Laboratory in Numerical Analysis, MA1207

3.5 ECTS credit points

Spring 2012

Instructions

1. You are allowed to work individual or in group of two persons. In any case, you need a group number to determine which sub exercise to solve, see the last page.

2. Last date for handing in your solutions is **June 8**. Advice: download this file in case *it's learning* is closed during the laboratory period.

3. Handwritten assignments will **not** be accepted. The solutions shall be handed in electronically via *it's learning*.

4. The solutions should be done on a computer, preferably with Matlab or Octave.

5. Write a short report, in English or Swedish, where you explain your solutions with you own words, and typeset the answers. Acceptable file formats are LaTeX, OpenOffice, PostScript, and PDF. Avoid Microsoft Word.

6. The report should come with a cover page. This page should contain your name and personal code number, the name of the course, the number of additional M-files submitted (preferably as a list of file names).

7. Save all code needed to solve each problem in one or several M-files, and comment your code. These M-files should be uploaded when the assignment is handed in.

8. To pass the laboratory all problems should be solved correctly. It will be possible to hand in complement.

Problem 1: LU factorization

Study the block matrix

$$A = \begin{pmatrix} B & 0 & B \\ 0 & B & 0 \\ B^2 & I & -B \end{pmatrix},$$

where I is the identity matrix of order 2, 0 is the zero matrix of order 2, and

(a) $B = \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix}$ (b) $B = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$ (c) $B = \begin{pmatrix} 1 & 7 \\ 1 & -4 \end{pmatrix}$.

Fins the matrices L, U and P such that $PA = LU$.

Problem 2: Non-linear equations

Find an approximation of the smallest positive solution of the equation

$$e^{-x}\sqrt{x^2+a} = \cos(x)$$

by using the secant method, with the accuracy $\varepsilon = 10^{-12}$.

(a) $a = 2$ (b) $a = 12$ (c) $a = 22$

Show all numbers in the sequence you generate.

Problem 3: Curve length

Let $f\colon \mathbb{R} \to \mathbb{R}$. The length of the curve $y = f(x)$, where $a \leq x \leq b$, is given by

$$\int_a^b \sqrt{1+[f'(x)]^2}\, dx.$$

Approximate the length of the curve $y = f(x)$, when

$$f(x) = \sin(x^2) \quad \text{and} \quad 0 \leq x \leq b,$$

by using the trapezoidal rule and divide the interval $[a, b]$ into 100 subintervals.

(a) $b = 2$ (b) $b = 3$ (c) $b = 4$

Problem 4: Ordinary differential equation

Approximate with the Runge-Kutta method of order $N = 4$ a solution $y = y(x)$ of the initial value problem

$$y' = y\sin(xy), \qquad y(0) = y_0,$$

over the interval $(0, b)$ and with 50 steps.

(a) $y_0 = 1$ and $b = 5$ (b) $y_0 = 2$ and $b = 4$ (c) $y_0 = 3$ and $b = 3$

Plot the solution and determine an approximation of $y(b)$.

Group parameters

Group	1	2	3	4	Group	1	2	3	4
1	(a)	(a)	(b)	(b)	26	(c)	(c)	(b)	(c)
2	(b)	(a)	(a)	(b)	27	(b)	(c)	(c)	(a)
3	(c)	(b)	(a)	(b)	28	(a)	(c)	(a)	(c)
4	(c)	(a)	(b)	(c)	29	(c)	(c)	(a)	(b)
5	(c)	(b)	(c)	(b)	30	(a)	(b)	(c)	(b)
6	(c)	(a)	(a)	(b)	31	(b)	(c)	(c)	(b)
7	(c)	(b)	(c)	(c)	32	(a)	(b)	(c)	(c)
8	(b)	(a)	(b)	(c)	33	(a)	(b)	(a)	(c)
9	(b)	(c)	(c)	(c)	34	(a)	(b)	(a)	(a)
10	(a)	(c)	(c)	(c)	35	(a)	(a)	(a)	(c)
11	(c)	(a)	(c)	(b)	36	(c)	(a)	(b)	(a)
12	(a)	(a)	(a)	(b)	37	(a)	(b)	(b)	(a)
13	(b)	(b)	(c)	(b)	38	(b)	(b)	(b)	(b)
14	(b)	(a)	(a)	(c)	39	(c)	(b)	(b)	(c)
15	(c)	(b)	(b)	(a)	40	(a)	(a)	(c)	(b)
16	(c)	(b)	(a)	(a)	41	(c)	(c)	(c)	(b)
17	(a)	(a)	(b)	(a)	42	(a)	(c)	(a)	(a)
18	(c)	(c)	(a)	(a)	43	(c)	(a)	(c)	(a)
19	(b)	(c)	(b)	(a)	44	(a)	(c)	(b)	(a)
20	(b)	(b)	(a)	(b)	45	(a)	(c)	(b)	(b)
21	(c)	(b)	(b)	(b)	46	(a)	(b)	(a)	(b)
22	(a)	(b)	(b)	(b)	47	(b)	(c)	(a)	(c)
23	(a)	(c)	(c)	(a)	48	(a)	(c)	(a)	(b)
24	(a)	(b)	(b)	(c)	49	(b)	(c)	(a)	(a)
25	(b)	(b)	(b)	(c)	50	(b)	(b)	(c)	(a)