

HW #1

Problem 2

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> restart

Helpful functions

> with(inttrans) :

> u := t → Heaviside(t) :

> PAR := (Za, Zb) → simplify($\frac{Za \cdot Zb}{Za + Zb}$) :

> SCS := X → sort(collect(simplify(expand(numer(X)) / expand(denom(X))), s), s) :

> IL := (X, s, t) → simplify(convert(invlaplace(convert(X, parfrac, s), s, t), expsincos)) :

> ILTS := (X, s, t) → simplify(convert(invlaplace(X, s, t), expsincos)) :

Part A

> eq1 := (R1 + R2 + L·s) · I1 - (R2 + L·s) · I2 - R1 · I3 = V
eq1 := (L s + R1 + R2) I1 - (L s + R2) I2 - R1 I3 = V (1)

> eq2 := $\left(R3 + \frac{1}{C2 \cdot s} + L \cdot s + R2\right) \cdot I2 - (R2 + L \cdot s) \cdot I1 - \left(R3 + \frac{1}{C2 \cdot s}\right) \cdot I3 = -Vr$
eq2 := $\left(R3 + \frac{1}{C2 s} + L s + R2\right) I2 - (L s + R2) I1 - \left(R3 + \frac{1}{C2 s}\right) I3 = -Vr$ (2)

> eq3 := $\left(\frac{1}{C1 \cdot s} + \frac{1}{C2 \cdot s} + R1 + R3\right) \cdot I3 - R1 \cdot I1 - \left(R3 + \frac{1}{C2 \cdot s}\right) \cdot I2 = 0$
eq3 := $\left(\frac{1}{C1 s} + \frac{1}{C2 s} + R1 + R3\right) I3 - R1 I1 - \left(R3 + \frac{1}{C2 s}\right) I2 = 0$ (3)

> eq4 := Vr = R4 · I2
eq4 := Vr = R4 I2 (4)

> solMesh := solve({eq1, eq2, eq3, eq4}, [I1, I2, I3, Vr]) :

> TFMesh := simplify($\frac{rhs(solMesh[][4])}{V}$)
TFMesh := $\left((L C1 C2 (R1 + R3) s^2 + (((R2 + R3) R1 + R2 R3) C1 + L) C2 + L C1) s + C2 R2 + C1 (R1 + R2)\right) R4 s / \left((L C1 C2 R4 (R1 + R3) s^3 + (((R2 + R3) R1 + R2 R3) R4 C1 + L (R1 + R3 + R4)) C2 + L C1 R4) s^2 + ((R2 + R3 + R4) R1 + R2 (R3 + R4)) C2 + R4 (R1 + R2) C1 + L) s + R1 + R2\right)$ (5)

> vals := R1 = 2, R2 = 2, R3 = 1, R4 = 3, C1 = $\frac{1}{2}$, C2 = $\frac{1}{3}$, L = 4 :

> simplify(subs(vals, TFMesh))
$$\frac{9 s^3 + 21 s^2 + 12 s}{9 s^3 + 27 s^2 + 25 s + 6}$$
 (6)

Part B

$$\begin{aligned} > eq1Node := \left(\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3} \right) \cdot V1 - \frac{1}{R1} \cdot Vs - \frac{1}{R3} \cdot V2 - \frac{1}{R2} \cdot V3 = 0 \\ &eq1Node := \left(\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3} \right) V1 - \frac{Vs}{R1} - \frac{V2}{R3} - \frac{V3}{R2} = 0 \end{aligned} \quad (7)$$

$$\begin{aligned} > eq2Node := \left(\frac{1}{R3} + C2 \cdot s \right) \cdot V2 - \frac{1}{R3} \cdot V1 - C2 \cdot s \cdot Vr = 0 \\ &eq2Node := \left(\frac{1}{R3} + C2 s \right) V2 - \frac{V1}{R3} - Vr C2 s = 0 \end{aligned} \quad (8)$$

$$\begin{aligned} > eq3Node := \left(\frac{1}{R2} + \frac{1}{L \cdot s} \right) \cdot V3 - \frac{1}{R2} \cdot V1 = 0 \\ &eq3Node := \left(\frac{1}{R2} + \frac{1}{L s} \right) V3 - \frac{V1}{R2} = 0 \end{aligned} \quad (9)$$

$$\begin{aligned} > eq4Node := \left(C1 \cdot s + C2 \cdot s + \frac{1}{R4} \right) \cdot Vr - C1 \cdot s \cdot Vs - C2 \cdot s \cdot V2 = 0 \\ &eq4Node := \left(C1 s + C2 s + \frac{1}{R4} \right) Vr - C1 s Vs - C2 s V2 = 0 \end{aligned} \quad (10)$$

$$> solNode := solve(\{eq1Node, eq2Node, eq3Node, eq4Node\}, [V1, V2, V3, Vr]) :$$

$$\begin{aligned} > TFNode := simplify\left(expand\left(\frac{rhs(solNode[][4])}{Vs} \right) \right) \\ &TFNode := \left((L C1 C2 (R1 + R3) s^2 + (((R2 + R3) R1 + R2 R3) C1 + L) C2 + L C1) s \right. \\ &\quad \left. + C2 R2 + C1 (R1 + R2) \right) R4 s \Big/ \left((L C1 C2 R4 (R1 + R3) s^3 + (((R2 + R3) R1 \right. \\ &\quad \left. + R2 R3) R4 C1 + L (R1 + R3 + R4)) C2 + L C1 R4) s^2 + ((R2 + R3 + R4) R1 \right. \\ &\quad \left. + R2 (R3 + R4)) C2 + R4 (R1 + R2) C1 + L) s + R1 + R2 \right) \end{aligned} \quad (11)$$

$$\begin{aligned} > simplify(subs(vals, TFNode)) \\ &\frac{9 s^3 + 21 s^2 + 12 s}{9 s^3 + 27 s^2 + 25 s + 6} \end{aligned} \quad (12)$$

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