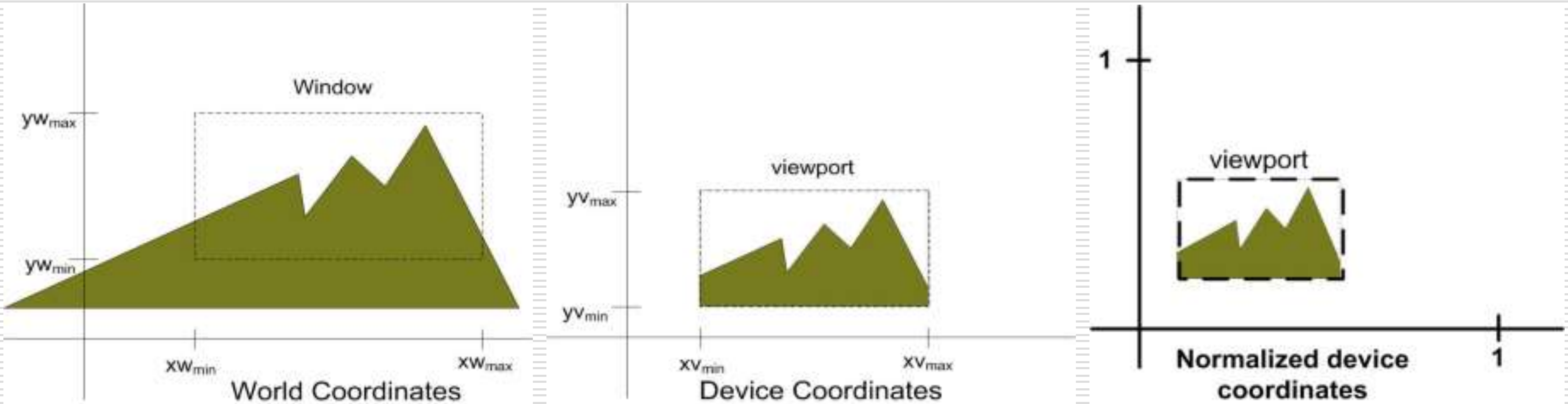
Computer Graphics (L10)
EG678EX

2-D Viewing

Viewing Transformation

- Transformation from world to viewport



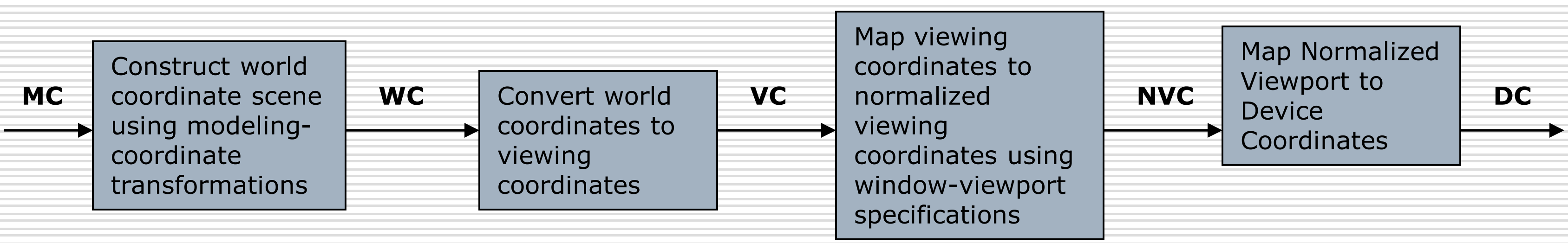
- For Fixed sized viewport, **zooming in** effect is attained if window size is decreased and **zooming out** effect if window size is increased.

- Panning effect is attained by successively changing the position of viewing window

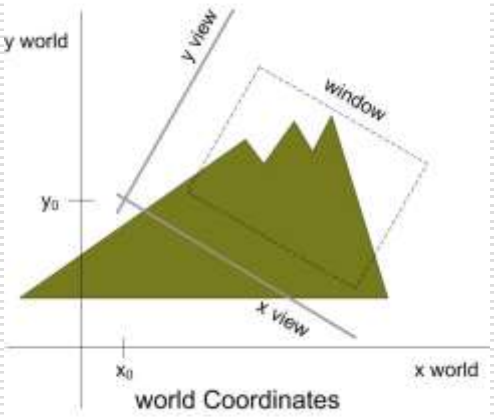
- The normalized device coordinates maps the world co-ordinate to the viewport of maximum size = unit square as shown in figure. The advantage is the mapping is display device independent

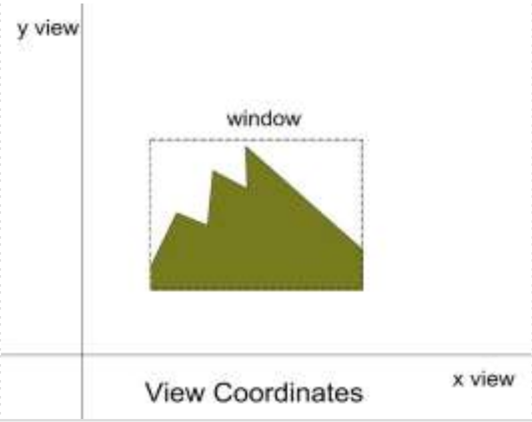
2-D Viewing pipeline

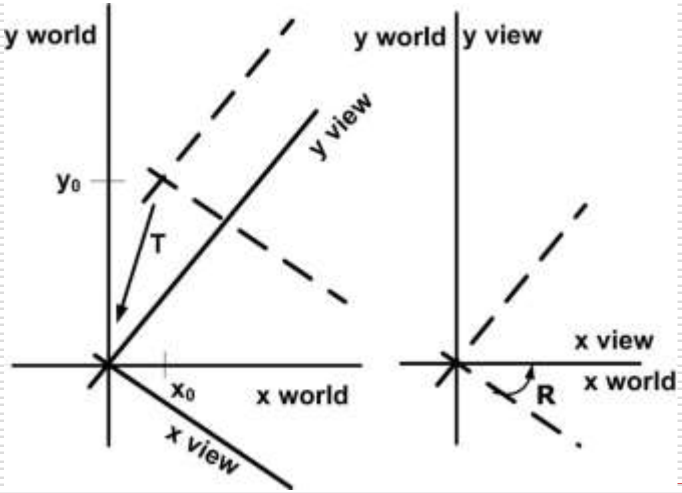
- Procedures for displaying views of a two-dimensional picture on an output device:
  - Specify which parts of the object to display (clipping window, or world window, or viewing window)
  - Where on the screen to display these parts (viewport).
- Clipping window is the selected section of a scene that is displayed on a display window.
- Viewport is the window where the object is viewed on the output device.



Viewing Reference Frame







Setting up viewing Reference Frame

The matrix is obtained in two steps:

1. Translate Viewing reference frame origin to world origin
2. Rotate Viewing reference frame to coincide with world coordinate axes

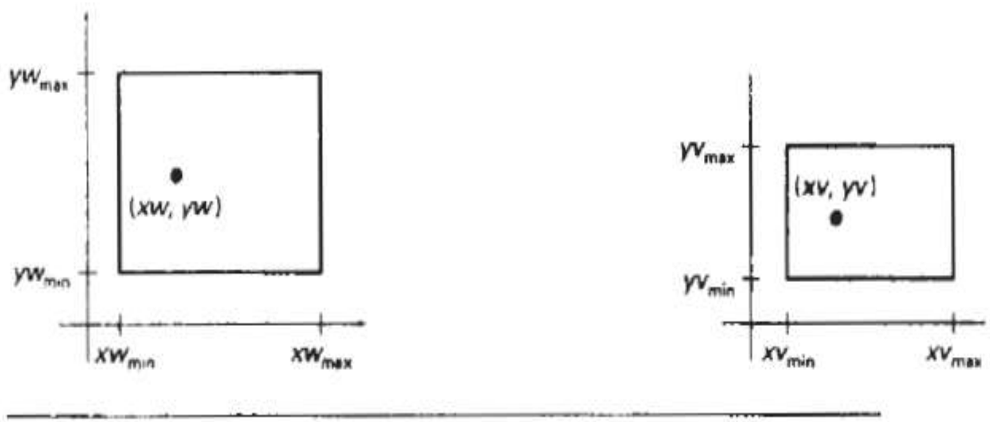
Thus we get the matrix as

$$\mathbf{M_{wc,vc} = R.T}$$

Windows To Viewport Co-ordinate Transformation

$$\frac{xv - xv_{min}}{xv_{max} - xv_{min}} = \frac{xw - xw_{min}}{xw_{max} - xw_{min}}$$

$$\frac{yv - yv_{min}}{yv_{max} - yv_{min}} = \frac{yw - yw_{min}}{yw_{max} - yw_{min}}$$



Solving these we get

$$xv = xv_{min} + (xw - xw_{min})sx$$

$$yv = yv_{min} + (yw - yw_{min})sy$$

Where scaling factors are

$$sx = \frac{xv_{max} - xv_{min}}{xw_{max} - xw_{min}}$$

$$sy = \frac{yv_{max} - yv_{min}}{yw_{max} - yw_{min}}$$

If sx = sy, the proportion is maintained otherwise the scene is stretched

Point Clipping

$xw_{min} \le x \le xw_{max}$

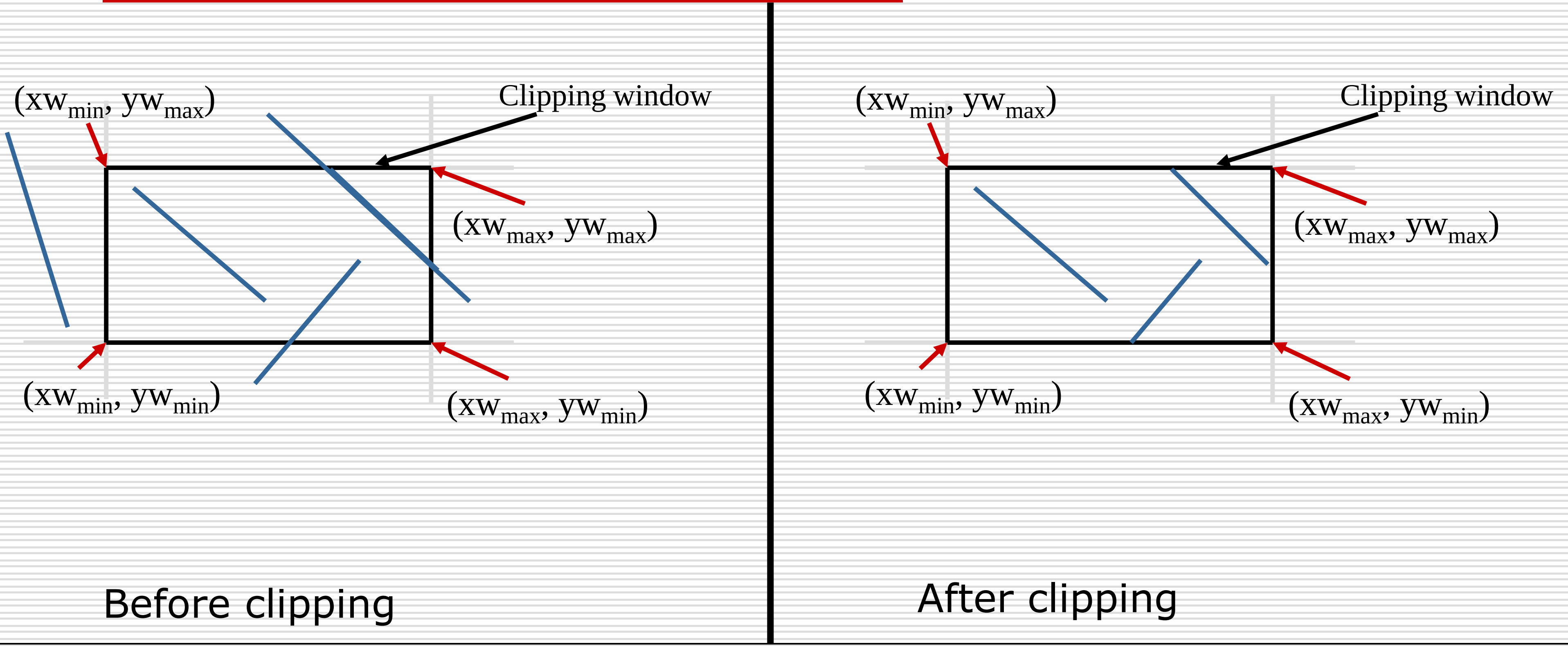
$yw_{min} \le y \le yw_{max}$

If all the four inequalities are satisfied for a point with co-ordinate (x,y), the point is accepted; i.e not clipped

Line Clipping

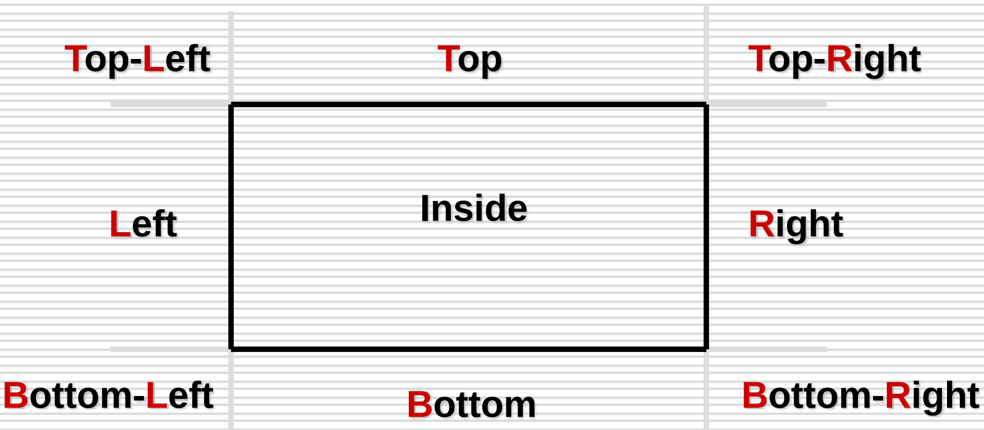
[Note: most of the slides about line clipping are from some internet resource]

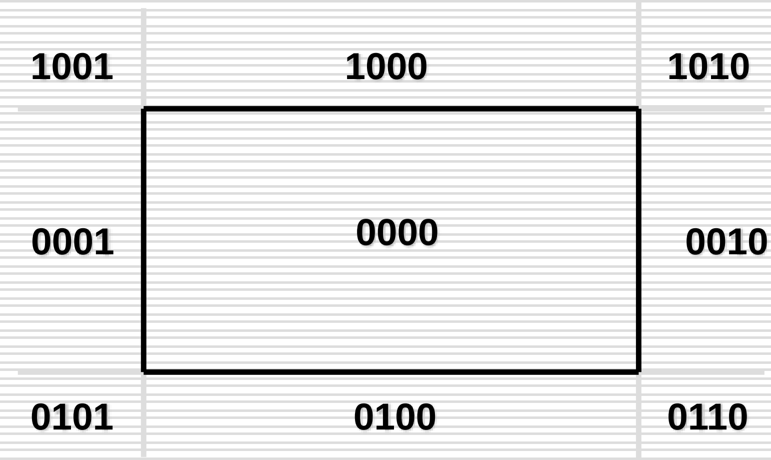
Line Clipping



What are the methods (algorithms) to perform clipping operations ?

1. Cohen-Sutherland line clipping





LRBT

Region codes	T	B	R	L
Bit position	1	2	3	4

Basic Idea

- ❑ label the Left, Right, Bottom and Top of clipping rectangle with region codes
- ❑ Test for Trivial Acceptance and Rejection (How?)
- ❑ If not trivially accepted or rejected successively clip out the portion of line outside the clip boundary and test whether it is trivially accepted or rejected

Cohen-Sutherland line clipping

Region coding

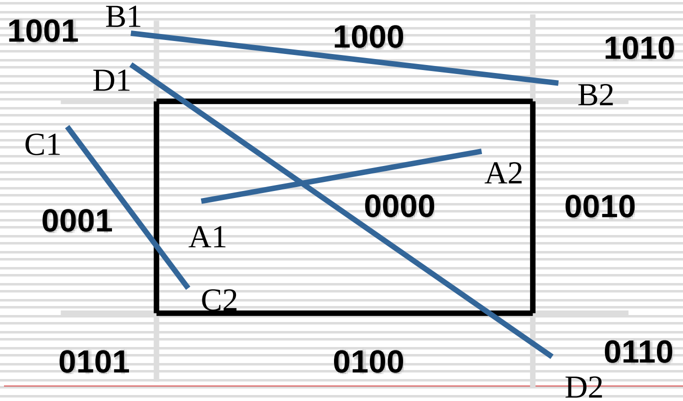
How would you decide which region an endpoint is in?

e.g

if $(x<xw_{min})$ && $(y>y_{max})$ → the point is at the **Top-Left**

Are there cases we can **trivially accept or reject**?

How would you test for those?



- A1=0000,A2=0000
  both (0000) → accept trivially
- B1=1001,B2=1010
  both NOT (0000)
  AND Operation
  B1 → 1001
  B2 → 1010
  Result 1000

  Result = NOT (0000) → reject trivially

Cohen-Sutherland line clipping

Algorithm Steps:

1. Assign a region code for each endpoints.
2. If both endpoints have a region code 0000 ---→ trivially accept these line.
3. Else, perform the logical AND operation for both region codes.
   - 3.1 **if** the result is **NOT** 0000 → trivially reject the line.
   - 3.2 **else** (i.e. result = 0000, need clipping)
     - 3.2.1. Choose an endpoint of the line that is outside the window.
     - 3.2.2. Find the intersection point at the window boundary (base on region code).
     - 3.2.3. Replace endpoint with the intersection point and update the region code.
     - 3.2.4. Repeat step 2 until we find a clipped line either trivially accepted or trivially rejected.
4. Repeat step 1 for other lines.

Cohen-Sutherland line clipping

Intersection calculations:

Intersection with vertical boundary

$y = y_1 + m(x-x_1)$

Where

$x = xw_{min}$ or xw_{max}

Intersection with horizontal boundary

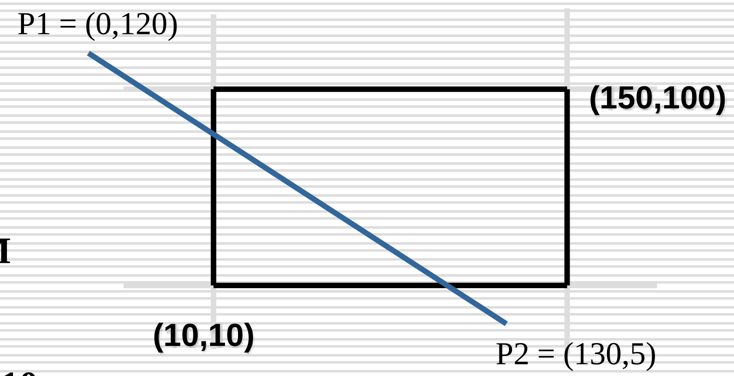
$x = x_1 + (y-y_1)/m$

Where

$y = yw_{min}$ or yw_{max}

Cohen-Sutherland line clipping

- Example:



1. P1=1001, P2=0100
2. (both 0000) – No
3. AND Operation

   P1 → 1001

   P2 → 0100

   Result 0000

   3.1 (not 0000) – no

   3.2 (0000) yes

   3.2.1 choose P2

   3.2.2 intersection with BOTTOM boundary

   m = (5-120)/(130-0) = -0.8846

   x = x1 + (y –y1)/m where y = 10;

   x = 130 + (10-5)/ -0.8846 = 124.35 = 124

   P2' = (124, 10)

   3.2.3 update region code P2' = 0000

   3.2. 4 repeat step 2

2. Liang-Barsky Line Clipping

- Based on parametric equation of a line:

  $x = x_1 + u.\triangle x$

  $y = y_1 + u.\triangle y \qquad 0 \le u \le 1$

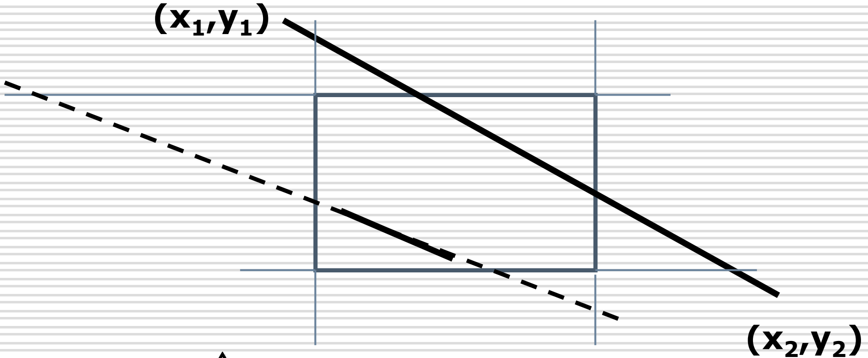
- Similarly, by adopting expressions for point clipping, the clipping window is represented by:

  $xw_{min} \le x_1 + u.\triangle x \le xw_{max}$

  $yw_{min} \le y_1 + u.\triangle y \le yw_{max}$

… or,

  $u.\ p_k \le q_k \qquad k = 1, 2, 3, 4$



- where:

k = 1 (is the line inside left boundary ?) $p_1 = -\triangle x$, $q_1 = x_1 - xw_{min}$

k = 2 (is the line inside right boundary ?) $p_2 = \triangle x$, $q_2 = xw_{max} - x_1$

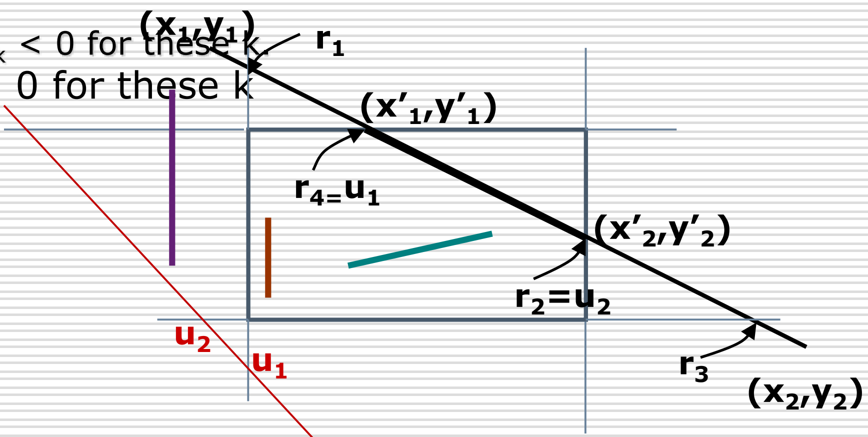
k = 3 (is the line inside bottom boundary ?) $p_3 = -\triangle y$, $q_3 = y_1 - yw_{min}$

k = 4 (is the line inside top boundary ?) $p_4 = \triangle y$, $q_4 = yw_{max} - y_1$

$P_k<0$ infinite extension of the line proceeds from outside to inside the infinitely extended boundary
$P_k>0$ infinite extension of the line proceeds from inside to outside of the infinitely extended boundary

Liang-Barsky Line Clipping

- Trivial rejection
  - Reject line with $p_k = 0$ for some k and one $q_k < 0$ for these k.
- For line with $p_k = 0$ for some k and all $q_k \geq 0$ for these k
  - Line is parallel to one of clip boundary
  - Some portion of line is inside



- For intersection with boundaries the parameters are supposed to be r_k given by

$$r_k = q_k / p_k$$

- Clipped line will be:

  $x_1' = x_1 + u_1 . \triangle x;$ $\quad u_1 \geq 0$
  $y_1' = y_1 + u_1 . \triangle y;$

  $x_2' = x_1 + u_2 . \triangle x;$ $\quad u_2 \leq 1$
  $y_2' = y_1 + u_2 . \triangle y;$

u_1 (For intersection with the boundaries to which line enters the boundary) = maximum value between 0 and r (for $p_k < 0$),

u_2 (For intersection with the boundaries to which line leaves the boundary) = minimum value between r and 1 (for $p_k > 0$),

Liang-Barsky Algorithm Steps

1. If $p_k = 0$ for some k then the line is parallel to a clipping boundary. Now test q_k :

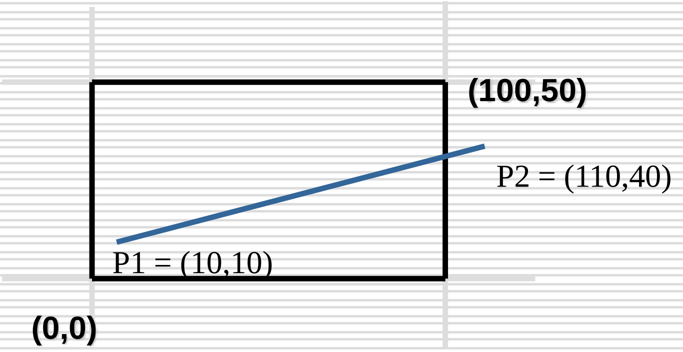
   if one $q_k < 0$ for these k then line is outside

   if all $q_k \geq 0$ for these k then some portion of line is inside
2. For all $p_k < 0$ (i.e. line proceeds from outside to inside the boundary) calculate $u = \max(0, \{r_k : r_k = q_k / p_k\})$ to determine intersection point with the possibly extended clipping boundary k and obtain a new starting point for the line at u_1.
3. For all $p_k > 0$ (i.e. line proceeds from inside to outside the boundary) calculate $u_2 = \min(1, \{r_k : r_k = q_k/p_k\})$ to determine intersection point with extended clipping boundary k and obtain a new end point at u_2.
4. If $u_1 > u_2$ then discard the line
5. The line is now between $[u_1, u_2]$

Liang-Barsky Algorithm Steps

- Example:

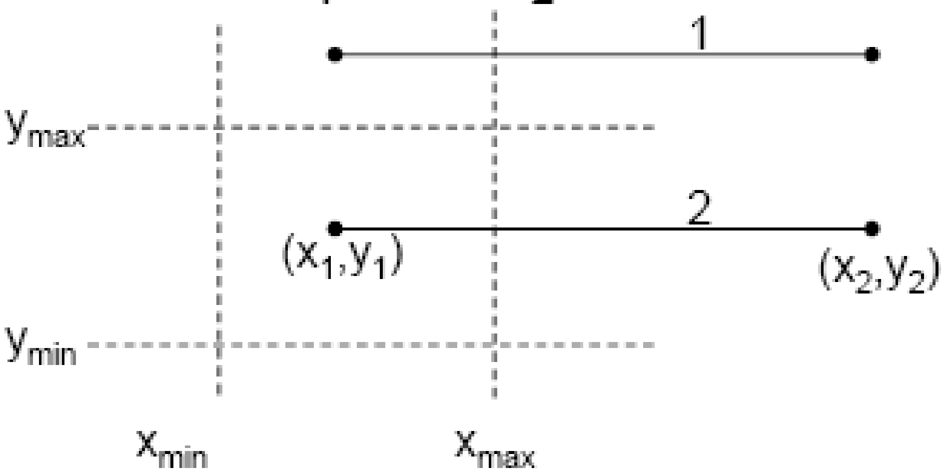
k	p_k	q_k	r_k
1	$-\triangle x$ $= -(110-10)$ $= -100$ i.e $p_k<0$	$x_1 - xw_{min}$ $= 10-0 = 10$	$r_1=10/(-100)$ $=-1/10$ u_1
2	$\triangle x$ $=110-10=100$ i.e $p_k>0$	$xw_{max}- x_1$ $= 100 - 10 = 90$	$r_2=90/100$ $=9/10$ u_2
3	$-\triangle y$ $= -(40-10)$ $=-30$ i.e $p_k<0$	$y_1 - yw_{min}$ $= 10-0 = 10$	$r_3=10/(-30)$ $=-1/3$ u_1
4	$\triangle y$ $= 40-10=30$ i.e $p_k>0$	$yw_{max} - y_1$ $= 50 - 10 = 40$	$r_4=40/30$ $=4/3$ u_2



We take
$u_1 = 0$
And
$u_2 = 0.9$

Line Clipping: Liang-Barsky

- Example 2: Consider horizontal lines with $\Delta y = 0$, $p_3 = p_4 = 0$.
- For line 1, $q_4 < 0$ and will be discarded.
- For line 2, $p_1 < 0$, $p_2 > 0$, $q_3 > 0$ and $q_4 > 0$. We proceed to calculate u_1 and u_2.



Line Clipping: Liang-Barsky

- $p_1 < 0$, note that $q_1 > 0$ hence $q_1/p_1 < 0$, and $u_1 = \max\{0, q_1/p_1\} = 0$
- $p_2 > 0$, calculate q_2/p_2 and $u_2 = \min(q_2/p_2, 1) = q_2/p_2$.
- $u_1 < u_2$ and the line is between $[u_1, u_2]$