Spatial kriging of radiation data

(Statistics for national security)

Alex Reinhart

Applied Research Labs

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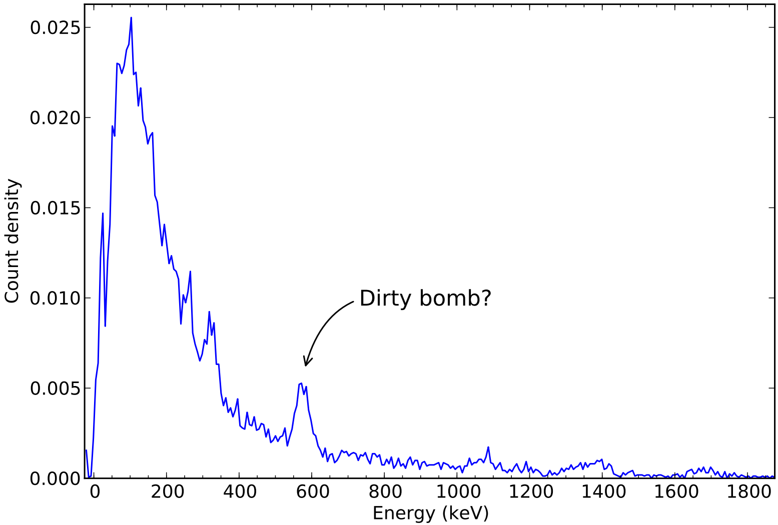

Figure: Gamma ray spectrum collected at Pickle Research Campus.

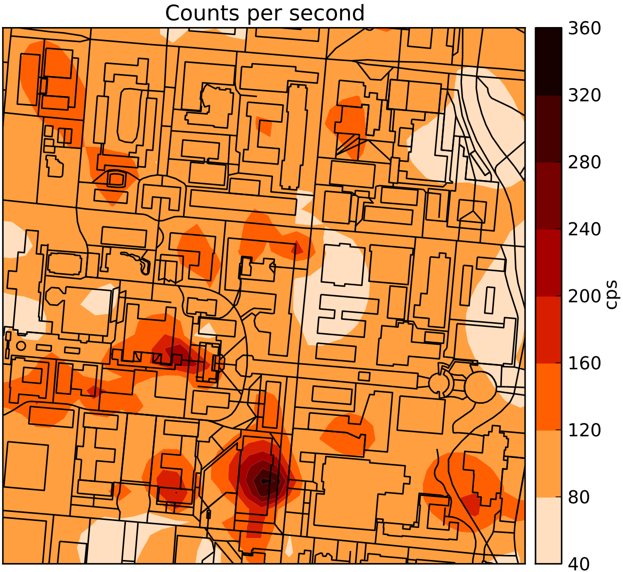

Figure: The University of Texas campus radiation map.

Kriging model

The model:

$$Z(\mathbf{s})|Y(\mathbf{s}) \sim \text{Poisson}(t(\mathbf{s})Y(\mathbf{s})) \tag{1}$$

$$C_Y(\mathbf{s}, \mathbf{s}') = \sigma_Y^2 - \gamma_Y(\mathbf{s}, \mathbf{s}') \tag{2}$$

We assume we can predict count rates with

$$\hat{Y}(\mathbf{s}_0) = \sum_{\alpha=1}^{n} \lambda_\alpha \frac{Z(\mathbf{s}_\alpha)}{t(\mathbf{s}_\alpha)}. \tag{3}$$

Kriging system

$$\sum_{\beta=1}^{n} \lambda_\beta C(\mathbf{s}_\alpha, \mathbf{s}_\beta) + \lambda_\alpha \frac{m}{t(\mathbf{s}_\alpha)} + \mu = C(\mathbf{s}_\alpha, \mathbf{s}_0) \qquad \text{for } \alpha = 1, \ldots, n \tag{4}$$

$$\sum_{\alpha=1}^{n} \lambda_\alpha = 1 \tag{5}$$

where m is the mean of Y and μ is a Lagrange multiplier.

Anomaly detection

- Estimate the variogram using a model:

$$\gamma_Y(h) = c\left(1 - \exp\left(-\left(\frac{h}{a}\right)^d\right)\right) \tag{6}$$

- Collect background data and use it to make a set of predictions
- Compare new data (aggregated into small spatial bins) to old predictions

Anomaly detection

$$Z(\mathbf{s}) | \hat{Y}(\mathbf{s}) t(\mathbf{s}) \sim \text{Poisson}(\hat{Y}(\mathbf{s}) t(\mathbf{s})) \tag{7}$$

$$\hat{Y}(\mathbf{s}) t(\mathbf{s}) \sim \text{Gamma}(a, (1-b)/b) \tag{8}$$

$$a = \frac{\hat{Y}^2}{\sigma^2_{\hat{Y}}} \tag{9}$$

$$b = \frac{t(\mathbf{s})\sigma^2_{\hat{Y}}}{\hat{Y} + t(\mathbf{s})\sigma^2_{\hat{Y}}} \tag{10}$$

Hence we can integrate (giving us a negative binomial) and determine $P(Z)$ to compare new data against.