Lab 2: Data Acquisition, Aliasing, and Quantization

Ryan St.Pierre, Doyong Kim, and Rohan Rajan

September 22, 2017

Abstract

The purpose of this experiment is to explore the subject of data acquisition (DAQ), including aliasing and quantization. Data acquisition is the process of measuring a signal and converting it to a voltage or current for computer import, at which point the signal can be stored and analyzed. This lab focuses on two topics of data acquisition - aliasing and quantization, both of which are described at length in the Introduction. In the aliasing portion of the lab the signal generator was used to pass two sinusoidal signals, one at 2 kHz and the other at 15 kHz, through a low pass filter circuit. Using the NI DAQ card (which has a sampling rate of 20 kHz) to record the output signal, aliasing was observed on the input signal - which was recorded as a 5 KHz, not a 15 kHz sine. The low pass filter was successfully used to observe this aliasing. In the data quantization portion of the lab a 50 Hz sine wave was applied to the system. Zooming on the recorded output of the system allowed us to verify our calculated value of the minimum voltage difference V_s, which was calculated to be 3.05 x 10^{-4} V.

1 Introduction

In the landscape of modern society, ubiquitous technology creates the need for accurate, representative digital signals. However, in the natural world signal controlling and responsive signals are purely analog. There are multiple problems that can arise in the conversion between this analog and digital world. The two problems that we explored in this lab are aliasing and quantization error. Each of these data subjects are described in greater depth below.

Data Acquisition

Data acquisition is the process of recording information about a signal so it can be stored or further analyzed at a later time. A signal, and the related way it is measured, can fall into one of two classifications, detailed below [1]:

1. Digital (Binary): the signal can fall as a 0 (low) or 1 (high). These signals are generally computer-generated.
2. Analog: continuous variation of a signal within some bounds over time. These signals cannot be created or interpreted by a computer.

A visual representation of these classifications is given below [2]:

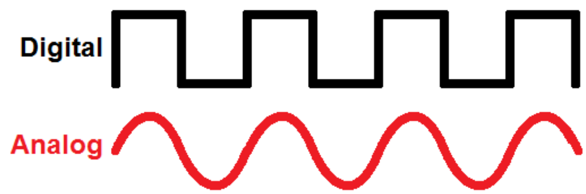

Figure 1: Digital signal (top) vs Analog signal (bottom)

As described above, when acquiring data from a natural system/signal the conversion to digital needs to be made for a computer to save and analyze this signal.

Aliasing

In order to convert an analog signal to a digital one, the analog signal's value has to be sampled at discrete time intervals. Aliasing is an effect that causes continuous signals with a high frequency to appear as lower-frequency signals when they are sampled with an insufficient sample rate [1]. This phenomenon is probably best explained through an example.

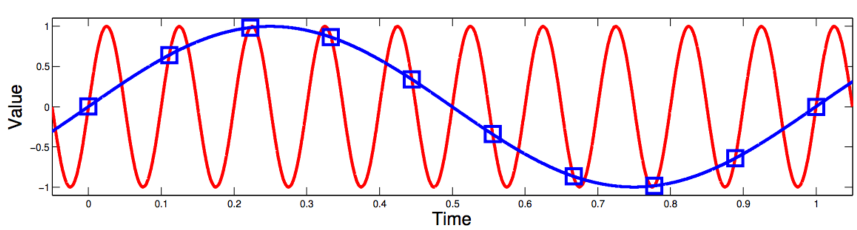

Figure 2: Example of aliasing

In Fig. 2 the higher frequency red signal is sampled at discrete time intervals indicated by the blue boxes. However, by simply taking the values at the blue boxes it appears as though the recorded signal is a lower frequency signal, indicated by the sinusoidal wave in blue. In other words, information is missed at the chosen sampling rate. This is clearly seen by studying the behavior between the first and second sample. The red signal rises then falls then rises again during this time frame. However, the second sample only perceives a rise from the first sample. Thus, this crucial information of oscillatory behavior is lost.

Aliasing can be difficult to detect. It occurs anytime the sampling rate is lower than the Nyquist frequency. The Nyquist frequency is half the true frequency of the signal. In other words, aliasing occurs anytime the sampling rate is less than half the true frequency of the signal. However, an engineer might forget this fact and aliasing would go undetected if precautions are not taken. In other words, aliasing is not a *loud* error. It does not cause the system to appear broken. One way to combat aliasing is to place a low pass filter. By doing such, any low frequency signal that has a lower magnitude once filtered would indicate its actually a high frequency signal that has been filtered but appears at lower frequency because of aliasing.

Mathematical insight to aliasing

A more mathematical explanation of aliasing, courtesy of the Lab 2 manual [1] is given below.

For the sake of example let's imagine there are two sine waves given by:

$$\begin{aligned} v_1(t) &= cos(2\pi f_1 t) \\ v_2(t) &= cos(2\pi f_2 t) \end{aligned}$$

If these two signals are sampled at a sampling rate f_s then the digital their digital representation is given by:

$$\begin{aligned} v_1(n) &= cos(\tfrac{2\pi n f_1 t}{f_s}) \\ v_2(n) &= cos(\tfrac{2\pi n f_2 t}{f_s}) \end{aligned}$$

...where n indicates the discrete time interval of a given sample, starting at 0 and proceeding until infinity (or the end of sampling).

It can be observed that these two discrete signals are equivalent if the following condition holds:

$$f_2 = f_1 + m * f_s$$

for values of $m = 1, 2, 3....$ The verification of this is further shown below. This equality is a direct consequence of the 2π repeating behavior of *cosine*.

$$v_2(n) \quad = \quad cos(\frac{2\pi n(f1+m*f_s)t}{f_s}) \quad = \quad cos(\frac{2\pi n f_1 t}{f_s} + 2\pi nm) \quad = \quad v_1(n)$$

Through the above contrived example it has been shown there are conditions in which aliasing can occur.

Quantization Error

In general computers or hardware components have no way of representation infinitesimally small changes. Rather, hardware components have a certain resolution that defines the smallest difference they can detect. This inability to described continuous signals perfectly and to approximate siganls with steps is called *quantization error*. This error is best seen below in Fig. 3 [4].

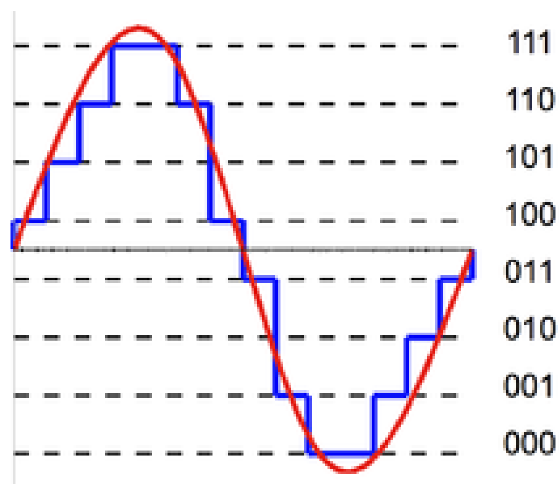

Figure 3: Quantization of an analog signal

In Fig. 3 the red signal is the analog signal. The blue signal is the sampled digital representation. Along the right side of the figure is the binary representation of each discrete voltage level that the hardware is capable of representation. In this example the hardware has 3-bits of resolution. Thus, it can represent 8 or 2^3 different voltage levels. More generally, hardware with a resolution of m will be able to represent 2^m voltage levels.

The minimum discrete voltage level that a given hardware setup can detect is given by the voltage range of hardware divided by the amount of voltage representations possible (2^m). Usually, the resolution of the hardware is large enough with respect to the input signal that quanitzation error is not an issue. In this case the digital signal will still look smooth, despite its digital representation.

Quantization error becomes an issue if the voltage range of the input signal becomes small or the resolution of the hardware becomes small. In these cases the step behavior of the signal might become more noticeable, as the hardware does not have enough resolution for the signal. In other words, the smaller the input voltage range of the signal, the larger the resolution of the hardware using the record it is needed (or hardware with a smaller voltage range).

2 Materials and Methods

Conducting the experiment involved using the following programs:

1. MATLAB
2. Controldesk 3.7

The experiment also required the following pieces of hardware:

1. National Instruments and dSPACE BNC breakout boxes
2. D74L4B low pass filter
3. Powered breadboard
4. BNC connector cables

MATLAB and Controldesk version 3.7 were used to measure and record the data needed to record aliasing as well as the receiver quantization errors.

With the circuit setup on the breadboard, data is then collected using MATLAB and Controldesk, first to study aliasing, then to study quantization.

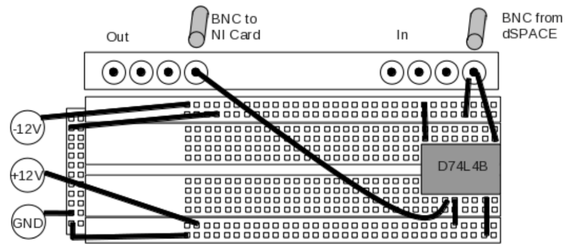

Figure 4: Setup of BNC cables and filter circuit [1]

2.1 Aliasing

We first collected aliasing data using two input sinusoidal functions, the frequencies of which are detailed in Table 1, below.

f_1	2 kHz
f_2	15 kHz

Table 1: Input frequencies f_1 and f_2, signal generator output sample rate of 40 kHz.

In this section we recorded the raw signals (the two sinusoidal functions produced from the function generator) and the output signal from our system. In this case our system was a low pass filter.

We created three different figures from these recorded signals:

1. Time domain plot of the original and filtered signals (overlapped).
2. Power spectrum plot of the original and filtered signals (overlapped).
3. Power spectrum plot of the original and filtered signals on side by side graphs.

2.2 Receiver Quantization Errors

The specific DAQ card equipment is detailed in Table 2 below.

The input to the signal generator was set to five times the minimum voltage difference (V_s). This minimum voltage difference was a signal at a magnification that exposed the discrete jumps between data points. Additionally, a low frequency was chosen. These values are documented below.

DAQ card	NI PCI-6036E
Sampling Rate	20 kHz
Voltage Range E_{FSR}	20 V (+10 V to -10V)
Resolution (bits)	16 bits [3]
Resolution (volts per step)	3.05 x 10^{-4} V per step
Minimum voltage difference V_s	3.05 x 10^{-4} V

Table 2: DAQ card characteristics.

Amplitude	1.53 x 10^{-3} V
Frequency	50 Hz

Table 3: Input signal characteristics, signal generator output sample rate of 40 kHz.

Like the Aliasing section, in this section we recorded the raw signal (a single sinusoidal function produced from the function generator) and the output signal from our system. To reiterate, our system was a low pass filter.

Again, we created three different figures from these recorded signals:

1. Time domain plot of the original and filtered signals (overlapped).
2. Power spectrum plot of the original and filtered signals (overlapped).
3. Power spectrum plot of the original and filtered signals on side by side graphs.

3 Results and Discussion

3.1 Aliasing

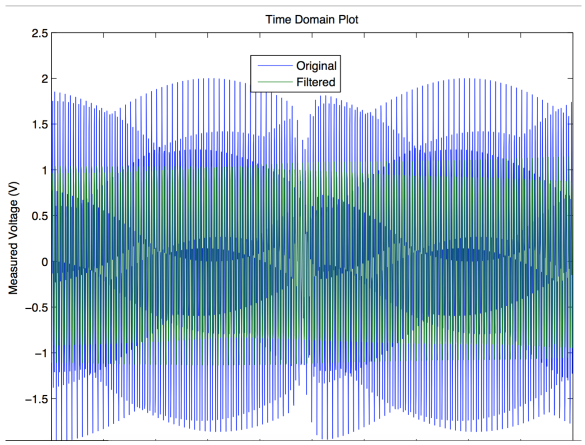

Figure 5: Time series for raw and filtered signal

Fig. 5 above gives an overlapped representation of the time series from the original and filtered signals. It is almost impossible to decipher any meaningful information from this figure.

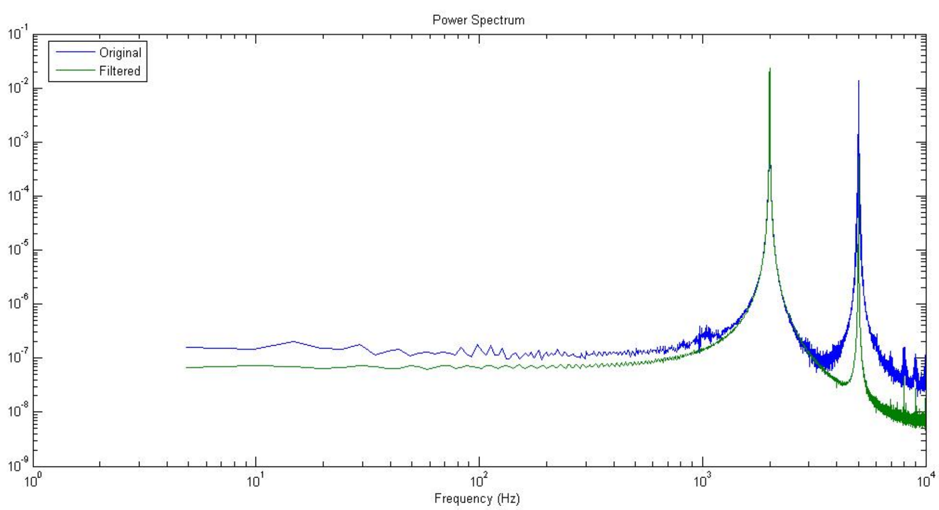

Figure 6: Power spectrum for the original and filtered signals

In Fig. 6 above the original signal is given in blue while the filtered signal is given in green. This power spectrum representation is much more meaningful in the context of aliasing. However, in order to understand its significance a discussion of the Nyquist frequency is first necessary. The Nyquist frequency calculation is given below

Nyquist Frequency

The lab manual defines the Nyquist, f^* frequency as,

$$f_{\text{sample}} = 2f^* \tag{1}$$

In other words, the Nyquist frequency is half the sampling frequency of the system. In our case the DAQ used has a sampling rate of 20 Hz [1]. The Nyquist frequency is given as,

$$\begin{aligned} f^* &= \tfrac{f_{\text{sample}}}{2} \\ f^* &= \tfrac{20 \text{ kHz}}{2} \\ f^* &= 10 \text{ kHz} \end{aligned}$$

Nyquist Frequency = k10 Hz

A Nyquist frequency of 10 Hz means the DAQ card is incapable of differentiating signals of multiples of 10 kHz. In other words, to the DAQ board a signal at 30 kHz, 20 kHz, and 10 kHz are indistinguishable and are all *perceived* as 10 kHz signals.

This phenomenon is observed in the Fig. 6 power plot. The input signals were set at 2 kHz and 15 kHz. The peaks of a power plot correspond to the frequencies at which a greater (in this case non-zero) amplitude is measured. However, the peaks of the power plot are not at 2 kHz and 15 kHz as expected. Rather, the peaks are at 2 kHz and 5 kHz. In other words, the 15 kHz input signal was perceived by the DAQ card as a 5 kHz signal.

Given aliasing, it is quite easy to explain why the peak in the power plot is at 5 kHz. This is simply the input frequency (15 kHz) minus the Nyquist frequency (10 kHz). This makes sense as the DAQ reader only has the sampling rate to interpret signals under 10 kHz, thus appearing to shift all frequencies greater than this value by multiples of the Nyquist frequency until it is within the required range.

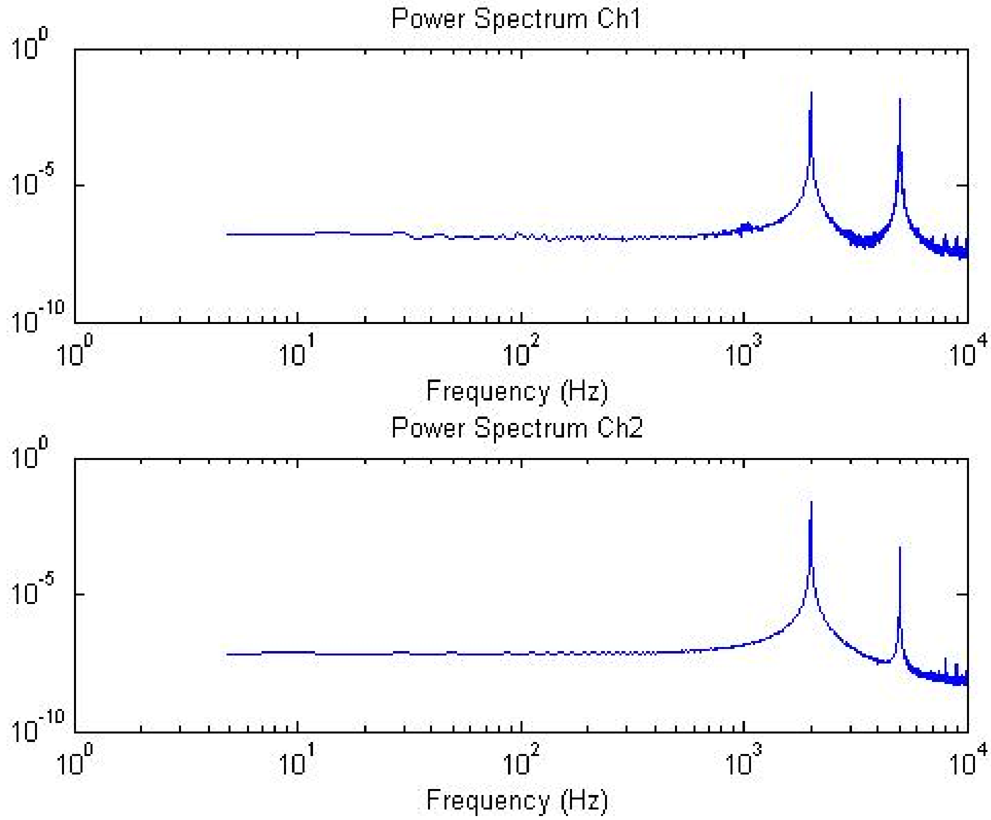

Figure 7: Power spectrum for the original (top) and filtered (bottom) signals

Fig. 7 above shows the power spectrum for the original and filtered signals one on top of the other. This is the same information as Fig. 6 displayed in a manner where the difference in magnitude of peaks can more clearly be seen. The 2 kHz signal appears the same on both plots. This means that the original signal is **not** filtered.

To the contrary the peak corresponding to the signal at 5 kHz is smaller on the filtered plot than the original signal plot. This means the original signal was filtered. However, the low pass filter is designed to allow signal under 10 kHz to pass through without less than unity gain. This means that although it appears the signal is at 5 kHz the signal is actually greater than 10 kHz and aliasing is present.

In this manner a way of identifying aliasing has been demonstrated. By using a low pass filter with a pass frequency equal to the Nyquist frequency of the system signal aquisition tool, the presence of aliasing can be detected by comparing the original and filter signals. If the original and filtered signals at a given frequency differ in magnitude then aliasing is present.

Of course in the above paragraph the behavior of a low pass filter has been idealized. In practice the low pass filter does not have one cut-off frequency. Thus, to be more precise the low-pass anti-aliazing filter should have the following specifications [1]:

$$\begin{aligned}\text{Filter Attenuation(dB)} &\equiv 20 \text{ x } \log_{10}\left(\frac{\mathrm{V_{Filtered}}}{\mathrm{V_{Unfiltered}}}\right) \quad \text{OR} \\ \text{Filter Attenuation(dB)} &\equiv 10 \text{ x } \log_{10}\left(\frac{\mathrm{P_{Filtered}}}{\mathrm{P_{Unfiltered}}}\right)\end{aligned}$$

Sources of Error

The setup for this aliasing experiment was relatively simple. Given the simplicity of the system there were little opportunities for sources of error to be introduced. It is likely that any sources of error could be attributed to the system itself, whether that be lose wires, a faulty low pass filter, or bad connection. It is also possible that the function generator introduced error.

From the figures given in this section it is also clear that there was a fair amount of noise. This noise can be seen best in the lack of smoothness in the Power Spectrum figure (Fig. 6). However, this noise is expected with any system and thus should be noted as noise, not error.

Extension

The higher frequency signal needs to be chosen less than 20 kHz because it is constrained by the sampling rate of the function generator. The function generator has a sampling rate of 40 kHz. Thus, it has a Nyquist frequency of 20 kHz. For the reasons observed and explained at great lengths above the output of the function generator needs to be at a frequency less than its Nyquist frequency. In other words, the function generator is unable to create a signal that is twice its sampling rate. If it attempts to do such it will only be able to sample at a high enough rate to which an outsider would observe a signal with frequency less than 20 kHz.

3.2 Receiver Quantization Errors

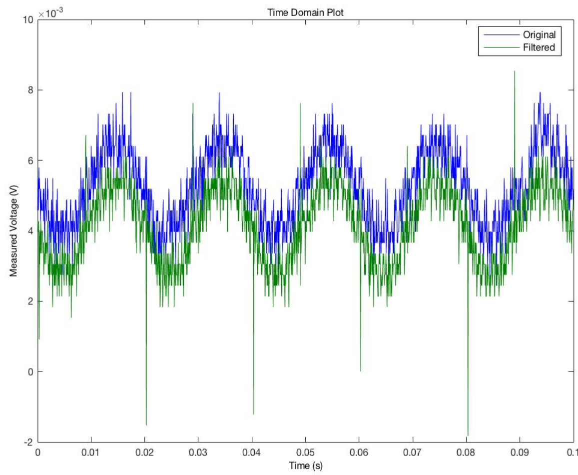

Figure 8: Time plot of the original and filtered signal

Fig. 8 shows the time plot for the original and filtered signal. It is important to observe the low amplitude of the signal used for this portion of the lab. However, this figure is not particular useful in explaining quantization error.

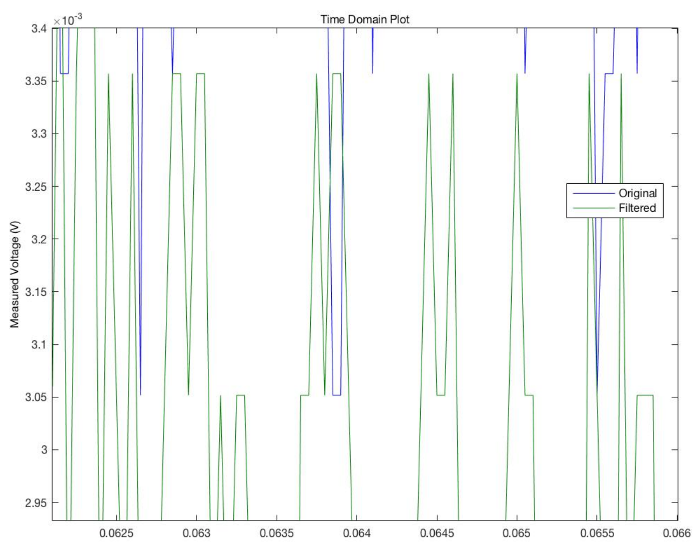

Figure 9: Time plot of the original and filtered signal zoomed to highlight quantization error

Fig. 9 is simply a zoomed in representation of Fig. 8. Although it is difficult to distinguish the noise from the quanitzation error, the step-like the behavior of the signal at small voltage levels can be seen. In Fig. 9 there are consistently repeated steps of approximate size 3 x 10^{-4}. This is on the scale of the quantization error calculated below.

Discrete Voltage Calculation

The DAQ card is a binary tool. In other words it represents the analog signal with a finite amount of bits. Thus, the card has a discrete (finite) number of of voltage levels that corresponds to the amount of bits the card has. In our case the card has 16 bits in which to represent the signal. Therefore, the calculation for number of finite voltage levels can be made:

$$\begin{aligned} \text{Discrete voltage levels} &= 2^{\text{number of bits}} \\ &= 2^{16} \\ &= 65536 \end{aligned}$$

The minimum detectable voltage difference, V_s, is simply the voltage range divided by the number of voltage levels. The DAQ card used in this lab has a voltage range of +10 V to -10 V [1].

$$\begin{aligned} V_s &= \tfrac{\text{Voltage range}}{\text{number of voltage levels}} \\ &= \tfrac{10-(-10)\ \text{V}}{65536} \\ &= \tfrac{20}{65536} \\ &= 3.05 \text{ x } 10^{-4}\ \text{V} \end{aligned}$$

Comparison

The smallest observable voltage discerned from observing the behavior in Fig. 9 closely matched the value determined analytically.

4 Conclusion

This experiment revealed two errors that can occur in data acquisition: aliasing and quantization. It was discovered that aliasing occurs for input signals with frequencies greater than the Nyquist frequency, which is half the sampling frequency of the data collection tool. Thus, engineers should ensure the largest frequency of the input is less than half the sampling frequency of the data collection tool. However, sometimes the input signal is not necessarily known. In this case aliasing must be detected. In this experiment it was shown that a low pass filter can be used to detect aliasing.

Quantization is a result of having to convert an analog signal to a complex world. Detecting quantization error does not make sense in the same context as detecting aliasing. Quantization error is always present, unlike aliasing. However, quantization error is often small and insignificant. Thus, engineers simply need to be aware that quantization exists and chose data acquisition tools with high enough bit resolution with respect to the signal that is being measured. In other words if an engineer wants to be able to measure a smaller voltage difference a higher bit resolution is needed. Thus, when it comes to quantization the engineer needs to choose a bit resolution that meets the specification of the degree of precision desired.

References

[1] *Laboratory 2: Data Acquisition, Aliasing and Quantization* lab manual.

[2] Spark Fun *https://learn.sparkfun.com/tutorials/logicblocks–digital-logic-introduction.*

[3] National Instrument PCI-60386E card website.
Hyperlink: http://sine.ni.com/nips/cds/view/p/lang/en/nid/11913

[4] Quantization Error Wiki.