Partial and Semipartial (part) Correlation

Contents

1 Correlations with 2 or more variables

- Correlations with more than 2 variables present a new challenge
- What if a third variable (X2) actually explains the relationship between X1 and Y?
- We need to find a way to figure out how X2 might relate to X1 and Y!

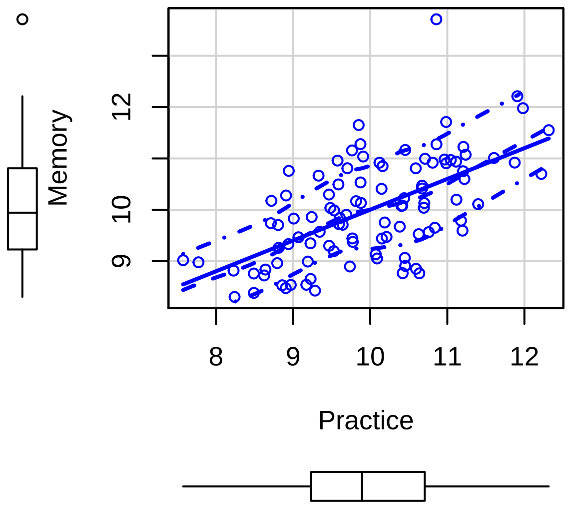
1.1 Example

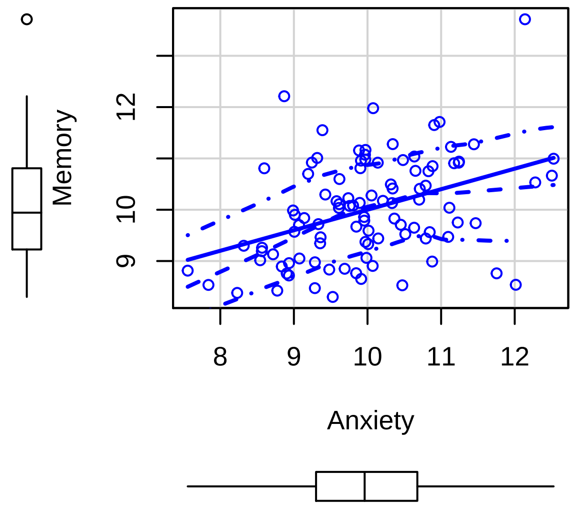
- Practice Time (X1)
- Performance Anxiety (X2)
- Memory Errors (Y)
- Question, how much does Practice Time and Performance Anxiety predict/explain Memory Errors in performance
- Lets simulate our dataset with the new variables and generate a specific correlation matrix

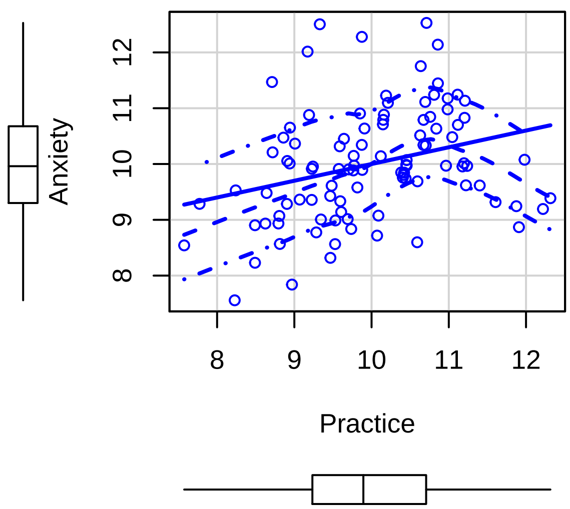
Variable	Practice Time (X1)	Performance Anxiety (X2)	Memory Errors (Y)
Practice Time (X1)	1	.3	.6
Performance Anxiety (X2)	.3	1	.4
Memory Errors (Y)	.6	.4	1

- Set the mean number for each variable to be 10

```r
#packages we will need to conduct to create and graph our data
library(MASS) #create data
library(car) #graph data
py1 =.6 #Cor between X1 (Practice Time) and Memory Errors
py2 =.4 #Cor between X2 (Performance Anxiety) and Memory Errors
p12= .3 #Cor between X1 (Practice Time) and X2 (Performance Anxiety)
Means.X1X2Y<- c(10,10,10) #set the means of X and Y variables
CovMatrix.X1X2Y <- matrix(c(1,p12,py1,
                            p12,1,py2,
                            py1,py2,1),3,3) # creates the covariate matrix
#build the correlated variables. Note: empirical=TRUE means make the correlation EXACTLY r.
# if we say empirical=FALSE, the correlation would be normally distributed around r
set.seed(42)
CorrDataT<-mvrnorm(n=100, mu=Means.X1X2Y,Sigma=CovMatrix.X1X2Y, empirical=TRUE)
#Convert them to a "Data.Frame" & add our labels to the vectors we created
CorrDataT<-as.data.frame(CorrDataT)
colnames(CorrDataT) <- c("Practice","Anxiety","Memory")
#make the scatter plots
scatterplot(Memory~Practice,CorrDataT, smoother=FALSE)
scatterplot(Memory~Anxiety,CorrDataT, smoother=FALSE)
scatterplot(Anxiety~Practice,CorrDataT, smoother=FALSE)
# Pearson Correlations
ry1<-cor(CorrDataT$Memory,CorrDataT$Practice)
ry2<-cor(CorrDataT$Memory,CorrDataT$Anxiety)
r12<-cor(CorrDataT$Anxiety,CorrDataT$Practice)
```







1.1.1 What the problem?

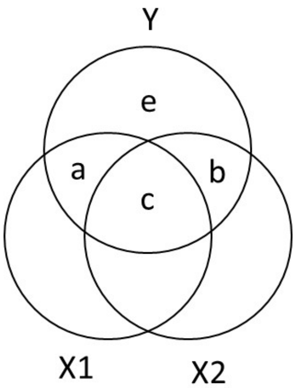
- Practice Time can explain Memory Errors, $r^2 = 0.36$
- Anxiety can explain Memory Errors, $r^2 = 0.16$
- But how we do know whether Practice Time and Anxiety are explaining the same variance? Anxiety and Practice Time explain each other a little, $r^2 = 0.09$

1.1.2 Multiple R

- We use the capital letter, R, now cause we have multiple variables
- $R_{Y.12} = \sqrt{\frac{r^2_{Y1}+r^2_{Y2}-2r_{Y1}r_{Y2}r_{12}}{1-r^2_{12}}}$
- $R_{Y.12} = 0.6427961$
- if we square that value, 0.3340659, we get the Multiple R^2
  - i.e., the total variance explained by these variables on Memory

2 Semipartial (part) correlation

- We need to define to contribution of each X variable on Y
- Semipartial (also called part) is one of two methods; the other is called partial
  - is called semi, cause it removes the effect of one IV relative to the other without removing the relationship to Y
- **Semipartial correlations indicate the "unique" contribution of an independent variable on the dependent variable**.
  - When we get to back to regression, "What is the contribution of this X above and beyond the other X variable?"



- $R^2_{Y.12} = a + b + c$
- $sr^2_1 : a = R^2_{Y.12} - r^2_{Y2}$
- $sr^2_2 : b = R^2_{Y.12} - r^2_{Y1}$

2.1 Calcuation

- Calculate unique variance

```
R2y.12<-sqrt((ry1^2+ry2^2 - (2*ry1*ry2*r12))/(1-r12^2))^2
a = R2y.12 -ry2^2
b = R2y.12 -ry1^2
```

- In other words,
  - In total we explained, 0.4131868 of the Memory Errors
  - Practice Time uniquely explained, 0.2531868 of Memory Errors
  - Anxiety uniquely explained, 0.0531868 of Memory Errors
  - We should not solve for c cause it can be negative (in some weird cases)

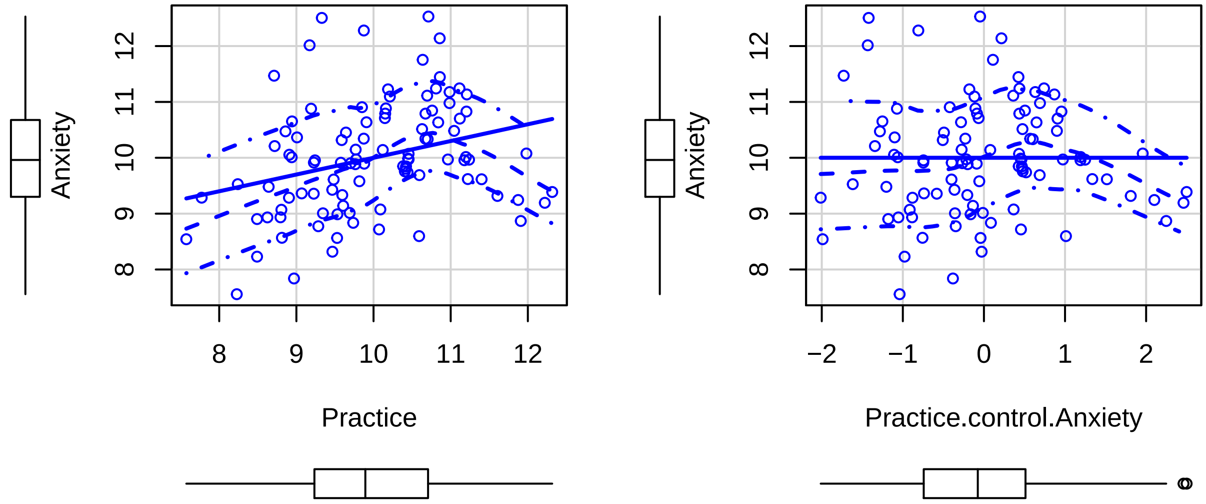
2.2 Seeing control in action in regression

- Another way to understand it:
  - What if you want to know about Memory Errors and how Practice Time uniquely explains it?
    - *Controlling the effect of Anxiety on Practice Time*

2.2.1 Memory (Y) ~ Practice(X1) [Control Anxiety (X2)]

1. We can remove the effect of Anxiety on Practice Time by extracting the residuals from *lm(X1~X2)*
2. Remember the residuals are the leftover (after extracting what was explainable)

```
scatterplot(Anxiety~Practice,CorrDataT, smoother=FALSE)
CorrDataT$Practice.control.Anxiety<-residuals(lm(Practice~Anxiety, CorrDataT))
scatterplot(Anxiety~Practice.control.Anxiety,CorrDataT, smoother=FALSE)
```



3. Next we can correlate Memory Errors with the residualized Practice Time, *cor(Memory,Practice[control Anxiety])*

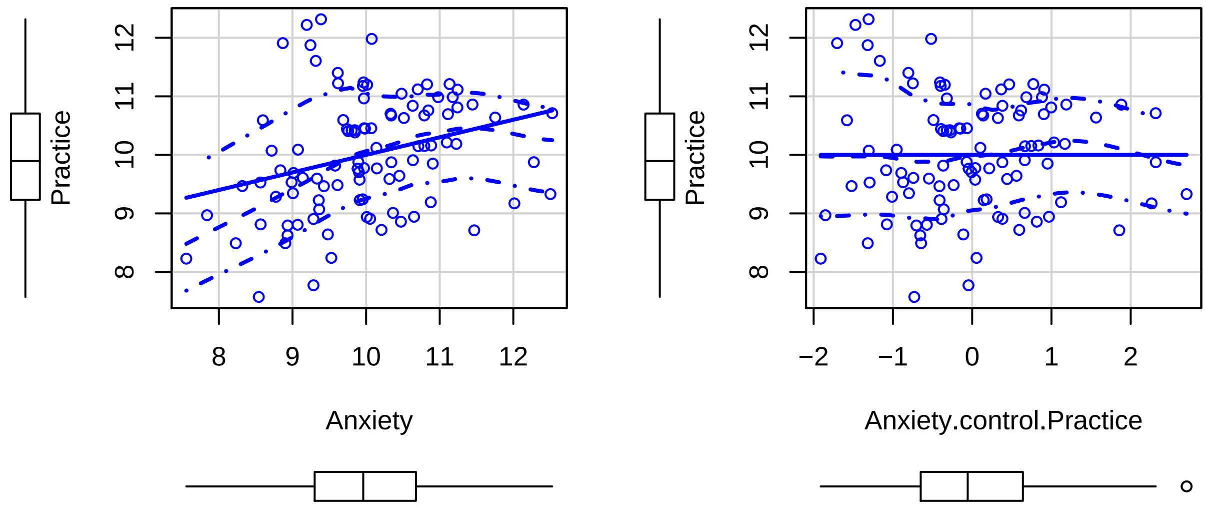
```
Sr1.alt<-cor(CorrDataT$Memory,CorrDataT$Practice.control.Anxiety)
```

If we square the correlation value we got 0.5031767, it becomes 0.2531868 which matches our *a* from the analysis above.

2.2.2 Memory (Y) ~ Anxiety(X2) [Control Practice (X1)]

1. We can remove effect of Practice on Anxiety by extracting the residuals from *lm(X2~X1)*
2. Remember the residuals are the leftover (after extracting what was explainable)

```
scatterplot(Practice~Anxiety,CorrDataT, smoother=FALSE)
CorrDataT$Anxiety.control.Practice<-residuals(lm(Anxiety~Practice, CorrDataT))
scatterplot(Practice~Anxiety.control.Practice,CorrDataT, smoother=FALSE)
```



3. Next we can correlate Memory Errors with the residualized Anxiety, *cor(Memory,Anxiety[control Practice])*

```
Sr2.alt<-cor(CorrDataT$Memory,CorrDataT$Anxiety.control.Practice)
```

If we square the correlation value we got 0.2306227, it becomes 0.0531868 which matches our b from the analysis above.

- **In regression when we have more than one predictors they are controlling for each other!**

2.3 Semipartial notes:

- It can be written as sr or more specifically, sr_1 for X1 (with X2 removed) and sr_2 (with X1 removed)
- correlations with no control variables are called the *zero-order* correlations
- in R you can calculate the sr rather quickly using the *ppcor* library
- The last variable in the list is the control

```
library(ppcor)
Sr1<-spcor.test(CorrDataT$Memory, CorrDataT$Practice, CorrDataT$Anxiety)
Sr2<-spcor.test(CorrDataT$Memory, CorrDataT$Anxiety, CorrDataT$Practice)
# Extract result call for Sr1$estimate
```

- The *Semi-partial* correlation between Memory and Practice (but controlling for Anxiety) was sr =0.503
  - If we square it becomes $sr^2 = 0.253$ which matches our a from the analysis above.
- The *Semi-partial* correlation between Memory and Anxiety (but controlling for Practice) was $sr =$ 0.231
  - If we square it becomes $sr^2 = 0.053$ which matches our b from the analysis above.

2.4 Relationship to Regression

- Lets say we want to report how practice is related to memory in performance, but we want to control for anxiety?

- The sr^2 for a variable tells us how much R^2 will decrease if that variable is removed from the regression equation
  - **Lets test it**
  - Lets run 3 regression models (I zscored everything to make the scales all the same)
    * lm(Memory ~ Practice)
    * lm(Memory ~ Anxiety)

    * lm(Memory ~ Practice + Anxiety)

```
# Center variables (if you said scale = TRUE it would zscore the predictors)
CorrDataT$Memory.Z<-scale(CorrDataT$Memory, scale=TRUE, center=TRUE)[,]
CorrDataT$Practice.Z<-scale(CorrDataT$Practice, scale=TRUE, center=TRUE)[,]
CorrDataT$Anxiety.Z<-scale(CorrDataT$Anxiety, scale=TRUE, center=TRUE)[,]
###############Model 1
M.Model.1<-lm(Memory.Z~ Practice.Z, data = CorrDataT)
M.Model.2<-lm(Memory.Z~ Anxiety.Z, data = CorrDataT)
M.Model.3<-lm(Memory.Z~ Practice.Z+Anxiety.Z, data = CorrDataT)
```

```
library(stargazer)
stargazer(M.Model.1,M.Model.2,M.Model.3,type="latex",
          intercept.bottom = FALSE, single.row=TRUE,
          star.cutoffs=c(.05,.01,.001), notes.append = FALSE,
          header=FALSE)
```

Table 2:

	Dependent variable:		
	Memory.Z		
	(1)	(2)	(3)
Constant	−0.000 (0.080)	0.000 (0.092)	−0.000 (0.077)
Practice.Z	0.600*** (0.081)		0.527*** (0.082)
Anxiety.Z		0.400*** (0.093)	0.242** (0.082)
Observations	100	100	100
R^2	0.360	0.160	0.413
Adjusted R^2	0.353	0.151	0.401
Residual Std. Error	0.804 (df = 98)	0.921 (df = 98)	0.774 (df = 97)
F Statistic	55.125*** (df = 1; 98)	18.667*** (df = 1; 98)	34.150*** (df = 2; 97)
Note:			*p<0.05; **p<0.01; ***p<0.001

Thus,

$$R^2_{model.3_{P+A}} - sr^2_{Anxiety} = R^2_{Practice.only}$$

- In R code:

```r
R2.Practice.only = (summary(M.Model.3)$r.squared) - (Sr2$estimate^2)
```

- So 0.36 = $R^2_{Model.1_{Practice}}$, as expected

Thus,

$$R^2_{model.3.P+A} - sr^2_{Practice} = R^2_{Anxiety.only}$$

- In R code:

```r
R2.Anxiety.only = (summary(M.Model.3)$r.squared) - (Sr1$estimate^2)
```

- So 0.16 = $R^2_{Model.2_{Anxiety}}$, as expected

2.4.1 Residualized into Regression

- Lets take our residualised effects: Practice (controlling for Anxiety) & Anxiety (controlling for Practice) and compare them to regression model where we enter the two variables into are regression
  - If our regression is semi-partialing we should get the same estimates (if we z-score everything cause residuals we are using are z-scored)

```r
M.Model.4<-lm(Memory.Z~ Practice.control.Anxiety, data = CorrDataT)
M.Model.5<-lm(Memory.Z~ Anxiety.control.Practice, data = CorrDataT)

stargazer(M.Model.3,M.Model.4,M.Model.5,type="latex",
          intercept.bottom = FALSE, single.row=TRUE,
          star.cutoffs=c(.05,.01,.001), notes.append = FALSE,
          header=FALSE)
```

Table 3:

	Dependent variable:		
	Memory.Z		
	(1)	(2)	(3)
Constant	−0.000 (0.077)	−0.000 (0.087)	−0.000 (0.098)
Practice.Z	0.527*** (0.082)		
Anxiety.Z	0.242** (0.082)		
Practice.control.Anxiety		0.527*** (0.092)	
Anxiety.control.Practice			0.242* (0.103)
Observations	100	100	100
R^2	0.413	0.253	0.053
Adjusted R^2	0.401	0.246	0.044
Residual Std. Error	0.774 (df = 97)	0.869 (df = 98)	0.978 (df = 98)
F Statistic	34.150*** (df = 2; 97)	33.224*** (df = 1; 98)	5.505* (df = 1; 98)
Note:			*p<0.05; **p<0.01; ***p<0.001

- The regression matches our results of when we by-hand residualized

3 Partial correlation

- Partial correlation asks how much of the Y variance, which is not estimated by the other IVs, is estimated by this variable.
- It removes the shared variance of the control variable (Say X2) from both Y and X1.
- $pr_1^2 := \frac{a}{a+e} = \frac{R^2_{Y.12} - r^2_{Y2}}{1 - r^2_{Y2}}$
- $pr_2^2 : \frac{b}{b+e} = \frac{R^2_{Y.12} - r^2_{Y1}}{1 - r^2_{Y1}}$

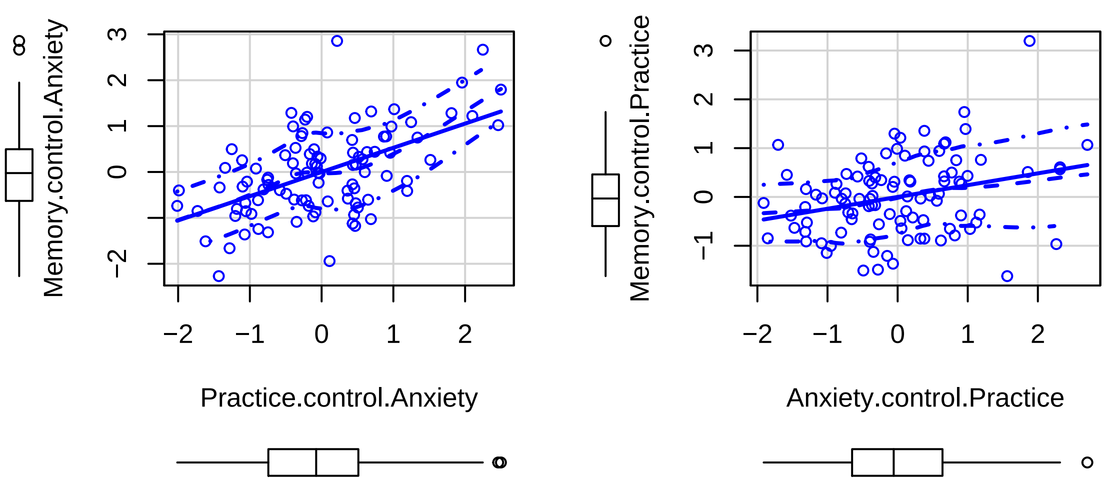
3.1 Seeing control in action

Another way to understand it:

- What if you want to know about Memory Errors and Practice Time while controlling for Anxiety on both Practice Time and Anxiety (cause Anxiety affect both Memory and Practice Time)
  - We take residuals of lm(Y~X2) and correlate it with the residuals of lm(X1~X2)
    - Remember the residuals are the leftover (after extracting what was explainable)
  - if you want to control for Practice Time you would: residuals of lm(Y~X1) with the residuals of lm(X2~X1)

```
# Control for Anxiety
CorrDataT$Memory.control.Anxiety<-residuals(lm(Memory~Anxiety, CorrDataT))
CorrDataT$Practice.control.Anxiety<-residuals(lm(Practice~Anxiety, CorrDataT))
scatterplot(Memory.control.Anxiety~Practice.control.Anxiety,CorrDataT, smoother=FALSE)

# Control for Practice Time
CorrDataT$Memory.control.Practice<-residuals(lm(Memory~Practice, CorrDataT))
CorrDataT$Anxiety.control.Practice<-residuals(lm(Anxiety~Practice, CorrDataT))
scatterplot(Memory.control.Practice~Anxiety.control.Practice,CorrDataT, smoother=FALSE)
```



3.1.1 Correlations based on residuals

```
library(apa)
Res.pr1<-cor_apa(cor.test(CorrDataT$Practice.control.Anxiety,CorrDataT$Memory.control.Anxiety
                 method = c("pearson")),format ="latex",print = FALSE)
Res.pr2<-cor_apa(cor.test(CorrDataT$Anxiety.control.Practice,CorrDataT$Memory.control.Practic
                 method = c("pearson")),format ="latex",print = FALSE)
```

- The pearson correlation Memory (controling for Anxiety) and Practice (controling for Anxiety) was $r(98)_{=}.55$, $p_{<}.001$
- The pearson correlation Memory (controling for Practice) and Anxiety (controling for Practice)was $r(98)_{=}.29$, $p_{=}.004$

3.1.2 Correlations based on R Functions

- in R you can calculate the *pr* directly via the functions

```
pr1<-pcor.test(CorrDataT$Memory, CorrDataT$Practice, CorrDataT$Anxiety)
pr2<-pcor.test(CorrDataT$Memory, CorrDataT$Anxiety, CorrDataT$Practice)
```

- The *Partial* correlation between Memory (controlling for Anxiety) and Practice (controlling for Anxiety) was pr =0.55
  - If we square it becomes $pr^2 = 0.3$
- The *Partial* correlation between Memory (controlling for Practice) and Anxiety (controlling for Practice) was $pr = 0.29$
  - If we square it becomes $pr^2 = 0.08$
- **These values all match our hand residualized calculations**

3.2 Partial Correlation in Regression?

- What if control for Anxiety?
  - We lose the effect of Anxiety

```
Partial.Model.1<-lm(Memory.control.Anxiety~ Practice.control.Anxiety+Anxiety.Z,
                    data = CorrDataT)

stargazer(Partial.Model.1,type="latex",
          intercept.bottom = FALSE, single.row=TRUE,
          star.cutoffs=c(.05,.01,.001), notes.append = FALSE,
          header=FALSE)
```

Table 4:

	Dependent variable:
	Memory.control.Anxiety
Constant	−0.000 (0.077)
Practice.control.Anxiety	0.527*** (0.082)
Anxiety.Z	0.000 (0.078)
Observations	100
R^2	0.301
Adjusted R^2	0.287
Residual Std. Error	0.774 (df = 97)
F Statistic	20.926*** (df = 2; 97)
Note:	*p<0.05; **p<0.01; ***p<0.001

- What if control for Practice?
  - We lose the effect of Practice

```
Partial.Model.2<-lm(Memory.control.Practice~ Anxiety.control.Practice+Practice.Z,
                data = CorrDataT)

stargazer(Partial.Model.2,type="latex",
      intercept.bottom = FALSE, single.row=TRUE,
      star.cutoffs=c(.05,.01,.001), notes.append = FALSE,
      header=FALSE)
```

Table 5:

	Dependent variable:
	Memory.control.Practice
Constant	0.000 (0.077)
Anxiety.control.Practice	0.242** (0.082)
Practice.Z	0.000 (0.078)
Observations	100
R^2	0.083
Adjusted R^2	0.064
Residual Std. Error	0.774 (df = 97)
F Statistic	4.396* (df = 2; 97)
Note:	*p<0.05; **p<0.01; ***p<0.001