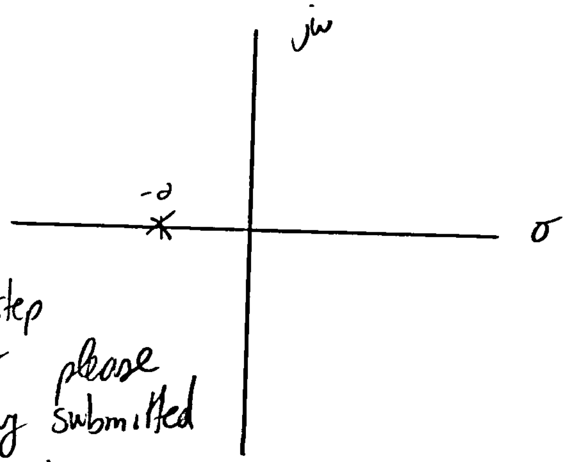
HW #3

ras70

Problem 1

a) $T(s) = \dfrac{2}{s+2}$

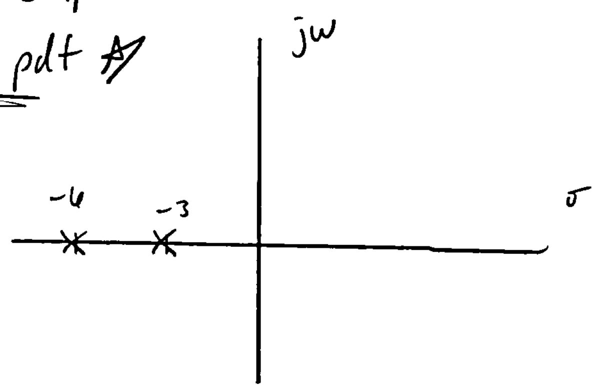


First order

* For actual step responses please see my submitted Maple script Problem1.pdf *

$S_R(t) = K_1 + K_2 e^{-2t}$

b) $T(s) = \dfrac{5}{(s+3)(s+6)}$



$(s+3),(s+6)$ both real

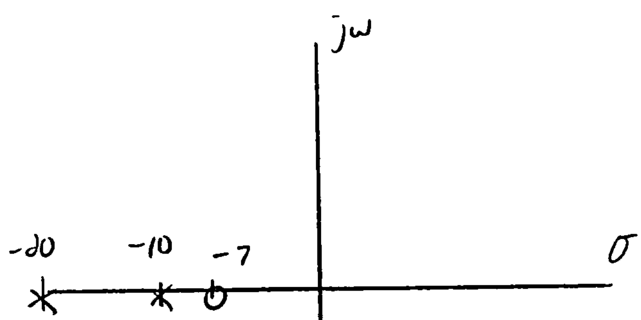
$s^2 + 9s + 18$

$b^2 - 4c = 81 - 72 = 9 > 0$

$\Rightarrow$ Overdamped

$S_R(t) = K_1 + K_2 e^{-3t} + K_3 e^{-6t}$

c) $T(s) = \dfrac{10(s+7)}{(s+10)(s+20)}$



Again 2 real roots

$\Rightarrow$ Overdamped

$S_R(t) = K_1 + K_2 e^{-10t} + K_3 e^{-20t}$

Problem 1 cont

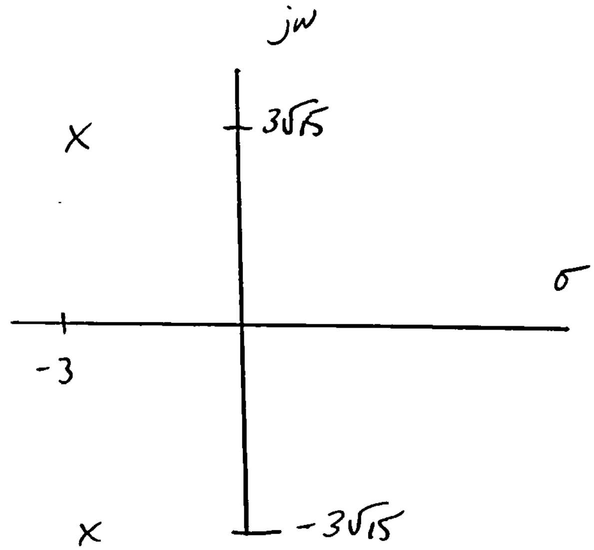
d) $T(s) = \dfrac{20}{s^2+6s+144}$

$\dfrac{-6 \pm \sqrt{36-4(144)}}{2}$

$\dfrac{-3 \pm \sqrt{-540}}{2}$

$-3 \pm 3\sqrt{15}$

$b^2 < 4c \Rightarrow$ Underdamped



$$\boxed{S_R(t) = K_1 + e^{-3t}\left(K_2 \cos(3\sqrt{15}t) + K_3 \sin(3\sqrt{15}t)\right)}$$

e) $T(s) = \dfrac{s+2}{s^2+9}$

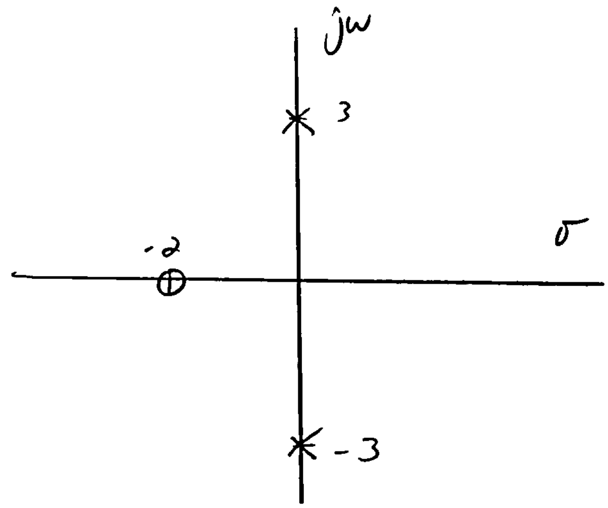
$b = 0$

$\boxed{\text{Undamped}}$

$(s^2+9) = 0$

$s^2 = -9$

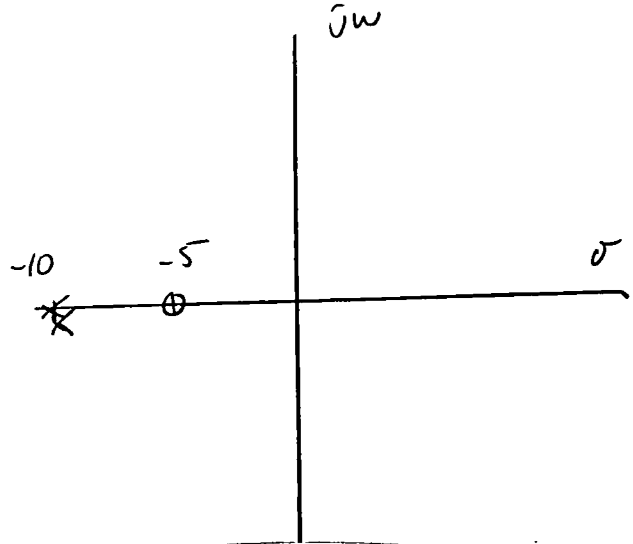
$s = \pm 3i$



$$\boxed{S_R(t) = K_1 + K_2 \cos(3t) + K_3 \sin(3t)}$$

Problem 1 cont

e) $T(s) = \dfrac{s+5}{(s+10)^2}$



$b^2 = 4b$

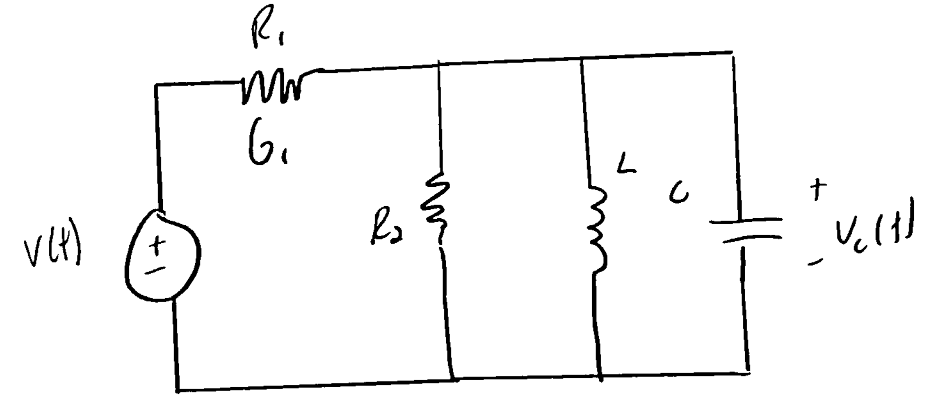
$20^2 = 4(100)$

$400 = 400$

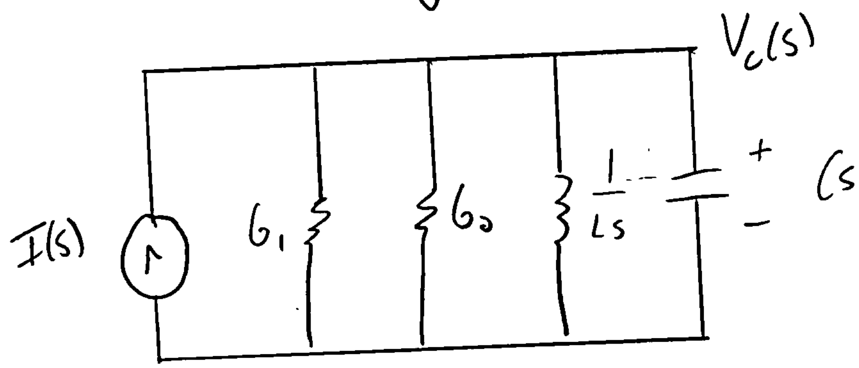
$\boxed{\text{Critically Damped}}$

$S_R(t) = K_1 + e^{-10t}(K_2 + K_3 t)$

Problem 2



$\Downarrow$



a)

$$I(s) = G_1 V(s)$$

$$V_c(s): \quad \left(G_1 + G_2 + \frac{1}{Ls} + Cs\right) V_c(s) = G_1 V(s)$$

$$\frac{V_c}{V_s} = \frac{G_1}{\left(G_1 + G_2 + \frac{1}{Ls} + Cs\right)} = \frac{1/R_1}{\left(1/R_1 + 1/R_2 + \frac{1}{Ls} + Cs\right)} = \frac{R_2 Ls}{R_2 Ls + R_1 Ls + R_1 R_2 + R_1 R_2 L C s^2}$$

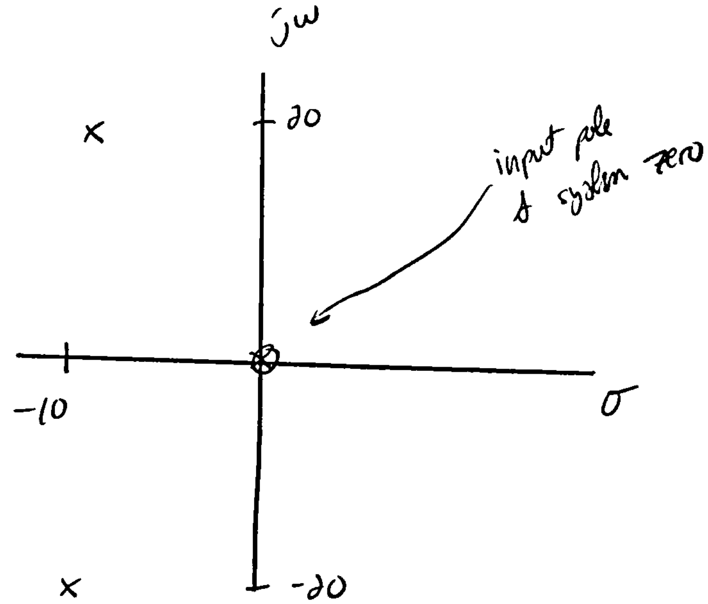
$$\boxed{\frac{V_c}{V_s} = \frac{R_2 Ls}{R_2 Ls + R_1 Ls + R_1 R_2 + R_1 R_2 L C s^2}}$$

$R_1 = 10k\Omega = R_2$, $L = 200H$, $C = 10\mu F$ $\rightarrow$ $$\frac{2\times10^6 s}{2\times10^5 s^2 + 4\times10^6 s + 1\times10^8}$$

Problem 2 cont

$$\boxed{\frac{V_c}{V_s} = \frac{10s}{s^2 + 20s + 500}}$$

b) Step input



$$\frac{-20 \pm \sqrt{20^2 - 4(500)}}{2}$$

$-10 \pm 20i$

↑ roots

c) Underdamped

$$\boxed{c(t) = K_1 + e^{-10t}\left(K_2 \cos(20t) + K_3 \sin(20t)\right)}$$

Problem 3

Please reference Sakai for submitted scripts.
↳ & diary

a) $T(s) = \dfrac{16}{s^2+3s+16}$

$\boxed{\begin{array}{ll} W_n = 4 \text{ rad/s} & \%OS = 28.06\% \\ \zeta = 0.3750 & \\ T_r = 3.56 \times 10^{-1} \text{ s} & \\ T_p = 8.47 \times 10^{-1} \text{ s} & \\ T_s = 2.67 \text{ s} & \end{array}}$

b) $T(s) = \dfrac{0.04}{s^2+.02s+0.04}$

$\boxed{\begin{array}{ll} W_n = 2 \times 10^{-1} \text{ rad/s} & \zeta = 0.0500 \\ T_r = 5.26 \text{ s} & T_p = 1.57 \times 10^{1} \text{ s} \\ T_s = 4.00 \times 10^{0} \text{ s} & \%OS = 85.45\% \end{array}}$

c) $T(s) = \dfrac{1.05 \times 10^7}{s^2 + 1.6\times10^3 s + 1.05\times10^7}$

$\boxed{\begin{array}{lll} W_n = 3.87 \times 10^3 \text{ rad/s} & T_r = 3.13 \times 10^{-4} \text{ s} & T_s = 5.00 \times 10^{-3} \text{ s} \\ \zeta = 0.2066 & T_p = 8.29 \times 10^{-4} \text{ s} & \%OS = 51.52\% \end{array}}$

Problem 4

a) Using Output1.fig

Final value $\rightarrow 2.5$

T_s = 98% of final value

Time to get to

98% of 2.5

$= 0.98(2.5) = 2.45$

$T_s \approx 1.956 s^*$

* for how I graphically got these values see my justification PDF

$$\boxed{\begin{array}{l} T_s \approx 1.956\,s \\ T_r \approx 1.097\,s \end{array}}$$

T_r = time 10% $\rightarrow$ 90% of final value

$= 0.25 \rightarrow 2.25$

$0.25 \rightarrow\ \approx 0.053^*$

$2.25 \rightarrow 1.15\,s^*$

$= 1.15 - 0.053 = 1.097\,s$

$$\boxed{a \approx 2}$$

$T_s = \frac{3.91}{a}$

$a = \frac{3.91}{T_s} = \frac{3.91}{1.956} \approx 2$

$\lim_{s \to 0} \frac{K}{s+2} = \frac{K}{2} = 2.5$

$K = 5$

$G(s) = \frac{K}{s+a} = \frac{K}{s+2}$

$$\boxed{G(s) = \frac{K}{s+2}}$$

$$\boxed{G(s) = \frac{5}{s+2}}$$

Problem 4 cont

b) From the Output 2.fig

* For all graphical approximations see justification PDF

$\boxed{T_p = 1.545\,s}$ *

The value at T_p is ≈ 0.3853

The final value is 0.361

$\boxed{\%OS = 6.31\%}$

$$\%\text{ overshoot} = \frac{0.3853 - 0.361}{0.361} \cdot 100 = 6.31\%$$

$\pm 2\%$ of final value $(0.354,\ 0.368)$

The second dip does not go below this value. Thus we can consider when the signal first enters this range

$\boxed{T_S \approx 2.05\ s}$

Rise time

$0.361(0.9) = 0.3249 \rightarrow \approx 0.9\ s$

$0.361(0.1) = 0.0361 \rightarrow \approx 0.187s$

$\boxed{T_r \approx 0.713\ s}$

$$\%\text{ Overshoot} = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \cdot 100 = 6.31$$

$$\frac{\zeta}{\sqrt{1-\zeta^2}} = -\ln\left(\frac{6.31}{100}\right)/\pi = 0.88$$

Problem 4b cont

$$\boxed{\zeta = 0.661}$$

$$\boxed{\omega_n = 2.69}$$

$$T_s = \frac{4}{\zeta \omega_n} \Rightarrow \omega_n = \frac{4}{\zeta T_s} = \frac{4}{(.661)(2.25)} = 2.68953$$

$$G(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{K}{s^2 + 3.56s + 7.24}$$

$$\lim_{s \to 0} G(s) = \frac{K}{7.24} = 0.361 = 2.61$$

$$\boxed{G(s) = \frac{2.61}{s^2 + 3.56s + 7.24}}$$

Problem 4c

For output3.fig

Final value = 0.77

Peak value $\approx$ 1.36 at 0.9 seconds

$$\% \text{ overshoot} = \frac{1.36 - 0.77}{0.77}(100) = 76.62\%$$

$\pm$ 2% of final value = (0.755, 0.785) → Graphically $\approx$ 14.5 s

$T_p = 0.9$ s

$\boxed{\% \text{ overshoot} = 76.62\%}$

$\boxed{T_s = 14.5 \text{ s}}$

Rise time

subtract = 0.32 s

$.77(.1) = 0.77 \rightarrow 0.13$

$.77(.9) = 0.693 \rightarrow 0.45\text{ s}$

$\boxed{T_r \approx 0.32 \text{ s}}$

$$\% \text{ overshoot} = e^{-(\xi\pi/\sqrt{1-\xi^2})} \cdot 100 = 76.62$$

$$\frac{\xi}{\sqrt{1-\xi^2}} = -\ln\left(\frac{76.62}{100}\right)/\pi = 0.08477$$

$\boxed{\xi = 0.084}$

$$T_s = \frac{4}{\xi\omega_n} \Rightarrow \omega_n = \frac{4}{\xi T_s} = \frac{4}{0.084(14.5)} = 3.28$$

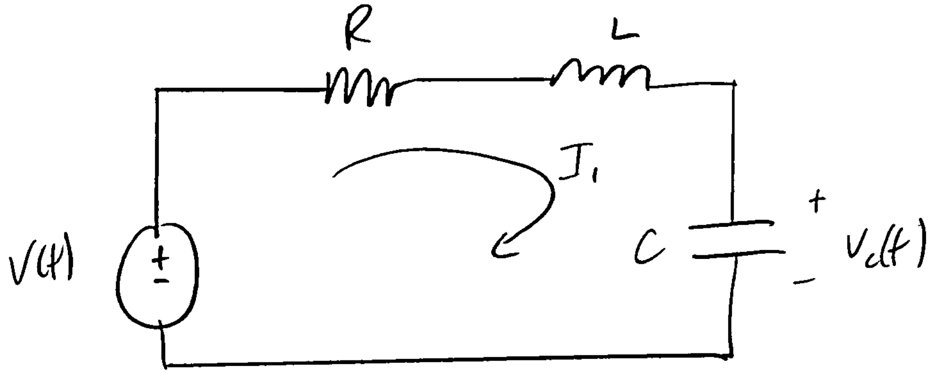
$\boxed{\omega_n = 3.28}$

$$G(s) = \frac{K}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{K}{s^2 + 1.8s + 10.8}$$

$$G(s)\Big|_{s \to 0} = \frac{K}{10.8} = 0.77 \Rightarrow K = 8.316$$

$$\boxed{G(s) \approx \frac{8.3}{s^2 + 1.8s + 10.8}}$$

Problem 5



a)
Mesh current

l_1: $\left(R + Ls + \frac{1}{Cs}\right) I_1(s) = V(s)$

Ohm's $V_c(s) = \frac{1}{Cs} I_1(s) \Rightarrow I_1(s) = Cs V_c(s)$

$$\left(R + Ls + \frac{1}{Cs}\right) Cs V_c(s) = V(s)$$

$$(RCs + CLs^2 + 1) V_c(s) = V(s)$$

$$\frac{V_c(s)}{V(s)} = \frac{1}{CLs^2 + RCs + 1}$$

b) $\frac{V_c(s)}{V(s)} = \frac{1/CL}{s^2 + \frac{RC}{CL}s + 1/CL} = \frac{1}{CL} \frac{1}{s^2 + R/L\, s + 1/LC}$

$\omega_n^2 = \frac{1}{LC}$

$\xi = \frac{R/L}{2\omega_n} = \frac{R/L}{2\,1/\sqrt{LC}} = \frac{R\sqrt{LC}}{2L} = \frac{R}{2}\frac{\sqrt{L}\sqrt{C}}{\sqrt{L}\sqrt{L}}$

$= \frac{R}{2}\sqrt{C/L}$

$$\boxed{\omega_n = \frac{1}{\sqrt{LC}}}$$

$$\boxed{\xi = \frac{R}{2}\sqrt{\frac{C}{L}}}$$

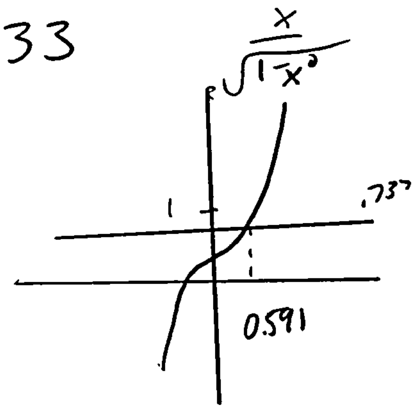
Problem 5c

$L = 100$

$$\% OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \cdot 100 = 10\%$$

$$e^{-(\zeta\pi/\sqrt{1-\zeta^2})} = 0.1$$

$$\frac{\zeta}{\sqrt{1-\zeta^2}} = \frac{-\ln(0.1)}{\pi} \approx 0.733$$



$\zeta \approx 0.591$ determined graphically

$T_r = 0.0321$ s

According to Figure 4.16, $\zeta \approx 0.6$ corresponds to a normalized rise time of $w_n T_r = 1.854$ ..

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$w_n T_r = 1.854$

$$w_n = \frac{1.854}{0.0321} = 57.757 \text{ rad/s}$$

$$R = 2\zeta / \sqrt{C/L} = \frac{2(0.591)}{\sqrt{\frac{3\times10^{-4}}{100}}}$$

$= 6824.28\ \Omega$

$$w_n = \frac{1}{\sqrt{LC}}$$

$$w_n^2 = \frac{1}{LC}$$

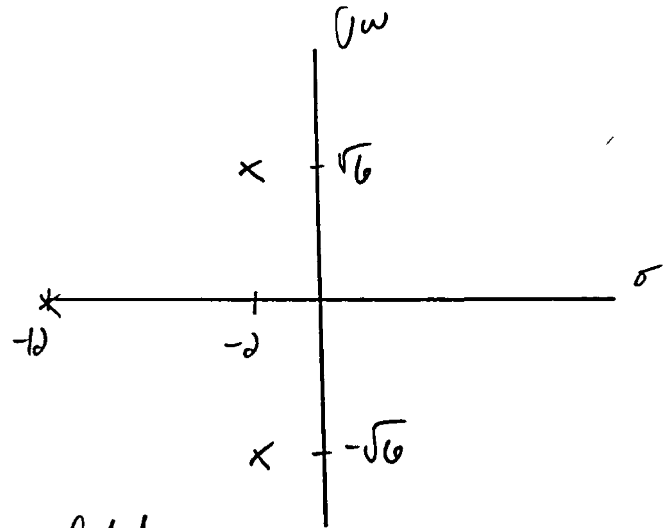
$$C = \frac{1}{Lw_n^2} = \frac{1}{100(57.757)^2} = 2.99771\ \mu F$$

$C \approx 3\mu F$
$R = 6824.28\ \Omega$

Problem 6

a) $T(s) = \dfrac{5}{(s+12)(s^2+4s+10)}$

$\dfrac{-4 \pm \sqrt{16-40}}{2} = -2 \pm \sqrt{6}\, i$



$12 > 2(5) = 10$, thus this pole can be neglected

$$\boxed{T(s) \approx \dfrac{5}{(s^2+4s+10)}}$$

b) $T(s) = \dfrac{s+2.1}{(s+2)(s^2+s+5)} = \dfrac{A}{s+2} + \dfrac{Bs+C}{s^2+s+5}$

$(s+2.1) = A(s^2+s+5) + (s+2)(Bs+C)$

$s = -2 \Rightarrow \quad +0.1 = 7A$

$A = 0.1/7$

s^2/ $\quad As^2 + Bs^2 = 0$

$A = -B$

$B = -0.1/7$

1's $\quad 5A + 2C = 2.1$

$C = \dfrac{2.1 - 5A}{2} = 1.01$

$A \approx 0.014$

$B \approx -0.014$

$C \approx 1.01$

About same magnitude

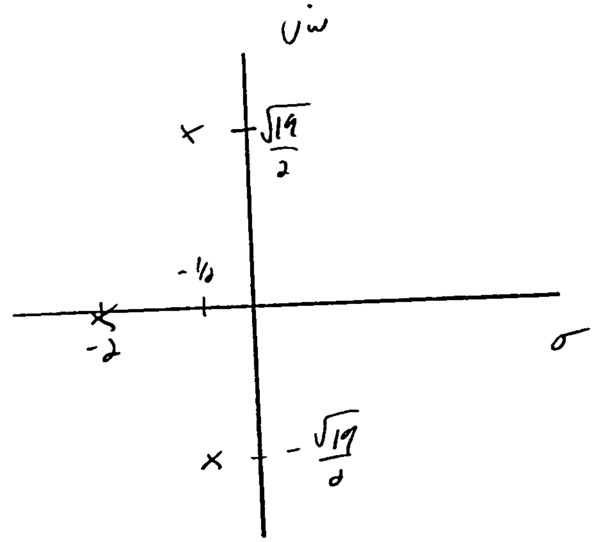
Cannot cancel zero & pole

s^2+s+5 roots

$\dfrac{-1 \pm \sqrt{1-20}}{2} = -\dfrac{1}{2} \pm \dfrac{\sqrt{19}}{2}$

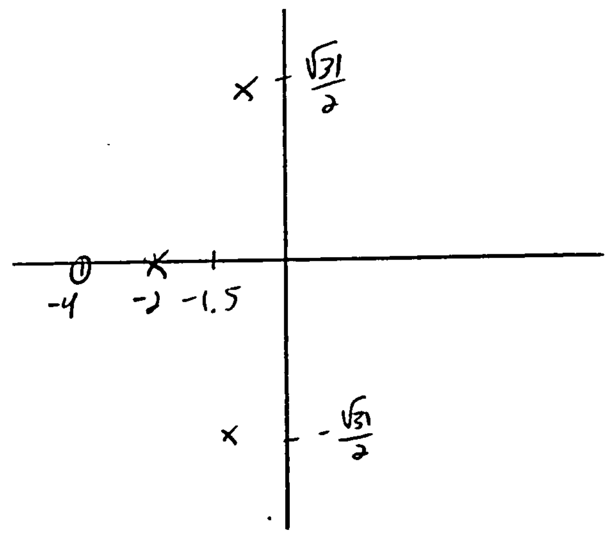
Problem 6b cont

-2 pole is <u>not</u> 5 times left of the other two poles. Thus, the two pole approximation cannot be made



c) $T(s) = \dfrac{s+4}{(s+2)(s^2+3s+10)}$

$$\frac{-3 \pm \sqrt{9-40}}{2} = -\frac{3}{2} \pm \frac{\sqrt{31}}{2} i$$



Again, 2 is not 5 times left of the other two poles. Thus, the two pole approximation cannot be made

$$\frac{s+4}{(s+2)(s^2+3s+10)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+3s+10}$$

A, B, & C are all of the same magnitude

↳ <u>cannot</u> cancel a zero & pole

$$s+4 = A(s^2+3s+10) + (s+2)(Bs+C)$$

$s=-2 \Rightarrow \quad 2 = A(8)$

$A = 1/4$

$10A + C = 4$

$C = 4 - 10/4 = \frac{3}{2}$

$A + B = 0$

$-A = B$

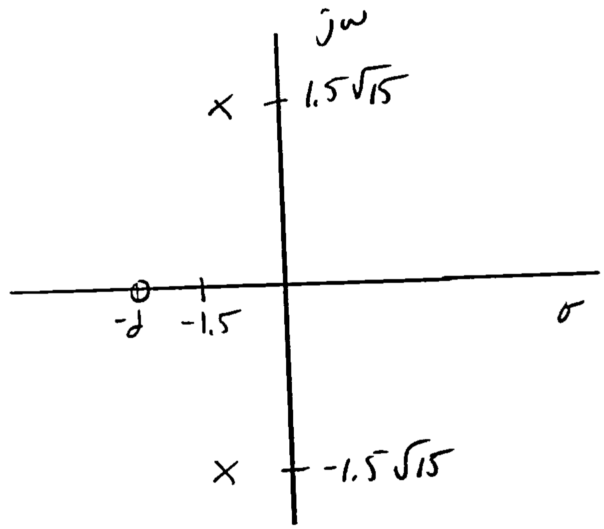
$B = -1/4$

Problem 7

Since $s^2+3s+36$ is the denominator for $T(s)$ in a-c let's analyze that first.

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-3 \pm \sqrt{9-4(36)}}{2} = -\frac{3}{2} \pm \frac{3}{2}\sqrt{15}\, i$$

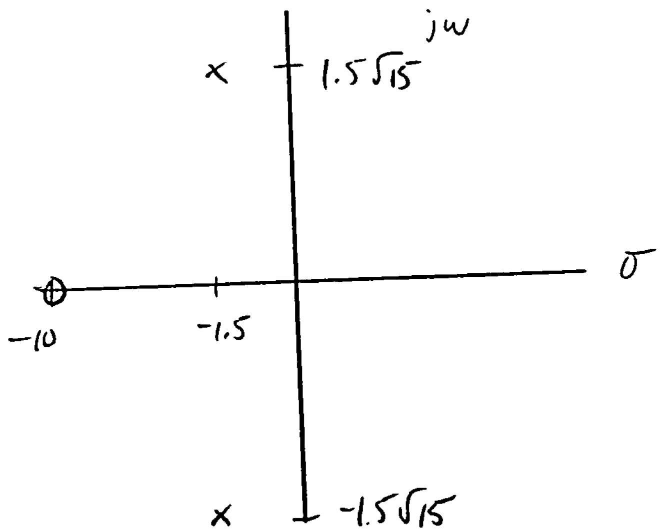
a) $T(s) = \dfrac{s+2}{s^2+3s+36}$



$2 \not> 5\left(\frac{3}{2}\right) = 7.5$

$\therefore$ zero approximation cannot be applied

b) $T(s) = \dfrac{s+10}{s^2+3s+36}$



-10 is much left of the poles

$10 > 5\left(\frac{3}{2}\right) = 7.5$

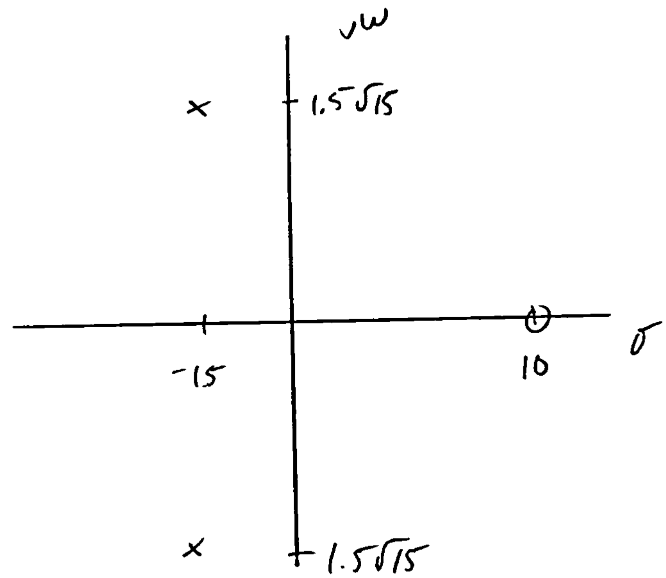
$\therefore$ zero approximation can be applied

$T(s)$ can be approximated with no zeros and a gain of 10

i.e $\dfrac{10}{s^2+3s+36}$

Problem 7 cont

c) $T(s) = \dfrac{s-10}{s^2+3s+36}$



Zero is far from poles. Therefore approximation can be applied. $K=-10$

$$T(s) \approx \frac{-10}{s^2+3s+36}$$

This is a nonminimum phase system

I have plotted T(s) vs its approximation for b & c. Please see my attached PDF's for this discussion.