POKHARA UNIVERSITY

Level: Bachelor | Semester: Spring | Year : 2015
Programme: BE | | Full Marks: 100
Course: Engineering Mathematics III | | Pass Marks: 45
| | | Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

a) Define consistency of the system of linear equations. Check consistency of : x + y + z= 8, x − y + z= 6 , 2x-y+z=8. If it is consistence, find its solution by Gauss Elimination method. 8

b) Define Eigen values and vectors of a square matrix with its characteristics equation. If the eigen values and the corresponding eigenvector of the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. 7

a) Show that the infinite series 8

$\sum_{n=1}^{\infty}\frac{1}{n^p}$ *is convergent if* $p > 1$ *and divergent if* $0 < p \leq 1$.

b) Find the centre, radius of convergence and interval of convergence of the power series $\sum_{n=0}^{\infty}\frac{(x-5)^n}{n5^n}$. 7

a) Using Simplex method, maximize $z=150x_1+300x_2$ subject to theconstant $2x_1 + x_2 \leq 16, x_1 + x_2 \leq 8, x_2 \leq 3.5, x_1 \geq 0, x_2 \geq 0$ 8

b) Define periodic function. Find the fourier series representation of the periodic function $f(x) = \frac{x^2}{2}$ for $-\pi \leq x \leq \pi$ and then show that $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \ldots\ldots\ldots = \frac{\pi^2}{6}$ 7

a) Prove that the necessary and sufficient condition for the vector 8 function $\vec{a}$ of scalar variable t to have constant direction is $\vec{a} \times \frac{d\vec{a}}{dt}$.

b) Define gradient of sclar function. If $\phi = x^3+y^3+z^3-3xyz$, find (gradϕ): and curl (gradϕ)

5. a) Evaluate $\iint_s \vec{F} \cdot \vec{n}\ dA$ if $\vec{F} = [x^2, e^y, 1]$

$S: x+y+z=1; x \geq 0, y \geq 0, z \geq 0.$

b) State Stokes theorem. Evaluate $\oint_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (z, x, y)$, S : hemisphere $z = (a^2 - x^2 - y^2)^{\frac{1}{2}}$.

6. a) Construct the dual problem corresponding to optimum problem minimize: $Z = 8x_1 + 9x_2$ subject to $x_1 + x_2 \geq 5$, $3x_1 + x_2 \geq 21$, $x_1 \geq$, $x_2 \geq 0$ and solve it by simplex method.

b) Find Fourier sine as well as cosine series representation of the half range function $f(x) = e^x$ for $0 < x < L$.

7. Write short notes on: **(Any two)**

a) Find the directional derivative of the scalar valued function $f(x) = x^2 + y^2$, at (1, 2) in the direction $\vec{a} = 2\vec{i} - \vec{j}$.

b) Prove, $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots\ldots\ldots\ldots$ is convergent

c) Let $T: R^2 \rightarrow R$ be defined by $T(x,y) = |x + y|$, check T is linear or not.

d) Show that the alternating series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$, is conditionally convergent series.

POKHARA UNIVERSITY

Level: Bachelor	Semester: Fall	Year : 2016
Programme: BE		Full Marks: 100
Course: Engineering Mathematics III		Pass Marks: 45
		Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Define basis of vector space. Check the following vectors form a basis of R^3 or not. 7

(1, 2, 1), (2,1,0), (1,-1, 2)

OR

Check whether the system of linear equations is consistent or not, if consistent solve it by using Gauss elimination method.

$$x + 6y + 2z = 0$$
$$7x + 3y + z = 13$$
$$x + 2y + 3z = 20$$

b) Find Eigen value and Eigen vector of the following matrix: 8

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

2. a) Show that the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots$ is conditionally convergent. 7

b) Find the center radius of convergence and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x-5)^n}{n\, 5^n}$ 8

OR

Find expansion of log (1 + sin x) as far as the term in x^4, by using Maclaurin expansion.

3. a) Find the Fourier series representation of the periodic function $f(x) = |x|\ for -\pi < x < \pi$. Using it show that 8

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\dots\dots=\frac{\pi^2}{8}$$

b) Find Fourier sine as well as cosine series of the function $f(x)=x$ for $0<x<L$

4. a) Define directional derivative of ϕ in the direction of $\vec{a}$. Find the directional derivative of $\phi = x^2 + 3y^2 + 4z^2$ in the direction $\vec{a} = -\vec{i} - \vec{j} + \vec{k}$ at P(1, 0, 0).

b) If $\vec{v} = x^2yz\,\vec{i} + xy^2z\,\vec{j} + xyz^2\,\vec{k}$, find div $(curl\,\vec{v})$ and curl $(curl\,\vec{v})$

5. a) Using Green's theorem, calculate $\int[(x^2+y^2)\vec{i} - 2xy\vec{j}].d\vec{r}$ along the rectangle bounded by y=0, y=b, x=0, x=a

b) State Guass Divergence Theorem and hence find $\iint \vec{F}.\vec{n}dA$, where $\vec{F}=(2x^2, \frac{y^2}{2}, -\cos \pi z)$ and S is the surface of the tetrahedron with vertices (0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1)

OR

State Stokes Theorem and using the theorem evaluate $\oint \vec{F}.d\vec{r}$ if $\vec{F} = (y^3, 0, x^3)$, along the boundry of the triangle (1, 0, 0), (0, 1, 0), (0, 0, 1)

6. a) Maximize $Z = 4x_1 + x_2 + 2x_3$ subject to the constraints $x_1 + x_2 +$ $x_3 \le 1, x_1 + x_2 - x_3 \le 0, x_1, x_2, x_3 \ge 0$

b) Minimize $z = 4x_1 + 3x_2$, subject to $2x_1 + 3x_2 \ge 1$, $3x_1 + x_2 \ge 4$, $x_1 \ge 0$, $x_2 \ge 0$, by using dual simplex method.

7. Attempt all

a) Show that vectors (1, 0, 0), (0, 1, 0) and (0, 0, 1) are linearly independent

b) Find the rank of $A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 0 & 3 \\ 0 & 8 & 7 \end{bmatrix}$

c) Check the following transformation is linear or not $T: R^2 \to R^2$ be defined by $T(x, y) = (x + 3, y)$

d) Test for convergence of the series $\sum \frac{n+1}{2n+3}$

POKHARA UNIVERSITY

Level: Bachelor
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Course: Engineering Mathematics III

Semester: Spring

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Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Define Eigen values and vectors of a square matrix with its characteristics equation. Find the Eigen values and the corresponding eigenvectors of the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. 8

b) Define consistency of the system of the linear equations. Check consistency of: x + y + z = 8, x − y + z = 6, 2x − y + z = 8. If it is consistence, find its solution by Gauss Elimination method. 7

2. a) Prove the necessary condition for the convergence of an infinite series Σu_n is $\lim_{n\to\infty} u_n = 0$ but this not sufficient. 8

b) Find the interval, center and radius of convergence of an infinite series $\sum_{n=1}^{\infty} \frac{2^n(x+4)^n}{n}$. 7

OR

Find the Maclaurin series representation of $y = e^{\sin^{-1}x}$, up to x^4 terms.

3. a) Find fourier series of $f(x) = x + |x|$ for $-\pi < x < \pi$. 8

b) Define periodic function with suitable example. Find the fourier series of the periodic function $f(x) = \frac{x^2}{2}$ for $-\pi < x < \pi$. Using it show that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\dots\dots\dots=\frac{\pi^2}{6}$. 7

4. a) Prove that the necessary and sufficient condition for a vector valued function $\vec{r}$ of scalar variable t to have a constant magnitude is 8

$\bar{r}\cdot\frac{d\bar{r}}{dt}=0.$

b) Define gradient of a scalar valued function. Find the directional derivative of the surface $f=xy^2+yz^3$ at P (2, -1, 1) along the direction of normal to the surface $x\log z-y^2+4=0$ at the point (1, 1, 1).

5. a) Evaluate the line integral $\oint_C [(x^3-3y)dx+(x+\sin y)dy]$, where C : the boundary of a triangle with vertices (0,0),(1,0),(0,2) along anticlockwise direction.

b) Evaluate $\iint_S (\vec{F}\cdot\vec{n})dA$, where $\vec{F}=4x\vec{i}-2y^2\vec{j}+z^2\vec{k}$ and S is the surface of the region bounded by $x^2+y^2=4$, z = 0, z = 3.

OR

State Stokes theorem. Evaluate $\oint_F \vec{F}\cdot d\vec{r}$,

where $\vec{F}=4z\vec{i}-2x\vec{j}+2x\vec{k}$, C is the circle $x^2+y^2=1$, z=y+1.

6. a) Maximize $z=x_1+x_2+x_3$ subjected to the constraints

$4x_1+5x_2+8x_3\le 12; 8x_1+5x_2+4x_3\le 12; x_1\ge 0; x_2\ge 0.$

b) Minimize $z=4x_1+3x_2$ such that

$2x_1+3x_2\ge 1; 3x_1+x_2\ge 4; x_1\ge 0; x_2\ge 0$, by constructing duality

7. Attempt all

a) Test the convergence and divergence of series $\frac{2}{9}+\frac{3}{16}+\frac{4}{25}+\cdots$

b) Let T: $R^2\to R$ be defined by T (x,y) = $|x+y|$, Check T is linear or not.

c) Find the rank of the matrix: $\begin{bmatrix}1&0&0\\2&2&0\\-3&-2&1\end{bmatrix}$

d) If $\vec{r}=\vec{a}e^{nt}+\vec{b}e^{-nt}$ where $\vec{a}$ and $\vec{b}$ are constant vectors. Show that

$\frac{d^2\vec{r}}{dt^2}-n^2\vec{r}=0.$

POKHARA UNIVERSITY

Level: Bachelor
Semester: Fall
Year : 2017
Full Marks: 100
Programme: BE
Pass Marks: 45
Course: Engineering Mathematics III
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) When a set of simultaneous equations is said to be inconsistent? Test for consistency and solve using Gauss elimination method. 8
-x+3y-2z =7, 3x+3z = -3, 2x+y+2z = -1

b) Find eigen values and eigen vector of the Matrix: 7

A = $\begin{pmatrix}2&0&1\\0&2&0\\1&0&2\end{pmatrix}$

2. a) Test the convergence and divergence of the infinite series $\sum[\sqrt{n^3+1}-\sqrt{n^3-1}\,]$ 8

b) Find the interval and radius of convergence of the power series $\sum_{n=0}^{\infty}\frac{x^{2n+1}}{n!}$ 7

OR

Find expansion of $e^{\sin^{-1}x}$ as far as the term in x^4, by using Maclaurin expansion.

3. a) Find the Fourier series of $f(x)=x^2$ for $-\pi<x<\pi$ and hence find the value of $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+$-------- 8

b) Find the Fourier sine and cosine series of the function $f(x)=x$ for $0<x<\pi$. 7

4. a) Maximize $z=4x_1+x_2+2x_3$, subject to $x_1+x_2+x_3\le 1$, $x_1+x_2-x_3\le 0$, $x_1\ge 0$, $x_2\ge 0$ and $x_3\ge 0$ by using simplex method. 8

b) Construct the dual problem corresponding to the optimum problem : 7

Minimize $z=4x_1+3x_2$ subject to
$4x_1+5x_2 \ge 1$; $3x_1+x_2 \ge 4$; $x_1 \ge 0$, $x_2 \ge 0$ and solve it by using simplex method.

5. a) A particle moves on the curve $x = t^3+1$, $y = t^2$, $z = 2t+5$ where t is the time. Find the component of velocity and acceleration at t = 1 in the direction of $\vec{i}-\vec{j}+3\vec{k}$.

b) Define Divergence and Curl of a vector. If $\phi = \log(x^2+y^2+z^2)$ find div (grad ϕ) and curl (grad ϕ).

6. a) State Greens theorem in plane. Evaluate $\oint_C [5xydx + x^3 dy]$, where C is the closed curve consisting of the graph of $y = x^2$ and $y = 2x$ between the points (0, 0) and (2, 4).

OR

State Stoke's theorem. Using the theorem evaluate $\oint_C \vec{F}\cdot d\vec{r}$, where $\vec{F} = [-5y, 4x, z]$, C: circle $x^2+y^2 = 25, z = 1$.

b) Evaluate $\iint_s \vec{F}\cdot\vec{n}\ dA$ if $\vec{F}=[x^2, e^y, 1]$

S: $x+y+z=1$, $x \ge 0, y \ge 0, z \ge 0$

7. Attempt all

a) Find the rank of $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 5 \\ 2 & 4 & 8 \end{bmatrix}$

b) Find the inverse of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

c) Check the following transformation is linear or not?
$T: R^2 \to R^2$ be defined by $T(x, y) = (x + 3, y)$

d) Test the convergence and divergence of infinite series
$$\frac{2}{9}+\frac{3}{16}+\frac{4}{25}+\cdots$$

POKHARA UNIVERSITY

Level: Bachelor
Semester: Spring
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Pass Marks: 45
Course: Engineering Mathematics III
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) State the condition for a set of simultaneous equations to be consistent? Show that the set of simultaneous equations is consistent and solve it using Gauss elimination method. 7
$3x-y+z=-2$, $x+5y+2z=6$, $2x+3y+z=0$

b) Find the eigen values and eigen vectors of the marix 8
$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

2. a) Find the directional derivative of $f = 4xz^3-3x^2yz^2$ in the direction of z-axis at P (2,-1,2). 7

b) Show that the vector $\vec{F} = (yz)\vec{i} + (xz)\vec{j} + (xy)\vec{k}$ is conservative vector field and find the function ϕ such that $\vec{F} = \nabla\phi$. 8

3. a) Find the flux integral of $\vec{F} = (x, y, z)$ through the surface S, where S is the portion of the plane $2x + 3y + z = 6$ in first octant. 7

b) State Stoke's Theorem and apply it to evaluate $\oint_C \vec{F}.dr$ where $\vec{F} = yz\,\vec{\imath} + xy\,\vec{\jmath} + xz\,\vec{k}$ 8

OR

Using divergence theorem find $\iint_S \vec{F}\cdot\hat{n}\,ds$ where $\vec{F} = y^2e^z\vec{\imath} - xy\vec{\jmath} + x\tan^{-1}y\,\vec{k}$ and S is the surface of the region bounded by the coordinate planes and the plane $x + y + z=1$.

4. a) State and prove P-test for hyperharmonic series. 7

b) Find the radius of convergence and interval of convergence of the infinite series: 8

POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Engineering Mathematics III

Semester: Fall

Year : 2018
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Check the consistency of the system of the equations and solve 7
$2x+5y+6z=13$
$3x+y-4z=0$
$x-3y-8z=-10$

b) Define Eigen values and vectors of a square matrix with its characteristics equation. Find the Eigen values and the corresponding eigenvectors of the matrix 8
$$\begin{bmatrix}3 & 1 & 4\\0 & 2 & 6\\0 & 0 & 5\end{bmatrix}$$

2. a) Define basis of vector space over a given field F. Show that $\{(1,1,1),(1,2,1),(2,3,3)\}$ forms a basis for R^3. 7

b) Apply simplex method to Minimize $Z = 5x_1 - 20x_2$ subjected to the constraints; $-2x_1+10x_2 \le 5$, $2x_1+5x_2 \le 10$, $x_1, x_2 \ge 0$ 8

3. a) Prove that if an infinite series $\sum u_n$ is convergent then $\lim_{n\to\infty} u_n = 0$. By taking suitable example show that $\sum u_n$ is not convergent even if $\lim_{n\to\infty} u_n = 0$. 7

b) Find the center radius of convergence and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n(2x-1)^n}{n6^n}$ 8

OR

Use Maclaurin's theorem to expand $f(x) = e^{\sin x}$ in powers of x upto four terms.

4. a) Find fourier series of $f(x) = x + |x|$ for $-\pi < x < \pi$. 7

b) Find the fourier cosine series as well as sine series of the function $f(x) = x^2$ for $0 < x < L$ 8

$$\sum_{n=0}^{\infty} \frac{(x-4)^n(n+1)}{10^n}$$

OR

Find expansion of $e^{\sin x}$ upto the forth power of x by using Maclaurin's expansion.

5. a) Find the fourier series of the periodic function $F(x) = x - x^2$ for $(-\pi < x < \pi)$

b) Find Fourier sine as well as cosine series of the function $f(x) = \pi - x$ for $0 < x < \pi$.

6. a) Maximize the total output $z = x_1 + x_2 + x_3$ subject to input constraints $4x_1 + 5x_2 + 8x_3 \le 12$, $8x_1 + 5x_2 + 4x_3 \le 12$

b) Construct the dual problem corresponding to the optimum problem
Minimize $z = 20x_1 + 30x_2$ subject to
$x_1 + 4x_2 \ge 8$; $x_1 + x_2 \ge 5$; $2x_1 + x_2 \ge 7$, $x_1 \ge 0$, $x_2 \ge 0$
and solve it by using simplex method

7. Attempt all questions

a) Prove that product of two odd functions is an even function

b) Check the following transformation is linear or not? $T: R^2 \to R^2$ defined by $T(x, y) = (x, -2y)$

c) If $f(x,y,z) = xyz$, show that $\nabla.(\nabla f) = 0$

d) Test the convergence of the series $1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \ldots$

5. a) Find the dual of given LPP and solve by using simplex me
Minimize $z = x_1+8x_2+5x_3$ subject to $x_1+x_2+x_3 \geq 8$, $-x_1+2x_2+x_3$
$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$.

b) Find the directional derivative of $f = xy^2 + yz^3$ at (2,-1,1) along direction of the normal to the surface $x\log z - y^2 + 4 = 0$ at (-1,2

6. a) Evaluate $\iint_S \vec{F}.\vec{n}dA$ where $\vec{F} = (x^2, e^y, 1)$ where S is the po of the plane $x + y + z = 1$ lying in the first octant.

OR

Using Stokes Theorem evaluate $\oint_C \vec{F}.\overrightarrow{dr}$ where $\vec{F} = (y, \frac{z}{2}, \frac{3y}{2})$ C is the circle $x^2 + y^2 + z^2 = 6z,\ z = x + 3$.

b) Show that the value under integral sign $\int_{(1,0,2)}^{(-2,1,3)} (6xy^3 + 2z^2)dx$ $9x^2y^2\,dy + (4xz + 1)dz$ is exact and evaluate it.

7. Attempt all questions.

a) Check the following transformation is linear or not?
$T:R^2 \to R^2$ be defined by $T(x, y) = (x + 2, y)$

b) Test the convergence and divergence of series $\frac{2}{9} + \frac{3}{16} + \frac{4}{25} + \cdots$

c) Find the acceleration of the curve $\vec{r} = (t, t^2, t^3)$ at t=1.

d) Check if the function $f(x) = x^3$ for $\frac{-\pi}{2} < x < \frac{3\pi}{2}$ is odd or even.

POKHARA UNIVERSITY

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Semester: Spring
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Programme: BE
Full Marks: 100
Course: Engineering Mathematics III
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) State condition for a set of simultaneous equations to be consistent? Show that the set of the equations are consistent and solve by Gauss Elimination method. $3x-y+z=0$, $x+5y+2z=6$, $2x+3y+z=0$. 7

b) State Cayley -Hemilton Theorem Find $A^{-1}$by using it. 8

$$A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$$

2. a) By using Simplex method, Maximize $z = 20x_1 + 20x_2$ subject to $-x_1 + x_2 \leq 1$, $x_1+3x_2 \leq 15$, $3x_1+x_2 \leq 21$, $x_1 \geq 0$, $x_2 \geq 0$. 7

b) Construct the dual problem corresponding to the optimum problem: Minimize $Z= 8x_1+9x_2$, subject to $x_1+x_2 \geq 5$, $3x_1+x_2 \geq 21$, $x_1, x_2 \geq 0$. Also solve it by using simplex method. 8

3. a) Show that the alternating series $u_1 - u_2 + u_3 - u_4 + \ldots$ in which each term is numerically less than the preceding term and $\lim_{n \to \infty} u_n = 0$, is convergent. And using it show that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$ is convergent. 7

b) Find the radius of convergence and interval of convergence of the infinite series: 8

$$\sum_{0}^{\infty} \frac{10^{n+1}}{3^{2n}} x^n$$

OR

Find expansion e^x secx, by using Maclaurin expansion.

4. a) Find the Fourier series of $f(x)=\frac{\pi x^3}{2}$ for $0<x<2\pi$

b) Find the fourier sine series for $f(x) = 2 - x$ for $o < x < 2$ and hence deduce $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots=\frac{\pi}{4}$

5. a) Show that a necessary and sufficient condition for a vector function of a scalar variable to have a constant magnitude is $\vec{a}.\frac{d\vec{a}}{dt}=0$

b) Define directional derivative of the function f in the direction $a^{\to}$. Find directional derivative of $f = xy^2 + yz^3$ at $(2,-1,1)$ along the direction of the normal to the surface

S: $x\log z - y^2 + 4 = 0$ at $(-1,2,1)$.

6. a) State Greens theorem in plane. Evaluate $\oint[5xydx + x^3dy]$, where C is the closed curve consisting of the graph of $y = x^2$ and $y = 2x$ between the points (0, 0) and (2, 4).

b) Evaluate $\oint_c (\vec{F}.d\vec{r})$ by using stokes theorem where $\vec{F} = -3y\vec{i} + 3x\vec{j} + z\vec{k}$ and circle (c): $x^2+y^2=4$, $z=1$

7. Attempt all the questions:

a) Check the linearly dependence and independence of the vectors {(1,1,1), (1,2,1), (2,3,3)}

b) Find the rank of Matrix A = $\begin{bmatrix}1 & 4 & 5\\ 2 & 0 & 3\\ 0 & 8 & 7\end{bmatrix}$

c) Find the Tangent vector of the curve $\vec{r} = (t, t^2, t^3)$ at t = 1

d) Prove , $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+$..................... is convergent

POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Engineering Mathematics III

Semester: Fall

Year : 2019
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Check whether the system of linear equations is consistent or not, if consistent solve it by using Gauss elimination method 8

$$x+6y+2z=0$$
$$7x+3y+z=13$$
$$X+2y+3z=20$$

b) Define eigen value and eigen vector. Find eigen values and eigen vectors of the matrix. $\begin{bmatrix}3 & 1 & 4\\ 0 & 2 & 0\\ 0 & 0 & 5\end{bmatrix}$ 7

2. a) Show that if the infinite series $\sum u_n$ is convergent then $\lim_{n\to\infty} u_n = 0$. With a suitable example, prove that the converse may not be true. 7

b) Find the interval of convergence of the power series $\sum_{n=0}^{\infty}\frac{10^{n+1}x^n}{3^{2n}}$. 8

OR

Find the expansion of $e^{\sin x}$ upto the fourth power of x by using Maclaurin Expansion.

3. a) If a particle is moving with acceleration $12\cos 2t\vec{i} - 8\sin 2t\vec{j} + 16t\vec{k}$ at time t, find its velocity $\vec{v}$ and displacement $\vec{r}$ at time t. Given that $\vec{v}=\vec{0}$ and $\vec{r}=\vec{0}$ when $t=0$. 7

b) Define Divergence and Curl of a vector. If $\phi = \log(x^2+y^2+z^2)$ find div (grad ϕ) and curl (grad ϕ). 8

4. a) Define surface integral of $\vec{F}$ on the surface S. Evaluate $\iint_S \vec{F}.\hat{n}\,ds$ where $\vec{F}=(x^2, e^x, 1)$, where S is the surface $x+y+z=1$, $x\geq 0, y\geq 0, z\geq 0$.

b) Find $\iint_S \vec{F}\bullet\vec{n}\;dA$ where $\vec{F}=(4x, x^2y, -x^2z)$, S is the surface of the tetrahedron with vertices (0,0,0), (1,0,0), (0,1,0), (0,0,1)

5. a) Find Fourier series of the function $f(x)=x^2\;for\;-\pi<x<\pi$ and hence deduce $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots=\frac{\pi^2}{6}$

b) Find the Fourier sine as well as cosine series representation of the half range function $f(x)=x^2$ for $0<x<1$.

6. a) Using simplex method, Maximize $z=5x_1+3x_2$ subject to $x_1+x_2\leq 2$, $5x_1+2x_2\leq 10$, $3x_1+8x_2\leq 12$, $x_1\geq 0$ $x_2\geq 0$.

b) Construct the dual problem corresponding to the optimum problem: Minimize $z=8x_1+9x_2$ subject to $x_1+x_2\geq 5$, $3x_1+x_2\geq 21$, $x_1\geq 0$, $x_2\geq 0$, and solve it by using simplex method.

7. Attempt all questions:

a) Find the rank of $A=\begin{pmatrix}1 & 4\\ 2 & 0\end{pmatrix}$.

b) Test the convergence and divergence of series $\frac{2}{9}+\frac{3}{16}+\frac{4}{25}+\ldots\ldots\ldots$

c) Check odd, even or neither of the function:

$$f(x)=\begin{cases}\frac{1}{7}+x & for\quad -\frac{1}{2}<x<0\\ \frac{1}{7}-x & for\quad 0<x<\frac{1}{2}\end{cases}$$

d) Evaluate $\int_{(-1,2)}^{(3,1)}[(y^2+2xy)dx+(x^2+2xy)dy]$

POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Engineering Mathematics III

Semester: Spring

Year : 2019
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Solve by using Gauss elimination method 7
$4x-8y+3z=16$, $-x+2y-5z=-21$, $3x-6y+z=7$

b) Using cayley Hamiltan Theorem find the inverse of 8

$$A=\begin{bmatrix}7 & -1 & 3\\ 6 & 1 & 4\\ 2 & 4 & 8\end{bmatrix}$$

2. a) State and prove the leibnitz's theorem for alternating series and hence test the convergence of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^3+1}$ 7

b) Find the centre, radius and interval of convergence : 8

$$\sum_{1}^{\infty}\frac{(-1)^n(2x-1)^n}{n6^n}$$

OR

Define Maclaurin series of function f(x) and find the expansion of **tanx** upto three terms and hence obtain the expansion of **$\sec^2$x.**

3. a) A function f(x) defined by $f(x)=x^2$ for $0\leq x\leq L$ Find the Fourier cosine series. 7

b) Find Fourier sine as well as cosine series of the function $f(x)=x$ for $0<x<1$ 8

4. a) Find the directional derivative of the function $f=x^2-y^2+2z^2$ at the point $P(1,2,3)$ in the directional of line PQ where Q is the point (5,0,4) 7

b) If θ is the acute angle between the surfaces $xy^2z=3x+z^2$ and

$3x^2-y^2+2z=1$ at the point (1,-2,1). Show that $Cos\,\theta=\frac{3}{7\sqrt{6}}$

5. a) Evaluate the line integral $\oint_C \vec{F}.d\vec{r}$, where $\vec{F}$ = (1, xy, yz) and S is the surface $x^2+y^2 \le z,\ y \ge 0,\ z \le 4$

b) State Gauss divergence theorem and using it, evaluate $\iint_s \vec{F}\cdot\hat{n}ds$, where $\vec{F}=2x^2y\vec{i}-y^2\vec{j}+4xz^2\vec{k}$ and S is the region bounded by the cylinder $y^2+z^2=3$ and $0 \le x \le 2$, $y \ge 0$ and $z \ge 0$.

6. a) Solve the following Linear programming problem using the simplex method.

Maximize $z=30x_1+20x_2$ subject to:

$-x_1+x_2 \le 5$

$2x_1+x_2 \le 10$

$x_1, x_2 \ge 0$

b) Solve the linear programming problem by Simplex method constructing its duality: Minimize $z = 20x_1 + 30x_2$ Subject to $x_1 + 4x_2 \ge 8$, $x_1+x_2 \ge 5$, $2x_1 + x_2 \ge 7$, $x_1 \ge 0$, $x_2 \ge 0$

7. Attempt all the questions.

a) Check the following transformation is linear or not ?

$T:R^2 \to R^2$ be defined by T(u, v) = (u , v +3)

b) Find the unit tangent vector to the curve $\vec{r} = (t, t^2, t^3)$ at t = 1

c) Test the convergence of the series: $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+\ldots\ldots$

d) Evaluate the integral

$$\int_{(-1,2)}^{(3,1)}[(y^2+2xy)dx+(x^2+2xy)dy]$$

POKHARA UNIVERSITY

Level: Bachelor	Semester: Fall	Year : 2020
Programme: BE		Full Marks: 100
Course: Engineering Mathematics III		Pass Marks: 45
		Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) What do you mean by consistency of system of linear equations? Is a system of linear equations $x+3y+6z=2$, $3x-y+4z=9$, $x-4y+2z=7$ consistent? If it is consistent system, find its solution. 8

b) State Cayley –Hemilton theorem. Find A^{-1} by using it. 7

$$A=\begin{pmatrix}2 & 5 & 3\\ 3 & 1 & 2\\ 1 & 2 & 1\end{pmatrix}$$

2. a) Prove that the necessary and sufficient condition for a vector valued function $\vec{r}$ of scalar variable t to have a constant magnitude is $\vec{r}.\frac{d\vec{r}}{dt}=0$. A particle moves on the curve $\vec{r}=2t^2\vec{i}+(t^2-4t)\vec{j}+(3t-5)\vec{k}$, where t is the time. Find the component of velocity and acceleration at time t = 1 in the direction of $\vec{i}-3\vec{j}+2\vec{k}$. 8

b) Define gradient of a scalar valued function. Find the directional derivative of the surface $f=xy^2+yz^3$ at P (2, -1, 1) along the direction of normal to the surface $x\log z-y^2+4=0$ at the point (1, 1, 1). 7

3. a) State Green's Theorem in plane. Evaluate $\oint[5xydx+x^3dy]$, where C is the closed curve consisting of the graph of $y=x^2$ and $y=2x$ between the point (0, 0) and (2, 4). 7

b) Evaluate the line integral $\oint_C \vec{F}\cdot d\vec{r}$ by using Stoke's theorem, where $\vec{F} = 4y\vec{i} - 2z\vec{j} + 6y\vec{k}$ and C is the circle $x^2 + y^2 + z^2 = 6z$, $z = x + 3$

OR

Evaluate the surface integral $\iint_S \vec{F}.\vec{n}\ dA$ where

$\vec{F}(x, y, z) = [x^2, y^2, z^2]$, and S is the surface given by

$\vec{r}(u, v) = (u\cos v, u\sin v, 3v)$, $0 \le u \le 1$, $0 \le v \le 2\pi$

4. a) State and prove D'Alembert's ratio test. Test the convergence and divergence of the infinite series

$$\sum_{n=1}^{\infty}\left[\sqrt{n^2+1} - n\right]$$

b) Find the centre, radius and interval of convergence :

$$\sum_{1}^{\infty}\frac{(-1)^n(2x-1)^n}{n6^n}$$

OR

Expand the function $e^{ax}\cos bx$ upto the third power of x in Maclaurin series by assuming the validity of expansion.

5. a) Solve the linear programming problem: Maximize $z = 300x_1 + 500x_2$ subject to $2x_1 + 8x_2 \le 60$, $2x_1 + x_2 \le 30$, $x_1 + x_2 \le 15$, $x_1 \ge 0$, $x_2 \ge 0$.

b) Solve the linear programing problem, Minimize $z = 8x_1 + 9x_2$ subject to $x_1 + x_2 \ge 5$, $3x_1 + x_2 \ge 21$, $x_2 \ge 0, x_1 \ge 0$ $\bar{x}_2 \ge 0$, using simplex method, by constructing the duality.

6. a) Write the Fourier coefficients of a function f(x). Find the fourier series representation of the periodic function $f(x) = \frac{x^2}{2}$ for $-\pi \le x \le \pi$ and hence show that $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots\dots\dots = \frac{\pi^2}{6}$ and

$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \dots\dots\dots = \frac{\pi^2}{12}$.

b) Find Fourier series of $f(x)=x-x^2$ for $-\pi < x < \pi$.

4×2.5

7. Attempt all questions:

a) Check the following transformation is linear or not?
$T: R^2 \to R^2$ be defined by $T(m, n) = (m, n+3)$

b) Show that the matrix $\begin{bmatrix} i & 2+i & 3-i \\ -2+i & 2i & 2 \\ -3-i & -2 & -i \end{bmatrix}$ is skew Hermitian.

c) Evaluate $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

d) Test the convergence of series $\sum \frac{n+1}{2n+3}$

POKHARA UNIVERSITY

Level: Bachelor
Program: BE
Course: Engineering Mathematics III

Semester – Spring

Year: 2020
Full Marks: 70
Pass Marks: 31.5
Time: 2 hrs.

Candidates are required to answer in their own words as far as practicable. The figures in the margin indicate full marks.

Group - A: Attempt all questions (5×10=50)

Q. N. 1 Let 2x+3y+5z = 9, 7x+3y-2z = 8 and 2x+3y+az = b be the system of equations. 1.5+1.5+6+1

(i) What is consistency and how is consistency determined?

(ii) What is the difference between rank of a vector and matrix?

(iii) In above case, what can be the values of a and b so that the system has unique solution, infinite solution and no solution?

(iv) How infinite solution is geometrically interpreted?

OR

1+2+3+4

Given the matrix $A=\begin{pmatrix} 8 & -6 & 2 \\ -6 & -7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$

(i) State Caley Hamilton theorem

(ii) Find the characteristics equation and the eigenvalues for A

(iii) Verify Cayley-Hamilton theorem

(iv) Also find an inverse of A using Cayley-Hamilton theorem

Q. N. 2 Let the infinite series be $f(x) = \frac{1}{2}x + x^2 + \frac{9}{8}x^3 + x^4 + \frac{25}{32}x^5 + ----$ 2+1+3+4

(i) What is convergence and divergence in a series? What will be its application in engineering?

(ii) Sate P test in testing the convergence and divergence foe a series?

(iii) Use appropriate method with its statement to test the convergence of the given series f(x)

(iv) Find the interval, center and radius of convergence of the power series $\sum_0^\infty \frac{(n+1)(x-2)^n}{10^n}$

Q. N. 3 3+2+5

Let $f(x) = \begin{cases} \frac{1}{2} + x; & -\frac{1}{2} < x \le 0 \\ \frac{1}{2} - x; & 0 < x < \frac{1}{2} \end{cases}$

(i) What is periodicity and how is it determined? Explain with an example. What is the periodicity of above function?

(ii) Test whether the given function is odd or even or neither.

(iii) Find the Fourier series expansion of the given function f(x)

Q. N. 4 Let $x_1 + x_2 + x_3 \ge 100$, $2x_1 + 3x_2 + 10x_3 \ge 100$, $x_1 \ge 0$; $x_2 \ge 0 x_3 \ge 0$, be the set of linear inequalities 2+3+5

(i) What is the literal meaning of constructing a dual in simplex problems?

(ii) If the minimizing function is $C = 10x_1 + 50x_2 + 20x_3$, What will be its dual?

(iii) Solve the dual problem and obtain the value of C.

Q. N. 5 Given the vector $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$,

(i) Determine the angle between the tangents of the curve $\vec{r}$ if $x = t^2, y = 3t^2 and\ z = -(t^2 + 1)$ at $t = \pm 1$. 3+3+

(ii) Find the velocity and acceleration if $x = 3t^2$, $y = t^2 - 4t$ and $z = 3t+4$.

(iii) What will be the velocity and acceleration at $t = 3$ in the direction of $\vec{i} = 2\vec{j} + 2\vec{k}$

Group - B: (1×20=20)

Q. N. 6 i) Define surface integral of $\vec{F}$ on the surface S. Evaluate $\iint_S \vec{F}.\hat{n}\,dA$ where $\vec{F} = (x, 2y, 4z)$ and S is the surface of the cone $\sqrt{x^2 + y^2} \leq z, 0 \leq z \leq 2$ 2+(

ii) Evaluate $\oint_C \vec{F}.d\vec{r}$ where $\vec{F} = \left(y, \frac{z}{2}, \frac{3y}{2}\right)$, C is the circle of $x^2 + y^2 + z^2 = 4z$, $z = x + 2$. 6

iii) If $F = (2x^2 - 4z)\vec{i} - 2xy\vec{j} - 8x^2\vec{k}$, then evaluate $\iiint \nabla \cdot F\, dV$ where V is bounded by the planes $x = 0, y = 0, x = 0, x + y + z = 1$ 6

Best of Luck!

POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Engineering Mathematics III

Semester: Fall

Year : 2021
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Define consistence and inconsistence of a system of linear equations. Check consistence of a system of linear equations, $5x+3y+7z = 4$, $3x+26y+2z = 9$ and $7x+2y+10z = 5$ and solve it if possible. 7

   b) Find inverse of A with the help of Cayley Hamilton theorem. 8

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{pmatrix}$$

2. a) State and Prove hyper-harmonic series (p-Test). 7

   b) Find the interval , center and radius of convergence of an infinite series 8

$$\sum[\frac{n}{2^n}(x+4)^n]$$

OR

Find the Maclaurian Series representation of $y = e^{sin^{-1}x}$ upto x^4 terms.

3. a) Find the Fourier series of the following: 7

$$f(x) = \begin{cases} x & if \ \frac{-\pi}{2} < x < \frac{\pi}{2} \\ \pi - x & if \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$$

   b) Find the Fourier cosine as well as sine series of 8

$f(x) = L-x$ for $0< x < L$

4. a) Maximize $Z = x_1 + x_2 + x_3$,Subjected to the Constraints $4x_1 + 5x_2 + 8x_3 \leq 12, 8x_1 + 5x_2 + 4x_3 \leq 12, x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$ 7

   b) Construct the dual problem. 8

   Minimize $Z = 4x_1 + 7x_2$ Subject to $4x_1 + 4x_2 \geq 20$, $3x_1 + x_2 \geq 21$, $x_1 \geq 0$, $x_2 \geq 0$, and solve by simplex method.

5. a) If $\vec{a} = 18\cos 3t\vec{i} - 8\sin 2t\vec{j} + 6t\vec{k}$ be the acceleration of a particle at time t. If the velocity $\vec{v}$ and displacement $\vec{r}$ be zero at $t = 0$, f[ind] $\vec{v}$ and $\vec{r}$ at any point t.

b) If $\phi = x^3 + y^3 + z^3 - 3xyz$, find out $curl(grad\phi)$.and $div(grad\phi)$.

6. a) Evaluate $\iint_S (\vec{F}\cdot\vec{n})ds$, where $\vec{F} = 3x\vec{i} + 3y\vec{j} + z\vec{k}$, and S be the part of the surface $z = 9 - x^2 - y^2$, with $z \geq 0$

b) State stoke's Theorem, Evaluate $\oint_C \vec{F}.d\vec{r}$, where $\vec{F} = (2y^2, x, -z^3)$, ... circle $x^2 + y^2 = a^2, (z = b > 0)$

7. Solve ali questions:

a) Show that $\sum U_n = \dfrac{n+3}{4n+3}$ is divergent

b) Check linearity of T
T: $R^2 \rightarrow R^2$ be defined by $T(m, n) = (m , n - 3)$

c) If $\vec{r} = 5t^2\vec{i} + t\vec{j} - t^3\vec{k}$. Find $\int_1^2 \left(\vec{r} \times \frac{d^2\vec{r}}{dt^2}\right)dt$.

d) Find the period of $\cos\left(\frac{2\pi}{k}x\right)$

POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Engineering Mathematics III

Semester: Spring

Year : 2021
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) What is the condition of systems of linear equation to have unique solution? Check the consistency and solve 8
$x + 3y + 6z = 2,\ 3x - y + 4z = 9,\ x - 4y + 2z = 7.$

b) State Cayley-Hamilton theorem .Find A^{-1} by using it if 7

$$A = \begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$$

OR

Define Eigen value and vector of a square matrix. Find the Eigen value and corresponding Eigen vector of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

2. a) The necessary condition for the convergence of an infinite series $\sum u_n$ is , $\lim_{n\to\infty} u_n = 0$, but this is not sufficient. Test the absolute convergence of the series $\sum_1^\infty (-1)^{n+1}\frac{n}{n^3+1}$ 8

b) Find the center, radius, and interval of convergence of $\sum_1^\infty \frac{(-1)^n(2x-1)^n}{n6^n}$. 7

OR

Find expansion of log (1 + tan x) by using Maclaurins expansion.

3. a) Find the Fourier series of the periodic functions $f(x) = |x|$ for $-\pi<x<\pi$ also show that 7

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}.$$

POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Engineering Mathematics III

Semester: Fall

Year : 2022
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) State the conditions for a system of linear equations to be consistent and inconsistent. Discuss the consistency of the following system of equations and solve if possible. 7

$2x+3y+4z=11,\ x+5y+7z=15,\quad 3x+11y+13z=25$

b) Find Eigen values and the corresponding Eigen vectors of the matrix 8

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

2. a) Show that given series is conditionally convergent 7

$$\sum_{1}^{\infty}(-1)^{n+1}\frac{n}{n^2+1}$$

b) Find the interval, radius and centre of convergence of the series 8

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n^2}$$

OR

Find the expansion of log (1+sinx) upto x^4 by Maclaurin expansion

3. a) A particle moves along the curve $x=t^3+1,\ y=t^2,\ z=2t+5$. Find the components of its velocity and acceleration at $t=1$ in the direction of $\vec{i}+2\vec{j}+2\vec{k}$. 7

b) If θ is the angle between the surfaces $xy^2z=3x+z^2$ and $3x^2-y^2+2z=1$ at the point$(1,-2,1)$. Show that $\cos\theta=\frac{3}{7\sqrt{6}}$. 8

b) Find Fourier sine as well as cosine series of the function $f(x)=\pi-x$ for $0<x<\pi$.

4. a) Maximize: $Z=5x_1+4x_2$ Subjected to constraints : $x_1+x_2\le 20$, $2x_1+x_2\le 35$, $-3x_1+x_2\le 12$. by regular simplex method.

b) Minimize $Z=x_1+8x_2+5x_3$ subject to $x_1+x_2+x_3\ge 8$, $-x_1+2x_2+x_3\ge 2$. $\quad x_1\ge 0, x_2\ge 0, x_3\ge 0$ by constructing duality method.

5. a) Define directional derivative of f in the direction of $\vec{a}$. Find the directional derivative of $f=4xz^3-3x^2yz^2$ in the direction of z-axis at P(2,-1,2).

b) State Greens theorem in a plane. By using it evaluate $\int_c \vec{F}\cdot d\vec{r}$, where $\vec{F}=\sin y\,\vec{i}+\cos x\,\vec{j}$ and C is the positively oriented triangle with vertices $(0,0),(\pi,0),(\pi,1)$.

6. a) Evaluate $\iint_S \vec{F}.\vec{n}dA$ where $F=(x^2, e^y, 1)$, $S: x+y+z=1, x\ge0, y\ge0, z\ge0$.

b) State Stokes Theorem, Evaluate $\oint_C \vec{F}.\overrightarrow{dr}$

Where, $\vec{F}=(y,\frac{z}{2},\frac{3y}{2})$, C is circle, $x^2+y^2+z^2=6z$ and $z=x+3$

7. Attempt all questions:

a) Check the following transformation is linear or not ? $T:R^2\to R^2$ be defined by $T(m,n)=(m,n+3)$

b) Test the convergence of series $\sum\frac{n+1}{2n+3}$.

c) Find the rank of $A=\begin{pmatrix} 1 & 2 & 4 \\ 3 & 1 & 5 \\ 2 & 4 & 8 \end{pmatrix}$

d) If $\phi=\log(x^2+y^2+z^2)$ Find $div(grad\phi)$

4. a) Evaluate $\iint_S (\vec{F}\cdot\vec{n})dA$ where $\vec{F} = [18z, -12, 3y]$, S is the surface of the plane $2x + 3y + 6z = 12$ in the first octant.

b) State Gauss divergence theorem for the surface integral. Evaluate $\iint_S (\vec{F}.\vec{n})\, ds$, where $\vec{F} = (e^x, e^y, e^z)$ and S is the surface of the cube $|x| \le 1$, $|y| \le 1$ *and* $|z| \le 1$.

5. a) Find the Fourier series of $f(x) = |x|$ for $-2 < x < 2$

b) Find the Fourier series of $f(x) = x - x^2$ for $-\pi < x < \pi$. Hence show that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots\dots\dots = \frac{\pi^2}{12}$.

6. a) Maximize $Z = 40x_1 + 88x_2$ subject to $2x_1 + 8x_2 \le 60$, $5x_1 + 2x_2 \le 60$, $x_1 \ge 0$, $x_2 \ge 0$ using simplex method.

b) Constructing the dual of the given LPP, Find the minimum value for

$Z = 8x_1 + 9x_2$ subject to

$x_1 + x_2 \ge 5$, $3x_1 + x_2 \ge 21$, $x_1 \ge 0$, $x_2 \ge 0$

7. Attempt all question:

a) Check the following transformation is linear or not?

$T: R^2 \to R^2$ be defined by $T(x, y) = (x+3, y)$

b) Test the convergence or divergence of the series

$$\frac{2}{3^2} + \frac{3}{4^2} + \frac{4}{5^2} + \cdots$$

c) Check even, odd or neither of the function $f(x) = 3x(\pi^2 - x^2)$ for $-\pi < x < \pi$.

d) If $\vec{r} = \vec{a}e^{nt} + \vec{b}e^{-nt}$, show that $\frac{d^2\vec{r}}{dt^2} = n^2\vec{r}$.