MATHEMATICAL FOUNDATION FOR COMPUTER SCIENCE

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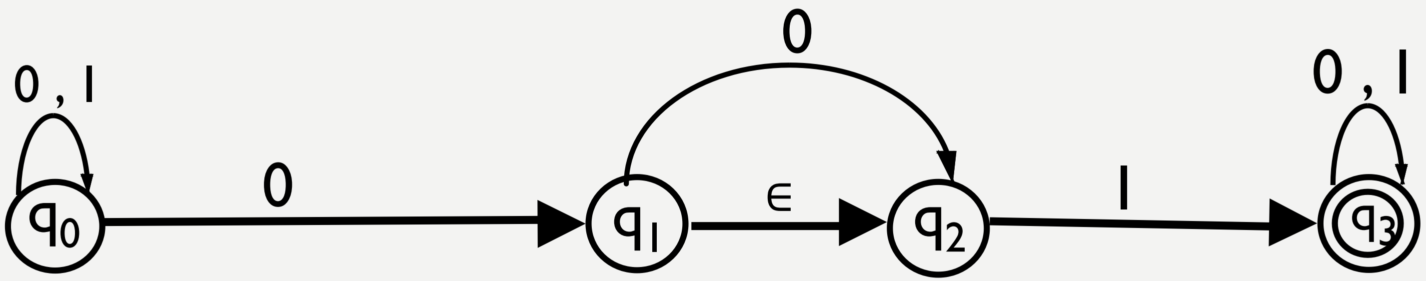
FINITE STATE AUTOMATA

- *Sequential Circuits and Finite state Machine*
- *Finite State Automata*
- *Non-deterministic Finite State Automata*
- *Language and Grammars*
- *Language and Automata*
- *Regular Expression*

NON-DETERMINISTIC FINITE STATE AUTOMATA(NFA):



Consider the machine shown in figure above. Like DFA it has finitely many states and transitions labelled by symbols from an input alphabet. However Above Figure has important difference when compared with DFA model:

- State q_0 has two outgoing transition labelled with 0.
- States q_1 and q_2 have missing transition. q_1 has no transition labelled 1, while q_2 has no transition labelled 0.
- State q_1 has transition that is labelled not by an input symbol but by ∈.

NON-DETERMINISTIC FINITE STATE AUTOMATA(NFA):

Key Difference Between NFA and DFA:

a) An NFA can have multiple transitions for a symbol from the same state but DFA can only have one transition for each symbol.

b) An NFA is not required to have a transition for each symbol where as for DFA there should be transition for each symbol.

c) NFA can have a transition for an empty string where as DFA cannot transition on empty string.

FORMAL DEFINITION OF NFA:

A NFA , N is a quin – Tuple(5 Tuple) defined as,

N = {I, S, f, σ, A} where,

I is the set of input symbols
S is the set of finite states
σ is an initial state
A is the final accepting state
$f:S*I \rightarrow 2^{s}$ is the next state transition function

A string 'w' is said to be accepted by NFA if there exist at least one transition path on which we start and ends at final state.

- *Every DFA is NFA and every NFA can be converted to DFA.*
- *Power of Both DFA and NFA is same.*

1.Construc a NFA which accepts a language of all strings stating with 'ab' over Σ={a, b}.

Solution:

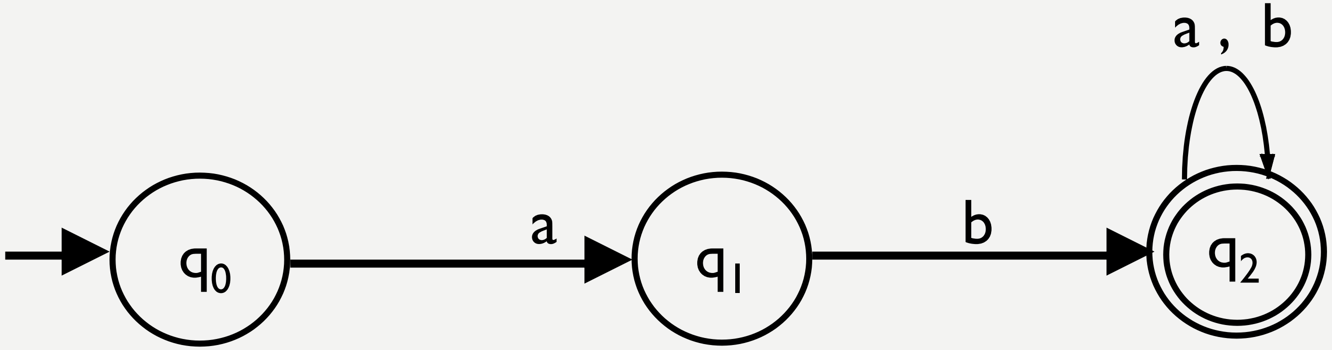


fig: Transition diagram

The required NFA is,

N = {I, S, f, σ, A} where,

I = {a , b} is the set of input symbols

S = {q_0 , q_1 , q_2 } is the set of finite states

σ = q_0 is an initial state

A = {q_2} is the final accepting state

$f:S*I \rightarrow 2^s$ is the next state transition function defined by following table

S I	a	b
q_0	q_1	Ø
q_1	Ø	q_2
q_2	q_2	q_2

2.Construc a NFA which accepts a language of all strings ending with 'bab' over Σ={a, b}.

Solution:

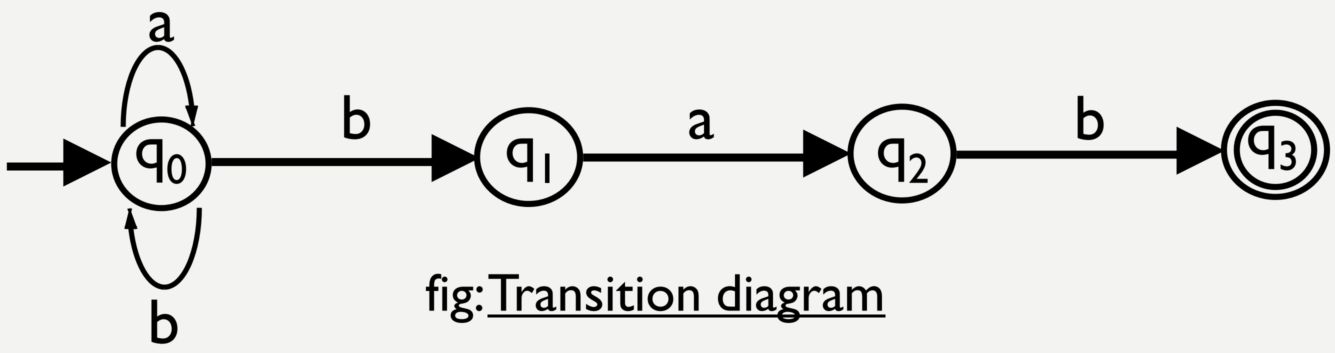


fig:Transition diagram

The required FSA is,

N = {I, S, f, σ, A} where,

I = {a , b} is the set of input symbols

S = {q_0 , q_1 , q_2 , q_3 } is the set of finite states

σ = q_0 is an initial state

A = {q_3} is the final accepting state

f:S*I→2^s is the next state transition function defined by following table

S \ I	**a**	**b**
q_0	q_0	q_0 , q_1
q_1	q_2	∅
q_2	∅	q_3
q_3	∅	∅

3.Construc a NFA which accepts a language of all strings starting and ending with ‘a’ over Σ={a, b}.

<u>Solution:</u>

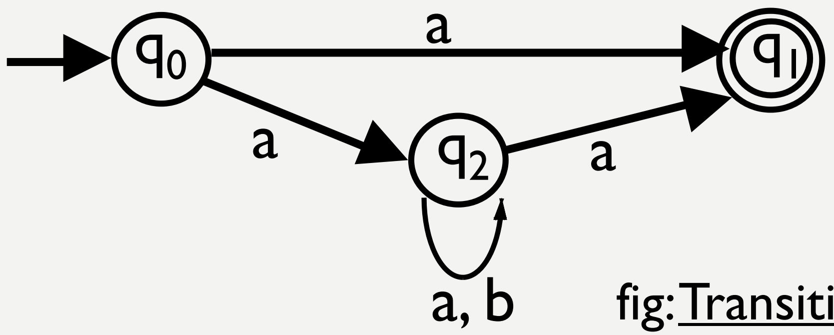


fig:<u>Transition diagram</u>

The required FSA is,

N = {I, S, f, σ, A} where,

I = {a , b} is the set of input symbols

S = {q_0 , q_1 , q_2 } is the set of finite states

σ = q_0 is an initial state

A = {q_1} is the final accepting state

f:S*I→2^s is the next state transition function defined by following table

S \ I	a	b
q_0	q_1, q_2	Ø
q_1	Ø	Ø
q_2	q_2 , q_1	q_2

4.Construc a NFA which accepts a language of all strings containing substring ‘abaab’ over Σ={a, b}.

Solution:

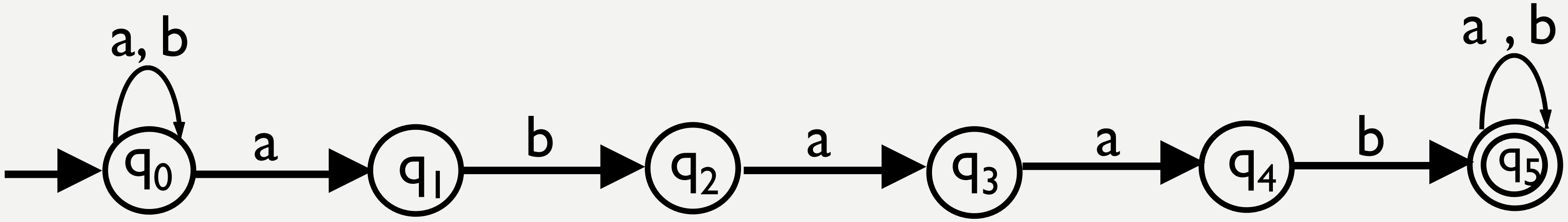


fig:Transition diagram

The required FSA is,

N = {I, S, f, σ, A} where,

I = {a , b} is the set of input symbols

S = {q_0 , q_1 , q_2 , q_3 , q_4 , q_5 } is the set of finite states

σ = q_0 is an initial state

A = {q_5} is the final accepting state

$f: S*I \rightarrow 2^s$ is the next state transition function defined by following table

S I	a	b
q_0	q_0 , q_1	q_0
q_1	∅	q_2
q_2	q_3	∅
q_3	q_4	∅
q_4	∅	q_5
q_5	q_5	q_5

CONVERSION OF NFA TO DFA:

We use subset construction method:

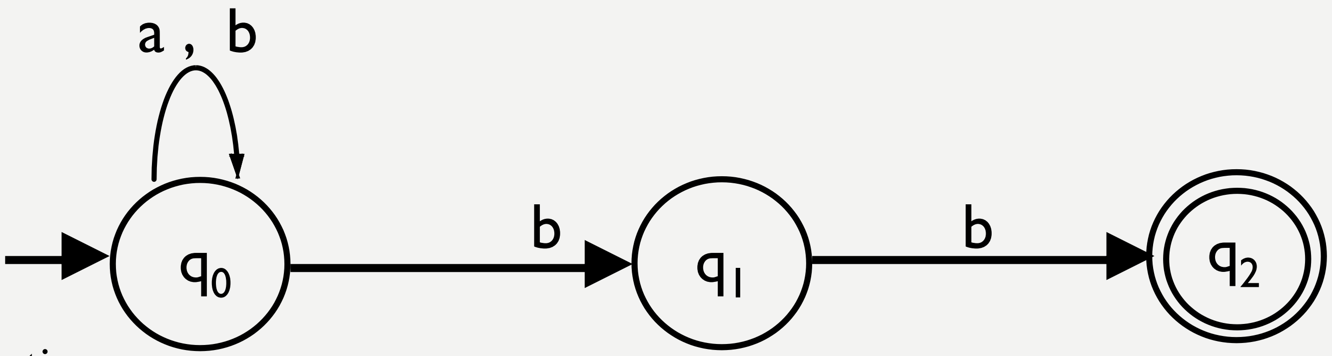
1) Construct a transition table of given NFA.

2) Identify all the new states from the transition table and find the transition for each new state in term of input symbols.

3) This process is continued until transaction for all the new states are identified.

4) Finally, draw a transition diagram by using all the states obtained.

1. Convert the following NFA to DFA:



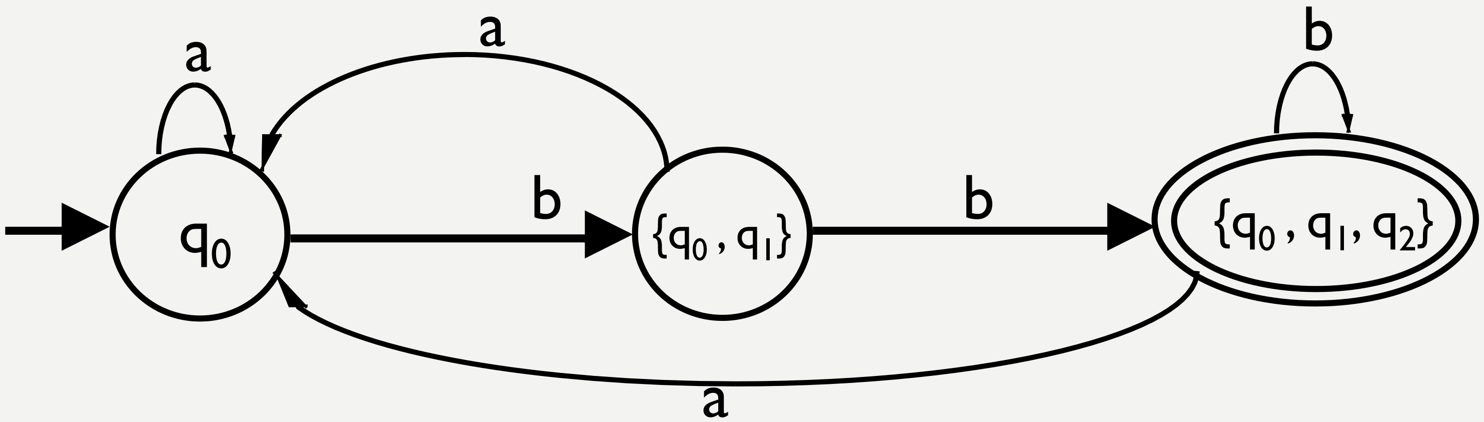
Solution:

a. Transition table for NFA:

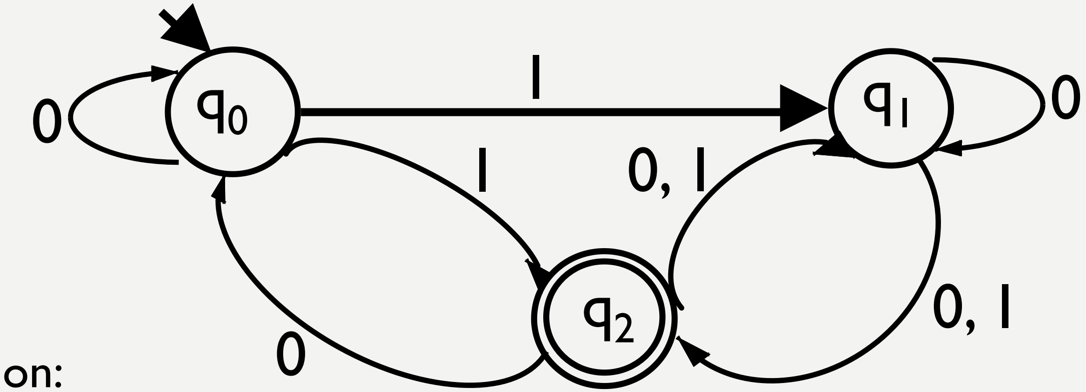
S\I	a	b
q_0	q_0	q_0 , q_1
q_1	Ø	q_2
q_2	Ø	Ø

b. Identifying new states and transition for DFA:

S\I	a	b
q_0	q_0	$\{q_0 , q_1\}$
$\{q_0 , q_1\}$	q_0	$\{q_0 , q_1, q_2\}$
$\{q_0 , q_1, q_2\}$ (Final)	q_0	$\{q_0 , q_1, q_2\}$



2. Convert the following NFA to DFA:



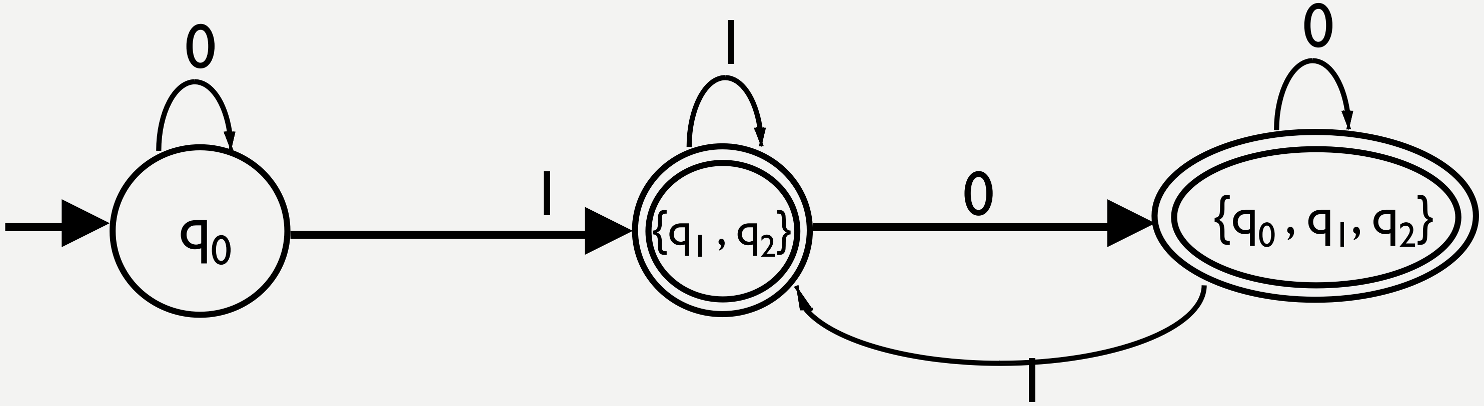
Solution:

a. Transition table for NFA:

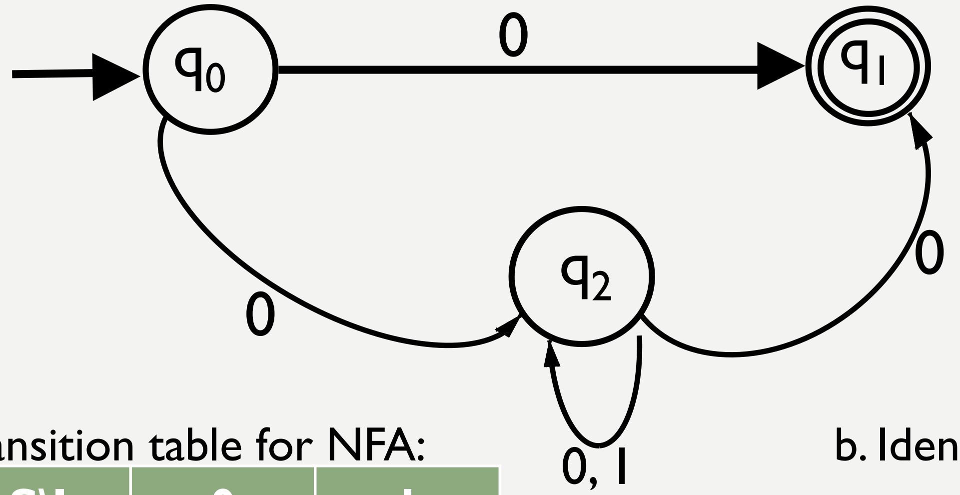
S\I	0	1
q_0	q_0	q_1 , q_2
q_1	q_1 , q_2	q_2
q_2	q_0 , q_1	q_1

b. Identifying new states and transition for DFA:

S\I	0	1
q_0	q_0	$\{q_1 , q_2\}$
$\{q_1 , q_2\}$	$\{q_0 , q_1, q_2\}$	$\{q_1, q_2\}$
$\{q_0 , q_1, q_2\}$	$\{q_0 , q_1, q_2\}$	$\{q_1, q_2\}$



3. Convert the following NFA to DFA:

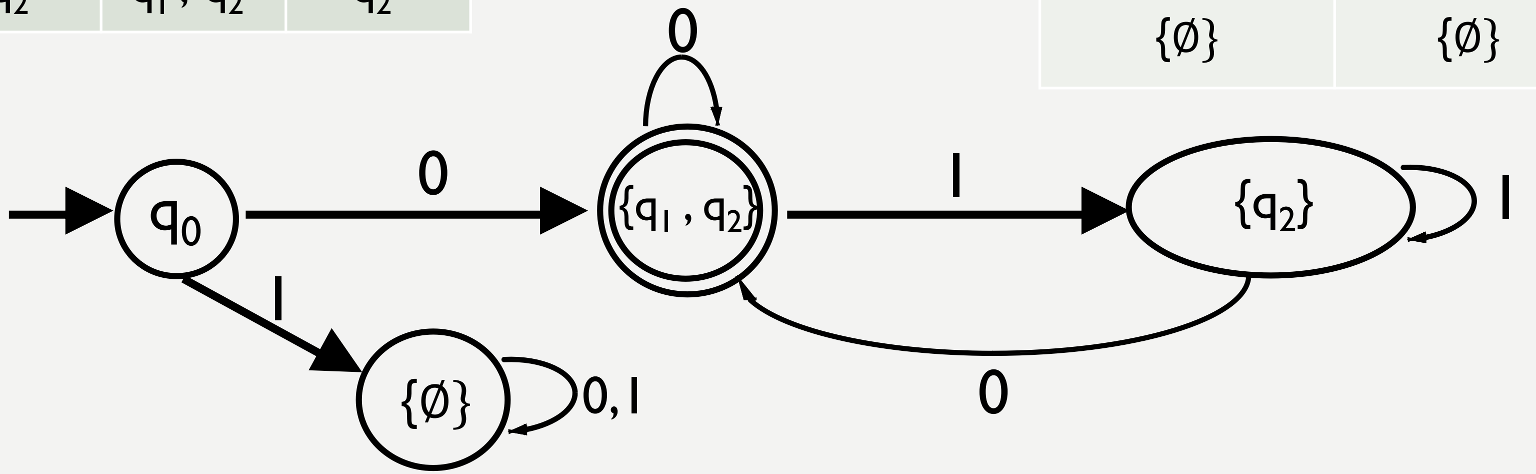


a. Transition table for NFA:

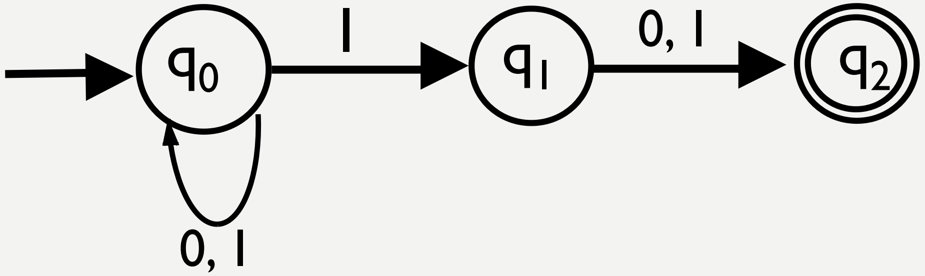
S\I	0	1
q_0	q_1 , q_2	$\emptyset$
q_1	$\emptyset$	$\emptyset$
q_2	q_1 , q_2	q_2

b. Identifying new states and transition for DFA:

S\I	0	1
q_0	$\{q_1 , q_2\}$	$\{\emptyset\}$
$\{q_1 , q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
$\{q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
$\{\emptyset\}$	$\{\emptyset\}$	$\{\emptyset\}$



4. Convert the following NFA That accepts all string in which second last bit is 1.



a. Transition table for NFA:

S\I	0	1
q_0	q_0	q_0 , q_1
q_1	q_2	q_2
q_2	∅	∅

b. Identifying new states and transition for DFA:

S\I	0	1
q_0	q_0	$\{q_0 , q_1\}$
$\{q_0 , q_1\}$	$\{q_0 , q_2\}$	$\{q_0 , q_1 , q_2\}$
$\{q_0 , q_2\}$	q_0	$\{q_0 , q_1\}$
$\{q_0 , q_1 , q_2\}$	$\{q_0 , q_2\}$	$\{q_0 , q_1 , q_2\}$