Measuring Conceptual Understanding Using Comparative Judgement

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Plan

- Conceptual Understanding
- Measuring conceptual understanding
- Study 1: Statistics (UG)
- Study 2: Derivative (UG)
- Study 3: Algebra (Year 7)

Procedural and conceptual

- **Procedural knowledge**
  - ability to execute actions to solve problems
  - tied to specific problems, not generalisable (Rittle-Johnson, Siegler & Alibali, 2001)
- **Conceptual understanding**
  - network of relationships between pieces of information in a domain (Hiebert & Lefevre, 1986)
  - understanding of fundamental principles (Rittle-Johnson et al., 2001)

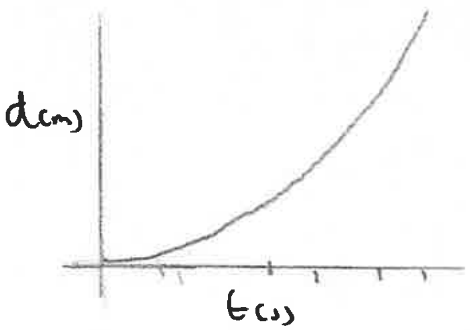
Measuring conceptual

- Evaluate teaching interventions (RCTs)
    - Clinical interviews (Piaget)
    - Instruments (implicit, explicit)
        - Difficult to define conceptual understanding
        - Time consuming
        - Resource intensive
        - Research based

A comparative judgement approach

Explain what a **derivative** is to someone who hasn't encountered it before. Use diagrams, examples and writing to include everything you know about derivatives. **Write only in the box below.**

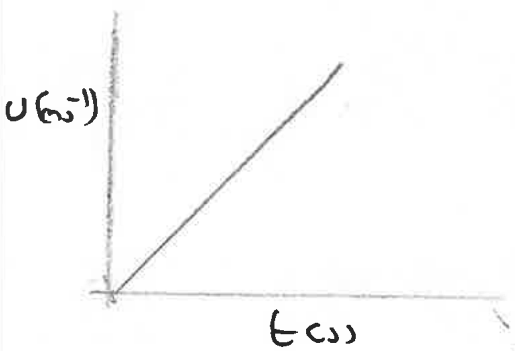
A derivative in ~~physical~~ algebraic terms is the change in variable x, over change in variable y. For example if we think of a car accelerating and plot a graph ~~at~~ for the distance covered ~~in each~~ per unit ~~each second~~ time.



We can see that the car is accelerating as the gradient is increasing per unit time this therefore means there is a larger increase in y per change in x, which is a derivative. So by finding the derivative of the equation we can find another variable or for this case the cars velocity at time t.

$x = s$ $\quad \dfrac{\Delta y}{\Delta x} = \dfrac{dy}{dx} = \dfrac{m}{s} = ms^{-1} = v$
$y = m$

We can differentiate it again to find another variable which is relevant to the problem for this case the case acceleration.



For the graph on the left the acceleration is constant hence the straight line and can be differentiated to find the gradient or acceleration of the car.

Can also be used to find the minimum & ~~maximum~~ maximum velocities or the ~~it~~ the graph is it steady.

$x = s$ $\quad \dfrac{\Delta y}{\Delta x} = \dfrac{ds}{dx} = \dfrac{ms^{-1}}{s} = ms^{-2} = a$
$y = ms^{-1}$

If an equation ~~it~~ models the cars displacement ~~per unit time~~ can be modelled as

$$s = x^2$$

then by differentiating we can find the equation for its velocity

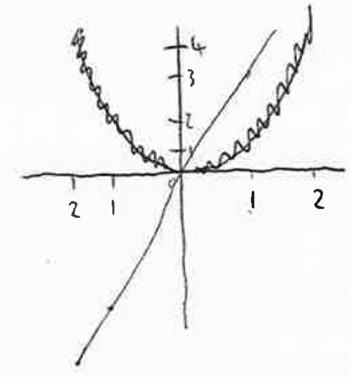
$$\frac{ds}{dx} = v = 2x$$

again for acceleration

$$\frac{dv}{dx} = a = 2$$

we can therefore put into the equation values for x to find the values for s, v and a.

Take a function of x: $f(x) = 3x$



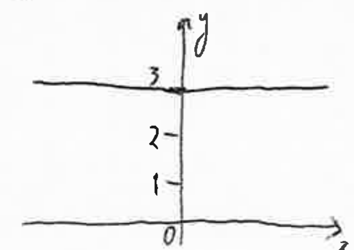
and the concept that a derivative is a change in something with respect to something else.

x	1	2	3	4
$f(x)=3x$	3	6	9	12

Note that the change between each $f(x)$ for each x value is 3

~~You can then~~

The change in y on the graph with respect to x is always 3 in this case. Plot $y = 3$.



The derivative of $f(x) = 3x$ is $f'(x) = 3$.

$$y = 3x$$

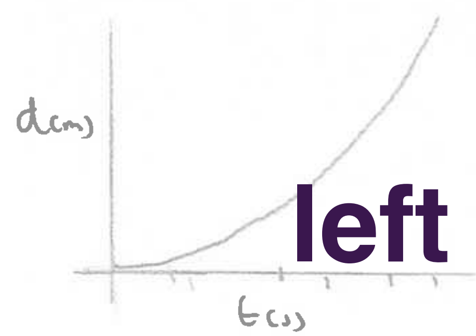
Change in y $\rightarrow$ $\dfrac{dy}{dx} = 3$
with respect to x $\rightarrow$

This can be applied to all functions, and you can find the change in y with respect to x (the gradient) at various points

Comparative Judgement

left

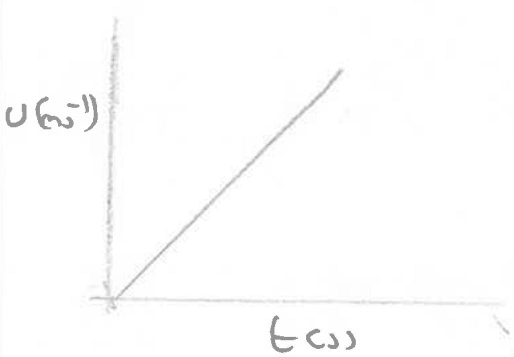
A derivative in ~~physical~~ algebraic terms is the change in variable x, over change in variable y. For example if we think of a car accelerating and plot a graph for the distance covered per unit time.



We can see that the car is accelerating as the gradient is increasing per unit time this therefore means there is a larger increase in y per change in x, which is a derivative. So by finding the derivative of the equation we can find another variable for this case the cars velocity at time t.

$x=s$
$y=m$
$\frac{\Delta y}{\Delta x}=\frac{dy}{dx}=\frac{m}{s}=ms^{-1}=v$

We can differentiate again to find another variable which is relevant to the problem for this case the case acceleration. For the graph on the left the acceleration is constant hence the straight line and can be differentiated to find the gradient or acceleration of the car. Can also be used to find the minimum or maximum velocities of the graph isn't steady.



$x=s$
$y=ms^{-1}$
$\frac{\Delta y}{\Delta x}=\frac{dy}{dx}=\frac{ms^{-1}}{s}=ms^{-2}=a$

If an equation models the cars displacement ~~per unit time~~ can be modelled as

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$$\frac{ds}{dx}=v=2x$$

again for acceleration

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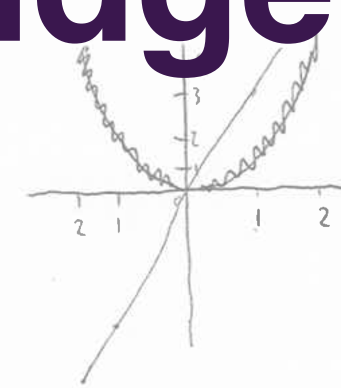
we can therefore put into the equation values for x to find the values for s, v and a.

right

Take a function $f(x)=3x$ and the concept that a derivative is a change in something with respect to something else.



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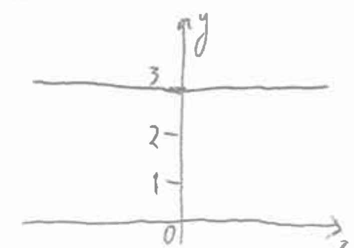


The derivative of $f(x)=3x$ is $f'(x)=3$.

Change in y → $y=3x$
with respect to x → $\frac{dy}{dx}=3$

This can be applied to all functions, and you can find the change in y with respect to x (the gradient) at various points

Which is best?

Advantages

- Quick and efficient
- Relative not absolute judgements
  (Thurstone 1927; Laming 2004)
- Collective expertise defines conceptual understanding
- Promising for measuring difficult-to-specify constructs

Questions

- Does it really assess what it's supposed to?
- Can we trust that the judges know what a "good answer" is?
- It is feasible to use with different age groups?

Study 1: UG Statistics

- Conceptual understanding of *p* values
- 20 UGs on an Applied Statistics module
- Completed:
  - CJ question
  - 13 items from RPASS scale
    (Lane-Getaz, 2013)
- 10 judges (psychology PhDs)

- CJ question

Explain what a ***p*-value** is and how it is used to someone who hasn't encountered it before. You can use words, diagrams and examples to make sure you explain everything you know about *p*-values. Write between half a page and one page.

- Example RPASS item

Scenario 1:

A research article reports that the mean number of minutes students at a particular university study each week is approximately 1000 minutes. The student council claims that students are spending much more time studying than this article reported. To test their claim, data from a random sample of 81 students is analysed using a one-tailed test. The analysis produces a *p*-value of .048.

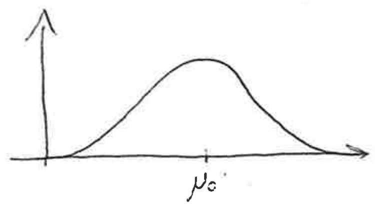
Question 1.1 Assume a student had conducted a two-tailed test instead of a one-tailed test on the same data, how would the *p*-value (.048) have changed?

a. The two-tailed *p*-value would be smaller (i.e., the *p*-value would be .024).
b. The two-tailed *p*-value be the same as the one-tailed (i.e., the *p*-value would be .048).
c. The two-tailed *p*-value would be larger than the one-tailed (i.e., the *p*-value would be .096

Explain what a **p-value** is and how it is used to someone who hasn't encountered it before. You can use words, diagrams and examples to make sure you explain everything you know about *p*-values. Write between half a page and one page.

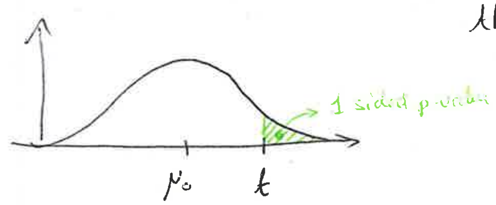
A p-value is the probability to get a result as or more unlikely that the one we got. It helps to assess if our null hypothesis (assumption about the statistic) is true.

For example, here is the underlying distribution of a statistic:



Where μ_0 is the mean of the statistic under the null hypothesis H_0.

If we get a result like "t":

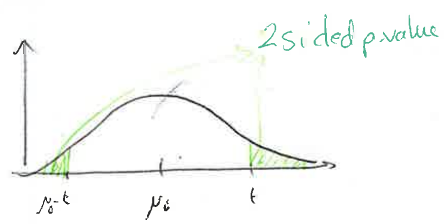


the p-value associated is the green area.

Ex: if the statistic is the height mean in a class, the p-value is the probability that we had had people taller than t on average.

We see that the closer we are from the mean μ_0, the bigger will be the p-value. Whereas, if we are far from the mean, we will end up with a small p-value. Generally, if the p-value is less than 0,05 (significant level), then it means that we had really few chances to get such a result under the H_0, an that we should revise our assumption.

Here was an example with a 1-sided p-value, But sometimes we are interested in a 2-sided p-value like below:



When having t is quite the same as having μ_0-t.

Ex: When you throw a coin, having 1 head on 10 flips means the same as having 1 head on 10 flips.

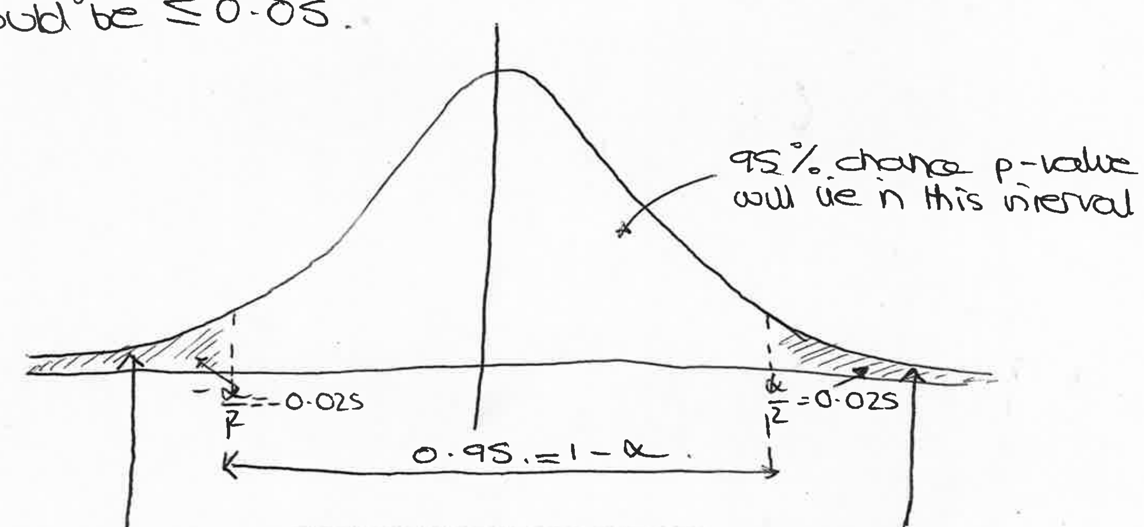
P1

Explain what a **p-value** is and how it is used to someone who hasn't encountered it before. You can use words, diagrams and examples to make sure you explain everything you know about *p*-values. Write between half a page and one page.

A p-value is used to determine whether we accept or reject our null hypothesis. A confidence interval will be provided, which is $(1-\alpha)\%$. If our p-value is less than or equal to α we accept the null hypothesis, ie significant, and if ~~the~~ the p-value is greater than α we reject the null hypothesis, ie insignificant.

The null hypothesis is what we are trying to determine ~~is true~~ to be true, and the other hypothesis if we reject our null hypothesis using our p-value is called the alternative hypothesis (what we are trying to determine is false).

Below is a diagram of a standard normal distribution, distributed along the 95% confidence interval, ie there is a 95% chance the p-value will lie in this interval leading to accepting the null hypothesis, so our p-value should be ≤ 0.05.



We reject the null hypothesis if our p-value lies in either one of these shaded regions.

P16

Results

- CJ reliability
  - Internal consistency measure = .882
  - Inter-rater reliability (split-halves median) = .762
- RPASS, Cronbach α = .539
- CJ validity
  - CJ vs. RPASS, $r = .721$ (attenuation correction)
  - CJ vs. module scores, $r = .555$, $p = .021$
  - (RPASS vs. module scores, $r = .553$, $p = .021$)

Study 2: UG Calculus

- Conceptual understanding of derivatives
- 42 UGs on an Mathematical Methods in Chemical Engineering module
- Completed:
    - CJ question
    - 10 items from Calculus Concept Inventory (Epstein, 2007)
- 30 judges (mathematics PhDs)

- CJ question

Explain what a **derivative** is to someone who hasn't encountered it before. Use diagrams, examples and writing to include everything you know about derivatives. **Write only in the box below.**

- Example Calculus Concept Inventory item

ANONYMISED

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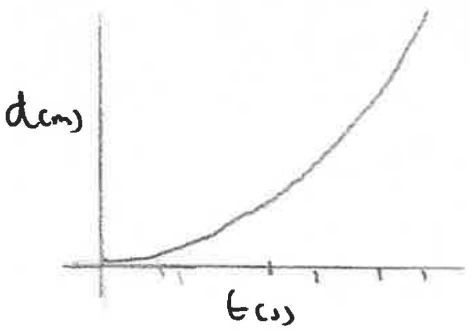

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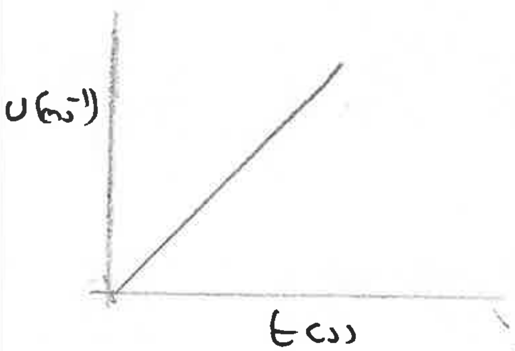

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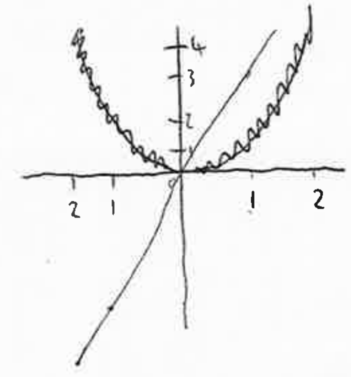

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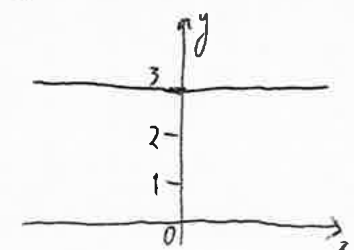

The derivative of $f(x) = 3x$ is $f'(x) = 3$.

$$y = 3x$$

Change in y → dy
with respect to x → dx

$$\frac{dy}{dx} = 3$$

This can be applied to all functions, and you can find the change in y with respect to x (the gradient) at various points

Results

- CJ reliability
  - Internal consistency measure = .938
  - Inter-rater reliability (split-halves median) = .871
- CCI, Cronbach α = .397
- CJ validity
  - CJ vs. CCI, Pearson's $r = .093$, $p = .568$
  - CJ vs. module scores, $r = .365$, $p = .021$
  - (CCI vs. module scores, $r = .277$, $p = .083$)

Judge group differences

- 3 groups of 10 judges
- Group 1 received guidance
- Groups 2 & 3 received no guidance
- $r_{12} = .849$, $r_{13} = .803$, $r_{23} = .898$
- ANOVA on misfit figures between groups, $F(2, 27) = 1.16$, $p = .328$

Study 3: Year 7 Algebra

- Conceptual understanding of letters in algebra
- 46 Year 7s
- Completed:
  - CJ question
  - 15 items from Algebra CSMS
    (Hart et al., 1981)
- 10 judges (mathematics PhDs)

- CJ question

Explain how letters are used in algebra to someone who has never seen them before. Use examples and writing to help you give the best explanation that you can. Write only in the box below.

- Example Algebra CSMS items

1. Write down the smallest and the largest of these: smallest .... largest ....

$n + 1$, $n + 4$, $n - 3$, n, $n - 7$

2. Which is larger, $2n$ or $n + 2$? ....

Explain: ....

In algebra they use letters to replace numbers so that it's easier to work out different equations.

Say if $x = 6$
$y = 7$
$z = 8$

$x + y + z = \underline{21}$

You can also square the letter: $x = 6$

x^2 = so this means $6 \times 6 = 36$.

You can times the ~~number~~ letter aswell: $x = 6$

$3x$ = this means $3 \times x = 3 \times 6 = 18$.

In Algebra you use letters because in Algebra letters represent the numbe e.g. $2n + 1n = 3n$ this shows that there ar 2 n's and 1n which add up to three n so the letters represent how many numbers there are to the letters so 2n means there are 2n's and 3x means there are 3x's.

Results

- CJ reliability
  - Internal consistency measure = .843
  - Inter-rater reliability (split-halves median) = .742
- CSMS, Cronbach $\alpha = .770$
- CJ validity
  - CJ vs. CSMS, Pearson's $r = .428$, $p = .003$
  - CJ vs. maths level, $r = .440$, $p = .002$
  - (CSMS vs. maths level, $r = .555$, $p < .001$)

Discussion

- Does it really assess what it's supposed to?
- Can we trust that the judges know what a "good answer" is?
- It is feasible to use with different age groups?

Summary

		Study 1 Statistics	Study 2 Calculus	Study 3 Algebra
Comparative Judgement				
Validity	Instrument	**.721**	**.093**	**.428**
	Achievement	**.555**	**.438**	**.440**
Reliability	Internal	**.882**	**.938**	**.843**
	Inter-rater	**.749**	**.869**	**.745**
Instrument				
Validity	Achievement	**.553**	**.277**	**.448**
Reliability	Cronbach	**.539**	**.397**	**.770**

Thank you

Funded by the Nuffield Foundation.

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