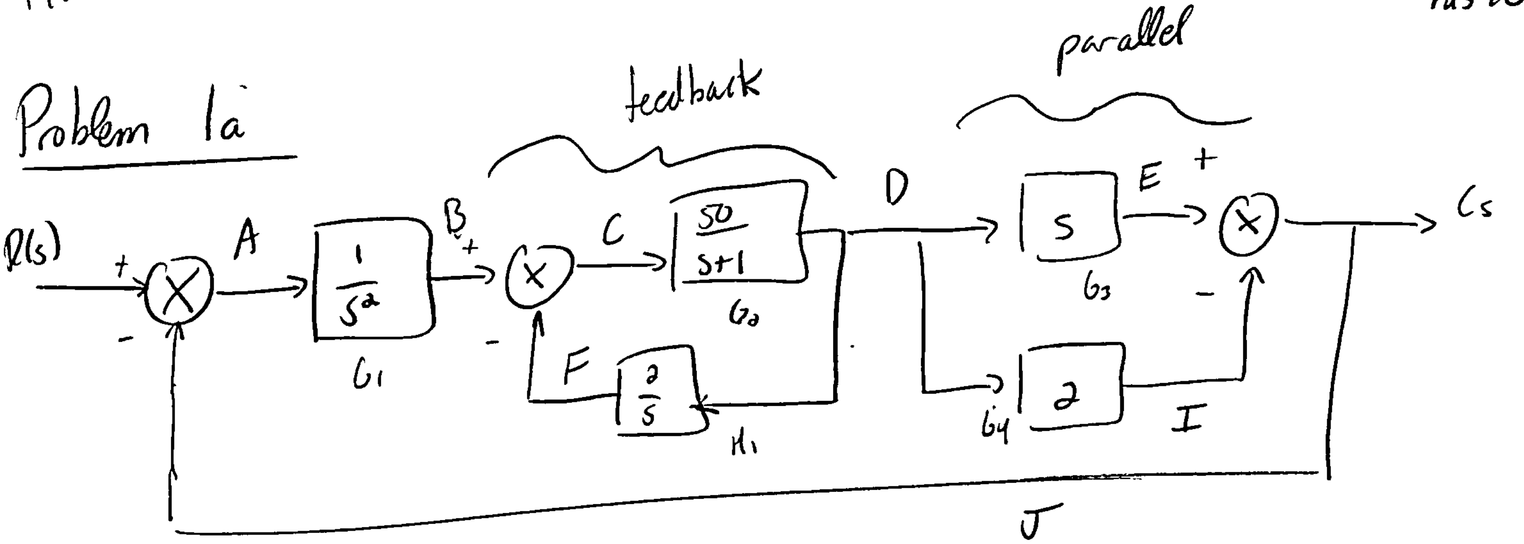
Hw #4

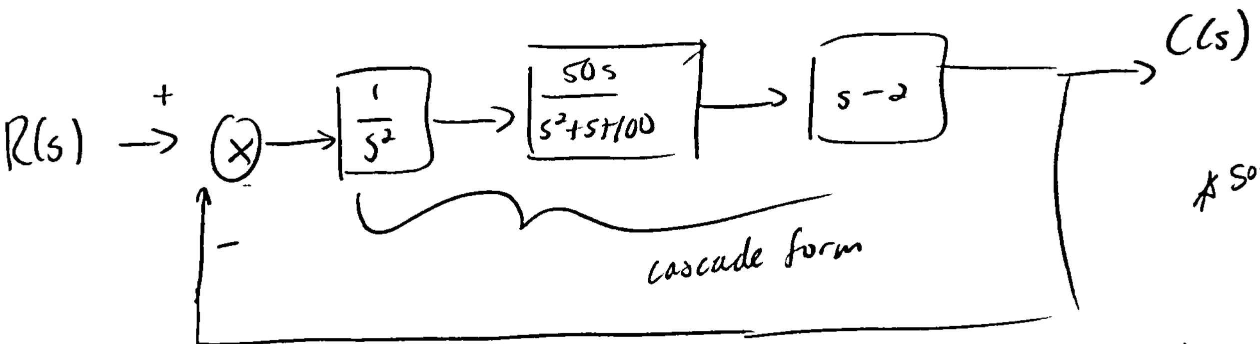
ms 20

Problem 1a



$$F_b(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{50}{s+1}}{1+\frac{2}{s}\frac{50}{s+1}} = \frac{50s}{(s+1)s+100} = \frac{50s}{s^2+s+100}$$

$$P(s) = G_1 + G_2 = s-2$$



* Sorry for being a bit messy

$$G(s) = \frac{1}{s^2}\left(\frac{50s}{s^2+s+100}\right)(s-2) = \frac{50(s-2)}{s^3+s^2+100s} = \frac{50(s-2)}{(s^2+s+100)s}$$

Final feedback

$$H(s) = 1$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{50(s-2)}{s(s^2+s+100)}}{1+\frac{50(s-2)}{s(s^2+s+100)}} = \frac{50(s-2)}{s(s^2+s+100)+50(s-2)}$$

$$= \frac{50(s-2)}{s^3+s^2+100s+50s-100} = \frac{50(s-2)}{s^3+s^2+150s-100} = \frac{50(s-2)}{s(s^2+s+150)-100}$$

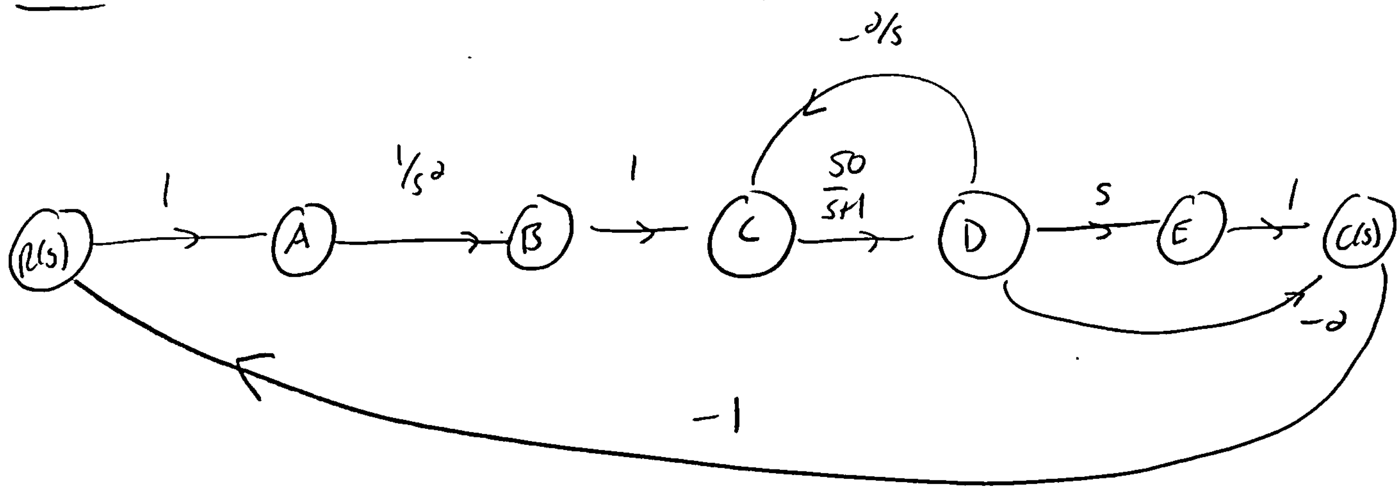
$$\boxed{\frac{C(s)}{R(s)} = \frac{50(s-2)}{s(s^2+s+150)-100}}$$

Problem 1B

Signals -> nodes

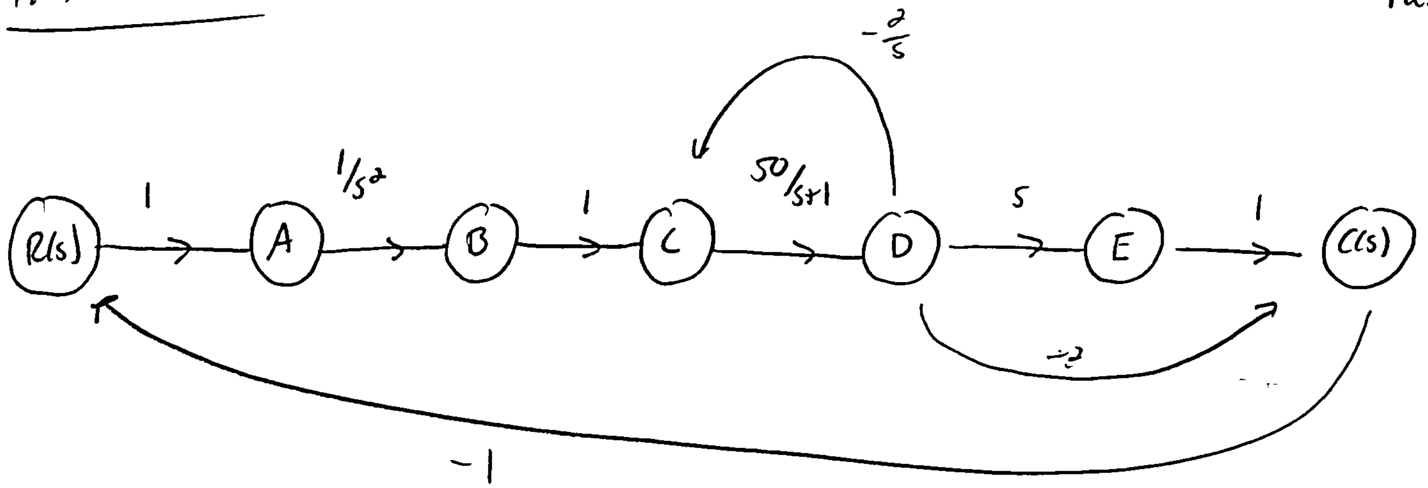
Blocks -> paths

Please reference Problem 1A for labeling of signals



Problem 1C

rks20



$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_K T_K \Delta_K}{\Delta}$$

2 forward loops

l_1 : $R(s) \to A \to B \to C \to D \to E \to C(s)$

l_2 : $R(s) \to A \to B \to C \to D \to C(s)$

Gain :

$$1/s^2 \left(\frac{50}{s+1}\right)(s) = \frac{50}{s(s+1)}$$

$$1/s^2 \left(\frac{50}{s+1}\right)(-2) = \frac{-100}{s^2(s+1)}$$

$\Delta = 1 - \sum$ loop gains $+ \sum NT\, l_s\, 2x$ - . . . . -

Loop Gains

$C \to D \to C$: $\frac{50}{s+1}\left(-\frac{2}{s}\right) = \frac{-100}{s(s+1)}$

$A \to D \to C(s) \to R(s) \to A$

Gain : $\frac{1}{s^2}\left(\frac{50}{s+1}\right)(-2)(-1)$

$$= \frac{100}{s^2(s+1)}$$

$A \to B \to C \to D \to E \to C(s) \to R(s) \to A = \frac{-50}{s(s+1)}$

No non-touching loop gains

$$\Delta = 1 - \left(\frac{-100}{s(s+1)} - \frac{50}{s(s+1)} + \frac{100}{s^2(s+1)}\right) = 1 - \left(\frac{100 - 150s}{s^2(s+1)}\right) \longrightarrow$$

Problem 1C cont

MS 20

$$\Delta = \frac{s^2(s+1) - 100 + 150s}{s^2(s+1)} = \frac{s^3 + s^2 + 150s - 100}{s^2(s+1)}$$

$k=1$

$$T_1 = \frac{50}{s(s+1)} \qquad \Delta_1 = 1$$

$k=2$

$$T_2 = \frac{-100}{s^2(s+1)} \qquad \Delta_2 = 1$$

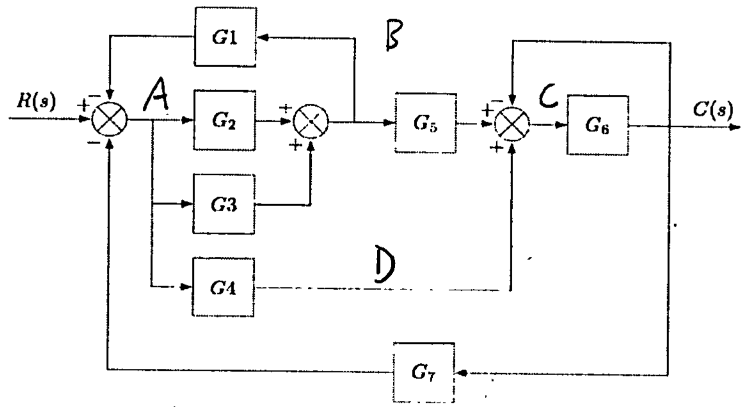
$$G(s) = \frac{T_1 + T_2}{\Delta} = \frac{\frac{50s - 100}{s^2(s+1)}}{\Delta} = \frac{50(s-2)}{s^2(s+1)} \cdot \frac{s^2(s+1)}{s(s^2+s+150)-100}$$

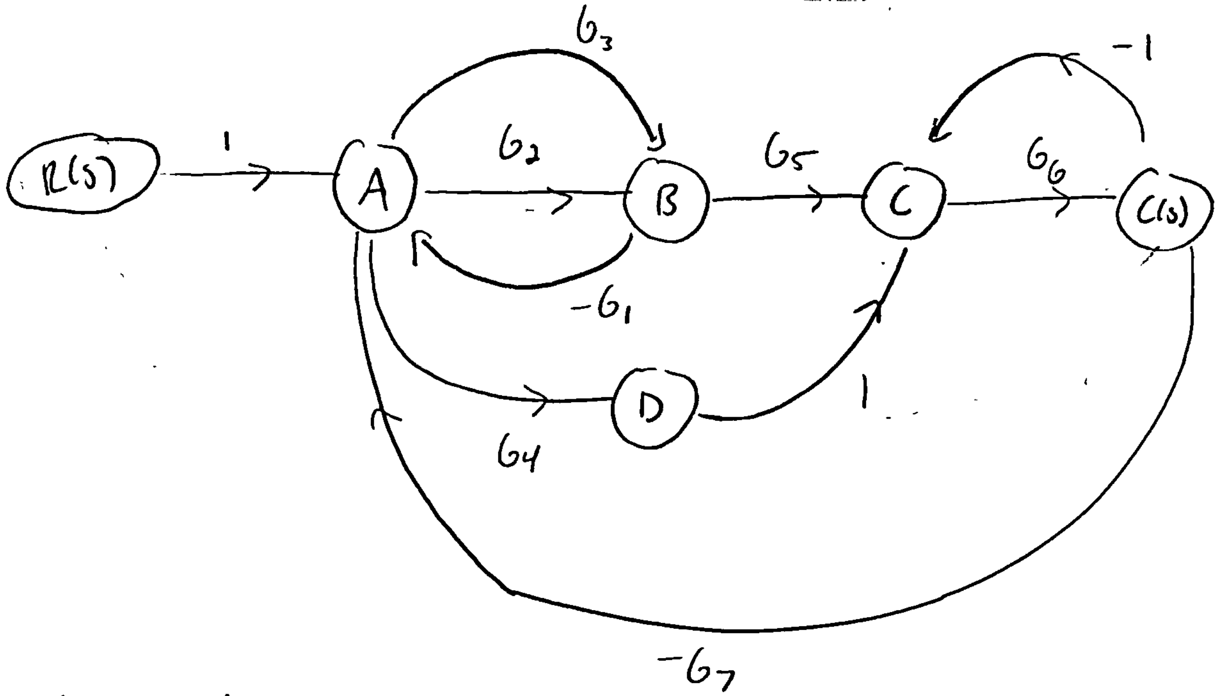
$$G(s) = \frac{50(s-2)}{s(s^2+s+150)-100}$$

← same as problem 1A

Problem 2

pts 20





3 forward loops

	gain
$R(s) \to A \to B_1 \to C \to C(s)$	$G_2 G_5 G_6$
$R(s) \to A \to B_2 \to C \to C(s)$	$G_3 G_5 G_6$
$R(s) \to A \to D \to C \to C(s)$	$G_4 G_6$

Loop Gains

Jcont ms20

l_1 : $-G_1 G_3$

l_2 : $-G_1 G_2$

l_3 : $-G_6$

l_4 : $-G_6 G_7 G_4$

l_5 : $-G_7 G_2 G_5 G_6$

l_6 : $-G_7 G_3 G_5 G_6$

Touch l_1, l_3

l_3 does not touch l_1 or l_2

NT LG x2 : $G_6 G_1 G_3 + G_1 G_2 G_6$

$$\Delta = 1 - \sum_{i=1}^{6} l_i + (G_6 G_1 G_3 + G_1 G_2 G_6)$$

<u>k=1</u>

$T_1 = G_2 G_5 G_6$ $\Delta_1 = 1$ (touches all loop paths)

<u>k=2</u>

$T_2 = G_3 G_5 G_6$ $\Delta_2 = 1$ (touches all loop paths)

<u>k=3</u>

$T_3 = G_4 G_6$ $\Delta_3 = 1$ (touches all lopp paths)

→

Problem 2 cont

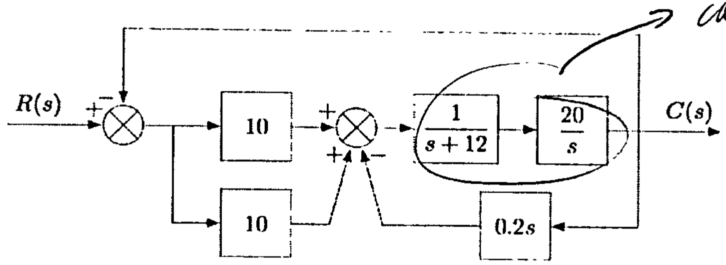
$$G(s) = \frac{\sum_K T_K \Delta_K}{\Delta}$$ → Maple script

$$G(s) = \frac{G_6\left((G_2+G_3)G_5+G_4\right)}{\left((G_2+G_3)G_1+G_2G_5G_7+G_3G_5G_7+G_4G_7+1\right)G_6+1+(G_2+G_3)G_1}$$

ns70

3. (Adapted from Nise P5.17) For the following block diagram:

cascade

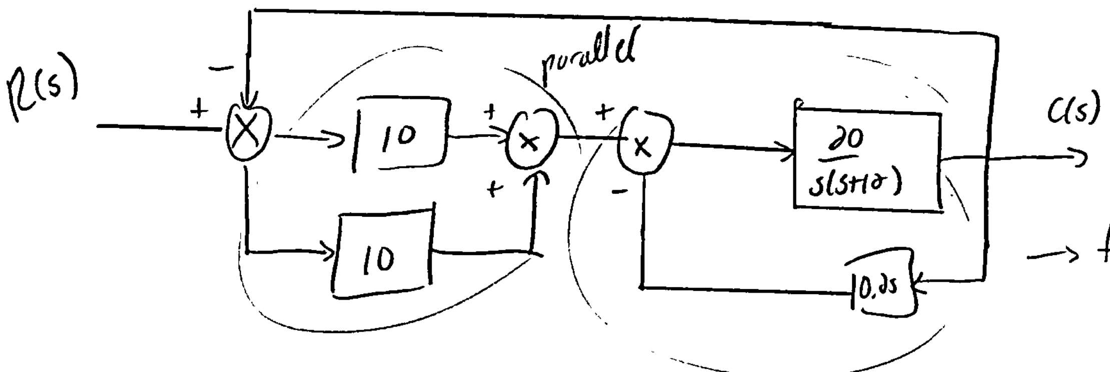


a) Reduce the block diagram to an equivalent transfer function.

b) Calculate the natural frequency, damping ratio, percent overshoot, settling time, peak time, rise time, and damped natural frequency. Check your work with the sysChar function you wrote for Homework 3.

a) Cascade :

$$\frac{1}{s+12}\left(\frac{20}{s}\right) = \frac{20}{s(s+12)}$$



→ feedback

$$G_1(s) = \frac{20}{s(s+12)}$$

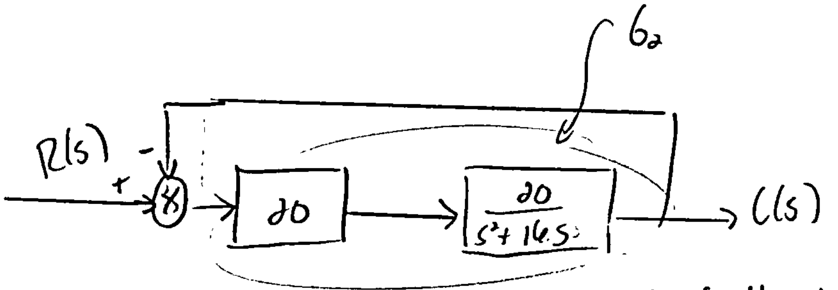
$$H_1(s) = 0.2s$$

$$G_2(s) = \frac{G_1(s)}{1 + H_1(s)G_1(s)} = \frac{\frac{20}{s(s+12)}}{1 + 0.2s\left(\frac{20}{(s+12)s}\right)} = \frac{20}{s(s+12) + 0.2s(20)}$$

$$= \frac{20}{s^2 + 12s + 4s}$$

$$= \frac{20}{s^2 + 16s}$$

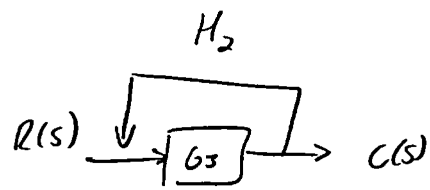
G_2



↳ cascade & feedback

→

Problem 3a cont



$$G_3 = 20\left(\frac{20}{s^2+16s}\right) = \frac{400}{s^2+16s}$$

$H_2 = 1$

$$G_{eq} = \frac{1}{1+G_3H_2} = \frac{\frac{400}{s^2+16s}}{1+\frac{400}{s^2+16s}} = \frac{400}{s^2+16s+400}$$

$$\boxed{G(s) = \frac{400}{s^2+16s+400}}$$

Problem 3b

rus20

$$G(s) = \frac{400}{s^2+16s+400}$$

$W_n = \sqrt{400} = 20$ rad/se

$\boxed{W_n = 20 \text{ rad/sec}}$

$16 = 2\zeta w_n$

$\zeta = \frac{16}{2w_n} = \frac{16}{40} = \frac{2}{5}$ $\boxed{\zeta = \frac{2}{5}}$

$T_s = \frac{4}{\zeta w_n} = \frac{4}{2/5(20)} = \frac{4}{8} = \frac{1}{2}$ $\boxed{T_s = \frac{1}{2} \text{ second}}$

$T_p = \frac{\pi}{w_n\sqrt{1-\zeta^2}} = \frac{\pi}{20\sqrt{1-\frac{4}{25}}} = 0.171$ $\boxed{T_p = 0.171 \text{ s}}$

% overshoot $= e^{-(\zeta\pi / \sqrt{1-\zeta^2})} \cdot 100 = e^{-(2/5\pi/\sqrt{1-\frac{4}{25}})}$

$\boxed{\% \text{ O.S} = 25.39 \%}$

Rise time

Using polynomial approx. given on page 177 of text

$w_n T_r = 1.76\zeta^3 - .417\zeta^2 + 1.039\zeta + 1$

$w_n T_r = 1.46$

$T_r = \frac{1.46}{w_n} = \frac{1.46}{20}$

$\boxed{T_r = 0.0731 \text{ s}}$

Damped frequency $=$ $W_d = w_n\sqrt{1-\zeta^2}$

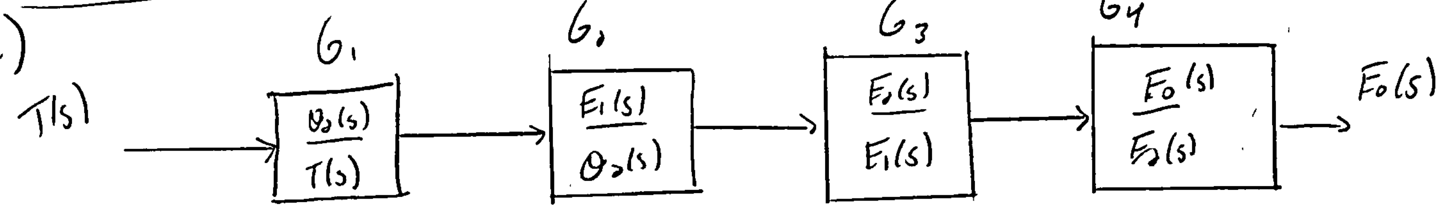
$= 20\sqrt{1-4/25}$

$\boxed{W_d = 4\sqrt{21} \text{ rad/sec}}$

Check diary on Sakai for checking this work w/ sysChar

Problem #4 mas 20

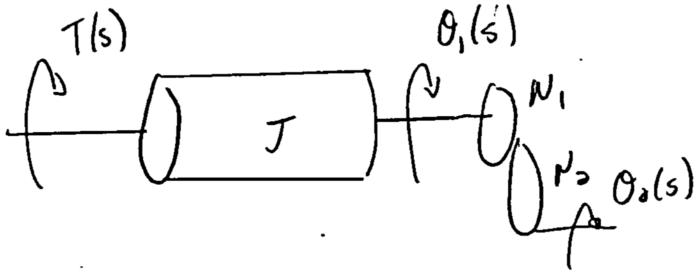
a)



Cascade

$$\frac{E_0}{T(s)} = G_1 G_2 G_3 G_4$$

①



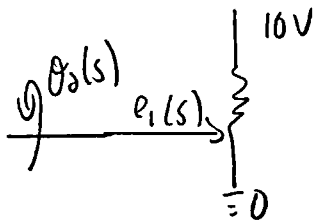
1 degree of freedom

$$Js^2\theta_1(s) = T(s) \qquad \frac{\theta_1}{\theta_2} = \frac{N_2}{N_1} \Rightarrow \theta_1 = \frac{N_2}{N_1}\theta_2$$

$$Js^2 \frac{N_2}{N_1}\theta_2 = T(s)$$

$$\frac{\theta_2}{T(s)} = \frac{N_1}{Js^2 N_2} = G_1$$

②



$\theta_2 = 0 \to 10V$ or vice versa, doesn't really matter

$\theta_2 = 2\pi \to 0V$

$$\text{gain} = \frac{\text{Voltage difference}}{\text{turn}} = \frac{10\ V}{2\pi} = \frac{5}{\pi}\ V/\text{rad}$$

$$G_2 = \frac{5}{\pi}\ V/\text{rad}$$

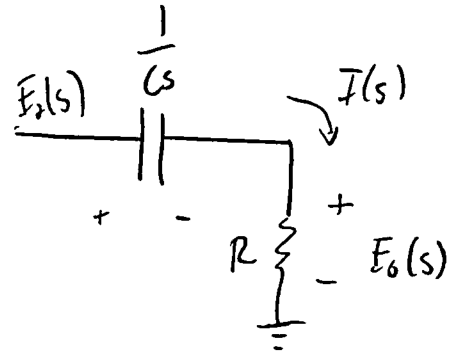
③ Buffer amplifier of gain 1 (unity)

$$G_3 = 1$$

$\longrightarrow$

20

(4)



Voltage division:

$$E_o = E_2\left(\frac{R}{\frac{1}{Cs}+R}\right) \Rightarrow \frac{E_o}{E_2} = \frac{RCs}{1+RCs}$$

✓

Current through capacitor = current through resistor

$V = IR$

$$I_c(s) = \frac{E_2(s) - E_o(s)}{1/Cs} = Cs\,(E_2(s) - E_o(s))$$

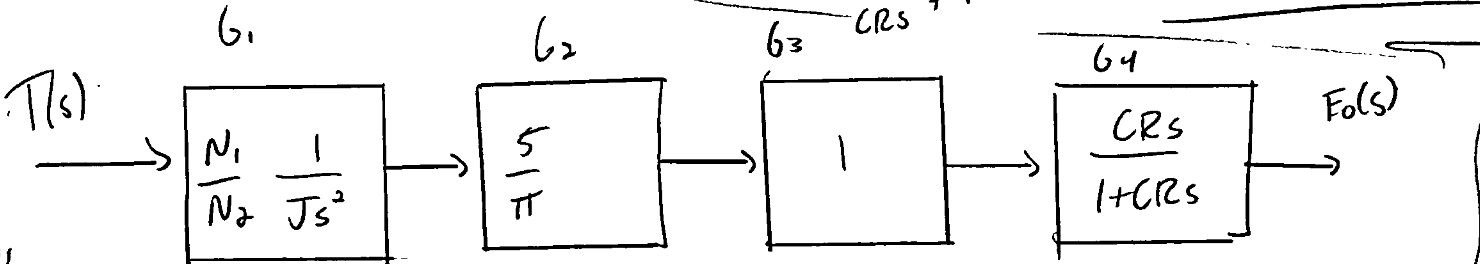
$$I_R(s) = \frac{E_o(s)}{R}$$

$$I_c(s) = I_R(s)$$

$$E_2(s) - E_o(s) = \frac{E_o(s)}{CRs}$$

$$E_2(s) = E_o(s)\left(\frac{1}{CRs} + 1\right)$$

$$\frac{E_o}{E_2} = \frac{1}{\frac{1}{CRs}+1} = \frac{CRs}{1+CRs} = G_3$$



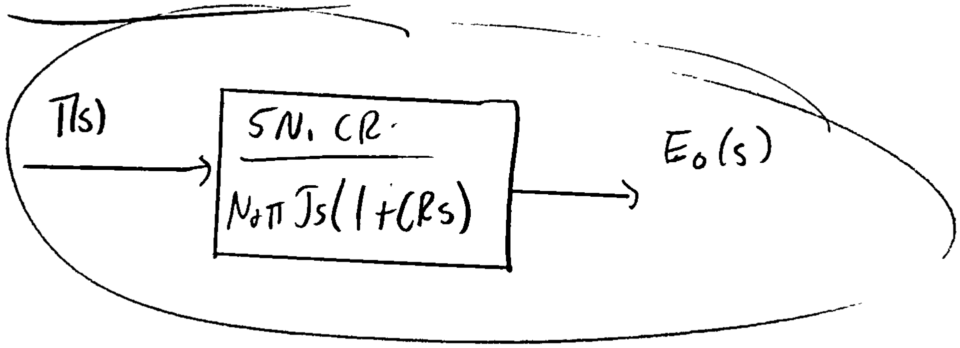
b)

$$\frac{E_o(s)}{T(s)} = G_1 G_2 G_3 G_4 \quad \text{(cascade)}$$

$$= \frac{N_1}{N_2}\frac{1}{Js^2}\cdot\frac{5}{\pi}\cdot 1\cdot\frac{CRs}{1+CRs} = \frac{5N_1CR\cancel{s}}{N_2\pi Js^{\cancel{2}}(1+CRs)}$$

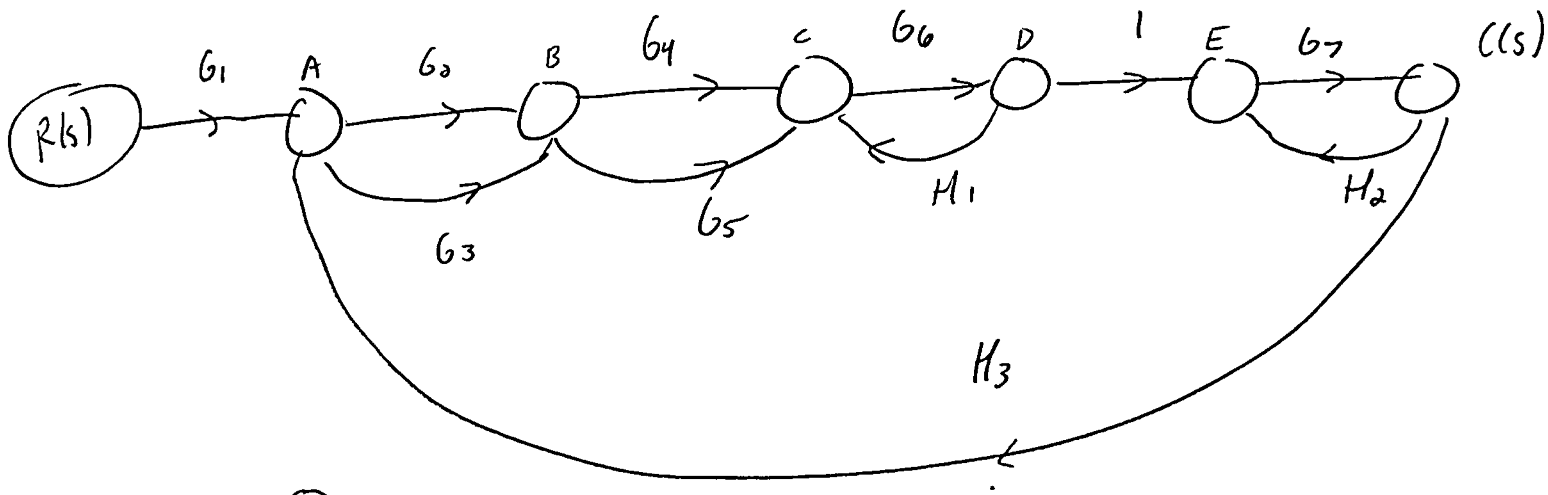
Problem 4b cont

ns 26



Problem #5

ms20



Forward Paths (4)

	gain
$R(s) \to A \to B \to C \to D \to E \to C(s)$	$G_1 G_2 G_4 G_6 G_7$
" "	$G_1 G_3 G_4 G_6 G_7$
" "	$G_1 G_2 G_5 G_6 G_7$
" "	$G_1 G_3 G_5 G_6 G_7$

Closed loop gains

l_1 $E \to C(s) \to E$ $\quad G_7 H_2$

l_2 $C \to D \to C$ $\quad G_6 H_1$

$A \to B \to C \to D \to E \to C(s) \to A$

↳ can do this 4 ways

1. $G_2 G_4 G_6 G_7 H_3$
2. $G_2 G_5 G_6 G_7 H_3$
3. $G_3 G_4 G_6 G_7 H_3$
4. $G_3 G_5 G_6 G_7 H_3$

$\Longrightarrow$

Problem 5 cont

ns20

Only l_1 & l_2 don't touch

N.T L.G x2 = $G_7 H_2 G_6 H_1$

$$\Delta = 1 - \sum_{i=1}^{6} \text{loop gain} + \sum \text{N.T LG x2}$$

$$= 1 - (G_7H_2 + G_6H_1 + G_6G_7H_3(G_2G_4 + G_2G_5 + G_3G_4 + G_3G_5)) - G_7H_2G_6H_1$$

$$T(s) = \frac{\sum_K T_K \Delta_K}{\Delta}$$

$\Delta_K = 1$ for all K

Touches all loops

↳ all forward paths touch all loops

K=1

$T_1 = G_1G_2G_4G_6G_7$ $\Delta_1 = 1$

K=2

$T_2 = G_1G_3G_4G_6G_7$

K=3

$T_3 = G_1G_2G_5G_6G_7$

K=4

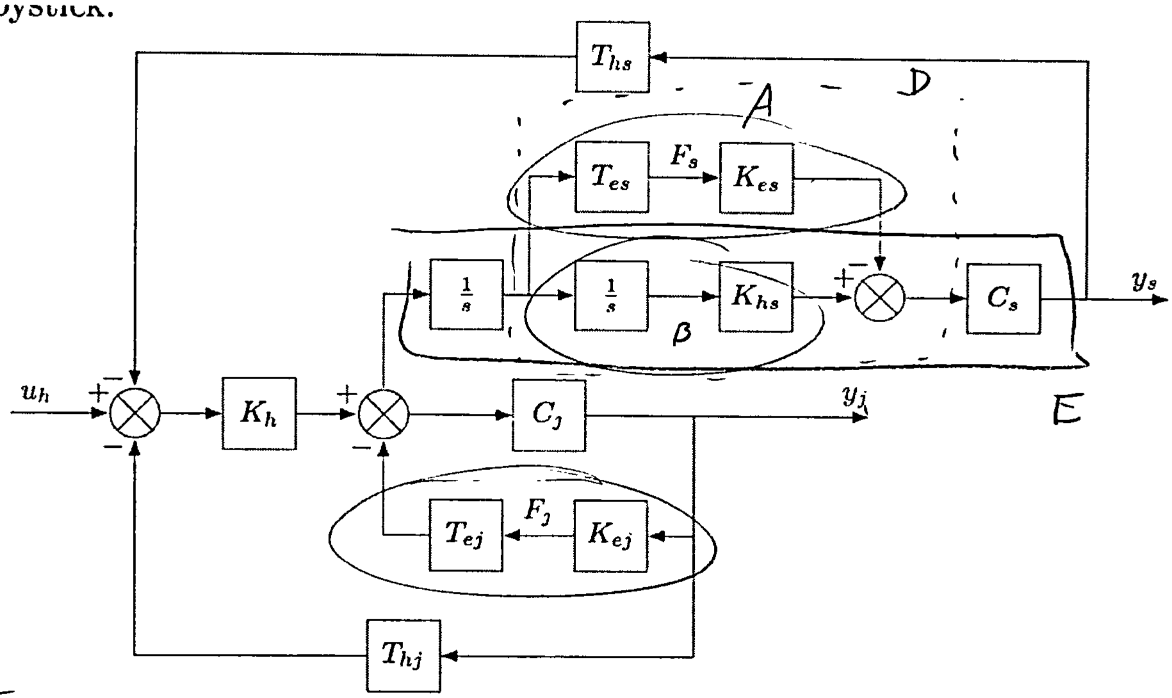
$T_4 = G_1G_3G_5G_6G_7$

$$T(s) = \frac{G_1G_6G_7(G_2G_4 + G_3G_4 + G_2G_5 + G_3G_5)}{1 - (G_7H_2 + G_6H_1 + G_6G_7H_3(G_2G_4 + G_2G_5 + G_3G_4 + G_3G_5)) - G_7H_2G_6H_1}$$

Problem 6

ns20

of the joystick.



Simplify Block

A : cascade $\rightarrow$ $T_{es} K_{es}$

B : cascade $\rightarrow$ $\frac{K_{hs}}{s}$

D : A & B parallel $\rightarrow$ $B - A = \frac{K_{hs}}{s} - T_{es} K_{es}$

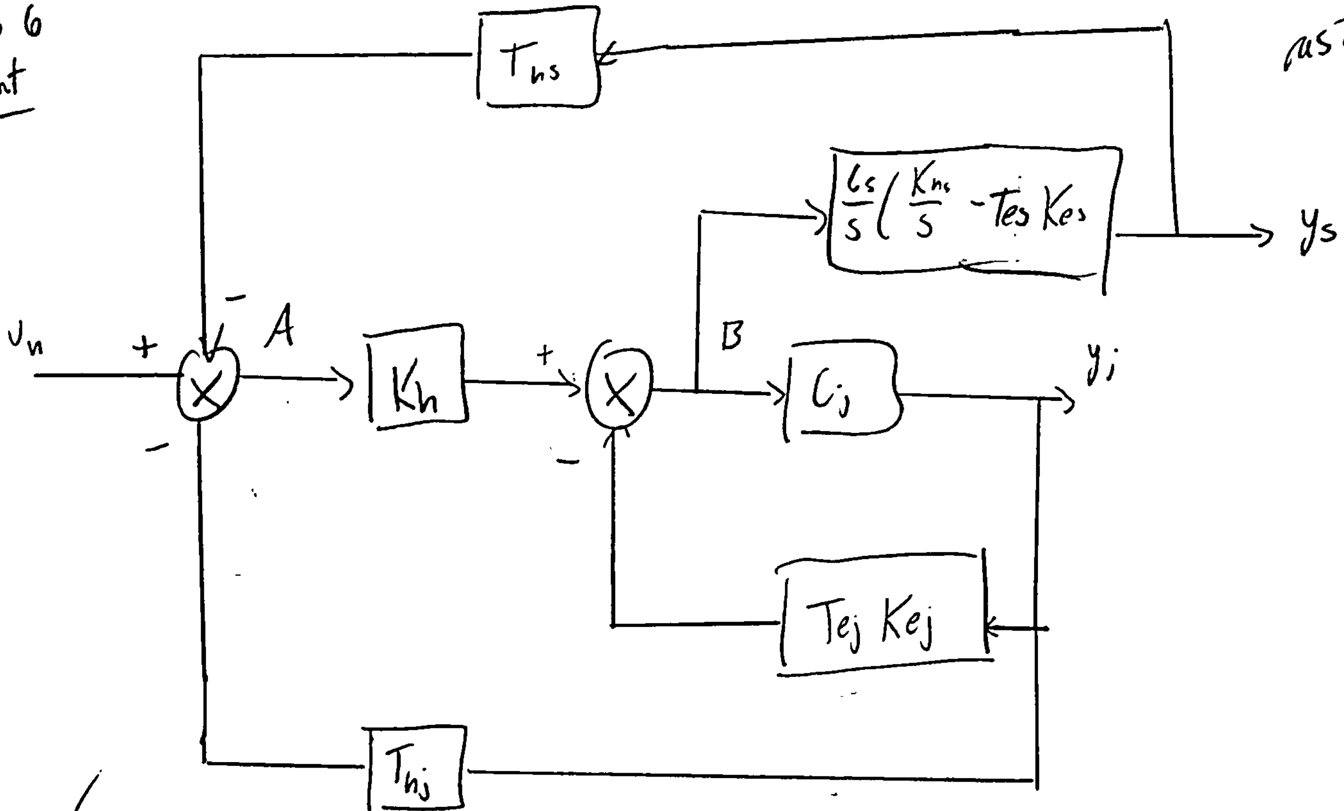
E : $\frac{1}{s}$, D, C_s cascade $\rightarrow$ $\frac{C_s}{s}\left(\frac{K_{hs}}{s} - T_{es} K_{es}\right)$

F : cascade $\rightarrow$ $T_{ej} K_{ej}$

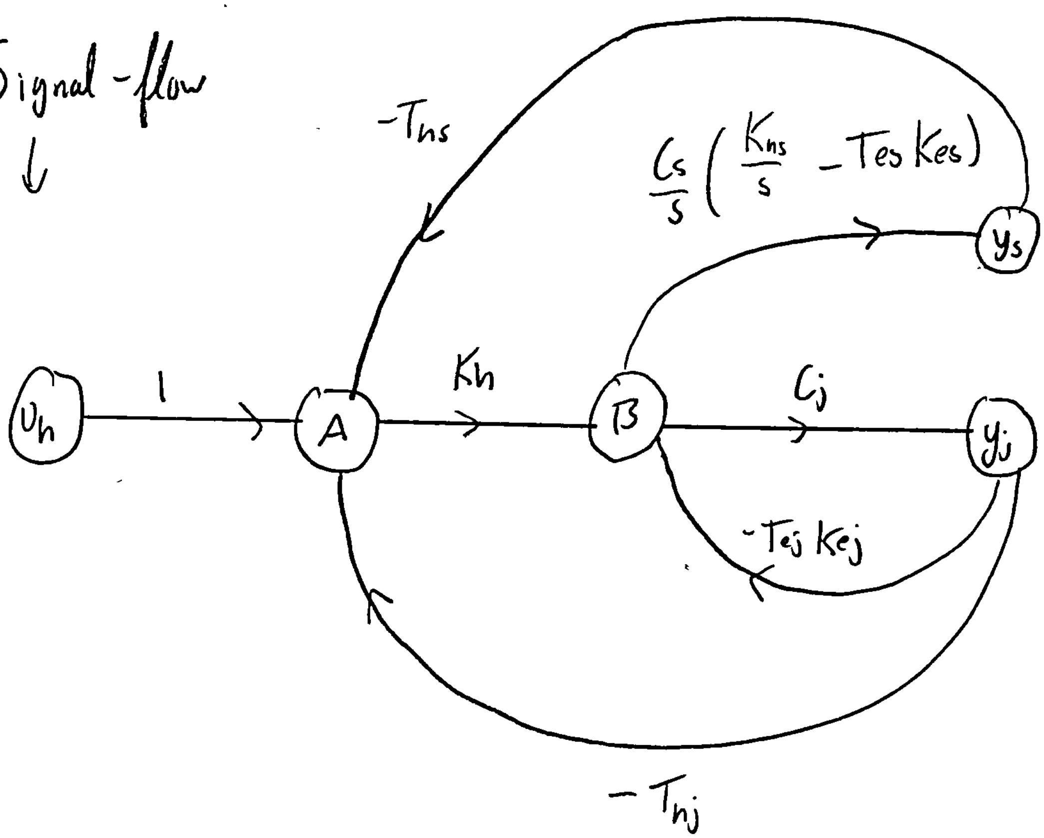
$\rightarrow$

Prob 6
cont

as26



Signal-flow



a) $\frac{Y_s(s)}{U_n(s)}$ Forward loop: only one $U_h \to A \to B \to y_j$

Gain $K_h C_j$

Prob 6A
cont

$\nearrow 20$

Loops: Gain

l_1 : $B \to y_j \to B$ $\quad -C_j T_{ej} K_{ej}$

l_2 : $A \to B \to y_j \to A$ $\quad -K_h T_{hj} C_j$

l_3 : $A \to B \to y_s \to A$ $\quad -\frac{T_{hs} K_h C_s}{s}\left(\frac{K_{hs}}{s} - T_{es}K_{es}\right)$

All loops touching (share node $\underline{B}$)

$$\frac{y_j}{U_h} = \frac{\sum_{k=1}^{1} T_k \Delta_k}{\Delta} \qquad \Delta = 1 - \sum_{i=1}^{3} l_i$$

$$\Delta = 1 + \frac{T_{hs} K_h C_s}{s}\left(\frac{K_{hs}}{s} - T_{es}K_{es}\right) + C_j T_{ej} K_{ej} + K_h T_{hj} C_j$$

$\underline{K=1}$

$T_1 = K_h C_j \qquad \Delta_1 = 1$ All loops touch forward path

$$\boxed{\frac{y_j}{U_h} = \frac{K_h C_j}{1 + \frac{T_{hs} K_h C_s}{s}\left(\frac{K_{hs}}{s} - T_{es}K_{es}\right) + C_j T_{ej} K_{ej} + K_h T_{hj} C_j}}$$

b) Forward path: $U_h \to A \to B \to y_s = \frac{K_h C_s}{s}\left(\frac{K_{hs}}{s} - T_{es}K_{es}\right)$

Same loops as a $\to$ Therefore same Δ as a

$\underline{K=1}$

$T_1 = \frac{K_h C_s}{s}\left(\frac{K_{hs}}{s} - T_{es}K_{es}\right) \qquad \Delta_1 = 1$ $\longrightarrow$

Prob 6B cont

$$\frac{y_s}{U_h} = \frac{T_1 \Delta_1}{\Delta} = \frac{\frac{K_h C_s}{s}\left(\frac{K_{hs}}{s} - T_{es} K_{es}\right)}{1 + \frac{T_{hs} K_h C_s}{s}\left(\frac{K_{hs}}{s} - T_{es} K_{es}\right) + C_j T_{ej} K_{ej} + K_h T_{hj} C_j}$$

$$= \boxed{\frac{K_h C_s \left(K_{hs} - T_{es} K_{es} s\right)}{s^2 + T_{hs} K_h C_s \left(K_{hs} - T_{es} K_{es} s\right) + C_j T_{ej} K_{ej} s^2 + K_h T_{hj} C_j s^2}}$$