Probability and Statistics

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Hypothesis Testing

Introduction

Testing of hypothesis is a statistical method of making inferences about the population under study from the sample observations by testing a hypothesis whether it is to reject or to accept. In other words, a statistical test which is used to make decision about population parameter on the basis of sample observations is called hypothesis testing. The main objective of the hypothesis testing is to make decision whether to reject or to accept the hypothesis being tested, on the basis of sample data. If the sample data provide sufficient evidence against the hypothesis tested, the hypothesis will be rejected, otherwise the hypothesis will be accepted.

Hypothesis

Hypothesis is a statement about the population parameter which may or may not be true, but needs to test using the sample observations.

Hypothesis is a tentative statement or supposition about the estimated value(s) of one or more parameter(s) of the population. The methods of inference used to support or reject the hypothesis about population based on sample data are known as tests of significance.

Some Examples:

1. The average age of the students of NCIT is 20.
2. The average pass percentage of NCIT is more than 70.
3. The size of pillars affect the strength of the building.
4. The mean breaking strength of wire is 2000.

Null Hypothesis and Alternative Hypothesis

Null Hypothesis

According to Prof. R.A. Fisher, a null hypothesis is the hypothesis which is tested for possible rejection under the assumption that it is true.

A null hypothesis is defined as the hypothesis of no difference, which is under the test. It is denoted by H_0. Every significance test begins with a null hypothesis(H_0). It is formulated for the rejection on the basis sample observations drawn from the population. If the sample data do not provide sufficient evidence against H_0, we accept the H_0. If the sample data provide sufficient evidence against H_0, we reject the H_0 and accept H_1.

Some examples:

1. The mean breaking strength of wire is 2000.

$H_0: \mu = 2000, i.e. The\ population\ mean\ breaking\ strength\ of$

$wire\ is\ 2000$

2. The average pass percentage of NCIT is more than 70.

$H_0: \mu = 70, i.e. The\ average\ pass\ percentage\ is$ 70.

Alternative Hypothesis

Any hypothesis which is mutually exclusive and complementary to the null hypothesis is called alternative hypothesis. The alternative hypothesis is accepted whenever the null hypothesis is rejected. It is denoted by H_1 or H_A. The null hypothesis H_0 is tested against the alternative hypothesis H_1. In fact, the alternative hypothesis is the research hypothesis which is the researcher's operational hypothesis.

Alternative hypothesis is of different types:

1. One – tailed alternative hypothesis
2. Two – tailed alternative hypothesis

One – tailed alternative hypothesis

1. Right – tailed alternative hypothesis
2. Left – tailed alternative hypothesis

1. The mean breaking strength of wire is 2000.

- $H_0 : \mu = 2000, i.e. The\ population\ mean\ breaking\ strength\ of$

$wire\ is$ 2000

Vs,

The mean breaking strength of wire is 2000.

- $H_1 : \mu \neq 2000, i.e. The\ population\ mean\ breaking\ strength\ of$

$wire\ is\ not$ 2000 (two – tailed alternative)

1. The mean breaking strength of wire is 2000.

- $H_0 : \mu = 2000, i.e. The\ population\ mean\ breaking\ strength\ of$

$wire\ is\ 2000$

Vs,

The mean breaking strength of wire is 2000.

- $H_1 : \mu > 2000, i.e. The\ population\ mean\ breaking\ strength\ of$ $wire\ is\ more\ than\ 2000$ (right tailed alternative)

1. The mean breaking strength of wire is 2000.

- $H_0: \mu = 2000, i.e. The\ population\ mean\ breaking\ strength\ of$

 $wire\ is$ 2000

Vs

The mean breaking strength of wire is 2000.

- $H_1: \mu < 2000, i.e. The\ population\ mean\ breaking\ strength\ of$

 $wire\ is\ less\ than$ 2000 (left tailed alternative)

Selection of an Appropriate Statistical Tool (Test Statistic)

There are many statistical tools while performing hypothesis testing. However, for the rational decision about the hypothesis, appropriate statistical tool should be selected.

Some important points to be noted for the selection of appropriate statistical tools are as follows:

1. The nature of the population from which a sample is drawn.
2. The nature of measurement scale used in the operational definition for the variables.
3. The nature of sampling distribution of statistics.

Continue..

Test statistic used in the hypothesis testing is relative value for making appropriate decision about the hypothesis. It is defined as

$$\textbf{Test Statistic} = \frac{\textbf{Statistic} - \textbf{Parameter}}{\textbf{Standard Error}} = \frac{\textbf{Difference}}{\textbf{SE}}$$

A test statistic is defined as the ratio of difference between estimate (statistic) and hypothesized value of parameter to the standard error of the estimator. The test statistic enables the researcher to specify the critical region which helps in making decision, whether to reject or accept H_0. The test statistic has a great role in hypothesis testing.

Errors in Hypothesis Testing

In hypothesis testing, when an investigator makes a decision to reject or to accept the null hypothesis H_0 on the basis of sample information, he/she is liable to commit two types of errors. The possible errors are:

a. Type I error

b. Type II error

Type I error

The rejection of null hypothesis, when it is true is called type I error. That is, if true null hypothesis H_0 is rejected, it is said to be type I error. The probability of making a type I error is denoted by α and thus this error is also called alpha error. The probability of making type I error is fixed. This α is also known as the size of the critical region or level of significance.

$$\therefore \boldsymbol{\alpha = \mathrm{P}(\text{Type I error}) = \mathrm{P}(\text{reject } \mathrm{H}_0 \,/\, \mathrm{H}_0 \text{is true})}$$

Type II error

The acceptance of null hypothesis H_0 when it is false is called type II error. That is, if false null hypothesis H_0 is accepted, it is said to be type II error. The probability of making a type II error is denoted by β.

$$\therefore \boldsymbol{\beta} = \mathbf{P(Type\ II\ error)} = \mathbf{P(accept\ H_0\ /\ H_0 is\ false)}$$

Note:

It is noted that as the accepting of rotten egg is more harmful than the rejecting of a good egg, the degree of impact of type II error is more dangerous or harmful than that of type I error.

Critical Region and Acceptance Region

- All possible outcomes obtained from all possible sample constitute a space, called a sample space which is denoted by S or Ω
- All possible values of the test statistic form a sample space (S)
- The region of the sample space where the null hypothesis is rejected when it is true is called critical region, also known as rejection region which is denoted by W.

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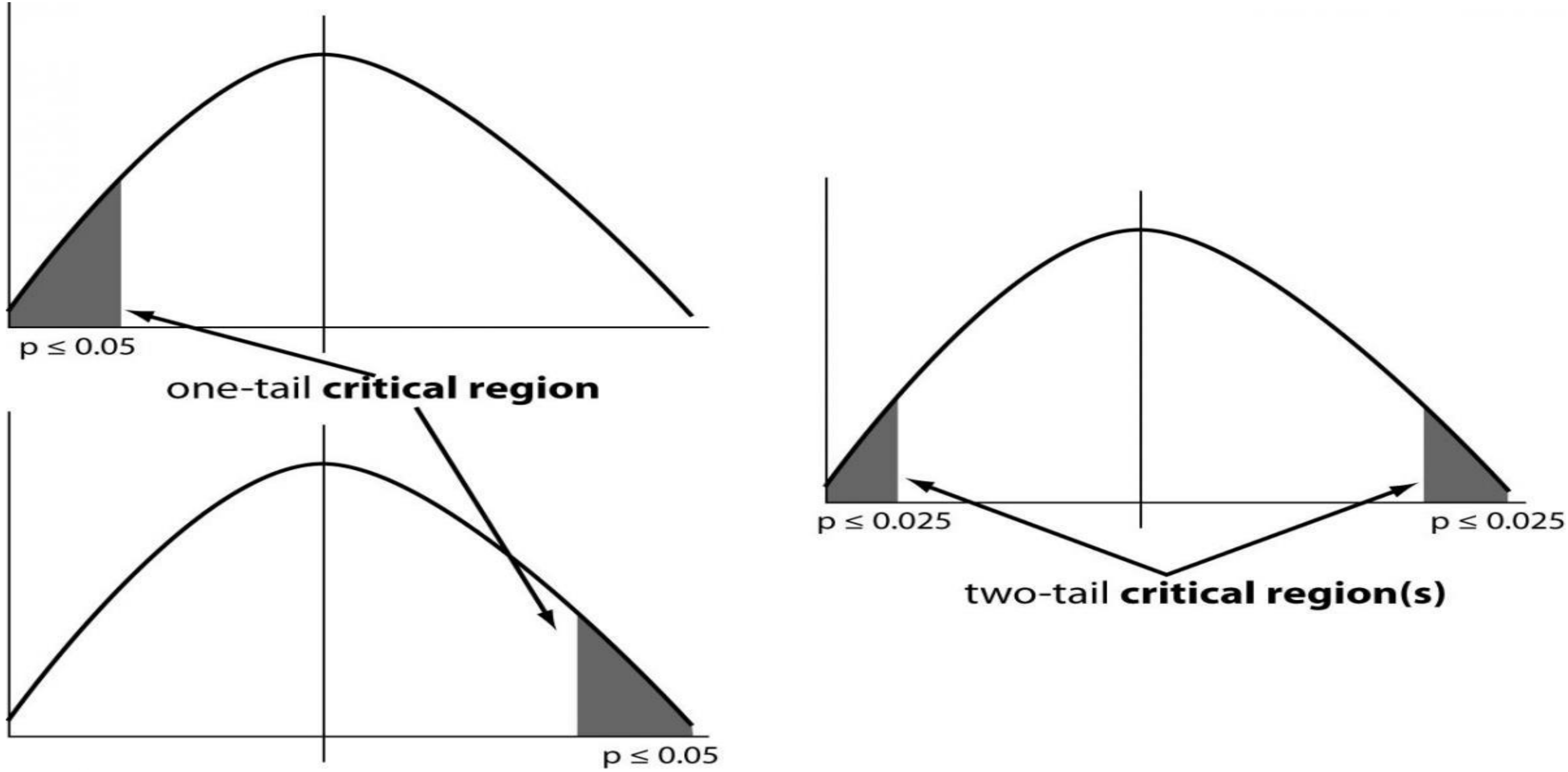
- Critical region plays a very important role in making decision whether to reject or to accept null hypothesis H_0 in testing of hypothesis. Since if the value of test statistic falls into the critical region, we reject H_0 otherwise accept H_0
- The mutually exclusive and complementary region to the critical region under sample space is acceptance region and is denoted by $\bar{W}$
- Therefore, S = W + $\bar{W}$
- The location of critical region depends upon the test employed in hypothesis testing.

Continue….

- When two-tailed test is employed, the critical region will be located on the two tails of the curve formed by the sampling distribution of the test statistic. In this case the critical region is equally divided on both sides of the curve.
- Similarly, when the one-tailed test (either right or left tailed) is employed, the critical region will be located on only one tail of the curve either on right tail or on left tail according to the test employed is right tailed or left tailed.



p ≤ 0.05
one-tail **critical region**
p ≤ 0.05
p ≤ 0.025
p ≤ 0.025
two-tail **critical region(s)**

Level of Significance

- The probability of rejecting null hypothesis H_0 when it is true is called level of significance.
- In other words, level of significance is the probability of type I error or the size of the critical region. That is, the level of significance is the probability used as the criterion for rejection of true null hypothesis H_0 and is denoted by α.
- Usually, the level of significance used in a test are 5% and 1%. Sometimes 10% also used.
- If the level of significance is fixed at 5%, then we are ready to take 5% risk of rejecting true H_0. In other words, α = 0.05 means the chances of making correct decision are 95 times out of 100 times and 5 times to make wrong decision.

One-Tailed Test and Two-Tailed Test

A test of statistical hypothesis depends upon the alternative hypothesis used in the test. A test is said to be one-tailed test if the alternative hypothesis used is one-tailed alternative hypothesis H_1. Similarly, a test is said to be two-tailed test if two-tailed alternative hypothesis is used in the test.

Precisely,

1. A test $H_0: \theta = \theta_0$
   Vs $H_1: \theta < \theta_0$ is called left tailed test.

2. A test $H_0: \theta = \theta_0$
   Vs $H_1: \theta > \theta_0$ is called right tailed test.

3. A test $H_0: \theta = \theta_0$
   Vs $H_1: \theta \neq \theta_0$ is called two tailed test.

Identification of One-Tailed and Two-Tailed Tests

Generally, if the direction of difference is not given in the statement of hypothesis, then we use two-tailed test. Similarly, if the direction of difference like at least, at most, increase, decrease, majority, minority, larger, taller, high, low, superior, inferior, improved, more than, less than, etc. is included in the statement of the hypothesis, then we use one-tailed test.

Critical Value

The points that divide the critical region and the acceptance region is known as critical values. The critical values depend upon the level of significance used and the alternative hypothesis which leads to one tailed and two tailed tests.

Steps involved in Hypothesis Testing

1. **Set up null hypothesis $\boldsymbol{H_0}$**
2. **Set up the alternative hypothesis $\boldsymbol{H_1}$**. (In alternative hypothesis whether we have to use one-tailed (right tailed or left tailed) test or two-tailed test).
3. **Selecting a significance level**. Generally, 1%, 5% and 10% are suggested level of significance. The level of significance is predetermined while performing hypothesis testing.
4. Identify the sample statistic to be used and its sampling distribution.

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4. Test Statistic. Define test statistic and compute it under H_0.

5. Obtain the critical value and critical region of the test statistic from the appropriate tables.

6. Conclusion Criteria.

a. If the calculated value of test statistic is less than the tabulated value at level of significance α% then H_0 is accepted and H_1 is rejected.

b. If the calculated value of the test statistic is greater than the tabulated value at α% then H_0 is rejected and H_1 is accepted.

Z – Test (large Sample Test i.e. $n \geq 30$)

Assumptions:

1. The population from which the samples are drawn is normally distributed.
2. The sample is randomly selected and independent.
3. The population variance is known.
4. The sample size is greater than or equal to 30 i.e. $n \geq 30$.

Z-test is used for

1. Test of significance for single mean
2. Test of significance of difference between two means (equality of two means)
3. Test of significance for single proportion
4. Test of significance of difference of two proportions (equality of two proportions)

Test of significance for single mean (μ)

Steps in Hypothesis Testing

Step 1: Setting hypothesis

$H_0: \mu = \mu_0, i.e. there\ is\ no\ difference\ between\ sample\ mean$ and population mean.

Against,

$H_1: \mu \neq \mu_0, (two - tailed\ test) i.e.$ There is some difference between sample mean and population mean.

$H_1: \mu > \mu_0, (right - tailed\ test)$

$i.e. the\ population\ mean\ is\ greater\ than\ specified\ mean\ \mu_0$

$H_1: \mu < \mu_0, (left - tailed\ test)$

$i.e. the\ population\ mean\ is\ smaller\ than\ specified\ value\ \mu_0$

<u>Step 2</u>: Test Statistic

Under the H_0, the test statistic is given by

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$|Z_{cal}| = \left|\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}\right|$ which follows standard normal distribution with mean 0 and variance 1.

<u>Note</u>: Modulus is taken for both one – tailed and two – tailed tests.

Step 3: Level of Significance

Usually we use 5% level of significance i.e. α = 0.05 otherwise given.

Step 4: Critical region or Critical value:

The critical value of Z-test at α = 0.05 for two – tailed test or One – tailed test is obtained from the SNT. $|Z_{tab}| = 1.96 \text{ } or \text{ } 1.645$ for 5% level of significance.

Step 5: Decision:

1. If calculated value $|Z_{cal}|$ is less than tabulated value $|Z_{tab}|$ $, we\ accept\ Ho$
2. If calculated value $|Z_{cal}|$ is more than tabulated value $|Z_{tab}|$, we reject H0 and accept H1.

Example:

A manufacturer of ball pens claims that **mean writing life of 400 pages with standard deviation of 20 pages**. A purchasing agent selects a sample **of 100 pens** and puts them for test. The mean writing life for the sample **was 390 pages**. Should the purchasing agent reject the manufacturer's claim at 5% level of significance?

Given, sample size, n = 100

Sample mean $\bar{x} = 390$

Population mean, $\mu = 400$

Population standard deviation, $\boldsymbol{\sigma}$ = 20

Step 1: Setting Hypothesis

$H_0: \mu = 400$

$i.e. the\ population\ mean\ writing\ life\ of\ ball\ pens\ is\ 400.$

Against,

$H_1: \mu \neq 400, (two - tailed\ test)$

$i.e. the\ population\ mean\ writing\ life\ is\ not\ 400$

<u>Step 2:</u> Test statistic

Under the H_0, the test statistic is given by

$$Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$$

$$Z = \frac{390-400}{20/\sqrt{100}} = -5$$

Or, $|Z_{cal}| = 5$, this is our calculated value.

Step 3: Level of significance.

For fixed $\alpha = 5\% = 0.05$

Step 4: Critical value or Critical region

The critical value of Z – test at $\alpha = 5\% = 0.05$ for two – tailed test is given by

$|Z_{tab}| = 1.96$, this is tabulated value.

Step 5: Decision

Since, calculated value $|Z_{cal}| = 5$ is greater than tabulated value $|Z_{tab}| = 1.96$, so we reject H0 and accept H1

Therefore, the purchasing agents rejects the manufacturer's claims.

Example:

A random sample of boots worn by 40 combat soldiers in a desert region showed an average life of 1.08 years with a standard deviation of 0.05 years. Under the standard conditions the boots are known to have an average life of 1.28 years. Is there reason to assert at a level of significance of 0.05 that use in the desert causes the mean life of boots to decrease?

Given, sample size, n = 40 (large Sample)

Sample mean $\bar{x} = 1.08$

Population mean, $\mu = 1.28$; (standard condition)

Sample Standard deviation, s = 0.05

<u>Step 1</u>: Setting Hypothesis

$H_0: \mu = 1.28$

$i.e. the\ average\ life\ of\ boot\ in\ standard\ condtion$

$is\ 1.28\ years.$

Against,

H_1: $\mu < 1.28$, $(One - tailed\ test$ or $left\ - tailed\ test)$

$i.e. the\ population\ average\ life\ of\ boot\ is\ less\ than\ 1.28\ years$

Step 2: Test statistic

Under the H_0, the test statistic is given by

$Z = \frac{\bar{x}-\mu}{s/\sqrt{n}}$ here, s is sample standard deviation (for large sample s is considered as unbiased estimate of $\boldsymbol{\sigma}$) .

$$Z = \frac{1.08-1.28}{0.05/\sqrt{40}} = -25.298$$

Or, $|Z_{cal}| = 25.298$, this our calculated value.

Step 3: Level of significance.
For fixed $\alpha = 5\% = 0.05$

Step 4: Critical value or Critical region
The critical value of Z – test at $\alpha = 0.05$ for one – tailed test is given by $|Z_{tab}| = 1.645$, this is tabulated value.

Step 5: Decision
Since, calculated value $|Z_{cal}| = 25.928$ is greater than tabulated value $|Z_{tab}| = 1.645$, so we reject H0 and accept H1
Therefore, we conclude that the use of boots in desert decreases the mean life of boots.

Example

The mean breaking strength of the cables supplied by the manufacture is 2000 with a standard deviation of 150. By a new technique in the manufacturing process, it is claimed that the breaking strength of the cables have been increased. In order to test his claim a sample of 100 cables was tested and the mean breaking strength turned out to be 2100. Can we support the claim at 1% level of significance?

Given, sample size, n = 100 (large Sample)

Sample mean $\bar{x} = 2100$

Population mean, $\mu = 2000$

Population Standard deviation, $\boldsymbol{\sigma}$ = 0.05

<u>Step 1</u>: Setting Hypothesis

$H_0 : \mu = 2000$

$i.e. the\ population\ mean\ breaking\ strength\ of\ the$

$cables\ is\ 2000.$

Against,

$H_1: \mu > 2000, (One - tailed\ test\ or\ right\ - tailed\ test)$

$i.e. the\ population\ mean\ breaking\ strength\ of\ the\ cables$

$is\ more\ than$ 2000.

<u>Step 2:</u> Test statistic

Under the H_0, the test statistic is given by $Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

$$Z = \frac{2100-2000}{150/\sqrt{100}} = 6.67$$

Or, $|Z_{cal}| = 6.67$, <u>this our calculated value.</u>

Step 3: Level of significance.
For fixed $\alpha = 1\% = 0.01$

Step 4: Critical value or Critical region
The critical value of Z – test at $\alpha = 0.01$ for one – tailed test is given by

$|Z_{tab}| = 2.33$, this is tabulated value.

Step 5: Decision
Since, calculated value $|Z_{cal}| = 6.67$ is greater than tabulated value $|Z_{tab}| = 2.33$, so we reject H0 and accept H1
Therefore, we conclude that the new technique increases the breaking strength of the cables.

Practice

1. A manufacturer claims that the mean life of batteries manufactured by his company is **44 months**. A random sample of 40 of these batteries was tested, resulting in a sample mean of 41 months with a sample standard deviation of 9 months. Test at $\alpha = 0.05$.
2. A moped manufacturer hypothesized that the mean miles per gallon for its moped is 115.2. It takes the samples of 49 mopeds and find the samples mean to be 117.4 miles per gallon. If the population standard deviation is known to 8.4, test the hypothesis that the true mean miles per gallon is 115.2 against the alternative hypothesis that it is greater than 115.2.

<u>Step 1</u>: Setting Hypothesis

$H_0: \mu = 44$

$i.e.\, the\ mean\ life\ of\ batteries\ is\ 44.$

Against

$H_1:\ \mu \neq 44,\ (two - tailed\ test\ )$

i.e. the mean life of batteries is not 44.

Step 2: Test statistic

Under the H_0, the test statistic is given by $Z = \frac{\bar{x}-\mu}{s/\sqrt{n}}$

$$Z = \frac{41-44}{9/\sqrt{40}} = -2.11$$

Or, $|Z_{cal}| = 2.11$

Step 3: level of significance

Fixed α = 0.05

Step 4: critical value

The critical value of Z – test at $\alpha = 0.05$ for two – tailed test is obtain from SNT as:

$|Z_{tab}| = 1.96$

Step 5: decision

Since, calculated value $|Z_{cal}| = 2.11$ is greater than tabulated value $|Z_{tab}| = 1.96$, so we reject H0 and accept H1.

Therefore, we can conclude that the mean life of batteries is not 44 months.

Given, sample size, n = 49

Sample mean, (xbar) = 117.4

Population mean, (u) = 115.2

Population standard deviation, $\boldsymbol{\sigma} = 8.4$

Step 1: setting hypothesis

H0: $u = 115.2$, i.e. the true mean miles per gallon is 115.2

Against,

H1 : $u > 115.2$, (one – tailed test) i.e. the true mean miles per gallon is greater than 115.2

Step 2: Test Statistic

Under H_0, the test statistic is given by $Z = \frac{\bar{x}-\mu}{\boldsymbol{\sigma}/\sqrt{n}}$

$|Z_{cal}| =$

Step 3 : level of significance

For fixed α = 0.05

Step 4 : critical value or critical region

The critical value of Z – test at α = 0.05 for one – tailed test is obtained from the SNT as:

$|Z_{tab}| = 1.645$

Step 5: Decision

Since cal

Test of significance of difference between two means (equality of two means)

Steps in Hypothesis Testing

<u>Step 1</u>: Setting Hypothesis

$H_0: \mu_1 = \mu_2$ $i.e.$ $there\ is\ no\ difference\ between\ two\ sample\ means.$

Against,

$H_1: \mu_1 \neq \mu_2$, $(two-tailed\ test)$ $i.e.$ $there\ is\ some\ difference\ between\ two\ sample\ means$

$H_1: \mu_1 > \mu_2$, $(right\ tailed\ test)$ $i.e.$ $the\ mean\ of\ first\ population\ is\ greater\ than\ the\ mean\ of\ second\ population.$

$H_1: \mu_1 < \mu_2, (left\ tailed\ test)\ i.e. the\ mean\ of\ first\ population\ is\ less\ than\ the\ mean\ of\ second\ population.$

Step 2: Test Statistic

Under H_0, the test statistic is given by

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$|Z_{cal}| = \left| \frac{\bar{x}_1 - \bar{x}_1}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \right|$ which follows standard normal distribution with mean 0 and variance 1.

Continue..

$\bar{x}_1$ is the mean of first sample

$\bar{x}_2$ is the mean of second sample

σ_1^2 is the variance of first population

σ_2^2 is the variance of second population

n_1 is the size of first sample

n_2 is the size of second sample

Sometimes, population variances are unknown i.e. σ_1^2 and σ_2^2 are unknown. In this case, we have

Continue…

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}, \text{ under } H_o$$

Here, s_1^2 is the variance of first sample

s_2^2 is the variance of second sample

When $\sigma_1^2 = \sigma_2^2 = \sigma^2$ (if sample is drawn from same population), then

$$Z = \frac{\bar{x}_1 - \bar{x}_1}{\sqrt{\sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}, \text{where } \sigma^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}$$

Remaining steps are same as previous.

Example

A consumer – research organization routinely selects several car models each year and evaluates their fuel efficiency. In this year's study of two similar subcompact models from two different automakers, the average gas mileage for 40 cars of brand A was 37.2 mpg, and the standard deviation was 3.8 mpg. The 50 brand B cars that are tested averaged 32.1 mpg, and the standard deviation was 4.3 mpg. At $\alpha = 0.01$, should it conclude that brand A cars have higher average gas mileage than brand B.

Solution

Given, for brand A

Sample size (n1) = 40

Sample mean, $\bar{x}_1 = 37.2$

Sample standard deviation, s1 = 3.8

Similarly, for brand B

Sample size, (n2) = 50

Sample mean, $\bar{x}_2 = 32.1$

Sample standard deviation, s2 = 4.3

Step 1: Setting hypothesis

$H_0 : \mu_1 = \mu_2$ $i.e. there\ is\ no\ difference\ between\ the\ average$ mileage per gallon of Brand A and Brand B.

Against,

$H_1 : \mu_1 > \mu_2, (one - \ tailed\ test)\ i.e. the\ average\ mileage\ of\ brand\ A\ is\ higher\ than\ average\ mileage\ of\ brand\ B.$

<u>Step 2</u>: Test Statistic

Under the H_0, the test statistic is given by

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$Z = \frac{37.2 - 32.1}{\sqrt{\frac{3.8\ \hat{}2}{40} + \frac{4.3\hat{}2}{50}}}$$

$$|Z_{cal}| = 5.96$$

Step 3: Level of significance

For fixed α = 0.01

Step 4: critical value

The critical value of Z – test at α = 0.01 for one – tailed test is obtained from SNT as

$|Z_{tab}| = 2.33$

Step 5: Decision

Since, calculated value $|Z_{cal}| = 5.96$ is greater than tabulated value $|Z_{tab}| = 2.33$, so reject H0 and accept H1.

Therefore, we can conclude that $the\ average\ mileage\ of$

$brand\ A\ is\ higher\ than\ average\ mileage\ of\ brand\ B.$

Practice

1. Two independent samples of observations were collected. For the first sample of 60 elements, the mean was 86 and the standard deviation 6. The second sample of 75 elements had a mean of 82 and standard deviation of 9. Test whether the two samples can reasonably be considered to have come from populations with the same mean. Use $\alpha = 0.01$
2. A company claims that its light bulbs are superior to those of the competitor on the basis of a study which showed that a sample of 40 of its bulbs had an average life time of 638 hours of continuous use with standard deviation of 27 hours. While the sample of 30 bulbs made by the competitor had an average life time of 619 hours of continuous use with standard deviation of 25 hours. Test whether this claim is justified.

Step 1: Setting hypothesis

$H_0: \mu_1 = \mu_2$ $i.e.$ the two samples drawn from the populations with same mean.

Against,

$H_1: \mu_1 \neq \mu_2, (two - \ tailed\ test)\ i.e. the\ two\ samples\ are\ drawn\ from\ the\ \ populations\ with\ different\ mean$

Solve yourself

Step 1: Setting hypothesis

$H_0: \mu_1 = \mu_2$ $i.e. there\ is\ no\ difference\ between\ the\ average$

Life time of the tube lights of two company.

Against,

$H_1: \mu_1 > \mu_2, (one - \ tailed\ test)$ $i.e. the\ average\ life\ time$

$of\ the\ tube\ lights\ of\ a\ company\ is\ superior\ than\ its\ competitor$.

Level of significance is not given in this case we use 5% level of significance.

Practice

1. The mean compressive strength of two independent large samples of steel is 7020 and 6980 psi (pound per square inch) with sample size 400 and 320 respectively. Can the samples be regarded as drawn from the same population of **standard deviation 554 psi**?
2. The mean yield of wheat from Lalitpur district was 210 lbs. with standard deviation 10 lbs. per acre from a sample of 100 plots. In Kathmandu district, the mean yield was 220 lbs. with standard deviation 12 lbs. from a sample of 150 plots. Assume that the standard deviation of the yield in the entire Bagmati zone was **11 lbs.** , test whether there is any significant differences between the mean yield of crops in the two districts.

Example 1

H0 : there is no difference between two sample means.

Vs

H1: there is some difference between two sample means.

Step 2: Test Statistic

$$Z = \frac{\bar{x}_1 - \bar{x}_1}{\sqrt{\sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}},$$

Remaining steps are same as previous

Example 2

Given, for Lalitpur

Sample size, n1 = 100

Sample mean, (xbar1) = 210

Sample standard deviation, s1 = 10

Similarly, for Kathmandu

Sample size, n2 = 150

Sample mean, (xbar2) = 220

Sample standard deviation, s2 = 12

Common standard deviation, $\boldsymbol{\sigma} = 11$

Step 1: Hypothesis setting

H0: u1 = u2, i.e there is no significant difference between the mean yields of wheat in this two districts.

Vs,

H1: u1≠u2 i.e. (two – tailed test) there is some significant difference between the mean yields of wheat in this two districts.

Step 2: Test Statistic

$$Z = \frac{\bar{x}_1 - \bar{x}_1}{\sqrt{\sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}},$$

Test of Significance For Single Proportion

- Population proportion (P) = $\frac{X}{N}$ = Proportion of success in Population.

Where, ‘X’ is the number of success in the population and ‘N’ is the size of population.

- And $Q = 1 - P$, known as Proportion of failure in population.
- Similarly, sample proportion (p) = $\frac{x}{n}$ = Proportion of success in sample.

Where, ‘x’ is the number of success in sample and ‘n’ is the size of sample

- And q = 1 – p , known as Proportion of failure in sample.

Steps in Testing of Hypothesis

Step 1 : Setting Hypothesis

$H_0: P = P_0, i.e. there\ is\ no\ difference\ between\ sample$

$proportion$ and population proportion.

Against,

$H_1: P \neq P_0, (two - tailed\ test) i.e.$ There is some difference between sample proportion and population proportion.

$H_1: P > P_0, (right - tailed\ test)$

$i.e. the\ population\ proportion\ is\ greater\ than\ specified$

$proportion\ P_0$

Continue..

$H_1: P < P_0, (left - tailed\ test)$

$i.e. the\ population\ proportion\ is\ smaller\ than\ specified\ proportion\ P_0$

<u>Step 2</u>: Test Statistic

Under the H_0, the test statistic is given by

$$Z = \frac{p-P}{\sqrt{\frac{PQ}{n}}}$$

$$|Z_{cal}| = \left|\frac{p-P}{\sqrt{\frac{PQ}{n}}}\right| \backsim N(0,1)$$

Remaining steps are same as previous.

Example

1. In a sample of 1000 people in Kathmandu district, 540 speak Nepali and rest speak Newari. Can we assume that both languages are equally popular in this district at 1% level of significance?
2. A member of public interest group concern with environment pollution asserts at public hearing that "fewer than 60% of the industrial plants in this area are complying with air pollution standards". The officials sample 60 plants and finds that 33 are complying with air pollution standard. Is the asserting by the member of public interest group a valid one? Test the hypothesis at 0.02 significance level.

Example 1

Given, sample size, n = 1000

No. of people who speak Nepali, x = 540

Sample proportion, p = x / n = 540 / 1000 = 0.54

Also, population proportion of people who speak Nepali, P = 0.5

Q = 1 – P = 0.5

Step 1 : setting hypothesis

$H_0: P = 0.5, i.e.\ the\ both\ languages\ are\ equally\ popular.$

$Vs,$

$H_1: P \neq 0.5, (two - tailed\ test) i.e. the\ both\ languages\ are\ not\ equally\ popular..$

Step 2: Test Statistic

Under H_0, the test statistic is given by

$$Z = \frac{p-P}{\sqrt{\frac{PQ}{n}}}$$

$$|Z_{cal}| = 2.53$$

Step 3: Level of significance

Fixed α = 0.01

Step 4 : critical region or critical value

The critical value of Z – test at α = 0.01 for two – tailed test is obtained from SNT as: $|Z_{tab}| = 2.58$

Step 5: Decision

Since, calculated value is less than tabulated value, so we accept H0 and reject H1.

Therefore, we can conclude that the both languages are equally popular.

Given, Population Proportion, P = 0.60 (assumed value)

Sample size, n = 60

No. of plants complying the air pollution standard, x = 33

Sample proportion, p = x / n = 33 / 60 = 0.55

Q = 1 – P = 1 – 0.6 = 0.4

Step 1 : Setting hypothesis

$H_0: P = 0.60\ i.e. the\ proportion\ of\ plants\ complying\ the\ air\ pollution\ standard\ is\ 0.60$

Against,

$H_1: P < 0.60\ (left\ - tailed\ test) i.e\ proportion\ of\ complying\ the\ air\ pollution\ standard\ is\ fewer\ than\ 0.60$

Step 2 : Test Statistic

Under H0, the test statistic is given by

$$Z = \frac{p-P}{\sqrt{\frac{PQ}{n}}}$$

$$|Z_{cal}| = 0.791$$

Step 3 : Level of significance

For fixed α = 0.02

Step 4: Critical value

The critical value of Z – test at α = 0.02 for one – tailed test is obtain from SNT as:

$$|Z_{tab}| = 2.05$$

Step 5 : Decision

Practice

1. Regardless of age, 20% of Nepalese adults participate in fitness activities at least twice a week. However, these fitness activities change as the people get older, and occasionally participants become non – participants as they aged. In local survey of 100 adults over 40 years old, a total of 15 people indicated that they participated in a fitness activity at least twice a week. Do these data indicate that the participation rate for adults over 40 years old age is significantly less than 20%?
2. A coin is tossed 600 times and heads appear 320 times. Does this result support the hypothesis that the coin is unbiased?

Given, sample size, n = 100

No. of adults who participates in fitness activity, x = 15

Sample proportion, p = x / n = 15 / 100 = 0.15

Also, population proportion of adults who participates in fitness activity, P = 0.20

Q = 1 – P = 1 – 0.20 = 0.8

Step1: Setting Hypothesis

$H_0: P = 0.20 \; i.e. \, the\ proportion\ of\ adults\ involved\ fitness$

$\text{acti}vities\ is\ 0.20$

Against,

$H_1: P < 0.20\ (left\ - tailed\ test) i.e\ proportion\ of\ adults\ involved\ in\ fitness\ activities\ is\ less\ than\ 0.20$

step 2: Test Statistic

Under H0, the test statistic is given by

$$Z = \frac{p-P}{\sqrt{\frac{PQ}{n}}}$$

$$|Z_{cal}| = 1.25$$

Step 3: Level of significance

For fixed α = 0.05

Step 4 : critical value

The critical value of Z – test at **α = 0.05 for one – tailed test** is obtained as

$|Z_{tab}| = 1.645$

Step 5: Decision

Since calculated value $|Z_{cal}| = 1.25$ is less than tabulated $|Z_{tab}| = 2.05$, so we accept H0.

Therefore, we can conclude that the proportion adults involved in fitness activities is 0.20.

Test of significance for difference of two proportions

Steps in Hypothesis Testing

Step 1: Setting Hypothesis

$H_0: P_1 = P_2$ $i.e. there\ is\ no\ difference\ between\ two$ $sample$ proportions.

Against,

$H_1: P_1 \neq P_2$, $(two\ -tailed\ test)$ $i.e. there\ is\ some\ difference\ between\ two\ sample\ proportions$

$H_1: P_1 > P_2, (right\ tailed\ test)$ $i.e. the\ proportion\ of\ first\ population\ is\ greater\ than\ the\ proportion\ of\ second\ population$.

Continue..

$H_1: P_1 < P_2, (left\ tailed\ test)\ i.e.\ the\ proportion\ of\ first\ population$
$is\ less\ than\ the\ proportion\ of\ second\ population.$

Step 2: Test Statistic

Under the H_0, the test statistic is given by

$$Z = \frac{p_1 - p_2}{\sqrt{\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}}}$$

$$|Z_{cal}| = \left|\frac{p_1 - p_2}{\sqrt{\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}}}\right| \backsim N(0,1)$$

Remaining steps are same as previous.

Continue..

$p_1 = \frac{x_1}{n_1}$, is the proportion of success in first sample

$p_2 = \frac{x_2}{n_2}$ is the proportion of success in second sample

$q_1 = 1 - p_1$

$q_2 = 1 - p_2$

x_1 is the number of success in first sample

x_2 is the number of success in second sample

n_1 is the size of first sample

n_2 is the size of second sample

Continue…

P_1 is the proportion of success in first population

P_2 is the proportion of success in second population

$Q_1 = 1 - P_1$

$Q_2 = 1 - P_2$

When Population proportions of success are unknown, then we use

$$Z = \frac{p_1 - p_2}{\sqrt{\hat{P}\hat{Q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Where, $\hat{P} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2}$ and $\hat{Q} = 1 - \hat{P}$

Example

1. In a certain large city 400 out of a random sample of 500 men was found to be smokers. After the tax on tobacco has been heavily increased, another random sample of 600 men in the same city included 400 smokers. Was the observed decrease in the proportion of smokers significant?
2. Random sample of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test whether the hypothesis that proportions of men and women in favours of the proposal are some against that they are not, at 5% level.

Example 1

Given, **<u>before tax in tobacco</u>**

Sample size, $n1 = 500$

No. of smokers, $x1 = 400$

Sample proportion, $p1 = x1 / n1 = 400 / 500 = 0.8$

‘$q1 = 1 – p1 = 1 – 0.8 = 0.2$

Similarly, **<u>after tax in tobacco</u>**

Sample size, $n2 = 600$

No. of smokers, $x2 = 400$

Sample proportion, p2 = x2 / n2 = 400 / 600 = 0.67

‘q2 = 1 – p2 = 1 – 0.67 = 0.33

Now,

$$\hat{P} = \frac{x_1+x_2}{n_1+n_2} = \frac{800}{1100} = 0.73$$

and $\hat{Q} = 1 - \hat{P}$ = 0. 27

Step 1: Setting Hypothesis

$H_0 : P_1 = P_2$ *i.e. there is no difference between the proportions of smokers before and after the tax in tobacco.*

Against,

$H_1 : P_1 > P_2, (right\ tailed\ test)\ i.e. the\ proportion\ of\ smokers$

$before\ the\ tax\ is\ more\ than\ the\ proportion\ of\ smokers\ after\ tax.$

<u>Step 2</u> : Test Statistic

Under H0, the test statistic is given by

$$Z = \frac{p_1 - p_2}{\sqrt{\hat{P}\hat{Q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$|Z_{cal}| = 4.835$

Remaining solve yourself.

Example 2

Given, **for men**

Sample size, $n1 = 400$

No. of men who are favour of proposal, $x1 = 200$

Sample proportion, $p1 = x1 / n1 = 200 / 400 = 0.5$

'$q1 = 1 – p1 = 0.5$

Similarly, **for women**

Sample size, $n2 = 500$

No. of women who favoured the proposal, $x2 = 325$

Sample proportion, $p2 = x2 / n2 = 325 / 500 = 0.65$

'$q2 = 1 – p2 = 1 – 0.65 = 0.35$

Example 2

Step 1 : setting hypothesis

H0 : P1=P2 i.e. there is no difference in favour of proposal between men and women.

Against,

H1 : P1 ≠ P2 (two – tailed test) there is difference in favour of proposal between men and women.

Step 2 : Test Statistic

Under H0,

$$Z = \frac{p_1 - p_2}{\sqrt{\hat{P}\hat{Q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$|Z_{cal}| =$

Step3 : level of significance

Step 4: critical value

Step 5 : decision

Practice

1. In two large populations there is 30 and 35 percentage respectively who read news paper. Is this difference likely to be hidden in samples of 1200 and 900 respectively from the two populations?
2. A machine puts out 16 imperfect articles in a sample of 500. after machine is overhauled, it puts out 3 imperfect articles in a batch of 100. Has the machine improved?
3. The records of hospital show that 52 men in a sample of 1000 men versus 23 women in a sample of 1000 women were admitted because of heart disease. Do these data present sufficient evidence to indicate higher rate of heart disease among men admitted to the hospital?

Given,

$P1 = 0.30$, is the proportion of people who reads newspaper for 1st population

$P2 = 0.35$, is the proportion of people who reads newspaper for 2nd population

'$n1 = 1200$

'$n2 = 900$

'$p1 = 0.30$

'$p2 = 0.35$

Step 1: setting hypothesis

H0 : P1 = P2 i.e. there is no difference

Vs,

H1 : P1 ≠P2 (two –tailed test) i.e. there is some difference

Step 2: test statistic

$$Z = \frac{p_1 - p_2}{\sqrt{\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}}}$$

H0 : P1 = P2, **there is no improvement in machine**.

H1: P1 > P2 , **there is improvement in machine. (one – tailed)**

Test statistics:

t – Test (small sample test n < 30)

Assumptions:

1. The population from which the samples are drawn is normally distributed.
2. The sample is randomly selected and independent.
3. The population variance is unknown.
4. The sample size is less than 30 i.e. $n < 30$.

Degree of freedom

- To determine the t – values , it depends up on the level of significance over a range of degree of freedom.
- Degree of freedom is the number of values in the sample that can be chosen freely.
- In some cases, the degree of freedom is determined as the sample size minus the number of population parameters that are estimated from the sample observations.
- For example, if the sum of three observations i.e. x, y, and z is 25 and if x = 10 and y = 7 then z must be 8. This shows that out of three values, two values can be specified freely. Thus degree of freedom is 2.

Test of significance of a Single Mean

Step 1: Setting hypothesis

$H_0: \mu = \mu_0, i.e. there\ is\ no\ difference\ between\ sample\ mean$ and population mean.

Against,

$H_1:\ \mu \neq \mu_0, (two - tailed\ test) i.e.$ There is some difference between sample mean and population mean.

$H_1: \mu > \mu_0, (right - tailed\ test)$

$i.e. the\ population\ mean\ is\ greater\ than\ specified\ mean\ \mu_0$

$H_1: \mu < \mu_0, (left - tailed\ test)$
$i.e. the\ population\ mean\ is\ smaller\ than\ specified\ value\ \mu_0$

<u>Step 2</u>: Test Statistic

Under the H_0, the test statistic is given by

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$$

$$|t_{cal}| = \left|\frac{\bar{x}-\mu}{s/\sqrt{n}}\right| \backsim t_{n-1}$$

Where,

Sample mean, $\bar{x} = \frac{\sum x}{n}$

s^2 is an Unbiased estimate of population variance is calculated as:

1. $s^2 = \frac{1}{n-1}\sum(x-\bar{x})^2$
2. By direct method, $s^2 = \frac{1}{n-1}\left[\sum x^2 - \frac{(\sum x)^2}{n}\right]$

When the biased estimate of the population variance is given then the test statistic is computed as:

$$t = \frac{\bar{x}-\mu}{s_b/\sqrt{n-1}} \backsim t_{n-1}$$

<u>Step 3</u>: Level of Significance

Usually we use 5% level significance i.e. $\alpha = 0.05$ otherwise given.

<u>Step 4:</u> Degree of freedom

DF = n – 1

<u>Step 5:</u> Critical region or Critical value:

The critical value of t-test at $\alpha = 0.05$ and df = n - 1 for two – tailed test or One – tailed test is obtained from the t - table. $|t_{tab}|$

Step 6: Decision:

1. If calculated value $|t_{cal}|$ is less than tabulated value $|t_{tab}|$ $, we\ accept\ Ho$
2. If calculated value $|t_{cal}|$ is more than tabulated value $|t_{tab}|$, we reject H0 and accept H1.

Examples

1. The nine items of a sample had the following values : 45, 47, 50, 52, 48, 47, 49, 53, and 51. Does the mean of the nine items differ significantly from the assumed mean of 47. 5? Test at 1% and 5% level of significance.
2. The height of 10 males of a given locality is found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, and 66. Is it reasonable to believe that the average height is greater than 64 inches? Use $\alpha = 0.05$
3. Ministry of Tourism and Civil Aviation has claimed that the average length of stay of tourist in Nepal is 13 days. To test this claim a researcher asked 9 tourists about their length of stay in Nepal and their length of stay in days were 10, 15, 11, 5, 7, 4, 8, 14, and 11. On the basis of this sample result, can we conclude that the average length of stay is 13 days?

1. Given, sample size, $n = 9$

Population mean, $\mu = 47.5$

Calculation for sample mean and unbiased sample variance

X	X - $\bar{X}$	$(X - \bar{X})^2$
45	-4.11	16.892
47	-2.11	4.452
50	0.89	0.792
52	2.89	8.352
48	1.11	1.232
47	2.11	4.452
49	0.11	0.0121
53	3.89	15.132
51	1.89	3.572
$\Sigma X = 442$		$\Sigma(X - \bar{X})^2 = 54.89$

Here, sample mean is given by

$$\bar{x} = \frac{\sum x}{n} = \frac{442}{9} = 49.11$$

Now, the unbiased sample variance is computed as:

$$s^2 = \frac{1}{n-1}\sum(x-\bar{x})^2 = \frac{54.89}{9-1} = 6.86$$

$\therefore$ s = 2.62, unbiased sample standard deviation

Step 1: setting hypothesis

H_0: $\mu = 47.5, i.e. there\ is\ no\ difference\ between\ sample\ mean\ and\ population\ mean$

Against,

$H_1: \mu \neq 47.5$ (two – tailed test) i.e. there is some difference between sample and population mean.

Step 2: Test Statistic

Under the H_0, the test statistic is given by

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$$

$|t_{cal}| = 1.8435$

<u>Step 3</u>: level of significance

For fixed, $\alpha = 0.01$

<u>Step 4</u>: Degree of Freedom

DF = n – 1 = 9 – 1 = 8

<u>Step 5</u>: Critical Value

The critical value of t – test at $\alpha = 0.01$ and DF = 8 for two – tailed test is obtained as

$|t_{tab}| = 3.355$

Step 6: Decision

Since calculated value $|t_{cal}| = 1.8435$ is less than tabulated value $|t_{tab}| = 3.355,$ so we accept H0.

Therefore, we can conclude that the assuming mean is not differ from the sample mean.

Test of Significance of Difference of Two Means (Independent t – test)

<u>Step 1</u>: Setting Hypothesis

$H_0: \mu_1 = \mu_2$ $i.e. there\ is\ no\ difference\ between\ two\ sample\ means$.

Against,

$H_1: \mu_1 \neq \mu_2$, $(two - tailed\ test)$ $i.e. there\ is\ some\ difference\ between\ two\ sample\ means$

$H_1: \mu_1 > \mu_2$, $(right\ tailed\ test)$ $i.e. the\ mean\ of\ first\ population\ is\ greater\ than\ the\ mean\ of\ second\ population$.

$H_1: \mu_1 < \mu_2, (left\ tailed\ test)\ i.e.\ the\ mean\ of\ first\ population$
$is\ less\ than\ the\ mean\ of\ second\ population.$

Step 2: Test Statistic

Under the H_0, the test statistic is given by

we assume that $\sigma_1^2 = \sigma_2^2 = \sigma^2$ i.e. the population variances are equal but unknown.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_P^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \backsim t_{n_1+n_2-2}$$

Where,

s_P^2 is the unbiased estimate of common population variance, it is calculated as

$$s_P^2 = \frac{1}{n_1+n_2-2}\left[\sum(x_1 - \overline{x_1})^2 + \sum(x_2 - \overline{x_2})^2\right]$$

Where,

$\overline{x_1} = \frac{\sum x_1}{n_1}$ is the mean of first sample

$\overline{x_2} = \frac{\sum x_2}{n_2}$ is the mean of second sample

- When biased sample variances (or biased standard deviations) is given then

s_P^2 is computed as

$$s_P^2 = \frac{\boldsymbol{n_1 s_1^2 + n_2 s_2^2}}{n_1 + n_2 - 2}$$

Step 3: Level of Significance

Usually we use 5% level significance i.e. $\alpha = 0.05$ otherwise given.

Step 4: Degree of freedom

DF = $n_1 + n_2 - 2$

Step 5: Critical region or Critical value:

The critical value of t-test at $\alpha = 0.05$ and DF = $n_1 + n_2 - 2$ for two – tailed test or One – tailed test is obtained from the t - table.

Step 5: Decision:

1. If calculated value $|t_{cal}|$ is less than tabulated value $|t_{tab}|$ $, we\ accept\ Ho$
2. If calculated value $|t_{cal}|$ is more than tabulated value $|t_{tab}|$, we reject H0 and accept H1.

Example:

1. Two different types of drugs were administered on certain patients for increasing weight at interval of one week time period. From the following observations, can you conclude that the second drug is more effective in increasing weight, use 1% level of significance.

D_1	8	12	13	9	3	8	10	9
D_2	10	8	12	15	6	11	12	12

2. A sample of scores on an examination given in Statistics are:

Boys	72	69	98	66	85	76	79	80	77
Girls	81	67	90	78	81	80	76		

Is there a significant difference in the scores of boys and girls?

Example 1

Given, n1 = 8

‘n2 = 8

Calculation for $\overline{x_1}$, $\overline{x_2}$ and S_p^2

x_1	x_2	x_1 - $\overline{x_1}$	$(x_1 - \overline{x_1})^2$	x_2 - $\overline{x_2}$	$(x_2 - \overline{x_2})^2$
8	10	-1	1	-0.75	0.5625
12	8	3	9	-2.75	7.5625
13	12	4	16	1.25	1.5625
9	15	0	0	4.25	18.0625
3	6	-6	36	-4.75	22.5625
8	11	-1	1	0.25	0.0625
10	12	1	1	1.25	1.5625
9	12	0	0	1.25	1.5625
Σx_1 = 72	Σx_2 = 86		$\Sigma(x_1 - \overline{x_1})^2$= 64		$\Sigma(x_2 - \overline{x_2})^2$=53.5

$$\overline{x_1} = \frac{\sum x_1}{n_1} = \frac{72}{8} = 9$$

$$\overline{x_2} = \frac{\sum x_2}{n_2} = \frac{86}{8} = 10.75$$

And the unbiased estimate of common population variance is given by

$$s_P^2 = \frac{1}{n_1+n_2-2}\left[\sum(x_1 - \overline{x_1})^2 + \sum(x_2 - \overline{x_2})^2\right]$$

$$s_P^2 = \frac{1}{8+8-2}[64 + 53.5]$$

$$s_P^2 = 8.39$$

Step 1 : Setting Hypothesis

$H_0: \mu_1 = \mu_2$ $i.e. the$ drug D1 and drug D2 are equally effective.

Against,

$H_1: \mu_1 < \mu_2, (left\ tailed\ test)\ i.e. the\ drug\ D2\ is\ more\ effective\ than\ drug\ D1.$

Step 2: Test statistic

Under H0, the test statistic is given by

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_P^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$|t_{cal}| = 1.21$

Step 3: level of significance

For fixed α = 0.01

Step 4: degree of freedom

DF = n1 + n2 – 2 = 8 + 8 – 2 = 14

Step 5: critical value

$|t_{tab}| = 2.624$

Practice

1. Two horses A and B were tested according to the time (in seconds) to run a particular track with the following results:

Horse A	28	30	32	33	33	29	34
Horse B	29	30	30	24	27	29	

Test whether the two horses have the same running capacity.

Use $\alpha = 0.05$

Paired t – test

The Paired Samples *t* Test compares the means of two measurements taken from the same individual, object, or related units. These "paired" measurements can represent things like:

- A measurement taken at two different times (e.g., pre-test and post-test score with an intervention administered between the two time points)
- A measurement taken under two different conditions (e.g., completing a test under a "control" condition and an "experimental" condition)
- Simply, Before-and-after observations on the same subjects (e.g. students' test results before and after coaching class).

Analysis steps for Paired t - test

Step 1: Setting Hypothesis

$H_0: \mu_x = \mu_y \; or \; \mu_D = 0 \;\; i.e. there\ is\ no\ difference$

$in\ the\ observations\ before\ and\ after\ treatment.$

$or\ the\ treatment\ is\ not\ effective.$

Against,

$H_1: \mu_x \neq \mu_y \;, or\ \mu_D \neq 0 \; (two\ - tailed\ test)\ i.e. there\ is$
$some\ difference\ in\ the\ observations\ before\ and\ after\ treatment$

$H_1: \mu_x > \mu_y, or\ \mu_D > 0 \; (right\ tailed\ test)\ i.e. there\ is$

$positive\ (negative) impact\ in\ the\ observation\ after\ treatment.$

Continue…

$H_1: \mu_x < \mu_y$ or $\mu_D < 0$, (left tailed test) i.e. there is negative (positive) impact in the observations after treatment.

Step 2: Test Statistic

Under H_0, the test statistic is given by

$$t = \frac{\bar{d}}{s_d / \sqrt{n}} \sim t_{n-1}$$

Where,

- $\bar{d} = \frac{\sum d}{n}$, is the mean of the difference.

Continue…

- $d = X - Y \; or \; Y - X$, is the difference between two pair of observation.
- $s_d^2 = \frac{1}{n-1}\sum(d-\bar{d})^2 = \frac{1}{n-1}\left[\sum d^2 - \frac{(\sum d)^2}{n}\right]$, is the sample variance of the difference

Step 3: Level of Significance

Usually we use 5% level significance i.e. $\alpha = 0.05$ otherwise given.

Step 4: Degree of freedom

DF = n – 1 , here n is the number of paired sample observations.

Step 5: Critical region or Critical value:

The critical value of t-test at α = 0.05 and DF = n - 1 for two – tailed test or One – tailed test is obtained from the t - table.

Step 6: Decision:

1. If calculated value $|t_{cal}|$ is less than tabulated value $|t_{tab}|$ $, we\ accept\ Ho$
2. If calculated value $|t_{cal}|$ is more than tabulated value $|t_{tab}|$, we reject H_0 and accept H_1.

Example

1. An I.Q. test was administered to 5 persons before and after they were trained. The results are given below:

Candidates	1	2	3	4	5
IQ before training	110	120	123	132	125
IQ after training	120	118	125	136	121

Is the training effective at 5% level of significance?

1. No, the training is not effective. Before training = after training
2. Yes , the training is effective. Before training < after training

Solution

$H_0: \mu_D = 0$ the training is not effective.

Vs,

$H_1: \mu_D > 0$, the training is effective (one – tailed Test).

Test statistic:

Under H_0, the test statistic is given by

$$t = \frac{\bar{d}}{s_d / \sqrt{n}} \sim t_{n-1}$$

Calculation for $\bar{d}$ and s_d

X	Y	$d = Y - X$	$d - \bar{d}$	$(d - \bar{d})^2$
110	120	10	8	64
120	118	-2	-4	16
123	125	2	0	0
132	136	4	2	4
125	121	-4	-6	36
		$\Sigma d = 10$		$\Sigma(d - \bar{d})^2 = 120$

$$\bar{d} = \frac{\sum d}{n} = \frac{10}{5} = 2$$, where, d is the difference between x and y.

The sample variance of difference is given by

$$s_d^2 = \frac{1}{n-1}\sum\left(d - \bar{d}\right)^2 = \frac{120}{4} = 30$$

$$s_d = 5.48$$

Also n = 5

The test statistic is given as

$$t = \frac{\bar{d}}{s_d/\sqrt{n}} = 0.82$$

Step 3: level of significance

Fixed $\alpha = 0.05$

Step4 : degree of freedom

$DF = n - 1 = 5 - 1 = 4$

Step5 : Critical Value

The critical value of t – test at $\alpha = 0.05$ and DF = 4 for one – tailed test is obtained as

$T_{tab} = 2.132$

Step 6: Decision

Example

1. Twelve college students were given a test in statistics. They were given a month's tuition and a second test was held at the end of it. Do the marks give evidence that the students have benefited by the extra coaching? Use $\alpha = 0.05$

Students	1	2	3	4	5	6	7	8	9	10	11	12
Marks in 1st test	23	20	19	21	18	20	18	17	23	16	19	24
Marks in 2nd test	24	19	22	18	20	22	20	20	23	20	18	22

Example

1. Eight sales executive trainees are assigned selling jobs right after their recruitment. After a fortnight they are withdrawn from their field duties and given a month's training for executive sales. Sales executed by them in thousands of rupees before and after the training, in the same period are listed below:

Sales before training	23	20	19	21	18	20	18	17
Sales after training	25	25	24	24	22	23	25	21

Do these data indicate that the training has contributed to their performance?

$H_0: \mu_x = \mu_y$ i.e. the training do not contribute in their performance.

against,

$H_1: \mu_x < \mu_y$ i.e. the training contribute in their performance.

Test Statistic:

Under H_0, the test statistic is given by

$$t = \frac{\bar{d}}{s_d / \sqrt{n}} \sim t_{n-1}$$

Calculation for $\bar{d}$ and s_d

X	Y	$d = Y - X$	d - $\bar{d}$	$(d - \bar{d})^2$
23	25	2	- 2.125	4.52
20	25	5	0.875	0.77
19	24	5	0.875	0.77
21	24	3	- 1.125	1.27
18	22	4	- 0.125	0.016
20	23	3	- 1.125	1.27
18	25	7	2.875	8.27
17	21	4	- 0.125	0.016
		$\Sigma\text{d} = 33$		$\Sigma(d - \bar{d})^2$=16.132

$$\bar{d} = \frac{\sum d}{n} = \frac{33}{8} = 4.125$$

And the variance of the difference is given by

$$s_d^2 = \frac{1}{n-1}\sum\left(d - \bar{d}\right)^2 = \frac{16.132}{7} = 2.30$$

Now, $s_d = 1.52$

Therefore, the test statistic is given by

$$t = \frac{\bar{d}}{s_d/\sqrt{n}} = 7.67$$

Level of significance

For fixed $\alpha = 0.05$

Degree of freedom

$DF = n - 1 = 8 - 1 = 7$

Critical value

The critical value of t – test at $\alpha = 0.05$ and $DF = 7$ for one – tailed test is obtained from table as:

$t_{tab} = 1.895$

Decision

Since calculated value is greater tabulated value, we reject null hypothesis and accept alternative hypothesis.

Therefore, we can conclude that the training contribute in their performance.

Example

1. Ten soldiers visit a rifle range for two consecutive weeks. For the first week their scores are:

67, 24, 57, 55, 63, 54, 56, 68, 33, 43

And during the second week they score in the same order:

70, 38, 58, 58, 56, 67, 68, 72, 42, 38

Examine if there is any significant difference in their performance.