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# Computer Graphics (L09)

EG678EX

## 3-D Transformations

# Translation

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$$x' = x + t_x$$

$$y' = y + t_y$$

$$z' = z + t_z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P' = T.P$$

# Rotation (about co-ordinate axes)

$$\begin{aligned}x' &= x \cos \theta - y \sin \theta \\y' &= x \sin \theta + y \cos \theta \\z' &= z\end{aligned}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P' = R_z(\theta).P$$

$$\begin{aligned}y' &= y \cos \theta - z \sin \theta \\z' &= y \sin \theta + z \cos \theta \\x' &= x\end{aligned}$$

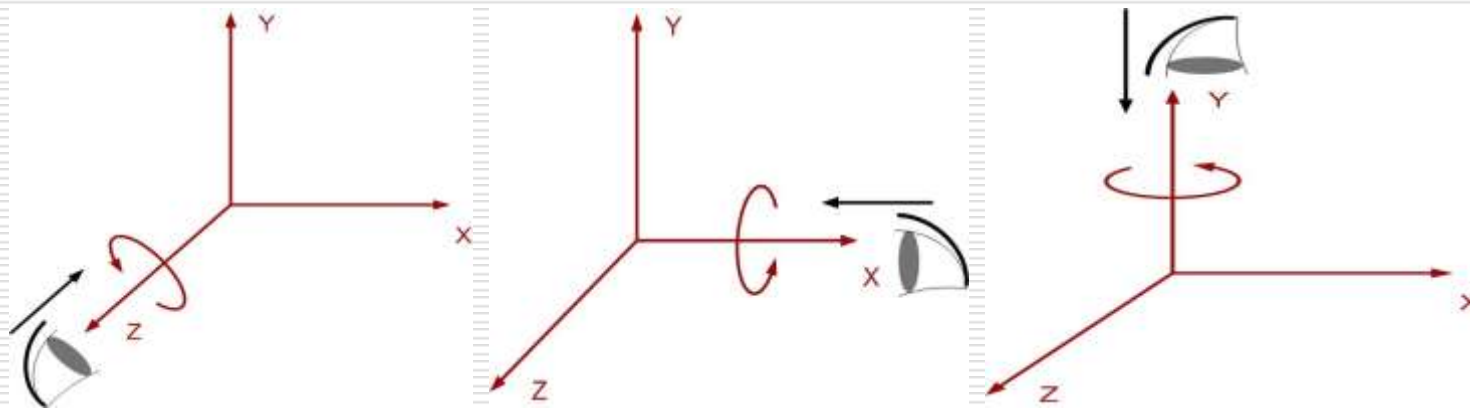
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P' = R_x(\theta).P$$

$$\begin{aligned}z' &= z \cos \theta - x \sin \theta \\x' &= z \sin \theta + x \cos \theta \\y' &= y\end{aligned}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

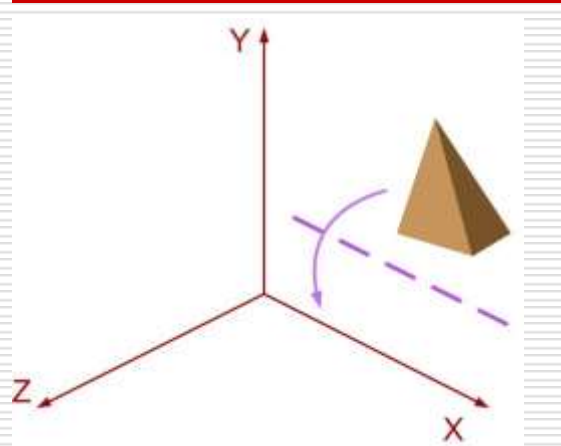
$$P' = R_y(\theta).P$$



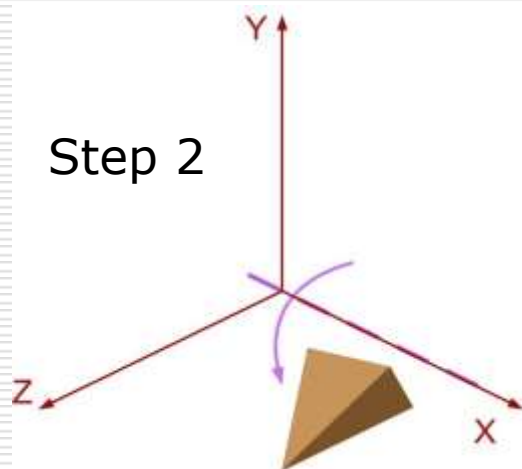
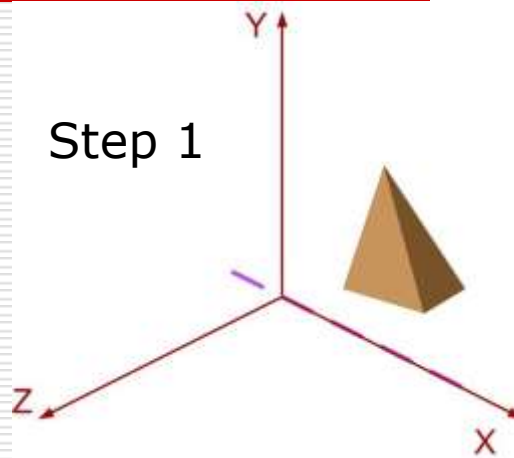
Remember The cyclic order:

$$x \rightarrow y \rightarrow z \rightarrow x$$

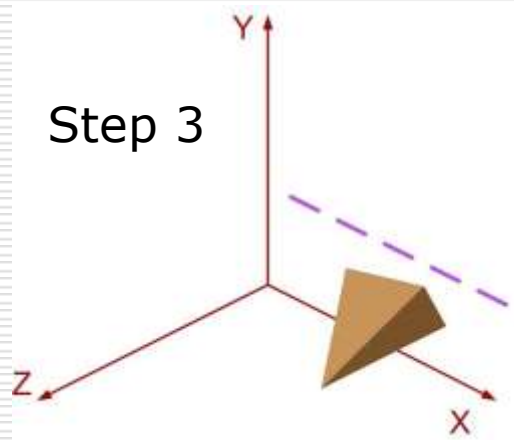
# Rotation about axis parallel to co-ordinate axis



Step 1



Step 2

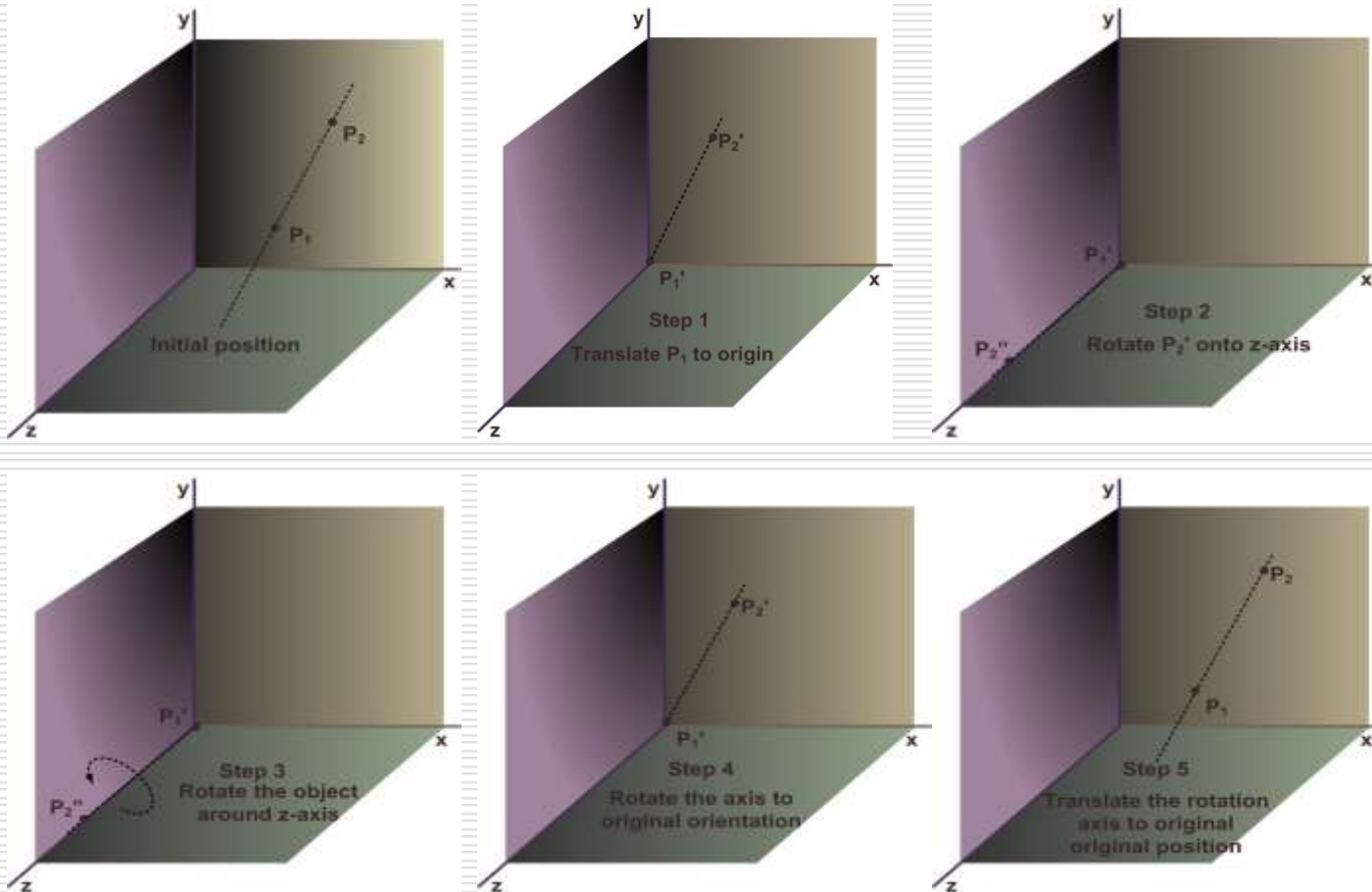


Step 3

## Steps

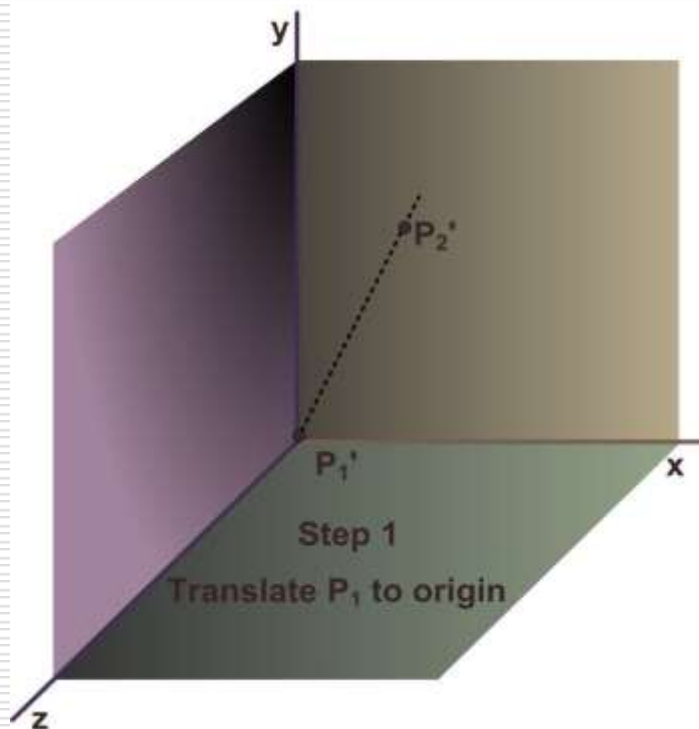
- ❑ Translate the object so as to coincide rotation axis to parallel co-ordinate axis
- ❑ Perform the rotation about the axis
- ❑ Translate back the object so as to move rotation axis to original position

# General 3-D Rotation



# General 3-D rotation Mathematics

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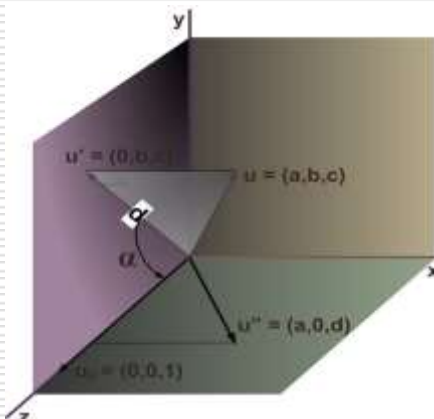
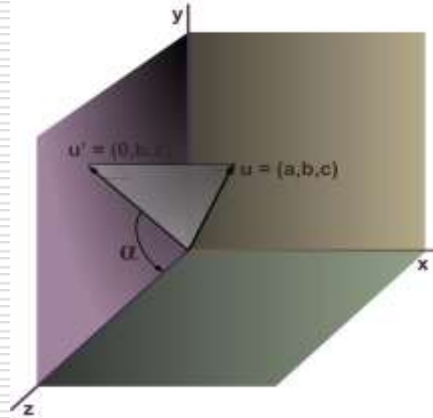
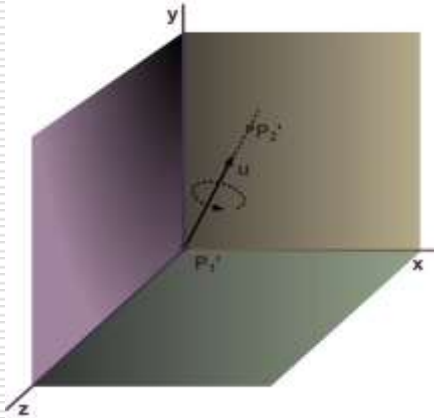
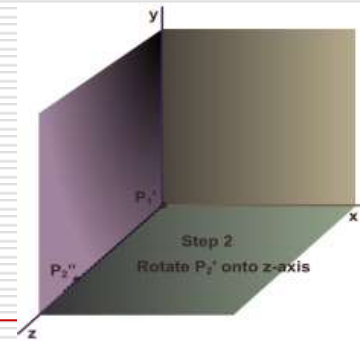


## Step 1

Translate object so as to coincide  $P_1$  to origin

$$T = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# General 3-D Rotation Mathematics



## Step 2

- Accomplished in two steps
- i. Perform x axis rotation with angle  $\alpha$  so as to bring rotation axis to zx plane
- ii. Perform y axis rotation with angle  $\beta$  so as to coincide the rotation axis with z-axis

## X- Axis rotation ( $\alpha$ )

$$\begin{aligned}\vec{V} &= \vec{p}_2 - \vec{p}_1 \\ &= (x_2 - x_1, y_2 - y_1, z_2 - z_1) \\ \vec{u} &= \frac{\vec{V}}{|\vec{V}|} = (a, b, c) \\ a &= \frac{x_2 - x_1}{|\vec{V}|}, \quad b = \frac{y_2 - y_1}{|\vec{V}|}, \quad c = \frac{z_2 - z_1}{|\vec{V}|}\end{aligned}$$

$$\cos \alpha = \frac{\vec{u}' \cdot \vec{u}_z}{|\vec{u}'| |\vec{u}_z|} = \frac{c}{d}$$

$$d = \sqrt{b^2 + c^2}$$

$$\vec{u}' \times \vec{u}_z = \vec{u}_x |\vec{u}'| |\vec{u}_z| \sin \alpha$$

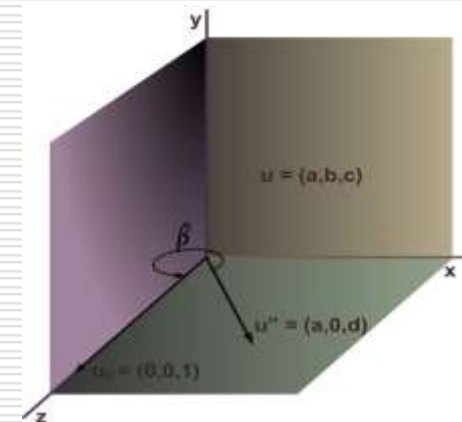
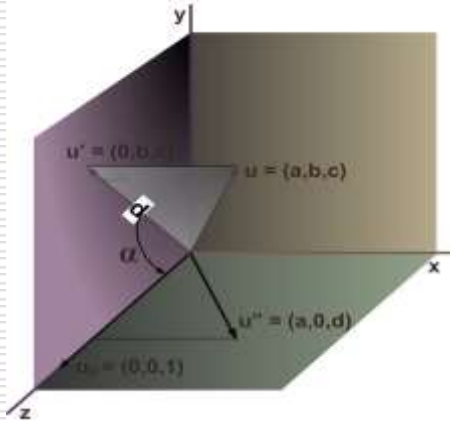
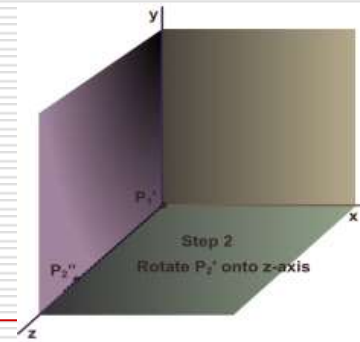
$$\begin{aligned}d \sin \alpha &= b \\ \sin \alpha &= \frac{b}{d}\end{aligned}$$

$$\vec{u}' \times \vec{u}_z = \begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ 0 & b & c \\ 0 & 0 & 1 \end{vmatrix} = \vec{u}_x b$$

$$\begin{aligned}|\vec{u}'| &= d \\ \text{and} \\ |\vec{u}_z| &= 1\end{aligned}$$

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c/d & -b/d & 0 \\ 0 & b/d & c/d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# General 3-D Rotation Mathematics



## Y-Axis Rotation ( $\beta$ )

$$\cos \beta = \frac{\vec{u}'' \cdot \vec{u}_z}{|\vec{u}''| |\vec{u}_z|} = d$$

$$\vec{u}'' \times \vec{u}_z = \vec{u}_y |\vec{u}''| |\vec{u}_z| \sin \beta$$

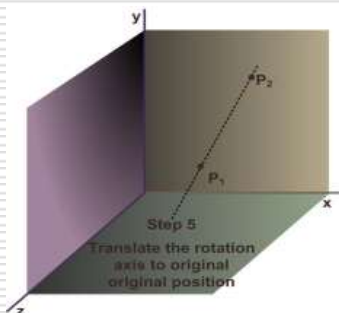
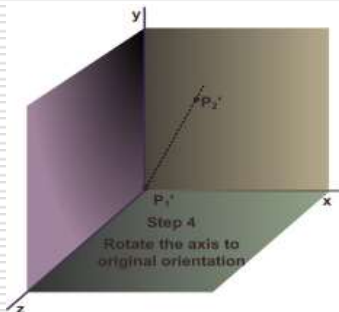
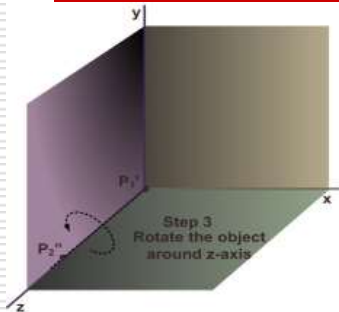
$$\vec{u}'' \times \vec{u}_z = \begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ a & 0 & d \\ 0 & 0 & 1 \end{vmatrix} = \vec{u}_y \cdot (-a)$$

$$\sin \beta = -a$$

$$R_y(\beta) = \begin{bmatrix} d & 0 & -a & 0 \\ 0 & 1 & 0 & 0 \\ a & 0 & d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



# General 3-D Rotation Mathematics



## Step 3

- Perform z-axis Rotation with angle  $\theta$  (the angle which the object is to be rotated about the given axis)

$$R_z(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Step 4

- Perform inverse Rotation and i.e  $R_x^{-1}(\alpha)$  and  $R_y^{-1}(\beta)$

## Step 5

- Perform inverse Translation i.e  $T^{-1}$

**Final composite Matrix is obtained as:**

$$R(\theta) = T^{-1} \cdot R_x^{-1}(\alpha) \cdot R_y^{-1}(\beta) \cdot R_z(\theta) \cdot R_y(\beta) \cdot R_x(\alpha) \cdot T$$

# 3-D Scaling

Scaling about origin

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$
$$S(s_x, s_y, s_z) = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fixed Point Scaling

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & (1-s_x)x_f \\ 0 & s_y & 0 & (1-s_y)y_f \\ 0 & 0 & s_z & (1-s_z)z_f \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$S(x_f, y_f, z_f, s_x, s_y, s_z) = T(x_f, y_f, z_f) \cdot S(s_x, s_y, s_z) \cdot T(-x_f, -y_f, -z_f)$$

# 3-D Reflection

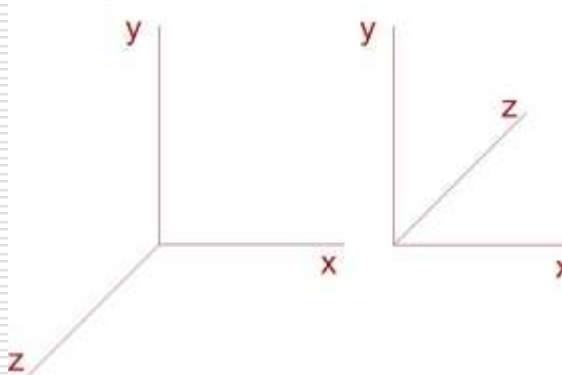
- ❑ Performed relative to a reflection axis or reflection plane
- ❑ Axis reflection → equivalent to 180 degree rotation about the axis in 3-D space
- ❑ Plane reflection → equivalent to 180 degree rotation in 4-D space
  - 4-D space ?? → not visualized in euclidian space
- ❑ Reflection about a plane converts right handed co-ordinate system to left handed co-ordinate system and vice versa

- ❑ Reflection in xy plane

$$\begin{aligned}x' &= x \\ y' &= y \\ z' &= -z\end{aligned}$$

- Matrix is as

$$RF_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



# 3-D Shear

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## □ Z-axis shear

$$x' = x + a.z$$

$$y' = y + b.z$$

$$z' = z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$SH_z = \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$