MATHEMATICAL FOUNDATION FOR COMPUTER SCIENCE

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RULES OF INTERFERENCE FOR QUANTIFIED STATEMENT

1. UNIVERSAL INSTANTIATION:

- P(c) is true , Where c is a particular member of the Domain, given the Premise $\forall_x [p(x)]$

$$\frac{\forall_x [p(x)]}{\therefore p(c)}$$

Example: We can conclude from the statement "All women are wise" that "Lisa is wise" where Lisa is a member of the domain of all women.

2. UNIVERSAL GENERALIZATION:

- $\forall_x [p(x)]$ is True, given the premise that P(c) is True for all elements c in the domain.

$$\frac{p(c) \quad \text{for an arbitrary } c}{\therefore \forall_x [p(x)]}$$

Example: The Domain consist of the dogs Fido, Ruby, Laika

"Fido is cute, Ruby is cute, Laika is cute".

Therefore, all dogs in the domain are cute.

3. EXISTENTIAL INSTANTIATION :

- There is an element c in the domain for which P (c) is true if we know that $\exists_x P(x)$ is true.

- We cannot select an arbitrary value of c here, but rather it must be a c for which P (c) is true. Usually we have no knowledge of what c is, only that it exists.

$$\frac{\exists_x [p(x)]}{\therefore p(c)\ \text{c}}$$

for some element

<u>Example:</u> "There is someone who got an A in the course."

"Let's call her c and say that c got an A"

4. EXISTENTIAL GENERALIZATION :

- That is, if we know one element c in the domain for which P (c) is true, then we know that $\exists_x P(x)$ is true.

$$\frac{p(c) \quad \text{for some element } c}{\therefore \ \exists_x [p(x)]}$$

Example: "Michelle got an A in the class."

"Therefore, there is someone who got an A in the class."

Q.1) "All King are men"

"All men are mortal"

∴ "All Kings are mortal

Solution:

Defining variables:

K(x): "x is king"

M(x): "x is man"

$M_0(x)$: "x is mortal"

Hypothesis: i) $\forall_x[K(x) \rightarrow M(x)$

ii) $\forall_x[M(x) \rightarrow M_o(x)$

Conclusion: $\therefore \forall_x[K(x) \rightarrow M_o(x)]$

STEPS	REASONS
1. $\forall_x[K(x) \rightarrow M(x)]$	GIVEN HYPOTHESIS
2. $K(c) \rightarrow M(c)$	UNIVERSAL INSTANTIATION ON 1
3. $\forall_x[M(x) \rightarrow M_o(x)]$	GIVEN HYPOTHESIS
4. $M(c) \rightarrow M_o(c)$	UNIVERSAL INSTANTIATION ON 3
5. $K(c) \rightarrow M_o(c)$	HYPOTHETICAL SYLLOGISM ON 2 & 4
6. $\forall_x[K(x) \rightarrow M_o(x)]$	UNIVERSAL GENERALIZATION ON 5

Q.2) Show that the premises *"Everyone in this discrete mathematics class has taken a course in computer science"* and *"Sita is a student in this class"* imply the conclusion *"Marla has taken a course in computer science."*

Solution:

Defining variables:

D(x): *"x is in this discrete mathematics class"*

C(x): *"x has taken a course in computer science."*

Hypothesis: i) $\forall_x[D(x) \rightarrow C(x)]$

ii) D(sita)

Conclusion: $\therefore$ C(sita)

STEPS	REASONS
1. $\forall_x[D(x) \rightarrow C(x)]$	GIVEN HYPOTHESIS
2. $D(sita) \rightarrow C(sita)$	UNIVERSAL INSTANTIATION ON 1
3. D(sita)	GIVEN HYPOTHESIS
4. C(sita)	MODUS PONENS ON 2 & 3

Q.3) Show that the premises *"A student in this class has not read the book,"* and *"Everyone in this class passed the first exam"* imply the conclusion *"Someone who passed the first exam has not read the book."*

<u>Solution:</u>

Defining variables:

C(x):"x is in this class"

B(x):"x has read the book"

P(x):"x passed the first exam"

<u>Hypothesis:</u> i) $\exists_x[C(x) \wedge \neg B(x)]$

ii) $\forall_x[C(x) \rightarrow P(x)]$

<u>Conclusion:</u> $\therefore \exists_x[P(x) \wedge \neg B(x)]$

STEPS	REASONS
1. $\exists_x[C(x) \wedge \neg B(x)]$	GIVEN HYPOTHESIS
2. $C(a) \wedge \neg B(a)$	EXISTENTIAL INSTANTIATION ON 1
3. $C(a)$	SIMPLIFICATION ON 2
4. $\forall_x[C(x) \rightarrow P(x)]$	GIVEN HYPOTHESIS
5. $C(a) \rightarrow P(a)$	UNIVERSAL INSTANTIATION FROM (4)
6. $P(a)$	MODUS PONENS FROM (3) AND (5)
7. $\neg B(a)$	SIMPLIFICATION ON 2
8. $P(a) \wedge \neg B(a)$	CONJUNCTION FROM (6) AND (7)
9. $\exists_x[P(x) \wedge \neg B(x)]$	EXISTENTIAL GENERALIZATION FROM (8)

Q.4) Show that the premises *"All rock music is loud"*, *"Some rock music exist"*, imply the conclusion *"Some Loud music exists"*

<u>Solution:</u>

Defining variables:

R(x): *"x is in rock music"*

L(x): *"x is loud music"*

<u>Hypothesis:</u> i) $\forall_x[R(x) \rightarrow L(x)]$

ii) $\exists_x[R(x)]$

<u>Conclusion:</u> $\therefore \exists_x[L(x)]$

STEPS	REASONS
1. $\forall_x[R(x) \rightarrow L(x)]$	GIVEN HYPOTHESIS
2. $R(c) \rightarrow L(c)$	UNIVERSAL INSTANTIATION ON 1
3. $\exists_x[R(x)]$	GIVEN HYPOTHESIS
4. R (c)	EXISTENTIAL INSTANTIATION ON 3
5. L(c)	MODUS PONENS FROM (2) AND (4)
6. $\exists_x[L(x)]$	EXISTENTIAL GENERALIZATION FROM (5)

Q.4) Show that the premises *"Every computer science student works harder than somebody", "Everyone who works harder than any other person gets less sleep than that person", "Maria is a computer science student"* Implies the conclusion *"Maria gets less sleep than someone else"*

Solution:

Defining variables:

C(x) : "*x is Computer science student*"

W(x, y) : "*x works harder than y*"

S(x, y) : "x gets less sleep than y"

C(m) : "Maria is a computer science student"

Hypothesis: i) $\forall_x \exists_y [C(x) \rightarrow W(x, y)]$

ii) $[\forall_x \exists_y W(x, y)] \rightarrow S(x, y)$

iii) $C(m)$

Conclusion: $\therefore \exists_y [S(m, y)]$

Hypothesis: i) $\forall_x \exists_y [C(x) \rightarrow W(x, y)]$

ii) $[\forall_x \exists_y W(x, y)] \rightarrow S(x, y)$

iii) $C(m)$

Conclusion: $\therefore \exists_y [S(m, y)]$

STEPS	REASONS
1. $\forall_x \exists_y [C(x) \rightarrow W(x, y)]$	GIVEN HYPOTHESIS
2. $\exists_y [C(m) \rightarrow W(m, y)]$	UNIVERSALINSTANTIATION ON 1
3. $C(m) \rightarrow W(m, c)$	EXISTENTIAL INSTANTIATION ON 2
4. $[\forall_x \exists_y W(x, y)] \rightarrow S(x, y)$	GIVEN HYPOTHESIS
5. $[\exists_y W(m, y)] \rightarrow S(m, y)$	UNIVERSAL INSTANTIATION ON 4
6. $W(m\ c) \rightarrow S(m, c)$	EXISTENTIAL INSTANTIATION ON 5
7. $C(m) \rightarrow S(m, c)$	FROM 3 AND 6
8. $C(m)$	GIVEN HYPOTHESIS
9. $S(m, c)$	MODUS TOLLENS ON 7 AND 8
10. $\exists_y [S(m, y)]$	EXISTENTIAL GENERALIZATION FROM 9

Students who pass the course either do the homework or attend lecture;” “Bob did not attend every lecture;” “Bob passed the course.” Therefore “ Bob must have done the homework.”