POKHARA UNIVERSITY

Level: Bachelor	Semester: Fall	Year : 2016
Programme: BE		Full Marks: 100
Course: Probability and Queuing Theory		Pass Marks: 45
		Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) A binary communication channel carries data as one of two types of signals denoted by 0 and 1. Owing to noise, a transmitted 0 is sometimes received as a 1 and a transmitted 1 is sometimes received as a 0. For a given channel, assume a probability of 0.94 that a transmitted o is correctly received as a 0 and a probability of 0.91 that a transmitted 1 is received as a 1. Further assume a probability of 0.45 of transmitting a 0. If a signal is sent, determine: 8

   i. Probability that a 1 is received
   ii. Probability that a 0 is received
   iii. Probability that a 1 was transmitted, given that a 1 was received
   iv. Probability that a 0 was transmitted, given that a 0 was received

   b) The joint probability density function of two random variables X and Y is given by 7

$$f(x,y) = \frac{9(1+x+y)}{2(1+x)^4(1+y)^4} \quad for\ x \geq 0, y \geq 0$$

   Find the marginal distribution of X and Y, and check the independence.

2. a) The time required to repair a machine is exponentially distributed with parameter ½. What is the probability that a repair time exceeds 2 hour? What is the conditional probability that a repair time takes at least 10 hours given that its duration exceeds 9 hours? 8

   b) The life time in hours of a certain electrical equipment has the normal distribution with mean 80 hours and standard deviation 16 hours 7

i. What is the probability that equipment lasts at least 100 hours?

ii. If the equipment has already lasted 88hours, what is the conditional probability that it will last at last another 12 hours?

3. a) How many times would you have to roll a fair dice in order to be at least 99% sure that the relative frequency of having a six come up is within 0.02 of the theoretical probability 1/6.

b) A discrete random variable X takes the values -1, 0, 1 with probabilities 1/8, 3/4, 1/8 respectively. Evaluate P {|X-E(X) |≥1} and compare it with probability given by chebyshev's inequality.

4. a) Find the characteristic function of the uniform distribution

$$f(x) = \begin{cases} \frac{1}{b-a} & if\ a < x < b \\ 0 & otherwise \end{cases}$$

Also find mean and variance.

b) The arrival of large jobs at a computer center forms a Poisson process with rate of 2 per hour. The service times of such jobs are exponentially distributed with mean 20 minutes. Only 4 large jobs can accommodated in the system at a time. Determine the probability that a large job will be turned away because of lack of storage.

5. a) Traffic to a message switching centre for one of the outgoing communication lines arrive in a random pattern at an average rate of 240 messages per minute. The line has a transmission rate of 800 characters per second. The message length distribution (including control characters) is approximately exponential with an average length of 176 characters. Calculate the following principal statistical measures of system performance, assuming that a very large number of message buffers are provided:

i. Average number of messages in the system

ii. Average number of messages in the queue waiting to be transmitted

iii. Average time a message waits for transmission

iv. Probability that 10 or more messages are waiting to be transmitted

b) Patient arrives for a physical examination according to a Poisson process at the rate of one per hour. The physical examination requires three stages, each one independently and exponentially distributed with a service time of 15 minutes. A patient must go through all three stages before the next patient is admitted to the treatment facility. Determine the average number of delayed patients and average length of queue(Lq) for this system. Also find expected time a patient spend in physical examination centre.

6. a) A message transmission system is found to be Markovian with transition probability of current message to next message as given by 7

$$P = \begin{pmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.7 \\ 0.6 & 0.3 & 0.1 \end{pmatrix}$$

The initial probabilities of the states are given by probability vector **P(0)**=(0.4, 0.3, 0.3). Find the probabilities of the next two messages.

b) Assume that a computer system is in one of three states: busy, idle, or undergoing repair, respectively denoted by states 0, 1, and 2. Observing its state at 2 P.M. each day, we believe that the system approximately behaves like a homogeneous Markov Chain with the transition probability matrix. 8

$$P = \begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0.0 & 0.4 \end{pmatrix}$$

Prove that the chain is irreducible, and determine the steady state probabilities.

7. Write short notes on: **(Any two)** 2×5

a) Bayes Rule

b) Memoryless property of Exponential Distribution

c) Central limit theorem

POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Probability and Queuing Theory

Semester: Spring

Year : 2016
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) A consulting firm rents cars from three agencies, 20% from agency D, 20% from agency E, and 60% from agency F. If 10% of the cars from D, 12% of the cars from E, and 4% of the cars from F have bad tires, what is the probability that the firm will get a car with bad tires? If a randomly selected tire is found to be a bad tire, what is the probability that it was rented from agency D? 8

   b) Show that for 1,000,000 flips of a balanced coin the probability is at least 0.99 that the proportion of heads will fall between 0.495 and 0.505. 7

2. a) Among the 24 invoices prepared by a billing department, 4 contain errors while the others do not. If we randomly check 2 of these invoices, what is the probability that 7
      i. both will contain errors
      ii. neither will contain error

   b) If two random variables X and Y have the joint probability density function 8

   $$f(x)=\frac{2}{3}(x+2y) \text{ for } 0<x<1, 0<y<1, 0 \text{ otherwise}$$

      i. Find the marginal density of X and Y.
      ii. Check the independence of X and Y.

3. a) A random variable has a normal distribution with $\mu = 62.4$. Find its standard deviation if the probability is 0.20 that it will take on a value greater than 79.2. 8

   b) At a checkout counter customers arrive at an average of 1.5 per minute. Find the probabilities that 7
      i. at most 4 will arrive in any given minute
      ii. at least 3 will arrive during an interval of 2 minute

4. a) If 2 independent random samples of size n1=9 and n2=16 are taken 7

from a normal population, what is the probability that the variance of the second sample will be at least 2.4 times the variance of the first sample?

b) What do you mean by memoryless property of Exponential Distribution? Explain with necessary derivation.

5. a) Telephone calls arrive at a booth according to poisson distribution with a mean tome of 9 minutes between two consecutive calls. The length of a call is assumed to be exponentially distributed with the mean of 3 minutes. Calculate:
   i. the probability that a person will have to wait to make a call
   ii. the average queue length

b) The arrival of large jobs at a computer center forms a Poisson process with rate 2 per hour. The service time of such jobs are exponentially distributed with mean 20 minutes. Only 4 large jobs can be accommodated in the system at a time.
   i. Determine the probability that a large job will be turned away because of lack of storage
   ii. What is the probability that the arriving customer has to wait?

6. a) A supermarket has two girls ringing up sales in the counters. Let the service time for each customer be exponential with mean 4 minutes and people arrive n the queue in Poisson fashion at the rate of 10 an hour.
   i. What is the probability that all servers are jobless?
   ii. What is the probability that an arriving customer has to wait?

b) Consider the cascade of binary communication channel as given by transition probability matrix

P= 3/4 ¼
1/2 1/2

Compute the n step transition probability matrix.

7. Write short notes on: (Any two)
   a) Conditional Probability
   b) Stochastic Process
   c) Markov Chain

POKHARA UNIVERSITY

Level: Bachelor | Semester: Spring | Year : 2017
Programme: BE | | Full Marks: 100
Course: Probability and Queuing Theory | | Pass Marks: 45
| | Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) A group of five printers in a sample consists of three good, labeled g_1, g_2 and g_3, and two defective, labeled d_1 and d_2. If three printers are selected at random from this group, what is the probability of the event E = "Two of the three selected printers are good"? 8

b) The Joint probability density function of two dimensional random variable (X,Y) is given by 7

$f(x) = 4xy$ 0<=x<=1, 0<=y<=1
=0 Otherwise
   i. Find marginal probability density function of X and Y.
   ii. Find conditional probability density of X given Y=y.
   iii. Examine whether X and Y are independent.

2. a) A random variable X has the following probability distribution. 8

X	0	1	2	3	4	5
p(x)	a	a/2	a/3	a/4	a/5	a/6

Find the following:
   i. value of a;
   ii. p(X<3)
   iii. P(X≥3)
   iv. p(0<X<5)

b) The probability that the noise level of a band pass amplifier will exceed 2 db is 0.1. Find the probability that among 100 such amplifiers the noise level of 7
   i. One will exceed 2 dB;
   ii. At most two will exceed 2 dB;
   iii. Two or more will exceed 2 dB.

3. a) A man leaving for work every morning is equally likely to step out of his door at any time between 8:00 to 8:12. If he works for 220 days each year, compute the probability that the average time he leaves for work lies between 8:04 to 8:08. 8

b) At a rail station, 3 passengers arrive per minute on the average. What is the probability that exactly 30 customers will arrive during on 5 minute span?

4. a) On any given day 4000 cars pass the Birgunj-Raxaul Boarder. The actual number of cars X is Poisson distributed with parameter $\lambda=3000$. Use the normal approximation to find P (X>=3300), P (X<=4400) and P (3600<=X<=4500).

b) What is Normal distribution? Describe the main features of normal curve. Also briefly explain the significance of normal distribution.

5. a) Assuming that a computer system is in one of the three states: busy, idle or undergoing repairs. Observing its state at 12 PM each day, we believe that the system behaves approximately like a homogenous MC with the tpm given by

$$\begin{pmatrix} 0.8 & 0.2 & 0.3 \\ 0.1 & 0.6 & 0.2 \\ 0.5 & 0 & 0 \end{pmatrix}$$

Show that the MC is irreducible.

b) In a university canteen it was observed that there were only two servers who takes on an average 4 minutes to serve a student. If students arrive in the canteen at an average of 30 per hour, how much time is a student expected to wait for his turn? How many students are expected to be in the queue at any time?

6. a) What is transition probability matrix? Given that transition probability matrix of markov chains as shown below. Obtain the n-step transition probability and then find two-step and three-step transition probabilities. $\begin{pmatrix} 1/6 & 5/6 \\ 1/12 & 11/12 \end{pmatrix}$

b) At TIA, it takes 8 minutes for a plane to land. Although incoming planes have scheduled arrival times the wide variability in arrival time produces an effect making the incoming planes appear to arrive in Poisson fashion at an average rate of 5 planes an hour. A pilot has to exhort to circling around the valley till it gets its turn. Calculate the average time that the plane has to circle around the valley in the air.

7. Write short notes on: **(Any two)**
   a) Exponential distribution
   b) Additional property of expectation
   c) Pure death process

POKHARA UNIVERSITY

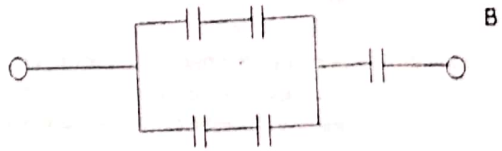
Level: Bachelor	Semester: Spring	Year : 2018
Programme: BE		Full Marks: 100
Course: Probability and Queuing Theory		Pass Marks: 45
		Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) For the circuit given below, the probability of closing each relay of the circuit is known to be 0.6. Assume that the relays act independently. What is the probability that a current will exist between the terminals A and B. 7



b) State Bayes' theorem.An aircraft emergency locator transmitter (ELT) is a device designed to transmit a signal in the case of a crash. The Altigauge manufacturing company makes 80% of the ELTs, the Bryant Company makes 15% of them, and the Chartair Company makes the other 5%. The ELTs made by Altigauge have a 4% rate of defects, the Bryant ELTs have a 6% rate of defects, and Chartair ELTs have a 9% rate of defects. 8

   i. If a randomly selected ELT is then tested and is found to be defective, find the probability that it was made by the Altigauge manufacturing company.
   ii. If a randomly selected ELT is then tested and is found to be no defective, find the probability that it was made by the Bryant manufacturing company.

2. a) A consulting firm rents cars from three agencies, 20% from agency D, 20% from agency E, and 60% from agency F. If 10% of the cars from D, 12% of the cars from E, and 4% of the cars from F have bad tires, what is the probability that the firm will get a car with bad tires?

b) It is known that 5% of the screws manufactured by an automatic machine are defective. If a sample of 20 screws is selected at random, find the probability that the sample contains (a) exactly 2 defectives, (b) at least 2 defective screws, (c) no defective screws

3. a) In a test administered to 1000 students, the average score was 60 and standard deviation 10. Find:

   i. The number of students exceeding a score 50.
   ii. The number of students lying between 44 and 74.
   iii. The value of score exceeded by the top 100 student.

b) What do you mean by memoryless property of Exponential Distribution? Explain with necessary derivation

4. a) A random sample of size 100 is taken from an infinite population having the mean μ=76 and variance σ^2= 256. What is the probability that sample mean (X) will be between 75 and 78?

b) A departmental store has two girls ringing up sales in the counters. Let the service time for each customer be exponential with mean 4 minutes and people arrive n the queue in Poisson fashion at the rate of 10 an hour.

   i. What is the probability that all servers are jobless?
   ii. What is the probability that an arriving customer has to wait?

5. a) A telephone exchange has two long distance operators. The telephone company finds that during the peak load, long distance calls arrive in a Poisson fashion at an average rate of 15 per hour. The length of service on these calls is approximately exponentially distributed with mean length 5 minutes. What is the probability that a subscriber will have to wait for his long-distance call during peak hour of the day?

b) A self-service store employs one cashier at its counter. Nine customers arrive on an average in every 5 minutes while the cashier can serve 10 customers in 5 minutes. If the arrival rate follows Poisson distribution and the service rate follows exponential distribution, find:

   i. The average number of customers in the queue.
   ii. Average time spent by a customer in the system.

Average time spent by a customer in the queue.

a) During 8:00 – 10:00 AM each morning, a train arrives every 20 minutes in a yard. The service times have an average of 36 minutes for each train. If the capacity of the yard is 4 trains only, find: 7

   i. The probability that the yard will be empty.
   ii. The average queue size.

b) In a factory, the customers have to pass through three counters. The customers buy coupons at the first counter, select and collect the snacks at the second counter and collect tea at third counter. The server at each counter takes on an average 1.5 minutes although the distribution of service time is approximately Poisson at an average rate of 6 per hour. Find the average time a customer spends waiting in cafeteria. Also find the most probable time spent in getting the service. 8

7. Write short notes on: (**Any two**) 2×5

   a) Conditional Probability
   b) Moments generating function
   c) Stochastic Process

POKHARA UNIVERSITY

Level: Bachelor	Semester: Fall	Year : 2019
Programme: BE		Full Marks: 100
Course: Probability and Queuing Theory		Pass Marks: 45
		Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

a) The chance that a doctor A will diagnose a disease X correctly is 60%. The chances that a patient will die by his treatment after correct diagnosis is 40%, and the chance of death by wring diagnosis is 70%. If a patient of doctor A, who had disease X died. What is the chance that his disease was diagnosed correctly? 7

b) An information source emits a six-digit message into a channel in binary code '0' or '1'. Each digit is chosen independently of the others and is a '1' with probability 0.3. If a message is sent, what is the probability that it contains: 8

i. Exactly three '1's
ii. '1's beween 2 and 4 (inclusive)
iii. No less than to '0's

a) A random variable X has the following probability distribution. 7

X	0	1	2	3	4	5	6	7	8
P(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

i. Find a
ii. Find $P(X < 3)$, $p(X >= 3)$, $P(0 < X < 5)$.

b) An information source emits a five digit message into a channel in a binary code. Each digit is chosen independently and is one with probability 0.3.Find the probability that the message contains: 8

i. Exactly Three ones
ii. Four ones and one zero
iii. At least two ones
iv. None of the ones

3. a) A blindfolded marksman finds that on the average he hits the target 4 times out of 5. If he fires 4 shots, what is the probability of
   i. More than 2 hits?
   ii. Atleast 3 misses?

   b) The time taken by a milkman to deliver milk is normally distributed with mean 12 minutes and standard deviation 2 minutes. He delivers milk every day. Estimate the number of days during the year when he takes
   i. longer than 17 minutes
   ii. less than 10 minutes
   iii. between 9 and 13 minutes

4. a) State the chebyshev's inequalities. Determine how many times an unbiased coin must be tossed in order that the probability will be at least 0.90 that the proportion of head will lie between 0.4 and 0.6.

   b) In a certain city the daily consumption of water (in millions of gallons) follows approximately a gamma distribution with α=2 and β=3. If the daily capacity of this city is 9 million gallons of water, what is the probability that on any given day the water supply is inadequate?

5. a) A message transmission system is found to be Markovian with probability of current message to next message as given by

$$p=\begin{pmatrix}0.2 & 0.3 & 0.5\\ 0.1 & 0.2 & 0.7\\ 0.6 & 0.3 & 0.1\end{pmatrix}$$

The initial probability of the states are given by probability vector p (0) = (0.4, 0.3, and 0.3).

Find the probabilities of the next two messages.

   b) Assume that a compute system is in one of the three states: busy, idle or undergoing repairs. Observing its state at 2 PM each day, we believe that the system behaves approximately like a homogeneous markov chain with tpm given by

$$\begin{bmatrix}0.6 & 0.2 & 0.2\\ 0.1 & 0.8 & 0.1\\ 0.6 & 0 & 0.4\end{bmatrix}$$

Show that the markov chain is irreducible.

6. a) Students of PU arrives at convocation at the rate of 30 per minutes in a Poisson manner. Gate doesn't contain more than 14 students at a time.

Service time per student is exponential with a mean rate of 20 per hour. Calculate the various OC for this queuing system. 7

   b) A barber shop has 2 barbers and 3 chairs for customers. Assume that the customers arrive in a Poisson fashion at a rate of 5 per hour and each barber serves a customer, the service times being exponential with a mean of 15 minutes. Furthermore, if a customer arrives and if there are no empty chairs, he will leave. Find
   i. probability that Shop is empty
   ii. probability that there are K customers in shop
   iii. Expected number of customers in shop

7. Write short notes on: **(Any two)** 2×5
   a) Stochastic process
   b) Conditional Probability
   c) Memory less Property of Exponential Distribution

POKHARA UNIVERSITY

Level: Bachelor
Semester: Spring
Year : 2019
Full Marks: 100
Programme: BE
Pass Marks: 45
Course: Probability and Queuing Theory
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

a) In a particular pain clinic, 10% of patients are prescribed narcotic pain killers. Overall, five percent of the clinic's patients are addicted to narcotics (including pain killers and illegal substances). Out of all the people prescribed pain pills, 8% are addicts. If a patient is an addict, what is the probability that they will be prescribed pain pills? 8

b) Nine items are taken at random from a box of 21 items. The box is rejected if more than 3 item is found to be faulty. If there are 5 faulty items in the box, find the probability that the box is accepted. 7

a) A credit accounts on store can be 0,1,2 months, paid up or bad debt can be given with tpm 8

$$P = \begin{matrix} 0.2 & 0.5 & 0.1 & 0.2 & 0 \\ 0.2 & 0.2 & 0.4 & 0.1 & 0.1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{matrix}$$

Find the total time the account will be kept alive.

b) Let X and Y have the joint probability mass function 7

$$f(x,y) = (x + y)/21, x = 1, 2, 3 \text{ and } y = 1, 2.$$

i. Find marginal probability of X and Y.

ii. Find conditional probability distribution of X given Y= y.

iii. Are X and Y independent?

3. a) A life insurance salesman sells on the average 3 life insurance policies per week. Use Poisson's law to calculate the probability that in a given week he will sell 8

i. Some policies

ii. 2 or more policies but less than 5 policies.

iii. Assuming that there are 5 working days per week, what is the probability that in a given day he will sell one policy?

b) The time taken by a delivery man to deliver milk is normally 7 distributed with mean 12 minutes and standard deviation 2 minutes. He delivers milk every day. Estimate the number of days in a year (take 1 year = 365 days) when he takes

i. Longer than 17 minutes   ii. between 9 and 13 minutes

4. a) A population of 29 year-old males has a mean salary of $29,321 with a standard deviation of $2,120. If a sample of 100 men is taken, what is the probability their mean salaries will be less than $29,000?

b) Consider a markov chain with 3 states, S={1,2,3} that has following tpm as :

¼	½	½
1/3	0	2/3
½	½	0

If we know P(X1=1)=P(X1=2)=1/4, find P(X1=3,X2=2,X3=1)

5. a) In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time (tie time taken to hump to train) distribution is also exponential with an avg. of 36 minutes. Calculate

i. Expected queue size (line length)

ii. Prob. that the queue size exceeds 10.

b) A super market has a single cashier. During the peak hour customers arrive at a rate of 20 customers per hour. The average of customers that can be processed by the cashier is 24 per hour. Find:

i. The probability that the cashier is idle.

ii. The average no of customers in the queue system

iii. The average time a customer spends in the system.

iv. The average time a customer spends in queue.

v. The any time a customer spends in the queue waiting for service

6. a) Trains arrive at the yard every 15 minutes and the services time is 33 minutes. If the line capacity of the yard is limited to 5 trains, fi the probability that yard is empty and the average no of trains in the system.

b) What is Kendell Expression? Describe each terms used in it. Write t OC of M/M/s:M/G/D model.

7. Write short notes on: (Any two)

a) Baye's Rule

b) Beta Distribution

c) Types of Random Variables

POKHARA UNIVERSITY

Level: Bachelor   Semester:Fall   Year : 2020

Programme:BE   Full Marks: 100

Course: Probability and Queuing Theory   Pass Marks: 45

Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) A binary communication channel carries data as one of the two types of signals denoted by 0 and 1.Owing to noise, a transmitted 0 is sometimes received as a 1 and a transmitted 1 is received as a 0 .For a given channel, assume a probability of 0.94 that a transmitted 0 is correctly received as 0 and a probability of 0.91 that a transmitted 1 is received as a 1 .Further assume a probability of 0.45 of transmitting a 0.If a signal is sent, find the probability that: 8

i. a 1 is received

ii. a 0 is received

iii. a 1 was transmitted given that a 1 was received

iv. a 0 was transmitted given that a 1 was received

b) In a particular country, there are three airports A, B, and C. Airport A handles 50% of all airline traffic, airport B handles 30% of all air traffic, and airport C handles rest of that. The detection rates for weapons at the three airports are 0.9, 0.5, and 0.4 respectively. A passanger is randomly selected at random of the airports. Then, what is the probability that he/she is carrying a weapon? If he/she is found to be carrying a weapon, what is the probability that airport A being used? 7

2. a). If two random variables X abd Y have the joint probability joint density function as 7

$$f(x,y)=\frac{6-x-y}{8}, 0<x<2, 2<y<4$$

$$=0, otherwise$$

Find,

i. $p(X < 1, Y < 3)$

ii. $p(X < 1 / Y < 3)$

b) The normal rate of COVID virus infection in certain city is known to be 35%. In an experiment with 10 persons, Calculate the probability that

i. None of them are infected
ii. at least 5 of them are infected
iii. at most 5 of them are infected

3. a) The time taken by a delivery man to deliver milk is normally distributed with mean 12 minutes and standard deviation 2 minutes. He delivers milk every day. Estimate the number of days in a year (take 1 year = 365 days) when he takes

i. longer than 17 minutes
ii. between 9 and 13 minutes

b) Cristiano Ronaldo has a mean salary of \$29,321 with a standard deviation of \$2,120. If a sample of 100 such athletes is taken, what is the probability their average salaries will be less than \$29,000?

4. a) In a certain city, the daily consumption of a electric power (in millions of kilowatt-hous) can be treated as a random variable having a gamma distribution with $\alpha = 3, \beta = 2$. If the power plant of the city has a daily capacity of 12 million kilowatt hours . What is the probability that this power supply will be inadequate on any given day.

b) Assume that a computer system is in one of the three states:busy,idle or undergoing repairs. Observing the state at 2 P.M each day, the system behaves approximately like a homogeneous markov chain with tpm given by:

$$P = \begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{pmatrix}$$

Determine the steady state probabilities.

5. a) There are three brands of beauty products A, B, and C in a shop. It has been found that a person when he purchases a beauty product of a particular brand, he will continue to buy the same brand or switch over to another brand during his next purchase of a beauty product in the shop. The transition probability matrix associated with the three brands

is given below:

$$P = \begin{pmatrix} 0.4 & 0.3 & 0.3 \\ 0.6 & 0.1 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{pmatrix}$$

If the initial distribution of purchase of the brands A, B and C is [0.4 0.4 0.2], determine the distribution of the brands after two purchases.

b) A supermarket has two girls ringing up sales in the counters. Let the service time for each customer be exponential with mean 4 minutes and people arrive in the queue in Poisson fashion at the rate of 10 an hour. 7

i) what is the probability that all servers are jobless?

ii) what is the probability that an arriving customer has to wait?

a) A petrol pump station has 4 pumps. The service time follows an exponential distribution with a mean of 6 minutes and cars arrive for service in a Poisson process at the rate of 30 cars per hour. Find the probability that an arrival will have to wait in the line? 8

b) Define the memoryless property of a distribution. Prove that an exponential distribution satisfies the memoryless property. 7

Write short notes on: **(Any two)** 2×5

a) Gamma Distribution

b) Stochastic Process

c) Types of random variables

POKHARA UNIVERSITY

Level: Bachelor | Semester – Spring | Year: 2020

Program: BE | Full Marks: 70

Course: Probability and Queuing Theory | Pass Marks: 31.5

Time: 2 hrs.

Candidates are required to answer in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all the questions.

Section - A: (5×10=50)

Q. N. 1 Define conditional probability with example. State Bayes' theorem. If three machines A , B and C produces 1000 , 2000 and 3000 articles per hour respectively. These machines are known to be producing 1% , 2% and 3% respectively the defective articles. One article is selected at random from an hour production of the three machines and found to be defective. What is the probability that articles is produced by machine A. 2+2+6

Q. N. 2 State and prove Chebyshev's inequality. Let X is a random variable with mean of 11 and variance of 9. Using Chebyshev's inequality, find the following: 10

i) A lower bound for $P(6 < X < 16)$.
ii) The value of c such that $P(|X-11| \geq c) \leq 0.09$.

Q. N. 3 Define random variables with its type and example. A printer ink has a life of X hours. The random variable X is modelled by the probability function 2+3+5

$$f(x) = \frac{720}{x^2}, 400 \leq x \leq 900$$
$$=0 \text{ otherwise}$$

Find:

i. Probability that printer ink has lifetime not more than 550 hours.
ii. Probability that printer ink has lifetime more than 700 hours given it has already printed more than 600 hours.

OR

Define mathematical expectation regarding discrete and continuous random variables with one example each. Suppose that the duration of telephone calls follows a distribution with pdf given by

$$f(x) = \frac{1}{5}e^{-\frac{x}{5}}; x \geq 0$$
$$0 \text{ otherwise}$$

Find

i) Probability that the duration of calls exceeds 5 minutes.
ii) Probability that the duration of call will be less than 6 minutes given that it was greater than 3 minutes.

Q. N. 4 Discuss normal distribution and its characteristics. Packages from a pacing machine have a mass, which is normally distributed with mean 200g and standard deviation 2g. Find the probability that a package from the machine weighs 10

(a) less than 197 g

(b) more than 200.5 g

(c) Between 198.5g and 199.5g.

Q. N. 5 Define queuing theory. What are the basic characteristics of the queuing models? A supermarket has a single cashier. During peak hours, customers arrive at a rate of 20 per hour. The average number of customers that can be processed by the cashier is 24 per hour. Calculate 10

a) The probability that the cashier is idle.
b) The average number of customers in the system.
c) The average time the customer spends in the system.
d) The average number of customers in the queue.
e) The average time a customer spends in the queue.

Section - B: (1×20=20)

Q. N. 6 Explain {(M/M/1) : (N/FCFS)} system and give its operating characteristics. Patients arrive at a doctor's clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 9 patients. Examination time per patient is exponential with mean rate of 20 per hour. Find the 20

a) Probability that an arriving patient will not wait.
b) Effective arrival rate.
c) Average number of patients in the clinic.
d) Expected time a patient spends in the clinic.
e) Average number of patients in the queue.
f) Expected waiting time of a patient in the queue.

POKHARA UNIVERSITY

Level: Bachelor Semester: Fall Year : 2021

Programme: BE Full Marks: 100

Course: Probability and Queuing Theory Pass Marks: 45

Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) A survey of magazine subscribers shows that 45.8% rented a car during the past 12 months for business reason, 54% rented a car during past 12 months for personal reasons and 30% rented a car during the past 12 months for both the reasons.
   i. What is the probability that a subscriber rented a car during past 12 months for business or personal reasons?
   ii. What is the probability that a subscriber did not rent a car for either business or personals reasons?

   b) A company has 2 machines that produces widgets. An older machine produces 23% defective widgets while new machines produces only 8% widgets. In addition, the new machine produces 3 times as many as widgets than older machines does, 8
   i. What is the probability that the randomly chosen widget produced by the company is defective?
   ii. Given that a randomly chosen widget was tested and found to be defective, what is the probability it was produced by new machine?

2. a) According to a front page article in " The Wall street" journal, 30 % of all the students in American universities miss classes due to drinking. If 10 students are randomly chosen, what is the probability that 7
   i. Exactly 3 students miss the class due to drinking.
   ii. At most 3 students miss class due to drinking.
   iii. None of them miss the class due to drinking.

   b) Given the p.d.f

   $$f(x) = kx(1-x)\ for\ 0 < x < 1$$ 8

   $$= 0, otherwise$$

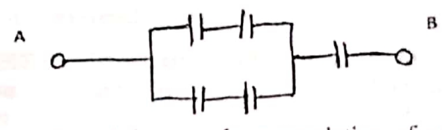
   Find the value of
   i. k
   ii. P(0.3<X<0.5)
   iii. P(X<0.4)

POKHARA UNIVERSITY

Level: Bachelor | Semester: Spring | Year : 2021
Programme: BE | Full Marks: 100
Course: Probability and Queuing Theory | Pass Marks: 45
Time : 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

a) For the circuit given below, the probability of closing each relay of the circuit is known to be 0.6. Assume that the relays act independently. What is the probability that a current will exist between the terminals A and B. 7



b) State Bayes' theorem. In a population of workers, suppose 40% are grade school graduates, 50% are high school graduates, and 10 % are college graduates. Among the grade school graduates, 10% are unemployed, among the high school graduates, 5 % are employed, and among the college graduates 2% are unemployed. 8

If a worker is chosen at random and found to be unemployed, what is the probability that he is a college graduate?

a) According to a front page article in " The Wall street" journal , 30 % oa all the students in American universities miss classes due to drinking. If 10 students are randomly chosen ,what is the probability that 7

i. Exactly 3 students miss the class due to drinking.
ii. At most 3 students miss class due to drinking.
iii. None of them miss the class due to drinking.

b) Given the p.d.f 8

$$f(x) = kx(1-x) for 0 < x < 1$$
$$= 0, otherwise$$

3. a) Agile is an organization whose members passes IQs that are in the t 2 % of the population. It is known that IQs are normally distribu with mean of 100 and standard deviation of 16. Find minimum needed to be an Agile member.

b) Define Chebyshev's inequality. Using it estimate the number of tim that a fair coin should be tossed to ensure that the probability between 48% and 52% of the results of heads is 0.99.

4. a) Differentiate between moment generating function and characteris function. Let X be a continuous random variable having followi probability function:

$$f(x) = \theta e^{-\theta x}; for\ x > 0$$

Find;

i. The moment generation function of X
ii. Also mean and variance of X.

b) State and prove Chebyshev's inequality. A random variable X has mean 10, a variance 4 and an unknown probability distribution. Fi the value of C such that $P(|X-10| \geq C) \leq 0.04$.

5. a) State CLT. A random sample of size 100 is taken from a populati whose mean is 60 and variance is 400. Using CLT, with wh probability can we assert that a mean of the sample will not diff from the population mean by more than 4?

b) Explain Kendall's notation of a Queuing system. In a given $M/M/$ Queuing system, the average arrivals is 4 customers per minu and $\rho = 0.7$. Find

i. Mean number of customers in the queue and system.
ii. Probability that the server is idle.
iii. Mean waiting time is the system.

6. a) Establish the memory less property of exponential distribution.

b) Write operating characteristics of $(M/M/s):(\infty/FIFO)$. A petr pump station has 4 pumps. The service time follows an exponenti distribution with a mean of 6 minutes and cars arrive for service in Poisson process at the rate of 30 cars per hour.

i. What is the probability that an arrival will have to wait in the line?
ii. Find the average waiting time in the queue.
iii For what percentage of time would the pumps be idle on a average?

7. Write short notes on: (Any two)

a) Conditional probability
b) Central Limit Theorem
c) Queue Discipline

Find the value of

i. k

ii. P(0.3<X<0.5)

iii. P(X<0.4)

3. a) Suppose that the joint probability function of random variables X and Y is given by

$$f(x,y) = 4xy \quad 0 < x < 1,\ 0 < y < 1$$

i. Verify that the joint probability function is a joint pdf.

ii. Are X and Y are independent?

b) Between the hours of 2 and 4 P.M, the average number of phone calls per minute coming into the switch – board of a company is 2.5. Find the probability that during one particular minute there will be

i. No phone call at all

ii. Exactly 3 calls

iii. At least 5 calls.

4. a) A large group of students took a test in Probability and the final grades have a mean of 70 and a standard deviation of 10. If we can approximate the distribution of these grades by a normal distribution, what percent of the students

i. Scored higher than 80?

ii. Should pass the test (grades≥60)?

iii. Should fail the test (grades<60)?

b) A random variable X has the following probability distribution:

X = x	-2	-1	0	1	2	3
P(X = x)	0.1	K	0.2	2K	0.3	K

Find the value of K, and calculate mean and variance of X.

5.

a) If one step TPM is $\begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{pmatrix}$, prove that it is irreducible and also find the steady state probabilities.

b) Given that number of trucks arrives follows a Poisson distribution with an average of 3 per hour and service to load truck by only one loading department gives service to 4 trucks in an hour in average follows exponential distribution. Then find

i) the loading department is free.

ii) the average number of trucks in a queue.

iii) the average waiting time of truck in queue.

6. a) A one person barber shop can accommodate maximum of 5 people at a time (4 waiting and 1 getting haircut) . Customers arrives in a shop at the rate of 8 per hour follows Poisson distribution and barber gives 6 minutes services to a customer follows exponential distribution. Find 7

i. Probability barber is free.

ii. Average time customer spent in a shop.

b) A travel agency has three service counters to receive people who visit to book air ticket. Customers arrives in a Poisson distribution with an average of 100 persons in 10 hours service day. It has been estimated that service time follow exponential distribution with an average of 15 minutes per customer. Find 8

i. Expected number of customer in the system.

ii. Expected number of customer in the queue.

7. Write short notes on: (**Any two**) 2×5

a) Binomial Distribution

b) Moments generating function.

c) Bayes' rule

2021-SP- PQT

POKHARA UNIVERSITY

Level: Bachelor | Semester: Fall | Year : 2022
Programme: BE | | Full Marks: 100
Course: Probability and Queuing Theory | | Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

a) From a faculty group of 5 Statisticians, 3 Engineers and 2 Physicists, a selection committee of 3 people is selected randomly. Find the probability that the selection committee will consist of 7

i. All Statisticians
ii. 2 Statisticians, 1 Engineer
iii. 1 Statistician, 1 Engineer and 1 Physicist.

b) State Bayes Theorem. A given lot of integrated circuit chips contains 2 % defective chips. Each chips is tested before delivery. The tester itself is not totally reliable. Probability of tester says the chip is good when it is actually good is 0.95 and the chip is defective when it is really defective is 0.94. If a tested device is indicated to be defective, what is the probability that it is actually defective? 8

a) The joint density function of X and Y is given by 7

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y); & 0< x <1 ;0 <y <1 \\ 0 & ; \text{elsewhere} \end{cases}$$

Find:

i. The marginal probability density functions of X and Y both.
ii. Find the conditional p.d.f of X given Y=y
iii. Are X and Y independent? Give reason for your answer.

b) At a checkout counter customers arrive at an average of 1.5per minute. Find the probability that: 8

i. At most four will arrive in any given minute
ii. At least three will arrive during an interval of 2 minutes
iii. Exactly two will arrive during an interval of 4 minutes

3. a) Let the joint probability density function of two random variables X and Y is given by

$$f(x,y) = \frac{1}{8}(6-x-y) \quad 0<x<2,\ 2<y<4$$

i. Find $P\left(X<\frac{3}{2} \middle/ Y<\frac{5}{2}\right)$

ii. Find the marginal distribution of X and Y.

b) Under what conditions Binomial distribution is used. The bank of Kathmandu has recently starts a new credit program. Customers meeting certain credit requirements can obtained a credit card accepted by participating area merchants that carries a discount. Past numbers show that 20% of all applicants for this card are rejected. If 10 applicants are selected, what is the probability that

i. Exactly 4 will be rejected?

ii. None of them are rejected?

iii. At least two are rejected?

4. a) Define standard normal variate. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and the standard deviation of the distribution.

b) A random variable X has the following probability distribution:

X	-3	-2	-1	0	1	2	3
P(x)	0.2	2a	0.1	a	0.4	a	0.1

Find:

i. the value of K,

ii. $P(X<1)$

iii. Calculate mean and variance of X.

5. a) Derive the memory–less property of exponential distribution.

b) Consider the cascade of binary communication channel as given by

$$P = \begin{bmatrix} 0.75 & 0.25 \\ 0.5 & 0.5 \end{bmatrix}$$

Compute the n^{th} step transition probability matrix (tpm) and hence find 3^{rd} and 4^{th} step tpm.

6. a) The arrivals of customers in the only teller counter of a bank is Poisson-distributed at the rate of 25 customers per hour. The teller

takes, on an average 2 minutes to cash the cheque. The service time has been shown to be exponentially distributed. Find

i. Percentage of time the teller is busy.

ii. Probability of at least 10 customers in the Bank.

iii. Average number of customers in the bank.

iv. Average waiting time of a customer in the queue.

b) During 8:00-10:00 AM in each morning, train arrives in every 20 minutes in a yard. The Service time have an average of 36 minutes for each train. If the capacity of the yard is 4 trains only, find: 8

i. The probability that the yard will be empty.

ii. The average queue size.

7. Write short notes on: (Any two) 2×5

a) Normal Distribution

b) Mathematical expectation

c) Components of Queuing system

PQT-2022 Fall

POKHARA UNIVERSITY

Level: Bachelor　　Semester: Spring　　Year : 2023
Programme: BE　　Full Marks: 100
Course: Probability and Queuing Theory　　Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define dependent and independent events with examples. In a group of 10 engineers, 5 doctors and 4 mathematicians 3 peoples are selected randomly. What is the probability that selected groups contain i) One from each profession, ii) At least 1 mathematician and iii) At most 2 doctors? 7

   b) State Bayes' theorem. In a population of workers, suppose 40% are grade school graduates, 50% are high school graduate and 10% college graduates. Among the grade school graduates, 90% are unemployed, among the high school graduates, 50% are employed, and among the college graduates 2% are unemployed. If a worker is chosen at random and found to be unemployed, what is the probability that he is a college graduate? 8

2. a) A random variable X has the following Probability distribution: 8

X	0	1	2	3	4	5	6	7
P(x)	a	4a	3a	7a	8a	10a	6a	9a

   i. Determine the value of a.
   ii. Find $P(X < 3)$, and $P(0 < X < 5)$
   iii. Find $E(X)$ and $V(X)$

   b) Let X and Y be two continuous random variables with joint probability density function 7

$$f(x, y) = 4e^{-2x}y \; ; \quad x > 0 \; and \; 0 < y < 1$$

   i. Find the marginal density functions of X and Y.
   ii. Find the conditional density function of Y given X = x.
   iii. Check the independency of X and Y.

3. a) Between the hours of 2 and 4 P.M, the average number of phone calls per minute coming into the switch – board of a company is 2.5. Find the probability that during one particular minute there will be 8

   i. No phone call at all
   ii. Exactly 3 calls
   iii. At least 5 calls.

   b) The weekly wages of 1000 workmen are normally distributed around a mean of Rs. 500 with a standard deviation of Rs. 50. Estimate the number of workers, whose weekly wages will be 7

   i. between Rs. 400 and Rs. 600.
   ii. less than Rs. 400
   iii. more than Rs. 600.

4. a) State and prove Chebyshev's Inequality. A random variable X has a mean 10, a variance 4 and an unknown probability distribution. Find the value of C such that 7

$$P[|X - 10| \geq C] \leq 0.04 .$$

   b) State central limit theorem. A random sample of size 100 is taken from an infinite population whose mean is 80 and variance is 400. Using CLT, with what probability can we assert that the sample mean will not differ from population mean $\mu = 80$ by more than 6? 8

5. a) Assume that a computer system is in any one of the three states: busy, idle and under repair, respectively, denoted by 0, 1 and 2. Observing its state at 2 P.M. each day, we get the transition probability matrix as 7

$$\begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{bmatrix}$$

   Determine the steady – state probabilities. Also check whether Markov chain is irreducible or not.

   b) A grocery store has a single cashier. It is known that the customers arrive at the rate of 20 customers per hour. The average number of customers that can be served by the cashier is 25 per hour. Assume that the arrivals are Poisson and the service time is exponentially distributed. Find: 8

   i. Probability that the cashier is idle
   ii. Average number of customers in the system
   iii. Average number of customers in the queue
   iv. Average time the customer spends in the system
   v. Average time a customer spends in the queue waiting for service

6. a) The arrivals for a service center are Poisson – distributed with a mean arrival rate of 4 units per hour. The mean service time has been shown to be exponentially distributed with a mean service time of 10 minutes per service of the unit. Find 7

i. the probability that the center will be idle.

ii. the probability of at least 6 units in the center.

iii. the expected number of units in the center.

iv. the expected waiting time of a unit in the center.

b) Customers arrive at a tailor shop with only one tailor. On average, customers arrive at the rate of 8 per hour and the tailor takes 10 minutes for serving each customer by taking measurements and the tailoring needs. The waiting room does not accommodate more than 3 customers. The arrival process is Poisson and the service time is an exponential random variable. Find 8

i. Probability that an arrival will not wait.

ii. Effective arrival rate at the tailor shop.

iii. Expected number of customers at the tailor shop.

iv. Expected time a customer spent in the tailor shop.

7. Write short notes on: (**Any two**) 2×5

a) Random Variable

b) Conditional Probability

c) Kendall's Notation of Queueing System