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1.5.3 Inverse Laplace Transforms

> $restart$

I'll start by defining some useful functions

> $with(inttrans):$

> $u := t \rightarrow \text{Heaviside}(t):$

> $PAR := (Za, Zb) \rightarrow simplify\left(\frac{Za \cdot Zb}{Za + Zb}\right):$

> $SCS := X \rightarrow sort(collect(simplify(expand(numer(X))/expand(denom(X))), s), s):$

> $IL := (X, s, t) \rightarrow simplify(convert(invlaplace(convert(X, parfrac, s), s, t), expsincos)):$

> $ILTS := (X, s, t) \rightarrow simplify(convert(invlaplace(X, s, t), expsincos)):$

Nise 2.9

a)

> $H2P9A := \frac{7}{s^2 + 5s + 10}$

$$H2P9A := \frac{7}{s^2 + 5s + 10} \quad \textbf{(1)}$$

> $h2p9a := IL(H2P9A, s, t)$

$$h2p9a := \frac{14\sqrt{15}\, e^{-\frac{5t}{2}} \sin\left(\frac{\sqrt{15}\, t}{2}\right)}{15} \quad \textbf{(2)}$$

> $evalf[4](simplify(expand(h2p9a)))$

$$3.615\, e^{-2.500\, t} \sin(1.936\, t) \quad \textbf{(3)}$$

b)

> $H2P9B := \frac{15}{(s + 10) \cdot (s + 11)}$

$$H2P9B := \frac{15}{(s + 10)\,(s + 11)} \quad \textbf{(4)}$$

> $h2p9b := IL(H2P9B, s, t)$

$$h2p9b := -15\, e^{-11\, t} + 15\, e^{-10\, t} \quad \textbf{(5)}$$

c)

> $H2P9C := \frac{s + 3}{s^3 + 11s^2 + 12s + 18}$

$$H2P9C := \frac{s + 3}{s^3 + 11s^2 + 12s + 18} \quad \textbf{(6)}$$

> $h2p9c := IL(H2P9C, s, t)$

(7)

$$h2p9c := -\frac{\left(\sum_{\_\alpha = RootOf(\_Z^3 + 11\_Z^2 + 12\_Z + 18)} (2\_\alpha^2 + 19\_\alpha + 5)\, e^{\alpha t}\right)}{190} \quad \textbf{(7)}$$

> $evalf[4](simplify(expand(h2p9c)))$

$$-0.07652\, e^{-9.978\, t} + (0.03827 + 0.1108\, I)\, e^{(-0.5109 - 1.242\, I)\, t} + (0.03827 - 0.1108\, I)\, e^{(-0.5109 + 1.242\, I)\, t} \quad \textbf{(8)}$$

Nise 2.7

a)

> $H2P7A := \frac{(s^2 + 3 \cdot s + 10) \cdot (s+5)}{(s+3) \cdot (s+4) \cdot (s^2 + 2 \cdot s + 100)}$

$$H2P7A := \frac{(s^2 + 3 s + 10)\,(s+5)}{(s+3)\,(s+4)\,(s^2 + 2 s + 100)} \quad \textbf{(9)}$$

> $h2p7a := IL(H2P7A, s, t)$

$$h2p7a := -\frac{e^{-t} \sin(3\sqrt{11}\, t)\sqrt{11}}{61182} + \frac{5203\, e^{-t} \cos(3\sqrt{11}\, t)}{5562} - \frac{7\, e^{-4\, t}}{54} + \frac{20\, e^{-3\, t}}{103} \quad \textbf{(10)}$$

> $evalf[4](simplify(expand(h2p7a)))$

$$0.00001634\, \left(-13.27 \sin(3.317\, t) \cos(3.317\, t)^2 e^{3.\, t} + 228900. \cos(3.317\, t)^3 e^{3.\, t} + 3.317 \sin(3.317\, t)\, e^{3.\, t} - 171700. \cos(3.317\, t)\, e^{3.\, t} + 11880.\, e^{t} - 7931.\right) e^{-4.\, t} \quad \textbf{(11)}$$

b)

> $H2P7B := \frac{s^3 + 4 \cdot s^2 + 2 \cdot s + 6}{(s+8) \cdot (s^2 + 8 \cdot s + 3) \cdot (s^2 + 5 \cdot s + 7)}$

$$H2P7B := \frac{s^3 + 4 s^2 + 2 s + 6}{(s+8)\,(s^2 + 8 s + 3)\,(s^2 + 5 s + 7)} \quad \textbf{(12)}$$

> $h2p7b := IL(H2P7B, s, t)$

$$h2p7b := -\frac{1}{336102} \Bigg(\bigg(44278\, e^{-\frac{5t}{2} + t\sqrt{13}} \sin\left(\frac{\sqrt{3}\, t}{2}\right) \sqrt{3} + 132122\, e^{2 t (\sqrt{13} - 2)} \sqrt{13} + 5070\, e^{-\frac{5t}{2} + t\sqrt{13}} \cos\left(\frac{\sqrt{3}\, t}{2}\right) - 483197\, e^{2 t (\sqrt{13} - 2)} + 961324\, e^{t (\sqrt{13} - 8)} - 132122\, e^{-4\, t} \sqrt{13} - 483197\, e^{-4\, t} \bigg) e^{-t\sqrt{13}} \Bigg) \quad \textbf{(13)}$$

> $evalf[4](simplify(expand(h2p7b)))$

$$-2.975\; 10^{-6}\, \left(76690.\, e^{9.106\, t} \sin(0.8660\, t) + 5070.\, e^{9.106\, t} \cos(0.8660\, t) - 6800.\, e^{11.21\, t} - 959600.\, e^{4.\, t} + 961300.\, e^{3.606\, t}\right) e^{-11.61\, t} \quad \textbf{(14)}$$

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