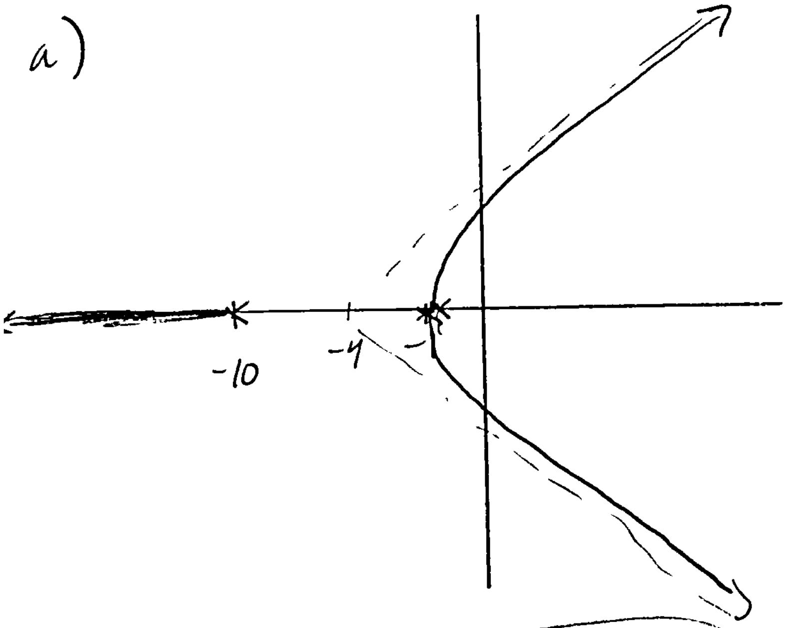
Problem 1

rus 20

a)



$$G(s) = \frac{K}{(s+1)^2(s+10)}$$

$$\sigma_a = \frac{(-1 + -1 + -10)}{3} = \frac{-12}{3} = -4$$

$$\theta_a = \frac{(2k+1)\pi}{3} = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

$\boxed{\text{Type Zero}}$

$\boxed{\text{OL Poles: } (-1, -1, -10)}$

$\boxed{K_p = \frac{K}{10}}$

$$K_p = \lim_{s \to 0} G(s) = \frac{K}{1^2(10)} = \frac{K}{10}$$

$$e_{ss}(\infty) = \frac{1}{1+K_p} = \frac{1}{1+\frac{K}{10}} = \frac{10}{10+K}$$

step $\boxed{e_{ss}(\infty) = \frac{10}{10+K}}$

b) From Matlab $j\omega$ crossing at $K = 242$. Therefore need $K < 242$

$\boxed{K < 242}$

c) From Matlab: $\boxed{K = 13.8 \quad \text{poles: } \begin{array}{l} -10.1643 \\ -0.9178 \pm 1.2241j \end{array}}$

Problem 1 cont

d) Graph attached

$$\boxed{e_s(\infty) = 0.42}$$

$$e_s(\infty) = \frac{10}{10+K} = \frac{10}{10+13.8} = 0.42$$

e) The estimate is valid. The third pole is far enough left from the other two dominant poles

$$|-10.16| > 5\,|.91781| \checkmark$$

PI Controller

$$G(s) = \frac{K(s+0.2)}{s(s+1)^2(s+10)}$$

a) From Matlab jω crossing at $K = 214$

Need $\boxed{K < 214}$

b) $\boxed{K = 13.3}$

$$\boxed{\begin{aligned} \text{poles} = & -10.1555 \\ & -0.8583 \pm 1.1456 \\ & -0.1278 \end{aligned}}$$

c) See attached

$e_{ss}(\infty) = 0$ System is now type 1 → zero error for step input

Problem 1 cont

d) The system can be approximated as second order.

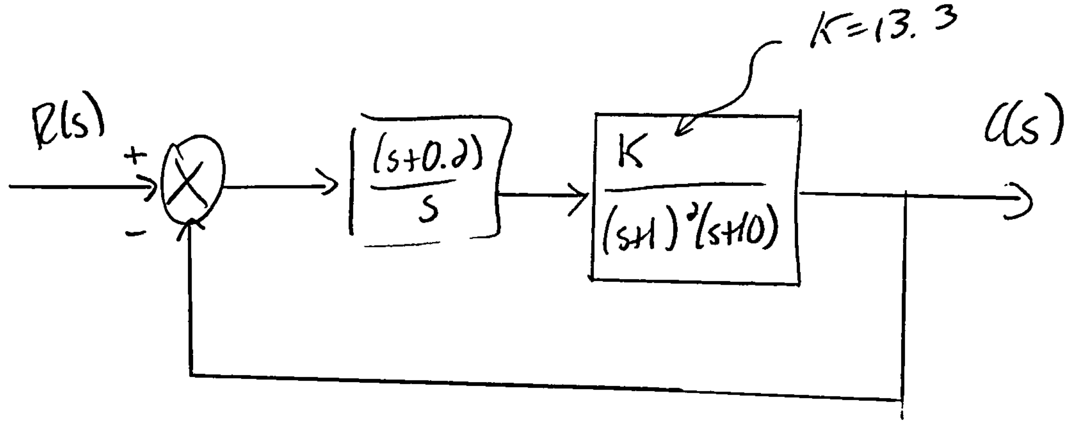
$-10.155 < 5\ (-.8583)$

$< 5\ (-0.1278)$

&

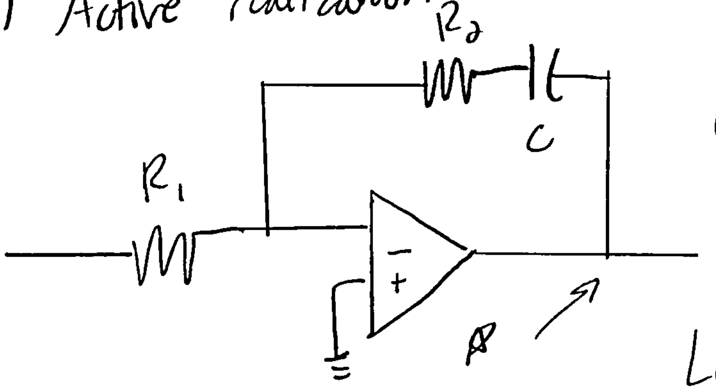
Pole $-0.1278 \approx -0.2$ so cancelled by zero at -0.2

e)



Cascade

f) Active realization → OP-AMP



$G_c(s) = -\frac{R_2}{R_1}\frac{\left(s + \frac{1}{R_2 C}\right)}{s}$

Need $R_1 = R_2$ & $\frac{1}{R_2 C} = 0.2$

Let $C = 1\ \mu F$

$R_2 C = 5$

$R_2 = \frac{5}{C} = \frac{5}{1\times10^{-6}} = 5000\,k\Omega = R_1$

$R_1 = 5000\ k\Omega$

$R_2 = 5000\ k\Omega$

$C = 1\ \mu F$

PI controller

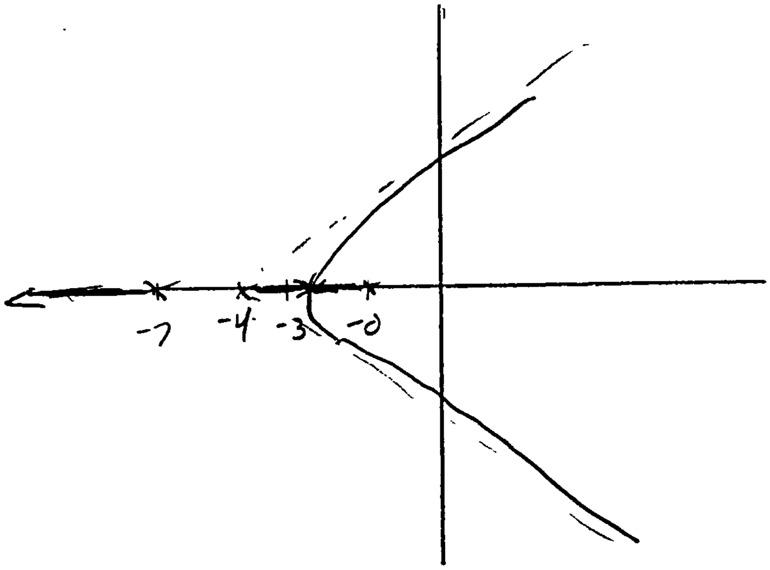
* Need to invert output

Problem 2

a) $G(s) = \dfrac{K}{(s+2)(s+3)(s+7)}$

$\boxed{\text{OL poles: } (-2, -3, -7)}$

$\boxed{\text{System Type: Zero}}$



$K_p = \lim_{s \to 0} G(s) = \dfrac{K}{2(3)(7)} = \dfrac{K}{42}$

$\boxed{K_p = \dfrac{K}{42}}$

$\sigma_a = \dfrac{(-7-3-2)}{3} = -4$

$\theta_a = \dfrac{\pi}{3}, \pi, 5\pi/3$

$e_s(\infty) = \dfrac{1}{1+K_p} = \dfrac{42}{42+K}$

$\boxed{e_s(\infty) = \dfrac{42}{42+K}}$

b) See attached $\boxed{K=41}$

$\zeta = 0.5912$

poles = $\boxed{\begin{matrix} -8.2497 \\ -1.8752 \pm 2.5583 \end{matrix}}$

c) See attached

$e_s = \dfrac{42}{42+K} = \dfrac{42}{42+41} = 0.506$ $\boxed{e_s(\infty) = 0.506}$ $\rightarrow$

Problem 2 cont

d) It cannot be approximated

$-8.25 \not\gg 5(-1.87)$

Lag Compensator

$$\frac{z_c}{p_c} = \frac{K_{pN}}{K_{po}} = \frac{4}{41/42} = 4.09756$$

If $p_c = 0.01$

$\hookrightarrow$ $z_c = 0.041$

$$G_c(s) = \frac{s + 0.041}{s + .01}$$

a) $\boxed{K = 41.1}$

poles: -8.2485

$-1.868 \pm 2.5572j$

-0.0255

c) Approximation cannot be made

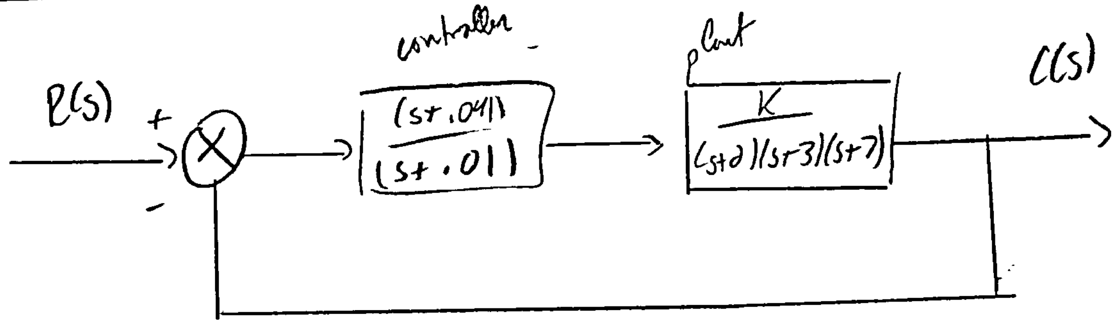
$-8.2485 \not\gg 5(-1.868)$

b) $K_p = \lim_{s \to 0} G(s) = \frac{41.1(.041)}{0.01(2)(3)(7)} = \boxed{4.012} \approx 4$

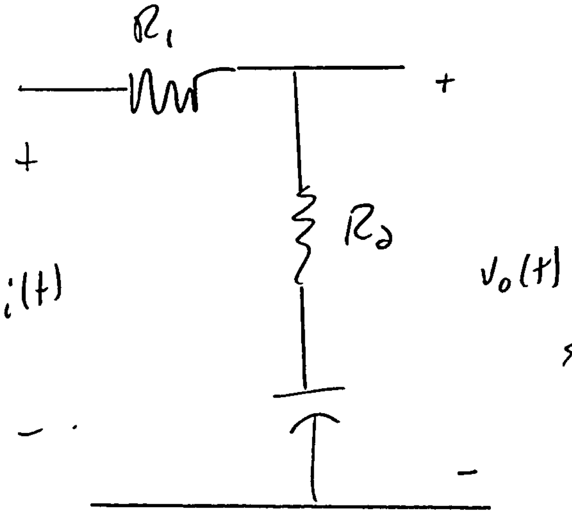
$e_s(\infty) = \frac{1}{1+K_p} \approx \frac{1}{1+4.01} = \boxed{0.1996}$

Problem 2 cont

d)



e)



$$G_{cont} = \frac{s+.041}{s+.01} = \frac{R_2}{R_1+R_2}\,\frac{s+\frac{1}{R_2C}}{s+\frac{1}{(R_1+R_2)C}}$$

$$\frac{1}{R_2C} = .041$$

$$\frac{1}{(R_1+R_2)C} = 0.01$$

If $C = 1\,\mu F$ $R_2 = 24390\ k\Omega$

$R_1 = 75610\ k\Omega$

This circuit adds a gain factor of $\frac{R_2}{R_1+R_2} = 0.25$

Therefore, K_{plant} must be changed to get a total K of 41.1

$K_{plant}\,K_{cont} = 41.1$

$K_{plant} = \frac{41.1}{.25} = 164.4$

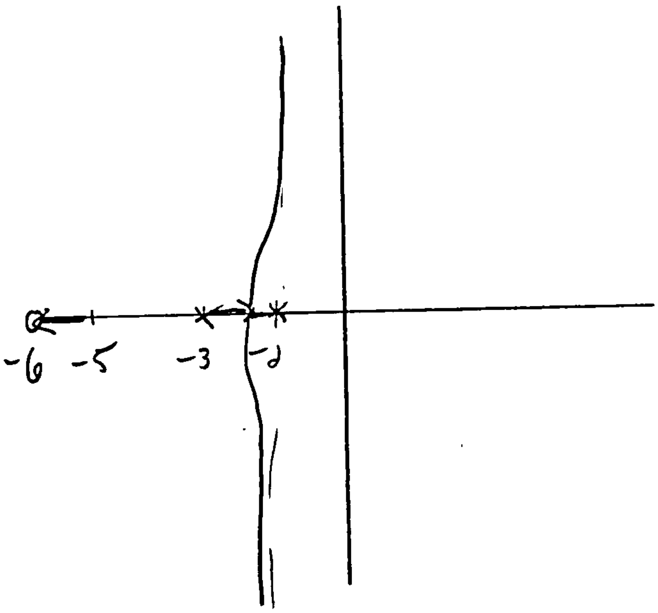
K_{plant} adjusted to 164.4

Problem 3

$$G(s) = \frac{K(s+6)}{(s+2)(s+3)(s+5)}$$

a)

Always stable



OL poles

$-2, -3, -5$

OL zeros

-6

$$\sigma_a = \frac{(-2-3-5) - -6}{2}$$

$$= -2$$

$$\theta_a = \frac{(2k+1)\pi}{2}$$

$$= \frac{\pi}{2}, \frac{3\pi}{2}$$

b) Desired damping $= 0.707$

$K = 4.6$

poles : -5.3661

$-2.3169 \pm 2.3164j$

c) Acc attained $\sigma_d = 2.3169$

$$T_s = \frac{4}{\sigma_d} = \frac{4}{2.3169} = 1.73\ s$$

$\boxed{T_s = 1.73\ s}$

This is just approximate since 2nd order approximation cannot be made

d) Cannot be approximated as second order

Pole on real axis not left enough

$|5.361| \not> 5(2.3169)$

Problem 3 cont

PD Controller

$$T_s = \tfrac{1}{2}(1.73\ s) = 0.865$$

$$T_s = \frac{4}{\sigma_d} \qquad \sigma_d = \frac{4}{T_s} = \frac{4}{0.865} = 4.62$$

$$\cos\theta = \xi$$

$$\tan\theta = \frac{\omega_d}{\sigma_d} \Rightarrow \omega_d = \sigma_d \tan(\cos^{-1}(\xi))$$

$$= 4.62 \tan(\cos^{-1}(.707))$$

$$= 4.62$$

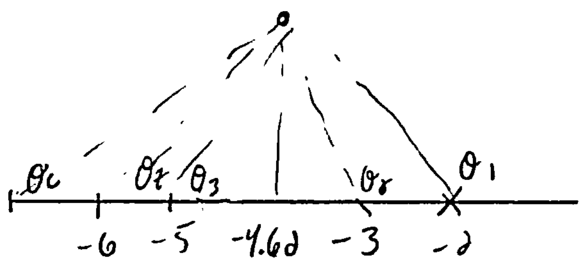
Need $-4.62 \pm 4.62j$ to lie on root locus

$$\theta_1 = 180^\circ - \tan^{-1}\left(\frac{4.62}{2.62}\right) = 119.558^\circ$$

$$\theta_2 = 180^\circ - \tan^{-1}\left(\frac{4.62}{1.62}\right) = 109.323^\circ$$

$$\theta_3 = \tan^{-1}\left(\frac{4.62}{0.38}\right) = 85.2979^\circ$$

$$\theta_z = \tan^{-1}\left(\frac{4.62}{1.38}\right) = 73.369^\circ$$

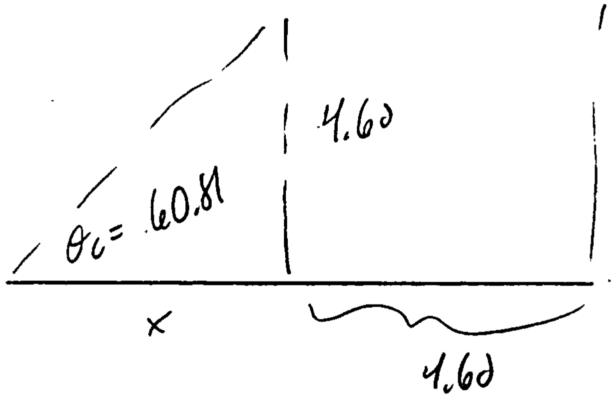


$$\theta_c + \theta_z - \theta_1 - \theta_2 - \theta_3 = (2k+1)\,180^\circ$$

$$\theta_c + -240.81 = (2k+1)\,180^\circ$$

$$\theta_c = 60.81$$

Problem 3 cont



SOH CAH TOA

$$\tan\theta_c = \frac{4.62}{x}$$

$$x = \frac{4.62}{\tan\theta_c} = 2.581$$

$$z_c = x + 4.62 \doteq 7.2$$

a) $K = 4.7$

$p = -4.6086 \pm 4.6114j$

-5.4828

See attached

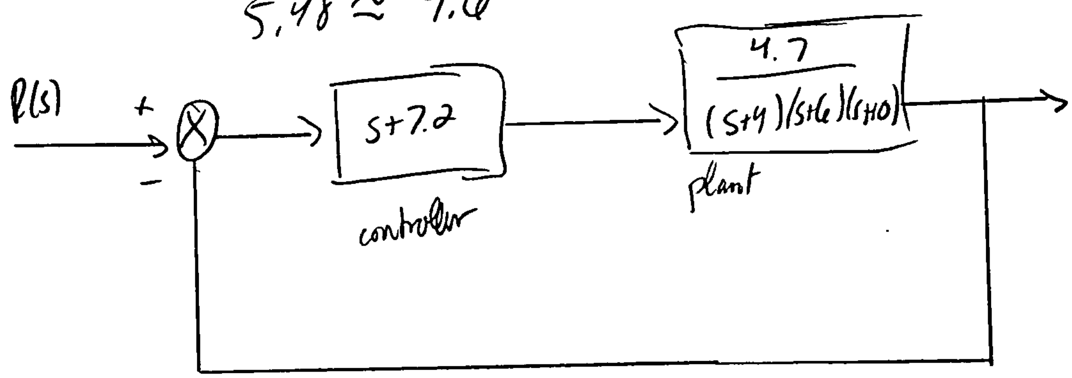
b) See Attached

$$T_s' = \frac{4}{\sigma_d'} = \frac{4}{4.6086} = 0.8695 \text{ s}$$ ✓

c) Again, not a great second order approximation

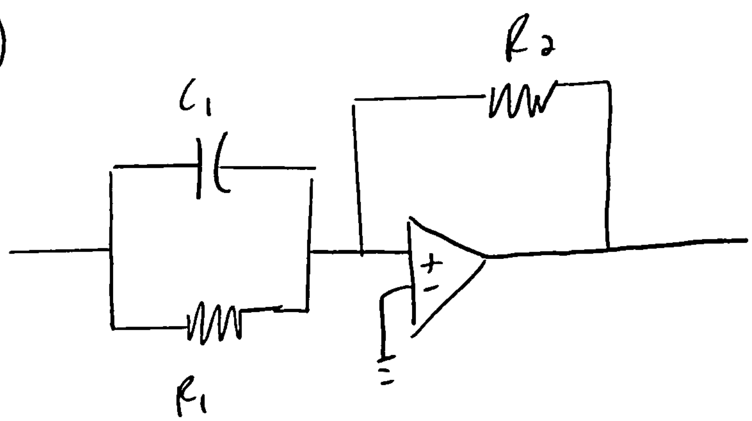
$5.48 \approx 4.6$

d)



Problem 3 cont

e)



$$-R_2C\left(\underbrace{s+\frac{1}{R_1C}}_{=s+7.2}\right)$$

$$\frac{1}{R_1C}=7.2$$

$$C = 1\,\mu F$$

$$\Rightarrow \quad R_1 = \frac{1}{7.2\,\mu F} = 138,889\,\Omega$$

Preferably want $R_2C = 1$

$$R_2 = \frac{1}{C} = \frac{1}{1\,\mu F} = 1000\,k\Omega$$

$$\boxed{\begin{aligned} &C = 1\,\mu F \\ &R_1 = 138,889\,\Omega \\ &R_2 = 1000\,k\Omega \end{aligned}}$$

Problem 4

$$G(s) = \frac{K}{(s+5)^3}$$

a) $K = 116$

poles: -9.877

$-2.5615 \pm 4.2236j$

$$T_s = \frac{4}{\sigma_d} = \frac{4}{2.5615} = 1.56159$$

$$\boxed{T_s \approx 1.56}$$

b) See attached

$FV = 0.48$

98% of $FV = 0.472 - .4896$

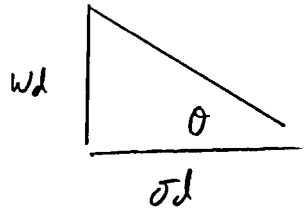
$T_s = 1.29$ → determined graphically

c) The second order approximation is poor. -9.877 & $5(-2.5615)$

This is why part a & b differ so drastically

Problem 4 cont

Lead Compensator



a) $T_s = 1.2\,s = \dfrac{4}{\sigma d}$ $\qquad \cos\theta = \zeta$

$\tan\theta = \dfrac{wd}{\sigma d}$

$\sigma d = \dfrac{4}{1.2} = 3.\overline{3}$ $\qquad wd = \sigma d \tan\theta = 3.\overline{3}\tan(\cos^{-1}(\zeta))$

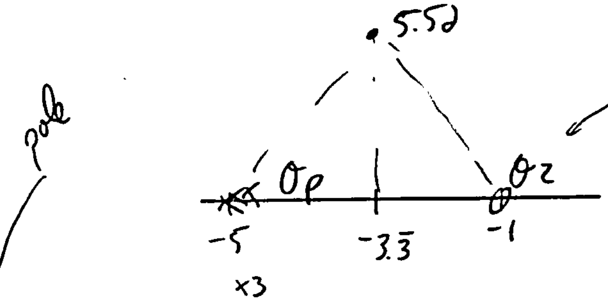
$= 5.51973$

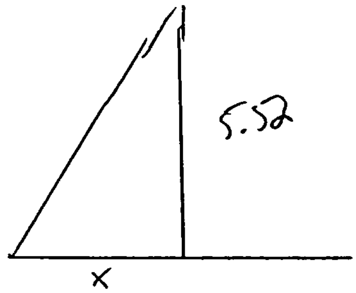
$\zeta = \dfrac{-\ln(.15)}{\sqrt{\pi^2 + \ln^2(.15)}} = 0.516931$

SOH CAH TOA

Need poles $\boxed{-3.\overline{3} \pm 5.52j}$ on root locus

zero @ -1





$x = \dfrac{5.52}{\tan(74°)}$

$x = 1.58$

~~$P_c = 3.3 + 1.58$~~

$P_c = 4.9$

$\theta_z = 180° - \tan^{-1}\left(\dfrac{5.52}{2.3}\right) = 112.62°$

$\theta_p = \tan^{-1}\left(\dfrac{5.52}{1.7}\right) = 72.8827°$

$-\theta_c + \theta_z - 3\theta_p = (2k+1)180°$

$-\theta_c - 106.028 = 180$

$-\theta_c = 286°$

$\theta_c = -286° = 74°$

$$\boxed{G_c = \frac{s+1}{s+4.9}}$$

Problem 4 cont

Lead Compensator

b) See Attached $\boxed{K = 182}$

c) See attached

$FV = .23$

$FV(1.02) = 0.234$

From the figure $\boxed{T_s = 2.3\ s}$

d) The poles of the system are

-11.6

$-3.3176 \pm 5.4952j$

-1.6648

This does not have a second order approx

-11.6 is not far enough left

e) The system specifications are not met. Settling time is not as desired. This is likely due to the fact the pole at -1.67 & the chosen zero at -1 do not cancel. Thus, another zero would have to be chosen

Problem 5

$$G(s) = \frac{K}{s(s+5)(s+11)}$$

a) See attached

For 30% OS $\zeta = 0.3579 \Rightarrow K = 219$

Using Rlocus command poles are : -13.0745

$-1.4608 \pm 3.8224j$

Dominating poles : $\omega_n = \sqrt{\sigma_d^2 + \omega_d^2}$

$= \sqrt{1.4608^2 + 3.8224^2}$

$= 4.09274$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$= \frac{\pi}{\omega_d} = \frac{\pi}{3.8224} = 0.82189\ s$$

$T_p = 0.82189\ s$

$K_v = \frac{219}{55}$ ← From Matlab

↳ see script

b) See attached

Lead Compensator

The book chose the lead compensator zero to arbitrarily be at -5. This may work well for us because there is a pole at -5 & this might help create a better second order approximation

Problem 5 cont

Lead Compensator

a) Want 15% OS & $T_p \downarrow 2$

$$T_{p_{new}} = \frac{.82}{2} = 0.41$$

$$T_p = \frac{\pi}{\omega_d} = 0.41 \Rightarrow \omega_d = \frac{\pi}{.41} = 7.66$$

$$\zeta = \frac{-\ln(.15)}{\sqrt{\pi^2 + \ln^2(.15)}} = 0.517$$

$$\cos\theta = \zeta$$

$$\tan\theta = \frac{\omega_d}{\sigma_d} \qquad \sigma_d = \frac{\omega_d}{\tan\theta} = \frac{7.66}{\tan\cos^{-1}(.517)} = 4.63$$

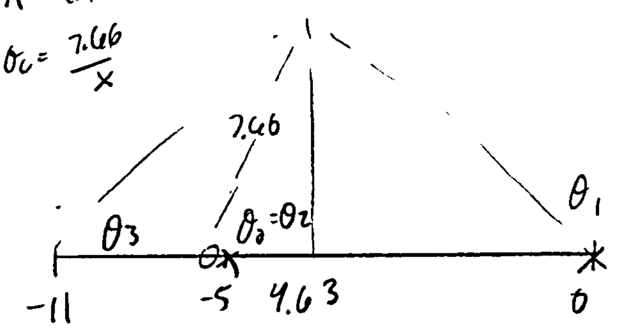
Want poles at $\boxed{-4.63 \pm 7.66j}$

$$\cancel{\theta_z} - \theta_1 - \cancel{\theta_2} - \theta_3 - \theta_c = (2k+1)180^\circ$$

$$\theta_z = \theta_2$$

SOH CAH TOA

$\tan\theta_c = \frac{7.66}{x}$



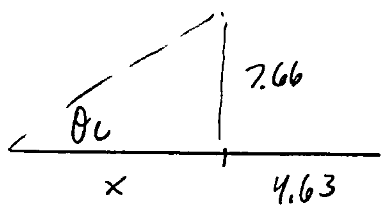
$$-\theta_1 - \theta_3 - \theta_c = (2k+1)180^\circ$$

$$-171.403 - \theta_c = \underbrace{(2k+1)180^\circ}_{-180^\circ}$$

$$\theta_c = 8.6^\circ$$

$$\theta_1 = 180^\circ - \tan^{-1}\left(\frac{7.66}{4.63}\right) = 121.15^\circ$$

$$\theta_3 = \tan^{-1}\left(\frac{7.66}{6.37}\right) = 50.2533^\circ$$



$$P_c = x + 4.63 = \frac{7.66}{\tan(8.6)} + 4.63 = 55.28$$

Problem 5 cont

Lead Compensator

a cont) Zero at -5, pole at -55.28

$$G_c = \frac{(s+5)}{(s+55.28)}$$

b) Need gain $= 4.6 \times 10^3$

See attached

c) The poles at $K = 4.6 \times 10^3$ are ...

-57.032 ← far left

-4.6239 ± 7.70 ← dominating poles

-5.00 ← cancels with zero

Second order approx, is good.

Lag Compensator

a) Want steady-state error ↓ 30

Need K_v ↑ 30

$K_{V_{new}} = 30 K_v$

Need to figure out lead compensator impact on steady-state error

$$K_{V_{LC}} = \lim_{s \to 0} s G_c G = \lim_{s \to 0} \frac{(\cancel{s+5})}{(s+55.28)} \frac{4.6 \times 10^3}{(\cancel{s+5})(s+11)}$$

$= 7.56753$

Problem 5 cont

Lag Compensator

Lead compensator increased K_v to 7.56

$$30\left(\frac{219}{55}\right) = X_{fact}\,(7.56)$$

$$X_{Fact} = 15.8$$

Lag compensator needs to increase K_v by 15.8

$$\frac{z_c}{p_c} = \frac{K_{vN}}{K_{vo}} = 15.8$$

If $p_c = 0.01 \Rightarrow$

$z_c = 0.158$

$$\boxed{G_{c_{LC}} = \frac{s + 0.158}{s + .01}}$$

b) Need to find K. Since $G_{c_{LC}}$ essentially cancels to 1 i.e $G_{c_{LC}} \approx 1$ and K is so large we can assume

K still $\approx 4.6 \times 10^3$

c) $$G_c = \frac{(s+5)(s+0.158)}{(s+55.28)(s+.01)} = \frac{(s+5)(s+.158)}{s^2 + 55.29s + 0.5528}$$

Problem 5 cont

c) $\frac{1}{R_1 C_1} = 5$ $\quad \frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_2 C_1} = 55.29$

$\frac{1}{R_2 C_2} = 0.158$ $\quad \frac{1}{R_1 R_2 C_1 C_2} = 0.5528$

Maple reveals this system has no solution

Therefore, must design passive lag & lead separately

Lag

$\frac{R_2}{R_1+R_2} \frac{s + \frac{1}{R_2 C}}{s + \frac{1}{(R_1+R_2)C}}$ $\quad \frac{1}{R_2 C} = 0.158$

$R_2 = \frac{1}{.158C} = 6.33 \times 10^6 \,\Omega$ if $C = 1\,\mu F$

$\frac{1}{(R_1+R_2)C} = .01$ $\quad \frac{R_2}{R_1+R_2} = 0.0633$

compensate for this in isolation

$R_1+R_2 = \frac{1}{.01C} = 1 \times 10^8 \,\Omega$

$R_1 = 9.37 \times 10^7 \,\Omega$

Lead

$\frac{s + \frac{1}{R_4 C_2}}{s + \frac{1}{R_4 C_2} + \frac{1}{R_5 C_2}}$ $\quad \frac{1}{R_4 C_2} = 5$

$\frac{1}{5 C_2} = R_4$

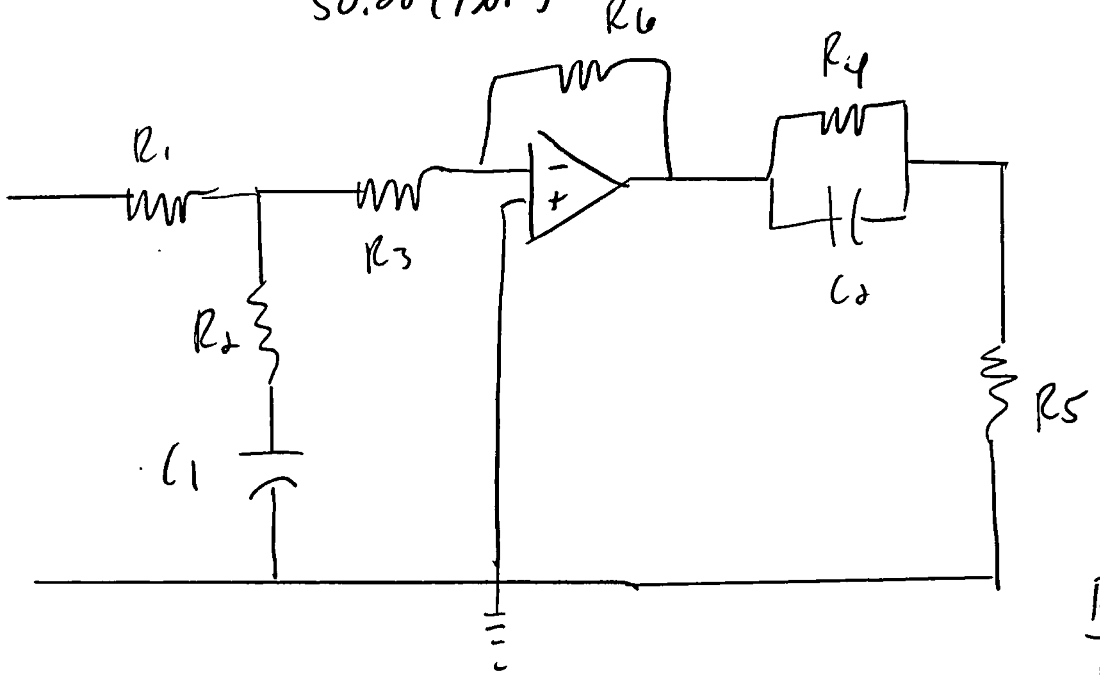
$R_4 = 2 \times 10^5 \,\Omega$ if $C_2 = 1\,\mu F$

Problem 5 cont

$$\frac{1}{\underbrace{R_4C_2}_{5}} + \frac{1}{R_5C_2} = 55.28$$

$$\frac{1}{R_5C_2} = 50.28$$

$$R_5 = \frac{1}{50.28(1nF)} = 19888.6\ \Omega$$



$$\frac{R_4}{R_3} = \frac{1}{.0633} = 15.8$$

$R_2 = 6.33\times10^{6}\ \Omega$ $\quad R_4 = 2\times10^{5}\ \Omega$

$C_1 = 1\,nF$ $\quad R_5 = 19888.6\ \Omega$

$R_1 = 9.37\times10^{7}\ \Omega$ $\quad C_2 = 1\,nF$

$R_3 = 1\ \Omega$ $\quad R_6 = 15.8\ \Omega$

Problem 6

$$G(s) = \frac{K}{(s+4)(s+6)(s+10)}$$

25% OS, 2 seconds T_s (max)

$$\zeta = \frac{-\ln(.25)}{\sqrt{\pi^2 + \ln(.25)^2}} = 0.4$$

$T_s = \frac{4}{\sigma_d}$ Need $\sigma_d > 2$

Chose $\zeta = 0.5$ & $\sigma_d = 2.25$ to be safe

$\cos\theta = \zeta$

$\tan\theta = \frac{\omega_d}{\sigma_d}$

$\omega_d = \sigma_d \tan\theta$

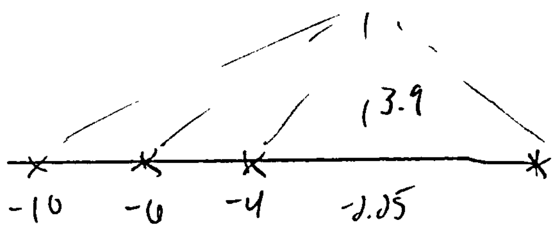
$= 2.25 \tan\cos^{-1}.5$

$= 3.9$

Dominant poles: $-2.25 \pm 3.9j$

$$\theta_{poles} = 180° - \tan^{-1}\left(\frac{3.9}{2.25}\right) + \tan^{-1}\left(\frac{3.9}{4-2.25}\right) + \tan^{-1}\left(\frac{3.9}{6-2.25}\right) + \tan^{-1}\left(\frac{3.9}{10-2.25}\right)$$

$= 258.615$



$\theta_c = (2K+1)180° + \theta_{poles} = (2K+1)180° + 258.615$

$\theta_c = 78.615°$

$$z_c = 2.25 + \frac{3.9}{\tan(78.6°)}$$

$= 3.0358$

zero at ≈ 3

See Matlab code & figures attached

Problem 6
cont