Mathematical Foundation for Computer Science

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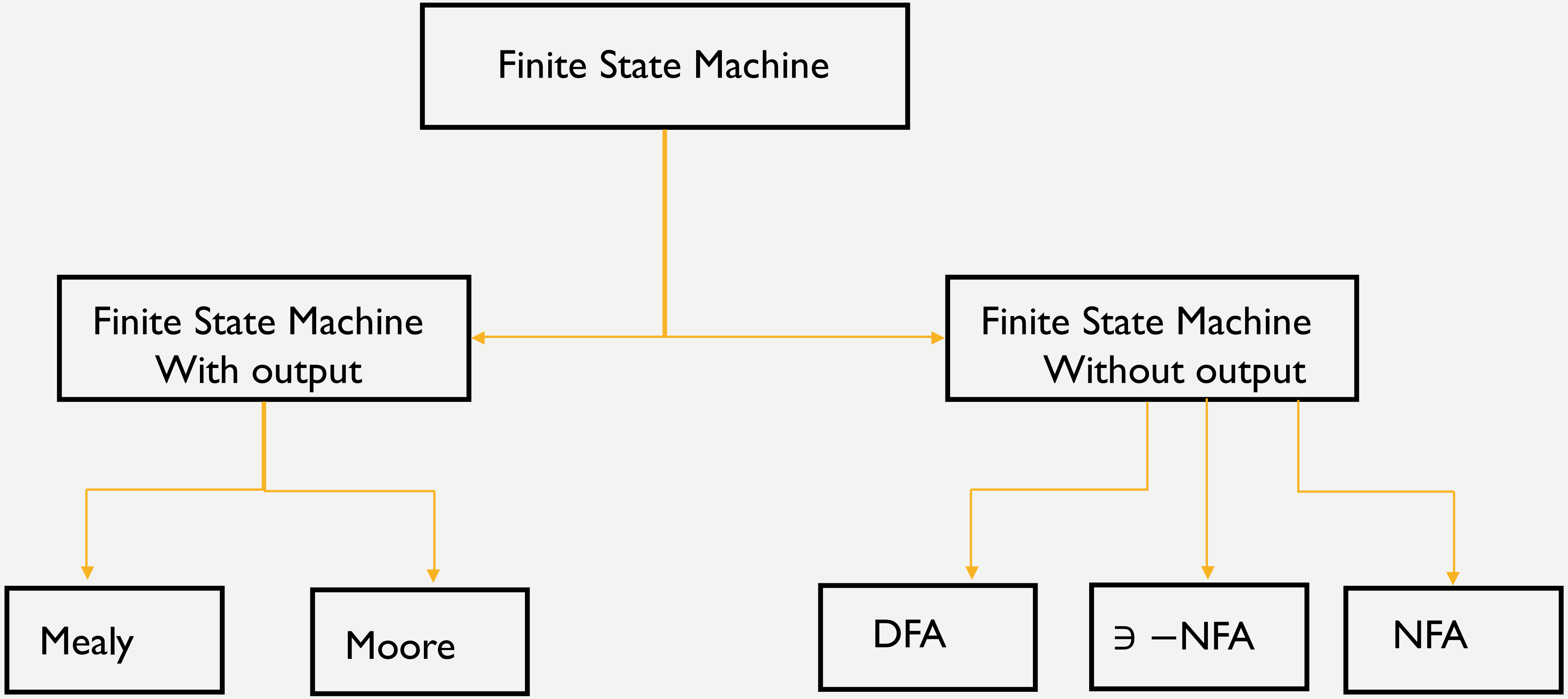
FINITE STATE AUTOMATA

- *Sequential Circuits and Finite state Machine*
- *Finite State Automata*
- *Non-deterministic Finite State Automata*
- *Language and Grammars*
- *Language and Automata*
- *Regular Expression*

FINITE STATE MACHINE(FSM):



FEW TERMINOLOGIES:

1. **Alphabet(Σ)**: It is a collection of input symbol. Example: Binary alphabet Σ= {1,0} , Σ= {a, b}

2. **String(w) :** It is a sequence of input symbol. Length of string is denoted by |w|.
   Example: w = aababa, w=01010101
   An empty string(Length 0) is denoted by ε or λ.

3. **Language(L):** It is a collection of string over given alphabet
   Example: Σ= {a, b} , L_1 = {set of all string of length 2 }, L_2= {set of all strong that begin with a}
   L_1 = {ab, ba , aa, bb} L_2 = {a , aa, ab, aaab, abab,……………..}

3. **Kleen closure(Σ^*):** The Kleene star, $\sum^*$, is a unary operator on a set of symbols or strings, $\sum$, that gives the infinite set of all possible strings of all possible lengths over $\sum$ including **λ**.
   Example − If $\sum$ = {a, b}, $\sum^*$ = {λ, a, b, aa, ab, ba, bb,………..}

4. **Positive closure(Σ^+):** The set $\sum^+$ is the infinite set of all possible strings of all possible lengths over $\sum$ excluding λ.
   Example − If $\sum$ = {a, b}, $\sum^+$ = {a, b, aa, ab, ba, bb,………..}

DETERMINISTIC FINITE STATE AUTOMATA(DFA):

- DFA is a Finite State Machine that accepts or rejects a given string of symbols, by running through a state sequence uniquely determined by the string. *Deterministic* refers to the uniqueness of the computation run.

- Formal Definition of Deterministic Finite State Automata(DFA)

  A deterministic finite state automata is defined as

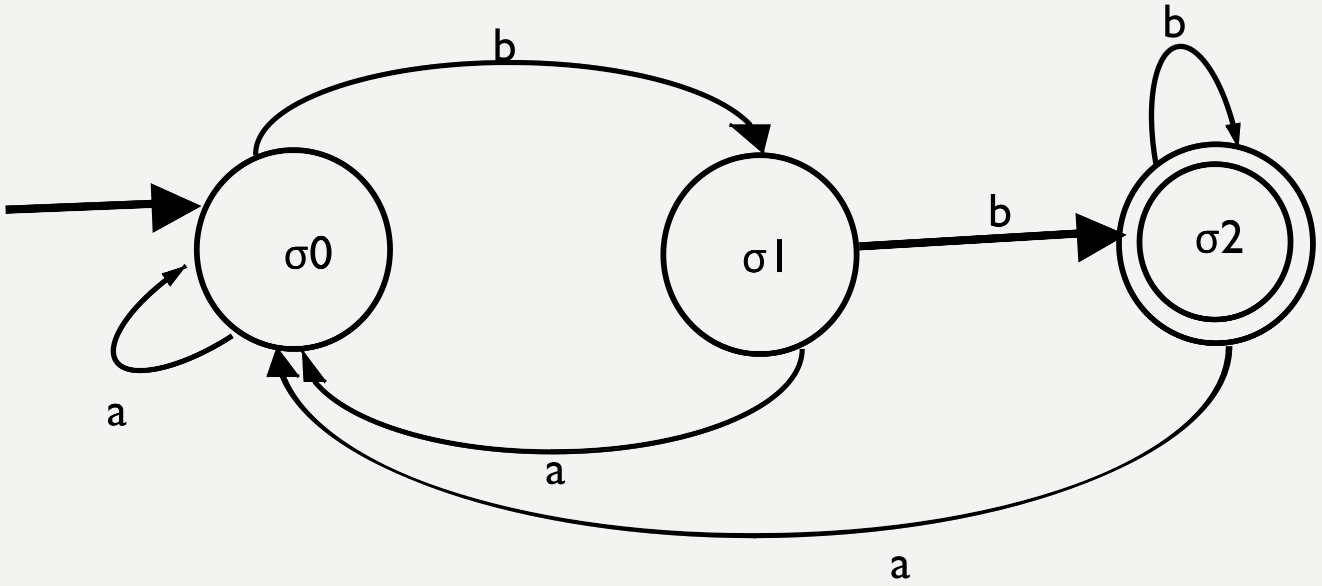
  M = (I, S, f, σ, A)

  (a) **I** = A finite set of input symbols.
  (c) **S** = A finite set of states.
  (d) **f** =A next-state Transition function defines as S × I into S.
  (e) **σ** =An initial state **σ** ∈ S.
  (f) **A** = set of accepting state or final state

DETERMINISTIC FINITE STATE AUTOMATA(DFA):

The transition diagram of the finite-state automaton A = (I, S, f, A, σ), where I = {a, b}, S = {σ0, σ1, σ2}, A = {σ2}, σ = σ0, and f is given by the following table

	f	
$\mathcal{S}$ \ $\mathcal{I}$	a	b
σ_0	σ_0	σ_1
σ_1	σ_0	σ_2
σ_2	σ_0	σ_2



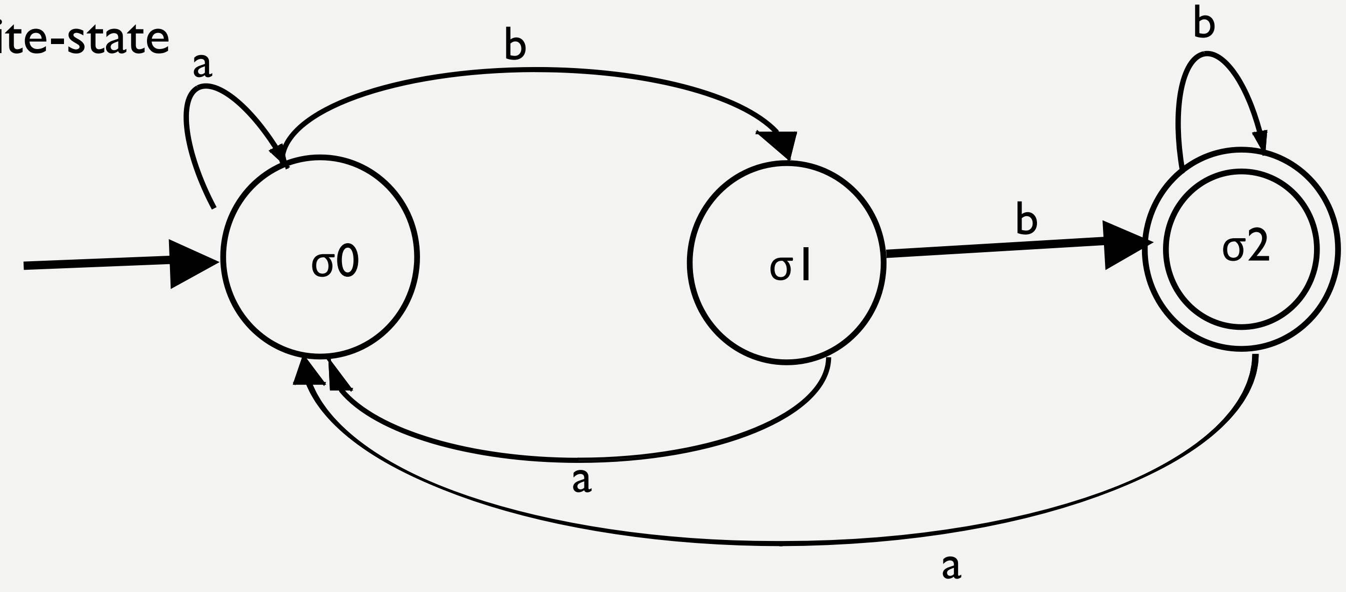
Is the string "abaa" accepted by the finite-state automaton?

ACCEPTANCE BY DFA:

Let $A = (I, S, f, A, \sigma)$ be a finite-state automaton.
Let $w = x_1 \cdots x_n$ be a string over I. If there exist states $\sigma_0, \ldots, \sigma_n$ satisfying
(a) $\sigma_0 = \sigma$
(b) $f(\sigma_{i-1}, x_i) = \sigma_i$ for $i = 1, \ldots, n$
(c) $\sigma_n \in A$,
we say that w is accepted by A. The null string is accepted if and only if $\sigma \in A$.

Is the string "abaa" accepted by the finite-state automaton?

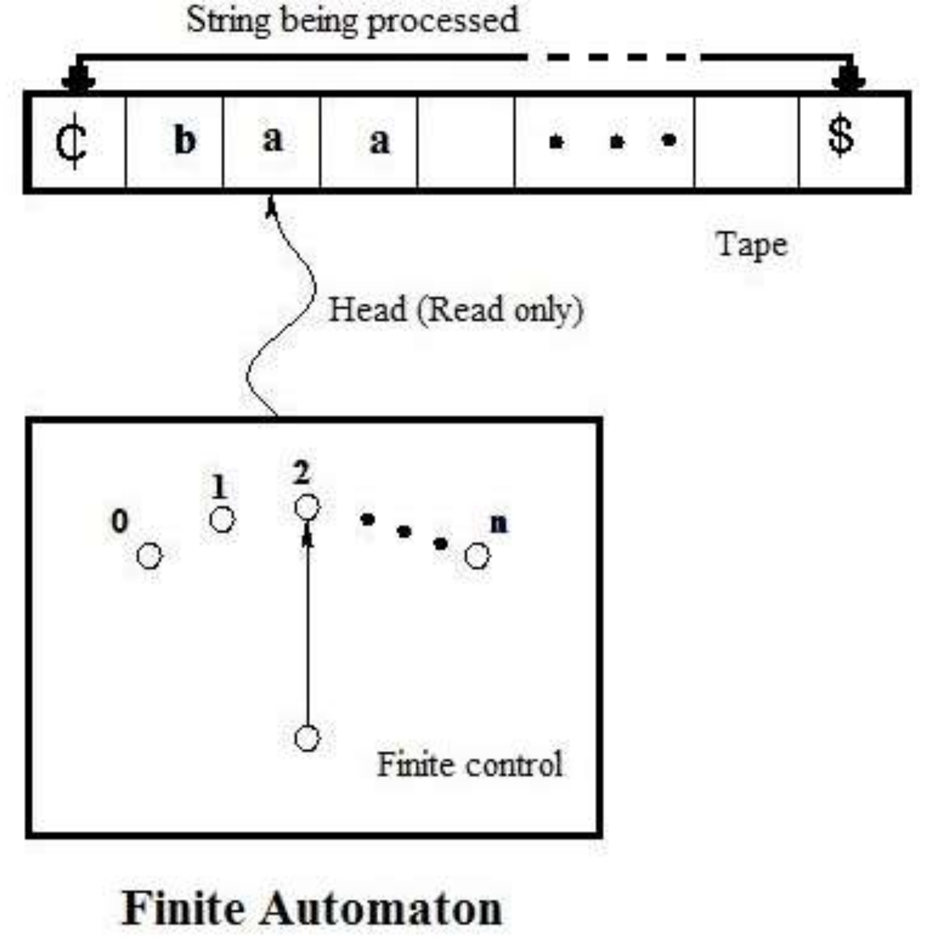
S\I	input	Next state
σ0	a	σ0
σ0	b	σ1
σ1	a	σ0
σ0	a	σ0



Since, σ0 is not the final state the string is not accepted

PROCESSING OF STRING BY DFA:

The thought process of a finite automaton can be graphically represented as;



Block diagram of Finite Automaton (FA)

PROCESSING OF STRING BY DFA:

The various components consist by a finite automata is as follows;

a) **Input tape:** The input tape has the left end and extends to the right end. It is divided into squares and each square containing a single symbol from the input alphabet ∑. The end squares of the tape contain the end markers ₵ at the left end and the end marker $ at the right end. The absence of endmarkers indicates that the tape is of infinite length.

b) **Read only Head:** The tape has a read only head that examines only one square at a time and can move one square either to the left or to the right. At the beginning of the operation the head is always at the leftmost square of the input tape. For further analysis, machine restrict the moment of the read only head from left to right direction only and one square every time when it reads a symbols.

c) **Finite control:** There is a finite control which determines the state of the automaton and also controls the movement of the head. The input to the finite control will usually be the symbol under the read head (assumed a) and the present state of the machine (assumed q) to give the following outputs;
A motion of Read head along the tape to the next square.
The next state of the finite state machine given by $\delta(q, a)$

1. Construc a DFA which accepts a language of all strings containing 'a' over Σ={a, b}.

Solution:

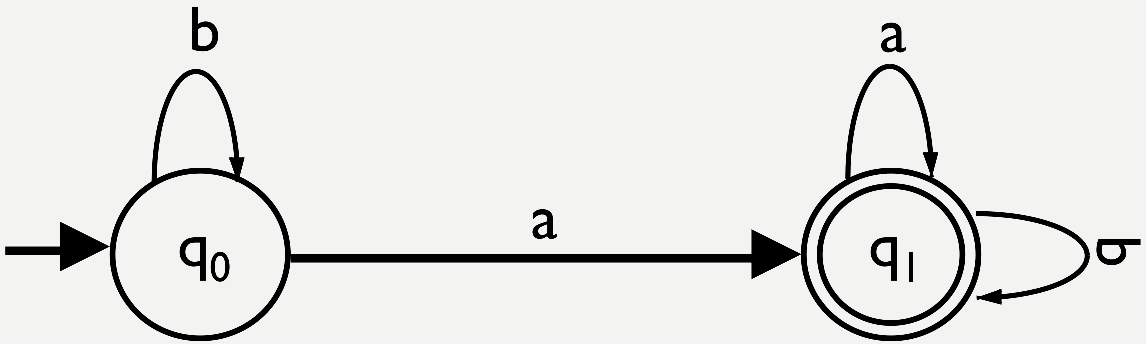


fig: Transition diagram

The required FSA is,

M = {I, S, f, σ, A} where,

I = {a , b} is the set of input symbols

S = {q_0 , q_1 } is the set of finite states

σ = q_0 is an initial state

A = {q_1} is the final accepting state

f:S*I→S is the next state transition function defined by following table

S \ I	a	b
q_0	q_1	q_0
q_1	q_1	q_1

2.Construc a DFA which accepts a language of all strings starting with 'a' over Σ={a, b}.

Solution:

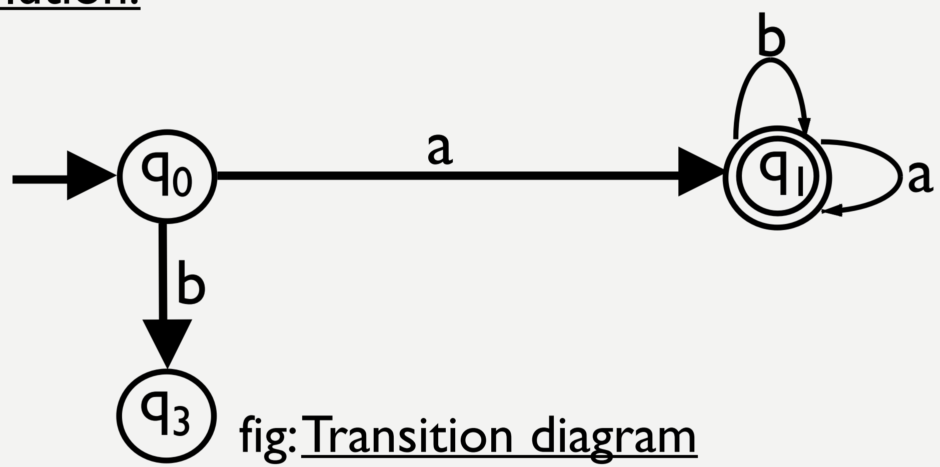


fig: Transition diagram

The required FSA is,

M = {I, S, f, σ, A} where,

I = {a , b} is the set of input symbols

S = {q_0 , q_1 } is the set of finite states

σ = q_0 is an initial state

A = {q_1} is the final accepting state

f:S*I→S is the next state transition function defined by following table

S \ I	a	b
q_0	q_1	q_3
q_1	q_1	q_1

3.Construc a DFA which accepts a language of all strings containing odd number of ‘a’ over Σ={a, b}.

<u>Solution:</u>

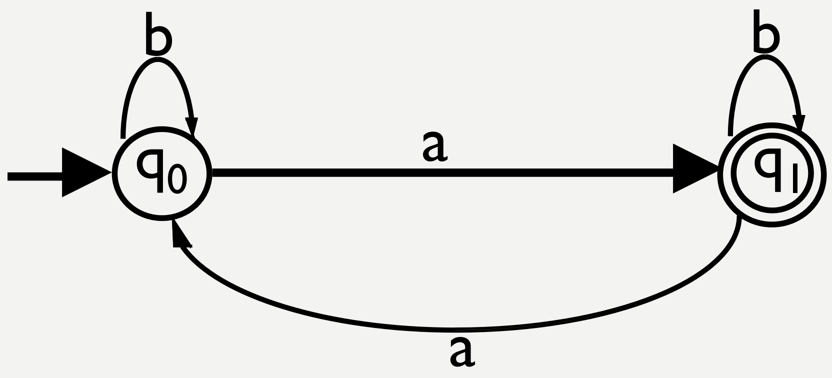


fig:<u>Transition diagram</u>

The required FSA is,

M = {I, S, f, σ, A} where,

I = {a , b} is the set of input symbols

S = {q_0 , q_1 } is the set of finite states

σ = q_0 is an initial state

A = {q_1} is the final accepting state

f:S*I→S is the next state transition function defined by following table

S \ I	a	b
q_0	q_1	q_0
q_1	q_0	q_1

4.Construc a DFA which accepts a language of all strings containing substring ‘abaab’ over Σ={a, b}.

Solution:

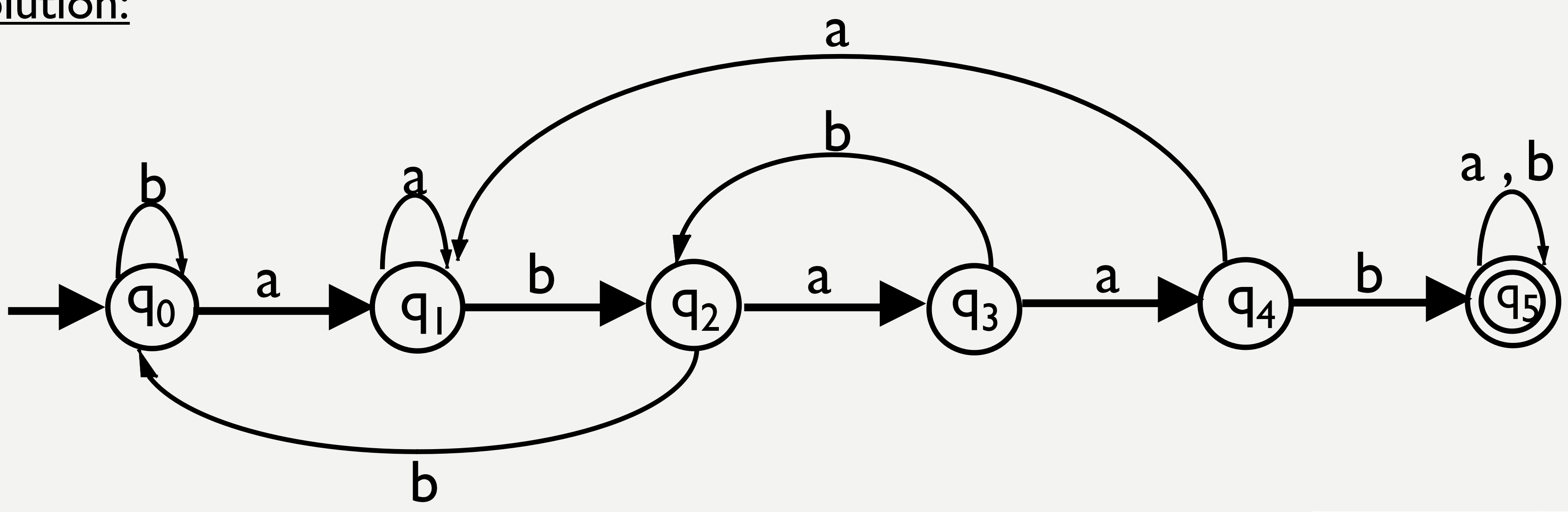


fig: Transition diagram

The required FSA is,

M = {I, S, f, σ, A} where,

I = {a , b} is the set of input symbols

S = {q_0 , q_1 , q_2 , q_3 , q_4 , q_5 } is the set of finite states

σ = q_0 is an initial state

A = {q_5} is the final accepting state

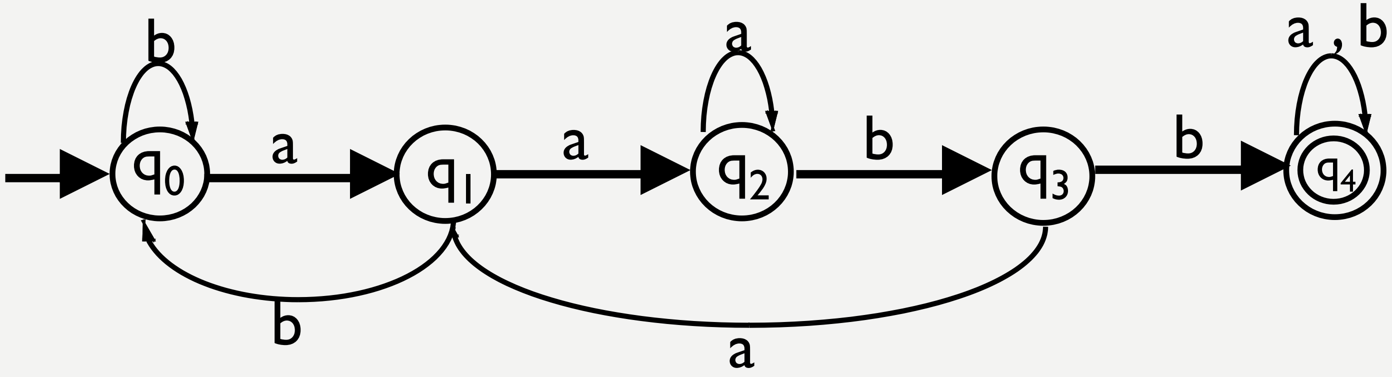
f:S*I→S is the next state transition function defined by following table

S \ I	a	b
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_3	q_0
q_3	q_4	q_2
q_4	q_1	q_5
q_5	q_5	q_5

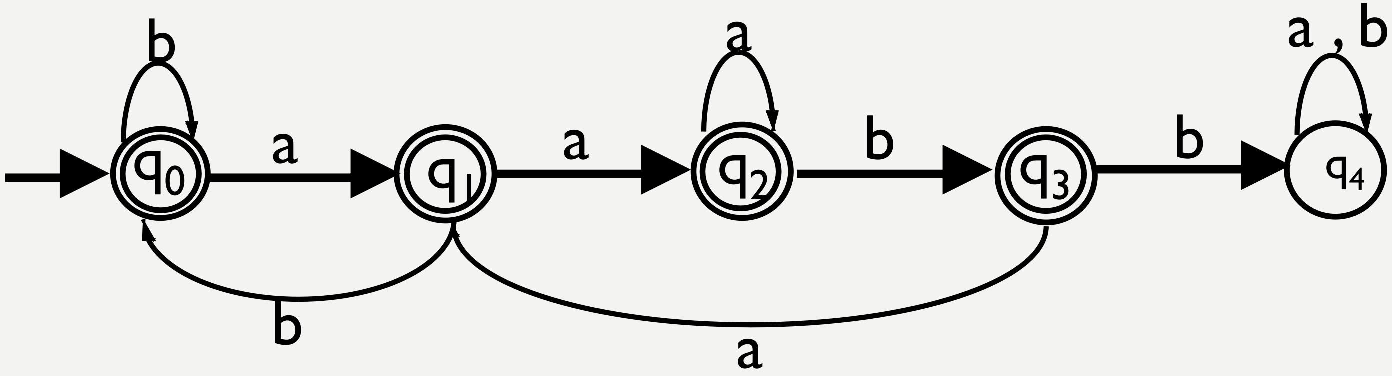
5.Construc a DFA which accepts a language of all strings that does not contain substring ‘aabb’ over Σ={a, b}.

Solution:

1. First construct the DFA that accepts string containing substring “aabb”:



2. Now, Flip the final and non final state:



5.Construc a DFA which accepts a language of all strings that ends with 111 and contains odd number of 1over Σ={0, 1}.

<u>Solution:</u>