Ryan St. Pierre
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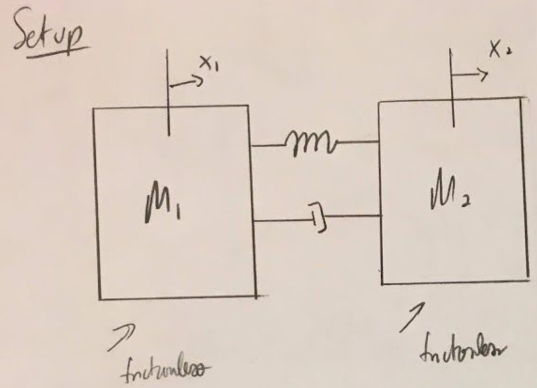
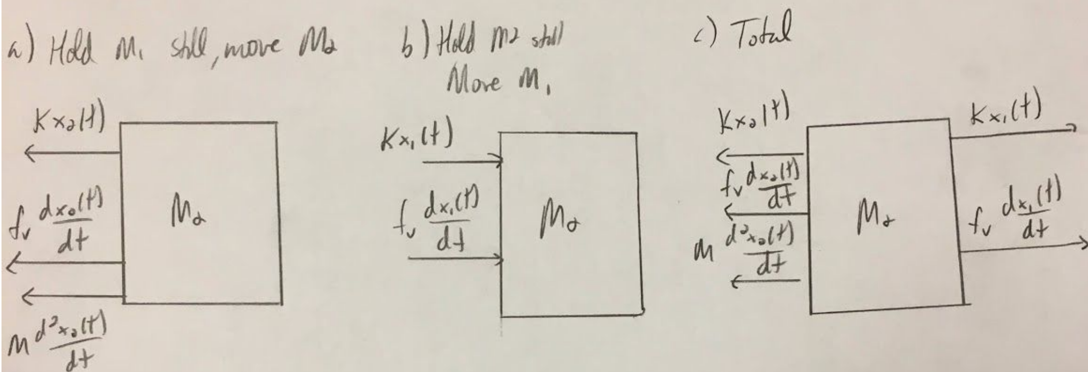
**Hand written work**

My work to derive the transfer function is given below:




$$M\frac{d^2x_2(t)}{dt} + f_v\frac{dx_2(t)}{dt} + kx_2(t) = f_v\frac{dx_1(t)}{dt} + kx_1(t)$$

$$Ms^2X_2(s) + f_v s X_2(s) + kX_2(s) = f_v s X_1(s) + KX_1(s)$$

$$(Ms^2 + f_v s + k)X_2(s) = (f_v s + k)X_1(s)$$

$$G(s) = \frac{X_2(s)}{X_1(s)} = \frac{f_v s + K}{Ms^2 + f_v s + K}$$

**Maple Plots**

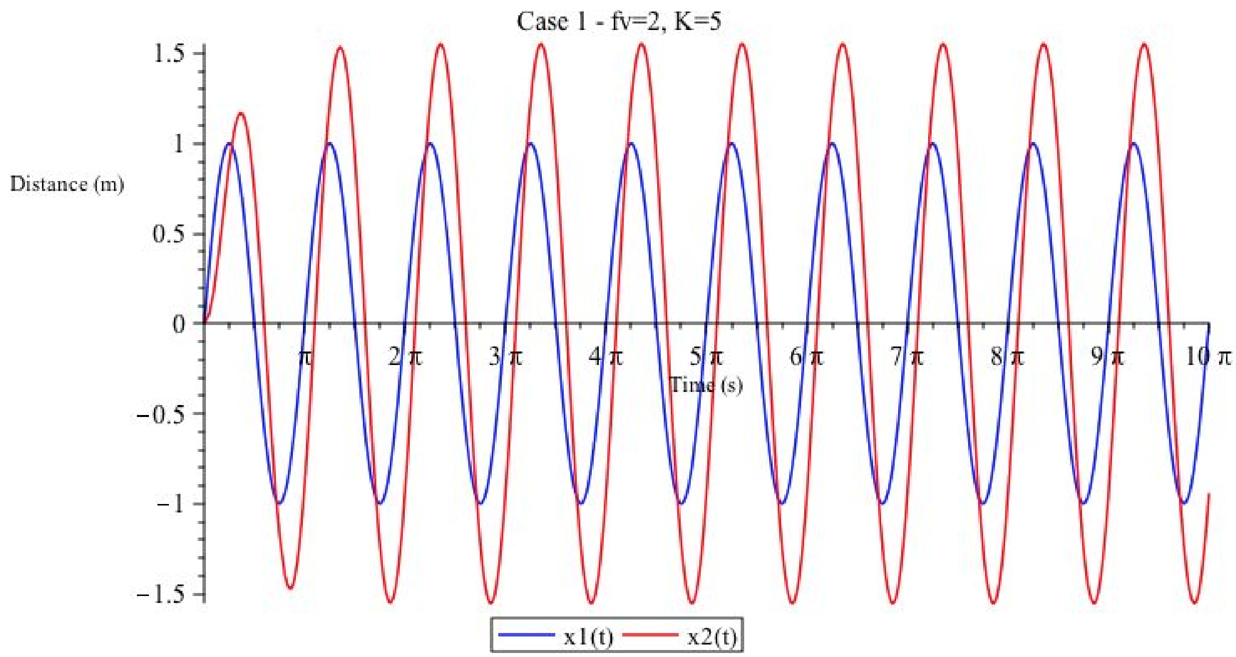



Figure 1- Case 1: $f_v = 2$, K=5

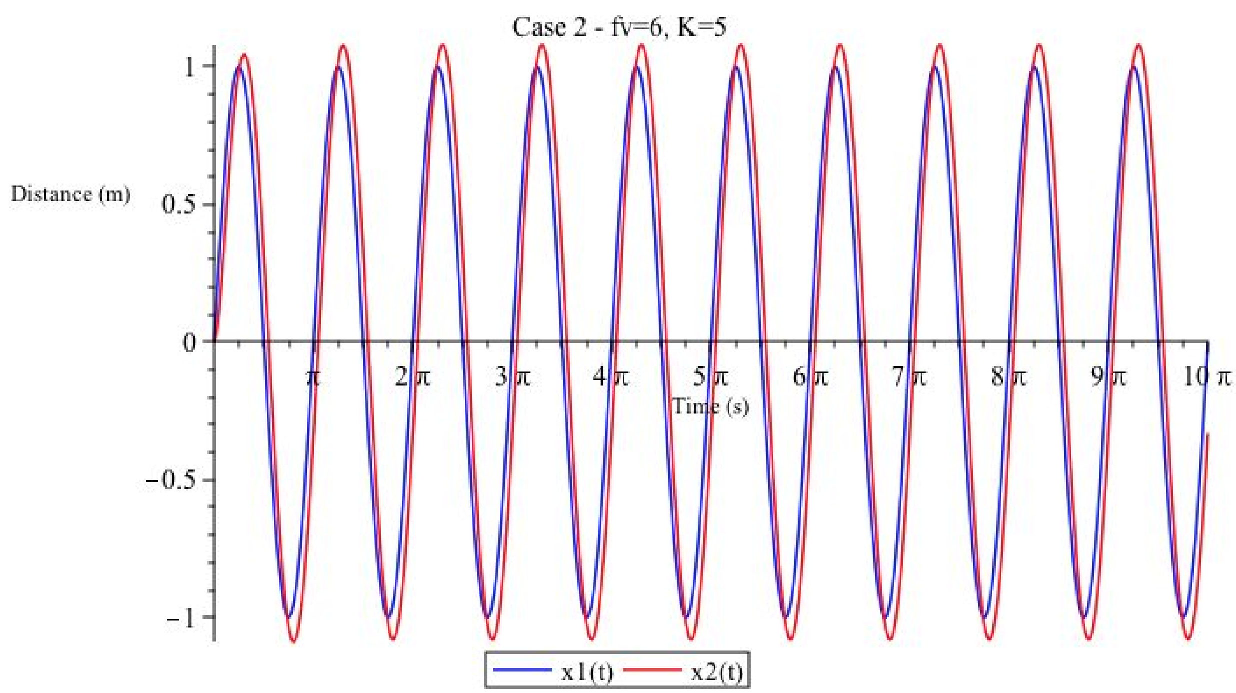



Figure 2- Case 2: $f_v = 6$, K=5

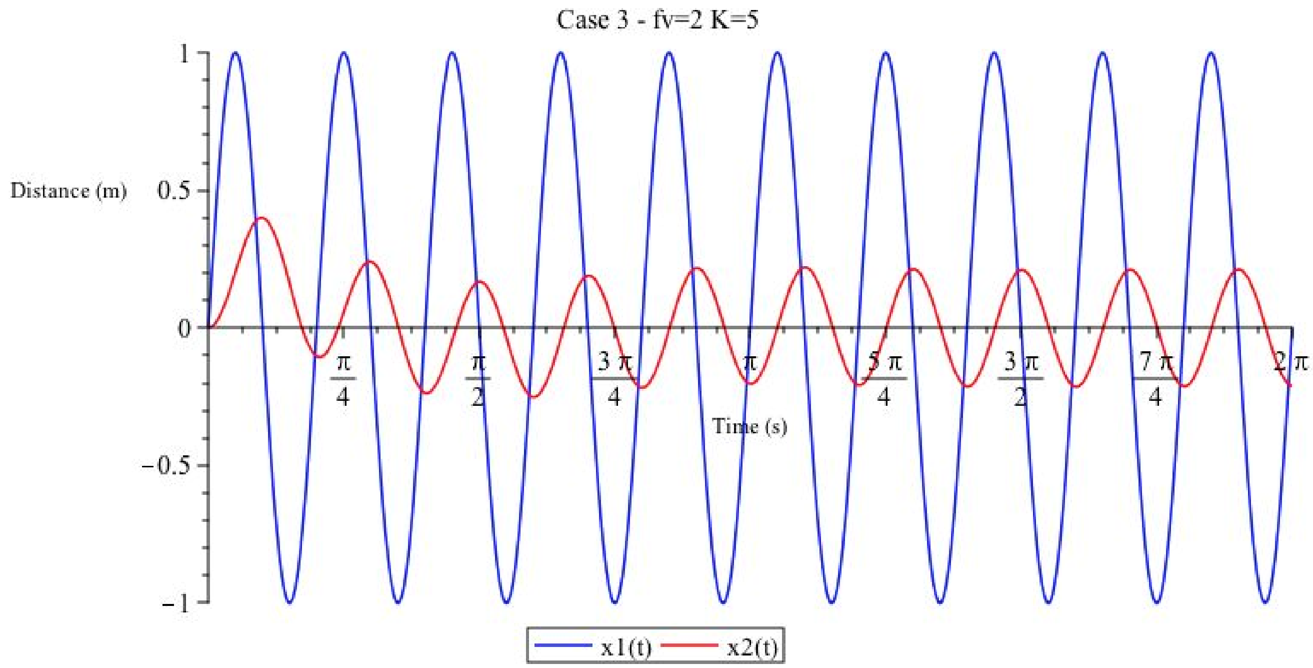



Figure 3- Case 3: $f_v$ = 2, K=5

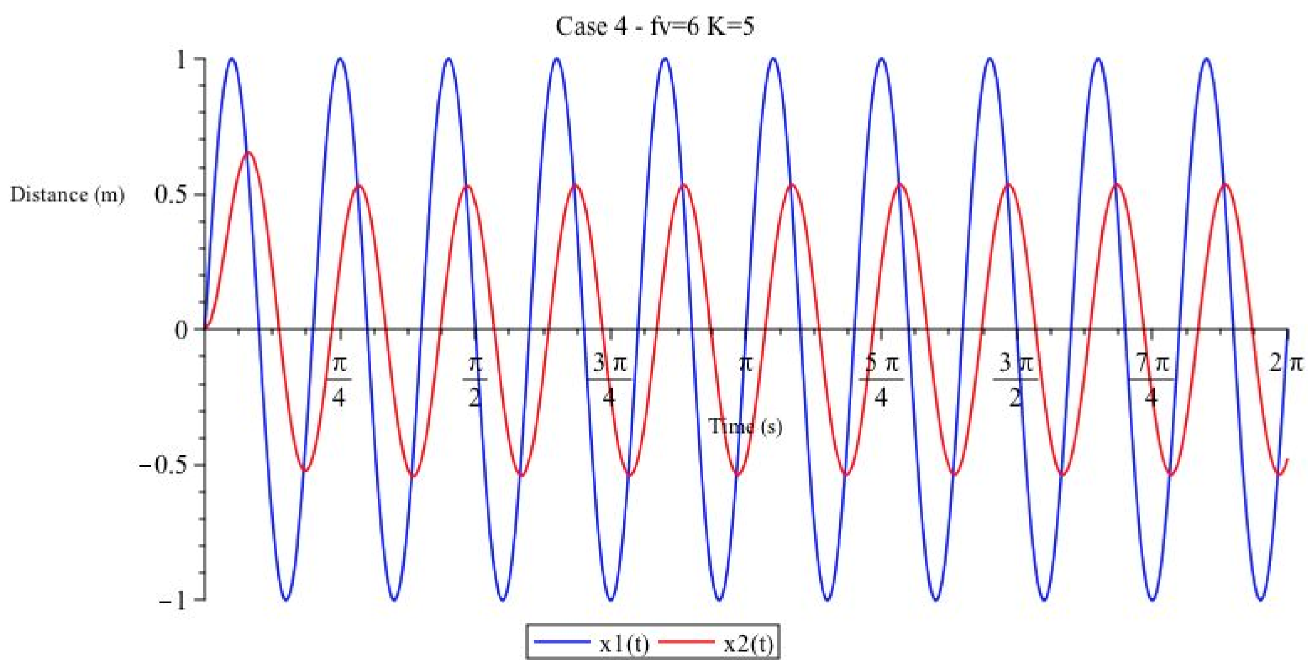



Figure 4- Case 4: $f_v$ = 6, K=5

**Matlab Plots**

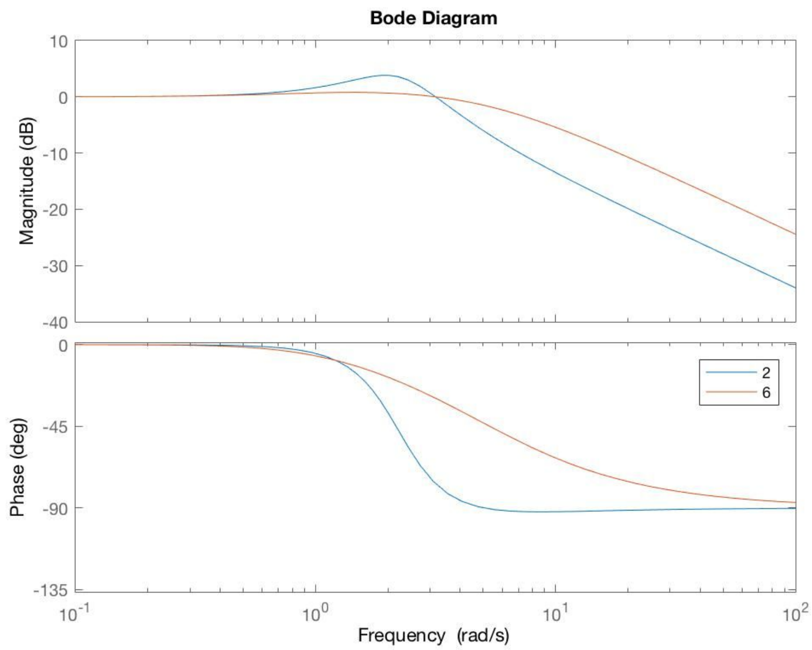



Figure 5- Bode diagram for $f_v = 2$ and $f_v = 6$ , K=5

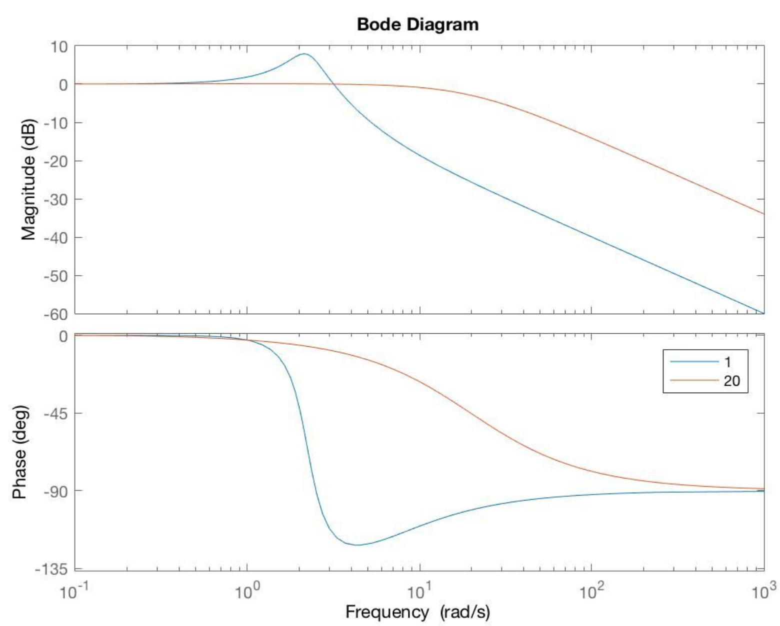



Figure 6- Bode diagram for $f_v = 1$ and $f_v = 20$ , K=5 [This figure was created to better exaggerate the outcome with difference damping coefficient]

**Further Discussion**

An in-depth discussion is given at the bottom of my Maple worksheet. However, I will augment this argument with a bode plot produced by Matlab. From the Maple and Matlab figures it is clear that the gain decreases as the frequency increases. This is best seen in Figures 5 and 6. Regardless of the damping coefficient the magnitude decreases with frequency.

By looking at Figures 5 and 6 it is slightly less clear the relationship between damping coefficient and transfer function magnitude and phase. For low frequencies it appears the the lower damping coefficient actually has a larger, non-negative gain. This is associated with the small “spike” at low frequencies of the bode plots. However, as the frequency increases the magnitude regardless of the damping frequency becomes negative. Furthermore, as the frequency increases the magnitude of the transfer function associated with the lower damping coefficient actually becomes more non-negative than the magnitude of the transfer function associated with the larger damping coefficient.

Regardless of the damping coefficient it appears that the phase shift starts at 0 degrees for negative and small frequencies and approaches 90 degrees as the frequency increases. However, the smaller the damping coefficient the faster this transition occurs (with respect to frequency). Additionally, at very small damping coefficients (reference Figure 6) the phase shift actually overshoots 90 degrees before settling back to that value as frequency increases. Thus, with a larger damping coefficient the transition to a 90 degree phase shift as frequency increases is more smooth and happens slower.

Additionally, it appears from Figures 5 and 6 that this system produces a negative magnitude for most frequencies. Thus, it is likely that what is being perceived in Figures 1-4 is not a small phase shift, but rather a flip about the x-axis (caused by the negative gain) and a more substantial phase shift. It is appropriate to note that a mirror about the x-axis and phase shift near 90 degrees can appear to be just a small phase shift for sinusoidal input.

**Matlab Code**

```
bode(tf([2 5], [1 2 5]), tf([6 5], [1 6 5]))
legend('2','6',0)
figure(2)
bode(tf([1 5], [1 1 5]), tf([20 5], [1 20 5]))
legend('1','20',0)
```