WRIGHT STATE UNIVERSITY

Chapter-9
Morphological Image Processing

In form and feature, face and limb,
I grew so like my brother,
That folks got taking me for him
And each for one another.

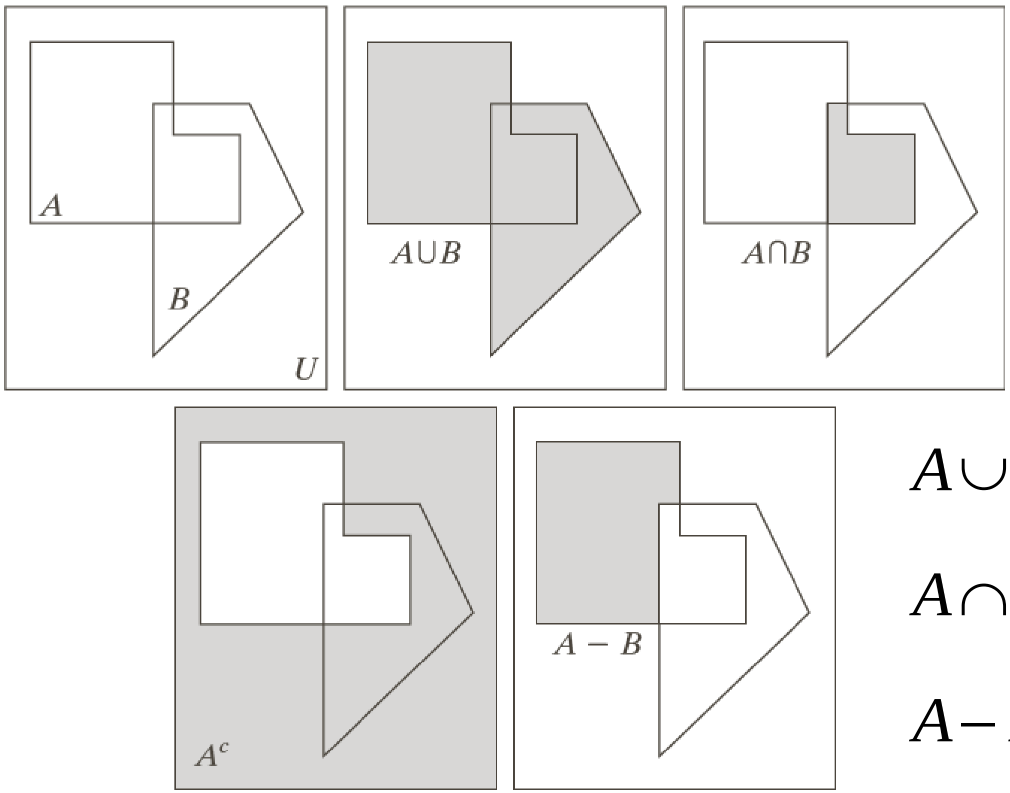
Henry Sambrooke Leigh,
Carols of Cockayne, The Twins

Contents

- Mathematical morphology provides tools for the representation and description of image regions (e.g. boundary extraction, skeleton, convex hull).
- It provides techniques for pre- and post-processing of an image (morphological thinning, pruning, filtering).
- The principles are based on set theory.
- Applications to both binary and grey level images.

Preliminaries

Basic set operations: (see Chapter 2)



$$A \cup B = \{w \mid w \in A \text{ OR } w \in B\}$$

$$A \cap B = \{w \mid w \in A \text{ AND } w \in B\}$$

$$A - B = \{w \mid w \in A,\ w \notin B\} = A \cap B^c$$

$$A^c = \{w \mid w \notin A\}$$

Objects as Sets, Graphical Image & Digital Image

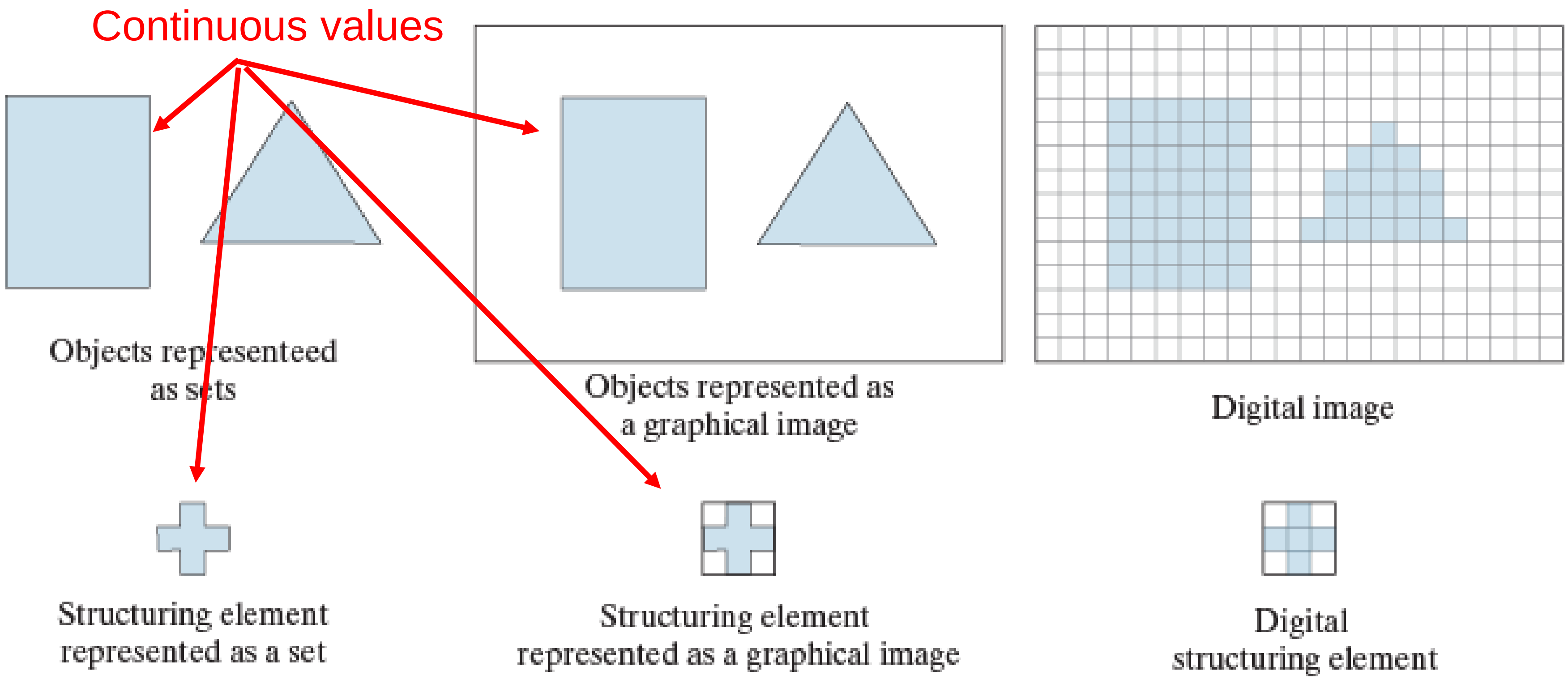
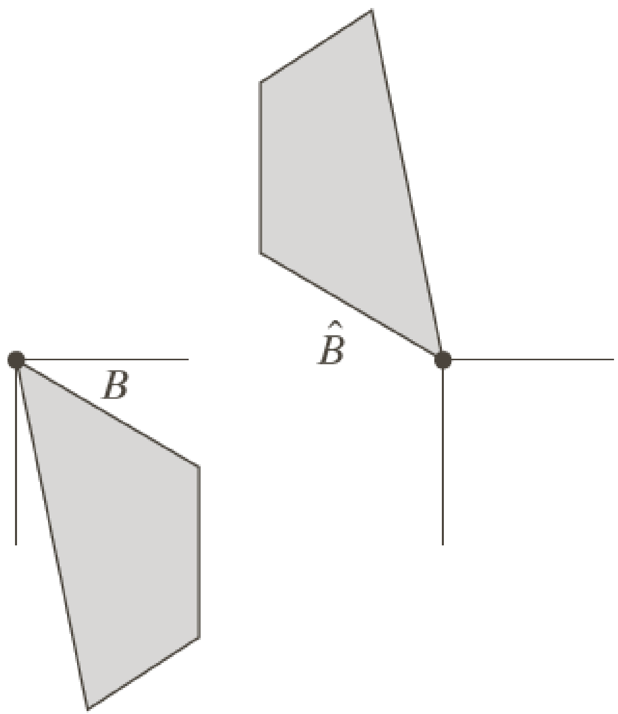


FIGURE 9.1 Top row. *Left:* Objects represented as graphical sets. *Center:* Objects embedded in a background to form a graphical image. *Right:* Object and background are digitized to form a digital image (note the grid). Second row: Example of a structuring element represented as a set, a graphical image, and finally as a digital SE.

Preliminaries (cont.)

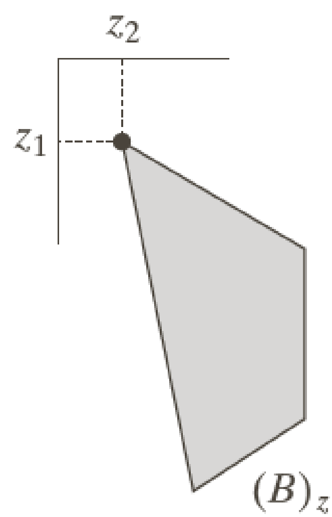
Set reflection:

$$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$$



Set translation by z:

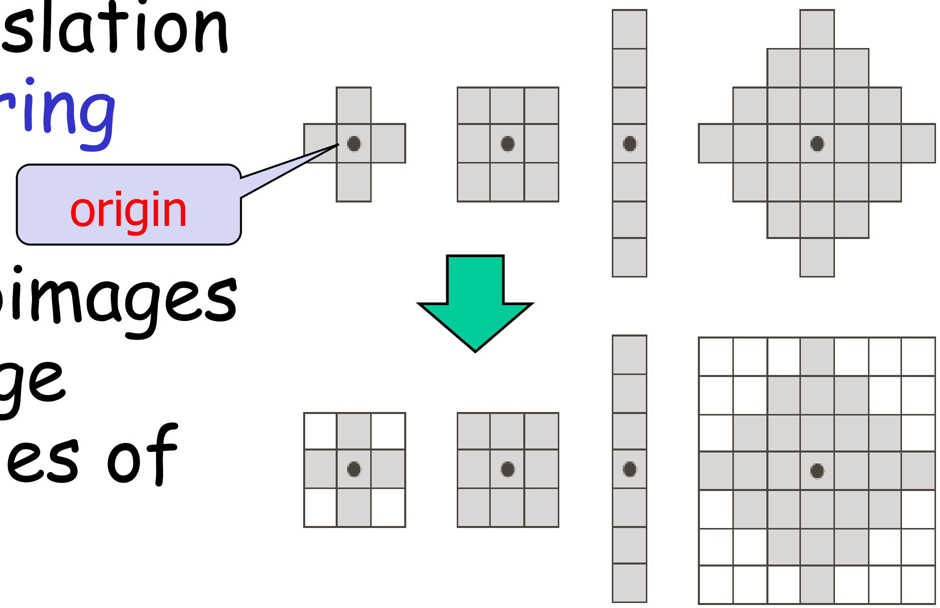
$$(B)_z = \{c \mid c = b + z, \text{ for } b \in B\}$$



Preliminaries (cont.)

- Set reflection and translation are employed to structuring elements (SE).
- SE are small sets or subimages used to examine the image under study for properties of interest.
- The origin must be specified.
- Zeros are appended to SE to give them a rectangular form.

Gray → 1; White → 0



Note: Gray represents a value of one and white a zero value.

Examples: Structuring Elements

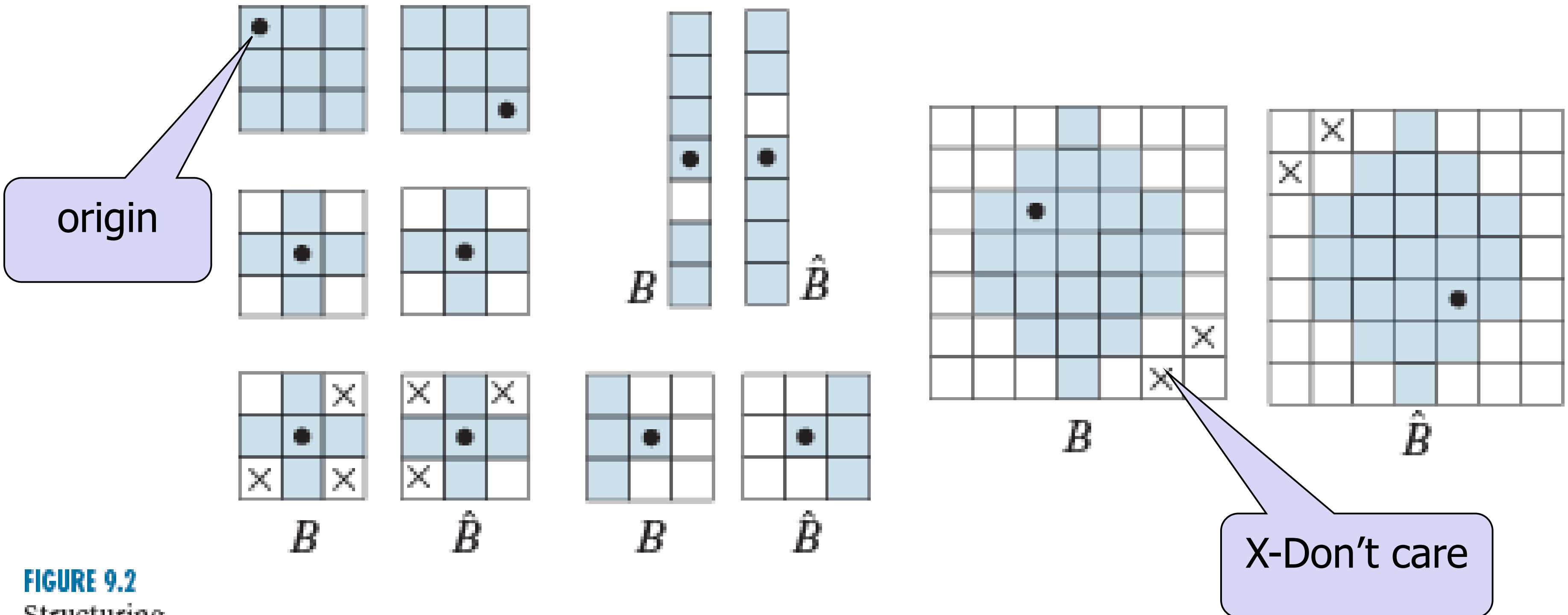


FIGURE 9.2 Structuring elements and their reflections about the origin (the ×'s are don't care elements, and the dots denote the origin). Reflection is rotation by 180° of an SE about its origin.

- Blue: A value of '1'
- White: A '0' value

Morphological Operation with a Structuring Element

- The **origin** of the SE B visits every pixel in an image A.
- It performs an operation (generally non-linear) between its elements and the pixels under it.
- It is then decided if the pixel will belong to the resulting set or not - based on the results of the operation.
- Zero padding is necessary (like in convolution) to ensure that all of the elements of A are processed.

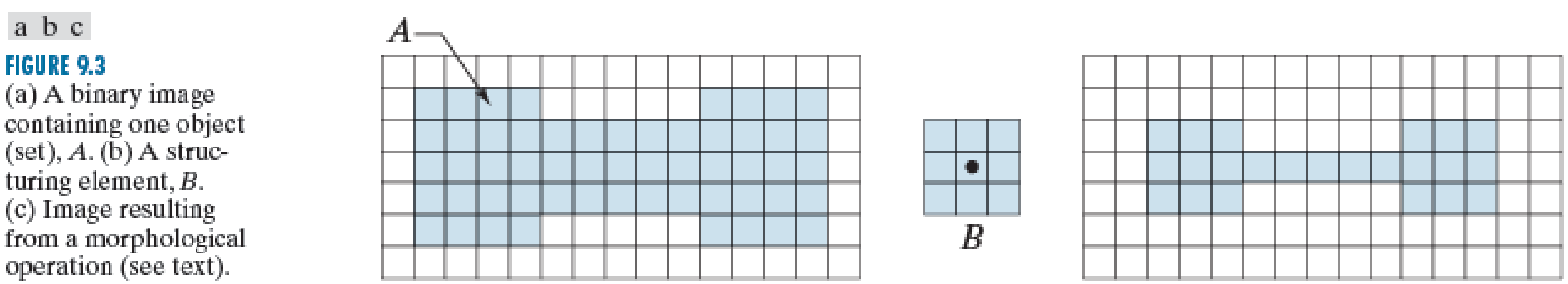


FIGURE 9.3 (a) A binary image containing one object (set), A. (b) A structuring element, B. (c) Image resulting from a morphological operation (see text).

Image I

Image after morphological operation

Preliminaries (cont.)

For example, **it marks the pixel under its center (i.e., the origin)** as belonging to the result if B is completely contained in A

a b c

FIGURE 9.3 (a) A binary image containing one object (set), A. (b) A structuring element, B. (c) Image resulting from a morphological operation (see text).

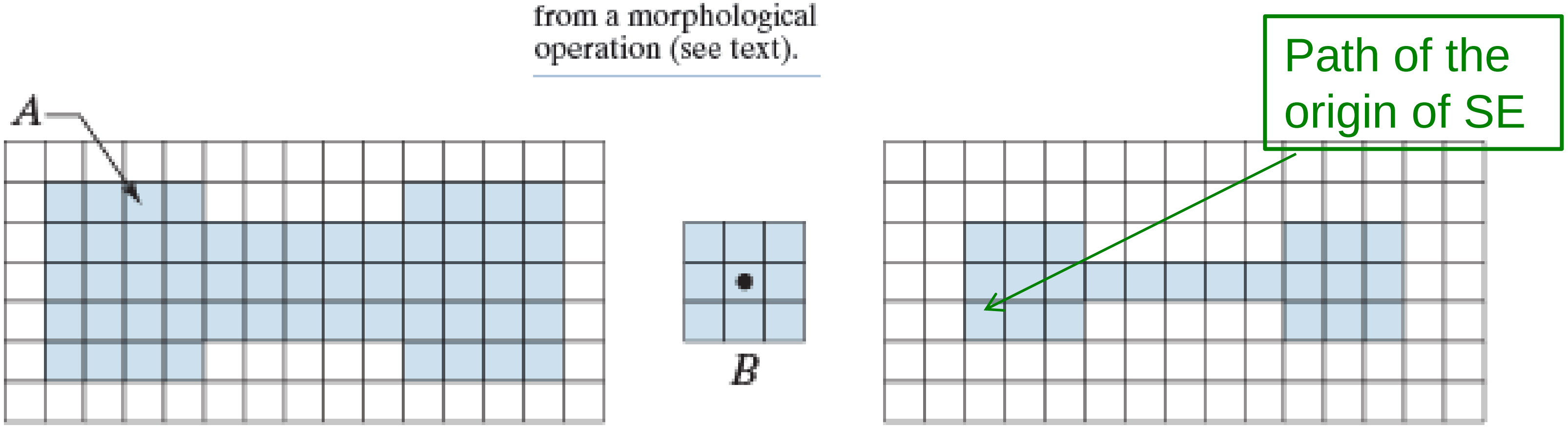
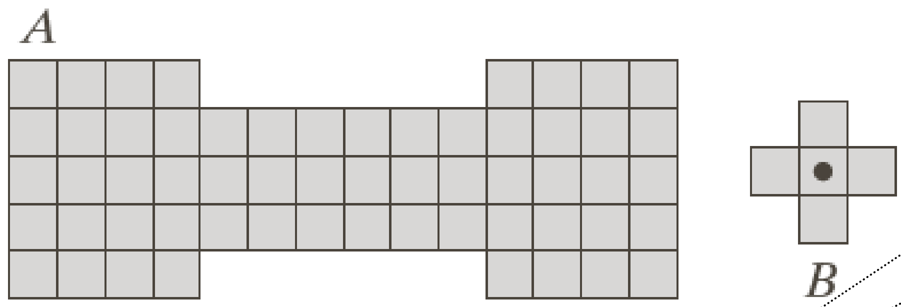


Image I Image after morphological operation

- This is Erosion: A shrinking operation

Structuring Elements (cont.)

Accommodate the entire structuring elements when its origin is on the border of the original set A



Origin of B visits every element of A

At each location of the origin of B, if B is completely contained in A, then the location is a member of the new set, otherwise it is not a member of the new set.

	a	b
c	d	e

FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

Erosion

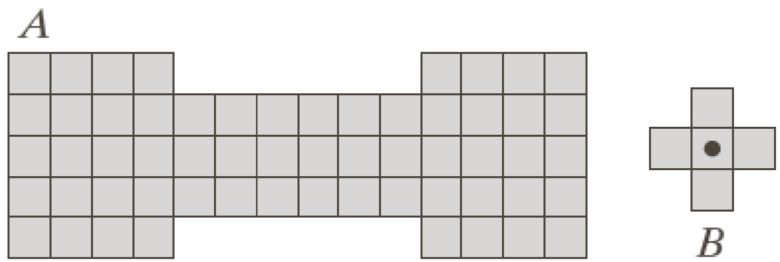
With A and B as sets in Z^2, the erosion of A by B, denoted $A \ominus B$, is defined as,

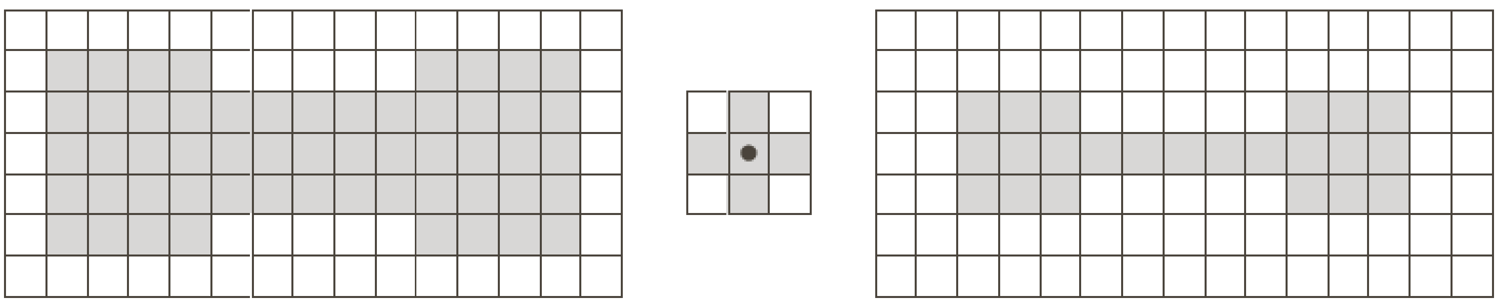
$$A \ominus B = \left\{ z \mid (B)_Z \subseteq A \right\}$$

- The result is the set of all points z such that B translated by z is contained in A.
- Equivalently: B does not share any common element with the background (= Complement of A)

$$A \ominus B = \left\{ z \mid (B)_Z \cap A^c = \varnothing \right\}$$

Erosion (cont.)



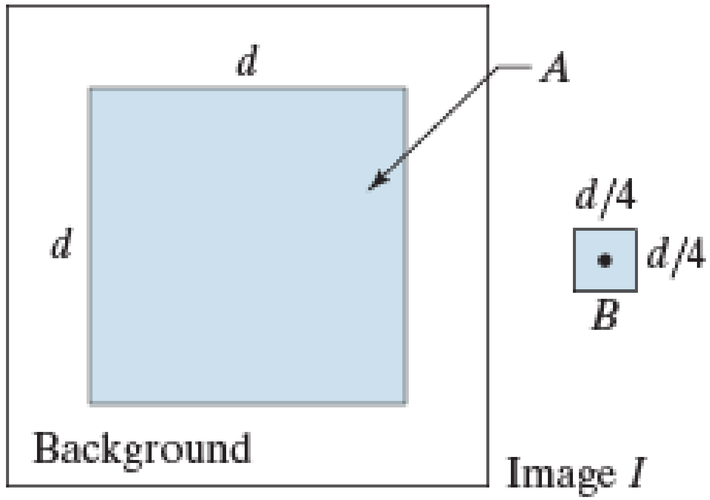


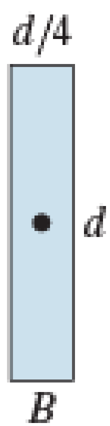
Erosion is a shrinking operation

Example of Erosion (1)

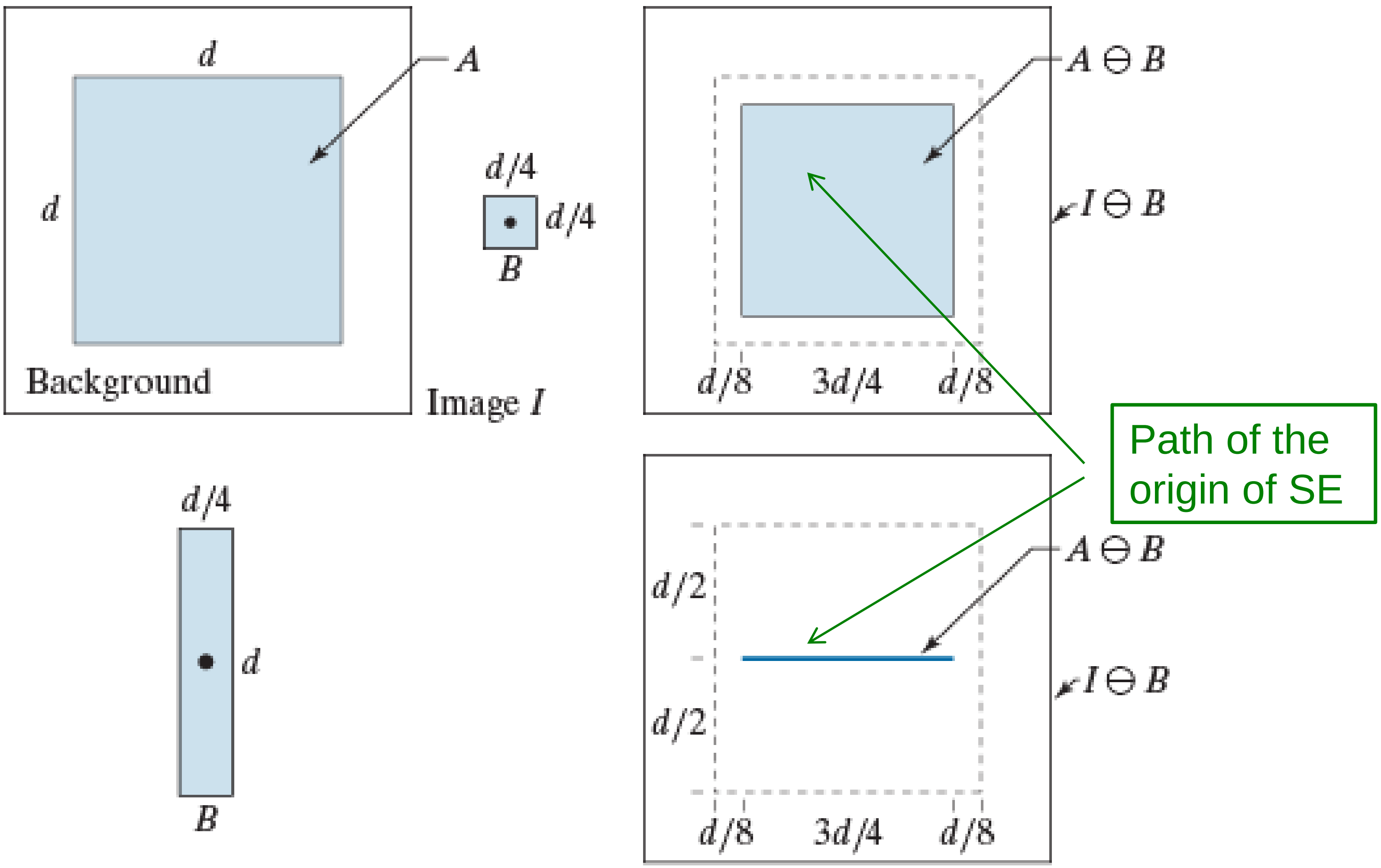
a b c
d e

FIGURE 9.4
(a) Image I, consisting of a set (object) A, and background.
(b) Square SE, B (the dot is the origin).
(c) Erosion of A by B (shown shaded in the resulting image).
(d) Elongated SE.
(e) Erosion of A by B. (The erosion is a line.) The dotted border in (c) and (e) is the boundary of A, shown for reference.





Erosion (cont.)



Erosion (cont.)

Erosion by a square SE of varying size

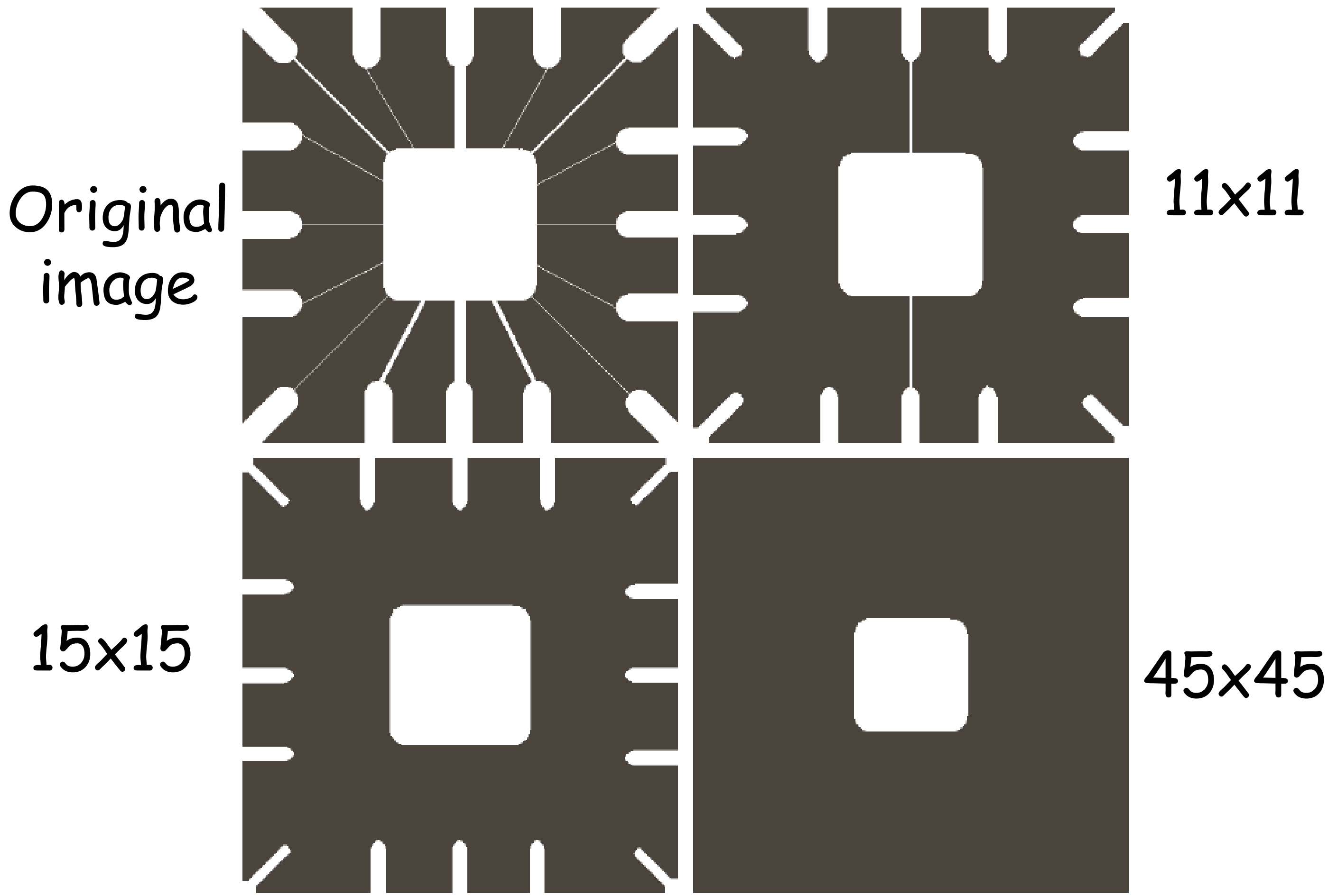
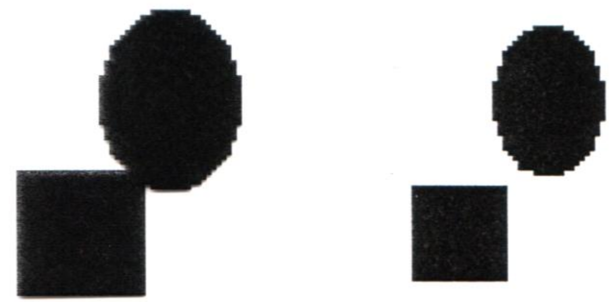


FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask in which foreground pixels are shown in white. (b)–(d) Image eroded using square structuring elements of sizes 11×11, 15×15, and 45×45 elements, respectively, all valued 1.

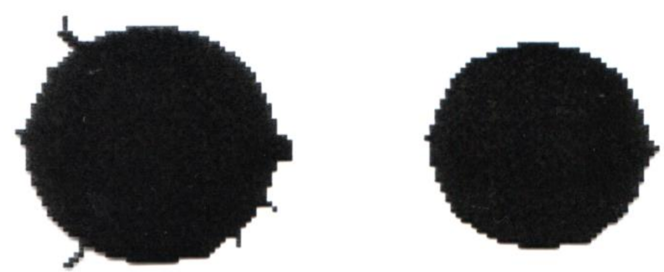
Erosion (cont.)

- Erosion can split apart joined objects

- Erosion can strip away extrusions



- Erosion shrinks objects



9 Dilation

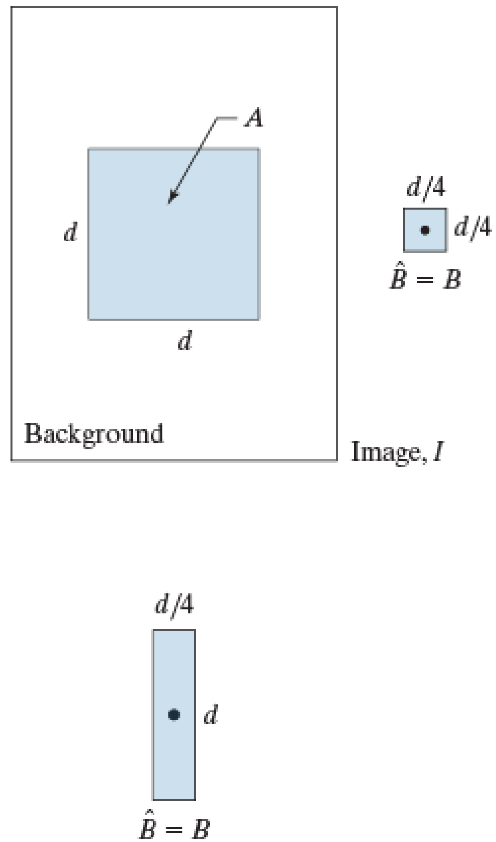
With A and B as sets in Z^2, the dilation of A by B, denoted $A \oplus B$, is defined as

$$A \oplus B = \left\{ z \mid \left(\hat{B} \right)_z \cap A \neq \varnothing \right\}$$

The set of all displacements z, the translated $\hat{B}$ and A **overlap by at least one element.**

$$A \oplus B = \left\{ z \mid \left[\left(\hat{B} \right)_z \cap A \right] \subseteq A \right\}$$

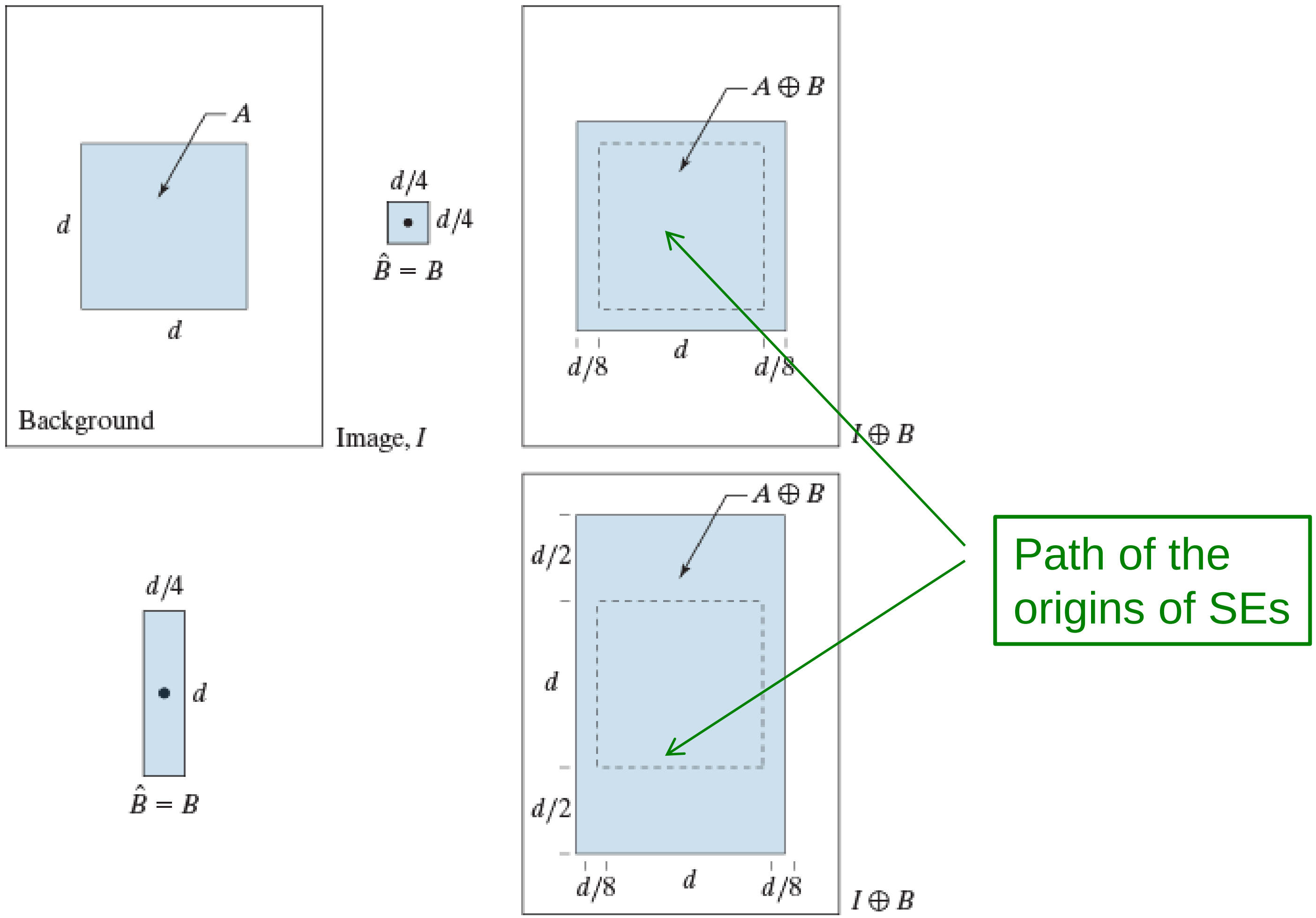
Examples of Dilation (1)



a b c
d e

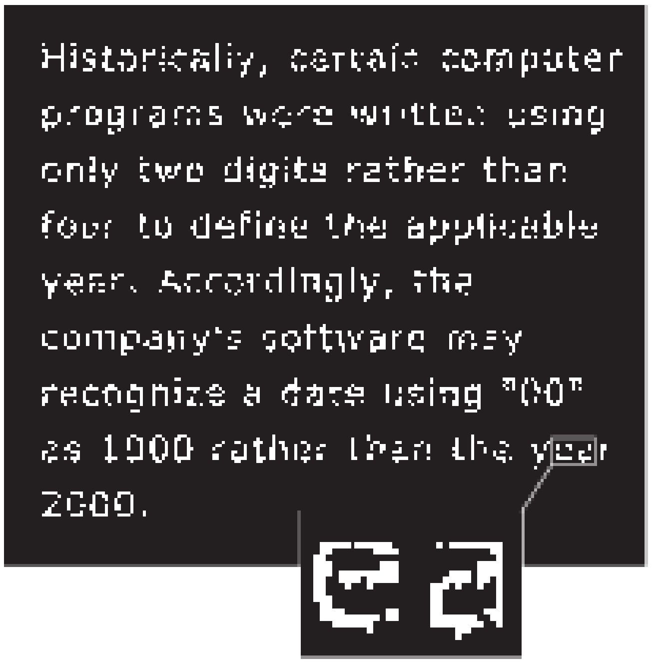
FIGURE 9.6
(a) Image I, composed of set (object) A and background.
(b) Square SE (the dot is the origin).
(c) Dilation of A by B (shown shaded).
(d) Elongated SE.
(e) Dilation of A by this element. The dotted line in (c) and (e) is the boundary of A, shown for reference.

Dilation (cont.)



Dilation is a thickening operation

Examples of Dilation (2)



1	1	1
1	1	1
1	1	1

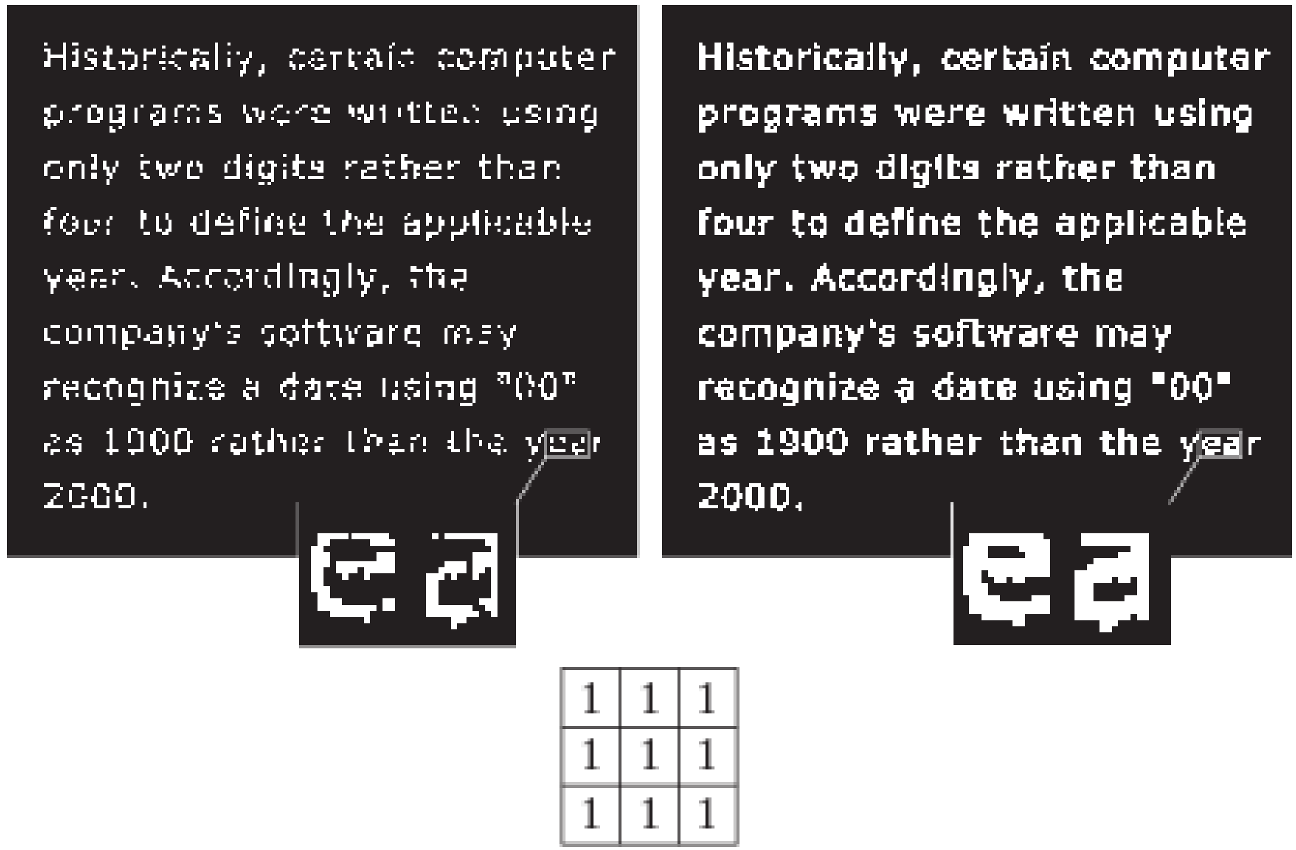
a c
b

FIGURE 9.7
(a) Low-resolution text showing broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Dilation (cont.)

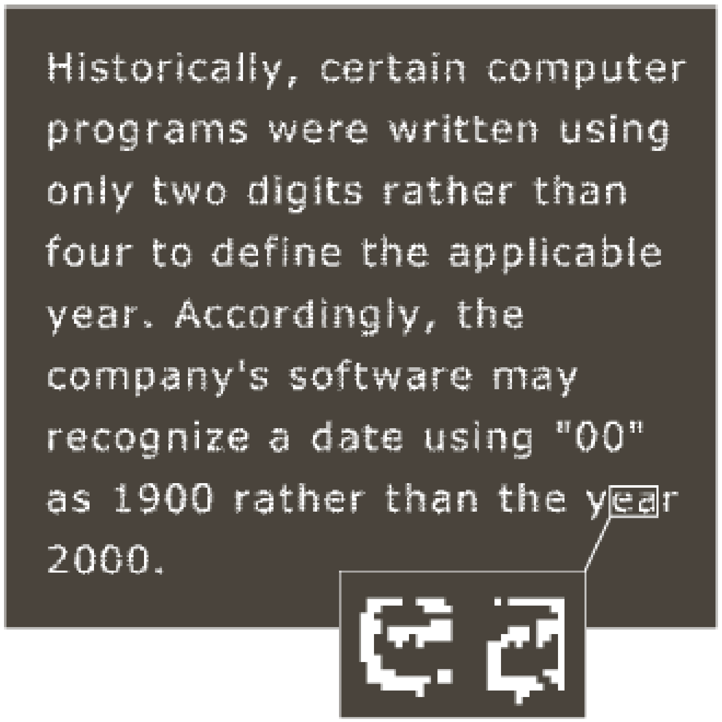
a b c

FIGURE 9.7
(a) Low-resolution text showing broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.



- Dilation bridges gaps.
- Contrary to low pass filtering it produces a binary image.

Examples of Dilation (2)



0	1	0
1	1	1
0	1	0

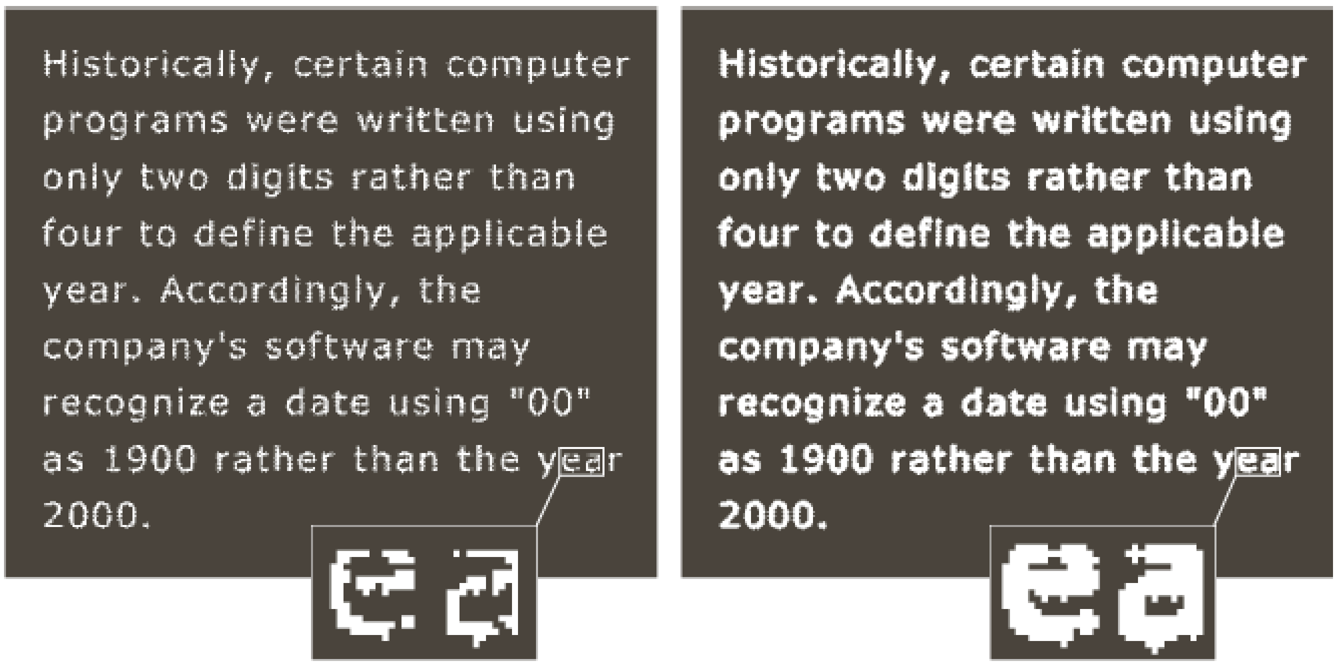
a c
b

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

3rd Edition

Dilation (cont.)

- Dilation bridges gaps.
- Contrary to low pass filtering it produces a binary image.



0	1	0
1	1	1
0	1	0

3rd Edition

Dilation (cont.)

- Dilation can repair breaks

- Dilation can repair intrusions
- Dilation enlarges objects

Duality

- Erosion and dilation are dual operations with respect to set complementation and reflection:

$$\left(A \ominus B\right)^c = A^c \oplus B$$

$$\left(A \oplus B\right)^c = A^c \ominus B$$

- Duality is useful when the SE is symmetric, $B = B$
- Then erosion of an image is the dilation of its background.

Duality (proof)

- Erosion and dilation are duals of each other with respect to set complementation and reflection

$$
\begin{aligned}
(A \oplus B)^c &= \left\{ z \mid \left(\hat{B}\right)_z \cap A \neq \varnothing \right\}^c \\
&= \left\{ z \mid \left(\hat{B}\right)_z \cap A^c = \varnothing \right\} \\
&= A^c \ominus \hat{B}
\end{aligned}
$$

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9.3 Opening And Closing

- More interesting morphological operations can be performed by combining erosions and dilations in order to reduce shrinking or thickening.
- The most widely used of these *compound operations* are:
  - Opening
  - Closing

Opening

- Opening generally
  - Smoothes the contour of an object
  - Breaks narrow isthmuses and
  - Eliminates thin protrusions

The opening of set A by structuring element B, denoted $A \circ B$, is defined as

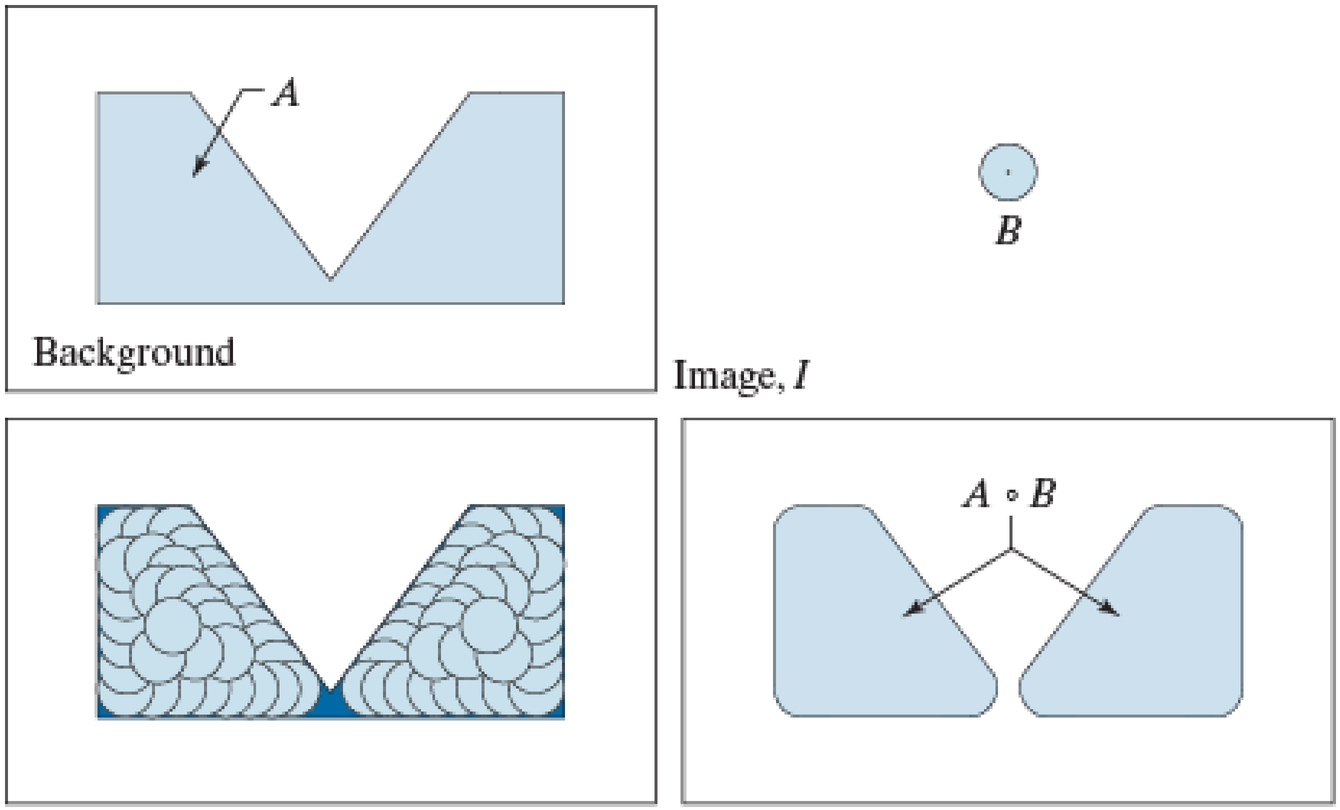
$$A \circ B = (A \ominus B) \oplus B$$

- An erosion of A by B followed by a dilation of the result by B.

Example: Opening

a	b
c	d

FIGURE 9.8
(a) Image I, composed of set (object) A and background.
(b) Structuring element, B.
(c) Translations of B while being contained in A. (A is shown dark for clarity.)
(d) Opening of A by B.



Opening (cont.)

- **Geometric Interpretation:** The boundary of the opening is defined by points of the SE that reach the farthest into the boundary of A as B is "rolled" inside of this boundary.

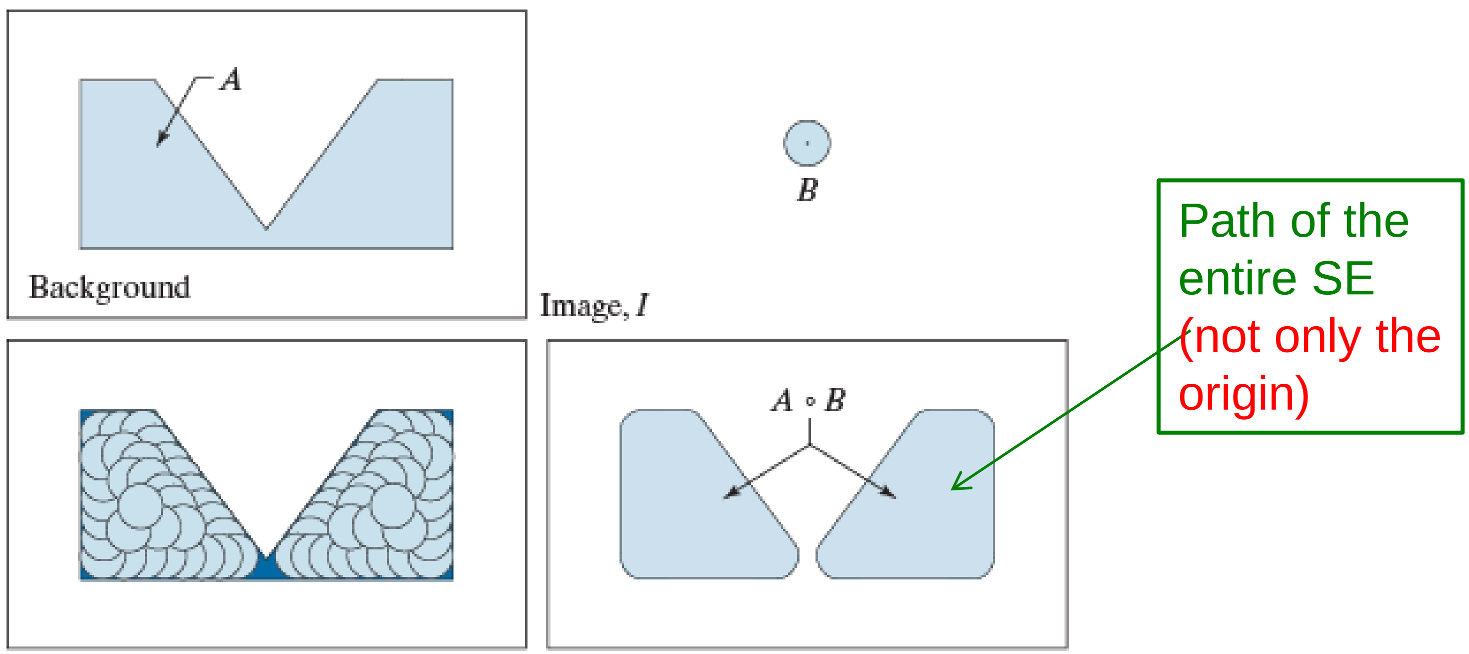


FIGURE 9.8 (a) Image I, composed of set (object) A and background. (b) Structuring element, B. (c) Translations of B while being contained in A. (A is shown dark for clarity.) (d) Opening of A by B.

$$A \circ B = \bigcup \{(B)_z \mid (B)_z \subseteq A\}$$

Opening (cont.)

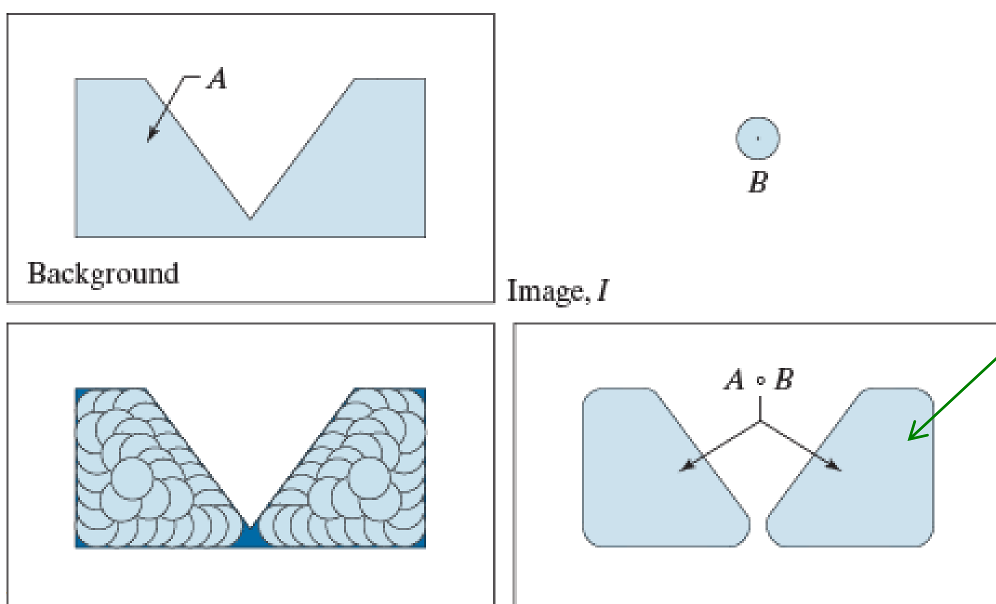
- Notice the difference with simple Erosion:

$$A \ominus B = \{z \mid (B)_z \subseteq A\} \qquad A \circ B = \bigcup \{(B)_z \mid (B)_z \subseteq A\}$$

- If B translated by z lies inside A, then the result contains the whole set of points covered by the SE and <u>not only its center as it is done in the erosion</u>.

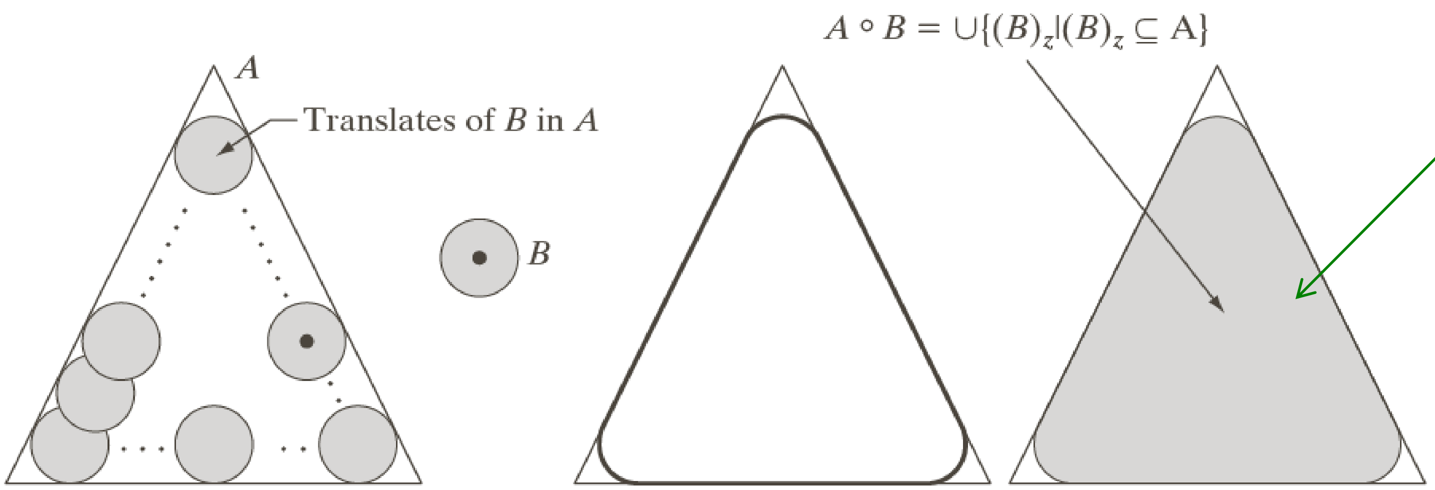
a b
c d

FIGURE 9.8
(a) Image I, composed of set (object) A and background.
(b) Structuring element, B.
(c) Translations of B while being contained in A. (A is shown dark for clarity.)
(d) Opening of A by B.



Path of the entire SE (not only the origin)

Opening (cont.)



Path of the entire SE (not only the origin)

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Closing

The closing of set A by structuring element B, denoted $A \bullet B$, is defined as

$$A \bullet B = \left(A \oplus B \right) \ominus B$$

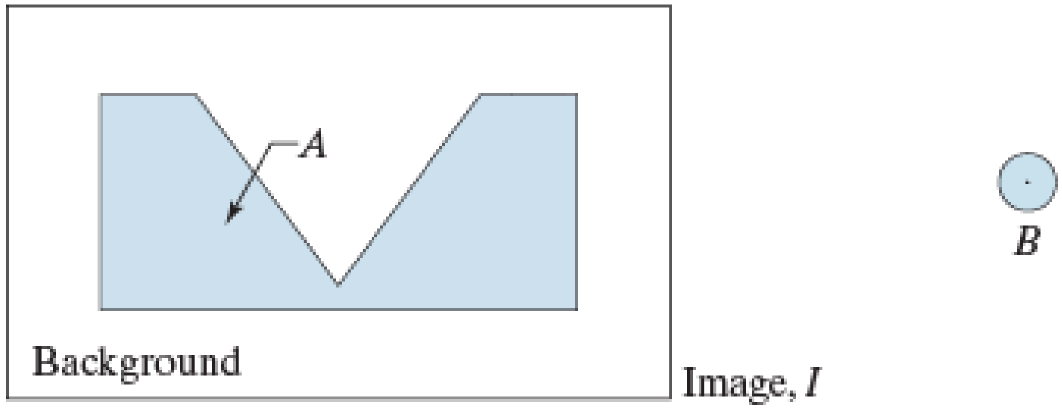
- Closing is dilation of A by B followed by an erosion of the result by B.

- Closing tends to smooth sections of contours but it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour

Example: Closing

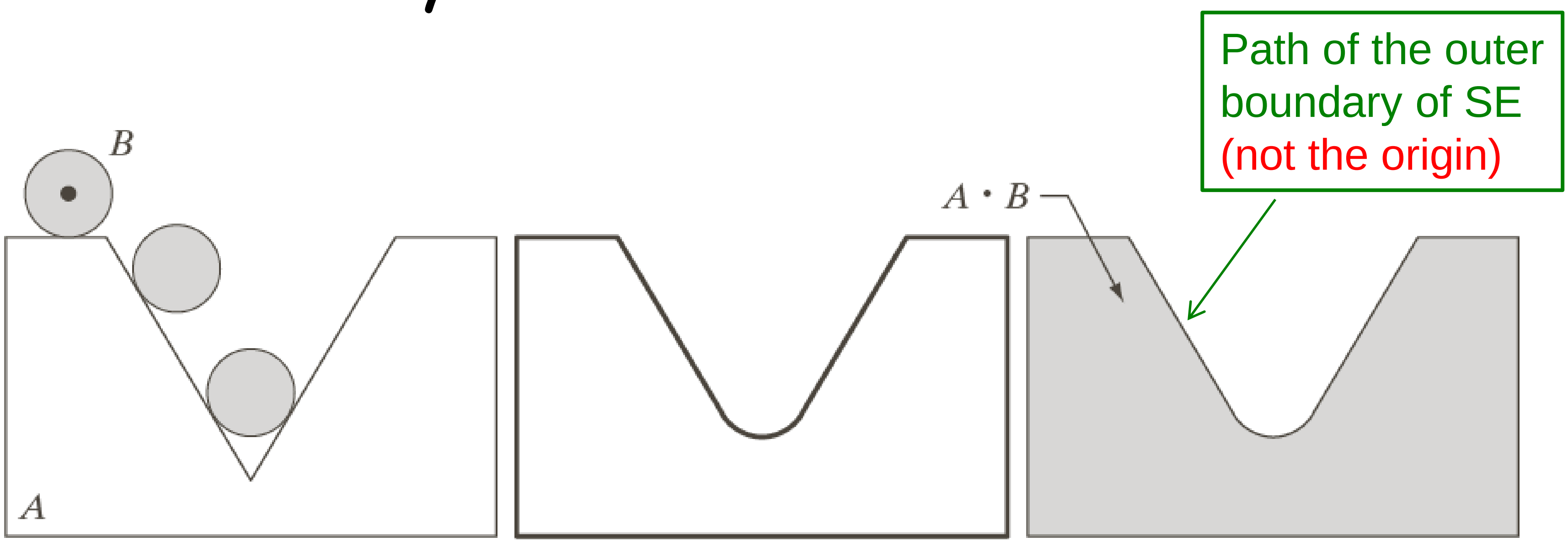
a b
c d

FIGURE 9.9
(a) Image I, composed of set (object) A, and background.
(b) Structuring element B.
(c) Translations of B such that B does not overlap any part of A. (A is shown dark for clarity.)
(d) Closing of A by B.

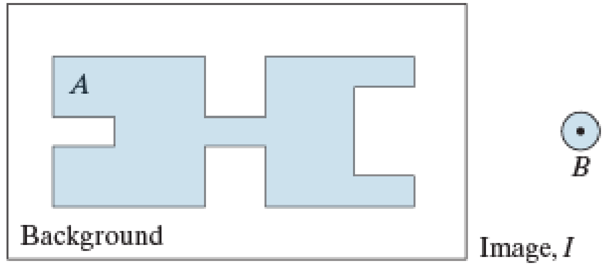


Closing (cont.)

It has a similar geometric interpretation except that B is rolled on the outside of the boundary:



$$A \bullet B = \{w \mid (B)_z \cap A \neq \varnothing, \text{for all translates of } (B)_z \text{ containing } w\}$$



a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. (a) Image *I*, composed of a set (object) *A* and background; a solid, circular structuring element is shown also. (The dot is the origin.) (b) Structuring element in various positions. (c)-(i) The morphological operations used to obtain the opening and closing.

Erosion

+ Dilation =

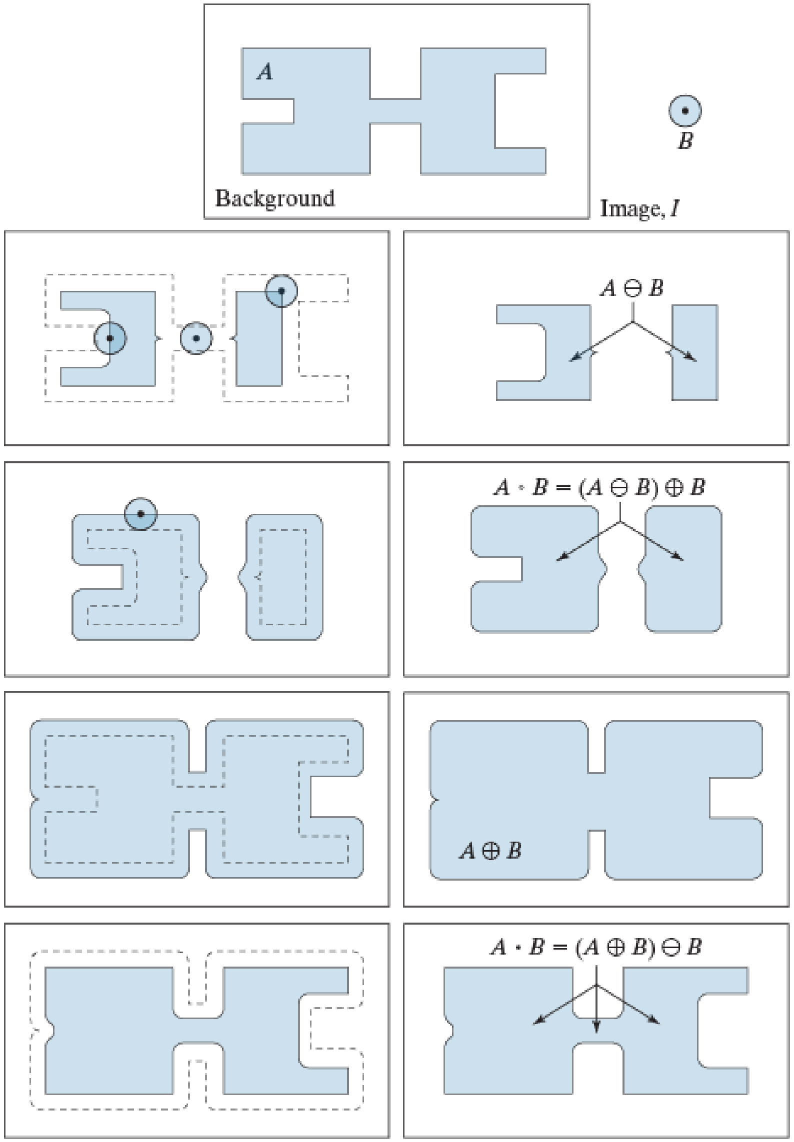
Opening

Dilation

+ Erosion =

Closing

Opening and Closing



Erosion: Elements where the disk can not fit are eliminated

Opening: Outward corners are rounded

Dilation: Inward intrusions are reduced in depth

Closing: Inward corners are rounded

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Duality

Opening and closing are duals of each other with respect to set complementation and reflection.

Erosion-Dilation duality

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

$$(A \oplus B)^c = A^c \ominus \hat{B}$$

Opening-Closing duality

$$(A \bullet B)^c = A^c \circ \hat{B}$$

$$(A \circ B)^c = A^c \bullet \hat{B}$$

Properties of Opening and Closing

- Properties of Opening

(a) $A \circ B$ is a subset (subimage) of A

- Properties of Closing

Properties of Opening and Closing

Opening:

$$A \circ B \subseteq A$$

$$C \subseteq D \Rightarrow C \circ B \subseteq D \circ B$$

$$(A \circ B) \circ B = A \circ B$$

Closing:

$$A \subseteq A \bullet B$$

$$C \subseteq D \Rightarrow C \bullet B \subseteq D \bullet B$$

$$(A \bullet B) \bullet B = A \bullet B$$

The last properties, in each case, indicate that multiple openings or closings have no effect after the first application of the operator

Morphological Filtering Example

1	1	1	B
1	1	1	
1	1	1	

The image contains noise:

- Light elements on dark background.
- Dark elements on the light components of the fingerprint.

Objective:

- Eliminate noise with as little distortion as possible
- We will apply an opening followed by closing

a b
d c
e f

FIGURE 9.11
(a) Noisy image. (b) Structuring element. (c) Eroded image. (d) Dilation of the erosion (opening of A). (e) Dilation of the opening. (f) Closing of the opening. (Original image courtesy of the National Institute of Standards and Technology.)

Morphological Filtering Example (cont.)

A

$A \ominus B$

- Erosion: Background noise is completely removed (noise components smaller than the SE).
- However, Size of the dark noise elements in bright fingerprint structure increased (inner dark structures).

Morphological Filtering Example (cont.)

$$A \ominus B$$

$$(A \ominus B) \oplus B = A \circ B$$

- Dilation of Erosion: Reduces the size of the inner noise dark spots or eliminated it completely
- However, new gaps were created by the opening between the fingerprint bright ridges

→ need to connect → Dilate again

Morphological Filtering Example (cont.)

$A \circ B$

$A \circ B \oplus B$

- The dilation reduces the new gaps between the ridges but it also thickens the ridges.

  → Need to Erode

Morphological Filtering Example (cont.)

$A \circ B \oplus B$

$[A \circ B \oplus B] \ominus B = (A \circ B) \bullet B$

- The final erosion (resulting to a closing of the opened image) makes the ridges thinner.

Morphological Filtering Example (cont.)

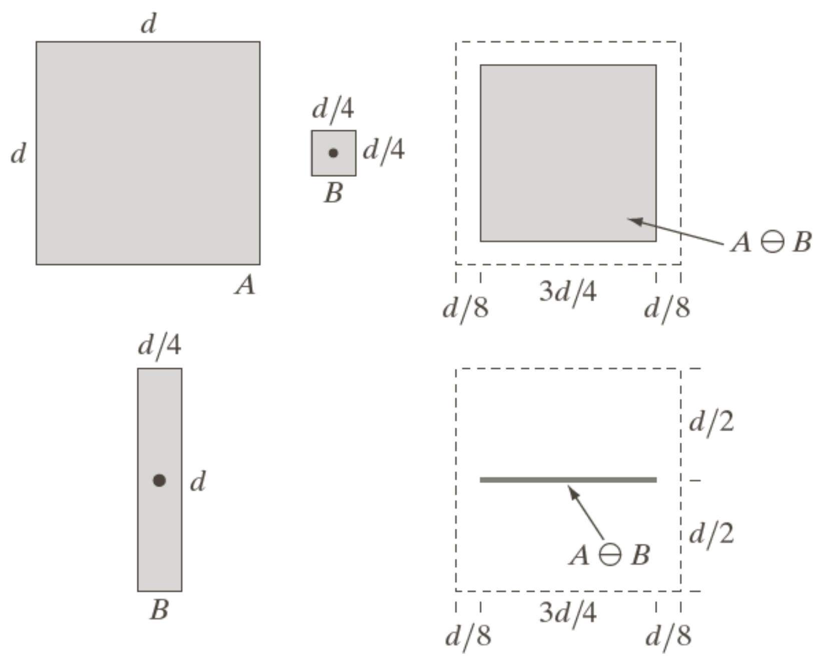
A

$(A \circ B) \bullet B$

- The final result is clean of noise but some ridges were not fully repaired.
- Conditions for maintaining the connectivity need to be imposed (Done in a more advanced algorithm - later).

9.4 Hit-or-Miss Transformation

- Basic tool for Shape Detection
- Erosion of A by B: The set of all locations of the <u>origin</u> of B so that B is completely contained in A.
- Alternate interpretation: It is the set of all locations that B found a match (i.e., hit) in A.



Hit-or-Miss Transformation

- There may be multiple disjoint locations for the shape (the SE!) being searched
- If we are looking for disjoint (disconnected) shapes it is natural to assume a background for it.
- Therefore, we seek to match B in A and simultaneously we seek to match the background of B (i.e., B_b) in A^c.
- Mathematically, the hit-or-miss transformation is:

$$A \circledast B = (A \ominus B) \cap (A^c \ominus B_b)$$

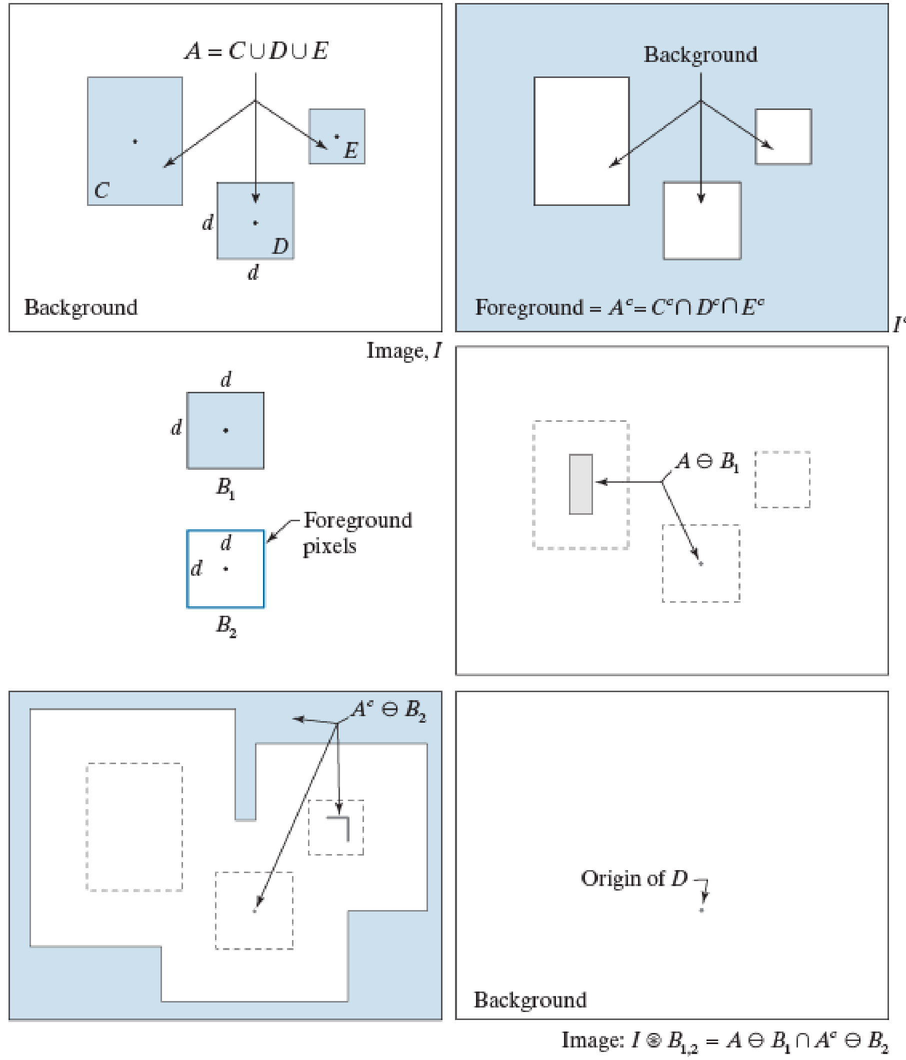
The Hit-or-Miss Transformation (cont.)

- Goal: Locate the shape D in the image A.

- Define a thin background B_2 for the shape.

- Take the intersection of the two results – The Hit!

$$A \circledast D = (A \ominus B_1) \cap \left[A^c \ominus B_2 \right]$$



a b
c d
e f

FIGURE 9.12 (a) Image consisting of a foreground (1's) equal to the union, A, of set of objects, and a background of 0's. (b) Image with its foreground defined as A^c. (c) Structuring elements designed to detect object D. (d) Erosion of A by B_1. (e) Erosion of A^c by B_2. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origin of their respective components. Each dot is a single pixel.

The Hit-or-Miss Transformation (cont.)

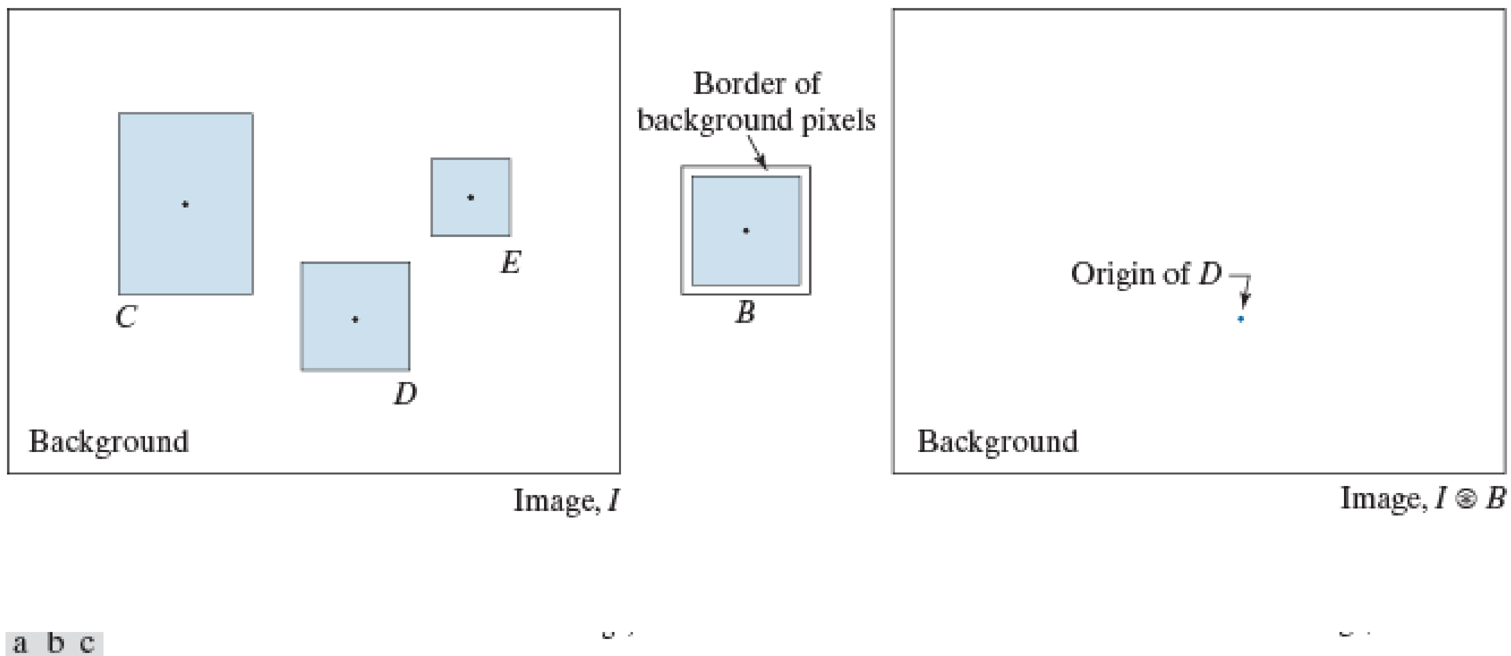


FIGURE 9.13 Same solution as in Fig. 9.12, but using Eq. (9-17) with a single structuring element.

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9.5 Morphological Algorithms

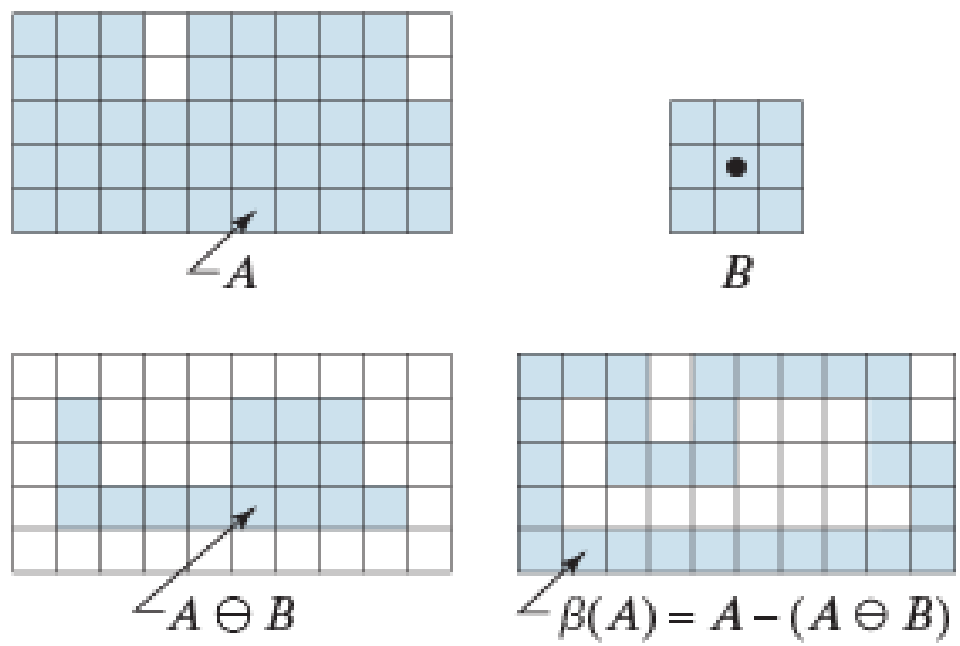
- Using these morphological operations image components can be extracted for shape representation:
  - Shape Boundaries
  - Region Filling
  - Connected Components
  - Convex Hull
  - Shape Thinning and Thickening
  - Skeletons
- Also, morphological image reconstruction

Boundary Extraction

- The boundary of a set A, denoted by $\beta(A)$, may be obtained by:

Erosion

$$\beta(A) = A - (A \ominus B)$$



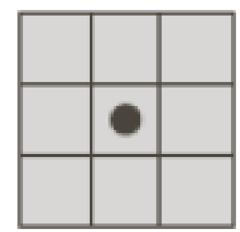
a b
c d

FIGURE 9.15
(a) Set, A, of foreground pixels.
(b) Structuring element.
(c) A eroded by B.
(d) Boundary of A.

Boundary Extraction Example

a b

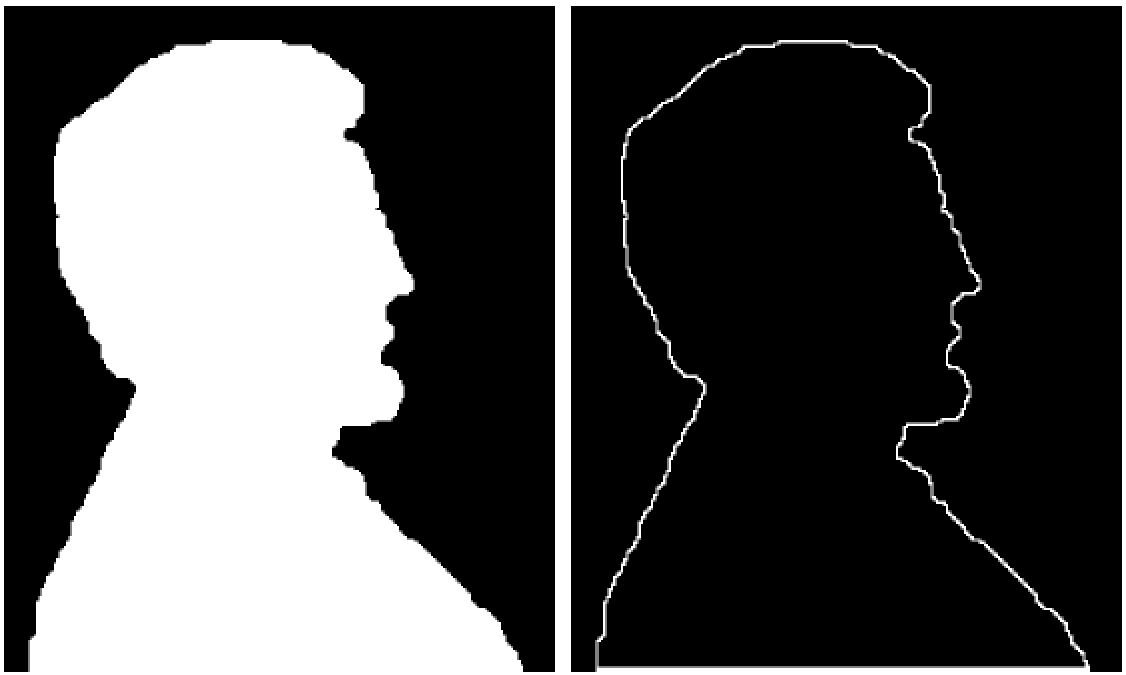
FIGURE 9.16
(a) A binary image.
(b) Result of using Eq. (9-18) with the structuring element in Fig. 9.15(b).



B

Boundary Extraction (cont.)

The boundary is one pixel thick due to the 3x3 SE. Other SE would result in thicker boundaries.



a b

FIGURE 9.16 (a) A binary image. (b) Result of using Eq. (9-18) with the structuring element in Fig. 9.15(b).

Original Image

Extracted Boundary

Hole Filling

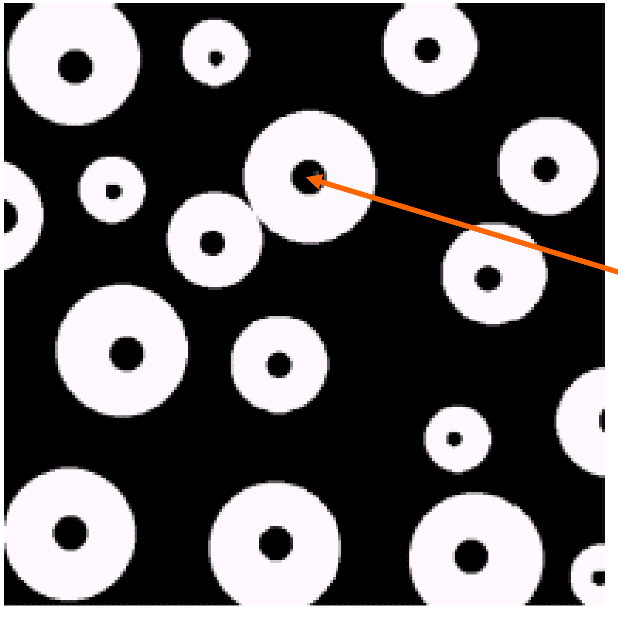
- A hole may be defined as a background region surrounded by a connected border of foreground pixels.

- Let A denote a set whose elements are 8-connected boundaries, each boundary enclosing a background region (i.e., a hole).

- Given a point in each hole, the objective is to fill all the holes with 1s.

Hole Filling

- Given a pixel inside a boundary, hole/region filling attempts to fill the area surrounded by that boundary with 1s.



Given a point inside here, can we fill the whole circle?

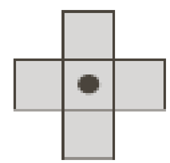
Hole Filling

1. Form an array X_0 of 0s (with same size as the array containing A), except the locations in X_0 corresponding to the given "seed" point in each hole, which is set to 1.

Dilation → Looks for overlap in at least one location

2. $$X_k = (X_{k-1} \oplus B) \cap A^c, \quad k = 1, 2, 3, \ldots$$

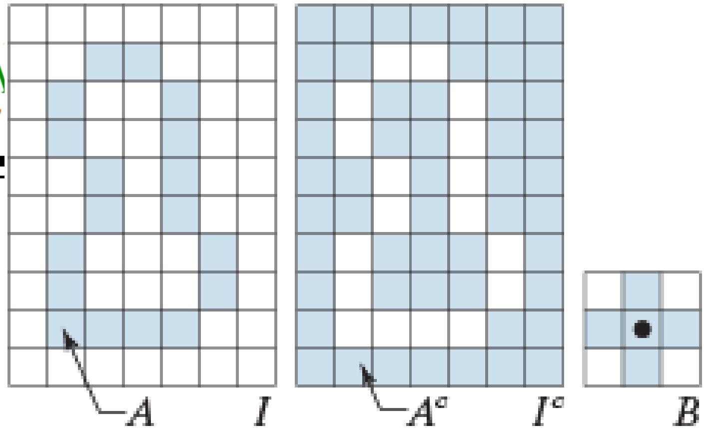
B : 3x3 cross-shaped SE.



3. Stop the iteration if $X_k = X_{k-1}$

- Final Step: The set union of X_k's and A contains all the filled holes and their boundaries.

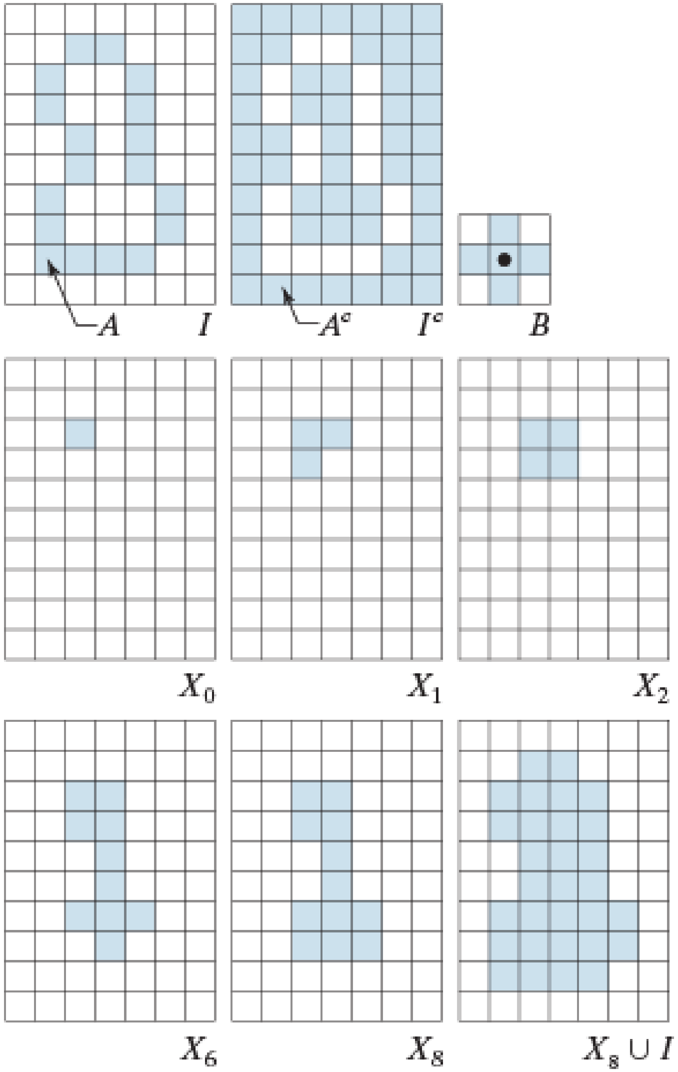
Example



a b c
d e f
g h i

FIGURE 9.17 Hole filling. (a) Set A (shown shaded) contained in image I. (b) Complement of I. (c) Structuring element B. *Only* the foreground elements are used in computations (d) Initial point inside hole, set to 1. (e)–(h) Various steps of Eq. (9-19). (i) Final result [union of (a) and (h)].

Hole/Region Filling (cont.)



a	b	c
d	e	f
g	h	i

FIGURE 9.17 Hole filling. (a) Set A (shown shaded) contained in image I. (b) Complement of I. (c) Structuring element B. *Only* the foreground elements are used in computations (d) Initial point inside hole, set to 1. (e)–(h) Various steps of Eq. (9-19). (i) Final result [union of (a) and (h)].

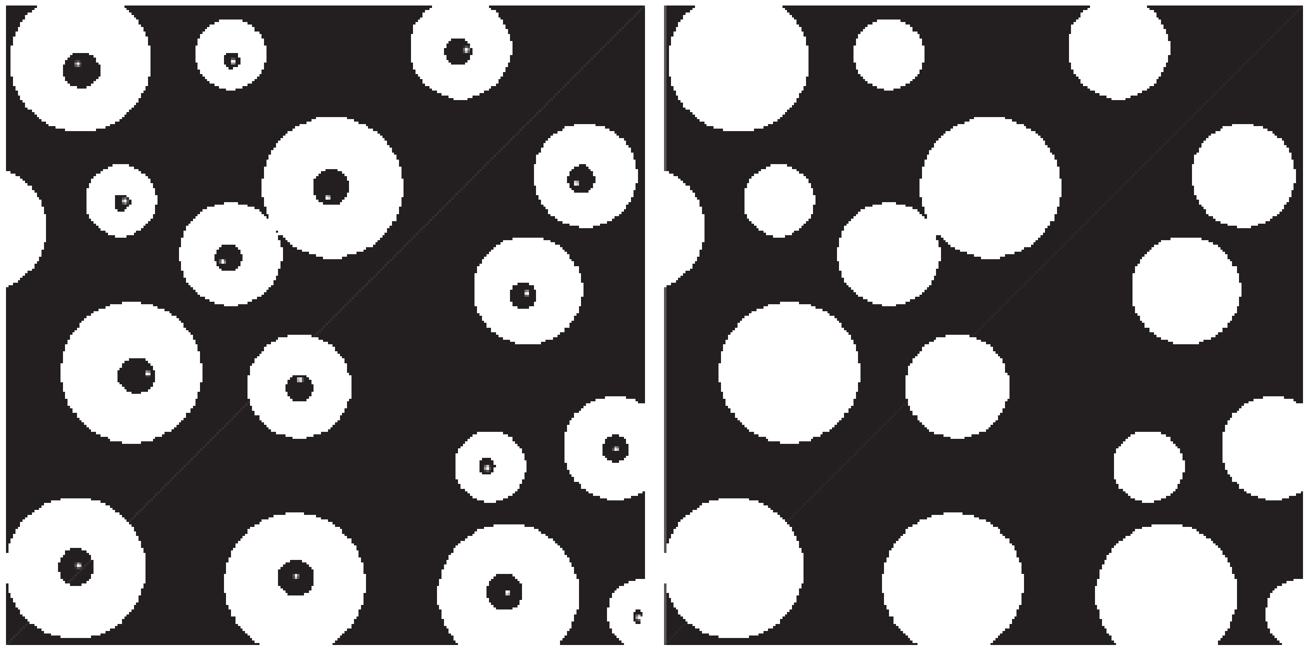
Hole/Region Filling (cont.)

- This is the first example where the morphological operation (dilation) is conditioned.
- Intersection of the result with A^c limits the result inside the region of interest.

$$X_k = (X_{k-1} \oplus B) \cap A^c, \quad k = 1, 2, 3, \ldots$$

- In this algorithm: Each seed-point has to be initiated manually
- Automated process is feasible

Hole Filling Example



a b

FIGURE 9.18 (a) Binary image. The white dots inside the regions (shown enlarged for clarity) are the starting points for the hole-filling algorithm. (b) Result of filling all holes.

Extraction of Connected Components

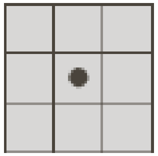
- Given a pixel on a connected component, find the rest of the pixels of that component.
- The algorithm may be applied to many connected components provided we know a pixel on each one of them.
- **Drawback:** We have to provide an initial pixel on the connected components.
- There are more sophisticated algorithms that detect the number of components without manual interaction.
- The purpose here is to demonstrate the flexibility of mathematical morphology.

Extraction of connected components (cont.)

- Form a set X_0 with zeros everywhere except at the seed point of the connected components.
- Then,

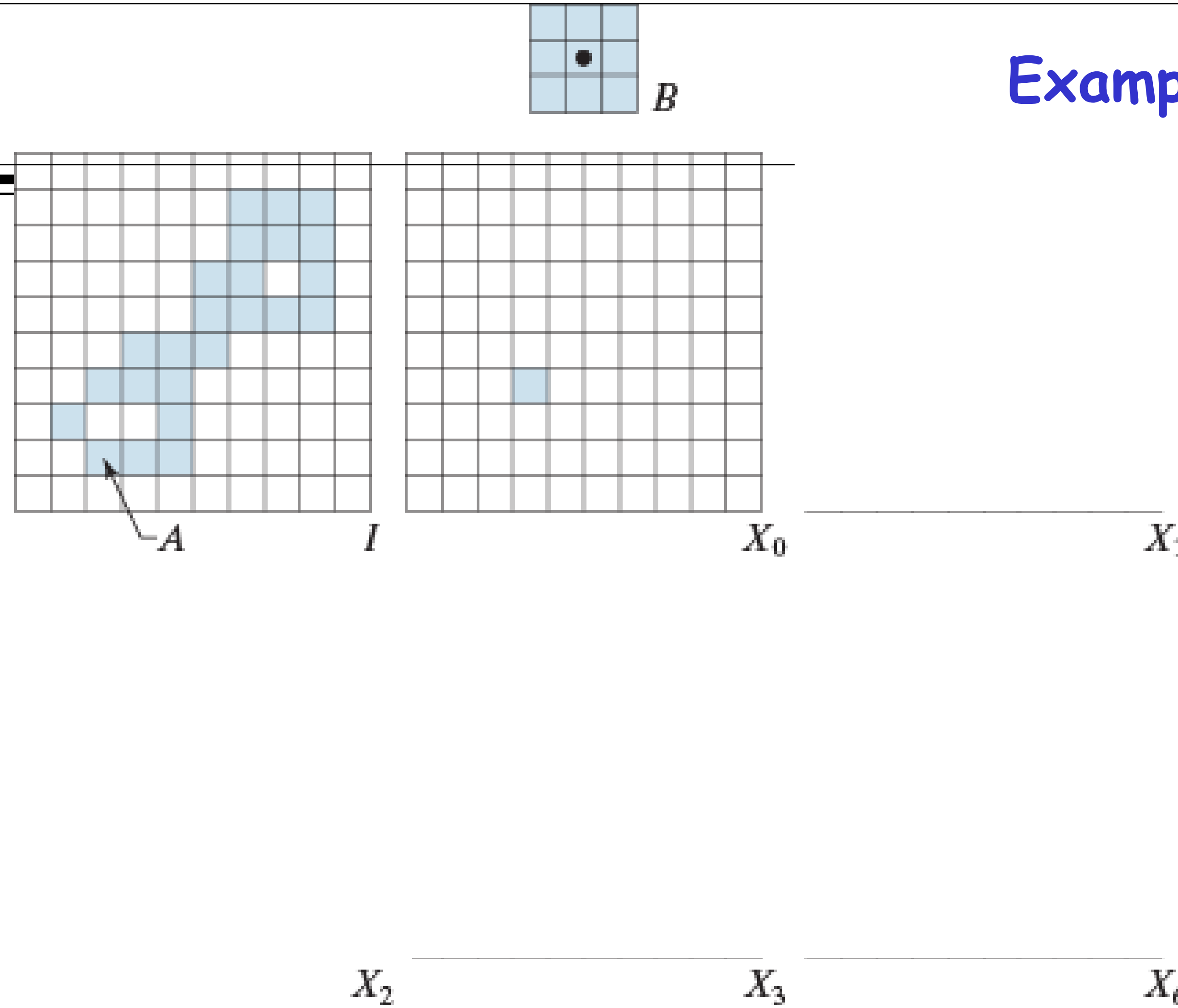
Dilation → Looks for overlap in at least one location

$$X_k = (X_{k-1} \oplus B) \cap A, \quad k = 1, 2, 3, \ldots$$



- B : 3x3 square-shaped SE.
- The algorithm terminates when $X_k = X_{k-1}$.
- X_k contains all the connected components.

Example

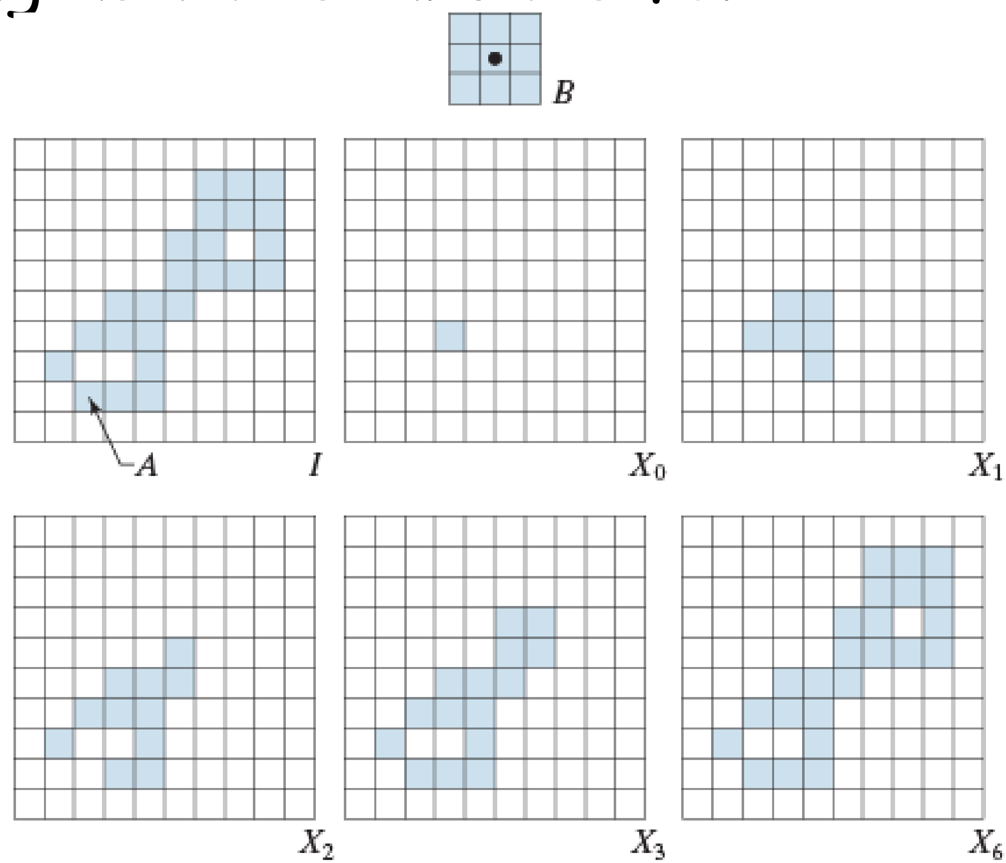


a
b c d
e f g

FIGURE 9.19
(a) Structuring element.
(b) Image containing a set with one connected component.
(c) Initial array containing a 1 in the region of the connected component.
(d)–(g) Various steps in the iteration of Eq. (9-20)

Extraction of Connected Components (cont.)

- Note the similarity with hole/region filling.
- The only difference is the use of A instead of A^c.
- This is not surprising as the search is for foreground objects.



a
b c d
e f g

FIGURE 9.19 (a) Structuring element. (b) Image containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)–(g) Various steps in the iteration of Eq. (9-20)

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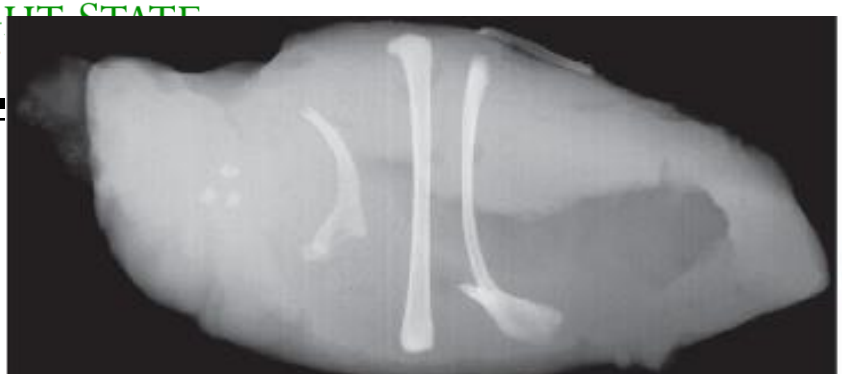
Example:

Automated Inspection



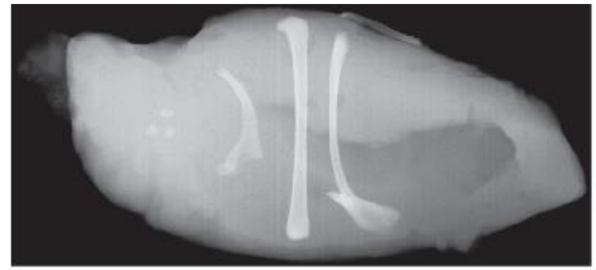
Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

a
b
c d

FIGURE 9.20
(a) X-ray image of a chicken filet with bone fragments.
(b) Thresholded image (shown as the negative for clarity).
(c) Image eroded with a 5×5 SE of 1's.
(d) Number of pixels in the connected components of (c).
(Image (a) courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Extraction of Connected Components (cont.)

• Image of chicken filet containing bone fragments



• Result of simple thresholding

•Image erosion retains only objects of significant size.

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

• 15 connected components detected with four of them being significant in size. This is an indication to remove the chicken filet from packaging.

Convex Hull

- A set A is convex if a straight line segment joining any two points in A lies entirely within A.
- The ***convex hull*** H of an arbitrary set S is the **smallest convex set containing** S.
- Set difference H-S is called ***convex deficiency***.
- The convex hull and the convex deficiency are useful quantities to characterize shapes.

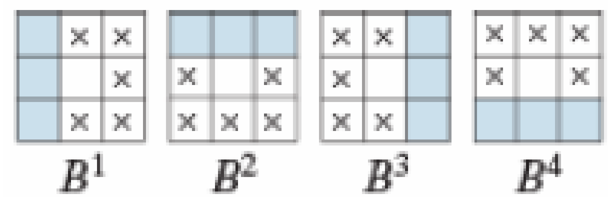
Convex Hull (cont.)

- The procedure requires four SE's: B^i, i =1, 2, 3, 4, and implements the following equation:

Hit or Miss (exact match with B^i s)

$$X_k^i = (X_{k-1} \circledast B^i) \cup A, \; i = 1, 2, 3, 4 \text{ and } k = 1, 2, 3, \ldots$$

$$\text{with } X_0^i = A$$



with i referring to the SE and k to the iteration.

- Then, let

$$D^i = X_k^i$$

- The convex hull of A is

$$C(A) = \bigcup_{i=1}^{4} D^i$$

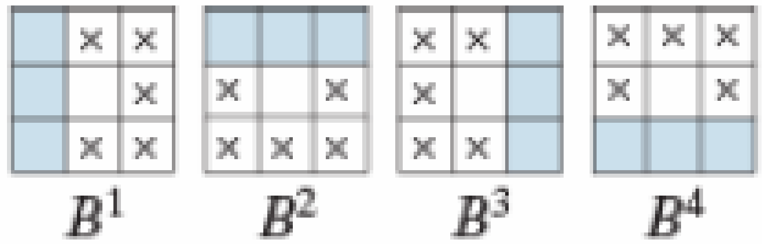
Convex Hull (cont.)

$$X_k^i = (X_{k-1} \circledast B^i) \cup A \quad i = 1, 2, 3, 4 \text{ and } k = 1, 2, 3, \ldots \text{ with } X_0^i = A$$

$$D^i = X_k^i, \; C(A) = \bigcup_{i=1}^{4} D^i$$

- The method consists of iteratively applying the hit-or miss transform to A with B^1.
- When no changes occur we perform the union with A and save the result to get D^1.
- The procedure is then continued with B^2 applied to A to get D^2 and so on.
- The union of the results is the convex hull of A.
- Note that a simple implementation of the hit or miss is applied (no background match required – simple erosion only).

Convex Hull (cont.)



- The hit-or-miss transform tries to find the match ("hit") with these structures in the image.
- x → Points on SE with "don't care" condition.
- For all SEs, a match is found in the image when the following conditions hold:
  - ➢ Center pixel in the 3x3 region in the image is 0 , AND
  - ➢ The three shaded pixels under the mask are 1s
- The remaining pixels do not matter

Convex Hull (cont.)

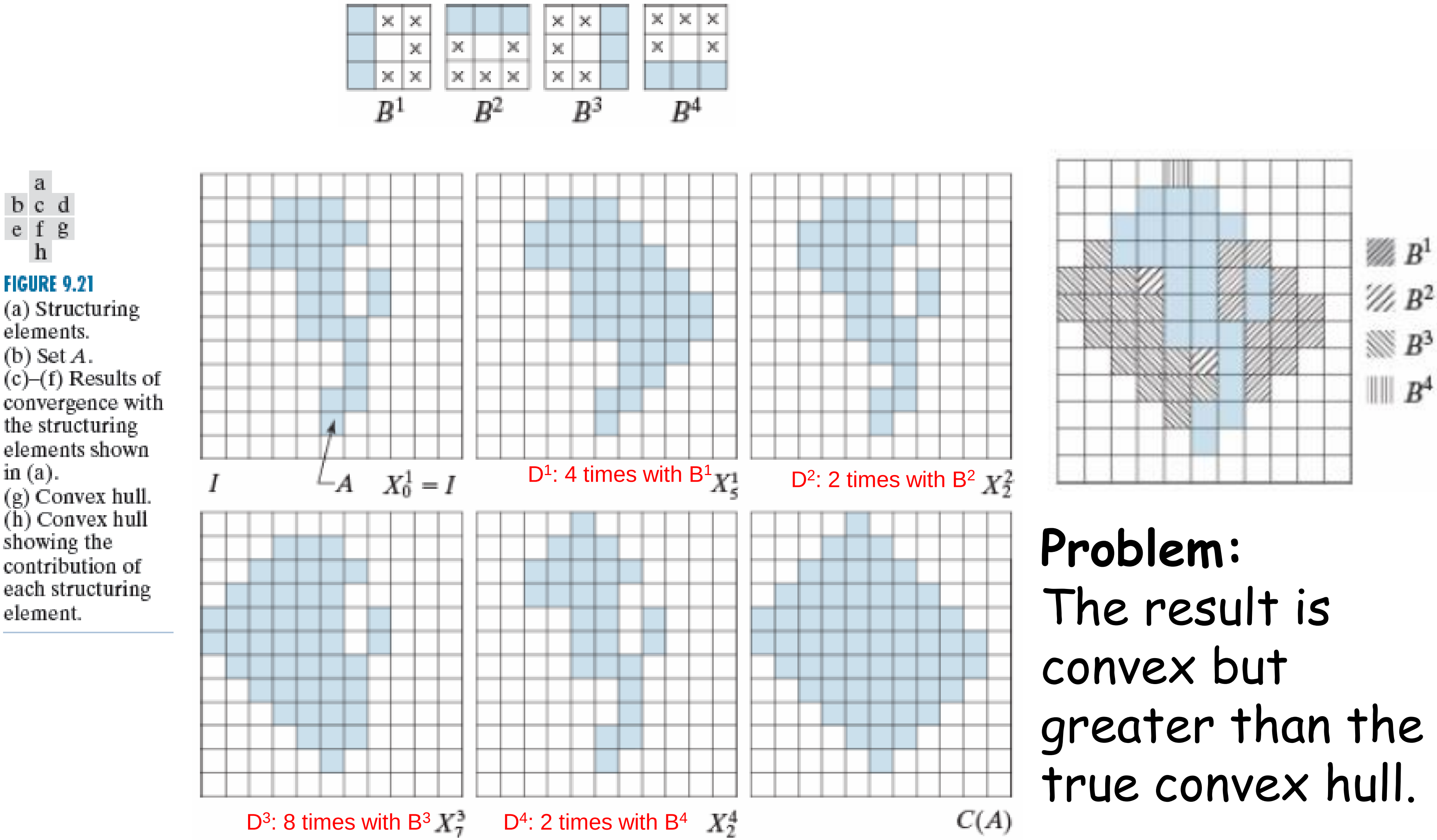
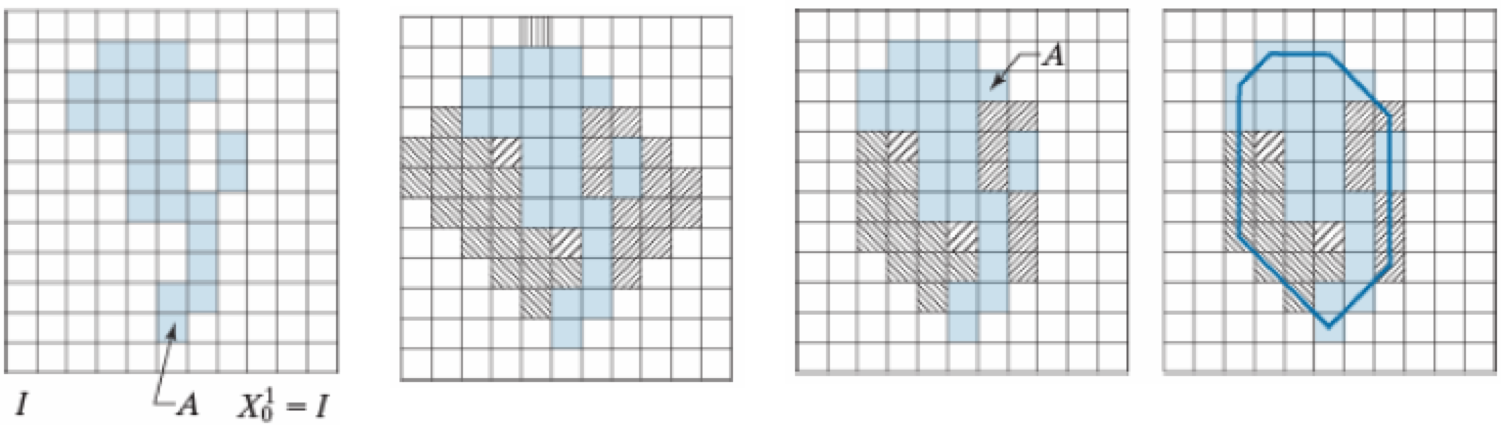


FIGURE 9.21
(a) Structuring elements.
(b) Set A.
(c)–(f) Results of convergence with the structuring elements shown in (a).
(g) Convex hull.
(h) Convex hull showing the contribution of each structuring element.

Problem:
The result is convex but greater than the true convex hull.

Convex Hull (cont.)

Solution: Limit the growth so that it does not extend past the horizontal and vertical limits of the original set of points.



a b

FIGURE 9.22
(a) Result of limiting growth of the convex hull algorithm.
(b) Straight lines connecting the boundary points show that the new set is convex also.

Original image Initial convex hull Refined convex hull

- More complex boundaries have been imposed to images with finer details in their structure (e.g. The maximum of the horizontal vertical and diagonal dimensions could be used).

Thinning

- The thinning of a set A, by a SE B may be defined in terms of the hit-or-miss transform:

Hit or Miss

$$A \otimes B = A - (A \circledast B) = A \cap (A \circledast B)^c$$

- Scoops out from A the "Hit" matched to the SE B
- No background match is required and the hit-or-miss part is reduced to simple erosion.

- A more advanced expression is based on a sequence of SE $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$,

where each B^i is a rotated version of B^{i-1}.

Thinning (cont.)

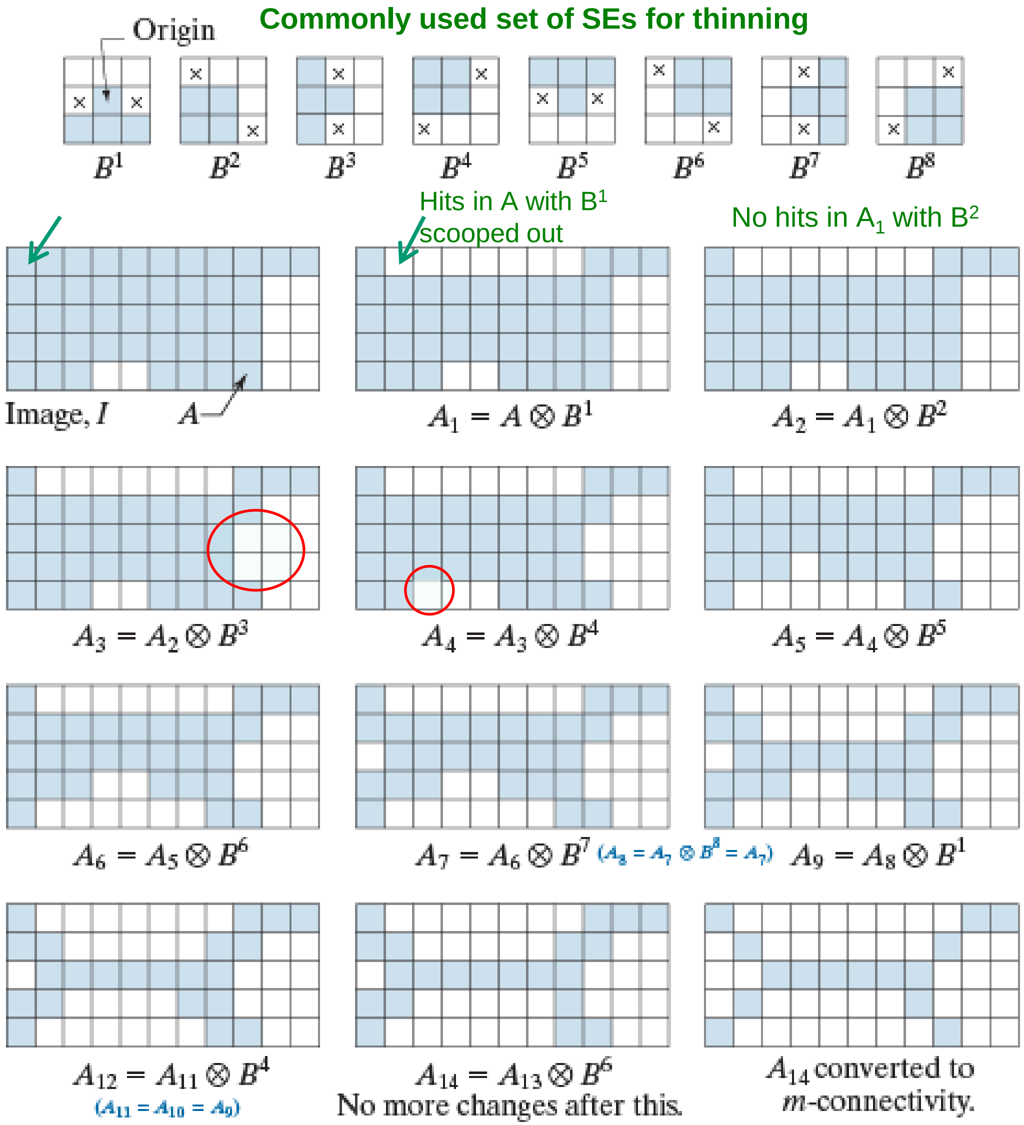
- Thinning by a sequence of SE is defined by:

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$

- The process is to thin A by one pass with B^1, then thin the result with one pass of B^2, and so on, until we employ B^n.
- The entire process is repeated until no further changes occur. Each individual thinning is performed by:

$$A \otimes B = A - (A \circledast B)$$

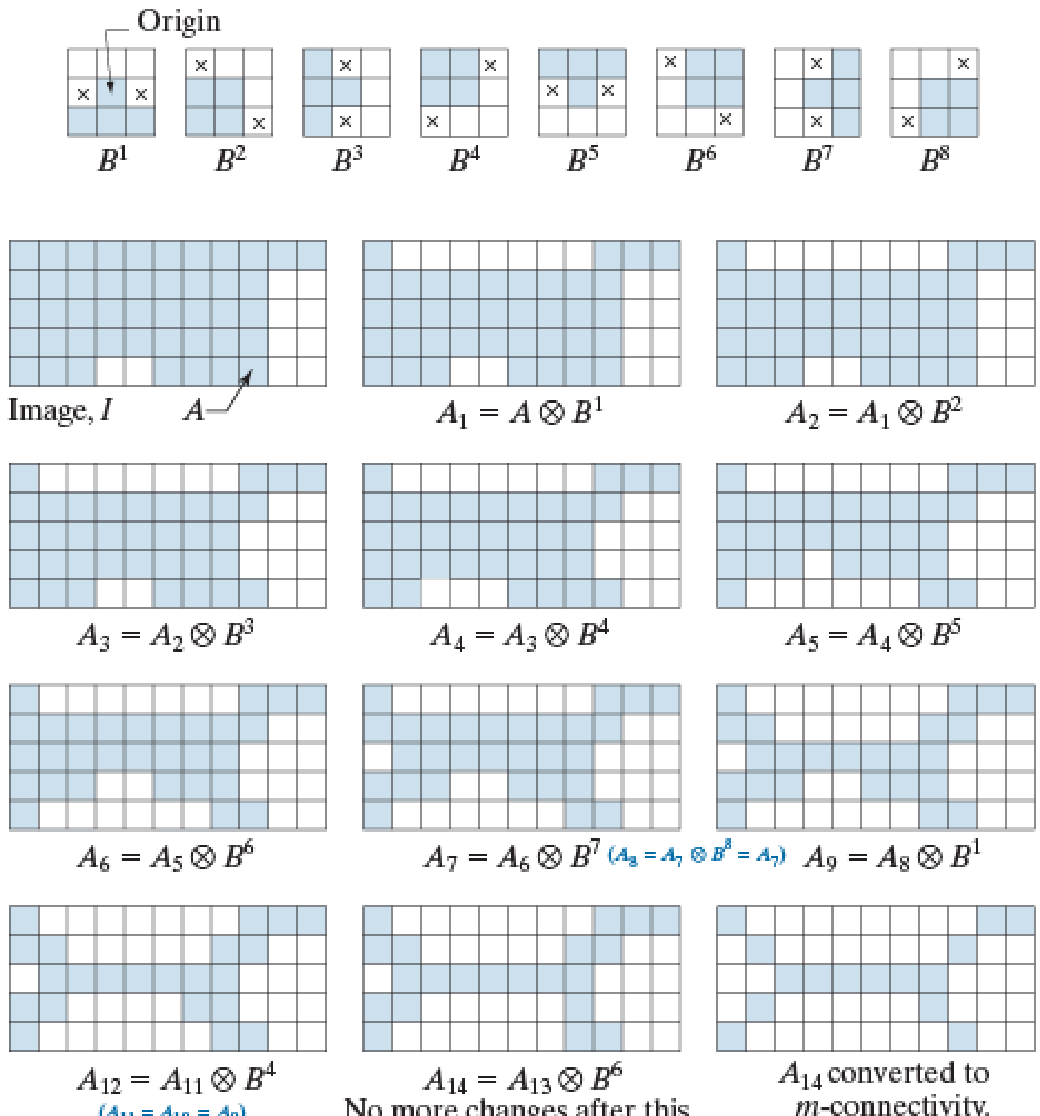
Thinning (cont.)



- Hits matched at origin of B^n
- No change between the result of B^7 and B^8 at the first pass.
- No change between the results of B^1, B^2, B^3, B^4 at the second pass.
- No change occurs after the second pass by B^6 .

- The final result is converted to *m*-connectivity to have a one pixel thick structure.

Thinning (cont.)



	a	
b	c	d
e	f	g
h	i	j
k	l	m

FIGURE 9.23
(a) Structuring elements.
(b) Set A.
(c) Result of thinning A with B^1 (shaded).
(d) Result of thinning A_1 with B_2.
(e)–(i) Results of thinning with the next six SEs. (There was no change between A_7 and A_8.)
(j)–(k) Result of using the first four elements again.
(l) Result after convergence.
(m) Result converted to m-connectivity.

Thickening

- Thickening is a morphological dual of thinning

Hit or Miss

$$A \odot B = A \cup (A \circledast B)$$

- The SEs have the same form as the ones used for thinning with the 1s and 0s interchanged.
- It may also be defined by a sequence of operations:

$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$$

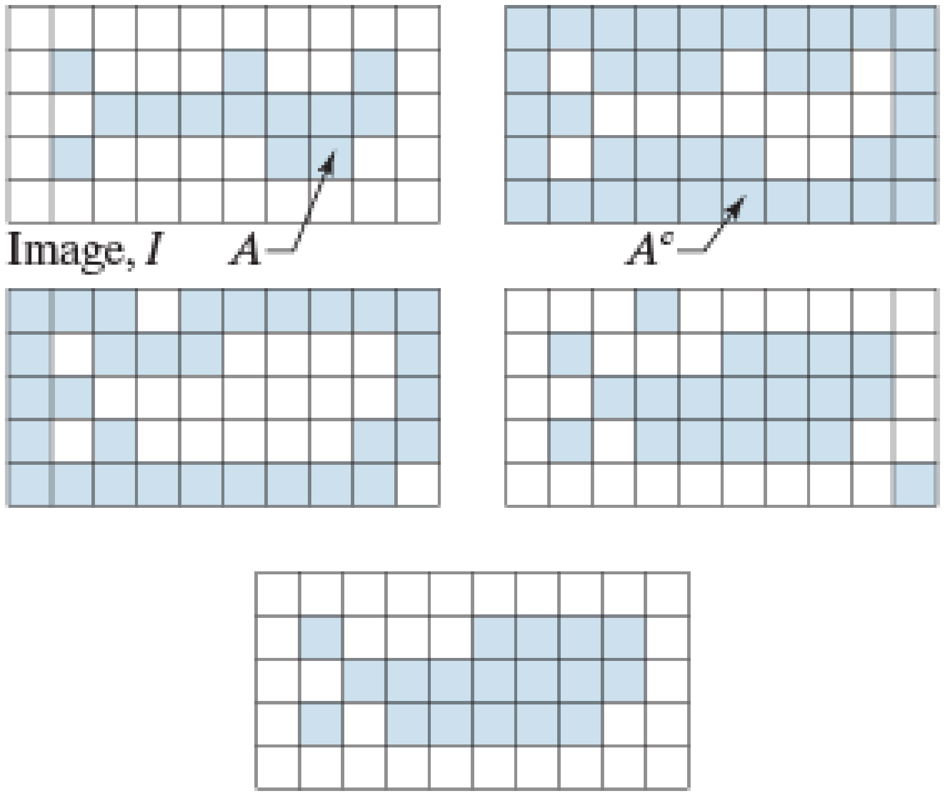
Thickening (cont.)

- In practice, a separate algorithm is seldom used for thickening.
- The usual process is to thin the background of the set in question and then take the complement of the result.
- The advantage is that the thinned background forms a boundary for the thickening process.
- Direct implementation of thickening has no stopping criterion.
- A disadvantage is that there may be isolated points needing post-processing.

Thickening (cont.)

Original set A

A^c



Thinning of A^c

Thickened set obtained by complementing the result of thinning.

Elimination of disconnected points.

a b
c d
e

FIGURE 9.24
(a) Set A.
(b) Complement of A.
(c) Result of thinning the complement.
(d) Thickened set obtained by complementing (c).
(e) Final result, with no disconnected points.

9.5.7 Skeletons

- The notion of a skeleton $S(A)$ of a set A, intuitively, has the following properties:
- A point z belongs to $S(A)$ if one cannot find a *larger disk* containing z and included in A

  → Maximum disk
- The maximum disk touches the boundary of A at two or more different points.

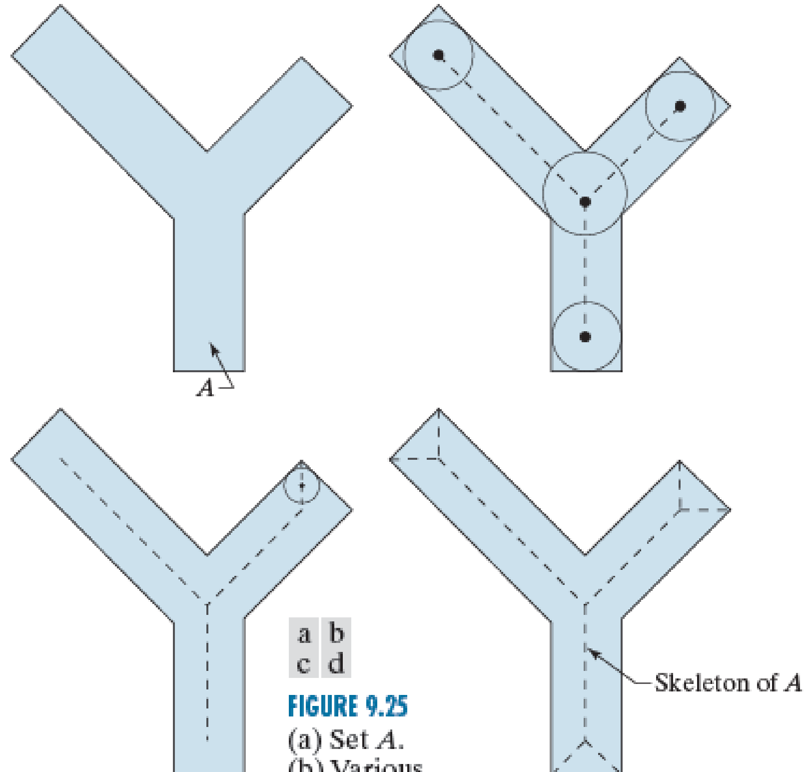


FIGURE 9.25 (a) Set A. (b) Various positions of maximum disks whose centers partially define the skeleton of A. (c) Another maximum disk, whose center defines a different segment of the skeleton of A. (d) Complete skeleton (dashed).

Acknowlegements

The slides are primarily based on the figures and images in the Digital Image Processing textbook by Gonzalez and Woods:

- http://www.imageprocessingplace.com/DIP-3E/dip3e_book_images_downloads.htm

In addition, slides have been adopted and modified from the following sources:

- http://www.cs.uoi.gr/~cnikou/Courses/Digital_Image_Processing
- http://www.comp.dit.ie/bmacnamee/gaip.htm
- http://baggins.nottingham.edu.my/~hsooihock/G52IIP/
- http://gear.kku.ac.th/~nawapak/178353.html
- https://cs.nmt.edu/~ip/index.html

Skeletons (cont.)

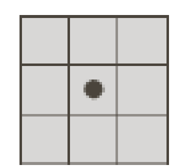
- It may be shown that a definition of the skeleton may be given in terms of erosions and openings:

$$S(A)=\bigcup_{k=0}^{K} S_k(A), \quad \text{with} \quad S_k(A)=\overset{\text{erosion}}{(A\ominus kB)}-\overset{\text{opening}}{(A\ominus kB)\circ B}$$

$$\text{with} \quad (A\ominus kB)=\underbrace{((\ldots(A\ominus B)\ominus B)\ominus\ldots)\ominus B)}_{k\text{ successive erosions}}$$

- K is the last iterative step before A erodes to an empty set:

$$K=\max\{k \mid A\ominus kB\neq\varnothing\}$$



B

Skeletons (cont.)

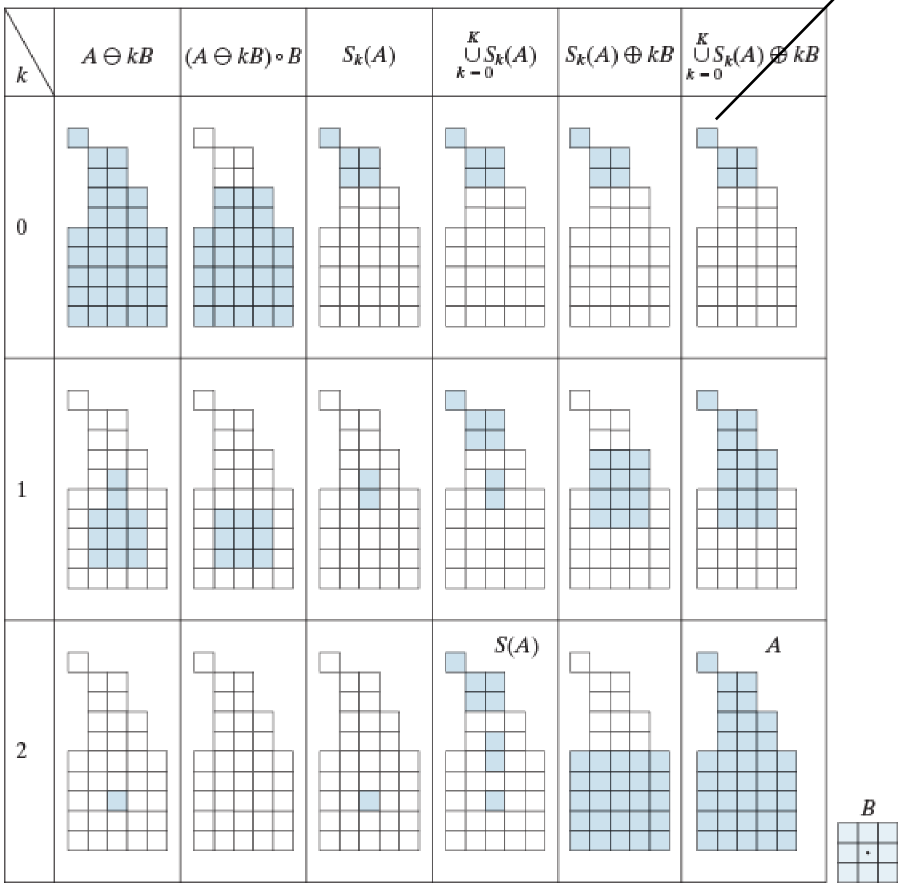
- The previous formulation allows the iterative reconstruction of A from the sets forming its skeleton by:

$$A = \bigcup_{k=0}^{K} \left(S_k(A) \oplus kB \right),$$

with $S_k(A) \oplus B = \underbrace{((\ldots(S_k(A) \oplus B) \oplus B) \oplus \ldots) \oplus B)}_{k \text{ successive dilations of the set } S_k(A)}$

Skeletons (cont.)

FIGURE 9.26 Implementation of Eqs. (9-28) through (9-33). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.



One more erosion → empty set

The skeleton is
- thicker than essential.
- disconnected.

The morphological formulation does not guarantee connectivity.

More assumptions are needed to obtain a maximally thin and connected skeleton.

Morphological Reconstruction

The morphological algorithms discussed so far involve an image and a SE.

Morphological reconstruction involves two images and a SE

- Marker Image: Contains the starting point of the transformation
- Mask mage: Constraints the transformation
- SE: Used to define connectivity

Morphological Reconstruction (cont.)

- The ***geodesic dilation*** of size 1 of a marker image F by a SE B, with respect to a mask image G is defined by:

Dilation B: SE

D: Dilation $$D_G^{(1)}(F) = (F \oplus B) \cap G$$ G: Mask

F: Marker Image

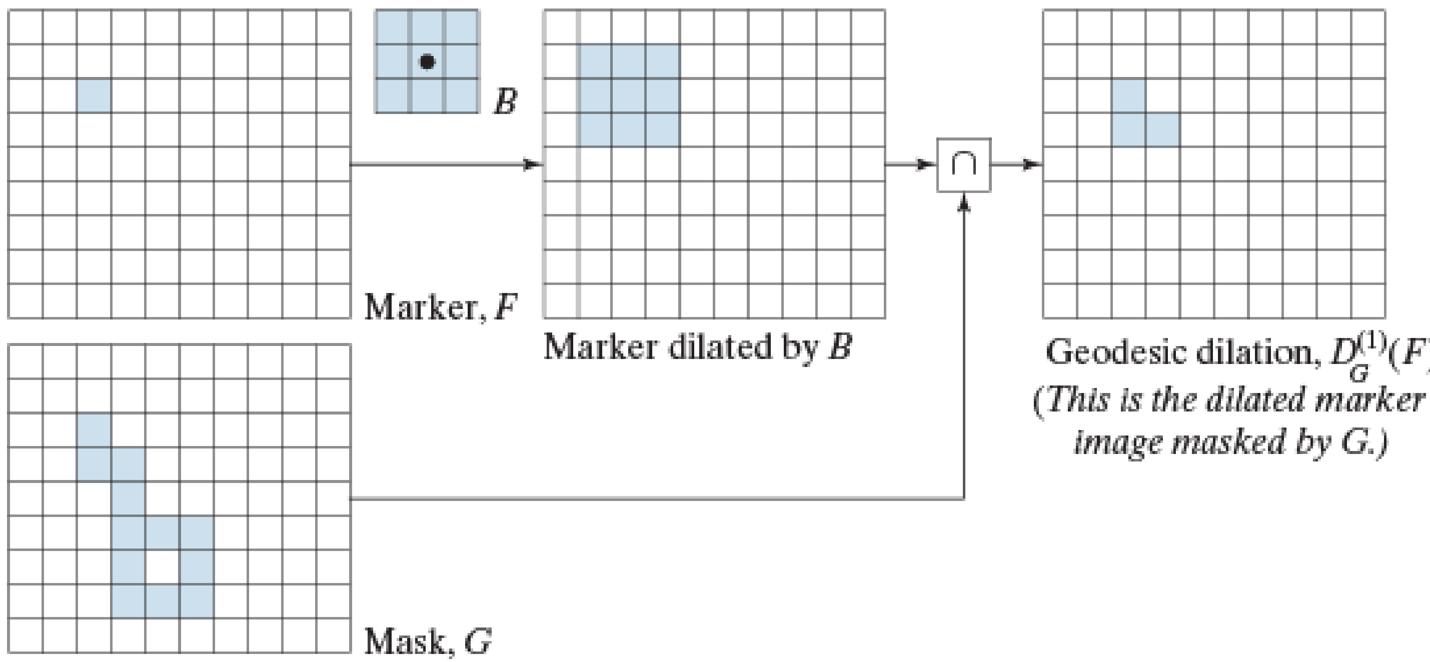
- Similarly, the ***geodesic dilation*** of size n is defined by:

$$D_G^{(n)}(F) = D_G^{(1)}\left[D_G^{(n-1)}(F)\right] \quad \text{with } D_G^{(0)}(F) = F$$

- The intersection operator at each step guarantees that the growth (dilation) of marker F is limited by the mask G.

Morphological Reconstruction (cont.)

FIGURE 9.28 Illustration of a geodesic dilation of size 1. Note that the marker image contains a point from the object in G. If continued, subsequent dilations and maskings would eventually result in the object contained in G.



- Geodesic dilation of size 1.
- The result will not contain elements not belonging to the mask G.

Morphological Reconstruction (cont.)

- The **geodesic erosion** of size 1 of a marker image F by a SE B, with respect to a mask image G is defined by:

Erosion

E: Erosion

$$E_G^{(1)}(F) = (F \ominus B) \cup G$$

- Similarly, the **geodesic erosion** of size n is defined by:

$$E_G^{(n)}(F) = E_G^{(1)}\left[E_G^{(n-1)}(F)\right] \quad \text{with } E_G^{(0)}(F) = F$$

- The union operator guarantees that the geodesic erosion of marker F remains greater than or equal to the mask G.

Morphological Reconstruction (cont.)

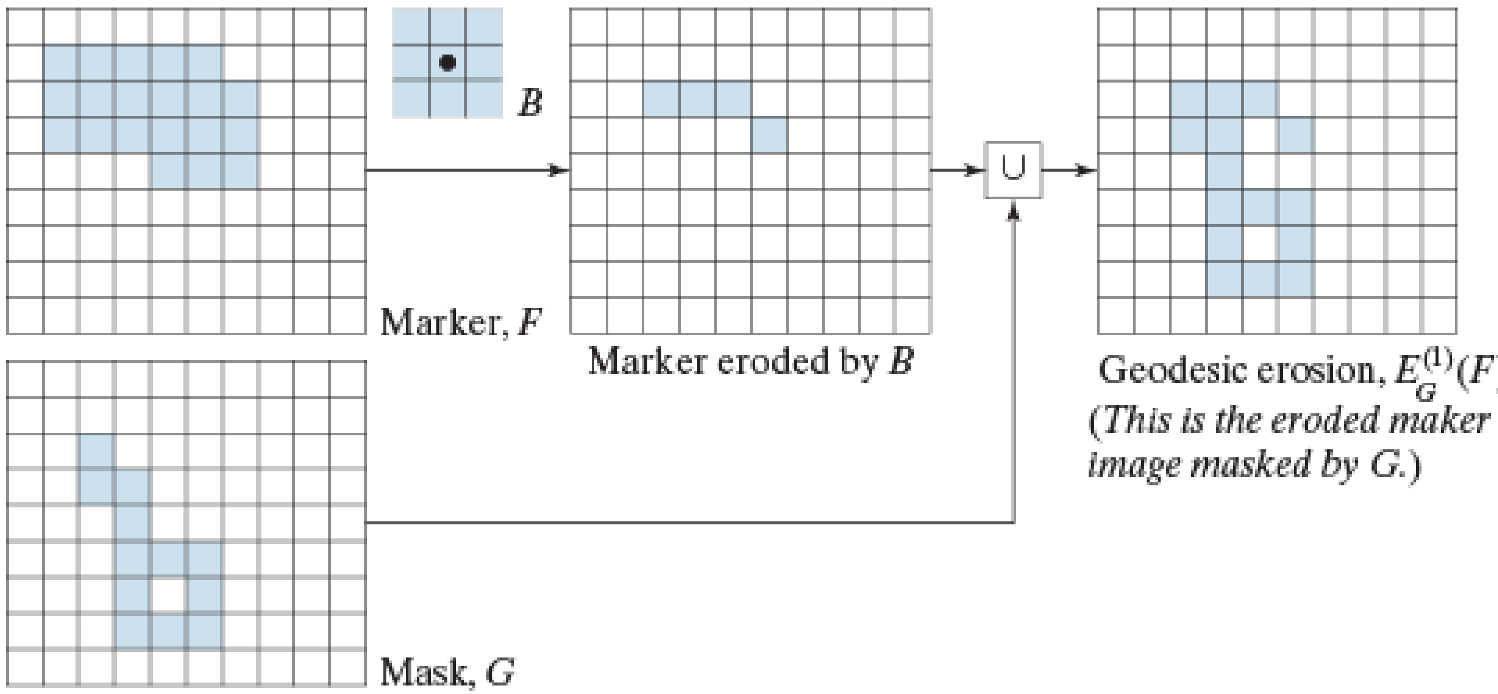


FIGURE 9.29 Illustration of a geodesic erosion of size 1.

- Geodesic erosion of size 1.
- The result will at least contain the mask G.

Morphological Reconstruction (cont.)

- The geodesic dilation and erosion are duals with respect to set complementation.

- They always converge after a finite number of steps:

  - Propagation of the marker (due to dilation), or
  - Shrinking of the marker (due to erosion)

  are constrained by the mask.

Morphological Reconstruction (cont.)

- The **morphological reconstruction by dilation** of mask image G from a marker image F is defined as the geodesic dilation of F with respect to G, iterated until stability is achieved:

D: Dilation F: Marker Image

R: Reconstruction

$$R_G^D(F) = D_G^{(k)}(F)$$

G: Mask

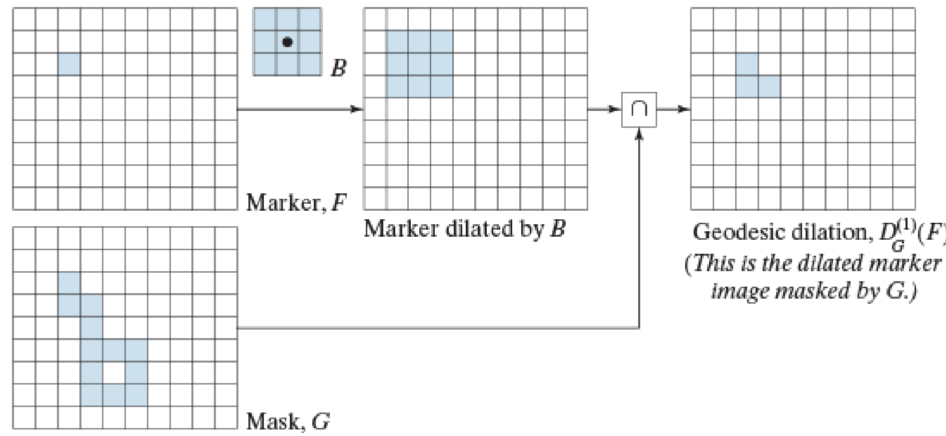
with k such that:

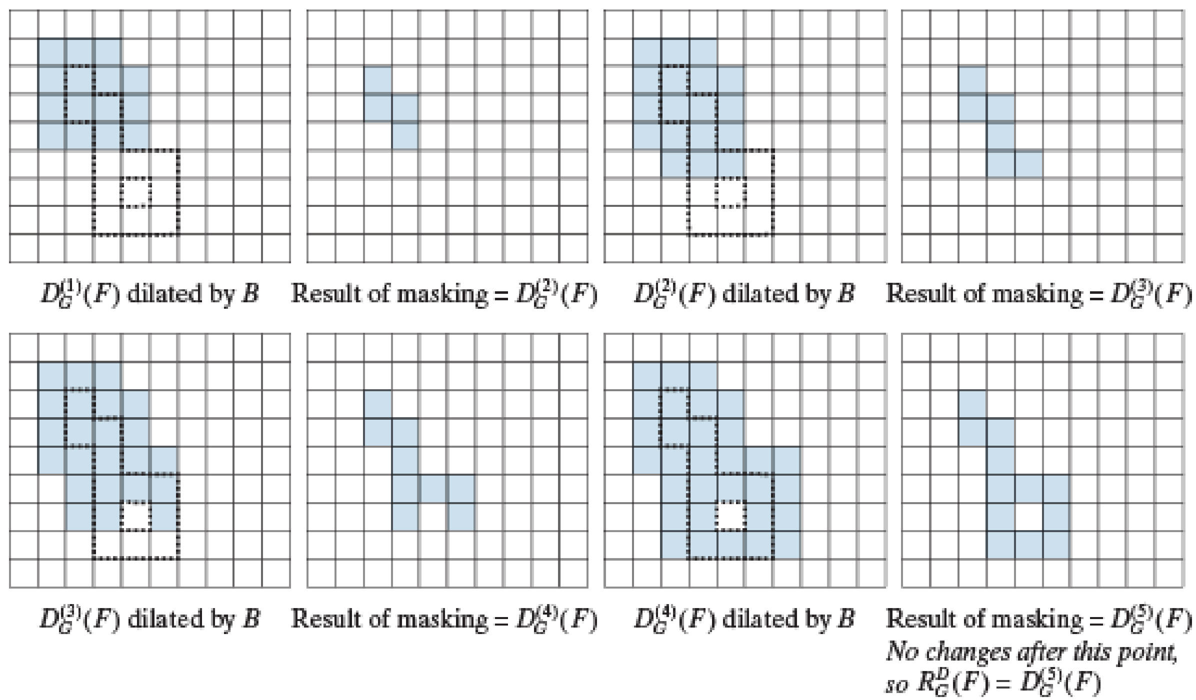
$$D_G^{(k)}(F) = D_G^{(k+1)}(F)$$

Morphological Reconstruction (cont.)

Example of morphological reconstruction by dilation.

The mask, marker, SE and the first step of the algorithm are from the example of geodesic dilation.





a	b	c	d
e	f	g	h

FIGURE 9.30 Illustration of morphological reconstruction by dilation. Sets $D_G^{(1)}(F)$, G, B and F are from Fig. 9.28. The mask (G) is shown dotted for reference.

Morphological Reconstruction (cont.)

- The **morphological reconstruction by erosion** of mask image G from a marker image F is defined as the geodesic erosion of F with respect to G, iterated until stability is achieved:

$$R_G^E(F) = E_G^{(k)}(F)$$

with k such that:

$$E_G^{(k)}(F) = E_G^{(k+1)}(F)$$

The example is left as an exercise!

Applications: Opening by Reconstruction

- In morphological opening, erosion removes small objects and dilation attempts to restore the shape of the objects that remain without the small objects.

- This is not accurate as it depends on the similarity between the shapes to be removed and the SE.

- **Opening by reconstruction** restores exactly the shapes of the objects that remain after erosion.

Opening by Reconstruction (cont.)

- The opening by reconstruction of size n of an image F is defined as the reconstruction by dilation of F from the erosion of size n of F:

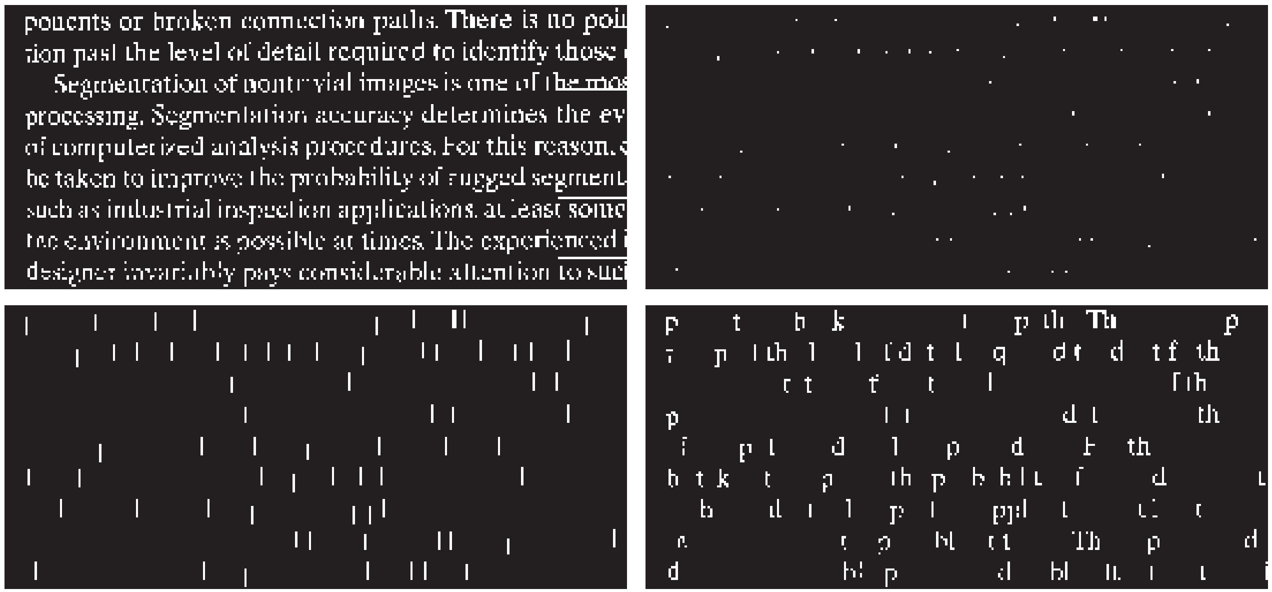
$$O_R^{(n)}(F) = R_F^D\left[(F \ominus nB)\right]$$

- The image F is used as the mask and the n erosions of F by B are used as the initial marker image.

Opening by Reconstruction (cont.)

- We are interested in extracted characters with long vertical strokes (~50 pixels high).

Original image | One erosion by a 51x1 SE



Opening | Opening by reconstruction

a	b
c	d

FIGURE 9.31 (a) Text image of size 918 × 2018 pixels. The approximate average height of the tall characters is 51 pixels. (b) Erosion of (a) with a structuring element of size 51 × 1 elements (all 1's). (c) Opening of (a) with the same structuring element, shown for comparison. (d) Result of opening by reconstruction.

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Applications: Region Filling

- No starting point is needed to be provided.
- The original image $I(x,y)$ is used as a mask.
- The marker image is

$$F(x,y)=\begin{cases}1-I(x,y) & \text{if } (x,y) \text{ is on the border of } I\\ 0 & \text{otherwise}\end{cases}$$

• Only dark pixels of $I(x,y)$ touching the border have a value of 1 in $F(x,y)$.

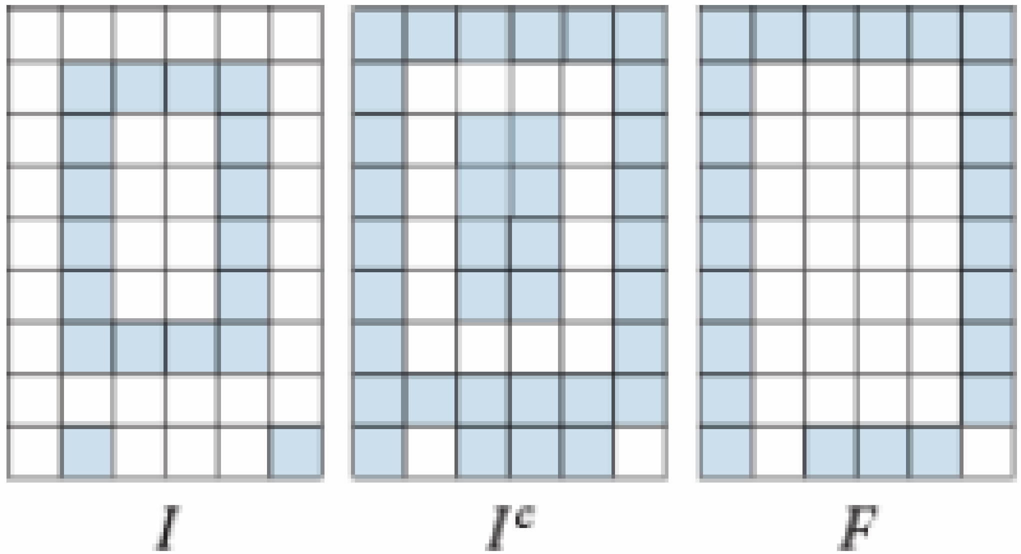
• The binary image with all regions (holes) filled is given by:

$$H=\left[R_{I^c}^{D}(F)\right]^c$$

Region Filling (cont.)

a b c d e f g

FIGURE 9.32 Hole filling using morphological reconstruction.



- We wish to fill the hole of the image I.
- The complement builds a wall around the hole.
- The marker image F is one at the border except from border pixels of the original image.

Region Filling (cont.)

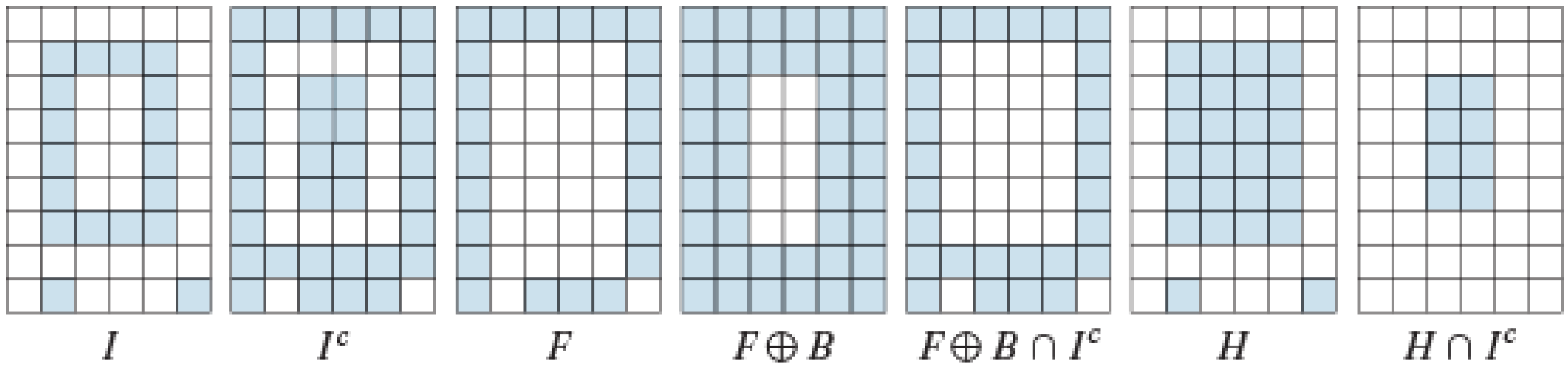


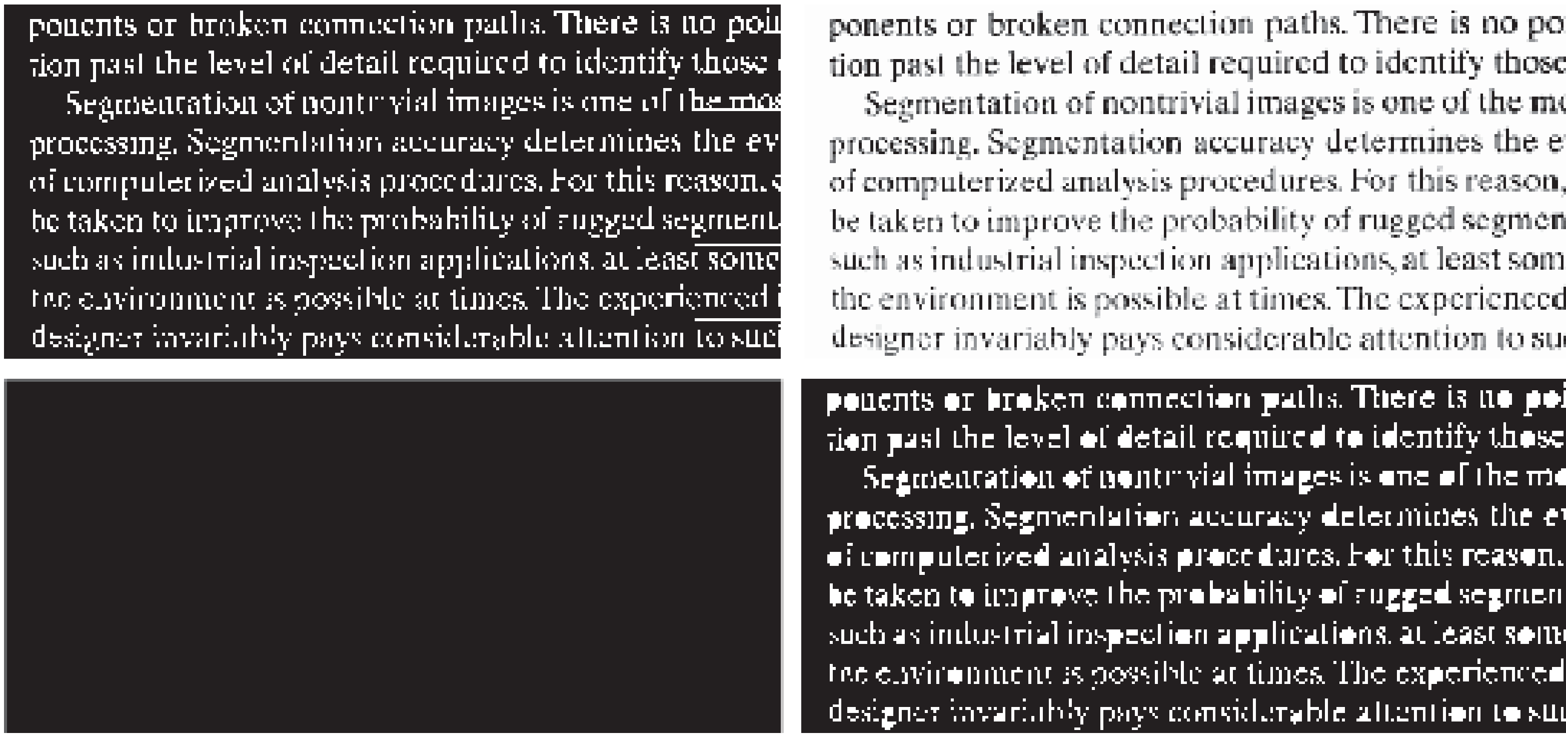
FIGURE 9.32 Hole filling using morphological reconstruction.

- The dilation of the marker F starts from the border and grows inward.
- The complement is used as AND mask: it protects all foreground pixels (including the wall) from changing during the iterations.
- The last operation provides only the hole points.

Region Filling (cont.)

Original image

Complement of original image



a b
c d

FIGURE 9.33
(a) Text image of size 918 × 2018 pixels.
(b) Complement of (a) for use as a mask image.
(c) Marker image.
(d) Result of hole-filling using Eqs. (9-45) and (9-46).

Marker image (1s almost everywhere apart of some points on the right border)

Result of hole filling

Applications: Border Clearing

- The extraction of objects from an image is a fundamental task in automated image analysis.
- An algorithm for removing objects that touch (are connected) to the image border is useful because
  - Only complete objects remain for further processing.
  - It is a signal that partial objects remain in the field of view.

Border Clearing (cont.)

- The original image is used as a mask.
- The marker image is

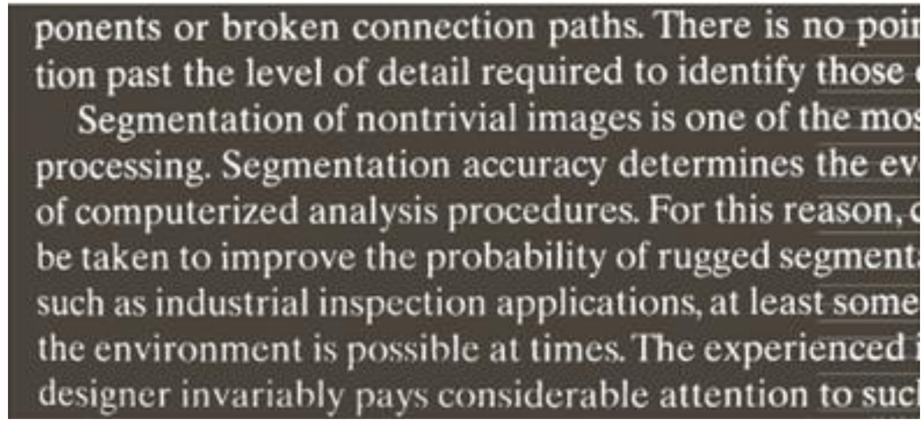
$$F(x,y)=\begin{cases} I(x,y) & \text{if } (x,y) \text{ is on the border of } I \\ 0 & \text{otherwise} \end{cases}$$

- The border clearing algorithm first computes the morphological reconstruction $R_I^D(F)$,

which simply extracts the objects touching the border and then obtains the new image with no objects touching the borders $I - R_I^D(F)$.

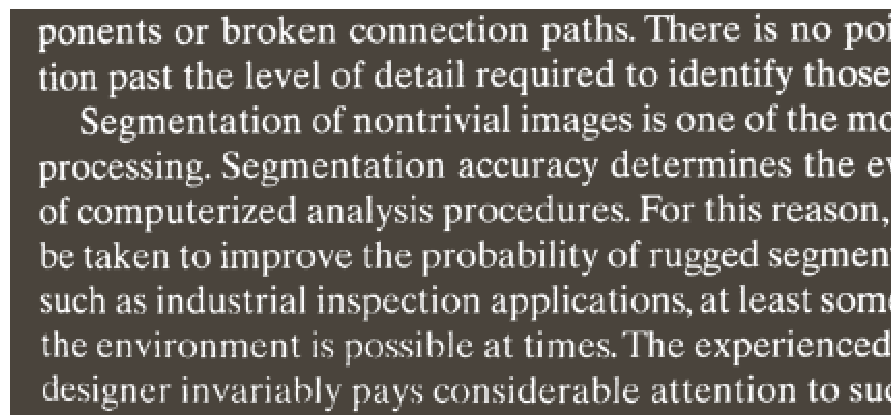
Border Clearing (cont.)

Original image I



a b

FIGURE 9.34 (a) Reconstruction by dilation of marker image. (b) Image with no objects touching the border. The original image is Fig. 9.31(a).



Marker image $F(x,y)$

Reconstructed image $I - R_I^D(F)$

9.6 Gray-Scale Morphology

- The image $f(x, y)$ and the SE $b(x, y)$ take real or integer values.
- SE may be flat or nonflat.
- Due to a number of difficulties (result interpretation, erosion is not bounded by the image, etc.) symmetrical flat SE with origin at the center are employed.
- Set reflection: $\hat{b}(x, y) = -b(x, y)$

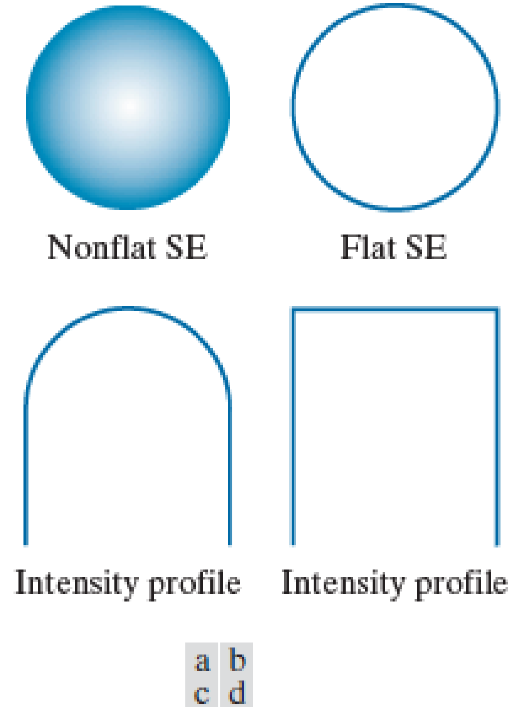


FIGURE 9.36 Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their centers. All examples in this section are based on flat SEs.

Gray-Scale Erosion

- The erosion of image f by a SE b at any location (x, y) is defined as the minimum value of the image in the region coincident with b when the origin of b is at (x, y):

$$[f \ominus b](x,y) = \min_{(s,t)\in b}\{f(x+s, y+t)\}$$

- In practice, we place the center of the SE at every pixel and select the minimum value of the image under the window of the SE.

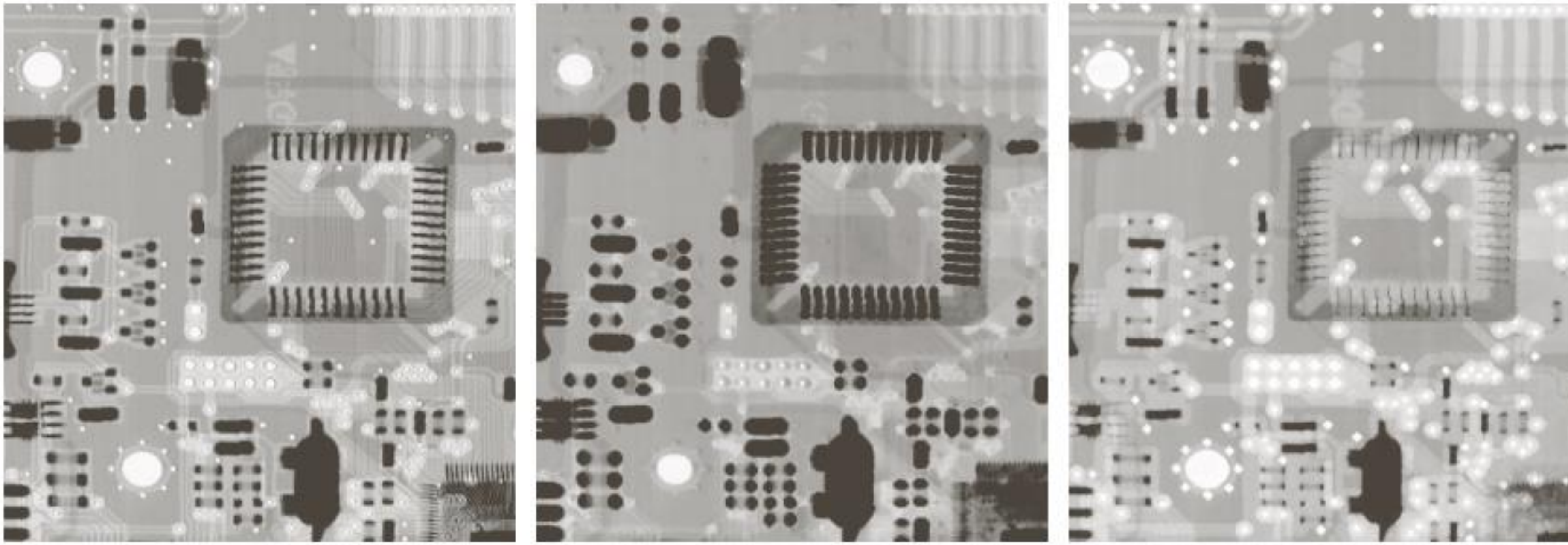
Gray-Scale Dilation

- The dilation of image f by a SE b at any location (x, y) is defined as the maximum value of the image in the window outlined by b:

$$[f \oplus b](x, y) = \max_{(s,t)\in b}\{ f(x-s, y-t)\}$$

The SE is reflected as in the binary case.

Gray-Scale Erosion and Dilation



Original image

Erosion by a flat disk SE of radius 2:
Darker background, small bright dots reduced, dark features grew.

Dilation by a flat disk SE of radius 2:
Lighter background, small dark dots reduced, light features grew.

a b c

FIGURE 9.37 (a) Gray-scale X-ray image of size 448 × 425 pixels. (b) Erosion using a flat disk SE with a radius of 2 pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

Gray-Scale Morphology (nonflat SE)

- The erosion of image f by a nonflat SE b_N is defined as:

$$[f \ominus b_N](x,y) = \min_{(s,t)\in b_N} \{ f(x+s, y+t) - b_N(s,t) \}$$

- The dilation of image f by a nonflat SE b_N is defined as:

$$[f \oplus b_N](x,y) = \max_{(s,t)\in b_N} \{ f(x-s, y-t) + b_N(s,t) \}$$

- When the SE is flat the equations reduce to the previous formulas up to a constant.

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Duality

- As in the binary case, erosion and dilation are dual operations with respect to function complementation and reflection:

$$(f \ominus b)^c(x,y) = (f^c \oplus \hat{b})(x,y)$$

- Similarly,

$$(f \oplus b)^c(x,y) = (f^c \ominus \hat{b})(x,y)$$

- In what follows, we omit the coordinates for simplicity.

9.6.2 Gray-Scale Opening and Closing

- Opening of image f by SE b is:

$$f \circ b = (f \ominus b) \oplus b$$

- Closing of image f by SE b is:

$$f \bullet b = (f \oplus b) \ominus b$$

- They are also duals with respect to function complementation and reflection:

$$(f \bullet b)^c = f^c \circ \hat{b} \qquad (f \circ b)^c = f^c \bullet \hat{b}$$

Gray-Scale Opening and Closing (cont.)

- Geometric interpretation of **opening**:
- It is the highest value reached by any part of the SE as it pushes up against the under-surface of the image (up to the point it fits completely).
- It removes small bright details.

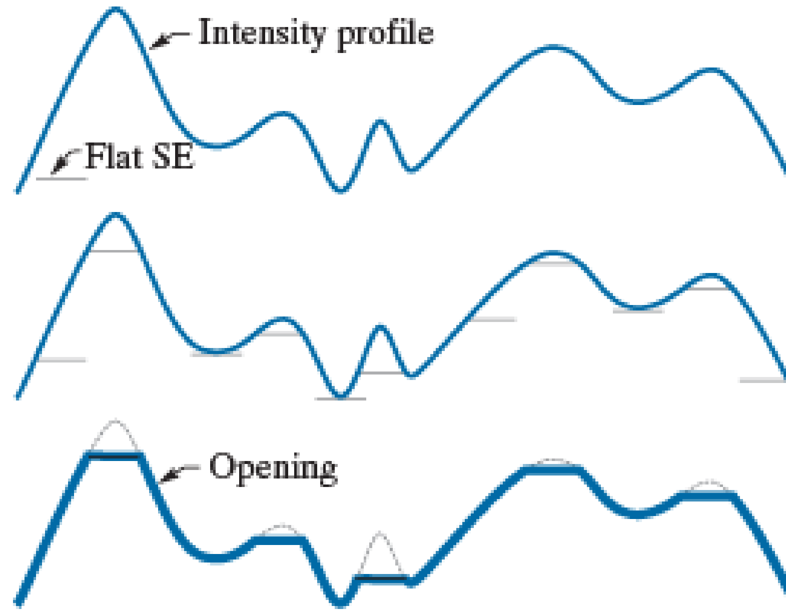
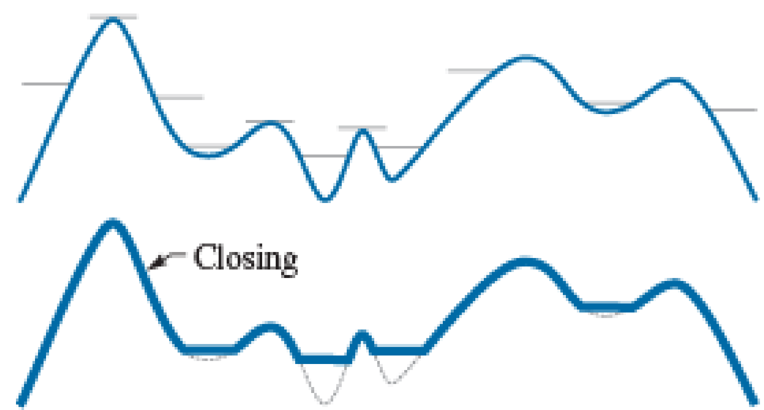


FIGURE 9.38 Grayscale opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal. (c) Opening. (d) Flat structuring element pushed down along the top of the signal. (e) Closing.

Gray-Scale Opening and Closing (cont.)

- Geometric interpretation of **closing:**

- It is the lowest value reached by any part of the SE as it pushes down against the upper side of the image intensity curve.



- It highlights small dark regions of the image.

Gray-Scale Opening and Closing (cont.)

Properties of opening:

(1) $f \circ b$ ↵ f

(2) If f_1 ↵ f_2, then $f_1 \circ b$ ↵ $f_2 \circ b$

(3) $(f \circ b) \circ b = f \circ b$

- The first property ↵ indicates that:

  - The domain of the opening is a subset of the domain of f and

$$[f \circ b](x, y) \leq f(x, y)$$

Gray-Scale Opening and Closing (cont.)

Properties of closing:

(1) $f \dashv f \bullet b$

(2) If $f_1 \dashv f_2$, then $f_1 \bullet b \dashv f_2 \bullet b$

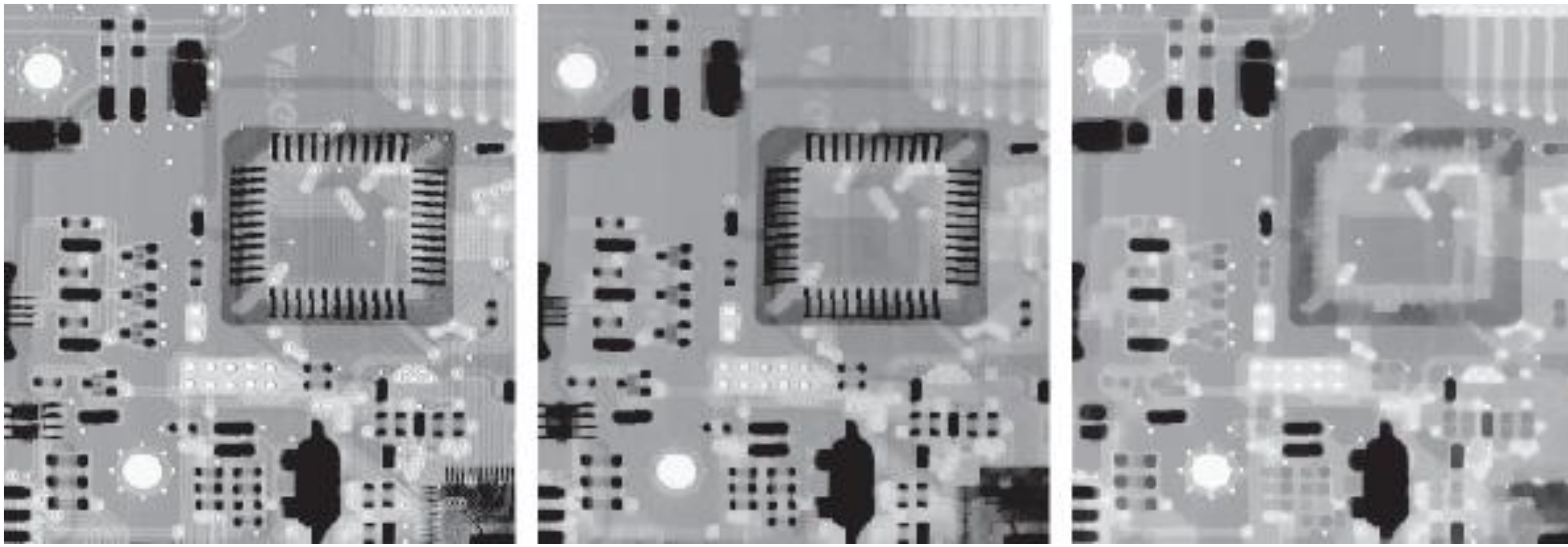
(3) $(f \bullet b) \bullet b = f \bullet b$

The first property indicates that:

- The domain of f is a subset of the domain of the closing and

$$f(x,y) \leq [f \bullet b](x,y)$$

Gray-Scale Opening and Closing (cont.)



Original image

Opening by a flat disk SE of radius 3:
Intensities of bright features decreased,
Effects on background are negligible (as opposed to erosion).

Closing by a flat disk SE of radius 5:
Intensities of dark features increased,
Effects on background are negligible (as opposed to dilation).

a b c

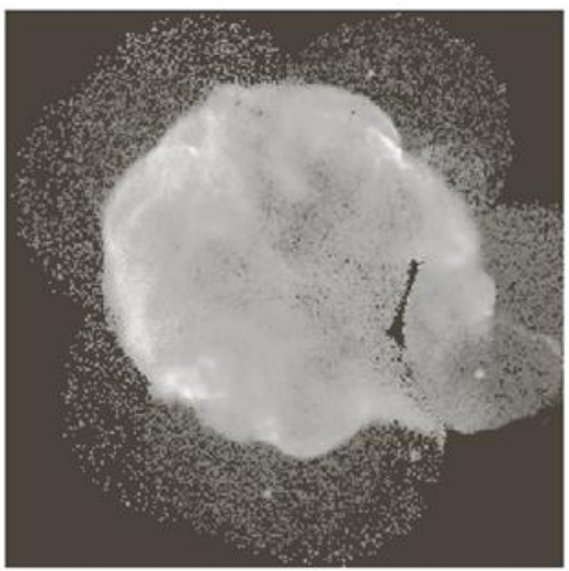
FIGURE 9.39
(a) A grayscale X-ray image of size 448×425 pixels.
(b) Opening using a disk SE with a radius of 3 pixels.
(c) Closing using an SE of radius 5.

9.6.3 Gray-Scale Morphological Algorithms

- Morphological smoothing
- Morphological gradient
- Top-hat transformation
- Bottom-hat transformation
- Granulometry
- Textural segmentation

Morphological Smoothing

- Opening suppresses light details smaller than the SE and closing suppresses (makes lighter) dark details smaller than the SE.
- They are used in combination as *morphological filters* to eliminate undesired structures.



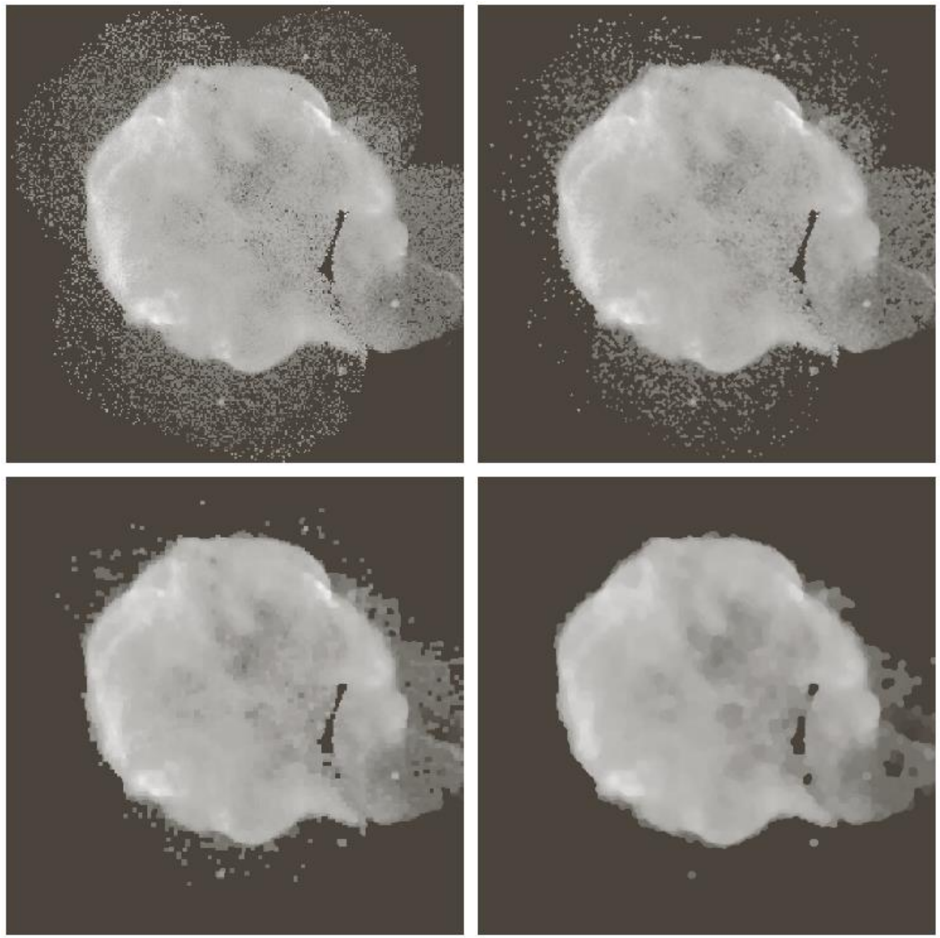
Cygnus Loop supernova.
We wish to extract the central light region.

Morphological Smoothing (cont.)

Opening followed by closing with disk SE of varying size

Original image

Radius 1



Radius 3

Radius 5

a b
c d

FIGURE 9.40
(a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

Morphological Gradient

- The difference of the dilation and the erosion of an image emphasizes the boundaries between regions:

$$g = (f \oplus b) - (f \ominus b)$$

- The difference of the dilation and the erosion of an image emphasizes the boundaries between regions.
- Homogeneous areas are not affected and the subtraction provides a derivative-like effect.
- The net result is an image with flat regions suppressed and edges enhanced.

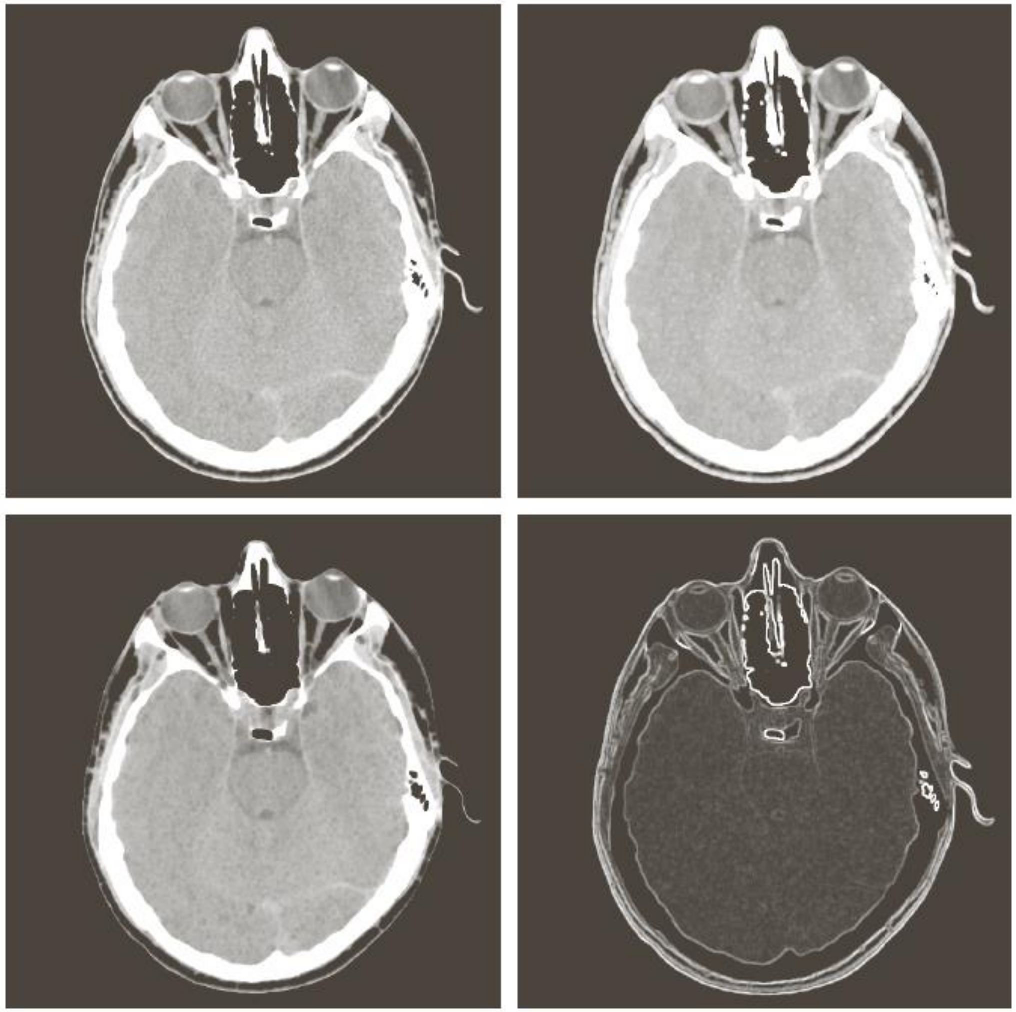
Morphological Gradient (cont.)

Original image

Dilation

Erosion

Difference



a b
c d

FIGURE 9.41
(a) 512 × 512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

Top-hat and Bottom-hat Transformations

- Opening suppresses light details smaller than the SE.
- Closing suppresses dark details smaller than the SE.
- Choosing an appropriate SE eliminates image details where the SE does not fit.
- Subtracting the outputs of opening or closing from the original image provides the removed components.

Top-hat and Bottom-hat Transformations (cont.)

- Because the results look like the top or bottom of a hat these algorithms are called **top-hat** and **bottom-hat** transformations:

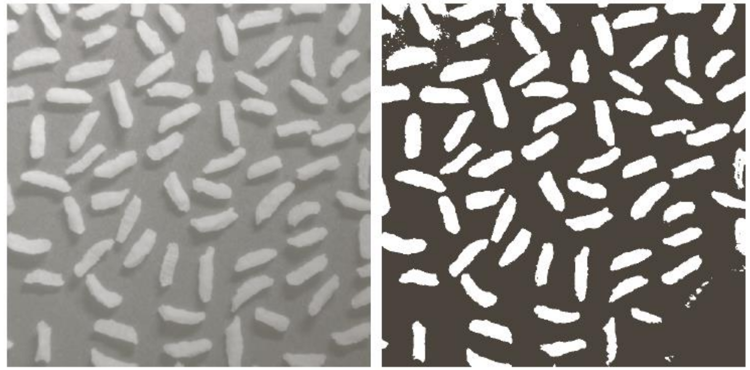
$$T_{\text{hat}}(f) = f - (f \circ b)$$ Light details remain

$$B_{\text{hat}}(f) = (f \bullet b) - f$$ Dark details remain

- An important application is the correction of nonuniform illumination which is a pre-segmentation step.

Top-hat and Bottom-hat Transformations (cont.)

Original image



Thresholded image (Otsu's method)

Opened image (disk SE r=40)
Does not fit to grains and eliminates them

Top-hat (image- opening)
Reduced nonuniformity

Thresholded top-hat

a b
c d e

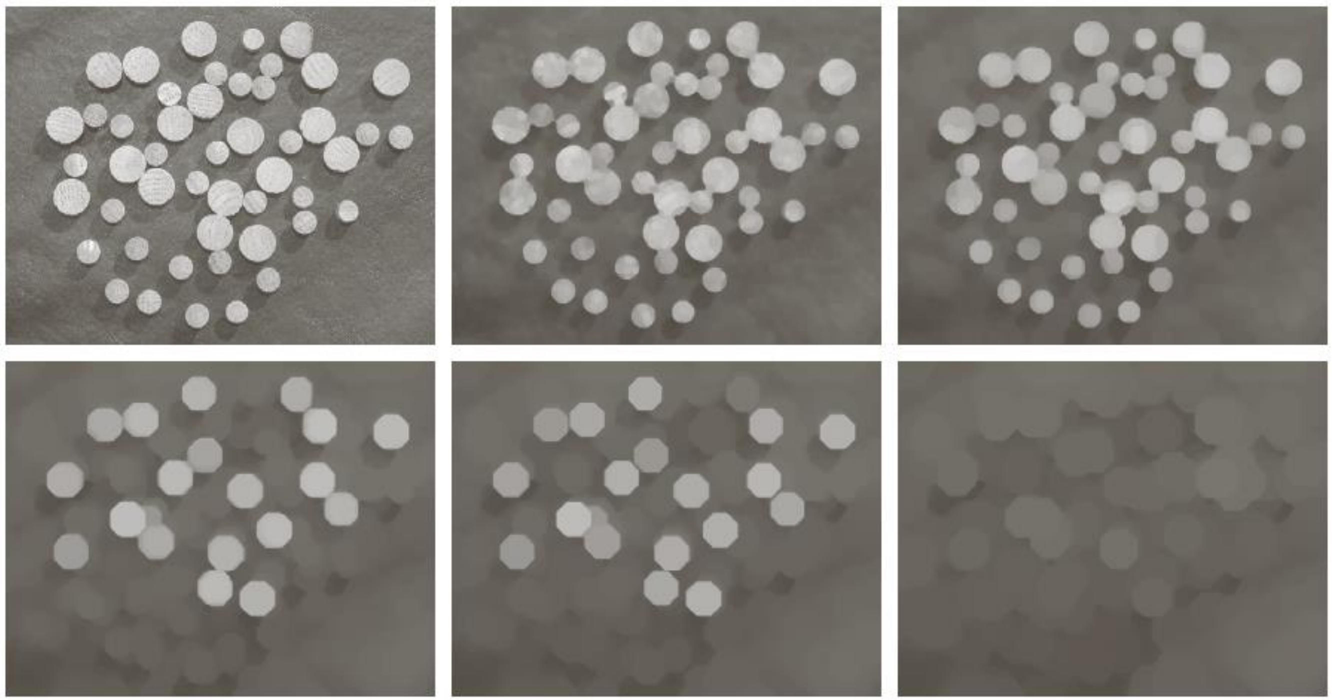
FIGURE 9.42 Using the top-hat transformation for *shading correction*. (a) Original image of size 600 × 600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

Granulometry

- Determination of the size distribution of particles in an image. Particles are seldom separated.
- The method described here measures their distribution indirectly.
- It applies openings with SE of increasing size.
- Each opening suppresses bright features where the SE does not fit.
- For each opening the sum of pixel values is computed and a histogram of the size of the SE vs the remaining pixel intensities is drawn.

Granulometry (cont.)

Image of wooden plugs | Smoothed image | Opening by SE of radius 10



a b c
d e f

FIGURE 9.43 (a) 531 × 675 image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, MathWorks, Inc.)

Opening by SE of radius 20. Small dowels disappeared.

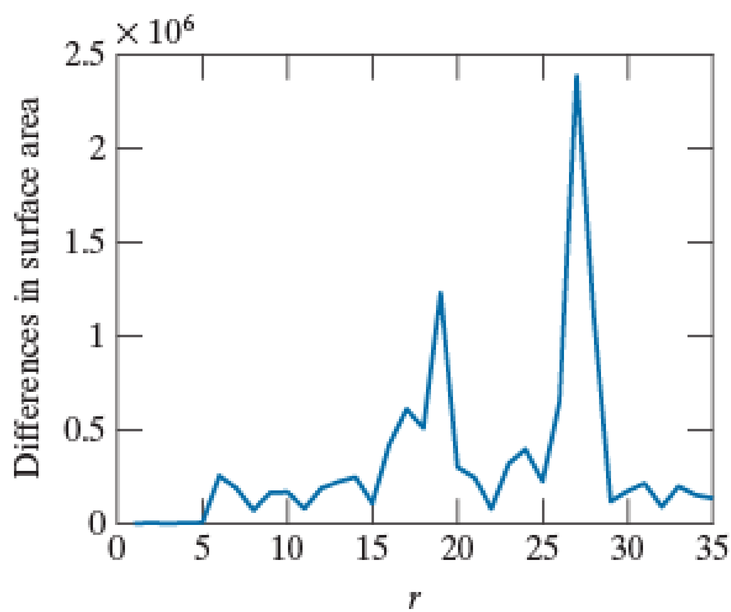
Opening by SE of radius 25

Opening by SE of radius 30 Large dowels disappeared.

Granulometry (cont.)

- Histogram of the differences of the total image intensities between successive openings as a function of the radius of the SE.
- There are two peaks indicating two dominant particle sizes (of radii 19 and 27).

FIGURE 9.44 Differences in surface area as a function of SE disk radius, *r*. The two peaks indicate that there are two dominant particle sizes in the image.



Textural Segmentation

- The objective is to find a boundary between the large and the small blobs (texture segmentation).
- The objects of interest are darker than the background.
- A closing with a SE larger than the blobs would eliminate them.

Textural segmentation (cont.)

- Closing with a SE of radius 30.
- The small blobs disappeared as they have a radius of approximately 25 pixels.

a b
c d

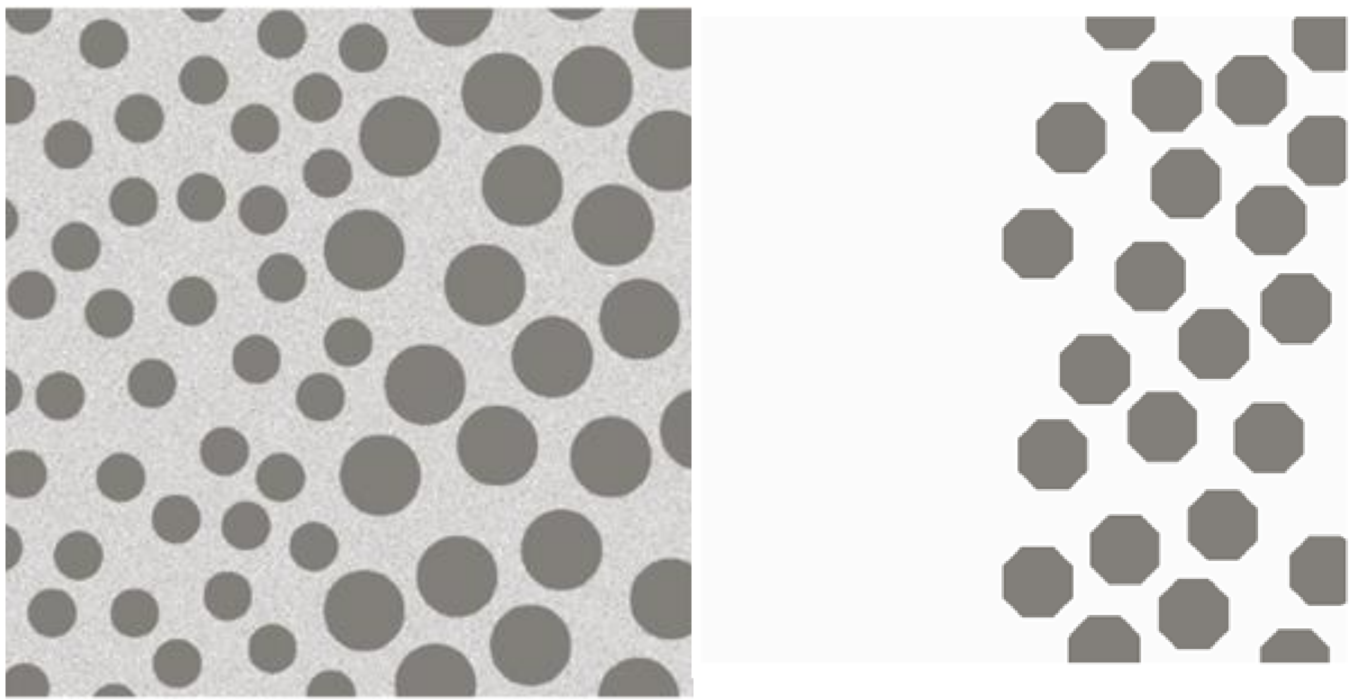


FIGURE 9.45 Textural segmentation. (a) A 600 × 600 image consisting of two types of blobs. (b) Image with small blobs removed by closing (a). (c) Image with light patches between large blobs removed by opening (b). (d) Original image with boundary between the two regions in (c) superimposed. The boundary was obtained using a morphological gradient.

Textural Segmentation (cont.)

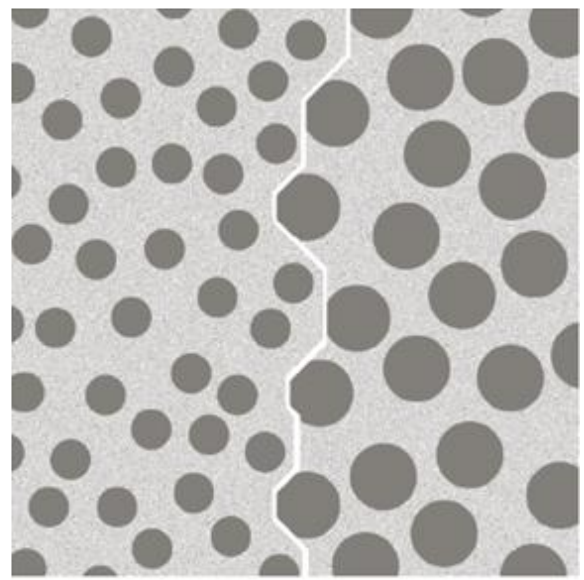
- The background is lighter than the large blobs.
- If we open the image with a SE larger than the distance between the large blobs then the blobs would disappear and the background would be dominant.

Textural segmentation (cont.)

- Opening with a SE of radius 60.
- The lighter background was suppressed to the level of the blobs.


Textural segmentation (cont.)

- A morphological gradient with a 3x3 SE gives the boundary between the two regions which is superimposed on the initial image.



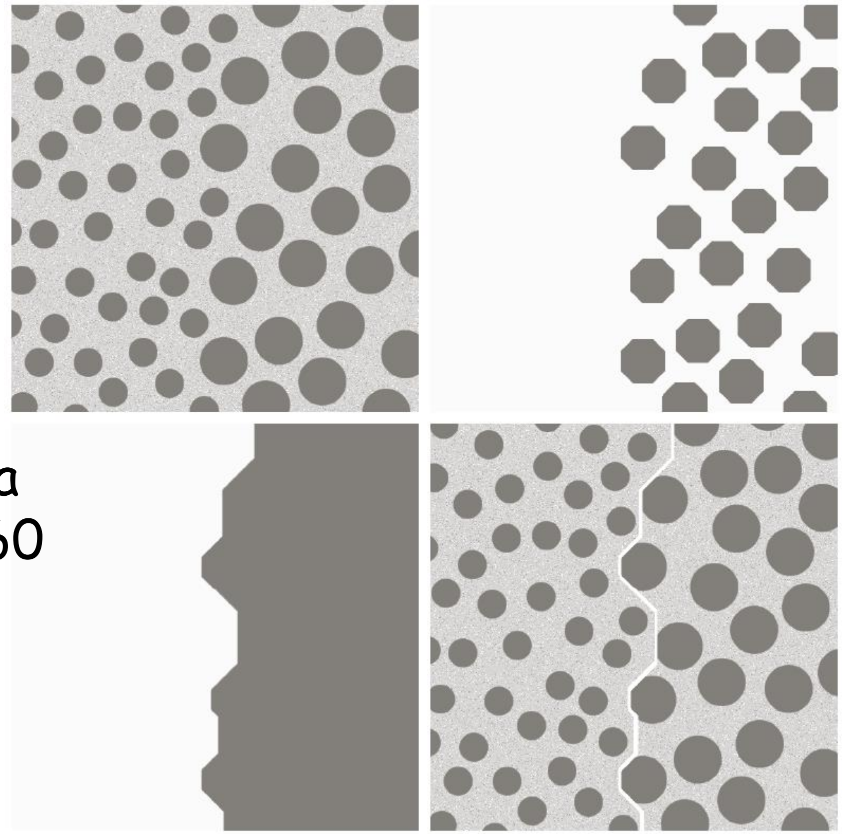
Textural segmentation (cont.)

a b
c d

FIGURE 9.45 Textural segmentation. (a) A 600 × 600 image consisting of two types of blobs. (b) Image with small blobs removed by closing (a). (c) Image with light patches between large blobs removed by opening (b). (d) Original image with boundary between the two regions in (c) superimposed. The boundary was obtained using a morphological gradient.

Original image

Closing with a SE of radius 30 (small blobs are removed)



Opening with a SE of radius 60 (large blobs flooded the background)

Morphological gradient superimposed onto the original image

9.6.4 Gray-Scale Morphological Reconstruction

- The **geodesic dilation** of size 1 of a marker image f by a SE b, with respect to a mask image g is defined by:

$$D_g^{(1)}(f) = (f \oplus b) \wedge g$$

where $\wedge$ is the point-wise minimum operator.

- This equation indicates that the geodesic dilation of size 1 is obtained by first computing the dilation of f by b and then selecting the minimum between the result and g at every point (x,y).

Gray-Scale Morphological Reconstruction (cont.)

- The **geodesic dilation** of size n of a marker image f by a SE b, with respect to a mask image g is defined by:

$$D_g^{(n)}(f) = D_g^{(1)}\left[D_g^{(n-1)}(f)\right]$$

with $D_g^{(0)}(f) = f$

Gray-Scale Morphological Reconstruction (cont.)

- The **geodesic erosion** of size 1 of a marker image f by a SE b, with respect to a mask image g is defined by:

$$E_g^{(1)}(f) = (f \ominus b) \vee g$$

where $\vee$ is the point-wise maximum operator.

- This equation indicates that the geodesic erosion of size 1 is obtained by first computing the erosion of f by b and then selecting the maximum between the result and g at every point (x,y).

Gray-Scale Morphological Reconstruction (cont.)

- The **geodesic erosion** of size n of a marker image f by a SE b, with respect to a mask image g is defined by:

$$E_g^{(n)}(f) = E_g^{(1)}\left[E_g^{(n-1)}(f)\right]$$

with $E_g^{(0)}(f) = f$

Gray-Scale Morphological Reconstruction (cont.)

- The **morphological reconstruction by dilation** of gray scale image g from a marker image f is defined as the geodesic dilation of f with respect to g, iterated until stability is achieved:

$$R_g^D(F) = D_g^{(k)}(F)$$

with k such that:

$$D_g^{(k)}(F) = D_g^{(k+1)}(F)$$

Gray-Scale Morphological Reconstruction (cont.)

- The **morphological reconstruction by erosion** of gray scale image g from a marker image f is defined as the geodesic erosion of f with respect to g, iterated until stability is achieved:

$$R_g^D(F) = E_g^{(k)}(F)$$

with k such that:

$$E_g^{(k)}(F) = E_g^{(k+1)}(F)$$

Gray-Scale Morphological Reconstruction (cont.)

The **opening by reconstruction** of size n of an image f is defined as the reconstruction by dilation of f from the erosion of size n of f:

$$O_R^{(n)}(f) = R_f^D\left[(f \ominus nB)\right]$$

- The image f is used as the mask and the n erosions of f by b are used as the initial marker image.
- Recall that the objective is to preserve the shape of the image components that remain after erosion.

Gray-Scale Morphological Reconstruction (cont.)

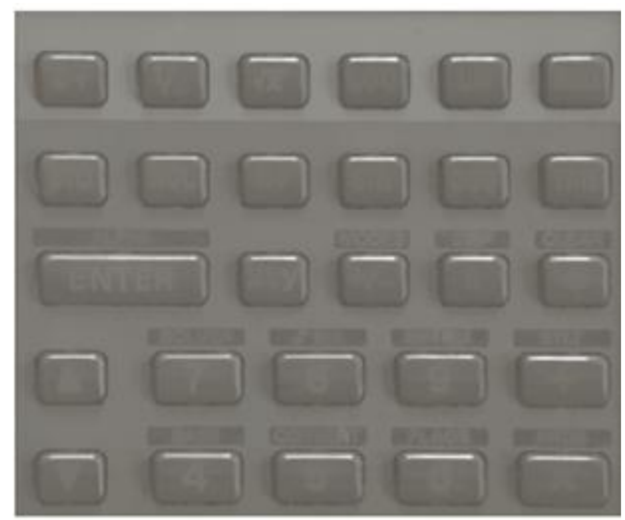
- The image has a size of 1134x1360.
- The target is to leave only the text on a flat background of constant intensity
- In other words, we want to remove the relief effect of the keys.

a b c
d e f
g h i

FIGURE 9.46 (a) Original image of size 1134 × 1360 pixels. (b) Opening by reconstruction of (a), using a structuring element consisting of a horizontal line 71 pixels long in the erosion. (c) Opening of (a) using the same SE. (d) Top-hat by reconstruction. (e) Result of applying just a top-hat transformation. (f) Opening by reconstruction of (d), using a horizontal line 11 pixels long. (g) Dilation of (f) using a horizontal line 21 pixels long. (h) Minimum of (d) and (g). (i) Final reconstruction result. (Images courtesy of Dr. Steve Eddins, MathWorks, Inc.)

Gray-Scale Morphological Reconstruction (cont.)

- At first we suppress the horizontal reflections on the top of the keys.
- The reflections are wider than any single character.
- An opening by reconstruction using a long horizontal line SE (1x71) in the erosion operation provides the keys and their reflections.



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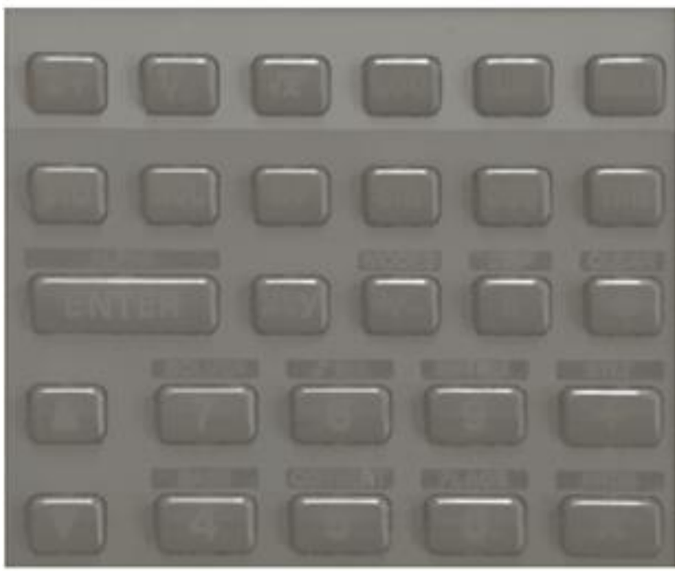
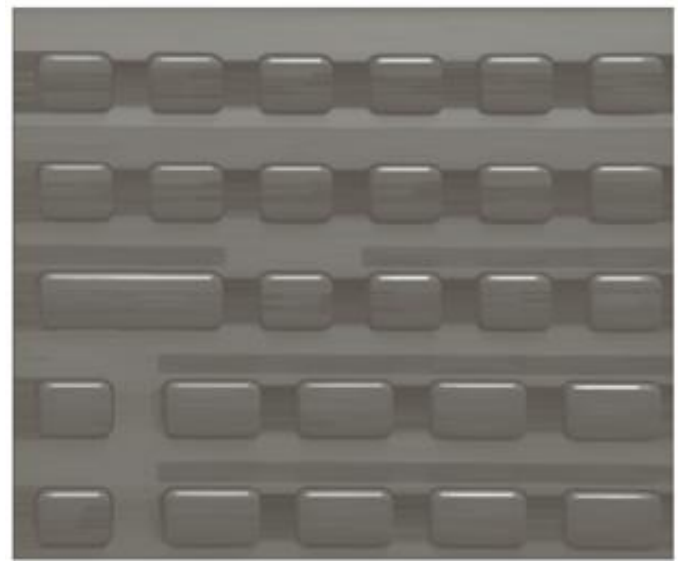
Gray-Scale Morphological Reconstruction (cont.)

- A standard opening would not be sufficient as the background would not have been as uniform (e.g. look at the regions between the keys horizontally).



Opening by reconstruction



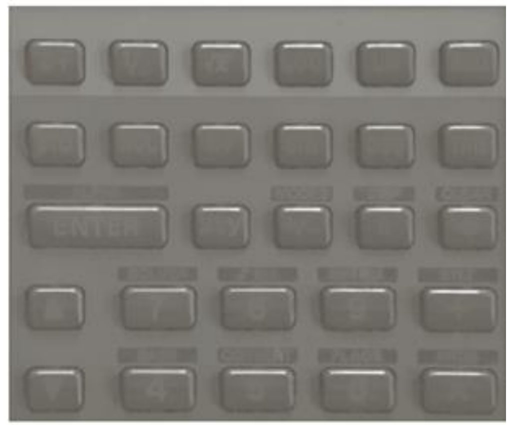
Standard opening

Gray-Scale Morphological Reconstruction (cont.)

- Then, subtracting this result from the original image (*top-hat by reconstruction*) eliminates the reflections.

Original Image

−



Opening by reconstruction

=

Top-hat by reconstruction

a	b	c
d	e	f
g	h	i

FIGURE 9.46 (a) Original image of size 1134 × 1360 pixels. (b) Opening by reconstruction of (a), using a structuring element consisting of a horizontal line 71 pixels long in the erosion. (c) Opening of (a) using the same SE. (d) Top-hat by reconstruction. (e) Result of applying just a top-hat transformation. (f) Opening by reconstruction of (d), using a horizontal line 11 pixels long. (g) Dilation of (f) using a horizontal line 21 pixels long. (h) Minimum of (d) and (g). (i) Final reconstruction result. (Images courtesy of Dr. Steve Eddins, MathWorks, Inc.)

Gray-Scale Morphological Reconstruction (cont.)

- A standard top-hat transformation would not be sufficient as the background is not as uniform as in the top-hat by reconstruction operation.

Top-hat by reconstruction

Standard top-hat

Gray-Scale Morphological Reconstruction (cont.)

- We now try to suppress the vertical reflections on the sides of the keys.

- An opening by reconstruction using a horizontal line SE (1x11) in the erosion operation provides the keys and their reflections (after subtracting the result from the previous image).

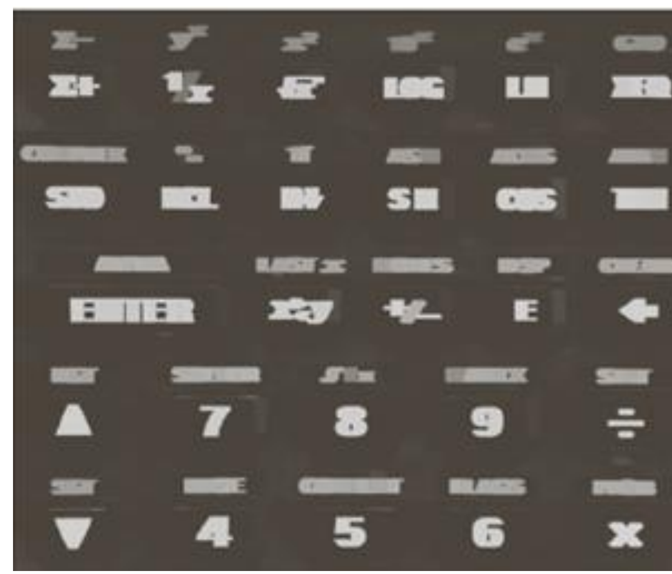
- Notice however that vertically oriented characters are eliminated (The "I" in the "SIN" key)

Gray-Scale Morphological Reconstruction (cont.)

- How can we restore the suppressed character?

→A dilation is not sufficient as the area of the suppressed character is now occupied by the expansion of its neighbors.

Dilation
(SE 1x21)

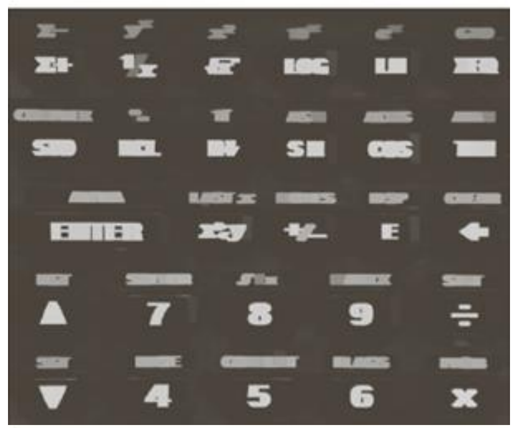


Gray-Scale Morphological Reconstruction (cont.)

- We form an image by taking the point-wise minimum between the top-hat by reconstruction image and the dilated image:

Top-hat by reconstruction

$\wedge$



Dilated image

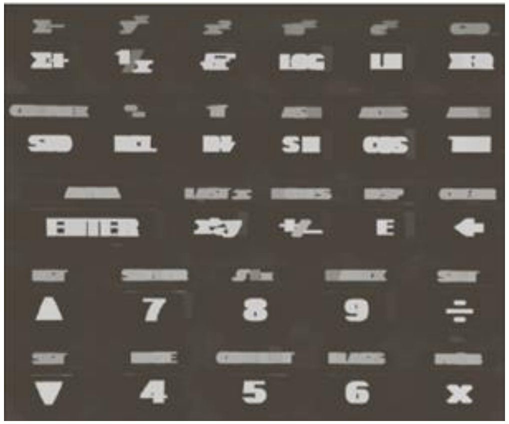
$=$

The result is close to our objective but the "I" is still missing

Gray-Scale Morphological Reconstruction (cont.)

- Using the last image as a marker and the dilated image as a mask we perform a gray-scale reconstruction by dilation and we obtain the desired result.

Marker



Mask

Result