Problem 3A

$$\vec{x} = \begin{bmatrix} c_1(L_2c_2+L_1) - s_1s_2L_2 \\ s_1(L_2c_2+L_1) + c_1s_2L_2 \\ q_1+q_2 \end{bmatrix}$$

$$J(q) = \begin{bmatrix} \partial x/\partial q_1 & \partial x/\partial q_2 \\ \partial y/\partial q_1 & \partial y/\partial q_2 \\ \partial \theta/\partial q_1 & \partial \theta/\partial q_2 \end{bmatrix}$$

$$x = L_2c_2\cos(q_1) + L_1\cos(q_1) - L_2s_2\sin(q_1)$$

$$\frac{\partial x}{\partial q_1} = -L_2c_2\sin(q_1) + L_1\sin(q_1) - L_2s_2\cos(q_1) = -s_1(L_2c_2+L_1) - c_1s_2L_2$$

$$x = L_2c_1\cos(q_2) + L_1c_1 - s_1L_2\sin(q_2)$$

$$\frac{\partial x}{\partial q_2} = -L_2c_1\sin(q_2) + 0 - s_1L_2\cos(q_2) = -L_2(c_1s_2 + s_1c_2)$$

$$\partial\theta/\partial q_1 = 1 \qquad \frac{\partial\theta}{\partial q_2} = 1$$

$$y = L_2c_2\sin(q_1) + L_1\sin(q_1) + L_2s_2\cos(q_1)$$

$$\frac{\partial y}{\partial q_1} = L_2c_2\cos(q_1) + L_1\cos(q_1) - L_2s_2\sin(q_1) = c_1(L_2c_2+L_1) - s_1s_2L_2$$

$$y = s_1L_2\cos(q_2) + L_1s_1 + L_2c_1\sin(q_2)$$

$$\frac{\partial y}{\partial q_2} = -s_1L_2\sin(q_2) + 0 + L_2c_1\cos(q_2) = -s_1L_2s_2 + L_2c_1c_2$$

Problem 3A cont

$$J(q) = \begin{bmatrix} -s_1(L_2c_2 + L_1) - c_1s_2L_2 & -L_2(c_1s_2 + s_1c_2) \\ c_1(L_2c_2 + L_1) - s_1s_2L_2 & -s_1s_2L_2 + c_1c_2L_2 \\ 1 & 1 \end{bmatrix}$$

Problem 3B

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The chain rule is given by $\frac{\partial f}{\partial t} = \frac{\partial f}{\partial q_1}\frac{\partial q_1}{\partial t} + \frac{\partial f}{\partial q_2}\frac{\partial q_2}{\partial t}$

$\frac{\partial f}{\partial q_1}$ and $\frac{\partial f}{\partial q_2}$ are values that were calculated in Problem 3A.

More specifically

$$\frac{\partial f}{\partial q_1} = \begin{bmatrix} -s_1(L_2c_2+L_1) - c_1s_2L_2 \\ c_1(L_2c_2+L_1) - s_1s_2L_2 \\ 1 \end{bmatrix} \qquad \frac{\partial f}{\partial q_2} = \begin{bmatrix} -L_2(c_1s_2+s_1c_2) \\ -s_1s_2L_2 + c_1c_2L_2 \\ 1 \end{bmatrix}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial q_1}\underbrace{\frac{\partial q_1}{\partial t}}_{\text{scalar}} + \frac{\partial f}{\partial q_2}\underbrace{\frac{\partial q_2}{\partial t}}_{\text{scalar}} =$$

$$\begin{bmatrix} \left(-s_1(L_2c_2+L_1) - c_1s_2L_2\right)\frac{\partial q_1}{\partial t} \\ \left(c_1(L_2c_2+L_1) - s_1s_2L_2\right)\frac{\partial q_1}{\partial t} \\ \frac{\partial q_1}{\partial t} \end{bmatrix} + \begin{bmatrix} \left(-L_2(c_1s_2+s_1c_2)\right)\frac{\partial q_2}{\partial t} \\ \left(-s_1s_2L_2 + c_1c_2L_2\right)\frac{\partial q_2}{\partial t} \\ \frac{\partial q_2}{\partial t} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial q_1}{\partial t}\left(-s_1(L_2c_2+L_1) - c_1s_2L_2\right) + \frac{\partial q_2}{\partial t}\left(-L_2(c_1s_2+s_1c_2)\right) \\ \frac{\partial q_1}{\partial t}\left(c_1(L_2c_2+L_1) - s_1s_2L_2\right) + \frac{\partial q_2}{\partial t}\left(-s_1s_2L_2 + c_1c_2L_2\right) \\ \frac{\partial q_1}{\partial t} + \frac{\partial q_2}{\partial t} \end{bmatrix}$$

$$= \begin{bmatrix} -s_1 (L_2 c_2 + L_1) - c_1 s_2 L_2 & -L_2 (c_1 s_2 + s_1 c_2) \\ c_1 (L_2 c_2 + L_1) - s_1 s_2 L_2 & -s_1 s_2 L_2 + c_1 c_2 L_2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial q_1}{\partial t} \\ \frac{\partial q_2}{\partial t} \end{bmatrix}$$

$$= J(q(t)) \; q'(t)$$

$$\therefore$$

$$\frac{\partial f}{\partial t} = J(q(t)) \; q'(t) \quad \blacksquare$$

Problem 3B cont

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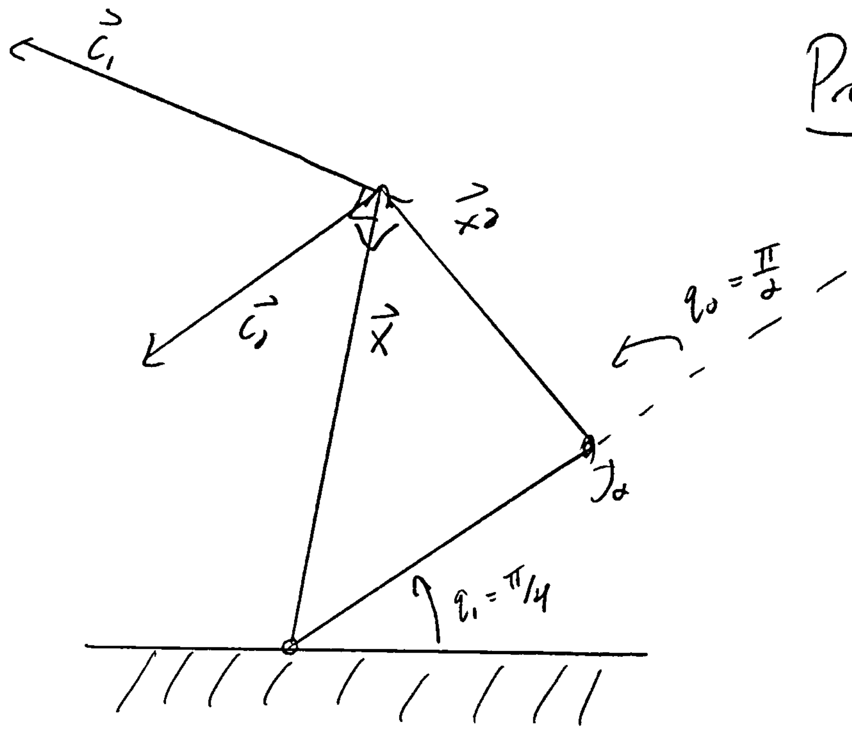
Problem 3C



$$\vec{x} = \begin{bmatrix} \frac{\sqrt{2}}{2} L_1 - \frac{\sqrt{2}}{2} L_2 \\ \frac{\sqrt{2}}{2} L_1 + \frac{\sqrt{2}}{2} L_2 \\ \frac{3\pi}{4} \end{bmatrix}$$

$c_1 = \cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$ $\quad c_2 = \cos(\frac{\pi}{2}) = 0$

$s_1 = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$ $\quad s_2 = \sin(\frac{\pi}{2}) = 1$

$$J\left(\begin{bmatrix} \pi/4 \\ \pi/2 \end{bmatrix}\right) = \begin{bmatrix} -\frac{\sqrt{2}}{2}(L_1 + L_2) & -\frac{\sqrt{2}}{2} L_2 \\ \frac{\sqrt{2}}{2}(L_1 - L_2) & -\frac{\sqrt{2}}{2} L_2 \\ 1 & 1 \end{bmatrix}$$

Let $\vec{c}_1$ & $\vec{c}_2$ be the two column vectors. $\vec{c}_1$ is the vector of instantaneous motion if q_1 was to move with q_2 still. Note: $||\vec{x}|| = ||\vec{c}_1||$ and $\vec{x} \perp \vec{c}_1$

$||\vec{x}|| = L_1^2 + L_2^2 = ||\vec{c}_1||$

$\vec{c}_2$ is the vector of instantaneous motion if q_2 was to move with q_1 still. Let $\vec{x}_2$ be the displacement vector from joint 2 (J_2) to the E.E

Then $\|\vec{x}_2\| = \|\vec{c}_2\|$ & $\vec{x}_2 \perp \vec{c}_2$

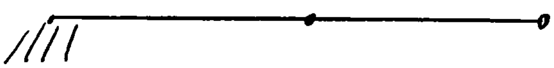
I have neglected the orientation in my discussion above. In 3D space (w/ orientation) the third component of the column vector gives θ's instantaneous change with a 1 rad/s increase in q_1 or q_2. Obviously this is 1.

Problem 3D

$$q = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad q_1 = q_2 = 0$$

$$S_1 = 0 \qquad S_2 = 0$$

$$C_1 = 1 \qquad C_2 = 1$$



$$J(q) = \begin{bmatrix} 0 & 0 \\ L_1 + L_2 & L_2 \end{bmatrix} \qquad \Delta = 0L_2 + 0(L_1 + L_2) = 0$$

$\Rightarrow$ linear dependence

The significance of this is the robot cannot move instantaneously in certain workplace dimensions.