

## Model question of calculus-I

F.M: 100

P.M: 45

### ATTEMPT ALL QUESTIONS

1(a) Define the continuity of the function  $f(x)$  at  $x=a$ . what value should be assigned to the constant 'a' to make the function

$$f(x) = \begin{cases} x^2 - 1 & \text{for } x < 3 \\ 2ax & \text{for } x \geq 3 \end{cases} \quad \text{is continuous at } x = 3 \quad [7]$$

(b) show that every differentiable function is continuous but converse may not always true [8]

Or

If  $y = (\sin^{-1} x)^2$  prove that  $(1-x^2)y_2 - xy_1 - 2 = 0$  and hence show that  $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$

2(a) State the mean value theorem also interpret geometrically. Verify Rolles theorem for the function  $f(x) = Ax^2 + Bx + C$  in  $(a, b)$  [8]

(b) Find the asymptotes of the curve

$$y^4 - 2xy^3 + 2x^3y - x^4 - 3x^3 + 3x^2y + 3xy^2 - 3y^3 - 2x^2 + 2y^2 - 1 = 0 \quad [7]$$

Or

Trace the curve  $x = a(t + \sin t)$ ,  $y = a(1 + \cos t)$

3. Attempt any three question  $\{3 \times 5 = 15\}$

(a) Prove that  $\int_0^1 \frac{\log x}{\sqrt{1-x^2}} dx = \frac{\pi}{2} \log\left(\frac{1}{2}\right)$

(b) Prove that  $\int_0^1 \frac{1}{(1-x^6)^{\frac{1}{6}}} dx = \frac{\pi}{3}$

(c) Evaluate  $\int_0^a \sqrt{\frac{a-x}{x}} dx$

(d) Obtained the reduction formula for  $\int \cos^n x \, dx$  and then evaluate

$$\int_0^{\frac{\pi}{2}} \cos^7 x \, dx$$

(4)a. Find the volume of solid generated by revolving the asteroid

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}} \quad [8]$$

b. Find the arc of the parabola  $y = x^2$  from (1,2) to (2,4) is rotated about the y-axis find the area of resulting surface. [7]

(5)(a) State and prove Euler's theorem for the homogeneous function of two variables. Verify If  $v = \log \frac{(x^2 + y^2)}{(x + y)}$  prove that  $x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} = 1$  [8]

(b) find the extreme value of  $f = x^2 + y^2 + z^2$  such that  $x + z = 1$  and  $2y + z = 2$ . [7]

(6) (a) solve:  $x \frac{dy}{dx} + y = y^2 \log x$  [7]

(b) solve  $y'' + y = e^x + x^3$ ,  $y(0) = 2$ ,  $y'(0) = 2$  [8]

7. short question

(a) solve  $\ln \left( \frac{dy}{dx} \right) = ax + by$

(b) solve  $y'' - y = 0$

(c) solve  $\frac{dy}{dx} = 10 - x$   $y(0) = -1$

(d) Define differential equation with examples.