Class 8: Calculating the Two-Way Analysis of Variance

Contents

1 Design Example

Kardas & O'Brien (2018) recently reported that with the proliferation of YouTube videos, people seem to think they can learn by *seeing* rather than *doing*. They "hypothesized that the more people merely watch others, the more they believe they can perform the skill themselves." They compared extensively watching the "tablecloth trick with extensively reading or thinking about it. Participants assessed their own abilities to perform the"tablecloth trick." Participants were asked to rate from 1 (I feel there's no chance at all I'd succeed on this attempt) to 7 (I feel I'd definitely succeed without a doubt on this attempt). The study was a 3 (type of exposure: watch, read, think) × 2 (amount of exposure: low, high) between-subjects design. They collected an N = 1,003 with small effect sizes ($n^2 < .06$), but to make this analysis doable by hand I will cut the same to 5 people per cell and inflate the effect size (but keep the pattern the same as their experiment 1).

Low Exposure Group	Watching	Reading	Thinking	**Row Means**
	1	3	3	
	3	2	3	
	3	3	2	
	4	1	3	
	2	4	1	
Cell Means	*2.6*	*2.6*	*2.4*	**2.53**
Cell SS	*5.2*	*5.2*	*3.2*	

High Exposure Group	Watching	Reading	Thinking	**Row Means**
	6	5	3	
	4	3	2	
	7	2	1	
	5	3	4	
	5	2	4	
Cell Means	*5.4*	*3.0*	*2.8*	**3.73**
Cell SS	*5.2*	*6.0*	*6.8*	
Column Means	**4.0**	**2.8**	**2.6**	G = 3.13

2 Logic of Two-Way ANOVA

We now have two factors, and we will test the main effects and interaction.

- Main effect (Factor A: Rows): The independent effects of Length of Exposure on Perceived Ability
- Main effect (Factor B: Columns): The independent effects of Type of Exposure on Perceived Ability
- Interaction: The joint effects of Length and Type of Exposure on Perceived Ability

2.1 Hypotheses

2.1.1 Main-Effects

H_0: All groups have equal means

H_1: At least 1 group is different from the other

2.1.2 Interaction

H_0: There is no interaction between factors A and B. All the mean differences between treatment conditions are explained by the main effects.

H_1: There is an interaction between factors A and B. The mean differences between treatment conditions are not what would be predicted from the overall main effects of the two factors.

2.2 Logic Variance Parsing

One-way ANOVA has Only one factor to test: $SS_T = SS_W + SS_B$

Two-way ANOVA has multiple factors to test: $SS_T = SS_W + (SS_{FactorA} + SS_{FactorB} + SS_{AxB})$

Note that in the two-way ANOVA: $SS_B = (SS_{FactorA} + SS_{FactorB} + SS_{AxB})$

2.2.1 F-Ratio Logic

Omnibus F (same as one-way)

$$F = \frac{MS_B^2}{MS_W^2}$$

This will still tell is if one cell is different from another, but we also have to calculate F-tests on Factors A, B, and the AxB

$$F = \frac{MS_{FactorA}^2}{MS_W^2}$$

$$F = \frac{MS_{FactorB}^2}{MS_W^2}$$

$$F = \frac{MS_{AXB}^2}{MS_W^2}$$

3 Analysis Procedures

3.1 One-way ANOVA Review (simplified for equal sample sizes)

With the formulas (formal equations)

Source	SS	DF	MS	F
Between	$n\sum_{i=1}^{k}(M_i - G)^2$	$K-1$	$\frac{SS_B}{df_B}$	$\frac{MS_B}{MS_W}$
Within	$\sum_{i=1}^{K}\sum_{j=1}^{n}(X_{i_j} - M_i)^2$	$N-K$	$\frac{SS_W}{df_W}$	
Total	$\sum_{j=1}^{N}(X_j - G)^2$	$N-1$		

With the formulas (conceptually simplified)

Source	SS	DF	MS	F
Between	$nSS_{treatment}$	$K-1$	$\frac{SS_B}{df_B}$	$\frac{MS_B}{MS_W}$
Within	$\sum SS_{within}$	$N-K$	$\frac{SS_W}{df_W}$	
Total	SS_{scores}	$N-1$		

Note: $SS_B + SS_W = SS_T$ & $df_B + df_W = df_T$

3.2 Two-way ANOVA Review (simplified for equal sample sizes)

Source	SS	DF	MS	F
Between	SS_B	df_B	MS_B	F
$Factor\ A_{Row}$	SS_r	df_r	MS_r	F
$Factor\ B_{Col}$	SS_c	df_c	MS_c	F
$Factor\ AxB$	SS_{rxc}	df_{rxc}	MS_{rxc}	F
Within	SS_W	df_W	MS_W	
Total	SS_T	df_T		

With the formulas (formal equations)

Source	SS	DF	MS	F
Between	$n\left(\sum_{w=1}^{r}(M_r - G)^2 + \sum_{u=1}^{c}(M_c - G)^2\right)$	$rc - 1$	$\frac{SS_B}{df_B}$	$\frac{MS_B}{MS_W}$
$A_{(Rows)}$	$n_r \sum_{w=1}^{r}(M_w - G)^2$	$r - 1$	$\frac{SS_r}{df_r}$	$\frac{MS_r}{MS_W}$
$B_{(Col)}$	$n_c \sum_{u=1}^{c}(M_u - G)^2$	$c - 1$	$\frac{SS_c}{df_c}$	$\frac{MS_c}{MS_W}$
AxB	$SS_B - (SS_r - SS_c)$	$(r-1)(c-1)$	$\frac{SS_{rxc}}{df_{rxc}}$	$\frac{MS_{rxc}}{MS_W}$
Within	$\sum_{w=1}^{r}\sum_{j=1}^{n}(X_{w_j} - M_w)^2 + \sum_{u=1}^{c}\sum_{j=1}^{n}(X_{u_j} - M_u)^2$	$nrc - rc$	$\frac{SS_w}{df_w}$	
Total	$\sum_{j=1}^{N}(X_j - G)^2$	$nrc - 1$		

With the formulas (conceptually simplified)

Source	SS	DF	MS	F
Between	$nSS_{all\,cell\,means}$	$rc - 1$	$\frac{SS_B}{df_B}$	$\frac{MS_B}{MS_W}$
$A_{(Rows)}$	$n_r SS_{row\,means}$	$r - 1$	$\frac{SS_r}{df_r}$	$\frac{MS_r}{MS_W}$
$B_{(Cols)}$	$n_c SS_{col\,means}$	$c - 1$	$\frac{SS_c}{df_c}$	$\frac{MS_c}{MS_W}$
AxB	$SS_B - (SS_r - SS_c)$	$(r-1)(c-1)$	$\frac{SS_{rxc}}{df_{rxc}}$	$\frac{MS_{rxc}}{MS_W}$
Within	$\sum SS_{within}$	$nrc - rc$	$\frac{SS_w}{df_w}$	
Total	SS_{scores}	$nrc - 1$		

Note for SS: $SS_B + SS_W = SS_T$ & $SS_B = SS_r + SS_c + SS_{rxc}$

Note for DF: $df_B + df_W = df_T$ & $df_B = df_r + df_c + df_{rxc}$

3.3 Effect size

3.3.1 Eta-Squared

Just like one-way ANOVA, we will use eta-squared = proportion of variance the treatment accounts for. Remember SS_B is the variation caused by treatment, and SS_T is the variation of Treatment + Noise.

$$\eta_B^2 = \frac{SS_B}{SS_B + SS_W} = \frac{SS_B}{SS_T}$$

$$\eta_r^2 = \frac{SS_r}{SS_T}$$

$$\eta_c^2 = \frac{SS_c}{SS_T}$$

$$\eta_{rxc}^2 = \frac{SS_{rxc}}{SS_T}$$

3.3.2 Partial Eta-Squared

Eta-squared the = proportion of the total (variance) pie this effect is accounting for. The nice thing is that eta-squared adds can be used to add up to 100%! However, the values calculated thus depend upon the number of other effects and the magnitude of those other effects.

People often prefer to report Partial Eta-Squared, η_p^2 = ratio: effect / error variance that is attributable to the effect. This "controls" for the other effects and reports what the effect is relative to the noise (not total). This is "ratio" where eta-squared was a "proportion". Thus you cannot sum them up to 100%.

$$\eta_{p_r}^2 = \frac{SS_r}{SS_T - (SS_c - SS_{rxc})}$$

$$\eta_{p_c}^2 = \frac{SS_c}{SS_T - (SS_r - SS_{rxc})}$$

$$\eta_{p_{rxc}}^2 = \frac{SS_{rxc}}{SS_T - (SS_r - SS_c)}$$

This Partial Eta-Squared formula for two-way ANOVA is equal to the more modern generalized-eta squared η_g^2 that R automatically generates (in the case that between subject treatment conditions were manipulated). When they are not-manipulated (like gender) that formulas change a bit which we will talk about next week along with ω_g^2 for bias correction.

4 Analysis in R

4.1 Build our data frame (or read in csv)

I build our data frame by hand in the long format.

```
n=5; r=2; c=3;
Example.Data<-data.frame(ID=seq(1:(n*r*c)),
          Level=c(rep("Low",(n*c)),rep("High",(n*c))),
          Type=rep(c(rep("Watching",n),rep("Reading",n),rep("Thinking",n)),2),
          Percieved.Ability = c(1,3,3,4,2,3,2,3,1,4,3,3,2,3,1,
                                6,4,7,5,5,5,3,2,3,2,3,2,1,4,4))

# Also, I want to re-set the order using the `factor` command and set the order I want.
Example.Data$Level<-factor(Example.Data$Level,
                              levels = c("Low","High"),
                              labels = c("Low","High"))

Example.Data$Type<-factor(Example.Data$Type,
                              levels = c("Watching","Reading","Thinking"),
                              labels = c("Watching","Reading","Thinking"))
```

4.2 Check the means and SS calculations with dpylr

```
library(dplyr)
Means.Table<-Example.Data %>%
  group_by(Level,Type) %>%
  summarise(N=n(),
            Means=mean(Percieved.Ability),
            SS=sum((Percieved.Ability-Means)^2),
            SD=sd(Percieved.Ability),
            SEM=SD/N^.5)
knitr::kable(Means.Table, digits = 2)
# Note remember knitr::kable makes it pretty, but you can just call `Means.Table`
```

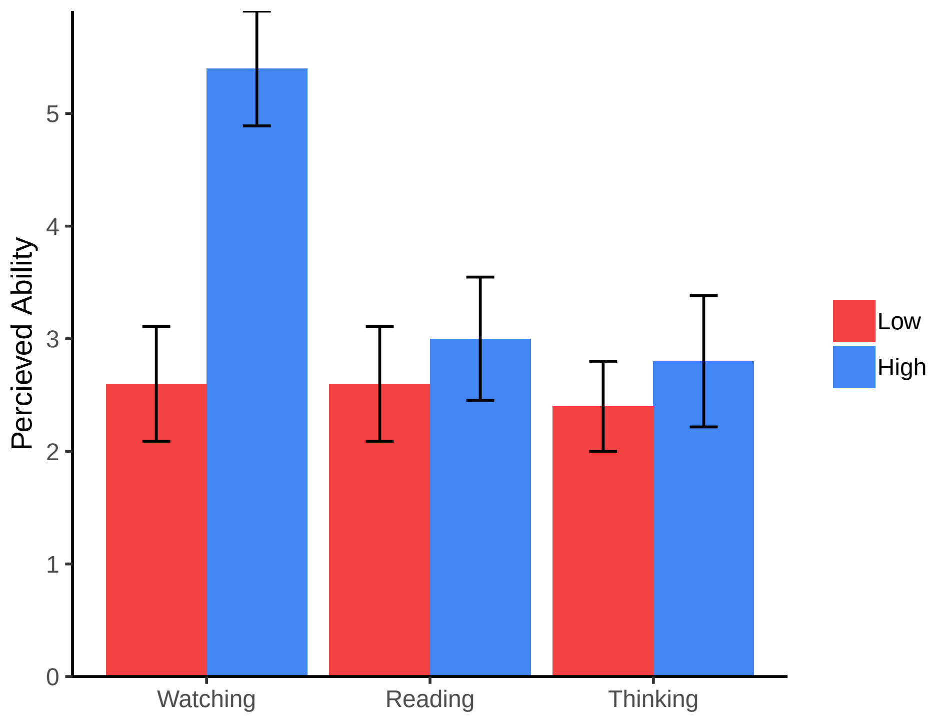
Level	Type	N	Means	SS	SD	SEM
Low	Watching	5	2.6	5.2	1.14	0.51
Low	Reading	5	2.6	5.2	1.14	0.51
Low	Thinking	5	2.4	3.2	0.89	0.40
High	Watching	5	5.4	5.2	1.14	0.51
High	Reading	5	3.0	6.0	1.22	0.55
High	Thinking	5	2.8	6.8	1.30	0.58

4.3 Plot Study

Normally we plot bar graphs with 1 SEM unit. I used the object I created `Means.Table` that has the means. A We will apply some fancy code to make our plot more APA!

```
library(ggplot2)
```

```r
Plot.1<-ggplot(Means.Table, aes(x = Type, y = Means, group=Level))+
  geom_col(aes(group=Level, fill=Level), position=position_dodge(width=0.9))+
  scale_y_continuous(expand = c(0, 0))+ # Forces plot to start at zero
  geom_errorbar(aes(ymax = Means + SEM, ymin= Means - SEM),
                position=position_dodge(width=0.9), width=0.25)+
  scale_fill_manual(values=c("#f44141","#4286f4"))+
  xlab('')+
  ylab('Percieved Ability')+
  theme_bw()+
  theme(panel.grid.major=element_blank(),
        panel.grid.minor=element_blank(),
        panel.border=element_blank(),
        axis.line=element_line(),
        legend.title=element_blank())
Plot.1
```



Which effects do you predict will be significant?

4.4 Run ANOVA

Again, we will use the `afex` package developed to allow people to do SPSS-like ANOVA in R (Type III).

4.4.1 ANOVA style output

- This style is close to our ANOVA source table, but instead its formatted like a regression F-source table.
- You can ignore the intercept line (we will cover that next semester).
- Level = SS_r: Main effect of Level of Exposure (note R will give the order you called them. I put rows first in the formula below).
- Type = SS_c: Main effect of Type of Exposure
- Level:Type = SS_{rxc}: Interaction between Level and Type of Exposure

- Residual = SS_w: Residual in regression means leftover (and this it the error term: chance)
- There is no total (you can do that with addition)

```
library(afex)

ANOVA.JA.Table<-aov_car(Percieved.Ability~Level*Type + Error(ID),
                  data=Example.Data, return='Anova')
ANOVA.JA.Table
```

```
## Anova Table (Type III tests)
##
## Response: dv
##              Sum Sq Df  F value    Pr(>F)
## (Intercept) 294.533  1 223.6962 1.157e-13 ***
## Level        10.800  1   8.2025  0.008553 **
## Type         11.467  2   4.3544  0.024353 *
## Level:Type    9.600  2   3.6456  0.041446 *
## Residuals    31.600 24
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

4.4.2 Nice style output

- This style reports everything you need to report in APA format (just leave the return argument off or write `nice`)

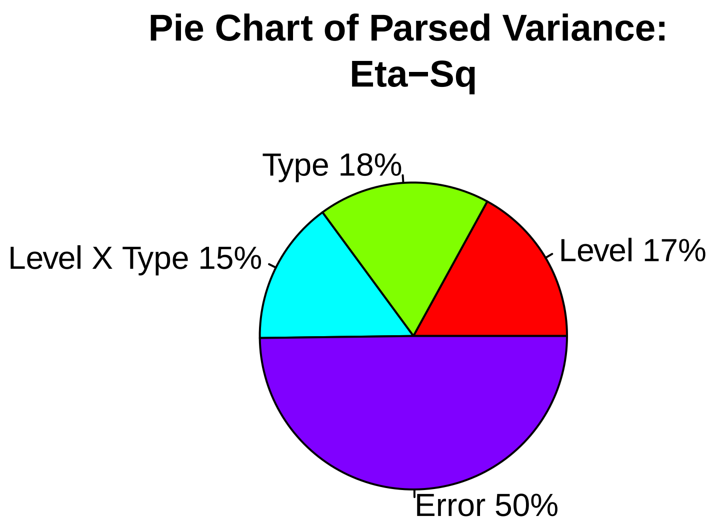
```
library(afex)

Nice.JA.Table<-aov_car(Percieved.Ability~Level*Type + Error(ID),
                  data=Example.Data)
Nice.JA.Table
```

```
## Anova Table (Type 3 tests)
##
## Response: Percieved.Ability
##       Effect    df  MSE       F ges p.value
## 1      Level 1, 24 1.32 8.20 ** .25    .009
## 2       Type 2, 24 1.32  4.35 * .27     .02
## 3 Level:Type 2, 24 1.32  3.65 * .23     .04
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '+' 0.1 ' ' 1
```

4.5 Understanding η^2

η^2 is a useful method for understanding how your ANOVA "parsed" your variance, but not always as useful to report in papers

Here we see that **Type**, **Level**, and **Type X Level** are "Explained Variance". In total, you explained about 50% of the variance. The error term is "Unexplained Variance" (the residual). It is important you do not confuse η^2 (the chart below) with η_p^2.



5 Assumptions

We assume HOV and normality. We can test for HOV using Levene's test formula, which is like the Brown-Forsythe test we used for one-way ANOVA, but we can apply it multiple factors.

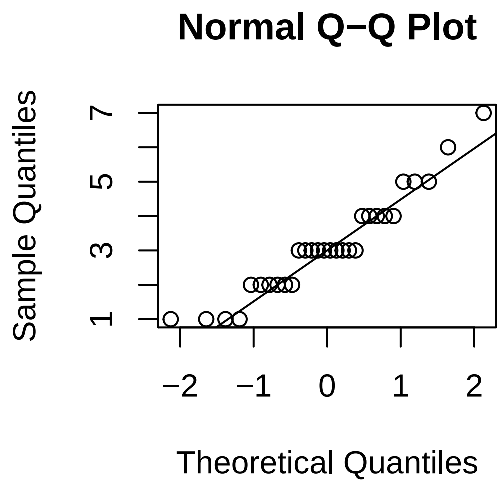
Using the `car` package, we can test our violations. Just like Brown-Forsythe, you do NOT want this test to be significant.

```
library(car)
leveneTest(Percieved.Ability ~ Level*Type, data = Example.Data)
```

```
## Levene's Test for Homogeneity of Variance (center = median)
##       Df F value Pr(>F)
## group  5  0.1171 0.9874
##       24
```

Often we examine QQ plots of the DV. If the data points are very different from the line, it suggests violations of normality.

```
qqnorm(Example.Data$Percieved.Ability)
qqline(Example.Data$Percieved.Ability)
```



Shapiro-Wilk test is a way of testing normality statistically, but this is the test is very sensitive especially in small samples. You do NOT want this test to be significant. In our case it is, but our sample is really small.

```
shapiro.test(Example.Data$Percieved.Ability)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  Example.Data$Percieved.Ability
## W = 0.92911, p-value = 0.04649
```

6 References

Kardas, M., & O'Brien, E. (2018). Easier Seen Than Done: Merely Watching Others Perform Can Foster an Illusion of Skill Acquisition. *Psychological science*, 29(4), 521-536.