Dynamic Equilibrium in Mechanical Systems with a Kinematic Lock: Classical, Exotic and Mixed Mode

Ranko Artuković
Independent researcher, Zadar, CROATIA
e-mail: bokola1@proton.me

Abstract

This paper presents a theoretical model of a mechanical system (planetary chain mechanism and differential) closed in a kinematic feedback loop. Three dynamic regimes are analysed. **Classical Mode (CM)** – excitation by external torques; the results are fully consistent with conservation laws. **Exotic Mode (EM)** – excitation solely by built-in tension (F_0); because of the kinematic lock the tension cannot relax, which results in continuous acceleration of all moving elements. In EM the carrier (C) exhibits a **negative dynamic contribution** – although its physical moment of inertia (I_C) is positive and constant throughout all regimes, in the dynamic equations it acts as an energy source. **Mixed Mode (MM)** – simultaneous action of external torques and built-in tension. It is proved that superposition holds for angular accelerations, while powers are not additive due to the quadratic dependence, revealing an interaction term as a measurable proof of the existence of the EM component. The paper concludes that the model is mathematically consistent and calls for experimental verification, with emphasis on the mixed mode as the most realistic framework for testing.

Keywords: kinematic lock, built-in tension, dynamic inertial equilibrium, negative dynamic contribution, critical moment of inertia, mixed mode

---

1. Introduction

Classical mechanics postulates that internal forces cannot change the state of motion of an isolated system. This postulate implicitly assumes the existence of paths for the relaxation of internal stresses – either through elastic deformations, friction or a change of configuration. However, the question arises: **What if the system is constructed so that all degrees of freedom required for relaxation are kinematically blocked?**

In this paper we analyse a system in which, owing to a specific coupling of a planetary chain mechanism and a differential, a **kinematic lock** is created. In such a lock, the tension

(F_0) introduced by an electromagnetic chain tensioner (ECT) cannot disappear by classical relaxation. The system is forced into a dynamic response – continuous acceleration.

The investigation is structured through three regimes:

1. **Classical Mode (CM)** – a verification model that confirms the validity of the formulated equations of motion.
2. **Exotic Mode (EM)** – the core subject of the research; it demonstrates that a built-in, non-relaxing tension can produce permanent acceleration.
3. **Mixed Mode (MM)** – the most general case; it enables a continuous transition between CM and EM and tests the limits of the system's linearity.

The aims of the paper are: (i) to formulate a consistent mathematical model for all three regimes using Newton's equations, (ii) to demonstrate that the negative dynamic contribution of the carrier in EM is a necessary consequence of the system's consistency, not a physical anomaly, (iii) to show that in MM the superposition principle holds for kinematics but not for energy quantities.

---

2. Description of the Mechanical Assembly and Kinematic Constraints

2.1. Component A – Planetary Chain Mechanism

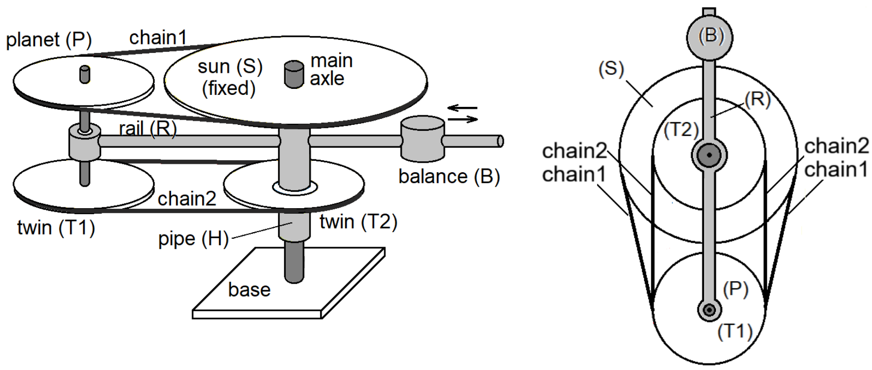

Figure 1. Planetary chain mechanism, left: perspective, right: top view

- **Sun (S)** – fixed sprocket, radius (R_S), ($\alpha_S = 0$).

- **Planet (P)** – sprocket of radius (R), pivotally mounted on the rail, angular acceleration (α_P).
- **Rail (R)** – rotates about the main shaft, moment of inertia (I_R), angular acceleration (α_R).
- **Twins (T1, T2)** – T1 rigidly connected to P, T2 rotatably mounted on the main shaft; all have the same (α_P). and a common moment of inertia (I_P).
- **Chain_1** – connects S and P, inextensible, no slip.
- **Chain_2** – connects T1 and T2, inextensible, no slip.

Kinematic relation A:

$$\alpha_P = \alpha_R \cdot (R - R_S) / R \qquad \text{(KM1)}$$

2.2. Component B – Differential Mechanism

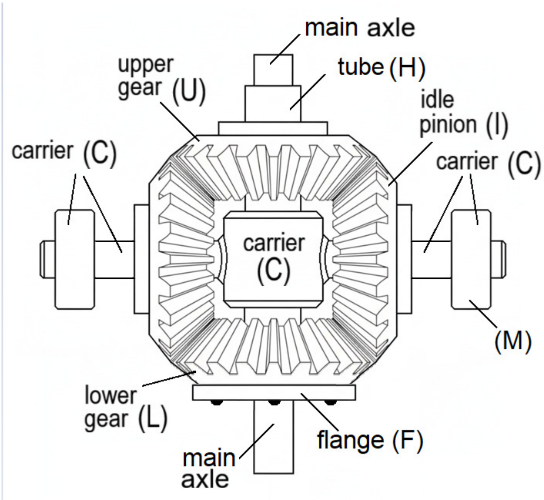

Figure 2. Differential mechanism with carrier C

- **Upper bevel gear (U)** and **lower bevel gear (L)** – radius (R), rotatably mounted on a vertical tube.
- **Idle pinion (I)** – radius (r), mounted on the carrier (C), simultaneously meshed with U and L. Its mass and moment of inertia are negligible ($m_I = 0$, $I_I = 0$).

- **Carrier (C)** – rotates about the main shaft, angular acceleration (α_C), moment of inertia (I_C) (physical quantity, positive).

Kinematic relations B:

$$\alpha_P = \alpha_C - \alpha_I \cdot r / R \qquad \text{(KM2)}$$

$$\alpha_P = \alpha_C - \alpha_I \cdot r / R \qquad \text{(KM3)}$$

2.3. Assembly and Kinematic Lock

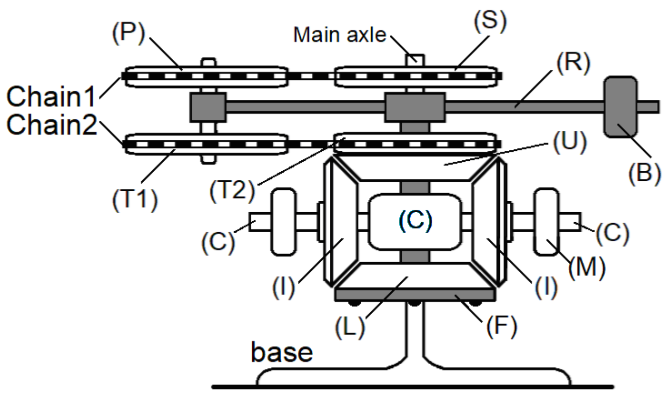

Figure 3. Assembly on a fixed base – components A and B together

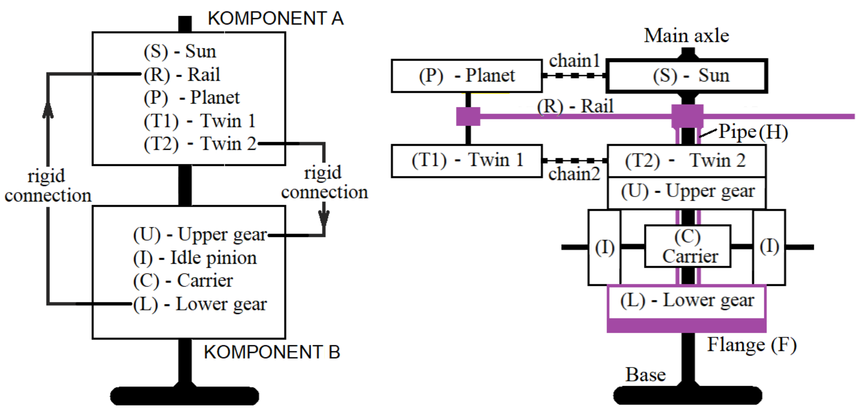

Figure 4. Schematic view of the entire assembly with mechanical feedback

By connecting the components (**T2 with U; L with R** via flange F and tube H) a **closed kinematic loop** is created. KM1, KM2 and KM3 form a system of three equations that uniquely determine all angular accelerations if any one of them is known. The **kinematic lock** arises because:

- any relative displacement of the sprockets requires a change in the length of the chains,
- chain_2 is permanently tensioned on one side by the force (F_0) (maintained by the ECT),
- any relaxation would require a change in geometry that is impossible without violating the kinematic constraints (tooth skipping, fracture).

2.4. Built-in Tension

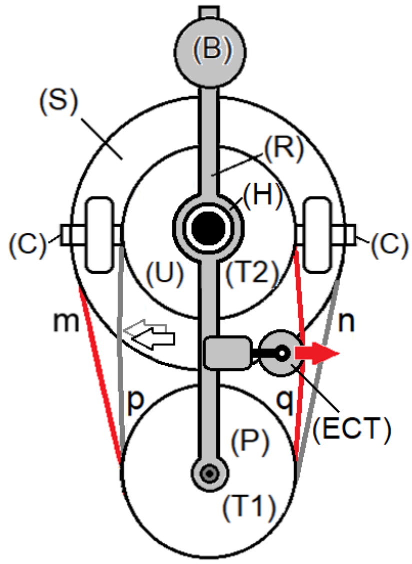

Figure 5. Top view – ECT tensions both chains, tensioned sides shown in red

An electromagnetic chain tensioner (ECT) mounted on the rail (R) maintains a constant tension force (F_0) on the right (q) side of chain_2. Because of the rigid connection P–T1 and sprocket T2, this tension also induces a tension (T) in chain_1. **The ECT does not supply energy continuously** – its role is only to maintain a constant force; the work done by the tensioner is zero because there is no displacement of its point of application (the tension is permanently trapped). Electrical energy is consumed only to sustain the magnetic field.

---

3. Generalised Dynamic Model

The equations of motion, applicable to **all three regimes**, are:

$$I_P \alpha_P = (F - T) R \tag{1}$$

$$I_R \alpha_R = F R - T (R_S - R) + \tau_R \tag{2}$$

$$I_C \alpha_C = -2 F R + \tau_C \tag{3}$$

$$\alpha_P = \alpha_R \cdot \frac{(R - R_S)}{R} \tag{4}$$

$$\alpha_P = \alpha_C - \alpha_I \frac{r}{R} \tag{5}$$

$$\alpha_R = \alpha_C + \alpha_I \frac{r}{R} \tag{6}$$

Remark on the sign in (3): Force (F) acts on carrier C via pinion I. Because of the geometry and the direction of the force (F_0), carrier C is "pushed" in the direction opposite to the positive one. The negative sign is not arbitrary – it is a direct consequence of the force orientation in Figure 5 and is necessary for the consistency of the kinematic relations.

---

4. Classical Mode (CM)

Condition: F=0, $\tau_R, \tau_C \neq 0$

System (1)–(6) is solved for the angular accelerations. The solutions are omitted for brevity, but have been fully verified:

Energy balance: $P_{IN} = \tau_C \alpha_C t + \tau_R \alpha_R t = I_P \alpha_P^2 \cdot t + I_R \alpha_R^2 \cdot t + I_C \alpha_C^2 \cdot t = P_{OUT}$

Torque balance: $M_1 = I_P \alpha_P + I_R \alpha_R + I_C \alpha_C = \tau_C + \tau_R - T \cdot R_S = M_2$

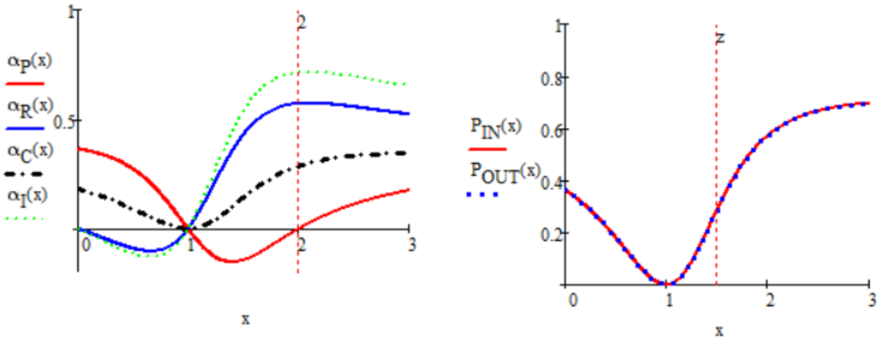

Figure 6. CM: graphs of angular accelerations (left), input/output power graph (right)

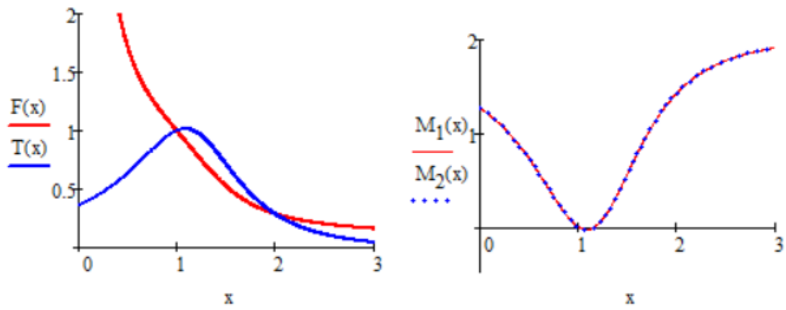

Figure 7. CM: tension forces F and T (left), torque balance (right)

Conclusion CM: Equations (1)–(6) are physically valid. The model is consistent with conservation laws. This eliminates the possibility that equations (1)–(6) are fundamentally wrong.

---

5. Exotic Mode (EM)

Condition: $\tau_R=\tau_C=0$, $F=F_0=\text{const.}$

System (1)–(6) becomes:

$$I_P\alpha_P=(F_0-T)R \tag{7}$$

$$I_R\alpha_R=F_0R-T(R_S-R) \tag{8}$$

$$I_C\alpha_C=-2F_0R \tag{9}$$

$$\alpha_P = \alpha_R \cdot \frac{(R - R_S)}{R} \tag{10}$$

$$\alpha_P = \alpha_C - \alpha_I \frac{r}{R} \tag{11}$$

$$\alpha_R = \alpha_C + \alpha_I \frac{r}{R} \tag{12}$$

5.1. Solutions for EM

$$\alpha_R^{EM} = \frac{F_0 R^2 (2R - R_S)}{D} \quad , \quad \alpha_C^{EM} = \frac{F_0\, R}{2} \cdot \frac{(2R - R_S)^2}{D}$$

$$\alpha_P^{EM} = \frac{F_0 R (R - R_S)(2R - R_S)}{D} \quad , \quad \alpha_I^{EM} = \frac{F_0 R^2 R_S}{2r} \cdot \frac{2R - R_S}{D}$$

$$T^{EM} = F_0 R \cdot \frac{I_R R - I_P (R - R_S)}{D}$$

where $$D = I_R R^2 + I_P (R - R_S)^2 \tag{13}$$

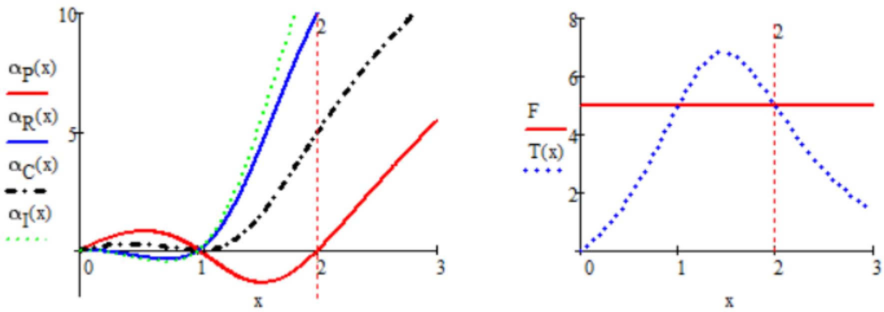

Figure 8. EM: graphs of angular accelerations (left), tension forces (F_0) and (T) (right)

5.2. Critical Moment of Inertia

The condition of kinematic consistency $\alpha_C = (\alpha_P + \alpha_R)/2$ (which follows from (4)–(6)) leads to:

$$I_C = -4 \cdot \frac{I_R R^2 + I_P (R - R_S)^2}{(2R - R_S)^2} \tag{14}$$

The physical moment of inertia of the carrier, (I_C), is positive and constant in all regimes. Formula (14) does not describe a change of a physical property, but rather **a condition imposed by the system** in order to be dynamically consistent. It shows that in EM the carrier C must *behave* as if it had a negative dynamic contribution. For the construction of a prototype to function, the actual moment of inertia of the carrier should be:

$$I_{C,CRIT} = |I_C| = 4 \cdot \frac{I_R R^2 + I_P (R - R_S)^2}{(2R - R_S)^2} \tag{15}$$

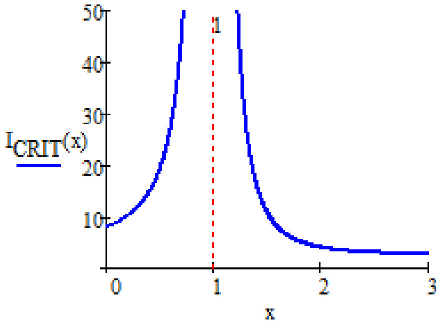

Figure 9. EM: Critical moment of inertia $I_{C,CRIT}$

5.3. Dynamic Properties of EM

1. **Negative dynamic contribution of the carrier:** Equation (9) reads $I_C \alpha_C = -2F_0 R$. The carrier does not decelerate the system – it accelerates *opposite* to the direction of the force (F_0). This is the definition of a **dynamic support**: an element that delivers energy to the rest of the system instead of absorbing it.

2. **Energy symmetry (50/50):**

$$P_P + P_R = I_P \alpha_P^2 \cdot t + I_R \alpha_R^2 \cdot t = I_{CRIT} \alpha_C^2 \cdot t = P_C$$
$$P_{TOT} = P_P + P_R + P_C = 2\,P_C$$

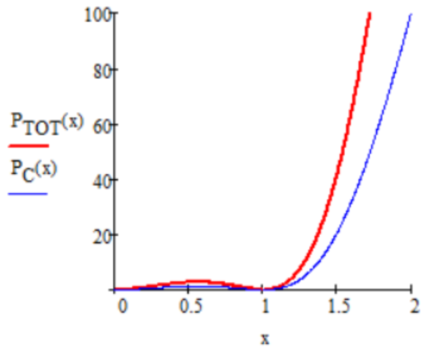

Figure10. EM: powers

3. **Torque balance:**

   - Dynamic torque: $M_1 = I_P\alpha_P + I_R\alpha_R + I_C\alpha_C = \tau_P + \tau_R + \tau_C$
   - Static reaction torque (on fixed S): $M_2 = T \cdot R_S$
   - **Always holds:** $M_1 + M_2 = 0$

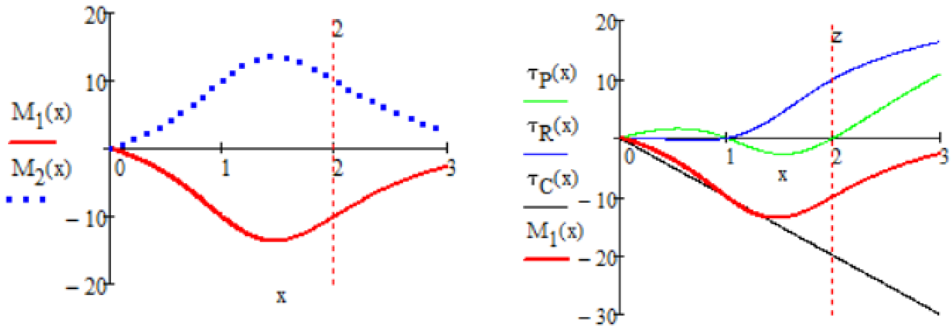

Figure 11. EM: torque balance (left), components of the dynamic torque (right)

This is dynamic inertial equilibrium: the inertial forces of the accelerated masses exactly balance the external static reaction. Energy is not drawn from the environment – it is a consequence of internal redistribution through the kinematic lock.

---

6. Mixed Mode (MM)

Condition: $\tau_R, \tau_C \neq 0$ i $F = F_0 = const.$ simultaneously.

6.1. Superposition for Angular Accelerations

System (1)–(6) is linear in the variables α_P, α_R, α_C, α_I, F, T. Let:

- $\alpha_i^{CM}(\tau_R,\tau_C)$ be the solutions for $\tau_R,\tau_C \neq 0,\ \ F=0$
- $\alpha_i^{EM}(F_0)$ be the solutions for $\tau_R=\tau_C=0,\ \ F=F_0$

Then the solution for MM is:

$$\alpha_i^{MM} = \alpha_i^{CM} + \alpha_i^{EM} \quad , \quad i = P,R,C,I \tag{16}$$

Proof: Substituting $F = F_0 + F'$ and $T = T_0 + T'$ into (1)–(6), where (F_0, T_0) are the EM solutions and (F', T') are the CM solutions, the system separates into two independent linear subsystems.

6.2. Nonlinearity of Powers – Interaction Term

The power on the carrier (C) in mixed mode (MM) is:

$$P_C^{MM} = I_C\left(\alpha_C^{MM}\right)^2 = I_C\left(\alpha_C^{CM} + \alpha_C^{EM}\right)^2$$

Expanding the square gives:

$$P_C^{MM} = \underbrace{I_C\left(\alpha_C^{CM}\right)^2}_{P_C^{CM}} + \underbrace{I_C\left(\alpha_C^{EM}\right)^2}_{P_C^{EM}} + \underbrace{2I_C\alpha_C^{CM}\alpha_C^{EM}}_{P_{INT}}$$

where:

P_C^{CM} – power that would be developed on the carrier in pure classical mode,

P_C^{EM} – power that would be developed on the carrier in pure exotic mode,

$P_{INT} = 2I_C\alpha_C^{CM}\alpha_C^{EM}$ – **interaction term**, representing additional power generated by the simultaneous action of external torques and built-in tension.

Physical interpretation:

The interaction term P_{INT} is contained neither in pure CM nor in pure EM. It is a **direct measurable consequence of the existence of the exotic component inside the mixed**

mode. If the exotic mode did not exist ($\alpha_C^{EM} = 0$) *then* $P_{INT} = 0$ and the powers would be simply additive. Any deviation from additivity of powers in MM constitutes an experimental proof of the existence of EM.

6.3. Significance of MM for Experiment

- MM is not a mere academic curiosity. In a real experiment: - It is impossible to perfectly eliminate friction and external influences – the system will always be close to MM.
- MM enables **regenerative braking**: by controlled application of τ_C *(braking), energy can be recovered, while (F_0) remains the driving source.*
- *The **interaction term** P_{INT} is the most sensitive indicator of the model's correctness – if EM does not exist,* $P_{INT} = 0$ and the powers would be additive.

---

7. Discussion

7.1. On the Critical Moment of Inertia and Negative Dynamic Contribution

Formula (14) is often misinterpreted as a "requirement for a negative moment of inertia". This is incorrect. The physical (I_C) is positive and constant. Formula (14) is a **mathematical consistency condition** that shows that in EM the carrier C behaves as a dynamic support. The negative sign in (9) is not an error – it is necessary for the system to satisfy the kinematic constraints (4)–(6).

Analogy: In electrical engineering, a negative differential resistance does not mean that the physical resistance is negative, but that the component (e.g. a tunnel diode) in a certain operating region delivers power to the circuit. Similarly, the carrier C in EM delivers energy to the mechanical system.

7.2. Why the Lagrangian Formalism Does Not Work for EM and MM

An attempt to apply Lagrangian mechanics to EM was unsuccessful. The reason is fundamental: the Lagrangian formalism assumes that potential energy can be uniquely defined and that the system tends to minimise it. In EM, the tension (F_0) is **non-relaxing and non-conservative** inside the considered kinematic loop. Energy is not stored as potential energy, but is continuously transformed into kinetic energy of acceleration. Newton's approach, which directly treats forces and constraints, is the only valid tool for such systems.

7.3. Consistency of the Model across Three Regimes

The strongest argument in favour of the model is its consistency:

- The same equations (1)–(6) with $F=0$, $\tau\neq0$ give classical solutions (CM) that obey all conservation laws.
- The same equations with $F=F_0$, $\tau=0$ give solutions (EM) that satisfy the same kinematic conditions, but with a negative dynamic contribution of the carrier.
- The same equations with $F=F_0$, $\tau\neq0$ give solutions (MM) for which superposition of accelerations holds.

One cannot reject EM while accepting CM, because they use identical physical laws and mathematical apparatus.

8. Conclusion

This paper has presented a consistent theoretical model of a mechanical system in a kinematic lock, analysed through three dynamic regimes.

1. **Consistency of the model:** The system of equations (1)–(6) is verified through the classical mode (CM), where it yields the expected solutions in accordance with conservation laws. The same system, applied to the exotic mode (EM), gives mathematically consistent solutions that describe continuous acceleration.
2. **Critical moment of inertia:** Formula (14) is a necessary mathematical consistency condition for EM. The physical moment of inertia of the carrier (I_C) is positive and constant in all regimes; what changes is the **dynamic contribution** of the carrier. In EM the carrier acts as a dynamic support with a negative effective contribution.
3. **Dynamic inertial equilibrium:** In EM, the inertial forces of the accelerated masses (M_1) exactly balance the static reaction torque of the environment (M_2). This is a new form of equilibrium, characterised by 50/50 energy symmetry and the absence of continuous external energy supply.
4. **Mixed mode (MM):** MM is formulated for the first time. Superposition is proved for angular accelerations, but not for powers. The interaction term $P_{INT} = 2I_C\alpha_C^{CM}\alpha_C^{EM}$ is a direct measurable consequence of the existence of the EM component and represents the most sensitive tool for experimental verification.

Call for experimental verification:

Theoretical analysis shows that the model is mathematically closed and consistent. The negative dynamic contribution is not an error, but a property. An experiment is now necessary.

A prototype with the following characteristics is proposed:

- Capability of precise regulation of the external torques τ_R, τ_C
- Maintenance of a constant built-in tension (F_0) via ECT
- Measurement of angular accelerations α_P, α_R, α_C
- Measurement of powers on the carrier and on the rest of the system

Special emphasis should be placed on the mixed mode (MM). Measuring the total power of the system at different ratios of (τ_C) and (F_0), *and comparing it with the predicted interaction term, will be the most direct proof of the correctness of the theory presented here. If it is experimentally confirmed that* $P_{INT} \neq 0$ and that its value corresponds to $2I_C\alpha_C^{CM}\alpha_C^{EM}$, this will be an unambiguous confirmation of the existence of the exotic mode.

References:

[1] Newton, I. (1687). *Philosophiæ Naturalis Principia Mathematica*.
[2] Goldstein, H., Poole, C., & Safko, J. (2002). *Classical Mechanics* (3rd ed.). Addison-Wesley.

Editor's Note (Author's Comment)

We are aware that the proposed model, especially the part concerning the negative dynamic contribution of the carrier in the exotic mode, represents a strong deviation from the usual interpretation of Newtonian mechanics. However, we insist that this model is derived *from* Newton's equations, not despite them. Rejecting the model without experimental verification would be equivalent to rejecting mathematical consistency. Therefore, it is experiment – not a priori theoretical convictions – that must deliver the final verdict.