Level: Bachelor semester: Fall	Year: 2022

Programme: SE   Full Marks:100
Course: Engineering mathematics 1   Pass marks: 45
   SET: B   Time   : 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all questions.

1 (a) Prove that differentiability of the function f(x) at x=a implies continuity at x=a. But converse is not always true.   [5]

OR

Check the continuity and differentiability of the function

$$f(x) = \begin{cases} x, \, for, x < 1 \\ 2 - x, \, for, 1 \le x < 2 \\ -2 + 3x - x^2 \end{cases}$$

At x=1 and x=2.

(b) Show that $\dfrac{b-a}{b} < \log\left(\dfrac{b}{a}\right) < \dfrac{b-a}{a}$ by using Lagrange's Mean value theorem.   [5]

(c) Find the expansion of $\sec x$ in Maclaurin's series.   [5]

2 (a)   Find asymptotes of the curve: $x^2(x-y)^2 - a^2(x^2 + y^2) = 0$ [8]

OR

Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at (0, 0).

(b) Find reduction formula for: $\displaystyle\int_0^{\frac{\pi}{2}} \cos^n x\, dx$ and evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \cos^7 x\, dx$ [7]

3 (a) Integrate: $\displaystyle\int \frac{xe^x}{(x+1)^2}\, dx$ [5]

(b) Evaluate: $\displaystyle\int_0^1 \frac{\log x}{\sqrt{1-x^2}}\, dx$ [5]

(c) Find the area bounded by $y + x = 2$ on the left $y = x^2$ and below by x-axis   [5]

4 (a) Find the volume of solid generated by revolution of the region bounded by $x^2 = 4y$ and $y = |x|$ about y= -2   [7]

<div align="center">OR</div>

 Find the area of the surface generated by revolving the curve $y^2 = x$, $4 \le x \le 9$ about x-axis.

 (b)Find absolute maxima and minima of $f(x, y) = x^2 + y^2 - 2x$ in triangular region with vertices (2, 0), (0, 2) and (0, -2).

OR

 If the sum of dimension of a rectangular pool is given. Prove that the amount of water in the pool is maximum when it is cube.   [8]

5 (a) Solve: $\dfrac{dy}{dx} + \dfrac{y}{x} \log y = \dfrac{y(\log y)^2}{x^2}$ [7]

(b) Define Riccati,s equation.   Solve $\dfrac{dy}{dx} = y^2 - \dfrac{y}{x} - \dfrac{1}{x^2}$; $y(1) = 2$ [8]

6 (a)   Solve: $x^2 y'' - xy' + 2y = x \log x$.   [7]

  (b)   Solve: $y'' + 2y' + y = e^{-x}$; $y(0) = -1, y'(0) = 1$.   [8]

7. Answer the following : (2×5)

(a) Find nth order derivative of $y = (a - bx)^m$

 (b) Plot parametric curve $x^2 y^2 = x^2 - 1$

(c) If $\sin u = \dfrac{x^2 - y^2}{x^2 + y^2}$.   Find value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$

(d). Verify Euler's theorem for $u = \log\left(\dfrac{x^2 + y^2}{xy}\right)$

(e) Find arc length of parabola $y = \cos x$, from x= 0 to $\dfrac{\pi}{2}$