

NEPAL COLLEGE OF INFORMATION TECHNOLOGY MODEL QUESTION

Level: Bachelor

Programme: BE

Course: Calculus I

Full Marks: 100

Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1a. A function is defined as follows

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$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 \leq x \leq 2 \\ x - \frac{x^2}{2} & \text{for } x > 2 \end{cases}$$

Is $f(x)$ continuous at $x = 2$? Does $f'(x)$ exist at this point?

b. If $y = (\sin^{-1} x)^2$, prove that $(1 - x^2) y_2 - xy_1 - 2 = 0$ and hence show that $(1 - x^2) y_{n+2} - (2n + 1) xy_{n+1} - n^2 y_n = 0$.

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c. State Lagranges mean value and give its geometrical meaning. Using it, show that, if $f(x) > 0$ in $[a, b]$ function $f(x)$ is increasing in $[a, b]$.

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2 a. Evaluate $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}}$

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b. Find the asymptotes for the curve $x^2(x-y)^2 - a^2(x^2+y^2) = 0$

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c. Define radius of curvature of the curve. Find radius of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at a point where it meets the line $y = x$.

OR

Trace the curve $y^2(2a-x) = x^3$

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3 Integrate for the following

3×5

a) $\int_0^1 \frac{\log x}{\sqrt{1-x^2}} dx$

b). Obtain the reduction formula for $\int \sec^n x \, dx$ and hence evaluate :
 $\int \sec^6 x \, dx$.

c). Show that $\int_0^\infty e^{-y^2} \sqrt{y} \, dy \times \int_0^\infty e^{-y^2} \frac{1}{\sqrt{y}} \, dy = \frac{\pi}{2\sqrt{2}}$.

4 a) Solve $\frac{dy}{dx} + \frac{1}{x} \sin 2y = x^3 \cos^2 y$ 5

b) Solve: $\frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + 2y = e^{2x}$ 5

c) Solve $y'' + 9y = \sec 3x$ by Wronskian method 5

5 a) Solve the following initial value problem.

$y'' + y' - 2y = 0, y(0) = 4, y'(0) = 1.$ 5

b)

6 a) Write down the criteria for a function $f(x, y)$ of two variables x & y to have maximum or minimum values at a point. Find the minimum value of $x^2 + y^2 + z^2$ such that $x + y + z = 3a$. 7

b) State and prove Eulers theorem for homogeneous function of two variables in x and y of degree n . If $u = \log \left(\frac{x^2 + y^2}{x + y} \right)$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 1$. 8

7 Solve the following: 5×2
.5=
10

a) Find domain and range of the function $y = \sqrt{(4-x)}$.

b) Evaluate $\lim_{x \rightarrow \pi/4} \frac{\sin x - \cos x}{x - \pi/4}$.

c) Find the arc length of the curve $y = x^2, -1 \leq x \leq 2$.

d) Evaluate $\int \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}} dx$