The Z Boson Resonance Parameters

Tord Riemann

DESY – Institut für Hochenergiephysik

Platanenallee 6, D-15738 Zeuthen, Germany

Email: riemann@ifh.de   ·   http://www.ifh.de/~riemann

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ABSTRACT

The Z line shape is measured at LEP/CERN with an accuracy at the per mille level. Usually it is described in the Standard Model of electroweak interactions with account of quantum corrections. Alternatively, one may attempt an S-matrix based model-independent approach in order to extract quantities like mass and width of the Z boson. I describe the formalism and its application to data.

1   Introduction

Our present understanding of weak interactions is completely described by the Standard Model [1], a spontaneously broken locally gauge invariant, anomaly-free, renormalizable quantum field theory of pointlike leptons and quarks, the latter in three colors. The model contains fermions, vector bosons, and a scalar particle:

fermions: $\begin{pmatrix} \nu_l \\ l \end{pmatrix}$ $l = e, \mu, \tau$

$\begin{pmatrix} U \\ D \end{pmatrix}$ $\left. \begin{matrix} U = u, c, t \\ D = d, s, b \end{matrix} \right\}$ in 3 colors

vector gauge bosons: $W^\pm,\ Z^0,\ \gamma$

scalar Higgs boson: H

The particle's masses (and mixing angles) are free parameters. Their interactions are determined from the invariance of the Lagrangean under local gauge transformations with gauge group $SU(2)_L \times U(1)$ and associated gauge fields $W^\pm, W^0, B$. One may parameterize the model in terms of masses and mixing angles plus electromagnetic coupling constant α_{em}. Often, instead of the W boson mass, the Fermi constant is used:

- $\alpha_{em} = 1/137.036$

- $G_\mu = 1.16634 \times 10^{-5}$ GeV^{-2}

- m_f, including $m_t = 175$ GeV [2]

- $M_Z = 91.186$ GeV [2]

- $M_H \ldots$ unknown

Discovery and study of the Z resonance are part of the long history of weak interactions and of unification of forces. First observations of virtual Z exchange lead to the discovery of weak neutral current reactions in the scattering of neutrinos off electrons, $\nu_\mu + e^- \to \nu_\mu + e^-$, and off nucleons, $\nu_\mu + N \to \nu_\mu + N$ in 1973 (Gargamelle Collab.: F.J. Hasert et al., A. Benvenuti et al., B. Aubert et al.) at the proton accelerator PS (CERN). The cross-section measurements may be interpreted in terms of the weak mixing angle θ_w. This angle characterizes not only the mixing of photon and Z boson but also the strength of the weak neutral interactions and the relations of the gauge boson masses to the Fermi constant (and among themselves). In the Standard Model[1]:

$$Z = \cos\theta_w \, W^0 - \sin\theta_w \, B \tag{1}$$

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[1] The relations get modified by radiative corrections; see e.g. [3].

$$\gamma = \sin\theta_w\, W^0 + \cos\theta_w\, B \tag{2}$$

$$g\sin\theta_w = e = \sqrt{4\pi\alpha_{em}} \tag{3}$$

$$a_{lept} = -\frac{1}{2} \tag{4}$$

$$v_{lept} = -\frac{1}{2}\left(1 - 4\sin^2\theta_w\right) \tag{5}$$

$$M_W = \sqrt{\frac{\pi\alpha}{G_\mu\sqrt{2}}}\,\frac{1}{\sin\theta_w} \geq 37.281 \text{ GeV} \tag{6}$$

$$M_Z = \frac{M_W}{\cos\theta_w} \tag{7}$$

The theory predicts the gauge boson masses as soon as there is a numerical estimate for the weak mixing angle[2]. Thus, after a few weak neutral current events were observed, a lot of information could gained from this. From the cross-sections of 1973, one may derive $0.1 < \sin^2\theta_w < 0.6$. This corresponds to $M_W = 118\cdots48$ GeV and $M_Z = 125\cdots75$ GeV. Both particles were discovered at the specially designed $p\bar{p}$ collider SPS (CERN) in 1983 (UA1 Collab.: G. Arnison et al., UA2 Collab.: P. Bagnaia et al., M. Banner et al.).

After the discovery of the Z boson, its detailed study by a dedicated tool, an e^+e^- collider with a center of mass energy corresponding to the Z mass, became a dream of particle physicists. Since the advent of the e^+e^- colliders LEP (CERN) and SLC (SLAC) in 1989, about sixteen millions of Z bosons have been produced at LEP and hundreds of thousands at SLC. They are produced as a resonance peak in the cross-section of the reaction

$$e^+ + e^- \quad \to (\gamma, Z) \to \quad \text{anything} \tag{8}$$

LEP finished operation as a Z factory in 1995 and is now running at higher energies for the study of W pair production and searches for Higgs, susy, and other particles while SLC goes yet on for a while.

Due to the impressive accuracy of the measurements it was possible to test the Standard Model at the level of quantum corrections. This raises the problem of the accurate description of unstable particles in a quantum field theory. In a quantitative sense, this has been done with great success. Practically all experimental results are described by the Standard Model consistently within the experimental errors. I should mention specially the recent discovery of the top quark with a mass of about 176 GeV at Fermilab (CDF Collab.: F. Abe et al. (1994), D0 Collab.: S. Abachi et al. (1995)). This value agrees nicely with that predicted from measurements of the Z resonance parameters when quantum corrections from virtual top quark exchange are taken into account in the Standard Model, $m_t \sim 147 - 167$ GeV [4]. The fits favor a light Higgs boson with $M_H = 121^{119}_{-68}$ GeV and the estimate $M_H < 430$ GeV at 95% C.L. However, one should note that there is no experimental hint for the existence of the Higgs boson, whose interactions are assumed to create all the particle masses.

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[2] An absolute lower limit is about 37 GeV, see (6).

In this contribution, the shape of the Z resonance excitation curve will be described[3]. It is analyzed to which extent the description is model-independent. Some emphasis will be given to an approach based on first principles as formulated in the S-matrix theory.

The Z resonance is part of our physical world. We are not faced with the problem of its existence but rather of its proper description. A rigid mathematical handling of unstable particles to which the efforts of many of the participants at this Symposium are devoted is certainly not developed within the framework of relativistic quantum field theory. I hope that my talk may serve as an introduction to the status of the study of the Z resonance. The presentation will reflect my working activities which are closely related to the interpretation of measured cross-sections and other observables in terms of theoretical quantities. This task deserves a close interaction of theoreticians and experimental physicists. For details about this cooperation interested collegues may consult e.g. [2, 3] and references therein.

With the advent of the high-precision data from LEP 1 on single Z boson production and the frequent W pair production at LEP 2, the problem of definition of their masses and widths in a renormalizable quantum field theory became an important issue. Experimentalists often use formulae in the on-mass-shell approach, while some theorists prefer the introduction of a complex particle pole prescription, proposed in [6, 7]. The first one is preferred by recent tradition and well-developed while the latter one looks more convincing from a conceptual point of view: the propagator may be constructed in an explicitly gauge invariant way. When used properly, both schemes will give gauge-invariant results in the relevant order of perturbation theory (see e.g. [6, 8, 9, 10]), but the numerical values for the Z mass differ significantly. This was observed first in [11, 12]. Quite recently, the relation of both schemes was discussed in detail [13]. Although I will not give an introduction to perturbative renormalization for unstable particles, few comments on it may be found in sections 5 and 7.

2 The Z Line Shape

Some of the predictions of the Standard Model have been mentioned in the Introduction. Particle masses are used as input parameters while their life times $\tau = 1/\Gamma, \Gamma$ being the decay width, may be predicted. The Z boson decays nearly exclusively into pairs $f\bar{f}$ of leptons $(e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau)$ or colored quarks $(3 \times d, u, s, c, b)$. The inverse life time (total Z width) is the incoherent sum of all partial widths of the different decay channels. From the Lagrangean for the $Zf\bar{f}$ interactions

$$\mathcal{L} = -\frac{ig}{2\cos\theta_w} Z_\mu \sum_f \bar{f}\gamma^\mu [v_f + a_f\gamma_5] f \tag{9}$$

one may derive [14, 3]

$$\Gamma_Z = \sum_f \Gamma_f = \sum_f N_{color}^f \frac{G_\mu M_Z^3}{6\pi\sqrt{2}} \left[(v_f^{eff})^2 + (a_f^{eff})^2 \right] = 2.4946 \pm 0.0027 \text{ GeV } [2] \tag{10}$$

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[3] For the description of hadron resonances see the contribution of G. López Castro [5] to this symposium.

The notations indicate that the (effective) couplings are slightly modified by radiative corrections.

Unfortunately, the width of a particle is not directly measurable. The Z width may be derived from an analysis of the Z resonance measured at the accelerators LEP1 and SLC. There, the most frequent reaction is

$$e^+e^- \rightarrow (\gamma, Z) \rightarrow \bar{f}f(+n\gamma) \tag{11}$$

The cross-section is shown as a function of the beam energy for a wide energy range in Figure 1.

The mass M_Z and width Γ_Z may be determined from cross-sections obtained in a small region around the Z peak ($s = 4E_{beam}^2$):

$$|\sqrt{s} - M_Z| < 3 \text{ GeV} \tag{12}$$

For energies off the resonance the cross-section falls down rapidly.

Without radiative corrections, the isolated Z resonance shape may be fitted with the following ansatz:

$$\sigma_0^{(Z)}(s) = \frac{4\pi\alpha^2}{3s}|\chi(s)|^2 \left(a_e^2 + v_e^2\right)\left(a_f^2 + v_f^2\right) N_{color}^f \tag{13}$$

with a Breit-Wigner shape function

$$\chi(s) = \frac{G_\mu M_Z^2}{\sqrt{2}2\pi\alpha} \times \kappa(s), \qquad \kappa(s) = \frac{s}{s - M_Z^2 + iM_Z\Gamma_Z(s)} \tag{14}$$

The natural appearance of an s dependence of the width function in a perturbative calculation was pointed out by Wetzel (1983) [16].

In Born approximation, the following expression in terms of partial Z widths is equivalent to (13):

$$\sigma_0^{(Z)}(s) = \frac{12\pi}{s}|\kappa(s)|^2 \frac{\Gamma_e \Gamma_f}{M_Z^2} \tag{15}$$

The cross-section values have to be related to the free parameters of the theory. In the Standard Model these are, e.g., $M_Z, M_H, m_t, \alpha_{strong}$ – and *not*, e.g., Γ_Z, or the partial widths Γ_f. When radiative corrections are taken into account – and they have to be – numerical differences may not be neglected.

Let me now mention some features of the Z line shape which make its analysis complicated. The first fact is that we want to study a $2 \rightarrow 2$ process with intermediate Z, but have also to take into account virtual photon exchange, see figure 2.

In addition, there are huge $2 \rightarrow 3, 4, \ldots$ contributions due to initial state radiation (ISR) and final state radiation (FSR), see figure 3.

Further, many virtual corrections are not shown here but have to be included:
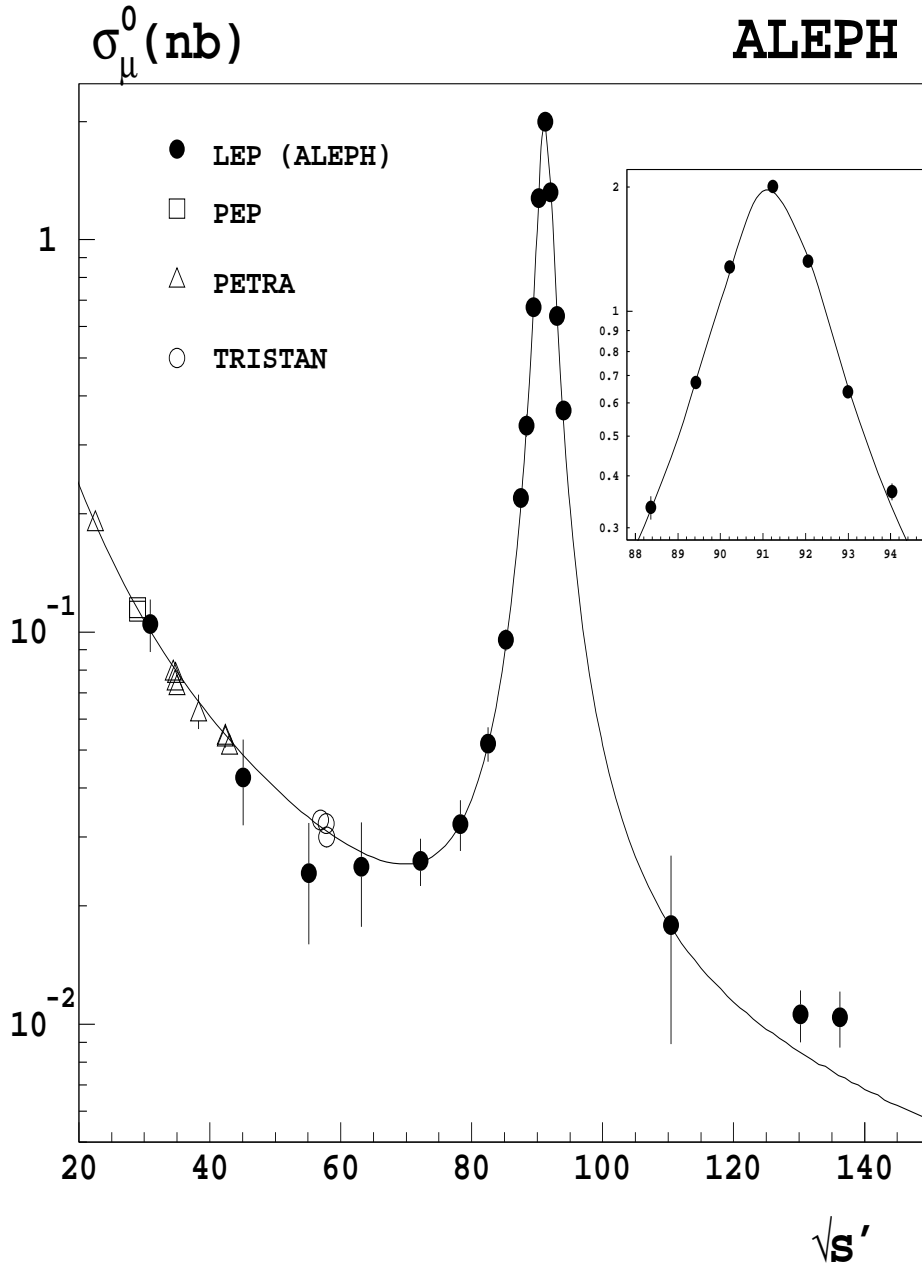
- vertex insertions

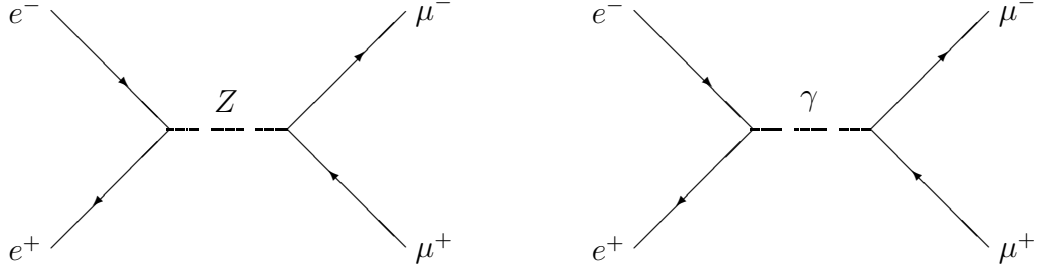Figure 1: *The muon production cross-section over a wide energy range [14]*

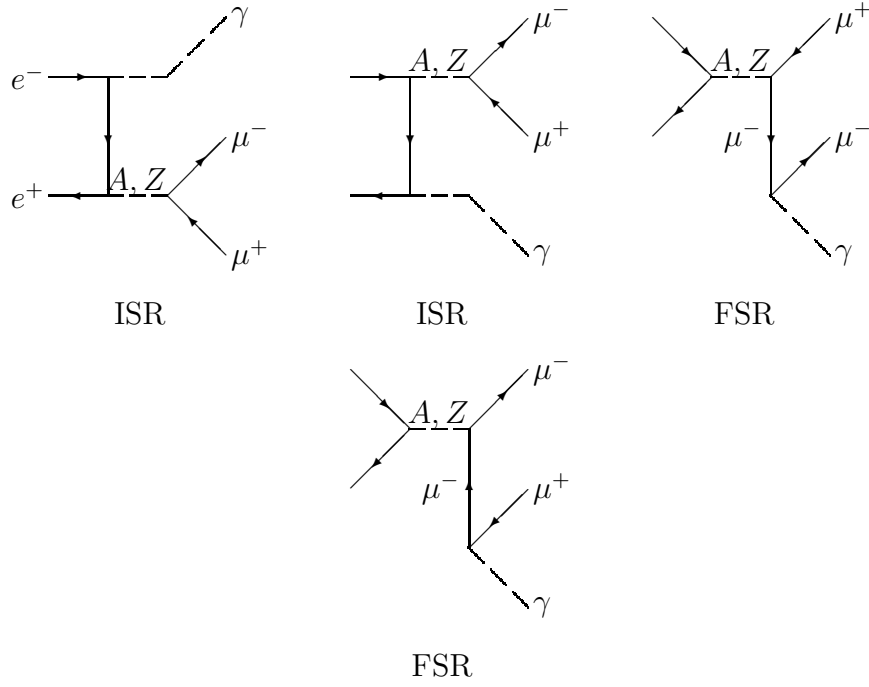Figure 2: *Born contributions to the Z resonance shape*

Figure 3: *QED corrections to the Z resonance shape*

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- self-energy insertions

- box diagrams

- for quarks: final state QCD corrections

In the next section, I will indicate the proper handling of the Z resonance as it is practiced by the LEP experimental groups ALEPH, DELPHI, L3, OPAL and the SLD group at SLC. Further details may be found in [2, 3] and, of course, in the articles of these collaborations.

3 Photonic Corrections to the Z Line Shape

Photonic corrections influence position and shape of the Z resonance heavily. An analysis of data without proper treatment of them leads to results which are numerically simply wrong. Fortunately, they may be taken into account in a generic way by the following convolution formula:

$$\sigma(s) = \int d\left(s'/s\right)\sigma_0(s')\,\rho\left(s'/s\right) + \int d\left(s'/s\right)\sigma_0^{int}(s,s')\,\rho^{int}\left(s'/s\right) \qquad (16)$$

with

- $\rho(s'/s)$ – a radiator describing initial and final state radiation of photons, including leading higher order effects and soft photon exponentiation;

- $\rho^{int}(s'/s)$ – taking into account the initial-final state interference effects which are comparatively small (a few per mille);

- $\sigma_0(s')$ – the basic scattering cross-section, which is the object of investigation;

- $\sigma_0^{int}(s,s')$ – a similar function, but often negligible since near the resonance peak numerically suppressed.

If ρ is known and ρ^{int} is also known and, more important, small, one may try to unfold the basic cross-section σ_0 from experimental data.

The dominant part of the photonic corrections is due to ISR (initial state radiation):

$$\rho(s'/s) = \beta(1 - s'/s)^{\beta-1}\delta^{soft+virtual} + \delta^{hard} \qquad (17)$$

where [3]

$$\beta = (2\alpha/\pi)\left[\ln\left(s/m_e^2\right) - 1\right] \qquad (18)$$

$$\delta^{soft+virtual} = 1 + (\alpha/(2\pi))\left[3\ln\left(s/m_e^2\right) + 2\pi^2/3 - 4\right] + \mathcal{O}\left(\alpha^2\right) \qquad (19)$$

$$\delta^{hard} = -(\alpha/\pi)(1 + s'/s)\left[\ln\left(s/m_e^2\right) - 1\right] + \mathcal{O}\left(\alpha^2\right) \qquad (20)$$

Near the resonance peak ISR leads to huge corrections of shape, position, and height of the peak and cannot be neglected. This means that any serious physical analysis is not only faced by higher order corrections but also by a substantial admixture of $2 \to 3$ (and higher order) processes which may not be experimentally extracted from the $2 \to 2$ process under investigation.

4   Approaching a Reasonable Formula for σ_0

In this section, I discuss the sensitivity of the determination of M_Z on the theoretical ansatz[4]. At first glance, one may expect that the peak of the Z resonance is at $\sqrt{s_{\max}} = M_Z$. This would reduce the determination of M_Z to a search of the peak location. This intuitive picture is modified by several effects. As already mentioned, photonic initial state radiation is rather influential. One may estimate the resulting shift of the Z resonance peak from (17):

$$\sqrt{s_{\max}} - M_Z \;=\; \delta_{QED} = \frac{\pi}{8}\beta\Gamma_Z + \text{small corr's.} \approx 106\,\text{MeV} \qquad (21)$$

A simple and reasonable ansatz for $\sigma_0^{(Z)}$ is a pure Breit-Wigner function

$$\sigma_0^{(Z)}(s) \sim \frac{M_Z^2 \cdot R}{|s - M_Z^2 + iM_Z\Gamma_Z|^2} \qquad (22)$$

It may be shown (and will be made plausible in the next two sections) that the following ansatz is more realistic[5]:

$$\sigma_0(s) = \frac{4}{3}\pi\alpha^2 \left[\frac{r^\gamma}{s} + \frac{s \cdot R + (s - M_Z^2) \cdot J}{|s - M_Z^2 + is\Gamma_Z/M_Z|^2}\right] \qquad (23)$$

This line shape is characterized by five parameters:

- $r^\gamma \sim \alpha_{em}^2(M_Z^2)/\alpha_{em}^2$ – this γ exchange term may be assumed to be known

- M_Z, Γ_Z

- R – measure of the Z peak height

- J – measure of the γZ interference

Besides (21), from the replacements

$$M_Z^2 \cdot R \to s \cdot R, \quad iM_Z\Gamma_Z \to is\Gamma_Z/M_Z \qquad (24)$$

additional shifts arise:

$$\sqrt{s_{\max}} - M_Z \;=\; \delta_{QED} \oplus \frac{1}{4}\frac{\Gamma_Z^2}{M_Z} \ominus \frac{1}{2}\frac{\Gamma_Z^2}{M_Z} \sim (90 + 17 - 34)\;\;\text{MeV} \qquad (25)$$

Additionally, there is the effect of the γZ interference J:

$$\sqrt{s_{\max}} - M_Z \;=\; \delta_{QED} \oplus \frac{1}{4}\frac{\Gamma_Z^2}{M_Z}\left(1 + \frac{J}{R}\right) \ominus \frac{1}{2}\frac{\Gamma_Z^2}{M_Z} \qquad (26)$$

$$\sim \;\; \left[90 + 17 \times \left(1 + \frac{J}{R}\right) - 34\right]\;\;\text{MeV}$$

If one wants to take into account the J, a model for its prediction is needed. Neglecting this interference (by setting $J{=}0$) leads to an erroneous systematic shift of the Z mass of 17 MeV$\otimes(J/R)$.

The value for hadron production in the Standard Model is, e.g., [21]:

$$\frac{J}{R} \otimes 17\;\;\text{MeV} = \frac{0.22}{2.97} \otimes 17\;\;\text{MeV} = 0.07 \otimes 17\;\;\text{MeV} = 1.26\;\;\text{MeV} \qquad (27)$$

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[4] More details may be found in [17].

[5]This or similar formulae have been proposed in [18, 19, 7, 20].

5 Z Boson Parameters (I): The Standard Model Approach

A realistic scan of the Z line shape may be performed with the following ansatz derived from the Standard Model, including higher order radiative corrections [3]:

$$\sigma_0(s) \;=\; \sigma_0^{(Z)}(s) + \sigma_0^{(\gamma Z)}(s) + \sigma_0^{(\gamma)}(s) \tag{28}$$

The dominating part is

$$\sigma_0^{(Z)}(s) \;=\; \frac{4\pi\alpha^2}{3s}|\chi(s)|^2|\rho_{ef}^{eff}|^2 \left(\frac{1}{16} + \frac{1}{4}|v_e^{eff}|^2 + \frac{1}{4}|v_f^{eff}|^2 + |v_{ef}^{eff}|^2\right) \tag{29}$$

with an s-dependent width function in the Breit-Wigner shape (14). The width is obtained in perturbation theory by summing an infinite Dyson series of self-energy insertions to the Z boson propagator. In order to prevent gauge violation (in the given order of perturbation theory), one has to add up a minimal set of Feynman diagrams that is necessary for the compensation of gauge dependences. The decay width of the Z boson in lowest order is given by the imaginary parts of fermion loops in the one loop self-energy. For single Z boson production we may use[6]:

$$\Gamma_Z(s) \;=\; \frac{s}{M_Z^2}\Gamma_Z \tag{30}$$

The simple s dependence is due to the smallness of the fermion masses allowing for the neglect of threshold effects. A complete two-loop calculation would also modify this. The virtual corrections are contained in the width Γ_Z and in four complex-valued form factors ρ_Z^{eff}, v_e^{eff}, v_f^{eff}, v_{ef}^{eff} which depend on beam energy and scattering angle.

To a good approximation, (28) agrees with (23). The cross-section values have to be related to the free parameters of the theory. A recent determination is [22]:

$$M_Z \;=\; 91.188 \pm 0.002 \;\; \text{GeV} \tag{31}$$
$$\alpha_{strong}(M_Z^2) \;=\; 0.126 \pm 0.007 \pm 0.002 \;(\text{Higgs}) \tag{32}$$
$$m_t \;=\; 189 \pm 15 \;(\text{exp.}) \pm 16 \;(\text{Higgs}) \;\;\; \text{GeV.} \tag{33}$$

The Higgs mass is varied from 60 to 1000 GeV with central values for 300 GeV. Quite similar values have been quoted in [2]. For comparison I quote also the direct measurements of m_t by CDF, $m_t = 175.6 \pm 4.4 \pm 4.8$ GeV, and D0 collaborations, $m_t = 169 \pm 8 \pm 8$ GeV [23].

A step towards a model-independent Z resonance analysis is the determination of some characteristic line shape parameters from table 6 of [2] (with indicated relative errors δ):

$$M_Z \;=\; 91.1863 \pm 0.0020 \;\; \text{GeV} \quad (\delta = 0.0022 \;\%) \tag{34}$$
$$\Gamma_Z \;=\; 2.4946 \pm 0.0027 \;\; \text{GeV} \quad (\delta = 0.11 \;\%) \tag{35}$$

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[6] For the by far more complex case of off-shell gauge boson pair production, the fermion-loop scheme [10, 13] solves the gauge problem satisfactorily.

$$\sigma_0^{had} \; = \; 41.508 \pm 0.056 \;\; \text{nb} \quad (\delta = 0.13 \; \%) \tag{36}$$

$$R_l = \frac{\sigma_0^{had}}{\sigma_0^{lept}} \; = \; 20.778 \pm 0.029 \quad (\delta = 0.14 \; \%) \tag{37}$$

$$A_{FB,0}^{lept} \; = \; 0.0174 \pm 0.0010 \tag{38}$$

Here, $M_Z, \Gamma_Z, \sigma_0^{had}$ are determined mainly from $\sigma^{had}(s)$, while R_l and A_{FB} from $\sigma^{lept}(s)$:

- $\sigma_0^{had(lep)}$ – hadronic (leptonic) peak cross-section

- $A_{FB,0}^{lept}$ – forward-backward asymmetry at the peak

These parameters are considered to be primary parameters in contrast to derived ones, e.g. the effective leptonic weak neutral current couplings of leptons (table 8 of [2]):

$$v_l \; = \; -0.03776 \pm 0.0062 \tag{39}$$

$$a_l \; = \; -0.50108 \pm 0.00034 \tag{40}$$

or the effective weak mixing angle (tables 1,4 of [2]):

$$\sin^2 \vartheta_W^{eff} \equiv \frac{1}{4} \left(1 - \frac{v_l}{a_l} \right) \; = \; 0.23165 \pm 0.00024 \tag{41}$$

The introduction of effective weak neutral couplings and the effective weak mixing angle comes back close to the language of the Standard Model.

6 Z Boson Parameters (II): The S-Matrix Approach

All the above results fit nicely with each other and strengthen the Standard Model's credit. Nevertheless, one may ask for an approach being independent of it. A tool with minimal assumptions is S-matrix theory [24, 25, 27]. The first application of S-matrix theory to the Z resonance is due to A. Martin (1985) [27] who studied the toponium-Z interference pattern assuming their masses to be of similar size. In 1991, R. Stuart proposed to consider the scattering matrix element for the process $e^+ e^- \to Z \to f \bar{f}$ as a Laurent series with the Z boson as resonance [7]. This allowed him to collect gauge invariant pieces of the cross-section in perturbation theory [28] and to derive a simple cross-section formula similar to (22), but with small perturbations. For an application to experimental data, a number of modifications have been added [20, 29] and the necessary software has been created [30]: consider the cross-section as an incoherent sum of four helicity scatterings; treat the photonic corrections properly, especially those due to initial state radiation; treat in the same manner as the total cross-section also asymmetries; try to include into the formula the fact that there is also photon exchange, i.e. that in reality one has the co-existence of two resonances. The first fit to LEP 1 data was performed in [20].

Consider four independent helicity amplitudes in the case of massless fermions f:

$$\mathcal{M}^{fi}(s) = \frac{R_\gamma^f}{s} + \frac{R_Z^{fi}}{s - s_Z} + \sum_{n=0}^{\infty} \frac{F_n^{fi}}{\overline{m}_Z^2} \left(\frac{s - s_Z}{\overline{m}_Z} \right)^n, \quad i = 0, \ldots, 3 \tag{42}$$

Without the first (photon) term, they are Laurent series. The position of the Z pole in the complex s plane is given by s_Z:

$$s_Z = \overline{m}_Z^2 - i\overline{m}_Z \overline{\Gamma}_Z \tag{43}$$

The R_γ^f and R_Z^{fi} are complex constants characterizing the photon and the Z boson, respectively. For practical purposes one may truncate the series:

$$\mathcal{M}^{fi}(s) = \frac{R_\gamma^f}{s} + \frac{R_Z^{fi}}{s - s_Z} + \frac{F_0^{fi}}{\overline{m}_Z^2} \approx \frac{R_\gamma^f}{s} + \frac{R_Z^{fi}}{s - s_Z} \tag{44}$$

There are four residua R_Z^{fi}:

$$\begin{aligned}
R_Z^{f0} &= R_Z(e_L^- e_R^+ \longrightarrow f_L^- f_R^+) \\
R_Z^{f1} &= R_Z(e_L^- e_R^+ \longrightarrow f_R^- f_L^+) \\
R_Z^{f2} &= R_Z(e_R^- e_L^+ \longrightarrow f_R^- f_L^+) \\
R_Z^{f3} &= R_Z(e_R^- e_L^+ \longrightarrow f_L^- f_R^+)
\end{aligned} \tag{45}$$

The amplitudes $\mathcal{M}^{fi}(s)$ give rise to four cross-sections σ_i:

$$\begin{aligned}
\sigma_T^0(s) &= &= &\; + \sigma_0 + \sigma_1 + \sigma_2 + \sigma_3 \\
\sigma_{lr\text{-}pol}^0(s) &= \tfrac{4}{3}\sigma_{FB}^0(s) &= &\; + \sigma_0 - \sigma_1 + \sigma_2 - \sigma_3 \\
\tfrac{4}{3}\sigma_{FB\text{-}lr}^0(s) &= \sigma_{pol}^0(s) &= &\; - \sigma_0 + \sigma_1 + \sigma_2 - \sigma_3 \\
\sigma_{lr}^0(s) &= \tfrac{4}{3}\sigma_{FB\text{-}pol}^0(s) &= &\; - \sigma_0 - \sigma_1 + \sigma_2 + \sigma_3
\end{aligned} \tag{46}$$

Here, it is

σ_T^0 – the total cross-section,

σ_{FB}^0 – numerator of the forward-backward asymmetry,

σ_{pol}^0 – numerator of the final state polarization,

$\sigma_{FB\text{-}pol}^0$ – that of the forward-backward asymmetry of the final state polarization etc.

All these cross-sections may be parameterized by the following master formula:

$$\begin{aligned}
\sigma_A^0(s) &= \frac{4}{3}\pi\alpha^2 \left[\frac{r_A^{\gamma f}}{s} + \frac{s r_A^f + (s - \overline{m}_Z^2) j_A^f}{(s - \overline{m}_Z^2)^2 + \overline{m}_Z^2 \overline{\Gamma}_Z^2} + \frac{r_A^{f0}}{\overline{m}_Z^2} \right] + \ldots \tag{47} \\
&\approx \frac{4}{3}\pi\alpha^2 \left[\frac{r_A^{\gamma f}}{s} + \frac{s r_A^f + (s - \overline{m}_Z^2) j_A^f}{(s - \overline{m}_Z^2)^2 + \overline{m}_Z^2 \overline{\Gamma}_Z^2} \right], \quad A = T, \ldots,
\end{aligned}$$

where the $\sigma_{FB\text{-}\ldots}^0(s)$ again get an additional factor 3/4.

Thus we have re-derived (23) with one modification: the Z width function is treated

as constant here. Now, the parameters r, j are related to the residua of the pole terms. The $r_A^{\gamma f}$ is the photon exchange term:

$$r_A^{\gamma f} = \frac{1}{4}c_f \sum_{i=0}^{3}\{\pm 1\} \left|R_\gamma^f\right|^2 R_{QCD}^A \tag{48}$$

It is known from QED for the total cross-section ($A = T$) and vanishes for all asymmetric cross-sections. Further, $c_f = 1, 3$ for leptons and quarks, respectively. QCD corrections for quarks are taken into account by the factor R_{QCD}. The Z exchange residuum r_A^f and the γZ interferences j_A^f are:

$$
\begin{aligned}
r_A^f &= c_f \left\{ \frac{1}{4}\sum_{i=0}^{3}\{\pm 1\} \left|R_Z^{fi}\right|^2 + 2\frac{\overline{\Gamma}_Z}{\overline{m}_Z}\Im m C_A^f \right\} R_{QCD}^A \\
j_A^f &= c_f \left\{ 2\Re e C_A^f - 2\frac{\overline{\Gamma}_Z}{\overline{m}_Z}\Im m C_A^f \right\} R_{QCD}^A \\
C_A^f &= (R_\gamma^f)^* \left(\frac{1}{4}\sum_{i=0}^{3}\{\pm 1\}R_Z^{fi} \right)
\end{aligned}
\tag{49}
$$

The factors $\{\pm 1\}$ in (48) and (49) indicate that the signs of $\left|R_\gamma^f\right|^2$, $\left|R_Z^{fi}\right|^2$, and of R_Z^{fi} correspond to the signs of σ_i in (46).

6.1 Asymmetries

Without QED corrections, asymmetries are defined by:

$$\mathcal{A}_A^0(s) = \frac{\sigma_A^0(s)}{\sigma_T^0(s)}, \quad A \neq T \tag{50}$$

They take a simple form around the Z resonance. For applications at LEP 1, they may be characterized by only two parameters [29]:

$$\mathcal{A}_A^0(s) = A_0^A + A_1^A \left(\frac{s}{\overline{m}_Z^2} - 1 \right) + A_2^A \left(\frac{s}{\overline{m}_Z^2} - 1 \right)^2 + \ldots \approx A_0^A + A_1^A \left(\frac{s}{\overline{m}_Z^2} - 1 \right) \tag{51}$$

The higher order terms may be neglected since $(s/\overline{m}_Z^2 - 1)^2 = \sigma^2 < 2 \times 10^{-4}$. The first coefficients are:

$$
\begin{aligned}
A_0^A &= \frac{r_A^f}{r_T^f + \gamma^2 r_T^{\gamma f}} \approx \frac{r_A^f}{r_T^f} \\
A_1^A &= \left[\frac{j_A^f}{r_A^f} - \frac{j_T^f - 2\gamma^2 r_T^{\gamma f}}{r_T^f + \gamma^2 r_T^{\gamma f}} \right] A_0^A \approx \left[\frac{j_A^f}{r_A^f} - \frac{j_T^f}{r_T^f} \right] A_0^A
\end{aligned}
\tag{52}
\tag{53}
$$

Here, the r_A^{0f} is neglected in both A_0 and A_1. Further, the definition $\gamma^2 = \overline{\Gamma}_Z^2/\overline{m}_Z^2 \approx 0.75 \times 10^{-3}$ is used. The non-vanishing of the slope of the asymmetry shape is due to the γZ interference. From figure 4 one may see that the linear rise is damped right of the peak. This is due to amplified QED corrections to the pure Z exchange cross-sections versus non-amplified QED corrections to the γZ interferences [29].
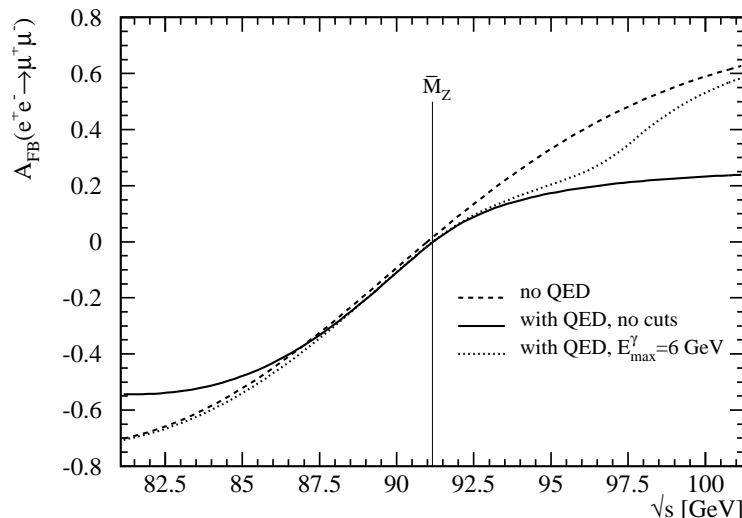
13

Figure 4: The forward-backward asymmetry for the process $e^+e^- \to \mu^+\mu^-$ near the Z peak [26].

6.2 Numerical Results in the S-matrix Approach

The first fit with the S-matrix approach to experimental data has been performed in 1991 [20]. The first experimental analysis of a LEP collaboration was due to L3 in 1993 [31]. Further systematic studies may be found in [32]. They helped to determine the appropriate number and location of energy points for a Z line shape scan. Recent experimental studies are e.g. [33, 34, 21, 35, 22]. Typically, results as in table 1 are obtained from the LEP1 and LEP1.5 Z line shape scans which were performed mainly in 1993 and 1995 (from table 6 of [21][7]). The biggest error correlations are shown in table 2 (from table 7 of [21]). We see here an essential difference to Standard Model fits which assume *fixed* relations among many of these parameters. They rely thus on stronger theoretical assumptions.

From the strong correlations in the S-matrix fit together with the excellent agreement of the central values of fitted parameters in both fit scenarios one may conclude that the two scenarios are highly compatible with each other.

Including into the analysis cross-sections measured at other energies may improve substantially e.g. the resolution of M_Z and j_T which are highly correlated (for a combination with data from the TOPAZ collaboration at KEK with $\sqrt{s} \sim 55$ GeV as shown in figure 1 see reference [36]; data from LEP 1.5 with $\sqrt{s} \sim 135$ GeV have been included already).

---

[7]Note that the table shows values of the on shell mass M_Z which were derived after the fit of the complex pole mass $\overline{m}_Z$ as explained in section 7.

Parameter	S-matrix fit	SM Prediction
M_Z [GeV]	91.1965±0.0048	–
Γ_Z [GeV]	2.4941±0.0033	2.4973
r_T^{had}	2.9644±0.0083	2.9681
j_T^{had}	0.22±0.25	0.22
r_T^{lept}	0.14245±0.00044	0.14268
j_T^{lept}	0.020±0.017	0.004
r_{FB}^{lept}	0.00315±0.00022	0.00271
j_{FB}^{lept}	0.793±0.016	0.799

Table 1: *Results from a combined LEP1 line shape fit*

Correlation	Value
M_Z–j_T^{had}	−0.89
M_Z–j_T^{lept}	−0.62
Γ_Z–r_T^{had}	0.77
Γ_Z–r_T^{lept}	0.69
r_T^{had}–r_T^{lept}	0.86
j_T^{had}–j_T^{lept}	0.62

Table 2: *Biggest correlations in the S-matrix fit*

7   Defining the Z Boson Mass

The complex Z pole definition in (42) with a constant width is natural in the S-matrix ansatz. It leads to different numerical values compared to the usual Standard Model, on mass shell approach as used in (28)–(30). The following discussion of this difference follows closely appendix D of [13] where more details may be found.

In the perturbative approach, the *complex pole* μ_Z of the propagator is defined as follows:

$$\mu_Z - \mu_Z^0 + \Sigma_Z^0(\mu_Z, \mu_Z^0) \;=\; 0 \tag{54}$$

$$\mu_Z \;=\; \overline{m}_Z^2 - i\overline{m}_Z\overline{\Gamma}_Z \tag{55}$$

The bare Z boson mass is denoted by μ_Z^0 and Σ_Z^0 is the bare self-energy. The perturbative solution of the above equations is:

$$\overline{m}_Z^2 \;=\; \mu_Z^0 - \Re e\Sigma_Z^0(\overline{m}_Z^2) - \left[\Im m\Sigma_Z^0(\overline{m}_Z^2)\right]\left[\Im m\Sigma_Z^{0\prime}(\overline{m}_Z^2)\right] + \ldots \tag{56}$$

$$\overline{m}_Z\overline{\Gamma}_Z \;=\; \Im m\Sigma_Z^0(\overline{m}_Z^2)\Bigg\{1 - \Re e\Sigma_Z^{0\prime}(\overline{m}_Z^2) + \left[\Re e\Sigma_Z^{0\prime}(\overline{m}_Z^2)\right]^2 \tag{57}$$

$$- \frac{1}{2}\left[\Im m\Sigma_Z^0(\overline{m}_Z^2)\right]\left[\Im m\Sigma_Z^{0\prime\prime}(\overline{m}_Z^2)\right] + \ldots\Bigg\}$$

The *on shell mass and width* are defined as:

$$M_Z^2 \;=\; \mu_Z^0 - \Re e\Sigma_Z^0(M_Z^2) \tag{58}$$

$$M_Z \Gamma_Z = \frac{\Im m \Sigma_Z^0(M_Z^2)}{1 + \Re e \Sigma_Z^{0\prime}(M_Z^2)} \tag{59}$$

One may relate the two definitions and see that they differ by two-loop and higher order corrections. This has been discussed first in 1986 [6]. The expected experimental accuracy was about 10 MeV [37] at that time. Since the authors of [6] restricted themselves to the one loop order, they failed to observe the numerical significance of the difference in the definitions of about 35 MeV. There are also bosonic corrections in one-loop approximation. A systematic Dyson summation of bosonic self-energy corrections may be attempted without violating Ward identities in the context of the background field method [38]. For references to the application of so-called pinch techniques see also there. Since the on shell mass and the complex mass definitions are uniquely related order by order in perturbation theory, it is fair to say that either both or none of them has a gauge invariance problem, provided it is used properly. One may argue that a mass definition should be related to a structure like $\text{Const}/(s - s_0)$, with s_0 being a constant, but as long as relations are unique to another definition, there is some freedom of choice. Of course, the (perturbative) complex mass definition is conceptually closest to what one has in the S-matrix theory.

Around $s = M_Z^2$ the Z decays only into light fermions and it is

$$\Im m \Sigma_Z^0(s) = s \, \Im m \Sigma_Z^{0\prime}(s) = s \, \frac{\Gamma_Z}{M_Z} \tag{60}$$

and thus

$$\overline{m}_Z^2 = M_Z^2 - \Gamma_Z^2 + \ldots \tag{61}$$

$$\frac{\overline{\Gamma}_Z}{\overline{m}_Z} = \frac{\Gamma_Z}{M_Z} \tag{62}$$

The resulting numerical differences may well be approximated by the following relations:

$$
\begin{aligned}
\overline{m}_Z &= M_Z - \frac{\Gamma_Z^2}{2M_Z} &\approx& \quad M_Z &-& \quad 34 \text{ MeV} \\
\overline{\Gamma}_Z &= \Gamma_Z - \frac{\Gamma_Z^3}{2M_Z^2} &\approx& \quad \Gamma_Z &-& \quad 1 \text{ MeV.}
\end{aligned}
\tag{63}
$$

Equations (63) and the numerical values of the shifts were derived in 1988 in [12], where also the γZ mixing was taken into account, and repeatedly discussed later (see e.g. [39, 8, 7, 40, 41]). The Z resonance peak shift due to the difference of the two treatments of the Z boson self energy was numerically observed independently in [11] and in [12]. Both papers did not point out the significance of the complex pole mass definition, although it became obvious immediately after and was frequently discussed during the 1989 LEP 1 workshop organized by CERN. Similar derivations to the above may be found in [6, 8, 7, 9]. Gauge problems have been studied also in [44].

The observation of the sensitive dependence of numerical mass values on the definitions chosen in the Breit-Wigner shape function was made for hadron resonances by Gounaris and Sakurai (1968) [42]. See also the recent studies of hadron resonances reviewed in [5].

8   The Z Resonance and the Photon

Being strict, one may develop the S-matrix into a Laurent series as a function of s around *one* resonance only. Otherwise the coefficients are not uniquely determined. This has been stressed recently [43].

In order to be rigorous, one has to replace the ansatz (42) by one with $R_\gamma^f = 0$. The essential physical consequence is that the photonic cross-section becomes part of the background:

$$\frac{r^\gamma}{s} \to \frac{r^\gamma}{M_Z^2} \tag{64}$$

What was understood so far as γZ interference J becomes a result of the background-Z interference.

I, personally, dislike this approach. The photon exists and we know how to describe it. So, I would prefer to see it treated as known input to an experimental analysis of the Z. But I agree that a detailed study of the resulting numerical differences between the rigorous S-matrix ansatz and that used by the LEP community at present could be of some interest. Perhaps it is worth to be mentioned that a consistent quantum mechanical description of two-resonance systems is possible; see e.g. chapter XX in [25].

9   Summary

Two different approaches to a numerical analysis of the Z boson line shape have been presented – the Standard Model of electroweak interactions and the S-matrix approach. The S-matrix approach allows to treat the γZ interference as an independent quantity, which enlarges the error for M_Z. Two different mass definitions may be used. Both agree in the numerical determination of the Z mass when the substantial difference in the mass definitions is taken into account. In the Standard Model the Z width is a derived quantity; the S-matrix approach allows a direct fit. Again, the two approaches agree numerically. The S-matrix approach shows that the Z line shape may be described by 4 independent parameters (per channel) – if QED is assumed to be a known phenomenon. Asymmetries may also be described by the S-matrix approach. They depend on two parameters (per channel). Their variation with s near the peak is due to the γZ interference. The S-matrix approach allows the combination of data from quite distinct kinematic regions. Finally, one should mention that the use of effective coupling constants leads to similarly reasonable numerical results.

Acknowledgements

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