Ryan St. Pierre
HW #2 Problem 2C

Problem 2C

One benefit of the (q1,q2) parameterization is that is has less charts necessary to cover the manifold. With the parameterization of (q1, q2) the problem is essentially reduced to one 2RIK problem (along with some geometry). As we studied in class the 2RIK problem has two charts necessary to cover the manifold (corresponding to "elbow up" and "elbow down"). By using q1 and q2 in the parameterization we are essentially removing from the ultimate solution set whether armature 1 and armature 2 are elbow up or elbow down because that information is given by the parameterization itself. To the contrary, the $\mathbf{x_2}$ parameterization does not encode this information about armature 1 and 2 in the parameterization itself. Thus, the $\mathbf{x_2}$ parameterization has 4 charts to cover its manifold, corresponding to the cases when armature 1 and 2 are in elbow up or down orientation and when armature 3 and 4 are in elbow up or elbow down orientation.

In reality the valid values for the q1,q2 in the (q1,q2) parameterization are actually infinite. However, to avoid redundancy we can refine the values of q1 and q2 to between 0 and 2π for each. Technically, for the $\mathbf{x_2}$ parameterization $\mathbf{x_2}$ can be any value in $\mathbf{R^2}$. However, given the constraint that all armature lengths are equal to one, $\mathbf{x_2}$ must actually fall within a circle of radius 2 around the origin. Thus, with both parameterizations the values of the parameterization variable can be limited. In this case I am not sure which parameterization is more preferred, as I believe it would be the preference of the user/coder and whether or not they prefer working in cartesian or angular coordinates. Since we are ultimately concerned with angular coordinates it is probably easier in interpolation and other tasks to deal with angular coordinates from the start in the parameterization choice.

From the work done in *Problem 2A* and *2B* it is clear that the (q1,q2) parameterization requires one inverse kinematics problem while the $\mathbf{x_2}$ parameterization requires two. It is prefered that during the procedure the number of inverse kinematics problems is limited. Additionally, it is quite easy to get the value of $\mathbf{x_2}$ given q1 and q2. This is done by a matrix multiplication, which is simpler than solving an inverse kinematics problem. Thus, it is easier to get $\mathbf{x_2}$ knowing q1 and q2 than to get q1 and q2 knowing $\mathbf{x_2}$.