Appendix 2.

Measurement procedure

Instead of a hole in the bottom (Fig.1 left), a suction tube (Fig.1 right) is more suitable for use. Right at the center of the tank is placed a vertical tube that reaches almost to the very bottom. For suction, we can use a pump or a natural drop.

1. Pour water up to the level h_0 into the stationary cylindrical tank. The tank is located on a rotating platform driven by a DC motor. After starting the DC motor, set by potentiometer the initial angular speed ω which corresponds to the Curve0 paraboloid. The angular speed can have any value in the interval $0<\omega<\omega_A$ where ω_A is described by (2)

2. As soon as drainage begins, the drive DC motor turns off and the mechanical connection to it is broken. The tank with the stand is still spinning due to inertia, and the drainage is done through a central suction pipe (Fig.1 right).

3. After a certain time (which depends on the diameter of the suction tube), the paraboloid touches the bottom and the water drainage stops (air bubbles appear in the pipe). At that moment, we measure the final angular speed ω_1 corresponding to the Curve1 paraboloid.

Again, pour water to the same initial level h_0 as before. We repeat the above procedure to obtain a new value pair ω and ω_1, etc. Using these value pairs, we draw a graph of the function $\Delta\omega = \omega_1(\omega) - \omega$ and compare it with a theoretical calculation (Fig.2 left) and describe the correlation between the theoretical and experimental results.

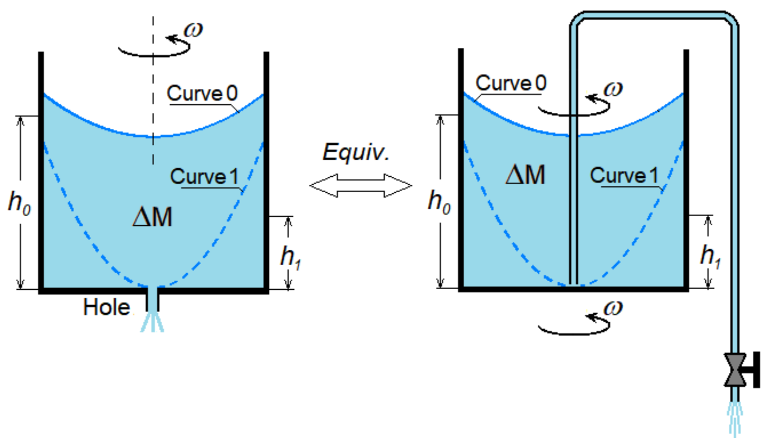

Fig.1 Both drainage methods are equivalent to each other. Two angular speeds are measured corresponding to the Curve0 and Curve1 paraboloids. During the drainage of water, the physical system must be closed with respect to external forces and torques. The suction tube is stationary (does not rotate).

Calculation of final angular speed

Let us try to find a relation between the final, ω_1, and initial, ω, angular speed. For a closed physical system (Fig.1) we can apply the law of conservation of angular momentum:

$$(I_1 + I_{00})\omega_1 = (I_0 + I_{00})\omega \tag{1}$$

Bottom-touching frequency is

$$\omega_A = \frac{2}{R}\sqrt{g\,h_0} \tag{2}$$

The initial mass of water in the tank is

$$M_0 = \rho\,\pi R^2 h_0 \tag{3}$$

The initial moment of inertia of a liquid paracylinder is

$$I_0 = \frac{M_0 R^2}{2}\left(1 + \frac{R^2\omega^2}{12\,g\,h_0}\right) \tag{4}$$

When drainage stops, the final angular speed of the tank, ω_1, becomes the new bottom-touching frequency, therefore

$$\omega_1 = \frac{2}{R}\sqrt{g\,h_1} \tag{5}$$

Thus

$$h_1 = \frac{R^2}{4g}\omega_1^2 \tag{6}$$

The mass of remaining water in the rotating tank is

$$M_1 = \rho\,\pi R^2 h_1 \tag{7}$$

Applying (6), we get

$$M_1 = \frac{\rho\pi R^4}{4g}\omega_1^2 \tag{8}$$

The final moment of inertia of a liquid paracylinder is

$$I_1 = \frac{M_1 R^2}{2}\left(1 + \frac{R^2\omega_1^2}{12\,g\,h_1}\right) = \frac{2}{3}M_1 R^2 \tag{9}$$

Thus

$$I_1 = \frac{\rho\pi R^6}{6g}\omega_1^2 \tag{10}$$

Equation (1) becomes

$$\left(\frac{\rho\pi R^6}{6g}\omega_1^2 + I_{00}\right)\omega_1 = \left[\frac{M_0 R^2}{2}\left(1 + \frac{R^2\omega^2}{12\,g\,h_0}\right) + I_{00}\right]\omega \tag{11}$$

After arranging, we get the cubic equation

$$\omega_1^3 + p\,\omega_1 + q(\omega) = 0 \tag{12}$$

where

$$p = \frac{6\,g\,I_{00}}{\rho\pi R^6} \qquad q(\omega) = -\frac{6\,g}{\rho\pi R^6}\left[\frac{M_0 R^2}{2}\left(1 + \frac{R^2\omega^2}{12\,g\,h_0}\right) + I_{00}\right]\omega \tag{13}$$

The discriminant is

$$D(\omega) = \left(\frac{q(\omega)}{2}\right)^2 + \left(\frac{p}{3}\right)^3 \tag{14}$$

The real solution of (12) is obtained using Cardano formula

$$\omega_1(\omega) = \sqrt[3]{\frac{-q(\omega)}{2} + \sqrt{D(\omega)}} + \sqrt[3]{\frac{-q(\omega)}{2} - \sqrt{D(\omega)}} \tag{15}$$

The change of the angular speed of the rotating tank is:

$$\Delta\omega = \omega_1(\omega) - \omega \tag{16}$$

The mass of leaked water is

$$\Delta M(\omega) = M_0 - M_1(\omega) = \rho\,\pi R^2 \left[h_0 - h_1(\omega)\right] \tag{17}$$

Using formula (6) we get

$$\Delta M(\omega) = \rho\,\pi R^2 \left(h_0 - \frac{R^2}{4g}\omega_1^2\right) \tag{18}$$

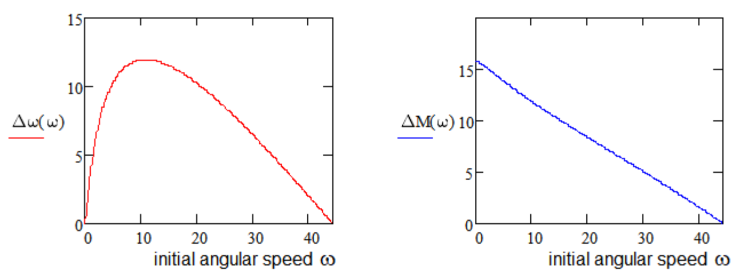

Fig.2 Input parameters are: *R=0.1m, h_0=0.5m, I_{00}=0.02kgm^2.*

Notice that during drainage the angular speed depends on the mass of remaining water in the tank and not on the time.

After measuring the angular speeds ω_1 and ω, the moment of inertia of the rotating apparatus can be calculated by using formula resulting from (1):

$$I_{00} = \frac{\rho\,\pi R^4}{2(\omega_1 - \omega)}\left[h_0\,\omega + \frac{R^2}{12g}\left(\omega^3 - 4\omega_1^3\right)\right] \tag{19}$$