CAN RELIC NEUTRINOS BE OBSERVED?

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ABSTRACT

Within the framework of big-bang cosmology and the standard electroweak model, it appears that the only possibly detectable interaction of relic neutrinos is their annihilation with cosmic ray neutrinos on the Z resonance. However, measurement will require the existence of a large density of $z \geqslant 5$ red-shifted sources with intense neutrino emission in the energy region 10^{21} eV^2/m_ν.

INTRODUCTION

The discovery in 1965 of an isotropic 2.7 °K photon gas permeating the universe marked the major triumph for big-bang cosmology. Big-bang theory had predicted an isotropic Planck-distribution of photons released from equilibrium when hydrogen formed at a temperature of 4000 °K (1/2 eV) and a time of roughly 10^5 years after the initial singularity. The present photon temperature reflects the red-shifting due to the universe's expansion over the past 10 to 20 billion years.

Standard big bang cosmology also predicts a 1.9 °K gas of neutrinos and antineutrinos. Due to the weakness of their interaction, these neutrinos last interacted when the universe's temperature was 10^{10} °K (1 MeV) and its age was but one second! Thus the properties of the relic neutrinos offer a glimpse of our embryonic universe at an age twelve orders of magnitude earlier than that afforded by the photon background. Yet, seventeen years after the discovery of the photon gas, the neutrino ether remains elusive. The reason is simple to divine: the very weakness of interaction which enabled neutrinos to decouple so early in cosmological evolution also enables them to defy detection today.

Current ideas for relic neutrino detection may be classified into one of three categories:

Flux detection proposes to measure directly the relic flux incident on earth. The predicted mean momentum of the neutrinos (assuming zero chemical potential) is 5×10^{-4} eV; the predicted

*Work supported by the Department of Energy, contract DE-AT03-81ER-40029.

number density of neutrino plus antineutrino is 100 cm^{-3} per flavor, down by a factor of four from the photon density. Given the weakness of the neutrino interaction strength, $G_F = 4 \times 10^{-33}$ cm^2, the feebleness of the relic momentum, and the moderate number density, it is clear that attempts to measure the relic flux must trigger on $\mathcal{O}(G_F)$ effects rather than $\mathcal{O}(G_F^2)$ cross sections. One is led to forward scattering effects.[1] However, $\mathcal{O}(G_F)$ forward scattering effects have recently been proven to vanish, or (for the single nonvanishing effect, torque transfer from relics with a nonzero chemical potential to a polarized target) be negligibly small.[2] In target detection the existence of the relic neutrinos is deduced from their interaction effects on another flux[3,4] (e.g. an accelerator beam, or a cosmic ray flux). We will argue in section three that the interaction need be an $\mathcal{O}(G_F)$ resonance production. In deductive detection, the indirect effects of the relics upon terrestrial phenomena are interpreted as evidence for the relics. Examples of indirect effects include alteration of end-point spectra in neutrino-emitting weak decays due to the existence of a Fermi sea of relic neutrinos,[5] and alteration of $K_L - K_S$ mass splitting or regeneration due to interactions with a nonzero net relic neutrino number (CP asymmetric).[6] Unless the bounds on relic densities to be derived in the next section are a gross underestimate, proposed indirect effects are completely immeasurable. Thus we need consider further only target detection.

In the next section the following assertion will be proven: Given big bang cosmology and the standard electroweak model of Glashow-Salam-Weinberg, the annihilation of a cosmic ray ν with a relic $\bar{\nu}$ (or vice versa) on the Z resonance is the unique process having sensitivity to the relic neutrino density. The magnitude of absorption of cosmic ray neutrinos from a single source is then derived in the following section. The absorption energy is $\mathcal{O}(10^{21}\ eV^2/m)$ for a neutrino of mass m. An anticipated paucity of flux at this energy then suggests a convolution of absorption per source over all sources. This is done for a realistic source distribution in the final section. Our conclusion is: detection of relic neutrinos appears possible only if there exist astrophysical objects with very intense neutrino emissions in the energy region $10^{21}\ eV^2/m$ and very large cosmological red-shifts. The content of this conclusion is not encouraging.

COSMOLOGICAL CONSIDERATIONS

In this section the properties of relic neutrinos expected in big bang cosmology[7] are summarized and used to prove the assertion in the preceding section. As mentioned in the introduction, the temperature T_d at which neutrinos decoupled from thermal equilibrium is approximately 1 MeV. Neutrinos with mass negligible on this temperature scale then approximately satisfy a massless Fermi-Dirac momentum distribution. After decoupling, momenta are red-shifted as $p \sim 1/R$, where R is the scale size of the Friedmann-Robertson-Walker universe. Hence the present relic number density for neutrino species i is

$$n_{\nu_i}(\xi_i) = (2\pi)^{-3}\int d^3\vec{p}\, f_{\nu_i}(p) \ ,$$

where (1)

$$f_{\nu_i(\bar{\nu}_i)}(p) = \frac{1}{e^{\beta p \mp \xi_i} + 1} \ ,$$

$\beta^{-1} \equiv T_d R_d/R = 1.9\,°K \times (T_\gamma/2.7\,°K)$, and ξ_i is the degeneracy parameter (chemical potential multiplied by β) characterizing a possible neutrino number asymmetry:[8]

$$n_{\nu_i} - n_{\bar{\nu}_i} = 96\,(\xi_i + \xi_i^3/\pi^2)\ \mathrm{cm}^{-3} \times (T_\gamma/2.7\,°K)^3 \ . \qquad (2)$$

T_γ is the present value of the relic photon temperature and R is the present scale size. From the integration measure, d^3p, it is clear that the number density falls as R^{-3} as expected. The present symmetric density value is $n_{\nu_i}(0) = n_{\bar{\nu}_i}(0) = 53 \times (T_\gamma/2.7\,°K)^3$ cm^{-3}. For $\xi_i \gg 1$,

$$n_{\nu_i} \simeq 0.18\,|\xi_i|^3\, n_{\nu_i}(0) \ \text{and}\ n_{\bar{\nu}_i} \simeq 1.1\, e^{-|\xi_i|}\, n_{\bar{\nu}}(0) \ . \qquad (3)$$

For $\xi_i \ll -1$, exchange ν_i and $\bar{\nu}_i$.

ξ and m are not known. A cosmological bound on $\xi(m)$ or $m(\xi)$ results from requiring that the neutrino energy density ρ_ν (a monotonically, increasing function of m and ξ) not exceed the total energy density of the universe,[7] $\rho_o \leqslant 4 \times 10^{-29}\ \mathrm{g/cm}^3$, i.e.

$$\sum_i \int \sqrt{p^2 + m_i^2}\, \frac{d^3\vec{p}}{(2\pi)^3}\, [f_{\nu_i}(p) + f_{\bar{\nu}_i}(p)] \le \rho_o \ . \qquad (4)$$

The sum is over light species. This gives $|\xi_i| \leqslant 60$ for $m_i \ll \beta^{-1}$, and $|\xi_i| \leqslant 10\,(m_i/\mathrm{eV})^{-1/3}$ for neutrino masses in the electron volt range (as suggested by grand unified models). Setting $\xi_i = 0$ yields a bound on the neutrino masses, viz. $\Sigma_i m_i \leqslant 200$ eV.

A more complicated and probably less trustworthy argument relates the observed He^4/H abundance ratio to the n/p ratio when nucleons decoupled, and leads to a more restrictive bound[9,8] on the electron neutrino degeneracy parameter, $|\xi_{\nu_e}| \leq 2$. Families other than ν_e are not bounded by the He^4/H ratio. Even a degeneracy parameter as small as 1 leads to a large neutrino asymmetry (in units of photon number), $[n_\nu(1) - n_{\bar{\nu}}(1)]/n_\gamma \sim 1/4$, as compared to the known baryon asymmetry, $\Delta B/n_\gamma \sim 10^{-10}$.

Armed with these results from standard cosmology, we now prove that the annihilation of a cosmic ray ν with a relic $\bar{\nu}$ (or vice versa) on the Z resonance is the unique particle-relic interaction sensitive to a reasonable density.[4] Consider first scattering of cosmic rays by relic neutrinos. The mean free path (mfp) of a particle through the relic neutrinos is roughly $1/n_\nu\sigma_W$. The weak cross section is $\sigma_W \simeq (G_F^2/\pi)[s/(1+s/M_W^2)] \leqslant (G_F^2/\pi)s$. M_W is the W-boson mass and $\sqrt{s}$ the center-of-mass energy. For a cosmic ray of energy E, impinging on a relic neutrino with mean energy $\langle \varepsilon \rangle$, $\sigma_W \leqslant (2G_F^2/\pi) E\langle\varepsilon\rangle$. But $\langle \varepsilon \rangle n_\nu$ is just the neutrino energy density, certainly less than the total energy density ρ_o. Therefore $\lambda_{mfp} > \pi/2G_F^2 E\rho_o$. For the scattering rate to be significant, the mfp must be less than or of order of the Hubble radius $H_o^{-1} = h^{-1} \times 10^{28}$ cm, with $1 \leq h^{-1} \leq 2$ the observational uncertainty. Thus one requires $E > \pi/2G_F^2\rho_o H_o^{-1}$, which in turn is $\geqslant 2 \times 10^{14}$ GeV. But the universe is opaque to electrons, nucleons and photons at such energies: radio and thermal photon backgrounds degrade electrons via inverse Compton scattering and e^+e^- pair creation,[10] the nucleons via photomeson production,[11] and absorb primary photons via e^+e^- production.[12] In addition, for the electron such energies are also disallowed due to synchrotron losses occurring inside or outside the galaxy, or even in the earth's magnetosphere.[13,10] The universe (but not the earth) is transparent to cosmic ray neutrinos, but at energies in excess of 10^{14} GeV, the flux may well be negligible. Thus the scattering of cosmic rays by relic neutrinos is infinitesimal, except possibly for primary neutrinos with energy exceeding 10^{14} GeV. We may also use[14] $\sigma_W \leqslant (G_F^2/\pi) M_W^2$ to deduce $\lambda_{mfp}/H_o^{-1} \geqslant \pi/G_F^2 M_W^2 n_\nu H_o^{-1} \geqslant 10^4/(n_\nu/50\ cm^{-3})$, which says that regardless of incident energy, cosmic ray scattering on relic neutrinos is negligible unless the relic density is several orders of magnitude larger than the big-bang value predicted for $\xi = 0$.

Primary photons and electrons at extreme energy may undergo absorption by the relic neutrinos via $\gamma\bar{\nu}_\ell \to \ell W$ and $e\nu_e \to \gamma W$.

The charged lepton exchange graphs imply a cross section comparable to the Compton value of $(2\pi\alpha^2/s)\ln(s/m_\ell^2)$; one then finds $\lambda_{mfp} \gtrsim M_W^2/n_\nu\, 2\pi\alpha^2 \ln(M_W^2/m_e^2) \gtrsim 10^3\, H_o^{-1}/(n_\nu/50\ \mathrm{cm}^{-3})$. A neutrino degeneracy could in fact make these processes significant, but again the universe is opaque to electrons and photons in the relevant energy range $E \gtrsim M_W^2/2\langle\epsilon\rangle$.

Now, consider resonant absorption of a cosmic ray lepton by a relic neutrino. Integration over the relic momenta or over the universe's expansion is equivalent, by a change of variable, to integration over the resonance width. Thus the relevant weighted cross section for a Breit-Wigner form is $\bar{\sigma} \equiv \int ds\, \sigma(s)/M_R^2 = 16\pi^2 S\, \Gamma(R \to \ell\nu)/M_R^3$. S is the ratio of resonance spin states to incident lepton spin states. This time, the condition $\lambda_{mfp} \lesssim H_o^{-1}$ becomes

$$\frac{\Gamma(R \to \ell\nu)}{M_R} \gtrsim \frac{G_F M_R^2}{(n_\nu/50\ \mathrm{cm}^{-3})S} \ . \tag{5}$$

We have replaced $10^{-5}\ \mathrm{GeV}^{-2}$ with G_F to make it clear that unless the relic neutrino density is several orders of magnitude larger than the value expected for $\xi = 0$, $\Gamma(R \to \ell\nu)/M_R$ must be of order $G_F M_R^2$ rather than $(G_F M_R^2)^2$. This leaves the $W^\pm$ and Z as the only significant resonance candidates. Since the universe is opaque to electrons near the resonant energy $E_R \sim M_R^2/2\langle\epsilon\rangle$, the Z is the only resonant candidate.

Finally, consider an accelerator beam of energy E and current j (number per unit time) impinging on a length ℓ of relic density. The number of scatterings per unit time is $n_\nu \sigma_W \ell j$. For $\sigma_W \lesssim G_F^2 s/\pi$ this is $\lesssim (2G_F^2/\pi)\rho_\nu E\ell j \lesssim 10^{-38}\ (E/\mathrm{GeV})(\ell/\mathrm{meter})j$. Even a one amp current of TeV particles over a distance of 100 meters yields only at most one scattering per 10^7 years! With massive neutrinos there exists the possibility of gravitational clustering of relic neutrinos around galaxies or galactic clusters. A density enhancement of 10^6 is possible, increasing the accelerator beam scattering rate by the same factor, but still leaving it negligible. Moreover, resonant Z or W production cannot help us here since an accelerator cannot attain the resonant energy. Thus we dismiss accelerator beam-relic interactions and our assertion is proven.

We now turn to a detailed calculation of cosmic ray neutrino absorption. Since the neutrino mean free path for annihilation on the Z resonance is comparable to the Hubble radius, the effects of an expanding universe must be included in the calculation.[15]

COSMIC RAY NEUTRINO ABSORPTION

Let $dn_\nu(E,t)$ be the number of primary neutrinos per unit volume, at time t, with energy in the range E to E+dE. Assuming relativistic velocities for the cosmic ray neutrinos, the rate of density loss due to absorption is

$$\frac{d}{dt}(dn_\nu) = -\left[\int \frac{d^3\vec{p}}{(2\pi)^3} f_{\bar{\nu}}(p)\,\sigma_Z(1 - v_{\bar{\nu}}\cos\theta)\right] dn_\nu \tag{6}$$

σ_Z is the annihilation cross section, $v_{\bar{\nu}}$ is the relic antineutrino velocity in units of c , and θ is the incident angle of collision. From Eq. (6) the present $(t=0)$ neutrino density is

$$dn_{\nu_o}(E_o) = e^{-\tau} dn_\nu(E(t),t), \quad \tau \equiv \int_o^t dt \int \frac{d^3\vec{p}}{(2\pi)^3} f_{\bar{\nu}}(p)\,\sigma_Z(1 - v_{\bar{\nu}}\cos\theta) \tag{7}$$

Here and hereafter a subscript zero will denote a present time value.

In an expanding universe, densities are diluted and momenta are red-shifted independent of the interaction. Introducing the red-shift variable for cosmological expansion from time t to present, $\omega(t) \equiv R_o/R(t) - 1$, one thus has

$$dn_{\nu_o}(E_o) = (\omega+1)^{-3} e^{-\tau} dn_\nu((1+\omega)E_o, t) \ . \tag{8}$$

Since the cosmological red-shift is a monotonic function of time, one may parameterize time by the red-shift. The relevant change of variable is $d\omega = -(\omega+1)H\,dt$, where $H \equiv \dot{R}/R$ is the Hubble parameter, itself a function of time. The Einstein equations for a matter-dominated (pressure $p \ll \rho$) era relate the Hubble parameter at time t to its present value (we assume zero cosmological constant): $H(t) = H_o(\omega+1)\sqrt{1+\Omega_o\omega}$. Ω_o is the present energy density of the universe in units of the critical value for closure. The bounds from observational astronomy are $0.02 \leqslant \Omega_o \leqslant 2$. Thus we have

$$d\omega = -(\omega+1)^2 \sqrt{1+\Omega_o\omega}\; H_o\,dt \ . \tag{9}$$

Let us concentrate on the calculation of τ, the neutrino depth, in the two limits of highly relativistic and nonrelativistic relic neutrinos. Since the relic mean momentum is $\langle p_{\nu_i}\rangle \sim \beta^{-1}(1+\xi_i)$, these limits correspond to $m_i << \beta^{-1}(1+\xi_i)$ and $m >> \beta^{-1}(1+\xi_i)$ respectively. For the Z resonance, we assume a Breit-Wigner form and a standard electroweak model coupling; thus $\int ds\, \sigma_Z(s) = 2\pi\sqrt{2}\, G_F M_Z^2$. For massless neutrinos, Eq. (7) becomes

$$\tau(E,\xi,z) = \frac{G_F M_Z^4}{4\pi\sqrt{2} H E^2} \int_0^z \frac{d\omega}{(\omega+1)^3\sqrt{1+\Omega\omega}} \int_0^\infty dp\, f_\nu(p)\, \Theta(p(\omega+1)^2 - M_Z^2/4E). \tag{10}$$

Subscript zeros have been dropped since every variable in (10) is either a present time variable or an integration variable. The maximum red-shift value, z, is the cosmological red-shift of the extragalactic neutrino source. The values of detected energy E for which some absorption may occur extend over the entire range from zero to infinity since for any cosmic ray energy there exists a value of relic energy that will guarantee the resonance CMS energy. Unfortunately, unless the degeneracy parameter is large, this smearing eliminates the possibility of an absorption dip. Furthermore, it is clear from the theta function of Eq. (10) that absorption reaches a maximum at energies of order $M_Z^2/(1+z)^2\langle p_\nu\rangle \gtrsim M_Z^2\beta/[(1+\xi)(1+z)^2]$, i.e. $E > 10^{13}$ GeV for $z \leqslant 4$ and $\xi \leqslant 50$. Transmission probabilities $e^{-\tau}$ are shown in Fig. 1. Prospects for detection of relativistic relic neutrinos appear poor.

In grand unified models, neutrino masses in the eV range arise naturally.[16] Furthermore, 30 eV neutrinos are a panacea for unresolved questions concerning the large scale structure of the universe.[17] Relic neutrinos with an eV mass satisfy the nonrelativistic criterion. The nonrelativistic limit of Eq. (7) is

$$\tau(\tilde{E},z) = \frac{\tau(1)\,\Theta(1-\tilde{E})\,\Theta(\tilde{E}(1+z)-1)}{\tilde{E}^{3/2}\sqrt{\Omega_o + \tilde{E}(1-\Omega_o)}} \tag{11}$$

Here $\tilde{E} = 2mE_o/M_Z^2$ is the energy incident at earth in units of the resonant energy $M_Z^2/2m$, and $\tau(1) = 2\pi\sqrt{2}\, G_F\, n_{\nu o} H_o^{-1} = 0.017\, h^{-1}$ $(n_{\nu o}/50\ \mathrm{cm}^{-3})$. The allowed range of $\tilde{E}$ has a simple interpretation: a neutrino received with energy E_o left its red-shifted source with energy $E_o(1+z)$, and was a candidate for annihilation only if the resonance energy, $E_o/\tilde{E}$, lay within this range. Equivalently, $M_Z^2/2m(1+z) \le E_o \le M_Z^2/2m$.

The transmission probability for nonrelativistic relics is plotted as a function of received energy in Fig. 2. An absorption dip is apparent beginning at an energy of 1/1+z. It is clear that an absorption dip of 15% to 50% can be expected for neutrinos

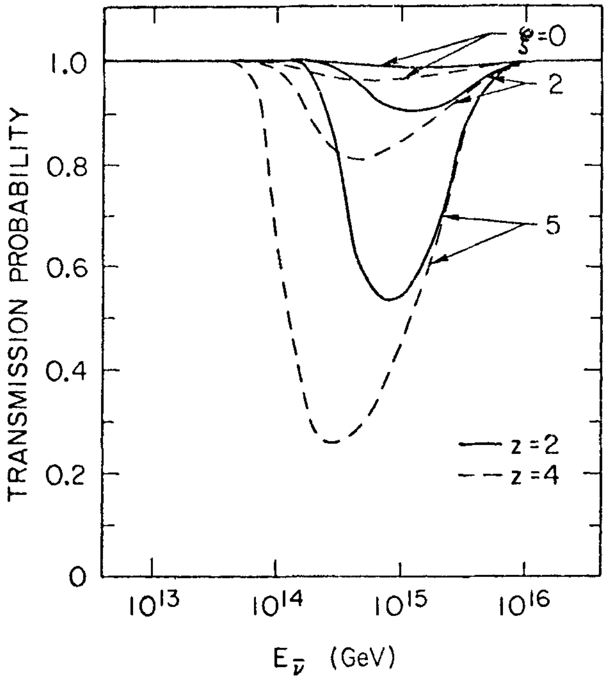

Fig. 1. Transmission probability for massless ($m \ll \beta^{-1}(1+\xi)$) cosmic ray antineutrinos from a single source (red-shift z) as a function of their energy. The assumed values of ($h^{-1}, \Omega, \xi, T_\gamma$) are (2, 1, 0, 2.7 °K) unless stated otherwise in the figure. $M_Z = 90$ GeV is assumed. Transmission probabilities for incident neutrinos are obtained by taking $\xi \to -\xi$.

from a $z = 3.5$ quasar source. If the degeneracy parameter is non-zero, the dip will be much larger. For a 90 GeV Z-boson mass and a $z = 3.5$ source, the dip energy is 10^{21} eV^2/m.

We have shown that a sizeable absorption dip at $\sim 10^{20}$ eV is unambiguously predicted for neutrinos emanating from a highly red-shifted source and having mass in the electron volt range. Detection feasibility depends on the magnitude of neutrino flux emitted by the source. Quasars, active galactic nuclei, pulsars, supernovae and accreting black holes are suggested sources.[18] Although their ultrahigh energy emission spectrum is unknown, it is very unlikely that a single source spectrum can be measured at the energies here required. Consider as a measurement criterion, detection of 10^3 events per year in the 10^{20} eV energy region by a 100% efficient DUMAND-type detector (one cubic kilometer of H_2O target). Since the ultrahigh energy neutrino-nucleon cross section is approximately[14]

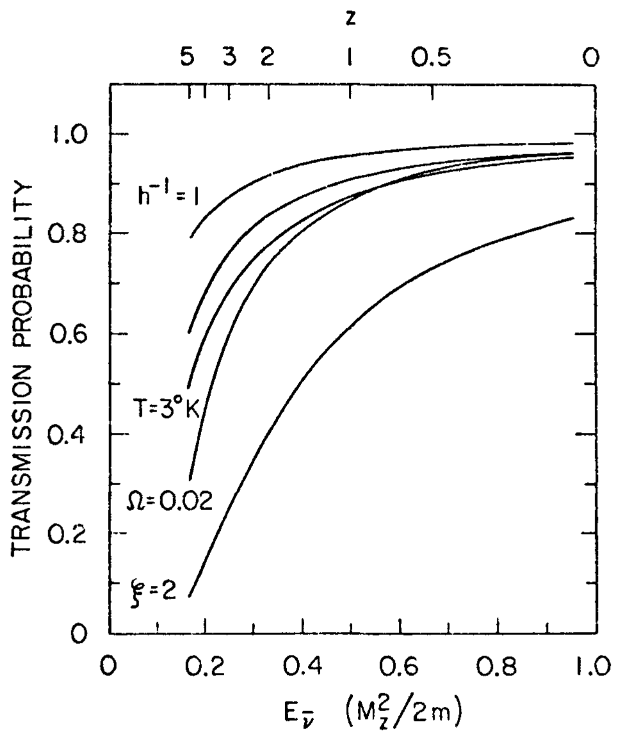

Fig. 2. Same as Fig. 1 but for massive $[m \gg (\xi+1)\beta^{-1}]$ neutrinos. The positions of the absorption dip for various z values are shown at the top of the figure.

$G_F^2 M_W^2 \ln(s/M_Z^2)/\pi \sim 10^{-33}$ cm^2 (the $\ln$ arises in QCD), an incident neutrino flux of $F \sim 10^{-10}/\text{cm}^2/\text{s}$ is required at 10^{20} eV. The distance to a highly red shifted sourced is a large fraction of the Hubble radius, so the power generating this minimum flux is $P \sim F \cdot E_\nu \cdot 4\pi H_o^{-2} \sim 10^{55}$ erg/s. This may be contrasted with a 10^{45} to 10^{49} erg/s optical emission from quasars, the most powerful sources observed to date. If the mass of a quasar is of order of that of a typical galaxy, the required neutrino flux at $E \sim 10^{20}$ eV represents a power conversion of 1% per year and an upper bound on the lifetime of such a source of a century. Thus the existence of a single source of sufficient flux to enable measurement of the dip appears untenable.

SUM OVER SOURCES

In order to enhance the flux of 10^{20} eV neutrinos incident on earth, a sum over sources will be carried out. Taking the energy density of the universe to be $\sim 10^{-29}$ g/cm, the necessary flux value results if 10^{-4} of the energy has been converted to 10^{20} eV neutrinos. This is possible, and we proceed to calculate the absorption of the neutrino flux integrated over sources. The summation over a distribution of source red-shifts will tend to smear the absorption dip, but it will now be shown that depending on the history of the sources, detectable absorption may survive.

Let the number of neutrinos emitted per unit volume in the time interval dt_e and in the energy increment dE be

$$d^2 n_\nu(E,z) = \left[\sum_s \rho_s(E,z)\, n_s(z)\right] dE\, dt_e \tag{12}$$

where $n_s(z)$ is the source density at "time" z, $\rho_s(E,z)$ is the source power (neutrino number emitted per unit time per unit energy) at energy E and time z, and the sum is over the various types of neutrino sources. Then the relativistic neutrino flux incident at earth (neutrino number per unit area, time, steradian and energy) is

$$\mathfrak{F}_o(E_o) = \frac{1}{4\pi} \int dt_e \frac{d^2 n_{\nu o}(E_o, z)}{dE_o\, dt_e} \tag{13}$$

Substituting from Eqs. (8) and (9) (with t replaced by t_e and ω replaced by z), and (12), gives for Eq. (13)

$$\mathfrak{F}_o(E_o) = \frac{1}{4\pi H_o} \sum_s \int_o^\infty dz \frac{\rho_s((1+z)E_o, z)\, n_s(z)\, e^{-\tau(E_o, z)}}{(1+z)^4 \sqrt{1+\Omega_o z}} \tag{14}$$

Then writing Eq. (11) as

$$e^{-\tau} = 1 + \Theta(1-\tilde{E})\,\Theta((1+z)\tilde{E}-1) \left\{\exp\left[\frac{-\tau(1)}{\tilde{E}^{3/2}\sqrt{\tilde{E}(1-\Omega_o)+\Omega_o}}\right] - 1\right\} \tag{15}$$

the flux incident at earth becomes

$$\mathfrak{F}_o(E_o) = \frac{1}{4\pi H_o} \sum_s \int_o^\infty dz \frac{\rho_s(1+z)E_o, z)\, n_s(z)}{(1+z)^4 \sqrt{1+\Omega_o z}} - \delta \mathfrak{F}_o(E_o) \,, \tag{16}$$

where the flux loss due to absorption is

$$\delta\mathcal{F}_o(\tilde{E}) = \frac{\Theta(1-\tilde{E})}{4\pi H_o}$$

$$\times\left\{1-\exp\left[\frac{-\tau(1)}{\tilde{E}^{3/2}\sqrt{\tilde{E}(1-\Omega_o)+\Omega_o}}\right]\right\}\sum_s\int_{1/\tilde{E}-1}^{\infty} dz\,\frac{\rho_s((1+z)E_o,z)\,n_s(z)}{(1+z)^4\sqrt{1+\Omega_o z}}\ . \tag{17}$$

The relative flux loss is simply the ratio $\delta\mathcal{F}_o/\mathcal{F}_o = \delta \ln \mathcal{F}_o$, and the transmission probability is $1 - \delta\mathcal{F}_o/\mathcal{F}_o$.

For further calculation, a source density distribution $n_s(z)$ and a source power distribution $\rho_s(E,z)$ are needed. Since extragalactic neutrino astronomy has not yet begun as an observational science, nothing certain is known about ultraenergy neutrino sources. However, the skies seem to favor power law spectra for galactic fluxes. Radio waves have a spectral index of ~ 0.7, while charged particles have an index of 2 to 3 (consistent with the synchrotron relation, $\alpha_{charge} = 2\alpha_{radio} + 1$). Power laws are scale invariant, since $A(E/\hat{E})^{-\alpha} = A' E^{-\alpha}$. Thus, an assumption of a power law for the neutrino emission spectrum leads to a factorizing power distribution,

$$\rho_s(E,z) = A_s E^{-\alpha_s} f_s(z) = A_s E_o^{-\alpha_s}(1+z)^{-\alpha_s} f_s(z)\ . \tag{18}$$

It is clear that luminosity evolution, characterized by nonconstant f(z), and source creation or destruction, characterized by nonconstant $(1+z)^{-3}n(z)$, mimic each other. We may therefore fix $(1+z)^{-3}n(z)$ to be constant, and parameterize all source evolution by $f_s(z)$. A useful and simple parameterization is

$$f_s(z) = (1+z)^{\beta_s}\,\Theta(z_s - z)\ . \tag{19}$$

The theta function defines the critical time, z_s, at which the source began emitting neutrinos, and $\beta \neq 0$ characterizes source evolution. Employing Eqs. (18) and (19), one has

$$\mathcal{F}_o(E_o) = \sum_s \frac{n_{so} A_s E_o^{-\alpha_s}}{4\pi H_o}\int_0^{z_s} dz\,\frac{(1+z)^{\beta_s-\alpha_s-1}}{\sqrt{1+\Omega_o z}} - \delta\mathcal{F}_o(E_o) \tag{20}$$

and

$$\delta\mathcal{F}_o(\tilde{E}) = \Theta(1-\tilde{E})$$

$$\times\left\{1 - \exp\left[\frac{-\tau(1)}{\tilde{E}^{3/2}\sqrt{\tilde{E}(1-\Omega_o)+\Omega_o}}\right]\right\}\sum_s \frac{n_{so} E_o^{-\alpha_s} A_s}{4\pi H_o}\int_{1/\tilde{E}=1}^{z_s} dz\,\frac{(1+z)^{\beta_s-\alpha_s-1}}{\sqrt{1+\Omega_o z}} \tag{21}$$

The values $\Omega_o = 0$ and 1 give an integrand probably bracketing the true value and lead to especially simple results:

$$\mathcal{F}_o(E_o) = \sum_s \frac{n_{so} A_s E_o^{-\alpha_s}}{4\pi H_o}\left[\frac{(1+z_s)^{\Gamma_s} - 1}{\Gamma_s}\right] - \delta\mathcal{F}_o(E_o)\ , \tag{22}$$

$$\delta\mathcal{F}_o(\tilde{E}) = \Theta(1-\tilde{E})$$

$$\times\left\{1 - \exp\left(\frac{-\tau(1)}{\tilde{E}^{3/2}\sqrt{\tilde{E}(1-\Omega_o)+\Omega_o}}\right)\right\}\sum_s \frac{n_{so} A_s E_o^{-\alpha_s}}{4\pi H_o}\left[\frac{(1+z_s)^{\Gamma_s} - (1/\tilde{E})^{\Gamma_s}}{\Gamma_s}\right] \tag{23}$$

with $\Gamma_s = \beta_s - \alpha_s - (0, 1/2)$ for $\Omega_o = (0, 1)$ in the integrand. For $\Gamma_s = 0$ replace the square brackets with $\ln(1+z_s)$ and $\ln[\tilde{E}(1+z_s)]$ respectively. The values in front of the square brackets are just what one gets by summing all sources out to the Hubble radius H_o^{-1}, ignoring red-shifting and source evolution. It is clear that such a naive sum is qualitatively correct. Significantly larger fluxes result only if evolution is important, and there exist sources at large red-shift (i.e. $z_s \gg 1$).

We will now assume that in the energy region of absorption ($\tilde{E} \sim 1$), one source type dominates. Then the sum on sources and the source subscripts may be dropped with the understanding that A, n_{s_o}, β, α and Γ refer to the dominant source. The relative flux loss is simply

$$\delta\ln\mathcal{F}_o(\tilde{E}) = \Theta((1+z_c)\tilde{E}-1)\,\Theta(1-\tilde{E})\,\mathcal{V}\left\{1 - \exp\left(\frac{-\tau(1)}{\tilde{E}^{3/2}\sqrt{\tilde{E}(1-\Omega_o)+\Omega_o}}\right)\right\} \tag{24}$$

with

$$\mathcal{W}(\tilde{E}, z_c, \Gamma) = \left[\frac{1 - [\tilde{E}(1+z_c)]^{-\Gamma}}{1 - (1+z_c)^{-\Gamma}}\right], \quad \Gamma \neq 0 \tag{25}$$

$$= \ln[\tilde{E}(1+z_c)]/\ln[1+z_c], \quad \Gamma = 0 .$$

z_c is the critical time at which the dominant source began neutrino emission. Note that except for the factor $\mathcal{W}$, Eq. (24) has the form of the familiar single source result with $z = z_c$ (c.f. Eq. (15)). Thus the whole effect of summing over source red-shifts is to reduce the single source result by the "wash-out" factor $\mathcal{W}$. For all choices of Γ and z_c, and $\tilde{E}$ consistent with the theta functions, $0 \leq \mathcal{W} \leq 1$. $\mathcal{W}$ rises from zero at $\tilde{E} = 1/1+z_c$, where only the most red-shifted source may contribute, to unity at $\tilde{E} = 1$, where all sources contribute. $\mathcal{W}$ increases with increasing Γ or increasing z_c. The infinite evolution limit, $\Gamma \to \infty$, pushes all sources back to $z = z_c$, thereby setting $\mathcal{W} = 1$ and restoring the single source result for relative flux loss.

The simple functional form of $\delta \ln \mathfrak{F}_0$ belies the fact that it depends on six ill-known parameters: Ω_0, T, ξ, h, z_c, Γ. $\mathfrak{F}_0$ depends on A, n_{s_0} and α in addition to these. Predictions for the transmission probability of the neutrino flux summed over source red-shifts, $1 - \delta \ln \mathfrak{F}_0(\tilde{E})$, are given in Fig. 3 for the standard set of (Ω, T, ξ, h^{-1}) values. Results for other values may be inferred by referring to Fig. 2.

The inference to be drawn from Fig. 3 is that unless the history of neutrino sources shows strong evolution (large Γ) and early emission (large z_c), the relative flux loss is negligible. What values of Γ and z_c might one expect? It is common belief that quasars emit ultraenergy neutrinos. The emission mechanism is presumed to be the collision of hadrons, accelerated by enormous magnetic fields, yielding pions with decay products $\nu_\mu e \nu_e$. Neutrinos produced in the quasar interior do not suffer the strong absorption and energy degradation in the high density environment that other quanta endure, and therefore escape with their ultraenergy intact. Quasar counts as a function of z show definite evolutionary effects, at least for $z \lesssim 2$, corresponding to $\beta \simeq 3.5$.[19] The apparent quasar density peaks around $z \sim 2$ (perhaps a selection effect), and vanishes abruptly above $z \sim 3.5$. This cut-off at $z \sim 3.5$ may be interpreted as signifying the age of quasar production, or alternatively, as signaling the end of a radio, optical and x-ray dense era. With the latter hypothesis, quasar birth occurred even earlier than $z = 3.5$, and neutrino astronomy will be sensitive to even larger z values. Also with the latter hypothesis,

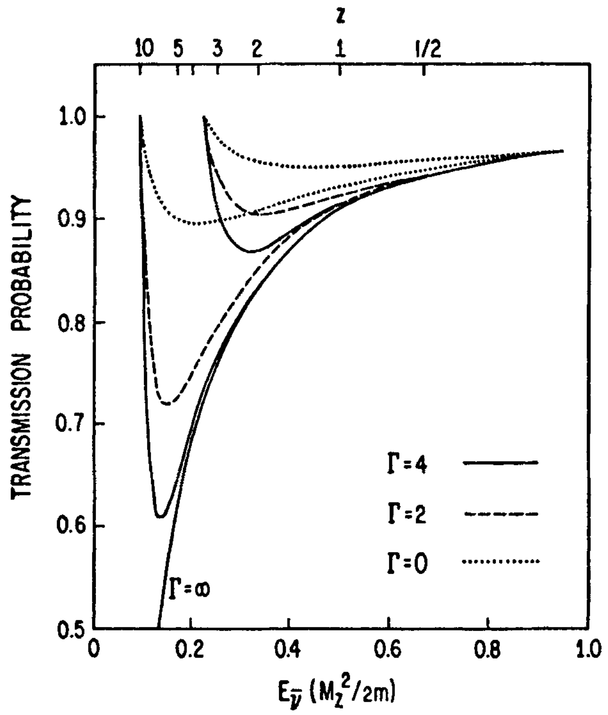

Fig. 3. Same as Fig. 2 but for a neutrino flux integrated over sources with red shift $\leq z_c$. Γ parameterizes source evolution and spectral index of neutrino emission (see text). Sets of curves for $z_c = 3.5$ and $z_c = 10$ are shown.

relic detection is perhaps possible if evolution continues to large z and the neutrino emission spectrum in the 10^{20} eV range is intense and not too steep, such that $\Gamma \gtrsim 2$. Otherwise, the only presently viable hope for relic detection, cosmic ray ν-relic $\bar{\nu}$ annihilation on the Z resonance, joins the prior proposals as well-intentioned immeasureabilia.

I wish to thank E. M. Burbidge, M. Schmidt and H. E. Smith for useful conversations.

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