CS330 Test 1 Regrade Request

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Problem 4a

I was awarded 2 points for this question. I spoke with Dr. Rong Ge after class and he said my work warranted more than 2 points. A picture of *Problem 4a* from my test is given at the end of this document and a description of our discussion and my approach is given below:

In the first three lines of my work I tried to give the probability that the first, second, and third ball land in different bins from each other. However, I became stuck using this approach. Therefore, I tried to pivot and calculate the probability that the balls land in the same bin, which corresponds to the next three lines of my work.

I calculated the probability that the first ball tossed lands in the same bin as another ball. This is clearly 0 since there are no balls to collide with. Then I calculate the probability the second ball does not collide with the first, which is $\frac{1}{n^2}$ since there is one spot occupied (by the first ball) and $m = n^2$ total spaces. Next I calculate the probability that the third ball is in the same bin as one of the first two balls given the first two balls don't collide. This is $\frac{2}{n^2}$ since there are two bins occupied (by the first and second ball) and still $m = n^2$ total spaces.

The probability of a collision by the first i balls can be bounded by the probability that the second ball collides with the first plus the probability the third ball collides with the first or the second given the first and second don't collide...and so forth. Therefore the probability of a collision for all n balls ($i = n$) is bounded by the summation:

$$\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} ... + \frac{n-1}{n^2} = \sum_{i=1}^{\infty} \frac{n-i}{n^2}$$

which is noted on my paper. It also follows, and I meant to show, $\sum_{i=1}^{\infty} \frac{n-i}{n^2}$ is bounded by $1/2$ since it equals $\frac{n-1}{2n}$ which is less than or equal to $1/2$ for all n. On my test I worked towards this, but dropped the square term while rushing.

Finally, I say that the probability the balls all fall in a different bins is 1 minus the probability of a collision. Since the probability of a collision is less than 1/2 then the probability the balls all fall in a different bin is as least 1/2, as desired.

I am not suggesting I should receive full credit for this problem. I understand that I filled in some of the gaps in my work in the description above, which is much easier to do in a Latex file after the test. I also realize that the intended approach leverages Markov's inequality (note the top right part of my paper), but struggled with this approach. However, I do believe I had the right idea and that a large percentage of the work needed for the proof is provided.

$Pr[X \geq 2\mathbb{E}[X]] \leq \frac{1}{2}$

2 balls → 1
3 balls → 3
4 balls →

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Problem 4 (Balls and Bins). In this problem we consider throwing n balls to m bins. Each ball will land randomly into one of the bins with equal probability. Different balls are completely independent, and they can land into the same bin. After throwing all n balls, we would like to check how many balls are in each bin.

(a) (10 points) Prove that, if $m = n^2$, then the probability that all balls are in different bins is at least 1/2.

(Hint: Let $X_{i,j}$ be 1 if balls i, j are in the same bin. Try to compute the expected number of "collisions" (two balls in the same bin) using $X_{i,j}$'s.)

The probability the first ball lands in a different bin = 1 ← $\frac{n^2}{n^2}$

The probability the second ball lands in a different bin = $\frac{n^2-1}{n^2}$ ← free bins

2 taken bins

" third " " = $\frac{n^2-2}{n^2}$

The probability first ball lands in same bin = 0

second ball in same bin as 1 = $1/n^2$

third ball in same bin as 2 = $2/n^2$

Probability of collision = $\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} \cdots \frac{n-1}{n^2} < \sum_{i=1}^{n} \frac{n-i}{n} \leq 1/2$

P[different bin] = 1 − probability of collision

$\geq 1 - 1/2$

$\geq 1/2$ ∎

Sorry this is messy

Ran out of time

$\frac{1}{n}\left[\sum_{i=1}^{n} 1 - \sum_{i=1}^{n} i\right]$

$n - \frac{n(n+1)}{2} =$

$= \frac{2n - n^2 - n}{2n} = \boxed{\frac{1}{2}}$

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