5, 2)
    10
    >>> choose(20, 6)
    38760
    """
```

2 Higher Order Functions

A function that manipulates other functions is called a **higher order function** (HOF), which is a function that takes functions as arguments, returns a function, or both.

2.1 Functions as Argument Values

Suppose we want to square or double every integer from `1` to `n` and print the result as we go. Fill in the functions `square_ints` and `double_ints` by using the `square` and `double` functions we have defined.

```
def square(x):
    return x * x
def square_ints(n):
    """Print out the square of every integer from 1 to n.
    >>> square_ints(3)
    1
    4
    9
    """
```

```
def double(x):
    return 2 * x
def double_ints(n):
    """Print out the double of every integer from 1 to n.
    >>> double_ints(3)
    2
    4
    6
    """
```

The only difference between `square_ints` and `double_ints` is the function called before printing (either `square` or `double`).

It would be nice to have a generalized function, `transform_ints`, that took care of the `while` loop and the incrementing for us. That way, we could `triple_ints` or `cube_ints` without repeating so much code:

```
def square_ints(n):
    transform_ints(square, n)

def double_ints(n):
    transform_ints(double, n)

def cube(x):
    return x * x * x

def cube_ints(n):
    transform_ints(cube, n)
```

2.2 Questions

1. Implement the function `transform_ints` that takes in a function `func` and a number `n` and prints the result of applying that function to each of the first n natural numbers.

```
def transform_ints(func, n):
    """Print out all integers from 1 to n with func applied
    on them.

    >>> def square(x):
    ...     return x * x
    >>> transform_ints(square, 3)
    1
    4
    9
    """
```

2.3 Functions as Return Values

Often, we will need to write a function that returns another function. One way to do this is to define a function inside of a function:

```
def outer(x):
    def inner(y):
        ...
    return inner
```

Note two things:

1. The return value of the `outer` function is `inner`. This is where a function returns a function.
2. In this case, the `inner` function is defined inside of the `outer` function. This is a common pattern, but it is not necessary; we could have defined `inner` outside of the `outer` and still use the same `return` statement.

2.4 Questions

1. Write a function `and_add` that takes a function `f` (such that `f` is a function of one argument) and a number `n` as arguments. It should return a function that takes one argument, and does the same thing as the function `f`, except also adds `n` to the result.

```
def and_add(f, n):
    """Return a new function. This new function takes an
    argument x and returns f(x) + n.

    >>> def square(x):
    ...     return x * x
    >>> new_square = and_add(square, 3)
    >>> new_square(4)    # 4 * 4 + 3
    19
    """
```

2. Draw the environment diagram that results from running the following code:

```
n = 7

def f(x):
    n = 8
    return x + 1

def g(x):
    n = 9
    def h():
        return x + 1
    return h

def f(f, x):
    return f(x + n)()

m = f(g, n)
```

2.5 Extra Questions

1. Implement a function `keep_ints`, which takes in a function `cond` and a number `n`, and only prints a number from `1` to `n` if calling `cond` on that number returns `True`:

```
def keep_ints(cond, n):
    """Print out all integers 1..i..n where cond(i) is true

    >>> def is_even(x):
    ...     # Even numbers have remainder 0 when divided by 2.
    ...     return x % 2 == 0
    >>> keep_ints(is_even, 5)
    2
    4
    """
```

2. The following code has been loaded into the Python interpreter:

```
def skipped(f):
    def g():
        return f
    return g
def composed(f, g):
    def h(x):
        return f(g(x))
    return h
def added(f, g):
    def h(x):
        return f(x) + g(x)
    return h
def square(x):
    return x*x
def two(x):
    return 2
```

What will Python output when the following lines are evaluated?

```
>>> composed(square, two)(7)


>>> skipped(added(square, two))()(3)


>>> composed(two, square)(2)
```

3. Draw the environment diagram for the following code:

```
from operator import add
def curry2(h):
    def f(x):
        def g(y):
            return h(x, y)
        return g
    return f

make_adder = curry2(add)
add_three = make_adder(3)
five = add_three(2)
```

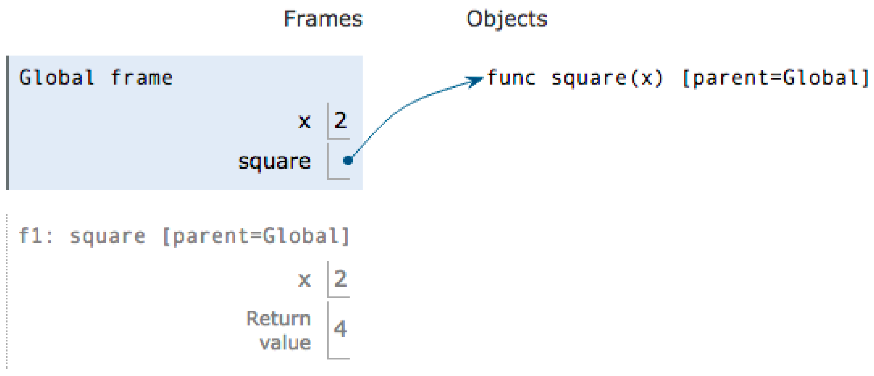
3 Addendum: Environment Diagrams

An **environment diagram** helps visualize the Python environment when a program is executed. The environment consists of a stack of frames, which contain variables and the values bound to them.

```
x = 2

def square(x):
    return x ** 2

square(2)
```



3.1 Questions

1. Draw the environment diagram that results from running the following code.

```
a = 1
def b(b):
    return a + b
a = b(a)
a = b(a)
```

2. Draw the environment diagram that results from executing the code below.

```
def this(x):
    return 2*that(x)

def that(x):
    x = y + 1
    this = that
    return x

x, y = 1, 2
this(that(y))
```

3.2 Extra Questions

1. Draw the environment diagram that results from executing the code below.

```
from operator import add, mul

six = 2

def ty(one, a):
    spring = one(a, six)
    return spring

def fif(teen):
    return teen ** 2

six = ty(add, mul(six, six))
spring = fif(six)
```