Computer Graphics (L06)
EG678EX

2-D Algorithms

Midpoint Circle Algorithm

- Circle function defined as:

$$f_{circle}(x,y) = x^2 + y^2 - r^2$$

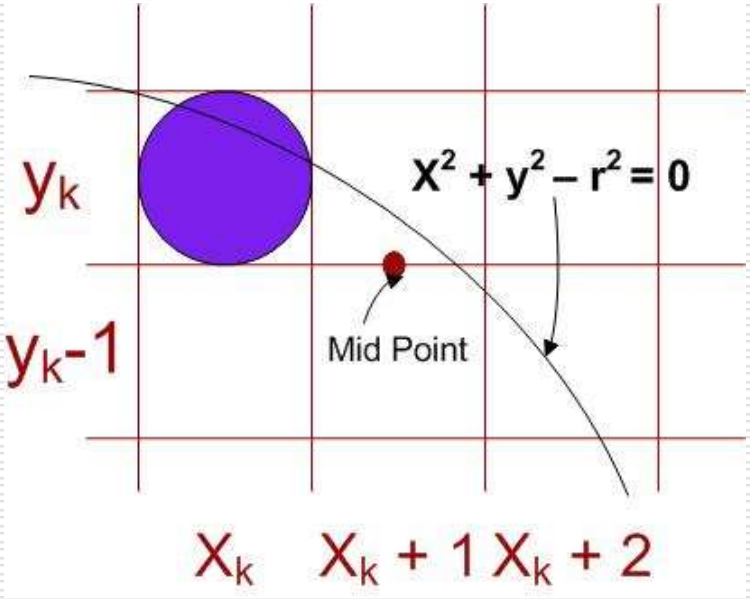
- Any point (x,y) satisfies following conditions

$$f_{circle}(x,y)\begin{cases} < 0, & if\ (x,y)\ \ is\ \ inside\ \ the\ \ circle\ \ boundary \\ = 0, & if\ (x,y)\ \ is\ \ on\ \ the\ \ circle\ \ boundary \\ > 0, & if\ (x,y)\ \ is\ \ outside\ \ the\ \ circle\ \ boundary \end{cases}$$

- Decision parameter is the circle function; evaluated as:

$$p_k = f_{circle}(x_k + 1, y_k - \frac{1}{2})$$

$$= (x_k + 1)^2 + (y_k - \frac{1}{2})^2 - r^2$$



$$p_{k+1} = f_{circle}(x_{k+1} + 1, y_{k+1} - \frac{1}{2})$$

$$= [(x_k + 1) + 1]^2 + (y_{k+1} - \frac{1}{2})^2 - r^2$$

$$p_{k+1} = p_k + 2(x_k + 1) + (y_{k+1}^{\ 2} - y_k^{\ 2}) - (y_{k+1} - y_k) + 1$$

$y_{k+1} = y_k$ *if* $p_k<0$

$y_{k+1} = y_k-1$ *otherwise*

- Thus

  $P_{k+1} = P_k + 2x_{k+1}+1$ *if* $pk<0$

  $P_{k+1} = P_k + 2x_{k+1}+1-2y_{k+1}$ *otherwise*

- Also incremental evaluation of $2x_{k+1}$ and $2y_{k+1}$

  $2x_{k+1} = 2x_k + 2$

  $2y_{k+1} = 2y_k - 2$ *if* $p_k >0$

- At start position $(x_0,y_0) = (0,r)$

  $2x_0 = 0$ *and* $2y_0 = 2r$

- Initial decision parameter

$$p_0 = f_{circle}(1, r - \frac{1}{2})$$
$$= 1 + (r - \frac{1}{2})^2 - r^2$$
$$= \frac{5}{4} - r$$

- For r specified as an integer, round p_0 to

$$P_0 = 1 - r$$

(because all increments are integers)

Algorithm

1. Input radius r and circle center (x_c, y_c) and obtain the first point on the circumference of a circle centered on the origin as
   $$(x_0, y_0) = (0, r)$$
2. Calculate the initial value of the decision parameter as
   $$P_0 = 5/4 - r$$
3. At each x_k position, starting at k = 0, perform the following test:
   If $p_k < 0$, the next point along the circle centered on $(0,0)$ is (x_{k+1}, y_k) and
   $$P_{k+1} = p_k + 2x_{k+1} + 1$$
   Otherwise, the next point along the circle is (x_k+1, y_K-1) and
   $$P_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$$
   Where $2x_{k+1} = 2x_k + 2$ and $2y_{k+1} = 2y_k - 2$
4. Determine the symmetry points in the other seven octants.
5. Move each calculated pixel position (x,y) onto the circular path centered on (x_c, y_c) and plot the co-ordinate values:
   $$x = x + x_c, \quad y = y + y_c$$
6. Repeat steps 3 through 5 until $x \geq y$