Chapter-8

Fundamentals of thermodynamics and heat transfer:

8.3. Heat transfer:

Heat is a common form of energy which is continuously being transferred from one body to another. In earlier time, during the development of concept of heat and temperature, people had only the concept of inequality for flow of heat. According to this concept, heat always flows from a body of higher temp. to a body of lower temp. Actually, heat always flows until there is difference in temp.

There are three different modes of transfer of heat. Transfer of heat due to actual motion of molecules in fluid (liquid and gas) is called convection, eg. boiling of water. Transfer of heat due to molecular vibrations in a solid is called conduction eg. heating of metal rod. Transfer of heat without support of any material medium is called radiation, eg. receiving heat from sun. These modes of heat transfers are described as following:

① **Conduction:**

Conduction is the process of transfer of heat from one point to another point of a body carried out by means of collisions between rapidly vibrating atoms at hotter region and slowly vibrating atoms at colder region. There is no actual transfer of particles during conduction.

when one end of a solid is heated, atoms at hotter end vibrate with greater amplitude and have more K.E. than neighbouring atoms at colder part. The atoms at hotter region collide with and give up some energy to the neighbouring atoms in colder regions. Similar process continues between each set of neighbouring atoms upto next end. As a result transfer of heat takes place and previously cold end also gets heated. This method of transfer of heat is called conduction.

(2) Convection:

Convection is the process of ~~heat~~ transfer of heat in fluids by means of actual motion of heated particles from higher temperature region to lower temp. region.

Heated particles carry heat and move from hot region to cold region but cold particles move in opposite direction. The current set up in the process is called convection current. This method is not possible in solid and vacuum. Heating of water, land breezes, sea breezes, wind etc. are some examples of convection.

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(3) Radiation:

Radiation is the process of transmission of heat from one point to another without need of any material medium. Radiation does not heat the medium through which heat energy passes.

In radiation, heat is transferred in the term of e.m. radiation (wave) and travel with speed of 3×10^{8} m/sec. in vacuum. Heat energy comming to the earth from the sun, and transmission of heat ~~aron~~ around fire are the examples of radiation. Heat energy radiated by an object is called radiant energy or thermal energy.

(II) Statement and assumption of Fourier law of thermal conductivity:

The ability of anybody to conduct heat is measured in term of thermal conductivity. Therefore, Ability which measures the thermal conduction of material is called thermal conductivity. Thermal conductivity generally occurs in solids.

Statement of Fourier law of Thermal conductivity:

The Fourier law states that the rate of heat flow in solids is directly proportional to the cross-section area perpendicular to the flow axis and negative of temp. gradient over the length of the path of conduction

Accordig to Fourier's law, the rate of heat flow Q through a homogeneous solid is directly proportional to the area A of the section at the right angles to the direction of heat flow, and to the temp. difference dT along the path of heat flow.

i.e. $Q = -KA\frac{dT}{dx}$ — (1)

where
Q = rate of heat transfer (watt)
K = Thermal Conductivity (W/m·K)
A = Area of cross-section (m^2)
dT = Change in temp. along the dirn. of heat flow
dx = Thickness of the object.

In the fourier heat conduction eqn. the -ve sign implies that heat is flowing from higher temp. to lower temp. therefore it is provided to compensate for the negative nature of the temp. gradient.

Assumption in fourier law of heat conduction:

Following are the assumptions of Fourier law of heat conduction:

1. The thermal conductivity of the material is constant throughout the material.
2. There is no internal heat generation that occurs in the body.
3. The temp. gradient is considered at constant.
4. The heat flow is unidirectional and takes place under steady-state conditions.
5. The surfaces are isothermal.

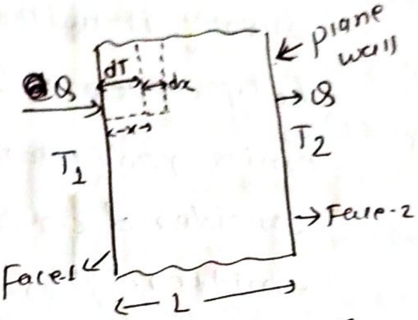
Temperature gradient

The rate of fall in temp. with distance along the dirn of heat flow is called the temp. gradient.

It is expressed as $-\frac{dT}{dx}$ and its unit is kelvin per meter (K/m). here dT is the small change in temp. over a small distance dx and -ve sign indicates that temp. falls with distance.

One dim^n steady state heat conduction through plane wall

Consider parallel sided plane wall of thickness L and uniform cross-section area A. as shown in figure. let K be the thermal conductivity of wall material through which heat is flowing only in x-direction.



let T_1 and T_2 are the temp. of higher temp. face-1 and lower temp. face-2 The small change of temp. dT when the particles conduct very small distance dx.

[Fig: plane wall]

According to fourier law, the rate of heat transfer is given by

$$\frac{dQ}{dt} = -KA\frac{dT}{dx}$$

$$\text{or } \frac{dQ}{dt}dx = -KA\,dT \quad \text{--- (1)}$$

To obtain the rate of flow of heat through this whole wall is obtained by integrating eq^n (1), we get

$$\int_0^L \frac{dQ}{dt}dx = \int_{T_1}^{T_2} -KA\,dT$$

$$\text{or } \frac{dQ}{dt}\int_0^L dx = -KA\int_{T_1}^{T_2} dT$$

$$\text{or } \frac{dQ}{dt}[L-0] = -KA[T_2-T_1]$$

$$\text{or } dQ/dt = \frac{KA(T_1-T_2)}{L}$$

$$\Rightarrow \frac{dQ}{dt} = \frac{(T_1-T_2)}{L/KA} \quad \text{--- (2)}$$

where $\frac{L}{KA}$ = Thermal resistance R_{th}

$$\therefore \frac{dQ}{dt} = \frac{T_1-T_2}{R_{th}} \quad \text{--- (3)}$$

This eq^n (3) gives the rate of flow of heat through the wall. If R_{th} increases, $\frac{dQ}{dt}$ decreases and vice-versa.

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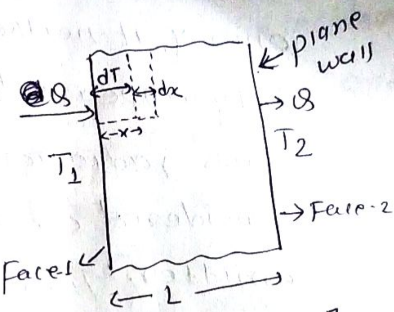

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To obtain the rate of flow of heat through this whole wall is obtained by integrating eq^n ①, we get

$$\int_0^L \frac{dQ}{dt}dx = \int_{T_1}^{T_2} -KA\,dT$$

or $$\frac{dQ}{dt}\int_0^L dx = -KA\int_{T_1}^{T_2} dT$$

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or $$\frac{dQ}{dt} = \frac{KA(T_1-T_2)}{L}$$

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This eq^n ③ gives the rate of flow of heat through the wall. If R_{th} increases, $\frac{dQ}{dt}$ decreases and vice-versa.

Black body and black body radiation

A perfect black body is one which absorbs heat radiation of all wavelengths falling on it and emits radiations of all possible wavelengths when heated.

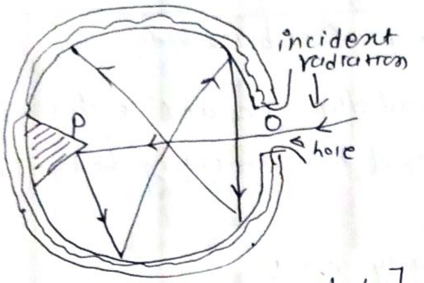
For a perfect black body absorptance (a) = 1 and emissivity (e) = 1. When a body absorbs all radiant energy, it neither reflect nor it transmit. As a result body appears black and known as black body. Sun emits radiations of all wavelengths. So it can be considered as a black body. A good absorber is a good emitter. Therefore, a black body is a good emitter and can emit the radiation of all possible wavelengths.

A perfect black body cannot be realized in actual practice. The nearest approach to a perfectly black body is a surface coated with lamp black or platinum black. Such a surface absorbs 96% to 98% of the incident radiation. Ferry's body is an example of perfectly black body realized in practice.

Ferry's black body:

Ferry's black body is a close approximation of perfectly black body which can be realized in practice. Ferry's body consists of a double walled hollow copper sphere with a small fine hole O and a conical projection P opposite to O as shown in figure.

A heating filament is kept in between two ~~walls~~ walls of the sphere and inner surface of sphere is coated with lamp black or platinum black.



[Fig: Ferry's black body]

When heat radiation falls on the conical projection P through hole O, projection P prevents the direct reflection of incident radiation rather it causes multiple reflections of radiations inside the sphere. most of the incident radiation is absorbed by lamp black or platinum black and remaining part is also lost by multiple reflection. When the sphere is heated by passing electric current to the heating filament, radiations of all possible wavelengths come out from the hole O. Hence, the hole O behaves as a black body.

Emissive power

It is the amount of heat radiations radiated by a unit area of a black body per unit time at a particular temp. It is denoted by E. Mathematically,

$$\text{Emissive power } E = \frac{\text{Energy radiated }(Q)}{\text{Area }(A) \times \text{time }(t)} = \frac{Q}{A \times t}$$

$$\therefore E = \frac{Q/t}{A} = \frac{P}{A}$$

$$\text{unit of } E = \frac{J/sec}{m^2} = W/m^2.$$

According to Stefan's-Boltzmann law,

$E = \sigma T^4$ where σ = Stefan-Boltzmann constant $= 5.67 \times 10^{-8}\ W/m^2 K^4$.

Emissivity (e):

Emissivity of a body is defined as the ratio of emissive power of a given body to the emissive power of a perfectly black body of same size at the same temp. It is denoted by e.

$\therefore e = \dfrac{\text{emissive power of body}}{\text{emissive power of perfect black body}} = \dfrac{E}{E_b}$

Emissivity has no unit and dim^n. Value of e depends upon the nature of the surface of the emitting body. Its values ranges from 0 to 1.

For a perfectly black body, $e = 1$

" a black body $e < 1$

For a perfectly reflecting surface $e = 0$.

Stefan's-Boltzmann's law of black body radiation:

Stefan's declared experimentally that energy radiated by a body depends only on its temp. and he put forward a law known as Stefan's law.

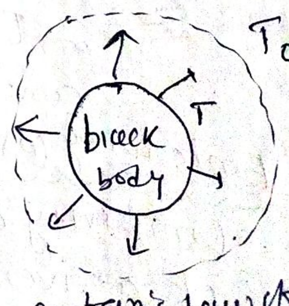
Stefan's-Boltzmann law states that "emissive power of a black body is directly proportional to the fourth power of absolute temp. of surface of the body".

Let E be the emissive power of a black body at absolute temp. T. Then, according to Stefan's-Boltzmann law $E \propto T^4$

or $E = \sigma T^4$ — (1) where σ is proportionality constant is called Stefan's Const. The value of σ in S.I. system is $5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$.

If a body is not perfectly black, and has its emissivity e. Then,

$E = e \sigma T^4$ — (2)



[Fig: Stefan's law of black body radiation] or $\frac{Q}{A \cdot t} = e\sigma T^4$ or $\frac{Q/t}{A} = e\sigma T^4 \Rightarrow \frac{power}{A} = e\sigma T^4$

$\therefore P = eA\sigma T^4$ —— (3)

This relations given above are valid only if the temp. of the surrounding is zero kelvin. But, if there is a surrounding at temp T_0 (more than 0K.) then, surrounding also emits heat radiation, which is absorbed by black body. Hence, relation is modified by Boltzmann for the net emissive power as

$E' = \sigma(T^4 - T_0^4)$ —— (4) This is called Stefan's-Boltzmann law.

If a body is not perfectly black body, the net emissive power $E' = e\sigma(T^4 - T_0^4)$

$$\frac{Q'}{A \times t} = e\sigma(T^4 - T_0^4)$$

$$\text{or } \frac{Q'/t}{A} = e\sigma(T^4 - T_0^4)$$

$$\therefore P' = eA\sigma(T^4 - T_0^4) \text{ —— (5)}$$

This gives power of radiated energy of a black body.

Some Numerical problems

(1) An insulating material having a thermal conductive of 0.08 W/m-K is used to limit the heat transfer of 80 W/m² for a temp. of 15°C across the opposite faces. Find the thickness of the material.

Soln. $K = 0.08\ W/m\text{-}K$, $\frac{Q}{A} = 80\ W/m^2$ $\Delta T = 15°C = 15K$

$dx = ?$ $\Rightarrow Q = KA\frac{dT}{dx}$ $\Rightarrow dx = \frac{Q}{AKdT}$ $\Rightarrow dx = 0.15m$