Problem 1 $KG(s)H(s) = \frac{K(s+2)}{(s+1)(s+3)(s+4)}$

HW #7

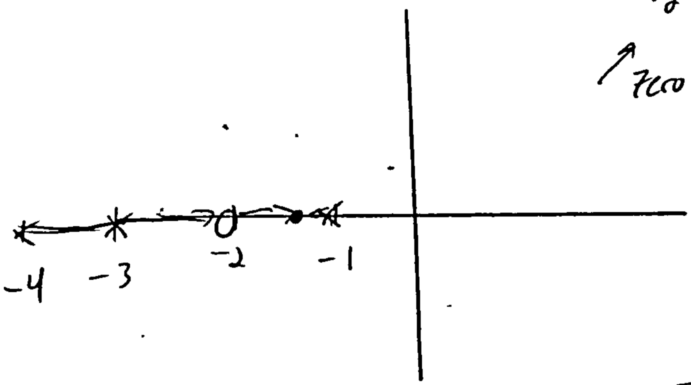
as20

a) $(-1.28, 0)$

$V_1 = 0.28\angle 180^\circ$ $V_3 = 1.72\angle 0^\circ$

$V_2 = .72\angle 0$ $V_4 = 2.72\angle 0^\circ$

↗ zero



$M = \frac{.72}{.28(1.72)(2.72)} \angle 0 - 180^\circ$

$= 0.5496 \angle -180^\circ =$

↗ odd multiple of -180 ✓

This point <u>is</u> on the root locus. | $K \approx \frac{1}{0.55} \approx 1.81$

This has to be on the root locus because -1.28 lies between -1 & -2 on the real axis. Thus, by the real axis rule (real axis segments exist to the left of an odd number of real axis finite open-loop poles/zeros) -1.28 must be on the root locus. -1 is an odd real axis finite open-loop pole.

b) $(-2.28, 0)$

-2.28 is to the left of an even real-axis pole. Thus it is <u>not</u> on the root locus.

$V_1 = 1.28\angle 180^\circ$ $V_3 = .72\angle 0^\circ$ $M = \frac{.28}{1.28(.72)(1.72)} \angle 180^\circ - 180^\circ$

$V_2 = .28\angle 180^\circ$ $V_4 = 1.72\angle 0^\circ$

↗ zero

$\boxed{\text{Not on root locus}}$

$0^\circ \neq (2k+1)(180^\circ)$

Problem 1 cont

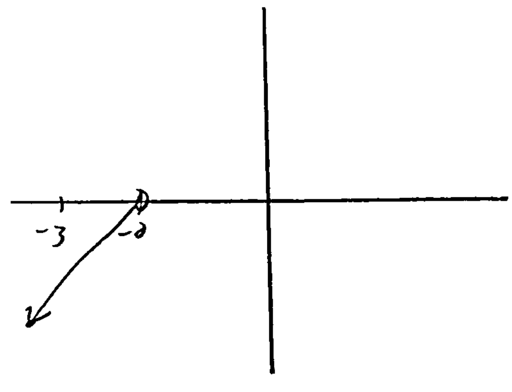
i) $(-3.35, -j1.04)$

Zero lengths

$(-3.35, -j1.04) - (-2, 0)$

$= (-1.35, -j1.04)$ $\leftarrow$ $\tan^{-1}\left(\frac{\text{imaginary}}{\text{real}}\right)$

$= 1.704 \angle 217.61^\circ$



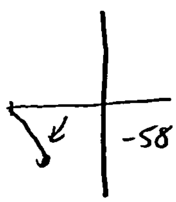
Pole lengths

(-1) $(-3.35, -j1.04) - (-1, 0)$

$= (-2.35, -j1.04) = 2.57 \angle 203.872^\circ$

(-3) $(-3.35, -j1.04) - (-3, 0) = (-0.35, -j1.04) = 1.097 \angle 251.4^\circ$

(-4) $(-3.35, -j1.04) - (-4, 0) = (0.65, -j1.04) = 1.22642 \angle -58^\circ$



$203.872 + 251.4 - 58$

$$M = \frac{1.704 \angle 217.61}{2.57 \cdot 1.097 \cdot 1.22642 \angle 397.272^\circ}$$

$\approx 0.505731 \angle -180^\circ$

$K = \frac{1}{0.49} \approx 2.03$

$$\boxed{\text{On root locus} \quad K = 2.03}$$

Indicates it is on root locus!

without rounding this is actually $\approx 0.492753 \rightarrow$ Put whole expression into calculator

Problem 1D

D) $(-3.05, j1.04)$

Zero length

$(-3.05, j1.04) - (-2,0) = (-1.05, j1.04) = 1.47787 \angle 135.274^\circ$

Pole lengths

(-1) $(-3.05, j1.04) - (-1,0) = (-2.05, j1.04) = 2.3 \angle 153.101^\circ$

(-3) $(-3.05, j1.04) - (-3,0) = (-.05, j1.04) = 1.04 \angle 92.7525$

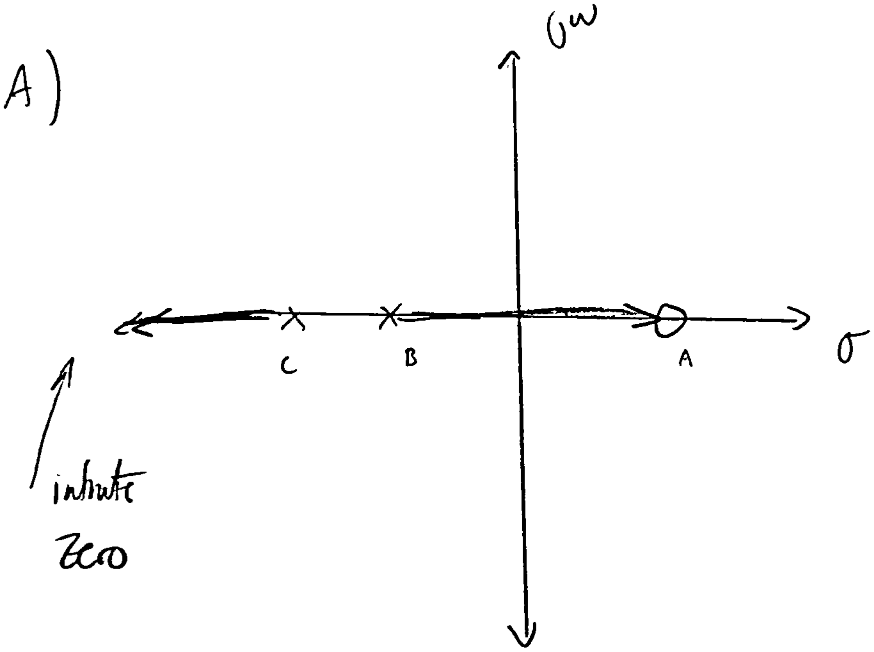
(-4) $(-3.05, j1.04) - (-4,0) = (.95, j1.04) = 1.40 \angle 47.5895^\circ$

$135.3^\circ - 153.1^\circ - 92.8^\circ - 47.6 = -158.2 \neq (2k+1)180^\circ$

Not on root locus

Problem 2

A)



Real axis

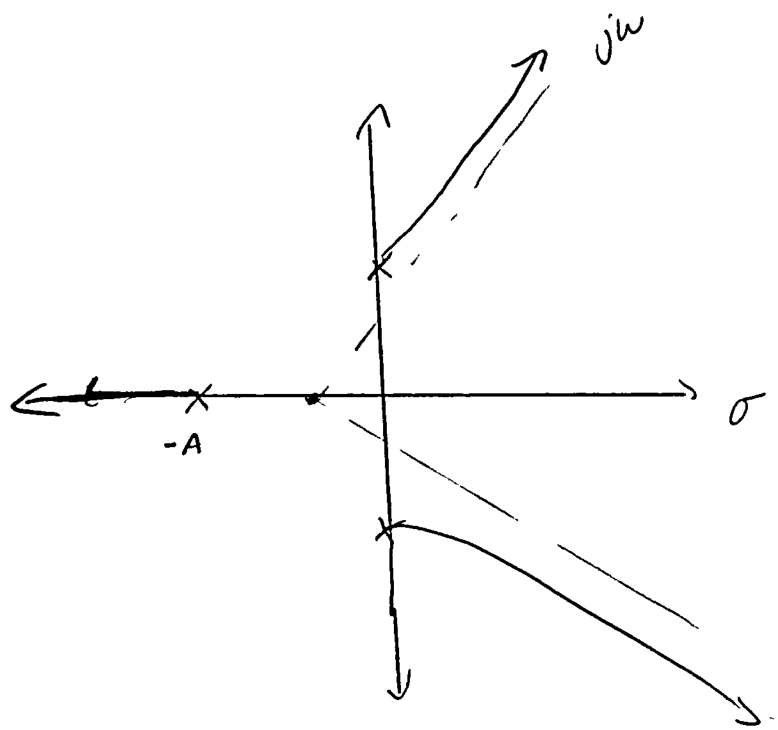
$A \to B$

$$\theta_a = \frac{(2k+1)\pi}{2}$$

$$= \frac{\pi}{2}$$

zero at $+180°$

B)



3 infinite zeros

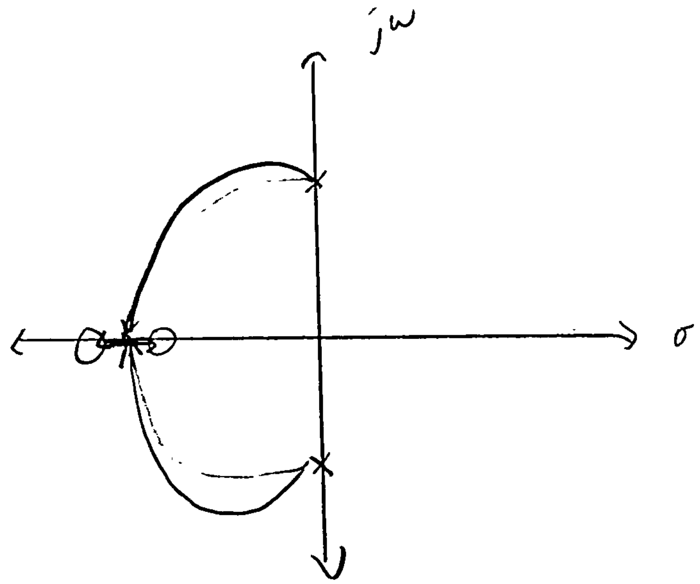
$$\sigma_a = \frac{-A}{3}$$

$$\theta_a = \frac{(2k+1)\pi}{3}$$

$$= \frac{\pi}{3},\ \pi,\ \frac{5\pi}{3}$$

Problem 2 cont

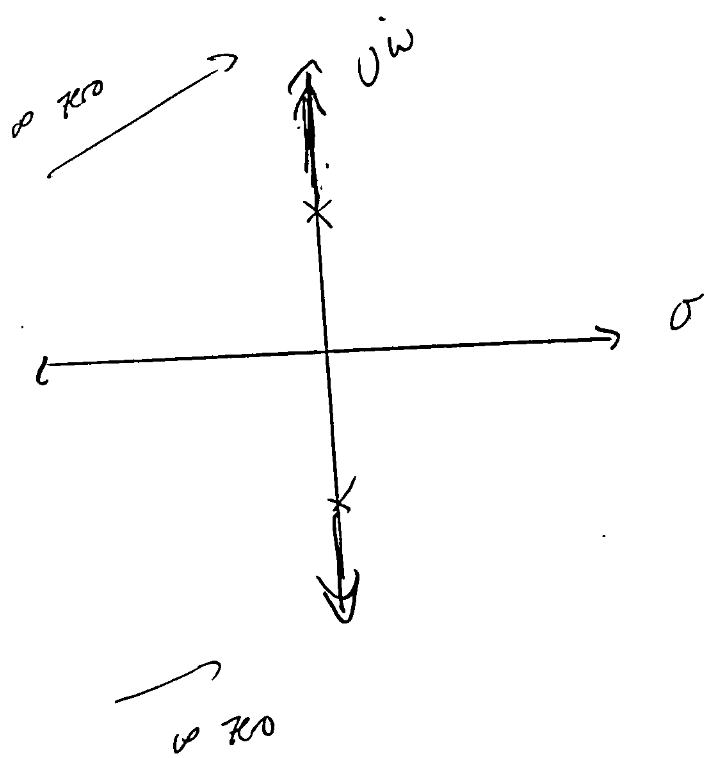
c)



No infinite poles/zeros

2 branches

d)



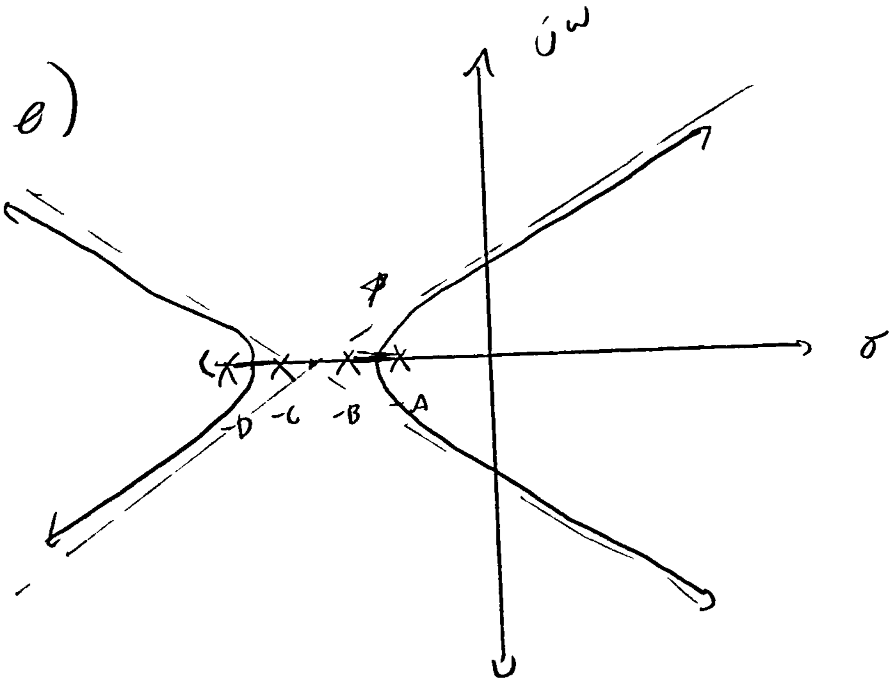
$\sigma_a = 0$

$$\theta_a = \frac{(2k+1)\pi}{2}$$

$$= \frac{\pi}{2}, \frac{3\pi}{2}$$

Problem 2 cont

e)



4 ∞ zeros

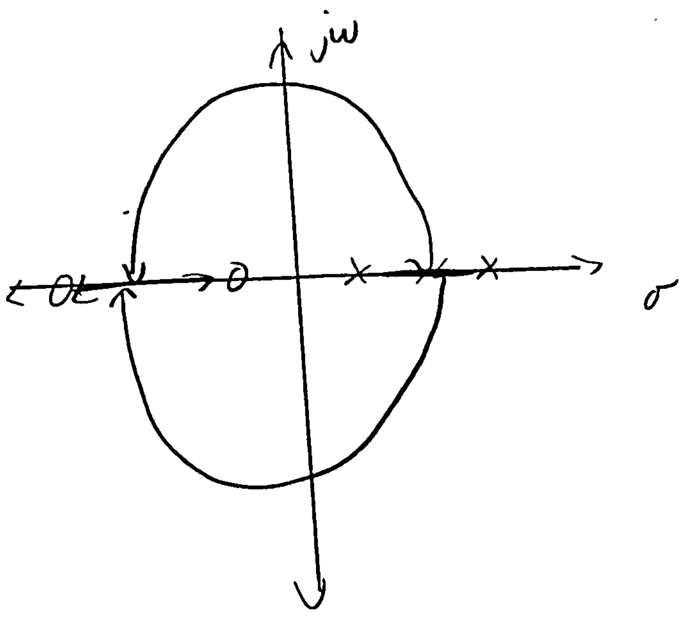
$\theta_a = \frac{(2k+1)\pi}{4}$

$= \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

* angles of departure $= 90°$

$\sigma_a = \frac{-A-B-C-D}{4}$ = weighted sum of $-A, -B, -C, -D$

f)



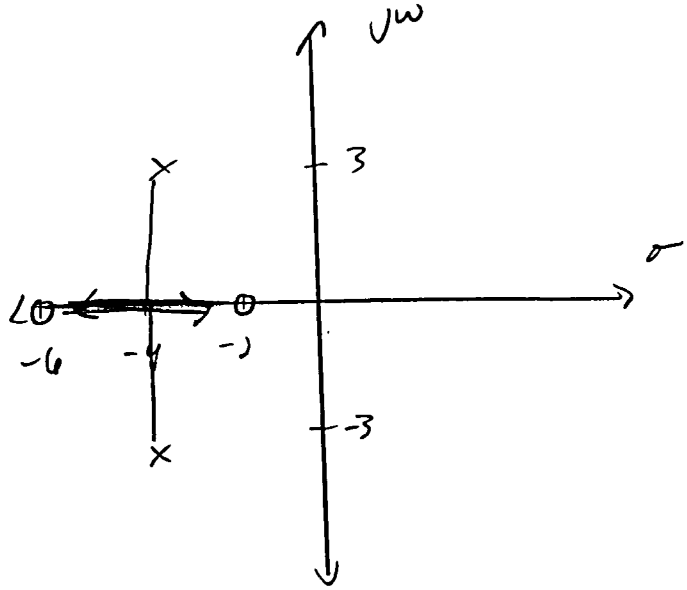
2 branches

No ∞ poles/zeros

Problem 3

a) $G(s) = \dfrac{K(s+2)(s+6)}{s^2+8s+25} \longrightarrow \dfrac{-8}{2} \pm \dfrac{\sqrt{64-100}}{2}$

$= -4 \pm 3i$



branches = 2

zeros = # finite poles ⇒ no infinite poles/zeros
^ finite

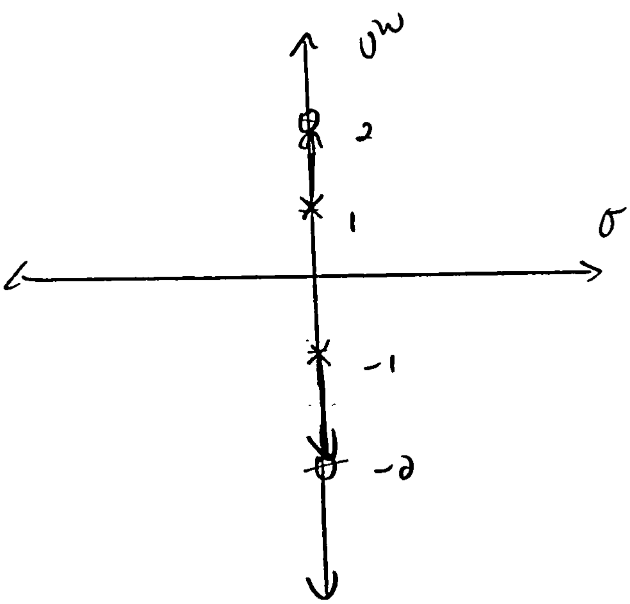
symmetric about real ✓

b) $G(s) = \dfrac{K(s^2+4)}{s^2+1}$

$s^2+4 \longrightarrow s = \pm 2i$

$s^2+1 \longrightarrow s = \pm i$

finite zeros = # finite poles



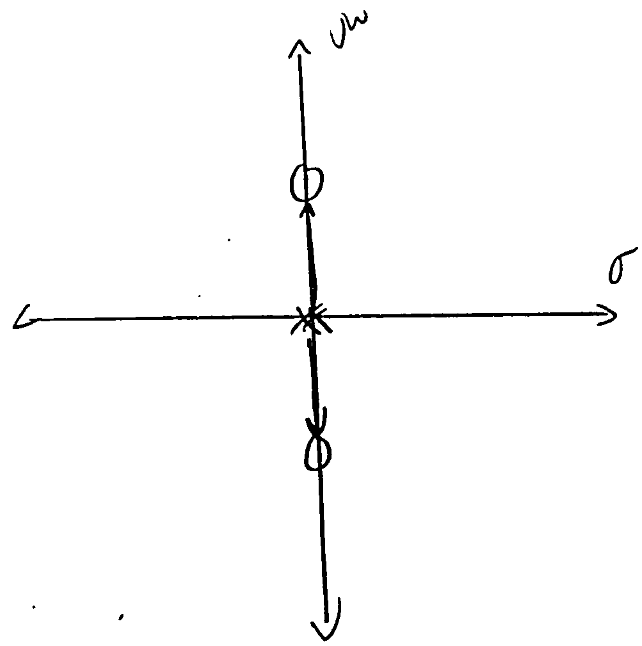
pole → zero

Nothing on real axis

Problem 3 cont

c)

$$G(s) = \frac{K(s^2+1)}{s^2} \quad \begin{array}{l} \rightsquigarrow s = \pm j \\ \rightsquigarrow s = 0 \text{ (multiplicity 2)} \end{array}$$



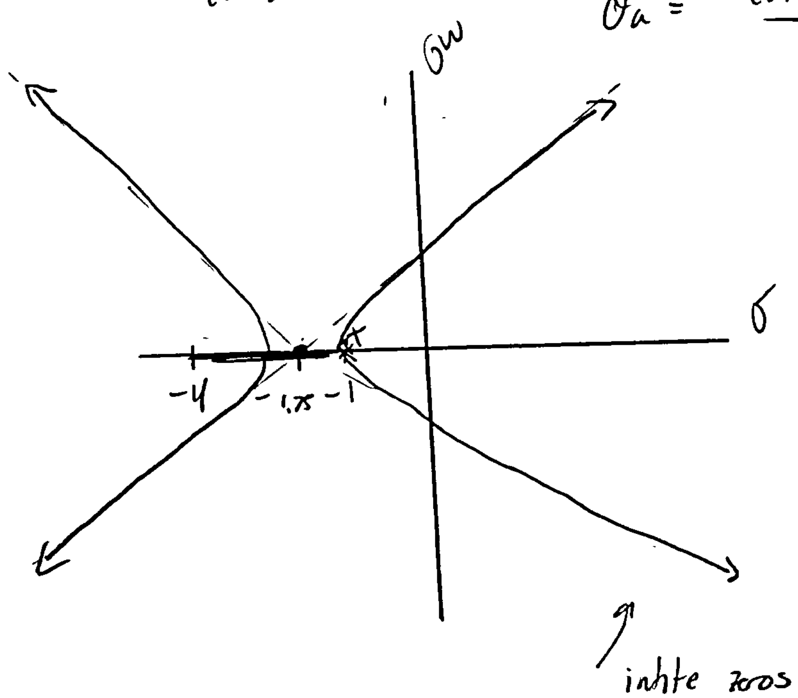
finite poles = # finite zeros

d) $G(s) = \dfrac{K}{(s+1)^3(s+4)}$

$$\sigma_a = \frac{(-1-1-1-4)}{4} = \frac{-7}{4} = -1.75$$

$$\theta_a = \frac{(2k+1)\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

4 infinite zeros



Problem 4

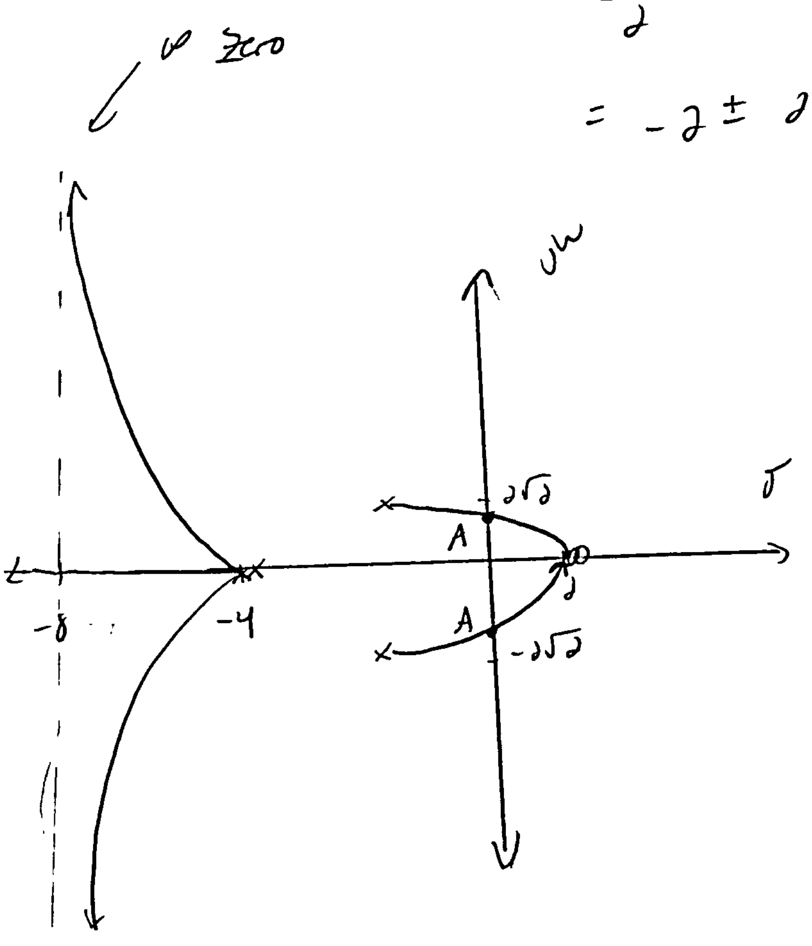
a) $KG(s)H(s) = \dfrac{K(s-2)^2}{(s^2+4s+12)(s+4)^2}$

$(s-2)^2 \to s=2$ (multiplicity 2)

$(s+4)^2 \to s=-4$ (mult 2)

$s^2+4s+12 \to \dfrac{-4}{2} \pm \dfrac{\sqrt{16-48}}{2}$

$= -2 \pm 2\sqrt{2}\,j$



4 branches

Real axis between 2 zeros

Real axis between 2 poles

$j\omega$ crossings $A = \pm 1.819$

see next page

$\sigma_a = \dfrac{\sum \text{finite poles} - \sum \text{finite zeros}}{\#\text{finite poles} - \#\text{finite zeros}} = \dfrac{(-2-2-4-4) - (2+2)}{4-2} = -8$

$\theta_a = \dfrac{(2k+1)\pi}{\#\text{finite poles} - \#\text{finite zeros}} = \dfrac{(2k+1)\pi}{2} = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

Problem 4a cont

To get $j\omega$ crossings need to first get Routh Table

$$T(s) = \frac{G(s)}{1+KG(s)H(s)} = \frac{G(s)}{1+\frac{K(s-2)^2}{(s^2+4s+12)(s+4)^2}} = \frac{G(s)(s^2+4s+12)(s+4)^2}{(s^2+4s+12)(s+4)^2+K(s-2)^2}$$

$$s^4+12s^3+(60+K)s^2+(160-4K)s+192+4K$$

From given Routh Table solver

s^4	1	$K+60$	$4K+192$
s^3	12	$160-4K$	0
s^2	$\frac{4K}{3}+\frac{140}{3}$	$4K+192$	0
s^1	$-\frac{(4K^2+16K-3872)}{K+35}$	0	0
s^0	$4K+192$	0	0

$$4K^2+16K-3872=0$$

$$K^2+4K-968=0$$

$$-\frac{4}{2} \pm \frac{\sqrt{16+4(968)}}{2} = 29.1269$$

only care about positive

Means

$$85.5692s^2+308.708=0$$

is a root

$$s = \pm\sqrt{-3.6}$$

$$\omega = \pm 1.899j$$

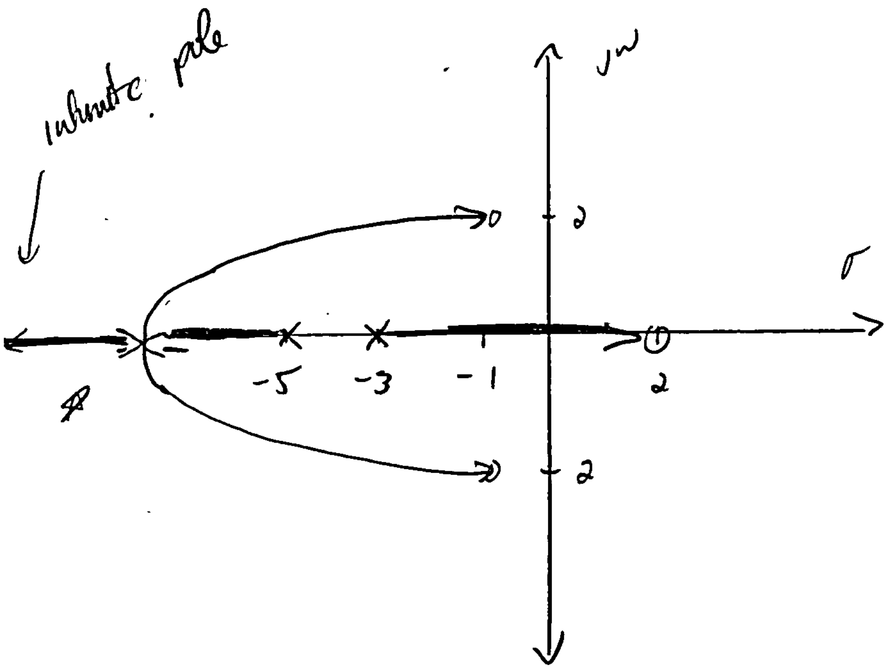
Problem 4b

$$K G(s) H(s) = \frac{K(s^2+2s+5)(s-2)}{(s+3)(s+5)}$$

$\rightarrow s = 2$

$s = -\frac{2}{2} \pm \frac{\sqrt{4-20}}{2} = -1 \pm 2j$

$\rightarrow -3, -5$



$$\sigma_a = \frac{(-3-5) - (-1-1+2)}{2-3} = \frac{-8}{-1} = 8$$

$$\theta_a = \underbrace{\frac{(2k+1)\pi}{-1} = -\pi}_{\text{infinite pole at } -180^\circ}$$

3 branches

* Must leave somewhere between pole at $-\infty$ and at -5, but not sure where

Departure initially around $90°$ & $-90°$