A conceptual approach to assessing mathematics

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Rasch Day

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How to make a maths exam

number

algebra

geometry

statistics

number

problem solving

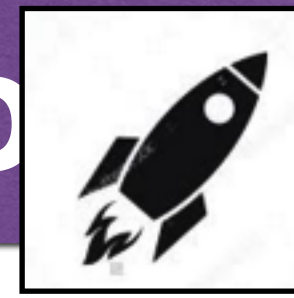
algebra

geometry

statistics

functional maths

nu 2 + 3 ber

pro  m solving

alge

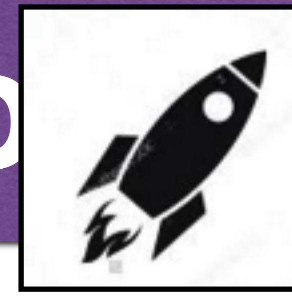
$a > c + 1$

geom sin θ y

stati P(A) = ½

func 30% discount aths

2 + 3



Procedural knowledge

“execute actions to solve problems, not generalisable”

(Rittle-Johnson, Siegler & Alibali, 2001)

$P(A) = \frac{1}{2}$

30% discount

number

Conceptual understanding

“fundamental principles,
network of relationships”

(Hiebert & Lefevre, 1986; Rittle-Johnson et al., 2001)

statistics

Left | Better understanding of "equation"? | 16 | Right

Question

What is an equation? Give examples of how equations can be useful.

An equation is like a sentence but ~~is~~ mathmatical and contains numbers and algebra. e.g $9+10=19$

$a=10$
$B=5$
$C=1$

$a+b=15$ $a \times b=50$
$b+c=6$ $b \times c=5$
$A+C=11$ $c \times A=10$
$A \div B=2$ $a-b=5$
$b-c=4$
$A-C=9$

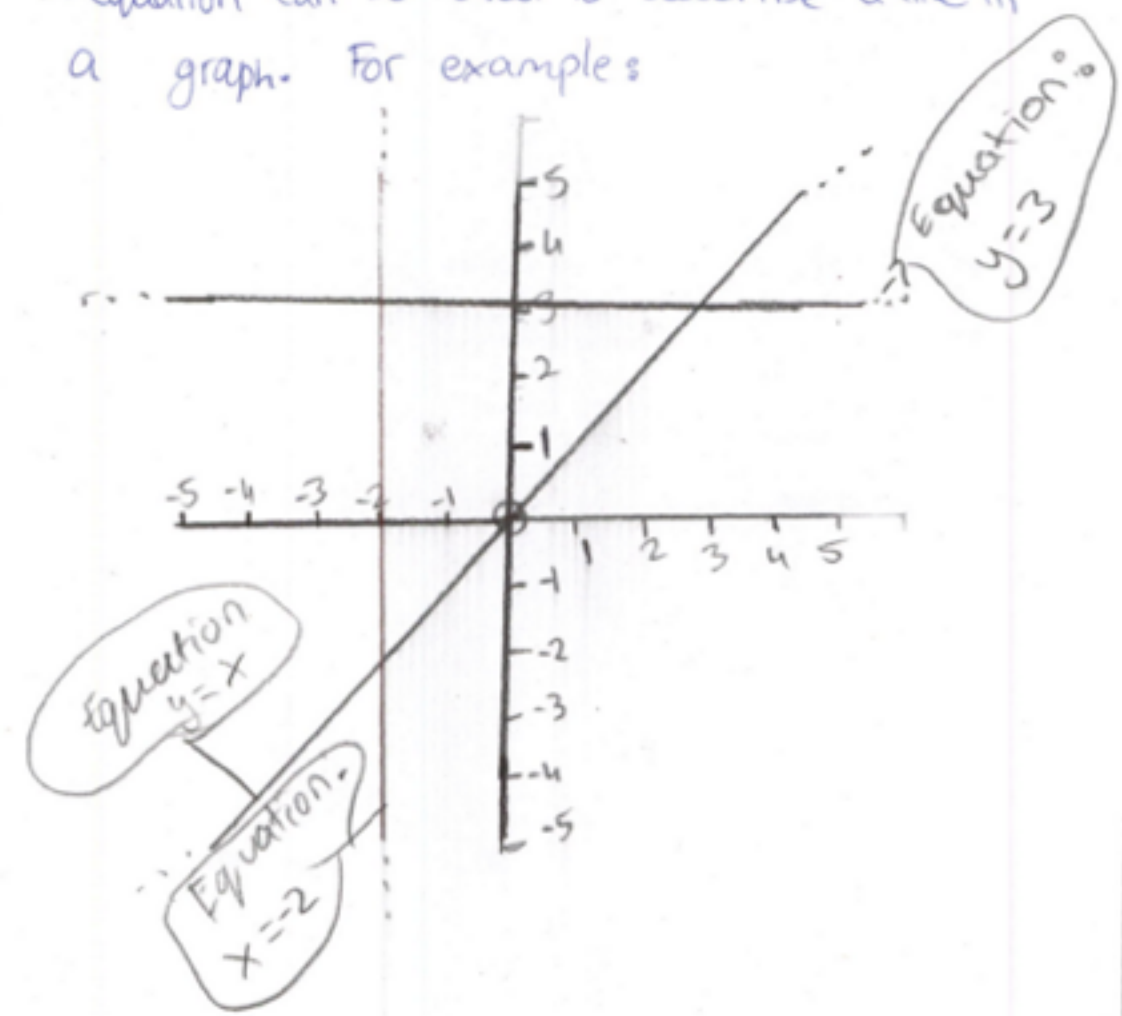
$a+4=14$

They can be useful in ~~big~~ our daily Lives to work out the cost of Something you buy or to see ~~which of~~ which thing is cheaper or more ~~expen~~ exspensive or to see which weighs more or less. Those are the few reasons of why equations are useful.

Question

What is an equation? Give examples of how equations can be useful.

An equation can be used to describe a line in a graph. For examples



Conceptual understanding

- Undergraduate calculus, statistics
  Bisson et al. (2016)
  Jones & Alcock (2014)
- Secondary algebra, calculus, fractions
  Bisson et al. (2016)
  Bisson et al. (in progress a)
  Jones et al. (2013)
- Primary algebra
  Bisson et al. (in progress b)
- Conceptual rather than procedural
  Bisson et al. (in progress a, b)
  Jones et al. (2013)

Can we infer general mathematical achievement from sampling concepts?

The better mathematician? 1082

Question

Write instructions to a friend on how to estimate the number of circles in the diagram. If you write more than one method say which is best and why.

You could just plain old count the circles and colour over the ones you've counted so you dont ~~&~~ count it again.

Or you could trap some circles into sections so when you count the sections just add them and its done.

The second pair of instructions I gave was better because it make it more simple and (or) accurate.

Question 2

Give examples of large fractions and small fractions. Show how some are larger than others.

$\frac{1}{2}$ =

$\frac{1}{4}$ = = so $\frac{1}{2}$ is bigger than $\frac{1}{4}$.

$\frac{3}{6}$ = they are both the same they are $\frac{1}{2}$.

$\frac{5}{8}$ = $\frac{4}{8}$

= so $\frac{5}{8}$ is bigger than $\frac{3}{6}$.

$\frac{2}{6}$ =

$\frac{3}{3}$ = ~~so~~ = so $\frac{3}{3}$ (1 hole) is bigger than $\frac{5}{7}$ because one hole is all of it.

$\frac{5}{7}$ =

$\frac{1}{8}$ = = so $\frac{1}{6}$ is bigger because $\frac{1}{8}$ is smaller.

$\frac{1}{6}$ =

The study

- 1887 Year 7 pupils across 10 schools
- Tested in September and June
- Judged in September and June
- Three questions at each time

The tests

What is an equation? Give examples of how equations can be useful.

Why do we need negative numbers? Give examples of how negative numbers can be useful.

Write one or more maths questions for a friend to solve that involves finding the area of a shape.

Set 1

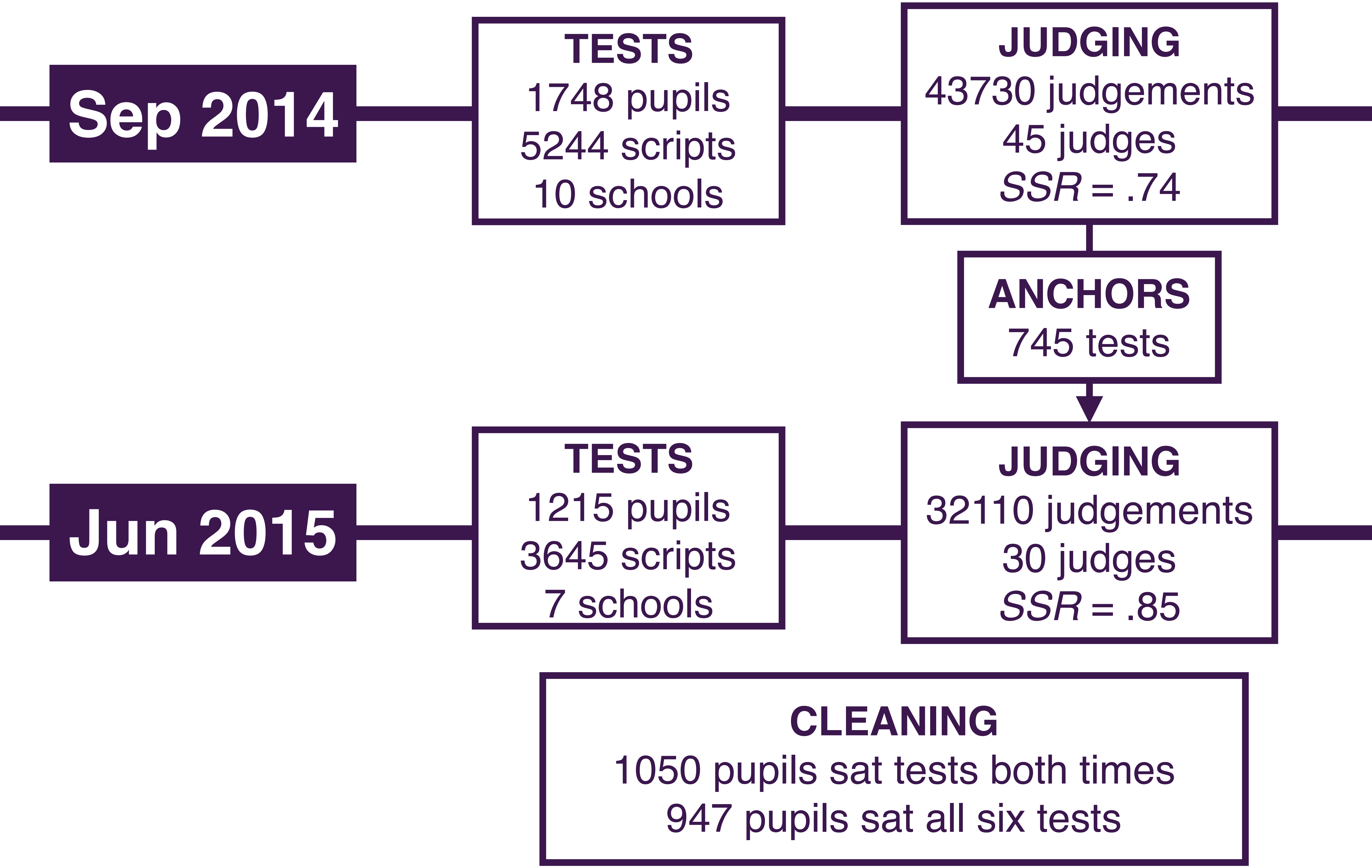
Give examples of large fractions and small fractions. Show how some are larger than others.

How can number lines can be useful for working out sums? Give examples of sums worked out using a number line.

Write instructions to a friend on how to estimate the number of circles in the diagram. If you write more than one method say which is best and why.

Set 2

The data



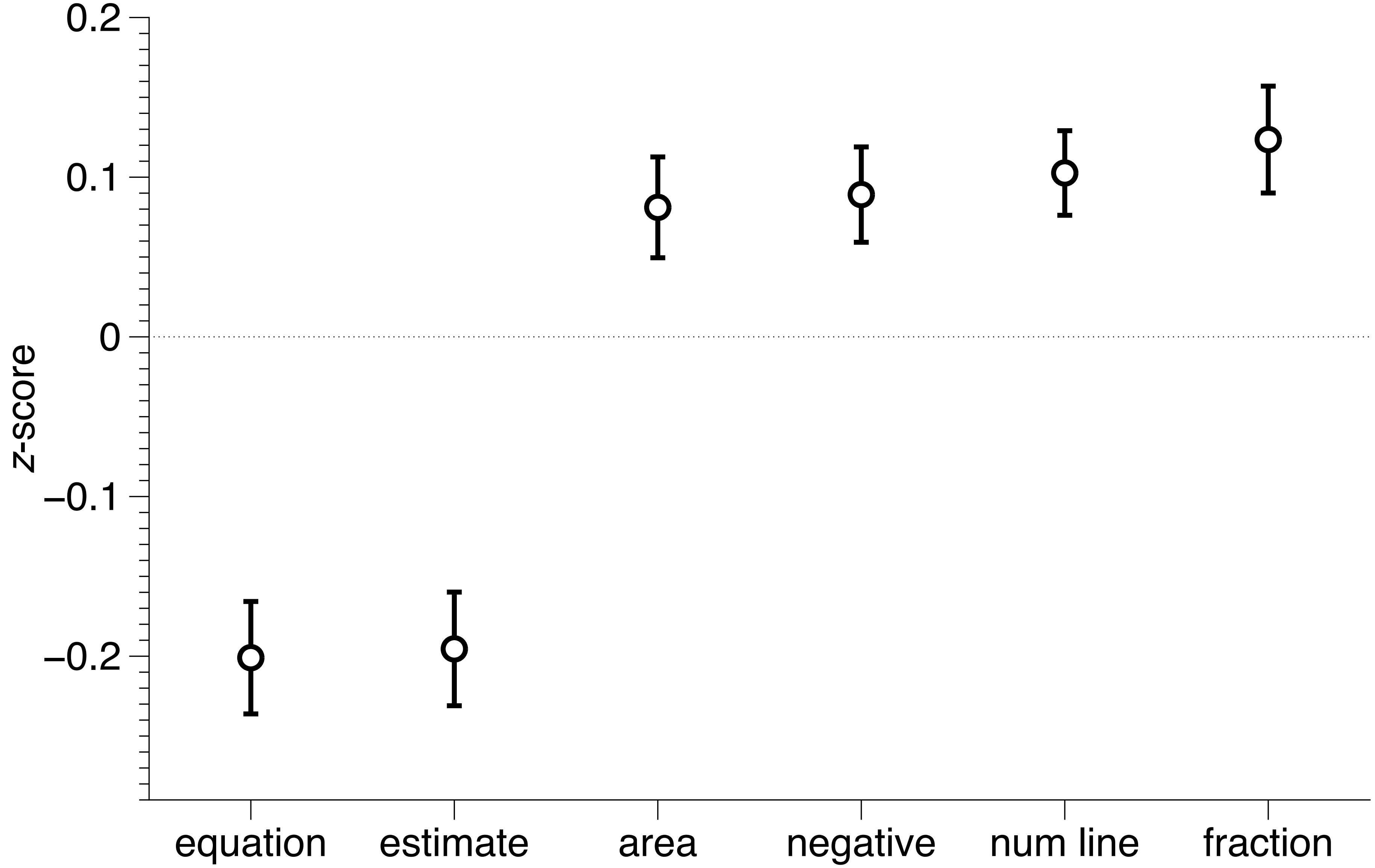
Sep 2014
TESTS
1748 pupils
5244 scripts
10 schools
JUDGING
43730 judgements
45 judges
SSR = .74
ANCHORS
745 tests
Jun 2015
TESTS
1215 pupils
3645 scripts
7 schools
JUDGING
32110 judgements
30 judges
SSR = .85
CLEANING
1050 pupils sat tests both times
947 pupils sat all six tests

‘Test’ performance

- 6 questions, 947 pupils
- Cronbach’s α = .762
- Correlations

	equation	area	negative	estimate	fractions
num line	0.30	0.32	0.31	0.33	0.43
equation		0.38	0.43	0.21	0.40
area			0.45	0.31	0.40
negative				0.27	0.40
estimate					0.36

Question performance

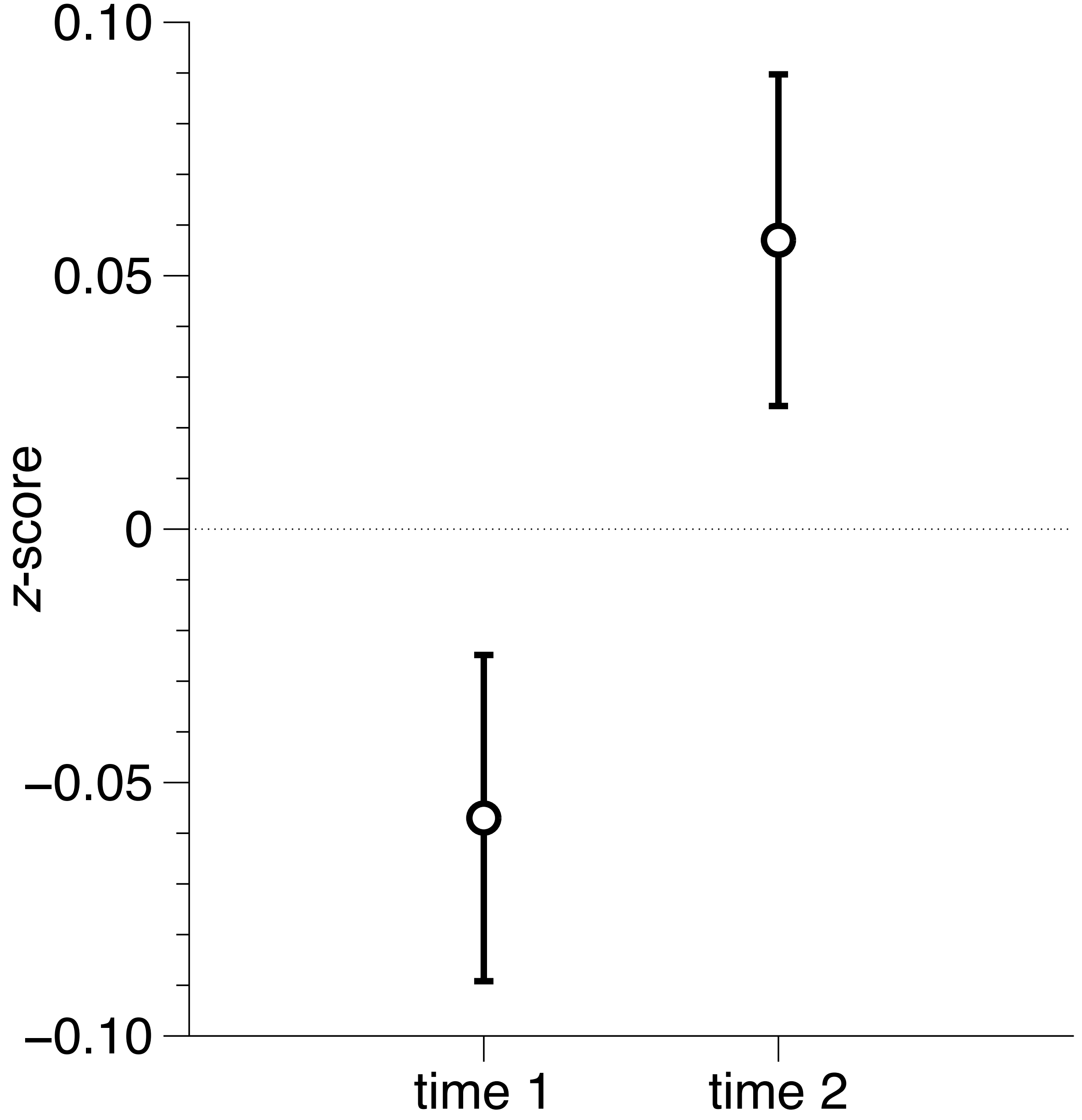


$F(5,5676) = 22.95$, $p < .001$, $\eta^2 = .02$

Criterion validity

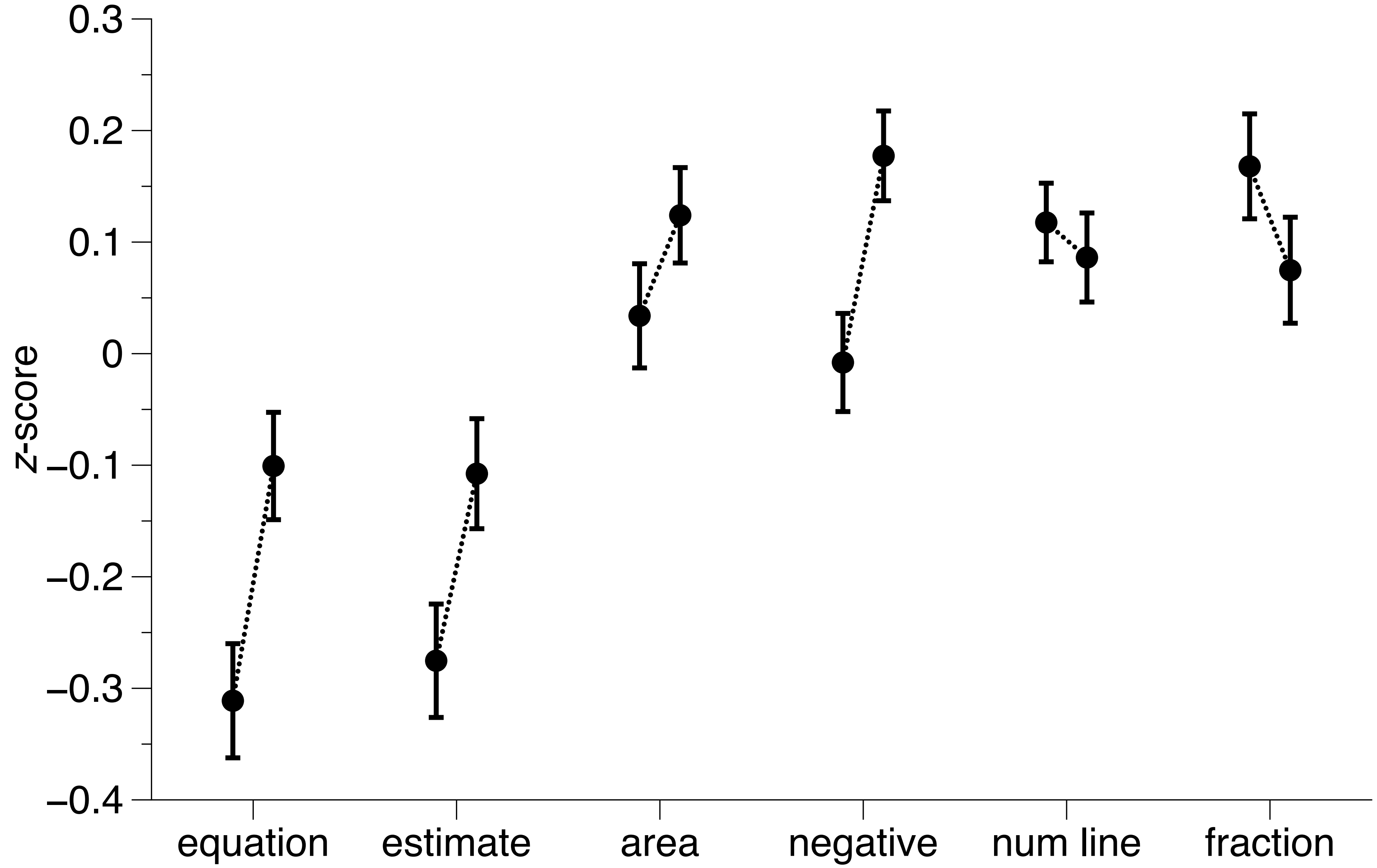
- Mean score across 6 questions for each pupil
- KS2 results for Maths & English as predictors (Level grades: 5.500, 4.212, 3.500, 5.681…)
- Available for 916 pupils
- $R^2 = .41$, $F(2,913) = 311.9$, $p < .001$
- Maths KS2: $\beta = 0.48$, $t(913) = 13.00$, $p < .001$
- English KS2: $\beta = 0.20$, $t(913) = 5.54$, $p < .001$

Evidence of progress



- Mean score at each time for each pupil
- $t(946) = -3.68$, $p < .001$
- Cohen's $d = 0.24$

Question analysis



- One-way between-groups ANOVA
- $F(11{,}5670) = 13.21$, $p < .001$, $\eta^2 = .03$

Conclusion

- Testing process worked satisfactorily
- Evidence of assessing general achievement
- Evidence of assessing learning over time
- But *what* is it assessing?

Further work

- Currently repeating with approximately 75 schools
- 23 test questions across curriculum areas (number, algebra, geometry)

Thank you

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