Maximum Likelihood Estimation (MLE) of Natural Parameter of Rayleigh Distribution

Let's assume that we have done sampling and drawn a sample of N observation from a population which is Rayleigh Distribution and all N observations are **Independent and Identically Distributed (IID)**.

$$L() = P(X = x_1 \cap X = x_2 \cap X = x_3 \cap \ldots \cap X = x_N)$$

$$= P(X = x_1).\, P(X = x_2).\, P(X = x_3). \ldots .\, P(X = x_N)$$

$$= \frac{x_1}{\sigma^{\wedge^2}} \mathrm{e}^{-\frac{x_1^2}{2\sigma^{\wedge 2}}} \cdot \frac{x_2}{\sigma^{\wedge^2}} \mathrm{e}^{-\frac{x_2^2}{2\sigma^{\wedge 2}}} \ldots . \frac{x_N}{\sigma^{\wedge^2}} \mathrm{e}^{-\frac{x_N^2}{2\sigma^{\wedge 2}}}$$

$$= (\frac{1}{\sigma^{\wedge^2}})^N . \prod_{i=1}^{n} x_i \mathrm{e}^{-\frac{\sum_{i=1}^{N} x_i^2}{2\sigma^{\wedge 2}}}$$

$$= (\prod_{i=1}^{n} x_i).\, (\frac{1}{\sigma^{\wedge^2}})^N \mathrm{e}^{-\frac{\sum_{i=1}^{N} x_i^2}{2\sigma^{\wedge 2}}}$$

$$-log_e L(\sigma^{\wedge}) = -log(\prod_{i=1}^{n} x_i) + log(\frac{1}{\sigma^{\wedge^2}})^N - \frac{\sum_{i=1}^{N} x_i^2}{2\sigma^{\wedge^2}}$$

$$= -[\sum_{i=1}^{N} log_e x_i + log_e(\frac{1}{\sigma^{\wedge^2}})^N - \frac{\sum_{i=1}^{N} x_i^2}{2\sigma^{\wedge^2}}]$$

$$= -\sum_{i=1}^{N} log_e x_i + log_e \sigma^{\wedge^2 N} + \frac{\sum_{i=1}^{N} x_i^2}{2\sigma^{\wedge^2}}$$

$$\frac{\delta}{\delta\sigma^{\wedge}}[-log_e L(\sigma^{\wedge})] = -\sum_{i=1}^{N} log_e x_i + log_e \sigma^{\wedge^2 N} + \frac{\sum_{i=1}^{N} x_i^2}{2\sigma^{\wedge^2}}$$

$$= -\sum_{i=1}^{N} log_e x_i + 2N.\, log_e \sigma^{\wedge} + \frac{\sum_{i=1}^{N} x_i^2}{2\sigma^{\wedge^2}}$$

$$= 0 + \frac{2N}{\sigma^{\wedge}} + \frac{\sum_{i=1}^{N} x_i^2}{\sigma^{\wedge^3}}$$

$$= \frac{\sum_{i=1}^{N} x_i^2}{\sigma^{\wedge^2}} = 2N$$

$$= \frac{\sum_{i=1}^{N} x_i^2}{2N} = \sigma^{\wedge^2}$$

Minimum Variance Unbaised (MVU) Estimator of Natural Parameter of Rayleigh Distribution

$$\sigma^{\wedge}_{ML} = \sqrt{\frac{\sum_{i=1}^{N} x_i^2}{2N}}$$

Here $\sigma = Mode$