Computer Graphics (L07)
EG678EX

2-D Algorithms

Ellipse Generating Algorithms

- Equation of ellipse:

$$d_1 + d_2 = constant$$

- F1→(x_1,y_1), F2→(x_2,y_2)

$$\sqrt{(x-x_1)^2+(y-y_1)^2}+\sqrt{(x-x_2)^2+(y-y_2)^2}=constant$$

- General Equation

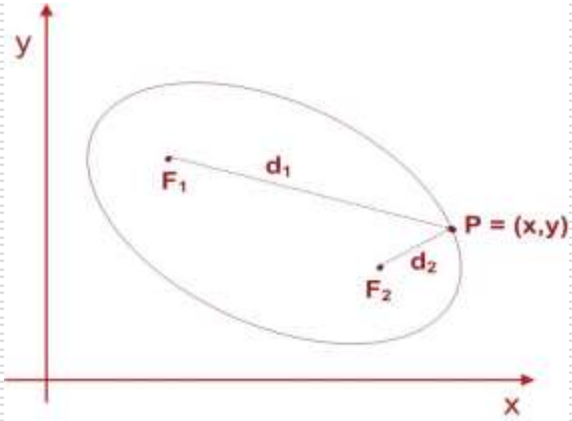
$$Ax^2+By^2+Cxy+Dx+Ey+F=0$$

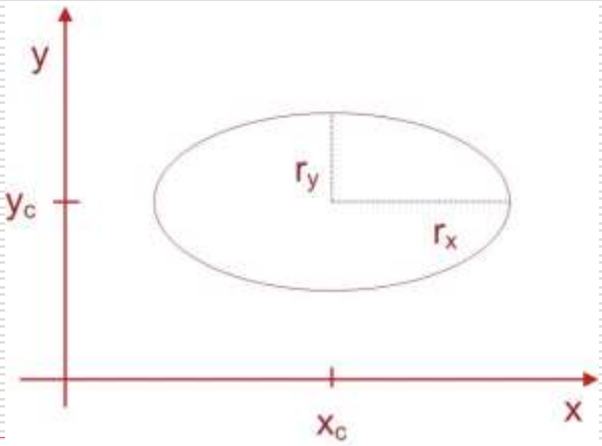
- Simplified Form

$$\left(\frac{x-x_c}{r_x}\right)^2+\left(\frac{y-y_c}{r_y}\right)^2=1$$

- In polar co-ordinate

$$x = x_c + r_x \cos\theta$$
$$y = y_c + r_y \sin\theta$$





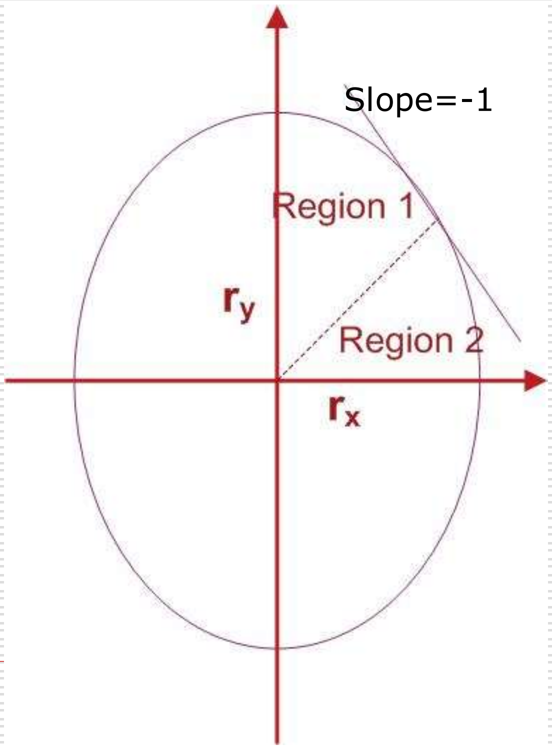
Ellipse function

$$f_{ellipse}(x,y) = r_y{}^2 x^2 + r_x{}^2 y^2 - r_x{}^2 r_y{}^2$$

$$f_{ellipse}(x,y)\begin{cases} < 0, & if\ (x,y) \quad is \quad inside \quad the \quad ellipse \quad boundary \\ = 0, & if\ (x,y) \quad is \quad on \quad the \quad ellipse \quad boundary \\ > 0, & if\ (x,y) \quad is \quad outside \quad the \quad ellipse \quad boundary \end{cases}$$

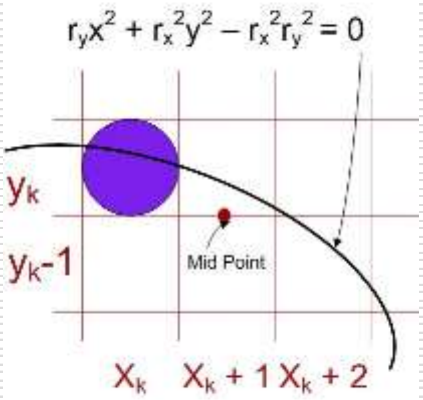
- From ellipse tangent slope: $\frac{dy}{dx} = -\frac{2r_y^2 x}{2r_x^2 y}$
- At boundary region (slope = -1): $2r_y^2 x = 2r_x^2 y$
- Start from $(0,r_y)$, take x samples to boundary between 1 and 2
- Switch to sample y from boundary between 1 and 2

  (i.e whenever $2r_y^2 x \geq 2r_x^2 y$)



- In the region 1

$$p1_k = f_{ellipse}(x_k + 1, y_k - \frac{1}{2})$$
$$= r_y^{\ 2}(x_k + 1)^2 + r_x^{\ 2}(y_k - \frac{1}{2})^2 - r_x^{\ 2} r_y^{\ 2}$$



- For next sample

$$p1_{k+1} = f_{ellipse}(x_{k+1} + 1, y_{k+1} - \frac{1}{2})$$
$$= r_y^{\ 2}\left[(x_k + 1) + 1\right]^2 + r_x^2 (y_{k+1} - \frac{1}{2})^2 - r_x^{\ 2} r_y^{\ 2}$$
$$p1_{k+1} = p1_k + 2r_y^2(x_k + 1) + r_y^2 + r_x^2\left[\left(y_{k+1} - \frac{1}{2}\right)^2 - \left(y_k - \frac{1}{2}\right)^2\right]$$

- Thus increment

$$increment = \begin{cases} 2r_y^{\ 2} x_{k+1} + r_y^2, & if \quad p1_k < 0 \\ 2r_y^2 x_{k+1} + r_y^2 - 2r_x^2 y_{k+1}, & if \quad p1_k \geq 0 \end{cases}$$

- For increment calculation; Initially:

$$2r_y^2 x = 0$$
$$2r_x^2 y = 2r_x^2 r_y$$

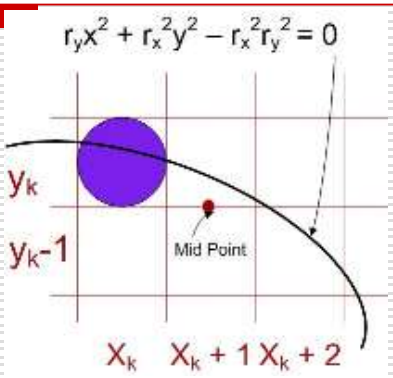
- Incrementally:
  Update x by adding $2r_y^2$ to first equation and update y by subtracting $2r_x^2$ to second equation

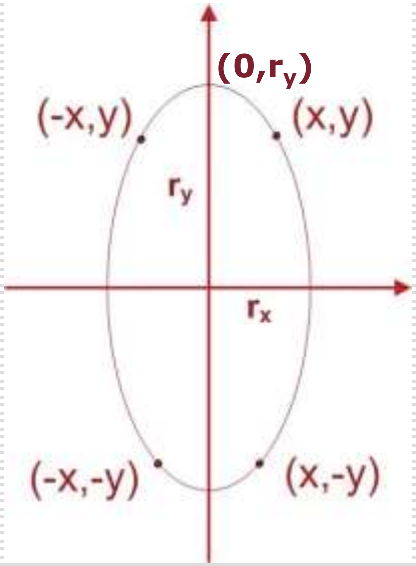
- Initial value

$$p1_0 = f_{ellipse}\left(1, r_y - \frac{1}{2}\right)$$

$$= r_y^2 + r_x^2\left(r_y - \frac{1}{2}\right)^2 - r_x^2 r_y^2$$

$$p1_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2$$





- In the region 2

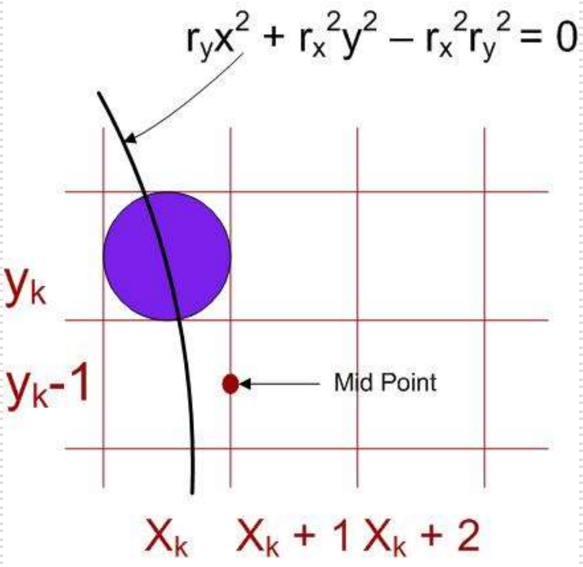
$$p2_k = f_{ellipse}(x_k + \frac{1}{2}, y_k - 1)$$
$$= r_y{}^2(x_k + \frac{1}{2})^2 + r_x{}^2(y_k - 1)^2 - r_x{}^2 r_y{}^2$$

- For next sample

$$p2_{k+1} = f_{ellipse}(x_{k+1} + \frac{1}{2}, y_{k+1} - 1)$$
$$= r_y{}^2\left(x_{k+1} + \frac{1}{2}\right)^2 + r_x^2[(y_k - 1) - 1]^2 - r_x{}^2 r_y{}^2$$
$$p2_{k+1} = p2_k + 2r_x^2(y_k - 1) + r_x^2 + r_y^2\left[\left(x_{k+1} + \frac{1}{2}\right)^2 - \left(x_k - \frac{1}{2}\right)^2\right]$$

- Initially

$$p2_0 = f_{ellipse}\left(x_0 + \frac{1}{2}, y_0 - 1\right)$$
$$p2_0 = r_y^2\left(x_0 + \frac{1}{2}\right)^2 + r_x^2(y_0 - 1)^2 - r_x^2 r_y^2$$



For simplification calculation of $p2_0$ can be done by selecting pixel positions in counter clockwise order starting at (r_x,0) and unit samples to positive y direction until the boundary between two regions

Algorithm

1. Input r_x,r_y, and the ellipse center(x_c,yc) and obtain the first point on an ellipse centered on the origin as

   $(x_0,y_0) = (0,r_y)$

2. Calculate the initial value of the decision parameter in region 1 as

$$p1_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2$$

3. At each x_k position in region 1, starting at k = 0, perform the following test: If $p1_k < 0$, the next point along the ellipse centered on (0,0) is (x_{k+1},y_k) and

$$p1_{k+1} = p1_k + 2r_y^2 x_{k+1} + r_y^2$$

   Otherwise, the next point along the ellipse is (x_k+1,y_k-1) and

$$p1_{k+1} = p1_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_y^2$$

   With

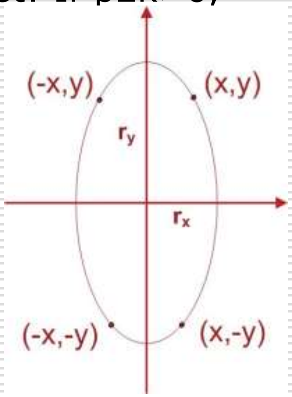
$$2r_y^2 x_{k+1} = 2r_y^2 x_k + 2r_y^2, \qquad 2r_x^2 y_{k+1} = 2r_x^2 y_k - 2r_x^2$$

   and continue until $2r_y^2 x \geq 2r_x^2 y$

4. Calculate the initial value of decision parameter in region 2 using the last point (x_0,y_0) calculated in region 1 as

$$p2_0 = r_y^2\left(x_0+\frac{1}{2}\right)^2 + r_x^2(y_0-1)^2 - r_x^2 r_y^2$$

5. At each y_k position in region 2, starting at k = 0, perform the following test: If p2k>0, the next point along the ellipse centered on (0,0) is (x_k,y_k-1) and



$$p2_{k+1} = p2_k - 2r_x^2 y_{k+1} + r_x^2$$

Otherwise the next point along the ellipse is (x_k+1,y_k-1) and

$$p2_{k+1} = p2_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_x^2$$

Using the same incremental calculations for x and y as in region 1.

6. Determine the symmetry points in the other three quadrants.
7. Move each calculated pixel position (x,y) onto the elliptical path centered on (x_c,y_c) and plot the co-ordinate values:

   $X = x + x_c$, $y = y + y_c$

8. Repeat the steps for region 1 until

$$2r_y^2 x \geq 2r_x^2 y$$