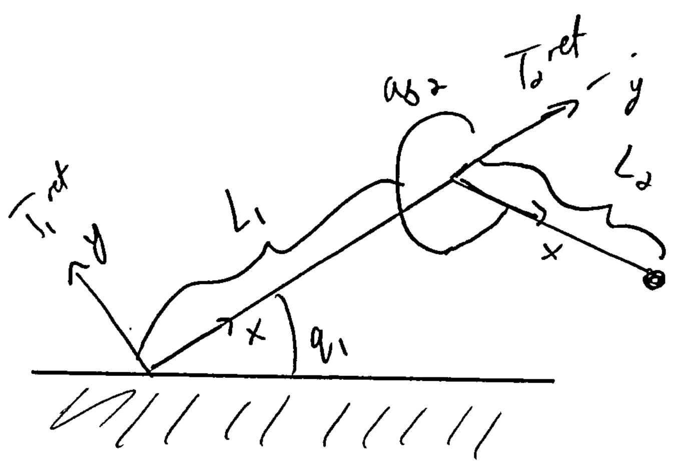
Problem 1 : Forward Kinematics

Ryan St. Pierre
ras70

A1



q_1 : World $\rightarrow$ T_1^{ref}

q_2 : T_1^{ref} $\rightarrow$ T_2^{ref}

E.E location w.r.t T_2^{ref}

A. $\vec{x}(q_1, q_2) = T(q_1, q_2)\,\vec{p}$

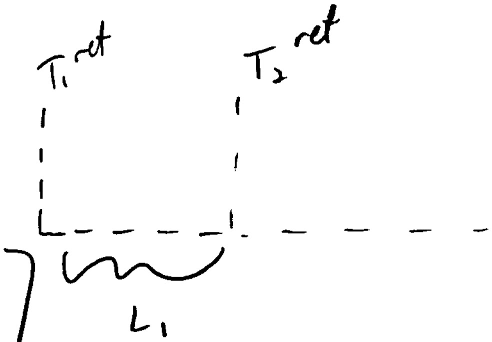
$T_1^{ref} \rightarrow$ world

conversion from T_2^{ref} to world reference frame after $R(q_2)$

$$T_1(q_1) = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$c_1 \equiv \cos q_1$

$s_1 \equiv \sin q_1$



$$T_2(q_1, q_2) = T_1(q_1) \cdot T_{2\rightarrow 1}^{ref}\, R(q_2)$$

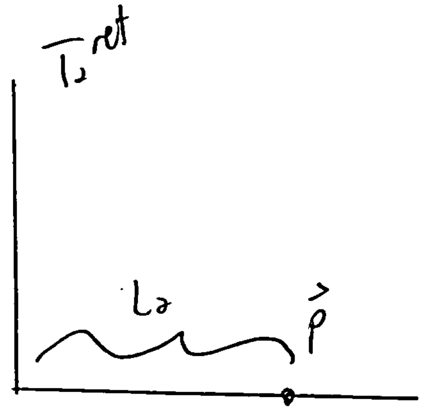
$$T_{2\rightarrow 1}^{ref} = \begin{bmatrix} 1 & 0 & L_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Point in T_2 w.r.t T_1

$$R(q_2) = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$c_2 \equiv \cos q_2$

$s_2 \equiv \sin q_2$



$$\vec{p} = \begin{bmatrix} L_2 \\ 0 \\ 1 \end{bmatrix}$$

$\rightarrow$

Problem 1A cont

$$\vec{x}(q_1, q_2) = T_1(q_1)\, T_{2\to1}^{ret}\, R(q_2) \begin{bmatrix} L_2 \\ 0 \\ 1 \end{bmatrix}$$

$$= T_1(q_1)\, T_{2\to1}^{ret} \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_2 \\ 0 \\ 1 \end{bmatrix}$$

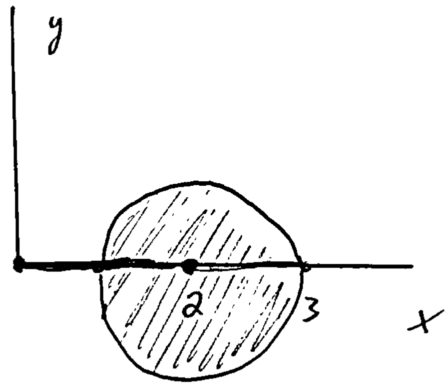
$$= T_1(q_1)\, T_{2\to1}^{ret} \begin{bmatrix} L_2 c_2 \\ s_2 L_2 \\ 1 \end{bmatrix} = T_1(q_1) \begin{bmatrix} 1 & 0 & L_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_2 c_2 \\ s_2 L_2 \\ 1 \end{bmatrix}$$

$$= T_1(q_1) \begin{bmatrix} L_2 c_2 + L_1 \\ s_2 L_2 \\ 1 \end{bmatrix} = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_2 c_2 + L_1 \\ s_2 L_2 \\ 1 \end{bmatrix}$$

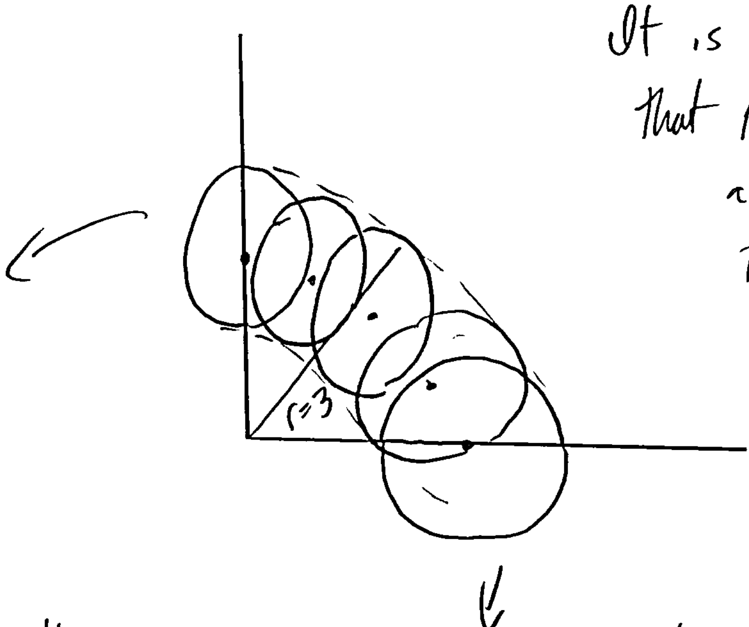
$$\boxed{\vec{x} = \begin{bmatrix} c_1 (L_2 c_2 + L_1) - s_1 s_2 L_2 \\ s_1 (L_2 c_2 + L_1) + c_1 s_2 L_2 \\ 1 \end{bmatrix}}$$

Problem 1B

Let $q_1 = 0$ and $q_2 \in [0°, 360°)$. This produces a workspace contained within a circle of radius 1 centered around the world point $(2, 0)$



We can iterate this process for $q_1 \in (0, 360°]$ to get a better idea of the complete workspace.

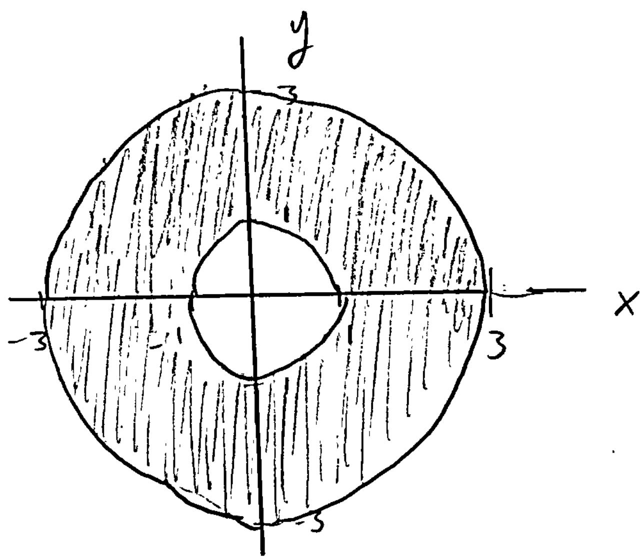


It is clear from this iterative process that the workspace is constrained by a circle of radius 3 around the origin. This makes sense, since when $q_2 = 0°$ the combined "length" of the two armatures is 3, which can be rotated anywhere is x, y space.

However, this is not the full picture. The circle of radius 1 around $(0,0)$ is not attainable.

Problem 1B cont

The final workspace is $L_2 - L_1 \le \sqrt{x^2+y^2} \le L_2 + L_1$



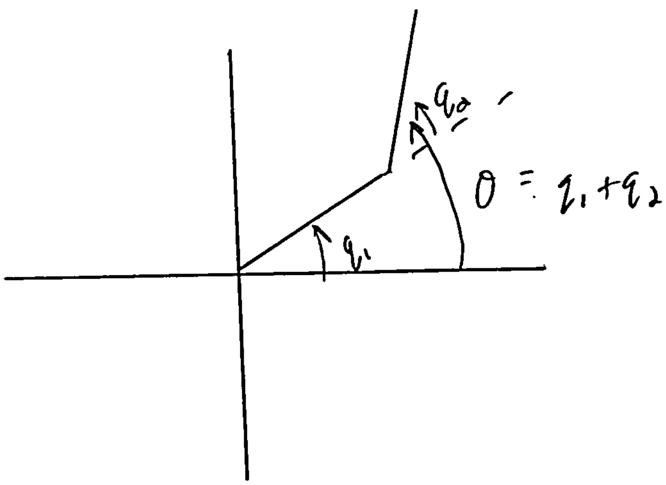
The workspace is given by the shaded region - between the circle with radius 1 and 3, both centered at the origin

In other words, $1 \le \sqrt{x^2+y^2} \le 3$ or $1 \le x^2+y^2 \le 9$

Problem 1C

θ = the angle of the second link in world coordinates

The angle of the second link in world coordinates is simply $q_1 + q_2$



$c_1 \equiv \cos q_1$

$s_1 \equiv \sin q_1$

$c_2 \equiv \cos q_2$

$s_2 \equiv \sin q_2$

From problem 1A

$$x = c_1(L_2 c_2 + L_1) - s_1 s_2 L_2$$
$$y = s_1(L_2 c_2 + L_1) + c_1 s_2 L_2$$

$\vec{x} = (x, y, \theta) \in SE(2)$

$$\vec{x} = \begin{bmatrix} c_1(L_2 c_2 + L_1) - s_1 s_2 L_2 \\ s_1(L_2 c_2 + L_1) + c_1 s_2 L_2 \\ q_1 + q_2 \end{bmatrix}$$

In homogeneous coordinates

$$\begin{bmatrix} c_1(L_2 c_2 + L_1) - s_1 s_2 L_2 \\ s_1(L_2 c_2 + L_1) + c_1 s_2 L_2 \\ q_1 + q_2 \\ 1 \end{bmatrix}$$

still a point

Problem 1D

The manifold of reachable values in the xy plane does not change, assuming L_2 is still 1 & L_1 is still 2. This means looking down on the xy axis, with z out of the page the workspace will look like a flat 'donut centered at the origin (the space between a circle of radius 3 and a circle with radius 1).

The introduction of θ in the z dimension adds a bit of spacial complexity. This arises because there is not a one to one mapping between θ and a (x,y) point. This is shown below

$$q_1 = 0 \quad q_2 = 180^\circ. \Rightarrow \theta = 180^\circ \quad x = 1 \quad y = 0$$

$$q_1 = 180^\circ \quad q_2 = 0^\circ \Rightarrow \theta = 180^\circ \quad x = -3 \quad y = 0$$

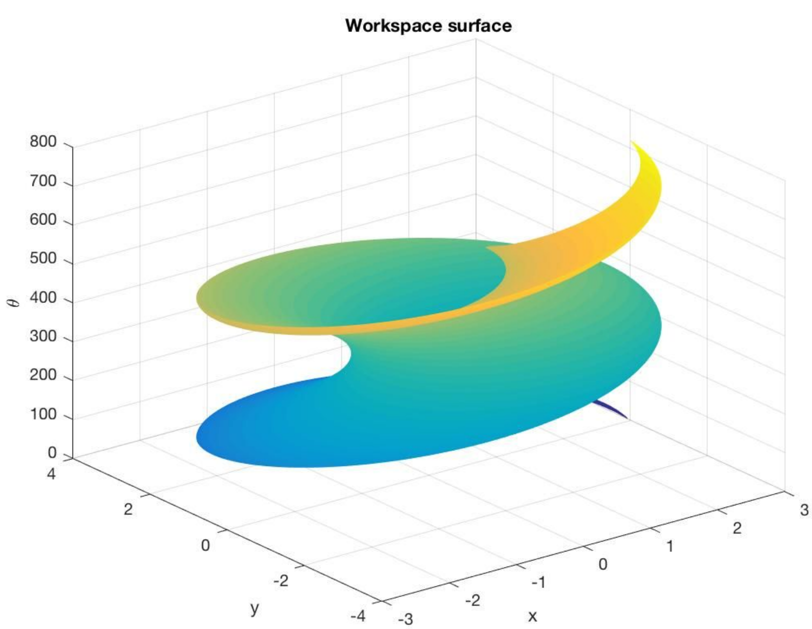
Above it is shown each theta maps to a series of points in $\mathbb{R}^2$ space. Generally this produces a donut in x,y space that follows a spiraling pattern as θ ranges from 0 to 720 degrees. I used Matlab to demonstrate this visually.

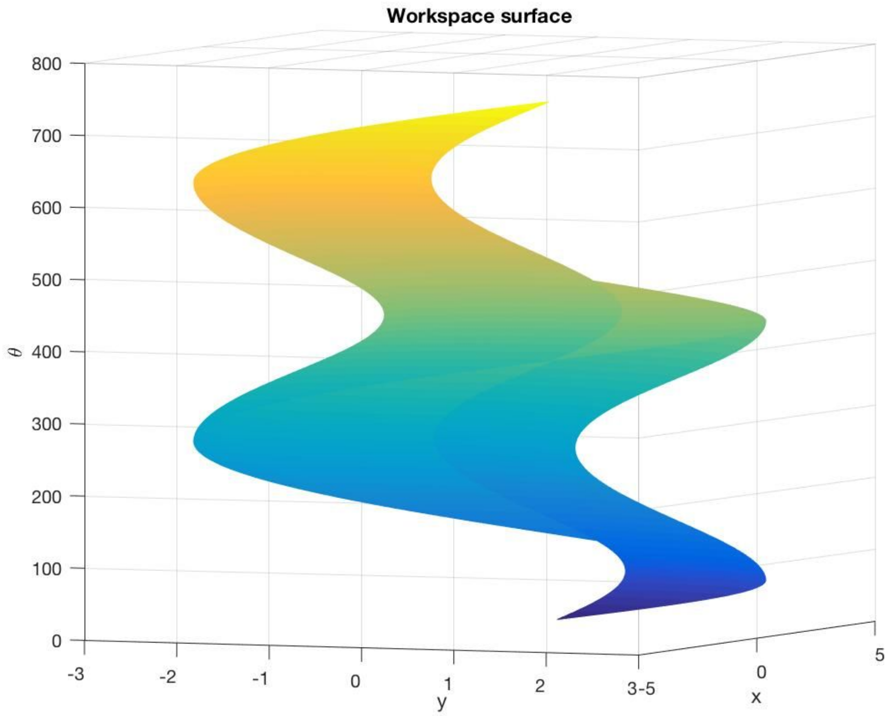
→

Ryan St. Pierre
HW #2 Problem 1D

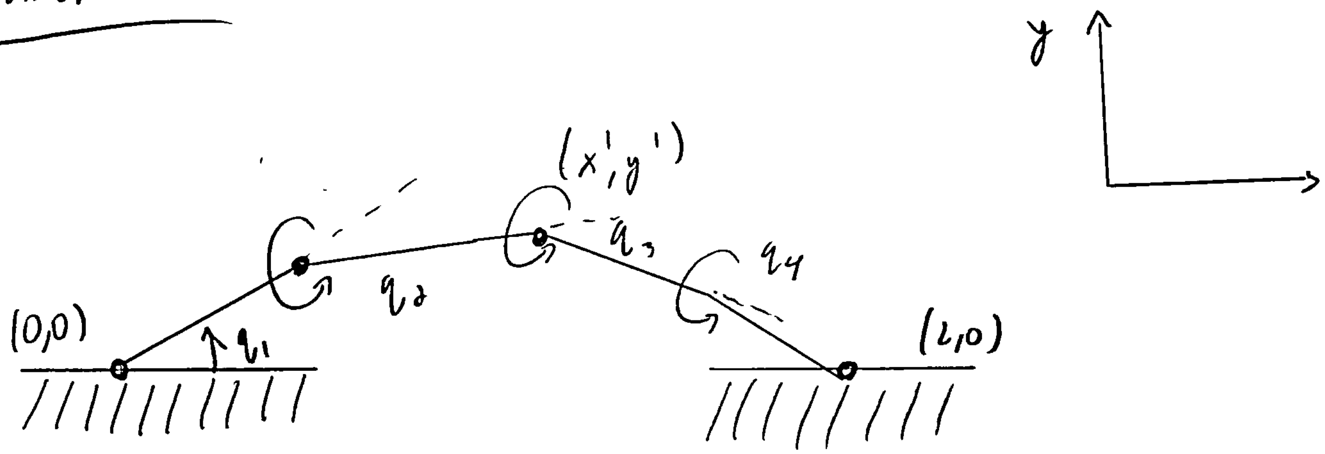
I used Matlab to visualize the workspace for 1C. The code and corresponding figures are given below:

```
[q1, q2] = meshgrid(1:0.3:360);
c1 = arrayfun(@(x) cosd(x),q1);
s1 = arrayfun(@(x) sind(x),q1);
c2 = arrayfun(@(x) cosd(x),q2);
s2 = arrayfun(@(x) sind(x),q2);

L1 = 2;
L2 = 1;

X = c1.*(L2 .* c2 + L1) - s1.*s2.*L2;
Y = s1.*(L2 .* c2 + L1) + c1.*s2.*L2;
Z = q1 + q2;

mesh(X,Y,Z)
xlabel('x')
ylabel('y')
zlabel('\theta')
title('Workspace surface')
```

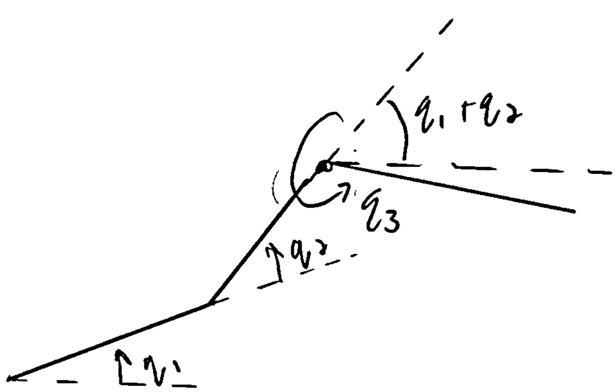



Workspace surface
800
700
600
500
400
300
200
100
0
θ
-3
-2
-1
0
1
2
3
y
-5
0
5
x

Problem 2A



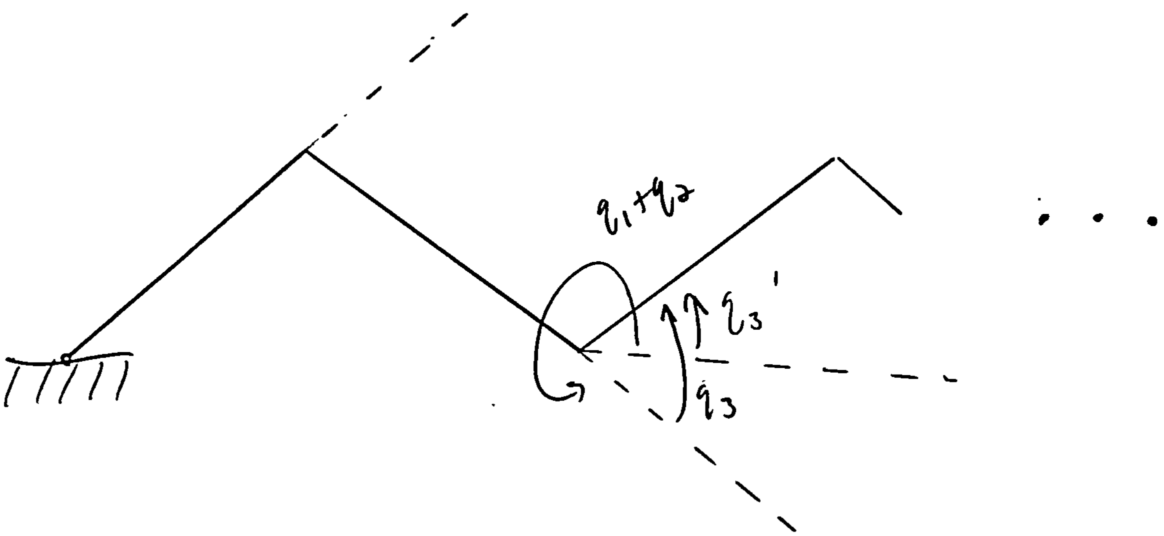
With the parameterization of q_1, q_2 we can solve for the location of the third joint. This is labeled (x', y') in the diagram above. Once this (x', y') location is calculated the problem is reduced to an inverse kinematics problem for the final two links (a 2R manipulator). However, assuming the 2RIK subfunction calculates the angles w.r.t the world xy coordinate system we must do a conversion from the solution to q_3 and q_4. In the diagram above q_3 is offset by $q_1 + q_2$. This correction needs to be made and is shown below



The returned answer of the subroutine 2RIK will return the angle $q_1 + q_2 + q_3$. Thus to find q_3 we need to take the solution and subtract q_1 and q_2

Problem 2A cont

However, the correction from q_3' to q_3 on the previous page only holds if $q_3' \geq q_1+q_2$.
If this case does not hold a different conversion needs to be made. This is shown below.



Here, when $q_1+q_2 > q_3'$ then $q_3 = 2\pi - (q_1+q_2) + q_3'$

Thus, the "correction" from q_3' (defined w.r.t the world x-axis) to q_3, defined w.r.t the second armature is given as:

if $(q_3' \geq q_1+q_2)$

$$q_3 = q_3' - (q_1+q_2)$$

else

$$q_3 = 2\pi - (q_1+q_2) + q_3'$$

The subroutine will return q_4 wrt the first armature, which in our case is the third armature. Thus, q_4 does not have to be changed.

Similar to Problem 1A the mapping of q_1, q_2 to x, y coordinates in the following way:

$$T_1(q_1) = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{matrix} c_1 \equiv \cos q_1 & c_2 \equiv \cos q_2 \\ s_1 \equiv \sin q_1 & s_2 \equiv \sin q_2 \end{matrix}$$

$$T_2(q_1, q_2) = T_1(q_1)\, T_{2\to 1}^{rel}\, R(q_2) \qquad R(q_2) = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{2\to 1}^{rel} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \; L_1$$

$$\vec{x}(q_1, q_2) = T(q_1, q_2) \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \; L_2$$

$$\vec{x}(q_1, q_2) = T_1(q_1)\, T_{2\to 1}^{rel}\, R(q_2) \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\vec{x}(q_1, q_2) = \begin{bmatrix} c_1(c_2+1) - s_1 s_2 \\ s_1(c_2+1) + c_1 s_2 \\ 1 \end{bmatrix}$$

Now we have all the necessary building blocks to build a routine that calculates q_3, q_4 given q_1 and q_2

→

Problem 2A cont

$4RIK(q_1, q_2)$

$c_1 = \cos q_1$

$s_1 = \sin q_1$

$c_2 = \cos q_2$

$s_2 = \sin q_2$

$x' = c_1(c_2+1) - s_1 s_2$

$y' = s_1(c_2+1) + c_1 s_2$

* $(q_3', q_4) = 2RIK(1, 1, L-x', 0-y')$ **

if $(q_3' \geq q_1+q_2)$ $q_3 = q_3' - (q_1+q_2)$

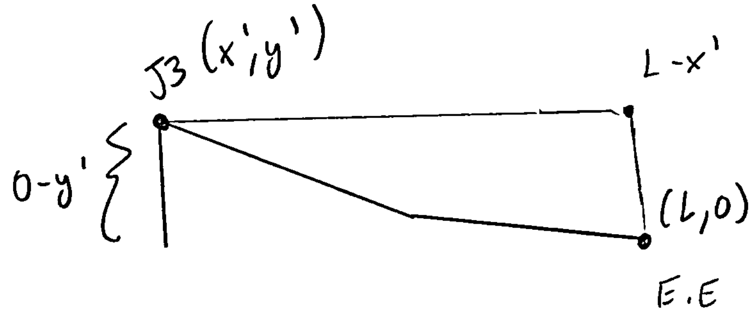
else $q_3 = 2\pi - (q_1+q_2) + q_3'$

return (q_3, q_4)

To the left is the routine to calculate all possible solutions for q_3 and q_4. It takes in the parameters q_1, q_2 and returns the touple of solutions for (q_3, q_4)

* Here (q_3', q_4) are all possible solutions to the 2R IK problem

** $L-x'$ and $0-y'$ are the x and y distance respectively that the 2R IK kinematics must solve to get joint 3 to align the E.E at $(L,0)$



Problem 2A cont

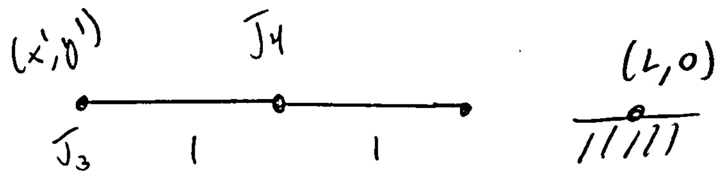
We will now consider the cases in which there are 0, 1, 2, or infinite solutions in the q_1, q_2 parameterization.

0 solutions

Since $L_3 = L_4$ the only case that can produce zero solutions is when the point $(L,0)$ is out of reach for the chosen q_1, q_2. The condition that must hold for this to be the case is given below.

$$x' = c_1(c_2+1) - s_1 s_2$$

$$y' = s_1(c_2+1) + c_1 s_2$$



$$\sqrt{(L-x')^2 + (-y')^2} > 2$$

$$(L-x')^2 + y'^2 > 4$$

$$\underbrace{\left(L - (c_1 c_2 + c_1 - s_1 s_2)\right)^2 + (s_1 c_2 + s_1 + c_1 s_2)}_{G(L,q_1,q_2)} > 4$$

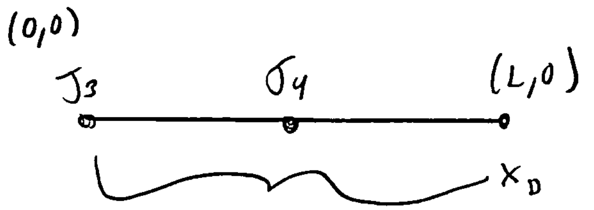
If we define G as the square of the distance between joint 3 (at a given q_1, q_2) and the point $(L,0)$, then the zero solution case is described by $G(L, q_1, q_2) > 4$ $\rightarrow$

Problem 2A cont

1 Solution

Intuitively, there is only 1 solution for (q_3, q_4) if q_1, q_2 are chosen such that $(L, 0)$ is just in reach. This is given by $G(L, q_1, q_2) = 4$.

Seen differently we can consider the case where armature 3 is along the x reference frame, without loss of generality. Given this we could calculate q_4 as follows.



$$\|x(q)\|^2 = \|x_D\|^2$$

$$\|x(q)\|^2 = (1 + c_4)^2 + (s_4)^2$$

$$= 1 + 2c_4 + c_4^2 + s_4^2$$

$$= 2 + 2c_4$$

$$c_4 = \frac{\|x_D\|^2 - 2}{2} = \frac{\|x_D\|^2}{2} - 1$$

If $\|x_D\| = 2$

then $c_4 = 1$ and

q_4 must be $0°$. With q_4 chosen q_3 must be chosen such that the armature points directly at $(L, 0)$, thus there is only one solution.

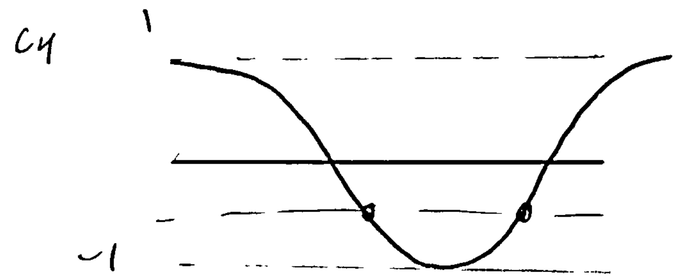
$\longrightarrow$

Problem 2A cont

2 Solutions

There are 2 solutions if $G(L, q_1, q_2) < 4$. Implicitly $G(L, q_1, q_2)$ is always larger than or equal to zero since it is defined as a distance.

$$c_4 = \frac{\|x_0\|^2}{2} - 1 \Rightarrow -1 \leq c_4 \leq 1$$



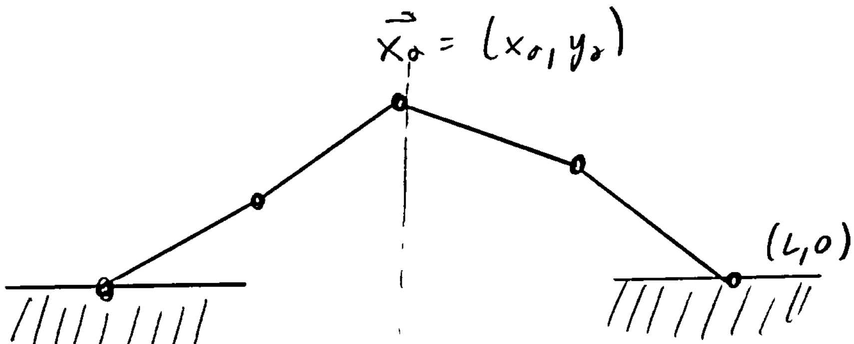
If c_4 falls between -1 and 1 2 q_4 values can be chosen.

∞ Solutions

There are infinite solutions if the third joint falls on $(L, 0)$ for the q_1 & q_2 values chosen. In this case q_4 can be made -180^0 and <u>any</u> q_3 can be chosen (since $L_3 = L_4$). This occurs when $c_1(c_2+1) - s_1 s_2 = L$ and $s_1(c_2+1) + c_1 s_2 = 0$

In the last couple of pages we have shown we can define a function G purely based on q_1, q_2, and L and use it to discern the size of the solution set

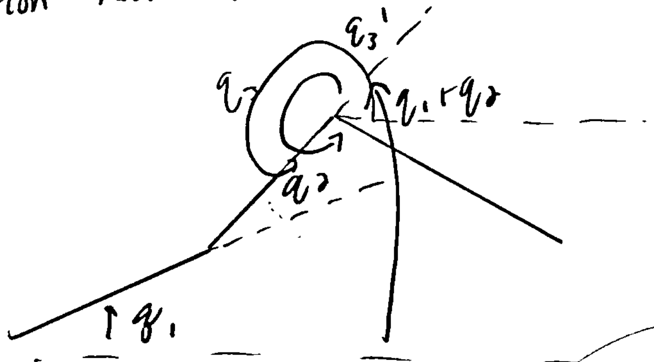
Problem 2B



Let $\vec{x}_2$ (the location of the third joint) be given by the point (x_2, y_2). We can divide this problem into 2 2RIK problems.

① $2RIK(1, 1, x_2, y_2)$ and ② $2RIK(1, 1, L - x_2, -y_2)$

Again, as in Problem 2A this will give us a q_3 with respect to the x-axis. However, we want it with respect to the second link. Thus, a conversion needs to be made.



Solution from ②

$q_3 = q_3' - q_1 - q_2$ if $q_3' \geq q_1 + q_2$

$q_3 = 2\pi - (q_1 + q_2) + q_3'$ $q_3' < q_1 + q_2$

(reference Problem 2A)

The routine to calculate $q_1, q_2, q_3, \& q_4$ given the position x_2, y_2 is given to the right

$4RIK(x_2, y_2, L)$
$(q_1, q_2) = 2RIK(1, 1, x_2, y_2)$
$(q_3', q_4) = 2RIK(1, 1, L - x_2, -y_2)$
if $(q_3' \geq q_1 + q_2)$ $q_3 = q_3' - (q_1 + q_2)$
else $q_3 = 2\pi - (q_1 + q_2) + q_3'$
return (q_1, q_2, q_3, q_4)

Problem 2B

We now consider the size of the solution set

0 solutions

This occurs $\vec{x}_2$ is out of range for q_1, q_2 or $\vec{x}_2$ is chosen such that ~~q_3 & q_4 cannot be chosen~~ to set to $(L,0)$

$$\|\vec{x}_2\| > 2 \quad \overset{(OR)}{\|} \quad \left\| \vec{x}_2 - \begin{bmatrix} L \\ 0 \end{bmatrix} \right\| > 2$$

1 solution

This occurs when $\vec{x}_2$ is just in range of $(0,0)$ & $(L,0)$

$$\|\vec{x}_2\| = 2 \quad \& \quad \left\| \vec{x}_2 - \begin{bmatrix} L \\ 0 \end{bmatrix} \right\| = 2$$

2 solutions

When $\vec{x}_2$ is just in range of $(0,0)$ and within range from $(L,0)$

or

$\vec{x}_2$ is within range from $(0,0)$ and just in range to $(L,0)$

$$\left(\|\vec{x}_2\| = 2 \ \& \ \left\| \vec{x}_2 - \begin{bmatrix} L \\ 0 \end{bmatrix} \right\| < 2 \right) \overset{(or)}{\|} \left(\|\vec{x}_2\| < 2 \ \& \ \left\| \vec{x}_2 - \begin{bmatrix} L \\ 0 \end{bmatrix} \right\| = 2 \right)$$

Problem 2B cont

4 solutions

$\vec{x}_2$ within range from $(0,0)$ & $(L,0)$

$$\|\vec{x}_2\| < 2 \quad \& \quad \left\|\vec{x}_2 - \begin{bmatrix} L \\ 0 \end{bmatrix}\right\| < 2$$

∞ Solutions

$\vec{x}_2 = (0,0)$

or

$\vec{x}_2 = (L,0)$

q_1 can be any value between 0 & 360°

q_2 can be any value between 0 & 360°

Ryan St. Pierre
HW #2 Problem 2C

Problem 2C

One benefit of the (q1,q2) parameterization is that is has less charts necessary to cover the manifold. With the parameterization of (q1, q2) the problem is essentially reduced to one 2RIK problem (along with some geometry). As we studied in class the 2RIK problem has two charts necessary to cover the manifold (corresponding to "elbow up" and "elbow down"). By using q1 and q2 in the parameterization we are essentially removing from the ultimate solution set whether armature 1 and armature 2 are elbow up or elbow down because that information is given by the parameterization itself. To the contrary, the $\mathbf{x_2}$ parameterization does not encode this information about armature 1 and 2 in the parameterization itself. Thus, the $\mathbf{x_2}$ parameterization has 4 charts to cover its manifold, corresponding to the cases when armature 1 and 2 are in elbow up or down orientation and when armature 3 and 4 are in elbow up or elbow down orientation.

In reality the valid values for the q1,q2 in the (q1,q2) parameterization are actually infinite. However, to avoid redundancy we can refine the values of q1 and q2 to between 0 and 2π for each. Technically, for the $\mathbf{x_2}$ parameterization $\mathbf{x_2}$ can be any value in $\mathbf{R}^2$. However, given the constraint that all armature lengths are equal to one, $\mathbf{x_2}$ must actually fall within a circle of radius 2 around the origin. Thus, with both parameterizations the values of the parameterization variable can be limited. In this case I am not sure which parameterization is more preferred, as I believe it would be the preference of the user/coder and whether or not they prefer working in cartesian or angular coordinates. Since we are ultimately concerned with angular coordinates it is probably easier in interpolation and other tasks to deal with angular coordinates from the start in the parameterization choice.

From the work done in *Problem 2A* and *2B* it is clear that the (q1,q2) parameterization requires one inverse kinematics problem while the $\mathbf{x_2}$ parameterization requires two. It is prefered that during the procedure the number of inverse kinematics problems is limited. Additionally, it is quite easy to get the value of $\mathbf{x_2}$ given q1 and q2. This is done by a matrix multiplication, which is simpler than solving an inverse kinematics problem. Thus, it is easier to get $\mathbf{x_2}$ knowing q1 and q2 than to get q1 and q2 knowing $\mathbf{x_2}$.

Problem 3A

$$\vec{x} = \begin{bmatrix} c_1 (L_2 c_2 + L_1) - s_1 s_2 L_2 \\ s_1 (L_2 c_2 + L_1) + c_1 s_2 L_2 \\ q_1 + q_2 \end{bmatrix}$$

$$J(q) = \begin{bmatrix} \partial x / \partial q_1 & \partial x / \partial q_2 \\ \partial y / \partial q_1 & \partial y / \partial q_2 \\ \partial \theta / \partial q_1 & \partial \theta / \partial q_2 \end{bmatrix}$$

$$x = L_2 c_2 \cos(q_1) + L_1 \cos(q_1) - L_2 s_2 \sin(q_1)$$

$$\frac{\partial x}{\partial q_1} = -L_2 c_2 \sin(q_1) + L_1 \sin(q_1) - L_2 s_2 \cos(q_1) = -s_1 (L_2 c_2 + L_1) - c_1 s_2 L_2$$

$$x = L_2 c_1 \cos(q_2) + L_1 c_1 - s_1 L_2 \sin(q_2)$$

$$\frac{\partial x}{\partial q_2} = -L_2 c_1 \sin(q_2) + 0 - s_1 L_2 \cos(q_2) = -L_2 (c_1 s_2 + s_1 c_2)$$

$$\partial \theta / \partial q_1 = 1 \qquad \frac{\partial \theta}{\partial q_2} = 1$$

$$y = L_2 c_2 \sin(q_1) + L_1 \sin(q_1) + L_2 s_2 \cos(q_1)$$

$$\frac{\partial y}{\partial q_1} = L_2 c_2 \cos(q_1) + L_1 \cos(q_1) - L_2 s_2 \sin(q_1) = c_1 (L_2 c_2 + L_1) - s_1 s_2 L_2$$

$$y = s_1 L_2 \cos(q_2) + L_1 s_1 + L_2 c_1 \sin(q_2)$$

$$\frac{\partial y}{\partial q_2} = -s_1 L_2 \sin(q_2) + 0 + L_2 c_1 \cos(q_2) = -s_1 L_2 s_2 + L_2 c_1 c_2$$

Problem 3A cont

$$J(q) = \begin{bmatrix} -s_1(L_2c_2 + L_1) - c_1s_2L_2 & -L_2(c_1s_2 + s_1c_2) \\ c_1(L_2c_2 + L_1) - s_1s_2L_2 & -s_1s_2L_2 + c_1c_2L_2 \\ 1 & 1 \end{bmatrix}$$

Problem 3B

The chain rule is given by $\frac{\partial f}{\partial t} = \frac{\partial f}{\partial q_1}\frac{\partial q_1}{\partial t} + \frac{\partial f}{\partial q_2}\frac{\partial q_2}{\partial t}$

$\frac{\partial f}{\partial q_1}$ and $\frac{\partial f}{\partial q_2}$ are values that were calculated in Problem 3A.

More specifically

$$\frac{\partial f}{\partial q_1} = \begin{bmatrix} -s_1(L_2c_2 + L_1) - c_1 s_2 L_2 \\ c_1(L_2c_2 + L_1) - s_1 s_2 L_2 \\ 1 \end{bmatrix} \qquad \frac{\partial f}{\partial q_2} = \begin{bmatrix} -L_2(c_1 s_2 + s_1 c_2) \\ -s_1 s_2 L_2 + c_1 c_2 L_2 \\ 1 \end{bmatrix}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial q_1}\underbrace{\frac{\partial q_1}{\partial t}}_{\text{scalar}} + \frac{\partial f}{\partial q_2}\underbrace{\frac{\partial q_2}{\partial t}}_{\text{scalar}} =$$

$$\begin{bmatrix} (-s_1(L_2c_2+L_1) - c_1 s_2 L_2)\frac{\partial q_1}{\partial t} \\ (c_1(L_2c_2+L_1) - s_1 s_2 L_2)\frac{\partial q_1}{\partial t} \\ \frac{\partial q_1}{\partial t} \end{bmatrix} + \begin{bmatrix} (-L_2(c_1 s_2 + s_1 c_2))\frac{\partial q_2}{\partial t} \\ (-s_1 s_2 L_2 + c_1 c_2 L_2)\frac{\partial q_2}{\partial t} \\ \frac{\partial q_2}{\partial t} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial q_1}{\partial t}(-s_1(L_2c_2+L_1) - c_1 s_2 L_2) + \frac{\partial q_2}{\partial t}(-L_2(c_1 s_2 + s_1 c_2)) \\ \frac{\partial q_1}{\partial t}(c_1(L_2c_2+L_1) - s_1 s_2 L_2) + \frac{\partial q_2}{\partial t}(-s_1 s_2 L_2 + c_1 c_2 L_2) \\ \frac{\partial q_1}{\partial t} + \frac{\partial q_2}{\partial t} \end{bmatrix}$$

$$= \begin{bmatrix} -s_1 (L_2 c_2 + L_1) - c_1 s_2 L_2 & -L_2 (c_1 s_2 + s_1 c_2) \\ c_1 (L_2 c_2 + L_1) - s_1 s_2 L_2 & -s_1 s_2 L_2 + c_1 c_2 L_2 \\ 1 \quad\quad 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial q_1}{\partial t} \\ \frac{\partial q_2}{\partial t} \end{bmatrix}$$

$$= J(q(t)) \; q'(t)$$

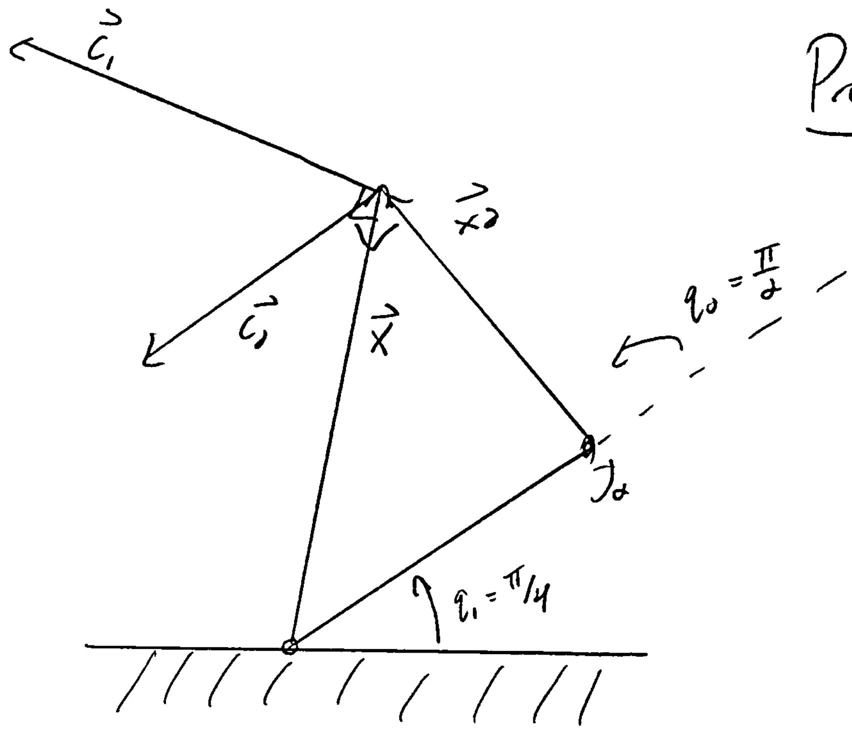
$$\therefore$$

$$\frac{\partial f}{\partial t} = J(q(t)) \; q'(t) \quad \blacksquare$$

Problem 3B cont

Ryan StPierre
ras70

Problem 3C



$$\vec{x} = \begin{bmatrix} \frac{\sqrt{2}}{2} L_1 - \frac{\sqrt{2}}{2} L_2 \\ \frac{\sqrt{2}}{2} L_1 + \frac{\sqrt{2}}{2} L_2 \\ \frac{3\pi}{4} \end{bmatrix}$$

$c_1 = \cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$ $\quad c_2 = \cos(\frac{\pi}{2}) = 0$

$s_1 = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$ $\quad s_2 = \sin(\frac{\pi}{2}) = 1$

$$J\left(\begin{bmatrix} \pi/4 \\ \pi/2 \end{bmatrix}\right) = \begin{bmatrix} -\frac{\sqrt{2}}{2}(L_1 + L_2) & -\frac{\sqrt{2}}{2} L_2 \\ \frac{\sqrt{2}}{2}(L_1 - L_2) & -\frac{\sqrt{2}}{2} L_2 \\ 1 & 1 \end{bmatrix}$$

instantaneous

Let $\vec{c}_1$ & $\vec{c}_2$ be the two column vectors. $\vec{c}_1$ is the vector of motion if q_1 was to move with q_2 still. Note: $||\vec{x}|| = ||\vec{c}_1||$ and $\vec{x} \perp \vec{c}_1$

$$||\vec{x}|| = L_1^2 + L_2^2 = ||\vec{c}_1||$$

$\vec{c}_2$ is the vector of instantaneous motion if q_2 was to move with q_1 still. Let $\vec{x}_2$ be the displacement vector from joint 2 (J_2) to the E.E

Problem 3C cont

Then $\|\vec{x}_2\| = \|\vec{c}_2\|$ & $\vec{x}_2 \perp \vec{c}_2$

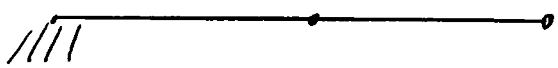
I have neglected the orientation in my discussion above. In 3D space (w/ orientation) the third component of the column vector gives θ's instantaneous change with a 1 rad/s increase in q_1 or q_2. Obviously this is 1.

Problem 3D

$$q = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad q_1 = q_2 = 0$$

$$S_1 = 0 \qquad S_2 = 0$$

$$C_1 = 1 \qquad C_2 = 1$$



$$J(q) = \begin{bmatrix} 0 & 0 \\ L_1 + L_2 & L_2 \end{bmatrix} \qquad \Delta = 0 L_2 + 0 (L_1 + L_2) = 0$$

$\Rightarrow$ linear dependence

The significance of this is the robot cannot move instantaneously in certain workplace dimensions.