MATHEMATICAL FOUNDATION FOR COMPUTER SCIENCE

Prepared by: Er. Ankit Kharel

Nepal college of information technology

LOGIC, INDUCTION AND REASONING

- Proposition and Truth function
- Propositional Logic
- Expressing statements in Logic Propositional Logic
- Rules of Inference
- The predicate Logic
- Validity
- Informal Deduction in Predicate Logic
- Proofs(Informal Proofs & Formal Proofs)
- Elementary Induction
- Complete Induction
- Methods of Tableaux
- Consistency and Completeness of the System

<u>Proposition:</u>

- Declarative statement that is either TRUE or FALSE.

- Symbol ‘T’ for TRUE and ‘F’ for FALSE.

  <u>Examples:</u>

  i) Paris is in France(T).
  ii) Delhi is in Nepal(F).
  iii) $2<4$(T).
  iv) $4=7$(F).

  <u>Example of statement that are not propositions:</u>

  i) What is your name? (This is a Question)
  ii) Do your Homework (This is a command)
  iii) “x” is even number (It depends on the value of x)

- Small alphabets like ‘p’, ’q’ , ’r’ are used to represent prepositions.

  p: Paris is in France.
  q: We live on Earth

Proposition Logic:

- ➢ Deals with proposition also known as Propositional Calculus.

- ➢ First developed by Aristotle.

I) Atomic Proposition:

❖ Which cannot be further broken down.

Example:

"Today is Friday"

II) Compound Proposition:

❖ Which can further be broken down.

❖ Logical operators are used.

Example:

"Ram is intelligent and diligent."

p: "Ram is intelligent"

q: "Ram is diligent"

1.Logical operators/connectives:

- Used to construct compound propositions.
- Some common logical connectives are:

1. NEGATION(NOT) ¬
2. CONJUCTION(AND) ∧
3. DISJUNCTION(OR) ∨
4. EXCLUSIVE OR(XOR) ⊕
5. IMPLICATION(IF-THEN) →

   (Inverse, Converse and Contrapositive)

6. BICONDITIONAL(IF AND ONLY IF) ←→

1.Negation(not):

- If ‘p’ is the proposition , then the negation of ‘p’ is denoted by ‘¬p’.
- ‘¬p’ means “it is not case that p” or simply “not p”.

Examples:

1) p: “Today is Friday”
¬p: “It is not the case that today is Friday”
¬p: “Today is not Friday”

2) p: “London is in Denmark”
¬p: “It is not the case that London is in Denmark”
¬p: “London is not in Denmark”

TRUTH TABLE

p	¬p
T	F
F	T

2.conjunction(and):

- If ‘p’ and ‘q’ are two proposition , then the conjunction of ‘p’ and ‘q’ is denoted by ‘p^q’.
- p^q is TRUE only when both ‘p’ and ‘q’ are TRUE, otherwise FALSE.

Examples:

1) p: “Today is Friday”
   q: “It is raining Today”
   p^q: “Today is Friday and it is raining Today”

Truth Table

p	q	p^q
T	T	T
T	F	F
F	T	F
F	F	F

3.disjunction(or):

- If ‘p’ and ‘q’ are two proposition , then the disjunction of ‘p’ and ‘q’ is denoted by ‘pVq’.
- pVq is FALSE when both ‘p’ and ‘q’ are FALSE, otherwise TRUE.

Examples:

1) p: “Today is Friday”
   q: “It is raining Today”
   pVq: “Today is Friday or it is raining Today”

Truth Table

p	q	pVq
T	T	T
T	F	T
F	T	T
F	F	F

4.Exclusive or (xor):

- If ‘p’ and ‘q’ are two proposition , then the Exclusive or of ‘p’ and ‘q’ is denoted by ‘$p \oplus q$’ which means “Either p or q but not both”
- $p \oplus q$ is TRUE when either ‘p’ or ‘q’ is TRUE and FALSE when both are TRUE or both are FALSE.

Examples:

1) p: “Today is Friday”
   q: “It is raining Today”
   $p \oplus q$: “Either today is Friday or it is raining today”

Truth Table

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

5.implication (if$\rightarrow$then):

- If ‘p’ and ‘q’ are two proposition then the statement “if p then q” is called an implication and denoted by $p \rightarrow q$.

- $p \rightarrow q$ is also called a conditional statement.

- ‘p’ is called ***hypothesis*** or ***antecedent*** or ***premise***.

- ‘q’ is called the ***conclusion*** or ***consequence***.

Some other terminologies used to express $p \rightarrow q$ are:

✓ If p , then q.
✓ p is sufficient for q
✓ q when p
✓ A necessary condition for p is q
✓ p only if q
✓ q unless $\neg p$
✓ q follows from p

5.implication (if→then):

Example:

p: “Today is holiday”

q: “The college is closed”

p→q: ”If today is holiday, then the college is closed”

TRUTH TABLE

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

inverse:

$p \rightarrow q$ → $\neg p \rightarrow \neg q$

"if p, then q" *"if not p, then not q"*

p: "Today is holiday"

q: "The college is closed"

¬p: "Today is not holiday"

¬q: "The college is not closed"

p→q: "If today is holiday, then the college is closed"

¬p→¬q: "If today is not holiday, then the college is not closed"

<u>converse:</u>

$p \rightarrow q$ → $q \rightarrow p$

"if p, then q" *"if q, then p"*

p: "Today is holiday"

q: "The college is closed"

$p \rightarrow q$: "If today is holiday, then the college is closed"

$q \rightarrow p$: "if the college is closed, then today is holiday"

Contra-positive:

$p \rightarrow q$ ⟶ $\neg q \rightarrow \neg p$

"if p, then q" *"if not q, then not p"*

p: "Today is holiday" ¬p: "Today is not holiday"

q: "The college is closed" ¬q: "The college is not closed"

$p \rightarrow q$: "If today is holiday, then the college is closed"

$\neg q \rightarrow \neg p$: "If the college is not closed, today is not holiday"

6.Biconditional(if and only if):

- If ‘p’ and ‘q’ are two proposition , then the biconditional statement $p \leftrightarrow q$ is the proposition “p if and only if q”

- $(p \rightarrow q) \wedge (q \rightarrow p) == p \leftrightarrow q$

- These are also called bi-implications.
- Some other Terminologies:
  “p is necessary and sufficient for q”
  “if p then q and conversely”

Examples:

p: “I am breathing”
q: “I am alive”
$p \leftrightarrow q$: “I am breathing if and only if I am alive”

Truth Table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Operator precedence:

Operator	Precedence (higher the number higher the precedence)
¬	1
∧	2
∨	3
→	4
↔	5

Examples:

1) ¬ p ∧ q {Given p =True and q=False}
=F^F
=F

2) p ∧ q ∨ r {Given p=True, q =False, r=True}
=F ∨ T
=T

3) p →q ∧ ¬p{Given p=True, q=False}
=T →F^F
=T →F
=F

4) (p^q)→((¬p)V q) {Given p=True, q=False}
=(T^F) →(FvF)
=F →F
=T

Truth table of compound preposition:

- Construct the truth table of compound proposition

(P∨¬Q)→(P∧Q)

P	Q	¬Q	P∨¬Q	P∧Q	(P∨¬Q)→(P∧Q)
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Truth table of compound preposition:

- Construct the truth table of compound proposition

1. $(P \vee \neg Q) \rightarrow (P \wedge Q)$

P	Q	¬Q	A=(PV¬Q)	B=(P^Q)	A→B
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Truth table of compound preposition:

- Construct the truth table of compound proposition

2. $(P \rightarrow Q) \wedge (Q \rightarrow R)$

P	Q	R	$(P \rightarrow Q)$	$(Q \rightarrow R)$	$(P \rightarrow Q) \wedge (Q \rightarrow R)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

TRANSLATING ENGLISH SENTENCES:

Examples:

1."You can access the internet from NCIT only if you are a masters student or you are a new student"

Let ,

p: You access the internet from NCIT

q: You are a masters student

r: You are a new student

$$p \rightarrow (q \wedge r)$$

2."The automated reply can not be sent when file system is full"

Let,

p:The automated reply can be sent

q:File system is full

$$q \rightarrow \neg p$$

Assignment 1 :

1. What are logical connectives explain each with example and truth table.
2. Construct truth table for
- **$\neg(p \wedge q) \vee (r \wedge \neg p)$**
- **$(p \vee \neg r) \wedge \neg((q \vee r) \vee \neg(r \vee p))$**
- **$((p \leftrightarrow q) \oplus (\neg p \rightarrow q)) \vee (q \rightarrow \neg r)$**

3. Let p, q ,r be:

    p="You have flu"
    q="You miss the final exam"
    r="You pass the course"

    Express each proposition as an English sentence and construct truth table:
    - **$p \rightarrow q$**
    - **$q \rightarrow \neg r$**
    - **$(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$**

4. Translate into mathematical expression
- You can't have voting right if you are mentally unfit and you are not over 18 years.
- Leaders will make correct decision only if you choose a good leader or you raise your voice against incorrect decision