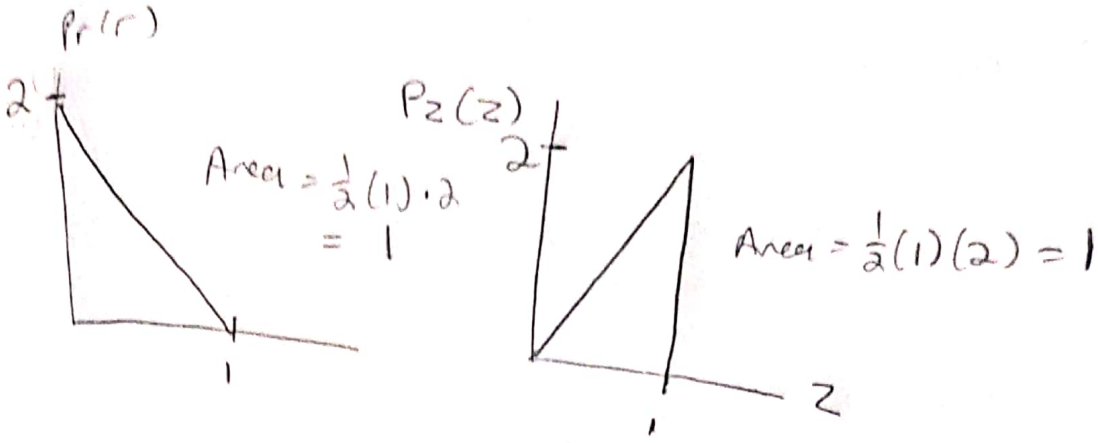
3.



Given: $r \in [0, 1]$ (normalized)

$P_r(r) = -2r + 2 \quad 0 \le r \le 1$ $\qquad P_z(z) = 2z$

$T(r) = G(z)$

$$T(r) = (L-1)\int_0^r P_r(w)\,dw$$

$$T(r) = (L-1)\int_0^r (-2w+2)\,dw$$

$$T(r) = (L-1)\left[-w^2+2r\right]_0^r$$

$$z = T(r) = (L-1)\left[-r^2+2r\right]$$

Mapping

$r = 0 \quad s = 0$

$r = 1 \quad s = L-1$

@ $r=0$ $T(r) = (L-1)(0+0) = 0$

if $0 \le r \le 1$

@ $r=1$ $T(r) = (L-1)(-(1)^2+2) = (L-1)$

Validation

$$\frac{dr}{ds} = \left[\frac{dz}{dr}\right]^{-1}$$

$$P_z(z) = P_r(r)\left[\frac{dz}{dr}\right]^{-1} = P_r(r)\left[(-2r+2)^{-1}(L-1)\right]^{-1}$$

$$P_z(z) = \frac{P_r(r)}{[-2r+2]} = \frac{[-2r+2]}{[-2r+2]}\frac{1}{(L-1)} = \frac{1}{(L-1)} \quad \text{for } 0 \le z \le L-1$$