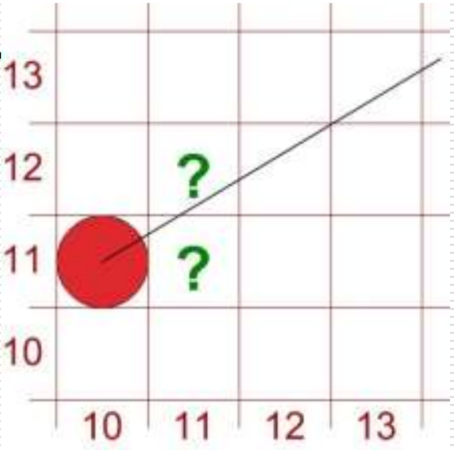
# Computer Graphics (L04)

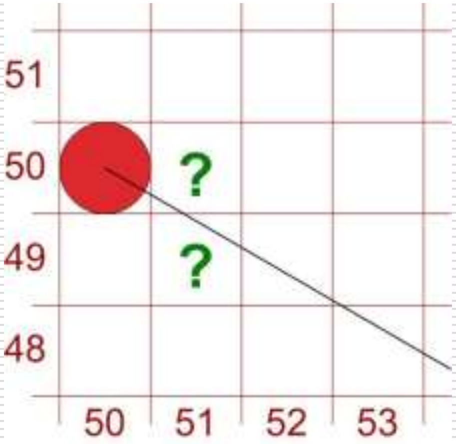
EG678EX

## 2-D Algorithms

# Bresenham's  Line Algorithm

- Uses only incremental integer calculations
- Which pixel to draw ?
  - (11,11) or (11,12) ?
  - (51,50) or (51,49) ?
  - Answered by Bresenham



- For $|m|<1$
  - Start from left end point $(x_0,y_0)$ step to each successive column (x samples) and plot the pixel whose scan line y value is closest to the line path.
  - After $(x_k,y_k)$ the choice could be $(x_k+1,y_k)$ or $(x_k+1,y_k+1)$

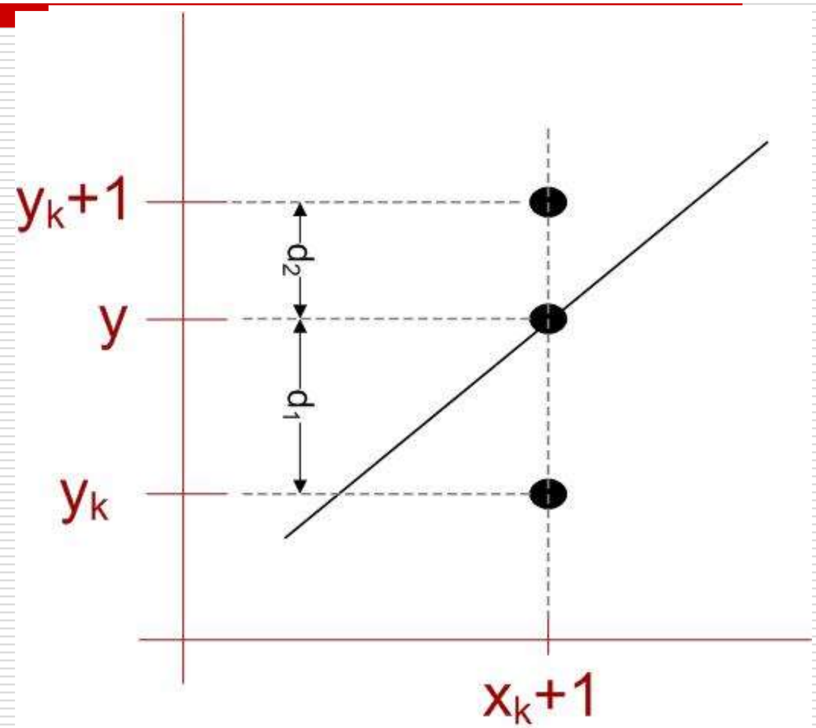
$$y = m(x_k + 1) + b$$

Then

$$d_1 = y - y_k$$
$$= m(x_k + 1) + b - y_k$$

And

$$d_2 = (y_k + 1) - y$$
$$= y_k + 1 - m(x_k + 1) - b$$




Difference between separations

$$d_1 - d_2 = 2m(x_k + 1) - 2y_k + 2b - 1$$

Constant=2Δy + Δx(2b-1) Which is independent of pixel position

# Defining decision parameter

$$p_k = \Delta x(d_1 - d_2) \quad [1]$$
$$= 2\Delta y.x_k - 2\Delta x.y_k + c$$

**Sign of $p_k$ is same as that of $d_1$-$d_2$** for Δx>0 (left to right sampling)

c eliminated here

$$p_{k+1} = 2\Delta y.x_{k+1} - 2\Delta x.y_{k+1} + c$$

$$p_{k+1} - p_k = 2\Delta y(x_{k+1} - x_k) - 2\Delta x(y_{k+1} - y_k)$$

because $x_{k+1} = x_k + 1$

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x(y_{k+1} - y_k)$$

$y_{k+1}$-$y_k$ = 0 if $p_k < 0$
$y_{k+1}$-$y_k$ = 1 if $p_k \geq 0$

For Recursive calculation, initially

$$p_0 = 2\Delta y - \Delta x$$

Substitute *b* = $y_0$ – m.$x_0$ and m = *Δy/Δx in [1]*

# Algorithm Steps (|m|<1)

1. Input the two line endpoints and store the left endpoint in $(x_0,y_0)$
2. Plot first point $(x_0,y_0)$
3. Calculate constants $\Delta x$, $\Delta y$, $2\Delta y$ and $2\Delta y - 2\Delta x$, and obtain $p_0 = 2\Delta y - \Delta x$
4. At each $x_k$ along the line, starting at k=0, perform the following test:

   If $p_k<0$, the next point plot is $(x_k+1,y_k)$ and

   $$P_{k+1} = p_k + 2\Delta y$$

   Otherwise, the next point to plot is $(x_k + 1, y_k+1)$ and

   $$P_{k+1} = p_k + 2\Delta y - 2\Delta x$$

5. Repeat step 4 $\Delta x$ times

# What’s the advantage?

- Answer: involves only the calculation of constants $\Delta x$, $\Delta y$, $2\Delta y$ and $2\Delta y - 2\Delta x$ once and integer addition and subtraction in each steps

# Example

Endpoints (20,10) and (30,18)

Slope m = 0.8

$\Delta x = 10$, $\Delta y = 8$

$P_0 = 2\Delta y - \Delta x = 6$

$2\Delta y = 16$, $2\Delta y - 2\Delta x = -4$

**Plot ?**

Plot $(x_0, y_0) = (20,10)$

| $k$ | $p_k$ | $(x_{k+1}, y_{k+1})$ | $k$ | $p_k$ | $(x_{k+1}, y_{k+1})$ |
|---|---|---|---|---|---|
| 0 | 6 | (21,11) | 5 | 6 | (26,15) |
| 1 | 2 | (22,12) | 6 | 2 | (27,16) |
| 2 | -2 | (23,12) | 7 | -2 | (28,16) |
| 3 | 14 | (24,13) | 8 | 14 | (29,17) |
| 4 | 10 | (25,14) | 9 | 10 | (30,18) |