POKHARA UNIVERSITY

Level: Bachelor Semester – Fall Year : 2012
Programme: BE Full Marks: 100
Course: Problem Solving Techniques Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Determine how many zeros end the number $300! + 12^{15} \times 25^{10}$. 5

   b) What is the last digit of 3^{65221}. 5

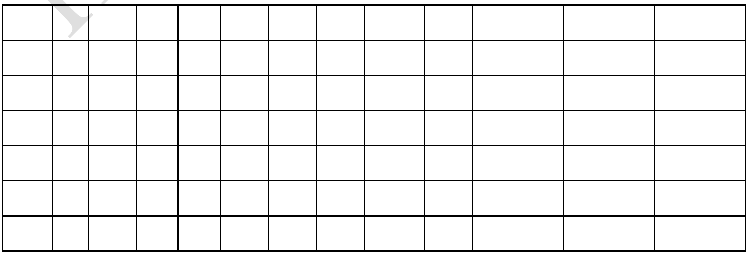
   c) Suppose you are told in advance that 10 of the cattle present are lame, and only have three feet. But the count yields 120 heads and 300 feet. How many cattle and how many people are there? 5

2. a) Prove the law of cosines: given a triangle Δ ABC, if α the angle determined by sides AB and AC, then $|BC|^2=|AB|^2+|AC|^2-2|AB||AC| \cos\alpha$. 8

   b) Consider a polyhedron with 5 triangular faces meeting at each vertex. How many faces, vertices and edges will it have? 7

3. a) A game is played by two players. They begin with a pile of 30 chips, all the same. For his or her move, a player may remove 1 to 6 chips. The player who removes the last chips wins. What strategy can the first player use so that he will always win? 7

   b) Draw a planar grid that is 13 squares wide and 9 square high. How many different non-trivial rectangles can be drawn, using the lines of the grid to determine the boundaries? 8



4. a) Suppose you have 9 pearls. They all look the same, but 8 of them have equal weight and one is different. The odd pearl is either lighter or heavier; you do not know which. The only equipment that you have at hand is balance scale. How can you use the scale to find the odd pearl with its weight in just three weighing? 8

b) Calculate the sum: 7

$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + 3 \cdot 4 \cdot 5 + \cdots + n(n+1)(n+2).$

5. a) Which is greater, sin(cosx) or cos(sinx)? 7

b) Suppose that x is an angle and that tan (x/2) is rational. Verify that then sin x, cos x are both rational. Also show that $|\cos x - \sin x| \leq \sqrt{2}$. 8

6. a) A general needs to take his troops across the river. He spies two boys with a small boat. He commandeers both the boats and the river. But the boat will hold two boys or one soldier. Everyone is capable of rowing the boat. He determines a method for getting his troops across. What could it be? 5

b) The following sequence has become known as the John H. Conway sequence. Explain how it has formed. Also write two more terms. 1,1,1,3,1,4,1,1,3,6,1,2,3,1,4,8,1,3,3,2,4,1,6.. ..? 5

c) You have a piece of paper with a circle of radius between 2’’ and 4’’ drawn on it. You have a plastic square of side 10’’. You have no ruler and compass. How can you find the center of the circle? 5

7. Write short notes on **any two:** 2×5

a) Pythagorean Theorem

b) Use of Problem Solving Techniques

c) Prove by Contradiction

d) Solid Geometry

POKHARA UNIVERSITY

Level: Bachelor | Semester: Fall | Year : 2014
Programme: BE | Full Marks: 100
Course: Problem Solving Technique | Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Derive the sum of first K square positive integer. 8

   b) There are M number of Pigeons holes and (M+1) number of Pigeon. Verify that one Pigeonhole must contain at least two Pigeons. 7

2. a) Prove the cosine law: $|BC|^2 = |AB|^2 + |AC|^2 - 2|AB||AC|COS\ a$ 8

   b) How many digits are used to number the pages of a book having 100 pages numbered from 1 to 100? 7

3. a) A 10 feet pole is randomly cut into 3 pieces. Work out the probability that the 3 pieces will form a triangle. 7

   b) Solve the given crypto-arithmetic, where * represent any digits. 8

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* * 9)  6 * 8 * * *  (* 5 3
        * * * 2
        -------
        * 9 * *
          * * 4 *
          -------
            * * 4 *
            * * * *
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4. a) Explain the process of creating magic square of 5×5 using rolling method with example. 7

   b) Prove : $COS(a/2).COS\ (a/4).COS\ (a/8) = \frac{\sin(a)}{8\sin(a/8)}$ 8

5. a) A martini is made by mixing K parts gin with 1 part vermouth. Gin is 8

usually 40% alcohol while vermouth is 20% alcohol. A martini is said to be "dry" if it contains relatively little Vermouth. For instance. if K= 15 then the martini is said to be dry. If instead K=5 then the martini is said to be "sweet". Now compare the percentage of alcohol in dry martini and sweet martini.

b) Write any three digit number. Write that three digit again adjacent to the first three number. So, you end with a six digit number. Divide that six digit number by 7, certain answer will come out. Divide the answer that you have just obtained by 11. The certain answer will come out. Divide the answer that you have just obtained by 13. Now the answer will be the three digit number which you wrote at first. Explain why this work does or happen? 7

6. a) Explain why, if two positive real number sum to 60, then their product cannot be 1000? 5

b) A right triangle has sides of length l, m, 10. But 10 is not the hypotenuse, both l and m are integer. Find l and m. 5

c) Suppose that you have another addition of encyclopedia with six volumes. The covers are ¼" thick, but the paper portion in each volume is 0.8" thick. then how far does the worm have to call to get from the very first page of volume 1 to the very last page of volume 4? 5

7. Write short notes on: (**Any two**) 2×5

a) Use of problem solving technique in computer fields

b) Proof by Contradiction

c) Is $10^{1/10} > 2^{1/3}$?

POKHARA UNIVERSITY

Level: Bachelor | Semester: Fall | Year : 2015
Programme: BE | Full Marks: 100
Course: Problem Solving Technique | Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Determine how many zeros end the number $300!+8^{15}\cdot 25^{10}$ 5

   b) Verify by using mathematical induction: 5

   For $n \geq 1$,

   $$2+2^2+2^3+2^4+\ldots+2^n = 2^{n+1}-2.$$

   c) What is the last digit of 31^{4568}? 5

2. a) Prove the law of sines: given a triangle Δ ABC, if α, β, γ are the angles at the vertices A,B,C respectively then 8

   $$\frac{\sin\alpha}{\overline{BC}} = \frac{\sin\beta}{\overline{AC}} = \frac{\sin\gamma}{\overline{AB}}.$$

   b) Explain why it is impossible to have a polyhedron with 6 triangular faces meeting at each vertex. 7

3. a) Draw a planar grid that is 31 squares wide and 17 squares high. How many different non-trivial rectangles can be drawn, using lines of the grid to determine the boundaries? 7

   b) Solve the given Crypto-arithmetic problem in which all different letters denote different digits. Identical letters denote the same digit. The problem is to identify all digits. 8

   LETS +WAVE =LATER.

   OR

   Examine the equations

   $$1 = 1$$

$$3+5 = 8$$
$$7+9+11 = 27$$
$$13+15+17+19 = 64$$
$$21+23+\ldots+29 = 125$$

Determine the pattern and prove the identity.

4. a) Game is played by two players. They begin with a pile of 30 chips, all the same. For his or her move, a player may remove 1 to 6 chips. The player who removes the last chip wins. What strategy can the first player use so that he will always win? 8

   b) Construct a 5×5 magic square with the description of the logical construction method. 5

   c) Explain why between any two rational numbers, there lies an irrational number. 5

5. a) Suppose that you have 9 pearls. They all look the same, but 8 of have equal weight and one is different. The odd pearl is either lighter of heavier; you do not know which. The only equipment that you have at hand is a balance scale. How can you use the scale to find the odd pearl in just three weighing? 7

   b) If n is a positive integer then show that $5^n + 2 \cdot 3^{n-1} + 1$ is divisible by 8. 5

6. a) Prove that there is no rational number whose square is 8. 5

   b) Show that with usual notation: $2 < \dfrac{1}{\log_2 \pi} + \dfrac{1}{\log_\pi 2}$ 5

   c) Say that a bottle has a round or square, flat, bottom and it has straight sides. The bottle is partially full (about half) of liquid. It is tapered at the top and has a screw-up cap. How can you accurately determine the volume of the bottle if you are equipped with only a ruler? 5

7. Write short notes on: (**Any two**) 2×5

   a) Use of problem solving technique in IT

   b) Magic Square

   c) Use of solid geometry

POKHARA UNIVERSITY

Level: Bachelor Semester – Fall Year : 2011
Programme: BE Full Marks: 100
Course: Problem Solving Techniques Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) In a PST class there are ‘k’ students. At the beginning of each class hour, each student shakes hands with each of the other students. If ‘k’ is even, then will the number of handshakes that takes place be even or odd? If ‘k’ is odd, then will the number of handshakes that takes place be even or odd? 8

   b) Show the sum of first K square positive integer. 7

2. a) What is greatest number of regions into which four straight lines can divide the plane? 7

   b) Find the number of vertex, edge and face if five triangles faces meets at each vertex. 8

3. a) Our problem is to inscribe a circle of radius 1 unit in an equilateral triangle. What will be the suitable height of the triangle? Furthermore there are three circles inscribed between the first circle and two sides of the triangle near each vertex, the process continue s indefinitely with progressively smaller circle, what is the sum of the radii of all the circles? 7

   b) A game is played by 2 players. They begin with a pile of 30 chips. For his move, a player may remove 1 to 6 chips. The player who removes the last chip wins. What strategy can a second player use so he will always win? 8

4. a) Solve the following crypto- arithmetic problem in which all different letters denote different integers. Identical letters denote the same integer. S T O P = (S+ T O P)^2 8

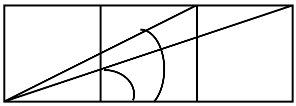
OR

Arrange the numbers from 1 to 100 in 4× 4 magic squares in such a way that sum of numbers in rows, columns and diagonals add to same result 94. 8

b) Imagine a 4 × 4 array of squares. The challenge is to put the consecutive odd integers 1 to 100, one in each square, so that each row each columns and each diagonal adds up to the same number. 7

5. a) How many different 5 card poker hands may be had from a deck of 52 cards? 8

b) In given unit squares show that the sum of given two angle is 45 degree. 7



6. a) Explain the process of creating magic square of 5 × 5 with example. 5

b) Prove: $\cos\left(a/2\right).\cos\left(a/4\right).\cos\left(a/8\right). = \dfrac{\sin(a)}{8\sin\left(a/8\right)}$ 5

c) An efficient expert is doing a study of a certain fast food restaurant. She observes that a particular clumsy waiter drops 40 % of of all the biscuits that he serves.What is the probability that he serves.What is the probability that he will drop exactly 3 out of 10? 5

7. Write short notes on **any two:** 2×5

a) Use of problem solving technique in computer fields

b) Use of geometry in computer

c) Explain impossible problems

POKHARA UNIVERSITY

Level: Bachelor   Semester: Fall   Year : 2015
Programme: BE   Full Marks: 100
Course: Problem Solving Techniques   Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) In a PST class there are 'k' students. At the beginning of each class hour, each student shakes hands with each of the other students. If 'k' is even, then will the number of handshakes that takes place be even or odd? If 'k' is odd, then will the number of handshakes that takes place be even or odd? 8

   b) There are M number of Computers and (M+1) number of Students in a "Programming in C" Lab. Verify that one computer must be shared by at least two Students. 7

2. a) Show the sum of first K square positive integer. 8

   b) Draw a 8×8 magic square such that each row, each column and each diagonal adds up to the same number. Explain in detail the process you used to draw such 8×8 magic square. 7

3. a) Find the number of vertex, edge and face if five triangles faces meets at each vertex. 8

   b) Prove the law of Sines: given a triangle Δ ABC where A, B, C are vertices and a, b, c are the sides opposite to the angles. 7

$$\frac{sin\ A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

4. a) Solve the Crypto arithmetic problem for: 8

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  S E N D
+ M O R E
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M O N E Y
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b) To number the pages of a large book, the printer uses 1890 digits. How many pages are in the book? 7

5. a) Explain the process of creating magic square of 5 × 5 with example. 5

b) Prove: $\cos\left(\frac{a}{2}\right).\cos\left(\frac{a}{4}\right).\cos\left(\frac{a}{8}\right). = \frac{\sin(a)}{8\sin\left(\frac{a}{8}\right)}$ 5

c) An efficient expert is doing a study of a certain fast food restaurant. She observes that a particular clumsy waiter drops 40 % of all the biscuits that he serves. What is the probability that he serves. What is the probability that he will drop exactly 3 out of 10? 5

6. a) Say that a bottle has a round or square, flat, bottom and it has straight sides. The bottle is partly full (about half) of liquid. It is tapered at the top (as bottles usually are) and has a screw on cap. How can you accurately determine the volume of the bottle if you are equipped with only a ruler? 5

b) A new car is equipped with three fuel saving devices. Device A by itself saves 40% on fuel; device B by itself saves 35% on fuel; and device C, itself saves 25% on fuel. Now suppose that the three devices are used together and that they act independently. Will the combination save 40+35+25=100% on fuel? 5

c) A martini is made by mixing K parts gin with 1 part vermouth. Gin is usually 40% alcohol while vermouth is 20% alcohol. A martini is said to be "dry" if it contains relatively little Vermouth. For instance. If K= 15 then the martini is said to be dry. If instead K=5 then the martini is said to be "sweet". Now compare the percentage of alcohol in dry martini and sweet martini 5

7. Write short notes on: (**Any two**) 2×5

a) Use of problem solving technique in computer fields

b) Use of Crypto- arithmetic problem

c) Explain impossible problems

POKHARA UNIVERSITY

Level: Bachelor | Semester: Spring | Year : 2012
Programme: BE | Full Marks: 100
Course: Problem Solving Techniques | Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Find the last digit of 2^{34526} 5

   b) A Math class has k students. At the beginning of each class hour, each student shakes hands with each of the other students. How many handshakes take place? If k is EVEN then will the number of handshakes that takes place be EVEN or ODD? If k is ODD then will the number of handshakes that takes place be EVEN or ODD? Justify. 10

2. a) A cow can clear a field in two days. A sheep can clear same field in three days. And a goat can clear it in four days. How long does it takes for all three animals together to clear the field? 7

   b) Imagine five planes in 3D space are kept in general position. Now find out into how many regions will these planes divide the space? 8

3. a) A right circular cone has a cube inscribed in it. If the radius of the cone is 1, and its height is 3 then what is the volume of the cube? 7

   b) A circle of radius 1 is inscribed in an equilateral triangle of suitable size. Then three more circles are inscribed between the first circle and the two sides of the triangle near each vertex. The process continues indefinitely, with progressively smaller circles. What is the sum of the radii of all the cicles? 8

4. a) Show that the following formula for the Fibonacci sequence is valid: 7

$$a_j = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^j - \left(\frac{1-\sqrt{5}}{2}\right)^j}{\sqrt{5}}$$

b) Examine the equation: 8

$$1=1$$
$$1-4= -(1+2)$$
$$1-4+9=1+2+3$$
$$1-4+9-16=-(1+2+3+4)$$

Determine the pattern and prove this identity.

5. a) Suppose you have 12 pearls, all appearing the same but with one having an odd weight. You do not know whether the odd pearl is heavier or lighter. How many weighings are needed to find the odd pearl? 7

b) There are two piles of chips. The piles may or may not have the same number of chips. There are two players. A legal move is either 8

i. To remove any number of chips from one pile, or

ii. To remove an equal number of chips from each pile.

The player who removes the last chip from the table wins. Give two examples of winning and losing positions.

6. a) 2 people at the opposite end of the road. Both started at the same time to walk at different constant speed. At first cross they crossed at 720m from RHS end. They reached to next end rested for 10 minutes and again started walking, now they crosses at 400m from LHS end. Now find the length of the road. 8

b) A 10 years old child puts \$100 in the bank. She intended to withdraw the money on her 21th birthday. Which one scheme is better for her: 7

i. An account with 5% interest compounded daily, or

ii. An account with 5.1% interest compounded weekly?

7. Write short notes on: (Any two) 2×5

a) Tower of Honai

b) Explain the use of geometry in computer field.

c) Mathematical Induction .

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POKHARA UNIVERSITY

Level: Bachelor | Semester – Fall | Year : 2005
Programme: BE | Full Marks: 100
Course: Problem Solving Technique | Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) In a PST class there are 'k' students. At the beginning of each class hour, each student shakes hands with each of the other students. If 'k' is even, then will the number of handshakes that takes place be even or odd? If 'k' is odd, then will the number of handshakes that takes place be even or odd? 5

   b) How many zeros end the number $\dfrac{30\times 15^{30}}{50!.20!} - 5^{20}.10^{60}$ 5

   c) What is greatest number of regions into which four straight lines can divide the plane? 5

2. a) Determine the validity case. To win a football match, players must be competent. If they do not win the match then either they are not competent or they are absent in the training. If they are not competent, they blame their coach. If they blame their coach or they are absent in the training, they will not get into the final. Therefore if they get into the final, they are competent. Verify the above statements, whether the conclusion is true. 7

   b) Our problem is to inscribe a circle of radius 1 unit in an equilateral triangle. What will be the suitable height of the triangle? Furthermore there are three circles inscribed between the first circle and two sides of the triangle near each vertex, the process continues indefinitely with progressively smaller circle, what is the sum of the radii of all the circles? 8

3. a) Of all parallelograms with a given perimeter, which has the greatest area? 7

   b) Take an equilateral triangle of side length L. Subdivide this triangle into smaller triangles by joining divided nodes its three sides. The division should be equal all the time. It means, the first division, the 8 mid points should be joined and you will have $2^2 = 4$ small triangles. In the next step, you divide the sides into three equal divisions and you will have $3^2 = 9$ small triangles. The pattern continues: the first n divisions have a n^2 smaller triangles. Explain why it happens.

4. a) Solve the following crypto-arithmetic problem, in which all different letters denote different integers. Identical letters denote the same integer. S T O P = (S + T O P)^2. 8

OR

Arrange the numbers from 1 to 100 in 4×4 magic square in such a way that sum of numbers in rows, columns and diagonals add to same result 94.

b) Imagine a 4 × 4 array of squares. The challenge is to put the consecutive odd integers 1 to 100, one in each square, so that each row each column and each diagonal adds up to the same number. 7

5. a) When I am as old as my father now. I will be five times as old as my son is now. But at that time my son will be eight years older than I am now. At present, the sum of the ages of my father and me is 100. How old is my son? 5

b) Suppose that you have 37 envelops and you address 37 letters to go with them. Closing your eyes, you randomly stuff one letter into each envelope. What is the probability that precisely two letters are in the wrong envelops and all others in the correct envelope? 5

c) A and B play game they begin with a pile of 40 chips all same. For his/her move a player may remove 1 to 6 chips. The player who removes the last chips wins. What strategy can A use so that he will always win? 5

6. a) What would be your strategy for finding the square of any numbers ending with digit 5. Explain upon your logic. 8

b) A general needs to take his troops across the river. He spies two boys with a small boat. He commandeers both the boat and the boys. Unfortunately, the boat will only hold two boys or one soldier. Yet he determines a method for getting his troops across. What could it be? 7

7. Write short notes on: (**Any Two**) 2×5

a) Aims and objectives of problem solving techniques

b) Crypto-arithmetic problem and its importance
c) Recreational problems
d) Impossible problems