Lab 2: Data Acquisition, Aliasing, and Quantization

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Abstract

The purpose of this experiment is to explore the subject of data acquisition (DAQ), including aliasing and quantization. Data acquisition is the process of measuring a signal and converting it to a voltage or current for computer import, at which point the signal can be stored and analyzed. This lab focuses on two topics of data acquisition - aliasing and quantization, both of which are described at length in the Introduction. In the aliasing portion of the lab the signal generator was used to pass two sinusoidal signals, one at 2 kHz and the other at 15 kHz, through a low pass filter circuit. Using the NI DAQ card (which has a sampling rate of 20 kHz) to record data aliasing was observed on the input signal, which was recorded as a 5 KHz, not a 15 kHz sinusoid. The low pass filter was successfully used to observe this aliasing. In the data quantization portion of the lab a 50 Hz sinusoid was applied to the system. Zooming on the recorded output of the system allowed us to verify our calculated value of the minimum voltage difference V_s, which was calculated to be 3.05 x 10^{-4} V.

1 Introduction

Write several paragraphs introducing the experiment and describing the theory behind aliasing and quantization error. You are welcome to use information from the lab document and appendices—just make sure you cite the source [1].

2 Materials and Methods

Describe the steps involved in the experiment, as well as the equipment you used. Use a figure where appropriate.

2.1 Aliasing

The frequencies chosen for the two input sinusoidal functions are detailed in, Table 1, below.

f_1	2 kHz
f_2	15 kHz

Table 1: Input frequencies f_1 and f_2, signal generator output sample rate of 40 kHz.

In this section we recorded the raw signals (the two sinusoidal function produced from the function generator) and the output signal from our system. In this case our system was a low pass filter.

We created three different figures from these recorded signals:

1. Time domain plot of the original and filtered signals (overlapped).
2. Power spectrum plot of the original and filtered signals (overlapped).
3. Power spectrum plot of the original and filtered signals on side by side graphs.

2.2 Receiver Quantization Errors

The specific DAQ card equipment is detailed in Table 2 below.

The input to the signal generator was set to five times the minimum voltage difference (V_s). Additionally, a low frequency was chosen. These values are documented below.

Like the Aliasing section, in this section we recorded the raw signal (a single sinusoidal function produced from the function generator) and the output signal from our system. To reiterate, our system was a low pass filter.

Again, we created three different figures from these recorded signals:

DAQ card	NI PCI-6036E
Sampling Rate	20 kHz
Voltage Range E_{FSR}	20 V (+10 V to -10V)
Resolution (bits)	16 bits [2]
Resolution (volts per step)	3.05 x 10^{-4} V per step
Minimum voltage difference V_s	3.05 x 10^{-4} V

Table 2: DAQ card characteristics.

Amplitude	1.53 x 10^{-3} V
Frequency	50 Hz

Table 3: Input signal characteristics, signal generator output sample rate of 40 kHz.

1. Time domain plot of the original and filtered signals (overlapped).
2. Power spectrum plot of the original and filtered signals (overlapped).
3. Power spectrum plot of the original and filtered signals on side by side graphs.

3 Results and Discussion

3.1 Aliasing

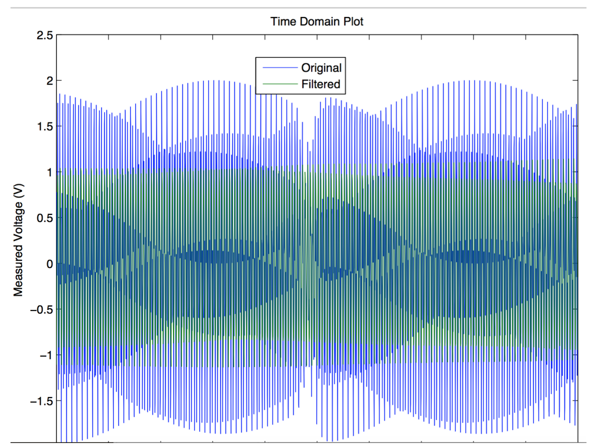

Figure 1: Time series for raw and filtered signal

Fig. 1 above gives an overlapped representation of the time series from the original and filtered signals. It is almost impossible to decipher any meaningful information from this figure.

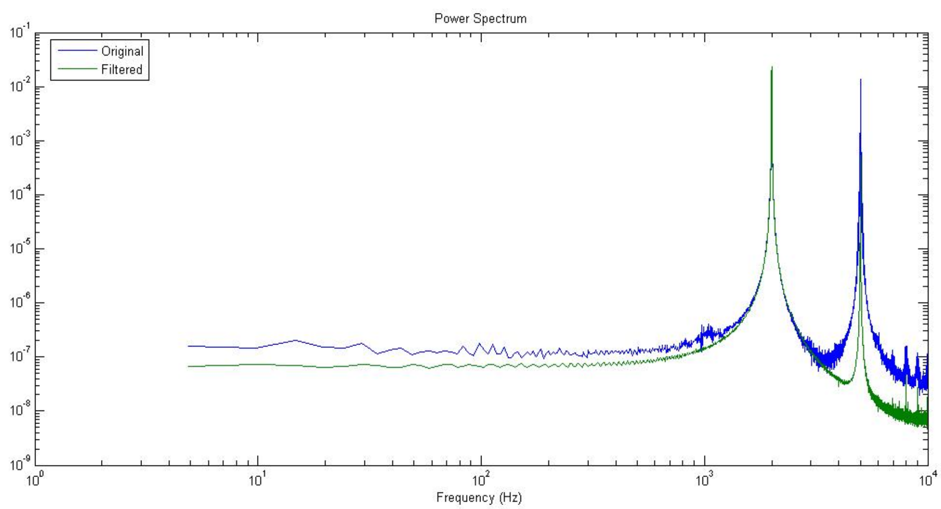

Figure 2: Power spectrum for the original and filtered signals

In Fig. 2 above the original signal is given in blue while the filtered signal is given in green. This power spectrum representation is much more meaningful in the context of aliasing. However, in order to understand its significance a discussion of the Nyquist frequency is first necessary. The Nyquist frequency calculation is given below

Nyquist Frequency

The lab manual defines the Nyquist, f^* frequency as,

$$f_{\text{sample}} = 2f^* \tag{1}$$

In other words, the Nyquist frequency is half the sampling frequency of the system. In our case the DAQ used has a sampling rate of 20 Hz [1]. The Nyquist frequency is given as,

$$\begin{aligned} f^* &= \tfrac{f_{\text{sample}}}{2} \\ f^* &= \tfrac{20\ \text{kHz}}{2} \\ f^* &= 10\ \text{kHz} \end{aligned}$$

Nyquist Frequency = k10 Hz

A Nyquist frequency of 10 Hz means the DAQ card is incapable of differentiating signals of multiples of 10 kHz. In other words, to the DAQ board a signal at 30 kHz, 20 kHz, and 10 kHz are indistinguishable and are all *perceived* as 10 kHz signals.

This phenomenon is observed in the Fig. 2 power plot. The input signals were set at 2 kHz and 15 kHz. The peaks of a power plot correspond to the frequencies at which a greater (in this case non-zero) amplitude is measured. However, the peaks of the power plot are not at 2 kHz and 15 kHz as expected. Rather, the peaks are at 2 kHz and 5 kHz. In other words, the 15 kHz input signal was perceived by the DAQ card as a 5 kHz signal.

Given aliasing, it is quite easy to explain why the peak in the power plot is at 5 kHz. This is simply the input frequency (15 kHz) minus the Nyquist frequency (10 kHz). This makes sense as the DAQ reader only has the sampling rate to interpret signals under 10 kHz, thus appearing to shift all frequencies greater than this value by multiples of the Nyquist frequency until it is within the required range.

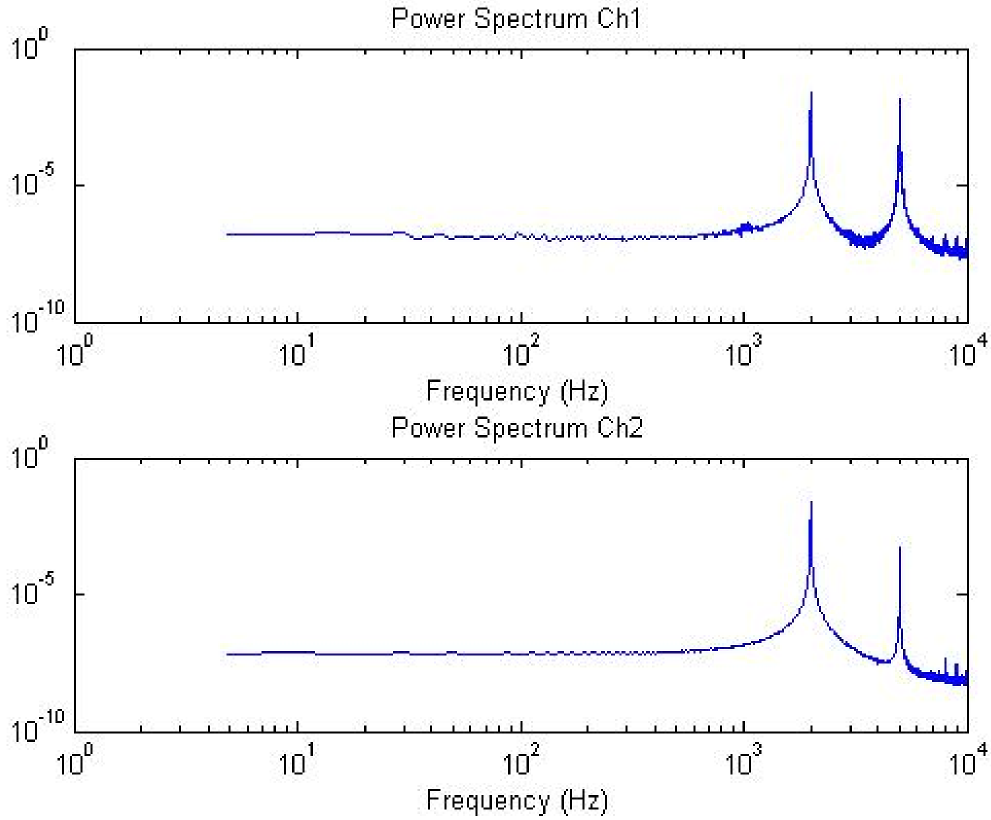

Figure 3: Power spectrum for the original (top) and filtered (bottom) signals

Fig. 3 above shows the power spectrum for the original and filtered signals one on top of the other. This is the same information as Fig. 2 displayed in a manner where the difference in magnitude of peaks can more clearly be seen. The 2 kHz signal appears the same on both plots. This means that the original signal is **not** filtered.

To the contrary the peak corresponding to the signal at 5 kHz is smaller on the filtered plot than the original signal plot. This means the original signal was filtered. However, the low pass filter is designed to allow signal under 10 kHz to pass through without less than unity gain. This means that although it appears the signal is at 5 kHz the signal is actually greater than 10 kHz and aliasing is present.

In this manner a way of identifying aliasing has been demonstrated. By using a low pass filter with a pass frequency equal to the Nyquist frequency of the system signal aquisition tool, the presence of aliasing can be detected by comparing the original and filter signals. If the original and filtered signals at a given frequency differ in magnitude then aliasing is present.

Of course in the above paragraph the behavior of a low pass filter has been idealized. In practice the low pass filter does not have one cut-off frequency. Thus, to be more precise the low-pass anti-aliazing filter should have the following specifications [1]:

$$\begin{aligned} \text{Filter Attenuation(dB)} &\equiv 20 \text{ x } \log_{10}\left(\frac{\mathrm{V_{Filtered}}}{\mathrm{V_{Unfiltered}}}\right) \quad \text{OR} \\ \text{Filter Attenuation(dB)} &\equiv 10 \text{ x } \log_{10}\left(\frac{\mathrm{P_{Filtered}}}{\mathrm{P_{Unfiltered}}}\right) \end{aligned}$$

Sources of Error

The setup for this aliasing experiment was relatively simple. Given the simplicity of the system there were little opportunities for sources of error to be introduced. It is likely that any sources of error could be attributed to the system itself, whether that be lose wires, a faulty low pass filter, or bad connection. It is also possible that the function generator introduced error.

From the figures given in this section it is also clear that there was a fair amount of noise. This noise can be seen best in the lack of smoothness in the Power Spectrum figure (Fig. 2). However, this noise is expected with any system and thus should be noted as noise, not error.

Extension

The higher frequency signal needs to be chosen less than 20 kHz because it is constrained by the sampling rate of the function generator. The function generator has a sampling rate of 40 kHz. Thus, it has a Nyquist frequency of 20 kHz. For the reasons observed and explained at great lengths above the output of the function generator needs to be at a frequency less than its Nyquist frequency. In other words, the function generator is unable to create a signal that is twice its sampling rate. If it attempts to do such it will only be able to sample at a high enough rate to which an outsider would observe a signal with frequency less than 20 kHz.

3.2 Receiver Quantization Errors

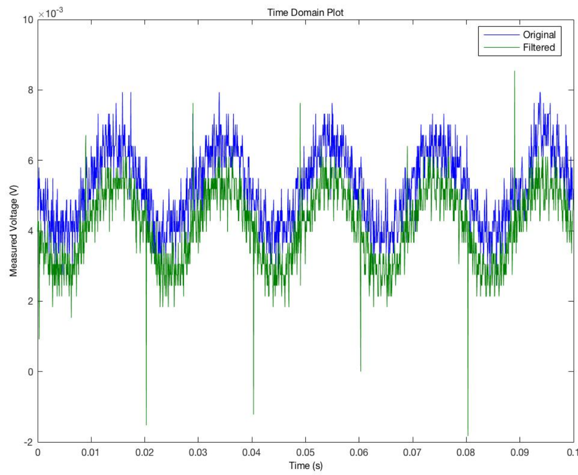

Figure 4: Time plot of the original and filtered signal

Fig. 4 shows the time plot for the original and filtered signal. It is important to observe the low amplitude of the signal used for this portion of the lab. However, this figure is not particular useful in explaining quantization error.

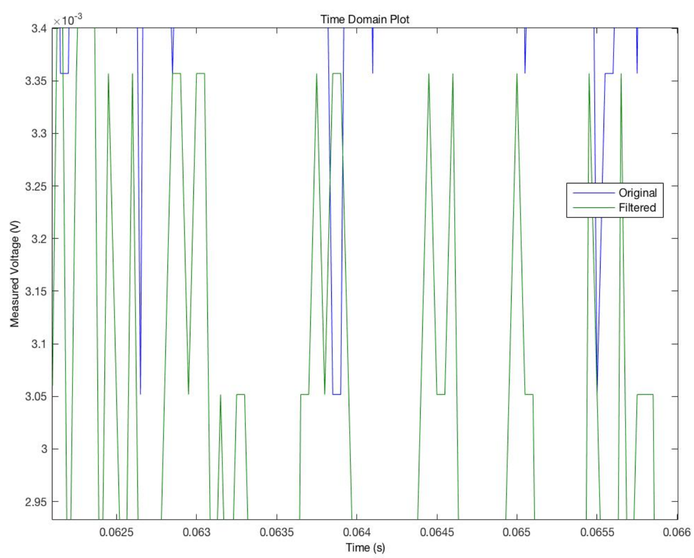

Figure 5: Time plot of the original and filtered signal zoomed to highlight quantization error

Fig. 5 is simply a zoomed in representation of Fig. 4. Although it is difficult to distinguish the noise from the quanitzation error, the step-like the behavior of the signal at small voltage levels can be seen. In Fig. 5 there are consistently repeated steps of approximate size 3 x 10^{-4}. This is on the scale of the quantization error calculated below.

Discrete Voltage Calculation

The DAQ card is a binary tool. In other words it represents the analog signal with a finite amount of bits. Thus, the card has a discrete (finite) number of of voltage levels that corresponds to the amount of bits the card has. In our case the card has 16 bits in which to represent the signal. Therefore, the calculation for number of finite voltage levels can be made:

$$\begin{aligned}
\text{Discrete voltage levels} &= 2^{\text{number of bits}} \\
&= 2^{16} \\
&= 65536
\end{aligned}$$

The minimum detectable voltage difference, V_s, is simply the voltage range divided by the number of voltage levels. The DAQ card used in this lab has a voltage range of +10 V to -10 V [1].

$$\begin{aligned}
V_s &= \tfrac{\text{Voltage range}}{\text{number of voltage levels}} \\
&= \tfrac{10-(-10)\text{ V}}{65536} \\
&= \tfrac{20}{65536} \\
&= 3.05 \text{ x } 10^{-4}\text{ V}
\end{aligned}$$

Comparison

The smallest observable voltage discerned from observing the behavior in Fig. 5 closely matched the value determined analytically.

4 Conclusion

This experiment revealed two errors that can occur in data acquisition: aliasing and quantization. It was discovered that aliasing occurs for input signals with frequencies greater than the Nyquist frequency, which is half the sampling frequency of the data collection tool. Thus, engineers should ensure the largest frequency of the input is less than half the sampling frequency of the data collection tool. However, sometimes the input signal is not necessarily known. In this case aliasing must be detected. In this experiment it was shown that a low pass filter can be used to detect aliasing.

Quantization is a result of having to convert an analog signal to a complex world. Detecting quantization error does not make sense in the same context as detecting aliasing. Quantization error is always present, unlike aliasing. However, quantization error is often small and insignificant. Thus, engineers simply need to be aware that quantization exists and chose data acquisition tools with high enough bit resolution with respect to the signal that is being measured. In other words if an engineer wants to be able to measure a smaller voltage difference a higher bit resolution is needed. Thus, when it comes to quantization the engineer needs to choose a bit resolution that meets the specification of the degree of precision desired.

References

[1] *Laboratory 2: Data Acquisition, Aliasing and Quantization* lab manual.

[2] National Instrument PCI-60386E card website.
Hyperlink: http://sine.ni.com/nips/cds/view/p/lang/en/nid/11913