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ras70
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ME 344

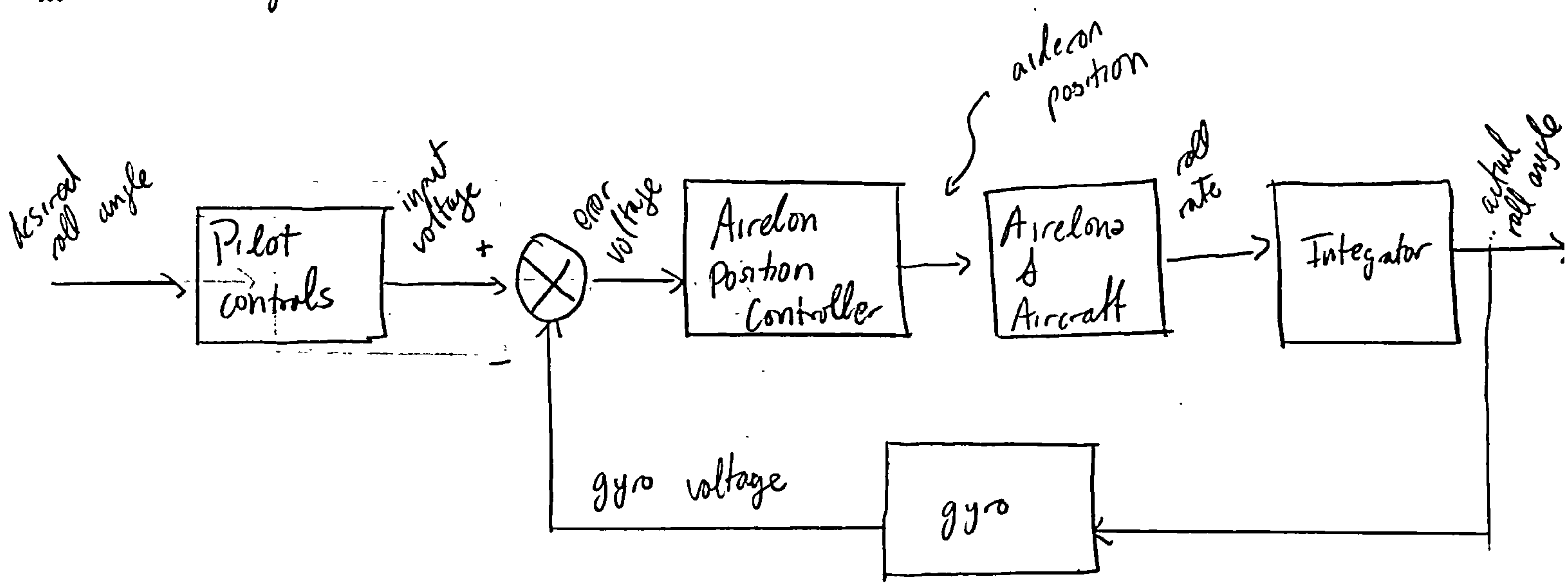
Problem 1

Subsystems

1. Gyro: roll angle → voltage ✓
2. ailerons & aircraft: plant? what needs to change/be controlled ✓
3. Integrator: rate $\frac{d\theta}{dt}$ → angle ✓
4. Pilot controls: ✓ sets desired roll angle / maybe able to convert to voltage
5. Aileron position ✓ controller: takes in voltage (error) → gives position of aileron (how much need to move)

Signals

- desired roll ∠ ✓
- roll rate ✓
- actual roll angle ✓
- input voltage ✓
- gyro voltage ✓
- error voltage ✓
- aileron position ✓



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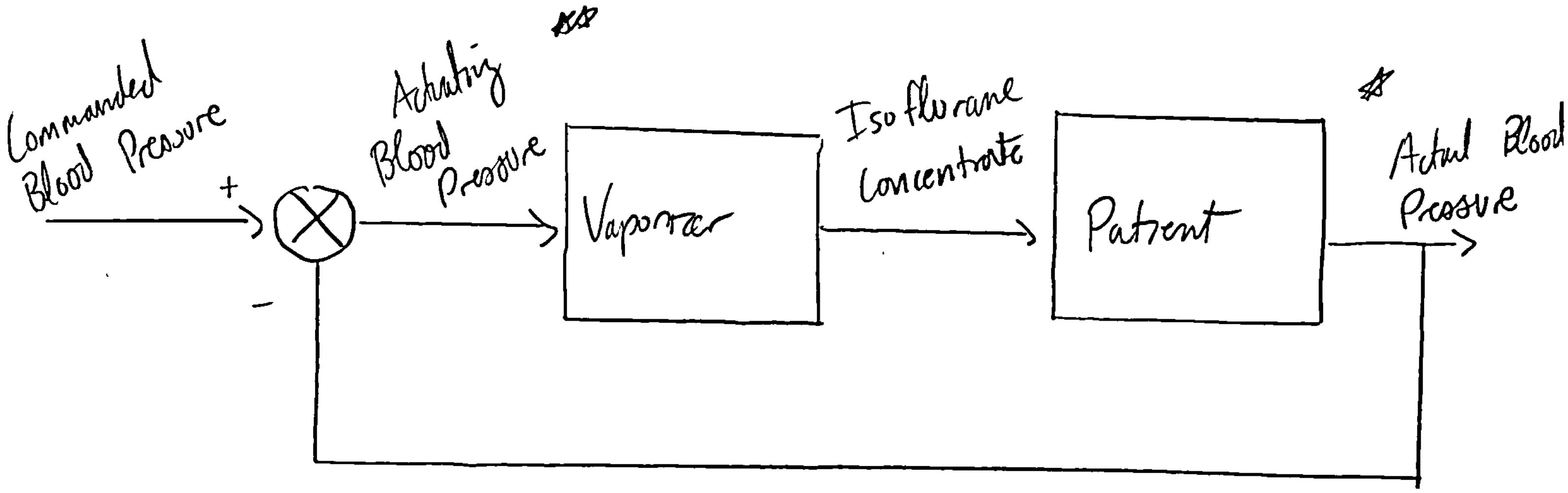
Problem 2

Signals

- Commanded blood pressure
- actual blood pressure
- isoflurane concentrate

Subsystems

- Patient → plant (is influenced)
- Vaporizer → controller (drives how much isoflurane concentrate the patient receives



Patient: plant

Vaporizer: controller

** Assuming the vaporizer takes in a blood pressure. If it needs a voltage a blood pressure → voltage subsystem would be needed.

* There needs to be some sort of device/subsystem to extract the blood pressure from the patient. I have assumed this is given and left this subsystem out

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Problem 3

Signals

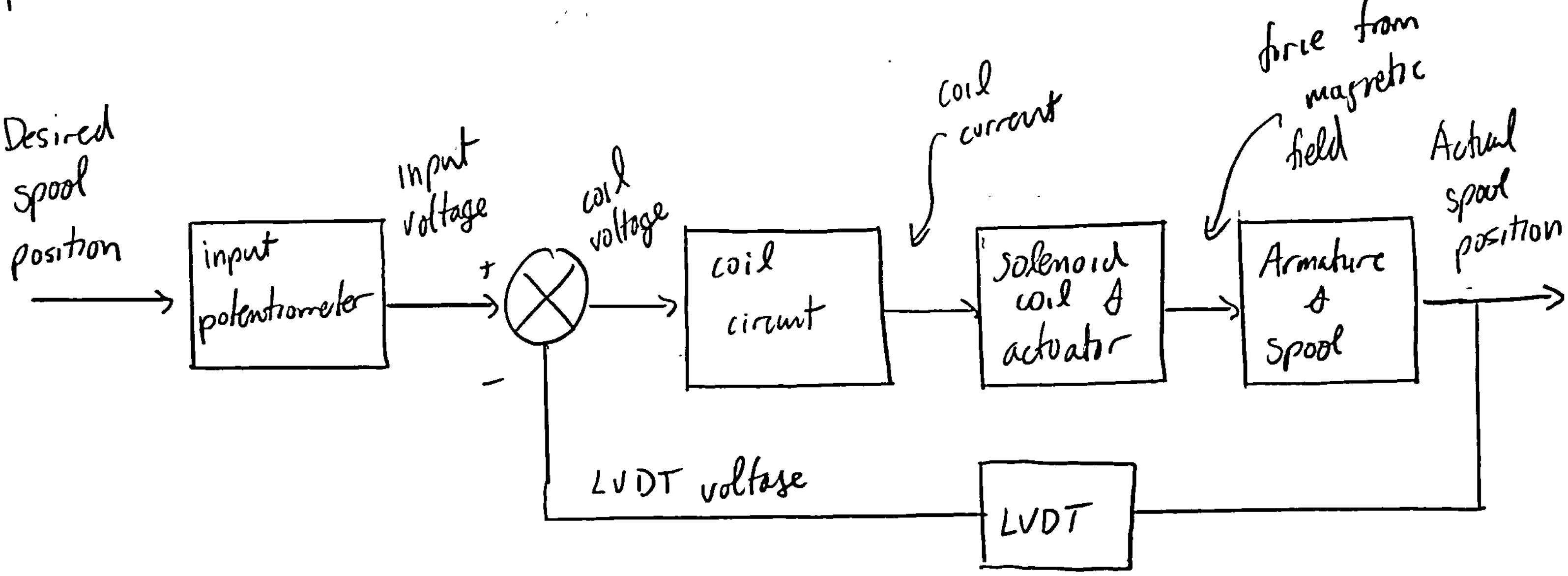
- coil voltage ✓
- coil current ✓
- desired spool position ✓
- actual spool position ✓
- LVDT voltage ✓
- force from magnetic field

Subsystems

- input potentiometer: ✓
  position → voltage
- coil & actuator ✓
  current → mag. field (force)
- LVDT ✓ spool position → voltage (feedback)
- armature & spool
- coil circuit ✓

Goal

position → voltage → coil → current → magnetic field → armature movement → position



4a) ras 70

$$\frac{dx}{dt} + 7x = 5\delta(t) \qquad y(0^-) = 0 \qquad y'(0^-) = 0$$

$$sY(s) - \cancel{y(0^-)} + 7Y(s) = 5(1)$$

*Assuming forcing function prior to $t=0$ is zero

$$Y(s)(s+7) = 5$$

$$Y(s) = \frac{5}{s+7}$$

$$\boxed{y(t) = 5e^{-7t}u(t)}$$

4b)

$$\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 25x = 10u(t) \qquad y(0)=0 \qquad y'(0)=0$$

$$s^2Y(s) - s\cancel{y(0^-)} - \cancel{y'(0^-)} + 8(sY(s) - \cancel{y(0^-)}) + 25Y(s) = \frac{10}{s}$$

* forcing function prior to $t=0$ is zero

$$s^2Y(s) + 8sY(s) + 25Y(s) = \frac{10}{s}$$

$$Y(s)(s^2+8s+25) = \frac{10}{s}$$

$$Y(s) = \frac{10}{s(s^2+8s+25)}$$

$s^2+8s+25$

$$s = \frac{-8 \pm \sqrt{64-4(25)}}{2}$$

$$= \frac{-8 \pm \sqrt{-36}}{2}$$

$$= -4 \pm 3i$$

Two imaginary roots

Partial-fraction expansion

$$Y(s) = \frac{10}{s(s^2+8s+25)} = \frac{A}{s} + \frac{Bs+C}{s^2+8s+25}$$

↓

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4b) cont

$$\frac{10}{s(s^2+8s+25)} = \frac{A}{s} + \frac{Bs+C}{s^2+8s+25}$$

$$10 = A(s^2+8s+25) + s(Bs+C)$$

$\underline{s=0}$

$10 = A \cdot 25$

$A = 10/25$

Balancing coefficients

s^2: $A + B = 0$

$B = -A = -10/25$

s: $8A + C = 0$

$C = -8A = -80/25$

$$Y(s) = \frac{10}{25}\left[\frac{1}{s} - \frac{s+8^{**}}{s^2+8s+25^{*}}\right]$$

$$\boxed{y(t) = \frac{10}{25}u(t)\left[1 - e^{-4t}\left(\cos 3t + \frac{4}{3}\sin 3t\right)\right]}$$

* Complete the square

$s^2+8s+25 = s^2+8s+16+9$

$= (s+4)^2+9$

$\omega = 3$

$a = 4$

**

$s+8 = s+4+4$

$= (s+4) + \frac{4}{3}(3)$

$$\mathcal{L}[Ae^{-at}\cos\omega t + Be^{-at}\sin\omega t] = \frac{A(s+a)+B\omega}{(s+a)^2+\omega^2}$$

$a = 4$

$\omega = 3$

$A = 1$

$B = \frac{4}{3}$

4c) $$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 5x = \frac{5}{2}\sin 2t$$

$y(0^-) = y'(0^-) = 0$

$\omega = 2$

$$s^2Y(s) - sy(0^-) - \cancel{y'(0^-)} + 4(sY(s) - \cancel{y(0^-)}) + 5Y(s) = \frac{5}{2}\,\frac{2}{s^2+4}$$

$$s^2Y(s) + 4sY(s) + 5Y(s) = \frac{5}{s^2+4}$$

$$Y(s) = \frac{5}{(s^2+4s+5)(s^2+4)}$$

Both have imaginary roots

↓

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4c cont) $Y(s) = \dfrac{5}{(s^2+4s+5)(s^2+4)} = \dfrac{As+B}{s^2+4s+5} + \dfrac{Cs+D}{s^2+4}$

$5 = (As+B)(s^2+4) + (Cs+D)(s^2+4s+5)$

$5 = As^3 + Bs^2 + 4As + 4B + Cs^3 + 4Cs^2 + 5Cs + Ds^2 + 4Ds + 5D$

$$\left.\begin{array}{lll} ① & 0: & 4B + 5D = 5 \\ ② & s: & 4A + 5C + 4D = 0 \\ ③ & s^2: & B + 4C + D = 0 \\ ④ & s^3: & A + C = 0 \end{array}\right\} \text{4 equations, 4 unknowns} \checkmark$$

From ④ $A = -C$

① $4B + 5D = 5$

② $-A + 4D = 0$

③ $B - 4A + D = 0$

$A = 4D$

① $4B + 5D = 5 \Rightarrow 12B + 15D = 15$

③ $B - 15D = 0$

$13B = 15$

$B = 15/13$

$D = 1/13$

$A = \frac{4}{13}$

$C = -\frac{4}{13}$

$Y(s) = \dfrac{\frac{4}{13}s + \frac{15}{13}}{s^2+4s+5} + \dfrac{-\frac{4}{13}s + \frac{1}{13}}{s^2+4}$

$= \dfrac{1}{13}\left[\dfrac{s+1}{s^2+4s+5}^{*} - \dfrac{4s+1}{s^2+4}\right]$

$= \dfrac{1}{13}\left[\dfrac{(s+2)-1}{(s+2)^2+1} - \dfrac{4s + \frac{1}{2}(2)}{s^2+4}\right]$

$w=1$, $A=1$, $a=2$, $B=-1$ $\qquad$ $w=2$, $B=1/2$, $A=4$, $a=0$

* $s^2+4s+5 = s^2+4s+4+1$
$= (s+2)^2+1$

$$\boxed{y(t) = \frac{1}{13}u(t)\left[e^{-2t}\cos t - e^{-2t}\sin t - 4\cos 2t - \tfrac{1}{2}\sin 2t\right]}$$

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5a) Find the step response for the system w/ the transfer function

$$\frac{X(s)}{F(s)} = \frac{4}{(s+3)(s+7)}$$

$f(t) = u(t) \qquad F(s) = \frac{1}{s}$

$$X(s) = F(s)\frac{4}{(s+3)(s+7)} = \frac{4}{s(s+3)(s+7)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+7}$$

$$4 = A(s+3)(s+7) + Bs(s+7) + C(s)(s+3)$$

$\underline{s=0}$

$21A = 4$

$A = 4/21$

$\underline{s=-3}$

$4 = B(-3)(4)$

$B = -1/3$

$\underline{s=-7}$

$4 = C(-7)(-4)$

$C = +1/7$

$$X(s) = \frac{4}{21s} - \frac{1}{3(s+3)} + \frac{1}{7(s+7)}$$

$$x(t) = \left(4 - \frac{1}{3}e^{-3t} + \frac{1}{7}e^{-7t}\right)u(t)$$

5b) $X(s) = F(s)\frac{5}{s^2+9} \qquad f(t) = u(t) \qquad F(s) = \frac{1}{s}$

$$X(s) = \frac{5}{s(s^2+9)} = \frac{A}{s} + \frac{Bs+C}{s^2+9}$$

$$5 = A(s^2+9) + s(Bs+C)$$

$\underline{s=0}$

$5 = 9A$

$A = 5/9$

$\underline{s^2}$

$A+B=0$

$B = -A = -5/9$

$\underline{s}$

$C=0$

↓

5b) cont) $X(s) = \frac{5}{9}\left[\frac{1}{s} - \frac{s}{s^2+9}\right]$ $\rightarrow$ $w=3$

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$$x(t) = \left(\frac{5}{9} - \cos 3t\right) u(t)$$

5c) $\frac{X(s)}{F(s)} = \frac{8}{(s+3)^2}$

$f(t) = t\,u(t)$

$F(s) = \frac{1}{s^2}$

$X(s) = F(s) \frac{8}{(s+3)^2}$

Both repeated

$X(s) = \frac{8}{s^2(s+3)^2}$

$X(s) = \frac{8}{s^2(s+3)^2} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+3} + \frac{D}{(s+3)^2}$

$8 = As(s+3)^2 + B(s+3)^2 + Cs^2(s+3) + Ds^2 = As(s^2+6s+9) + B(s^2+6s+9) + Cs^2(s+3) + Ds^2$

$\underline{s=0}$

$9B = 8$

$B = 8/9$

$\underline{s=-3}$

$8 = 9D$

$D = 8/9$

$\underline{s^3}$: $A+C=0$ $\quad A=-C$

$\underline{s^2}$: $6A+B+3C+D=0$

$\Rightarrow 6A+3C = -16/9$

$\Rightarrow 3A = -16/9$

$A = \frac{-16}{27}$

$C = \frac{16}{27}$

$X(s) = -\frac{16}{27}\frac{1}{s} + \frac{8}{9}\frac{1}{s^2} + \frac{16}{27(s+3)} + \frac{8}{9(s+3)^2}$

$$x(t) = \left(-\frac{16}{27} + \frac{8}{9}t + \frac{16}{27}e^{-3t} + \frac{8}{9}e^{-3t}t\right)u(t)$$

6) TF → ODE

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$$R(s) \longrightarrow \boxed{\dfrac{s^4+2s^3+4s^2+s+4}{s^5+7s^4+3s^3+2s^2+s+5}} \longrightarrow C(s)$$

$$G(s) = \frac{C(s)}{R(s)} = \frac{s^4+2s^3+4s^2+s+4}{s^5+7s^4+3s^3+2s^2+s+5}$$

$$(s^5+7s^4+3s^3+2s^2+s+5)\,C(s) = (s^4+2s^3+4s^2+s+4)\,R(s)$$

↓ inverse LaPlace

$$\boxed{\begin{aligned}&\frac{d^5c(t)}{dt^5} + \frac{7d^4c(t)}{dt^4} + \frac{3d^3c(t)}{dt^3} + \frac{2d^2c(t)}{dt^2} + \frac{dc(t)}{dt} + 5c(t) \\ &= \frac{d^4r(t)}{dt^4} + \frac{2d^3r(t)}{dt^3} + \frac{4d^2r(t)}{dt^2} + \frac{dr(t)}{dt} + 4r(t)\end{aligned}}$$

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7) $\frac{d^2c}{dt^2} + 4\frac{dc}{dt} + 5c(t) = r(t)$

a) $s^2 C(s) - s\,C(0) - c'(0) + 4(s\,C(s) - c(0)) + 5C(s) = R(s)$

$$s^2 C(s) - s + 1 + 4s\,C(s) - 4 + 5C(s) = R(s)$$

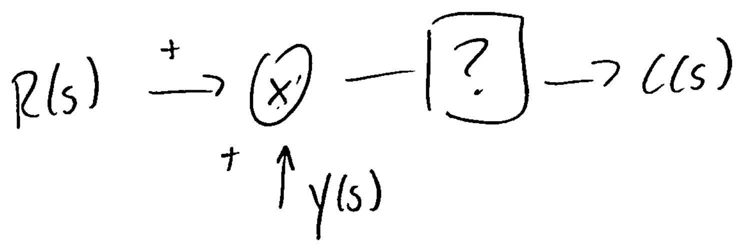
$$\boxed{(s^2+4s+5)\,C(s) - s - 3 = R(s)}$$

b) $s^2 C(s) + 4s\,C(s) + 5C(s) = R(s)$

$$C(s)\,(s^2+4s+5) = R(s)$$

$$\boxed{\frac{C(s)}{R(s)} = G(s) = \frac{1}{s^2+4s+5}}$$

c)



Need to solve for $Y(s)$

$$C(s) = \frac{R(s) + s + 3}{s^2+4s+5} = \frac{R(s)}{s^2+4s+5} + \frac{s+3}{s^2+4s+5}$$

where $G(s) = \frac{1}{s^2+4s+5}$

$$= R(s)\,G(s) + (s+3)\,G(s)$$

$\longrightarrow$

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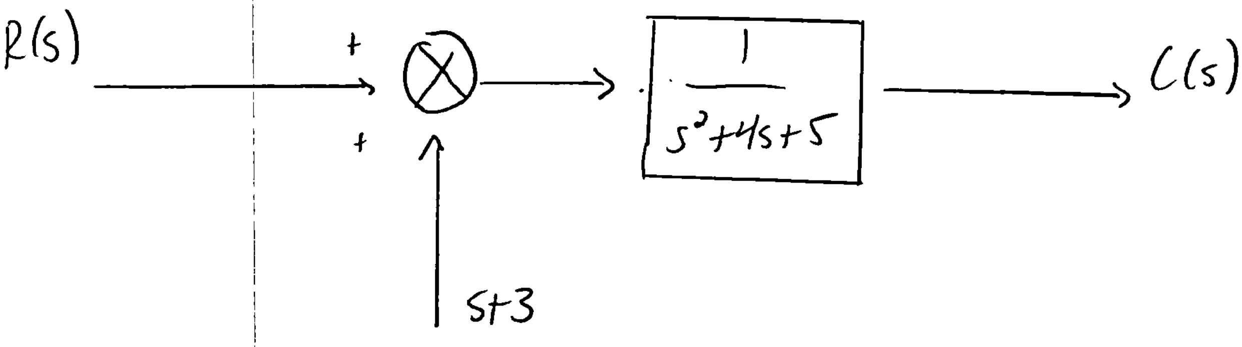
7(cont)



$$C(s) = \frac{R(s) + s+3}{s^2+4s+5}$$ ✓