N.C.I.T.
Sample questions

Level: Bachelor semester: Fall Year:2022

Programme : SE Full Marks:100
Course: Engineering mathematics 1 Pass marks: 45
SET: A Time : 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all questions.

1 (a) If $y = \log\left(x + \sqrt{a^2 + x^2}\right)$, show that

$(i).(x^2 + a^2)y_2 + xy_1 = 0, (ii).(x^2 + a^2)y_{n+2} + (2n+1)xy_{n+1} + n^2 y_n = 0$ [5]

(b) Evaluate $\lim_{x \to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$ [5]

(c) Find the expansion of $\log \sec x$ in Maclaurin's series . [5]

2 (a) Find assymptotes of of the curve: $(x^2 - y^2)^2 - 2(x^2 + y^2) + x - 1 = 0$ [8]

OR

Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at (0, 0).

(b) Find reduction formula for : $\int \sin^n x dx$ and evaluate $\int \sin^7 x dx$ [7]

OR

Evaluate: $\int_0^1 x^6 (1 - x^2)^{\frac{1}{2}} dx$

3 (a) Integrate: $\int \frac{1}{3\sin x + 4\cos x} dx$ [5]

(b) Evaluate: $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$ [5]

(c) Find the area bounded by $y^2 + x = 0$ and $3y^2 + x = 2$ [5]

4 (a) Find the volume of solid generated by revolution of the region bounded be $y^2 = x$, $y = 1, x = 4$ about y=1 [7]

OR

Find the area of the surface generated by revolving the curve $y=\sqrt{x}$, $4\le x\le 9$ about x-axis .

(b) Find the dimension of rectangular box open at the top of maximum volume whose surface area is 432 square centimeter [8]

OR

Find the shrtest and longest distance from the point (1, 2, -1) to the sphere $x^2+y^2+z^2=24$

5 (a) Solve: $\frac{dy}{dx}+\frac{y}{x}\log y=\frac{y(\log y)^2}{x^2}$ [7]

(b) A tank initially contains 40 kg of salt dissolved in 200 liters of water.Suppose that the salt solution 2kg of salt per liter is allowed to enter the tank at a rate of 5 liter/min and the uniform solution is drained from the tank at the same rate. Find the amount of salt in the tank after 30 minutes. [8]

6 (a) Solve: $y''-2y'+y=3e^x x^{\frac{3}{2}}$. [7]

(b) Solve: $y''-6y'+13y=4e^{3x};\ y(0)=2,\ y'(0)=4$. [8]

7. Answer the followig . (2×5)

(a) Find nth order derivative of $y=\frac{1}{3x+2}$

(b) Plot parametric curve $x=\cos 2t,\ y=\sin 2t, 0\le t\le 2\pi$

(c) If $\sin u=\frac{x^2-y^2}{x^2+y^2}$. Find value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$

(d). Verify Eulr's theorm for $f(x;y)=\sqrt{x^3+y^3}$

(e) Find arc length of parabola $y=x^2$, from x= -4 to x=4.