

# NUMBER SYSTEM

BINARY, DECIMAL, OCTAL AND HEXADECIMAL NUMBERS, CONVERSION BETWEEN  
DIFFERENT NUMBER SYSTEMS

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## WHAT IS NUMBER SYSTEM ?

A number system defines how a number can be represented using distinct symbols. A number can be represented differently in different systems. For example, the two numbers  $(2A)_{16}$  and  $(52)_8$  both refer to the same quantity,  $(42)_{10}$ , but their representations are different.

## COMPUTER NUMBER SYSTEMS

- **Decimal Numbers.**
- **Binary Numbers.**
- **Octal Numbers.**
- **Hexadecimal Numbers.**

## DECIMAL NUMBER SYSTEM

- ❑ The prefix “deci-” stands for 10
- ❑ The decimal number system is a Base 10 number system:
  - There are 10 symbols that represent quantities: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
  - Each place value in a decimal number is a power of 10.

### DECIMAL NUMBER SYSTEM

- Each position represents a power of 10:

$$\bullet \quad 401 \quad = 4 \cdot 10^2 + 0 \cdot 10^1 + 1 \cdot 10^0 = 400 + 1$$

$$\bullet \quad 130 \quad = 1 \cdot 10^2 + 3 \cdot 10^1 + 0 \cdot 10^0 = 100 + 30$$

$$\begin{aligned} \bullet \quad 9786 &= 9 \cdot 10^3 + 7 \cdot 10^2 + 8 \cdot 10^1 + 6 \cdot 10^0 = \\ &= 9 \cdot 1000 + 7 \cdot 100 + 8 \cdot 10 + 6 \cdot 1 \end{aligned}$$

## BINARY NUMBERS

- ❑ The prefix “bi-” stands for 2
- ❑ The binary number system is a Base 2 number system:
- ❑ There are 2 symbols that represent quantities 0, 1
- ❑ Each place value in a binary number is a power of 2.

### BINARY NUMBERS

- ❑ Representation of binary Number.

Each position represents a power of 2:

$$\bullet \quad 101_b \quad = 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 100_b + 1_b = 4 + 1 = 5$$

$$\bullet \quad 110_b \quad = 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 100_b + 10_b = 4 + 2 = 6$$

## OCTAL NUMBERS

- Octal (base 8) was previously a popular choice for representing digital circuit numbers in a form that is more compact than binary. Octal is sometimes abbreviated as “oct”.
- Octal counting goes:  
0, 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, and so on.

## HEXADECIMAL NUMBERS

- ❑ The prefix “hexa-” stands for 6 and the prefix “deci-” stands for 10
- ❑ The hexadecimal number system is a Base16 number system:
  - There are 16 symbols that represent quantities:  
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
  - Each place value in a hexadecimal number is a power of 16.
- ❑ We use hexadecimal numbers as “shorthand” for binary numbers
- ❑ Each group of four binary digits can be represented by a single hexadecimal digit.

## CONVERSIONS & THEIR USES

### ➤ Computer Number Systems

- |                                   |                                  |
|-----------------------------------|----------------------------------|
| ❑ Convert decimal to binary.      | ❑ Convert hexadecimal to binary. |
| ❑ Convert decimal to hexadecimal. | ❑ Convert hexadecimal to decimal |
| ❑ Convert decimal to Octal.       | ❑ Convert hexadecimal to Octal.  |
| ❑ Convert binary to decimal.      | ❑ Convert Octal to decimal.      |
| ❑ Convert binary to hexadecimal.  | ❑ Convert Octal to binary.       |
| ❑ Convert binary to Octal.        | ❑ Convert Octal to hexadecimal.  |

## CONVERTOR TABLE TO UNDERSTAND THE NUMBER SYSTEM

Decimal Number Convertor Table			
Decimal Number	Binary Number	Hexadecimal Number	Octal Number
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	8	10
9	1001	9	11
10	1010	a	12
11	1011	b	13
12	1100	c	14
13	1101	d	15
14	1110	e	16
15	1111	f	17

## CONVERTING DECIMAL TO BINARY

- ❑ Converting a number from decimal to binary is quite easy. All that is required is to find the binary value.
- ❑ Technique
  - Divide by two, keep track of the remainder
  - First remainder is bit 0 (LSB)
  - Second remainder is bit 1 because if any digit not perfectly divide so we take 1 as bit.

### CONVERTING DECIMAL TO BINARY

Example: convert  $(68)_{10}$  to binary

$$68/2 = 34 \text{ remainder is } 0$$

$$34/2 = 17 \text{ remainder is } 0$$

$$17/2 = 8 \text{ remainder is } 1$$

$$8/2 = 4 \text{ remainder is } 0$$

$$4/2 = 2 \text{ remainder is } 0$$

$$2/2 = 1 \text{ remainder is } 0$$

$$1/2 = 0 \text{ remainder is } 1$$

**Answer = 1 0 0 0 1 0 0**

## CONVERTING DECIMAL TO OCTAL

Method of successive division by 8

- To convert integer decimals to octal, divide the original number by the largest possible power of 8 and successively divide the remainders by successively smaller powers of 8 until the power is.

## CONVERTING DECIMAL TO OCTAL

- Example: convert  $(177)_{10}$  to octal

$$177 / 8 = 22 \text{ remainder is } 1$$

$$22 / 8 = 2 \text{ remainder is } 6$$

$$2 / 8 = 0 \text{ remainder is } 2$$

$$\text{Answer} = 2\ 6\ 1$$

Note: the answer is read from bottom to top as  $(261)_8$ , the same as with the binary case.

## CONVERTING DECIMAL TO HEXADECIMAL

### Steps:

1. Divide the decimal number by 16. Treat the division as an integer division.
2. Write down the remainder (in hexadecimal).
3. Divide the result again .
4. Repeat step 2 and 3 until result is 0.
5. The hex value is the digit sequence of the remainders from the last to first.

## CONVERTING DECIMAL TO HEXADECIMAL

Example:

1. convert  $(4768)_{10}$  to hex.

$$4768 / 16 = 298 \text{ remainder } 0$$

$$298 / 16 = 18 \text{ remainder } 10\ (A)$$

$$18 / 16 = 1 \text{ remainder } 2$$

$$1 / 16 = 0 \text{ remainder } 1$$

$$\text{Answer: } 1\ 2\ A\ 0$$

Note: the answer is read from bottom to top

## CONVERTING BINARY TO DECIMAL

Steps with example

1. let's convert the binary number  $10011011_2$  to decimal.
2. Write first the binary number below the list.
3. Draw lines, starting from the right, connecting each consecutive digit of the binary number to the power of two that is next in the list above it.
4. Move through each digit of the binary number.
5. Add the numbers written below the line.

### Example

Multiply each numeral by its exponent:

$$\begin{array}{rclclcl}
 \bullet \quad 1001_b & = 1 \cdot 2^3 & + 1 \cdot 2^0 & = 1 \cdot 8 & + 1 \cdot 1 & = 9 \\
 \bullet \quad 0111_b & = 0 \cdot 2^3 & + 1 \cdot 2^2 & + 1 \cdot 2^1 & + 1 \cdot 2^0 & = \\
 & = 100_b & + 10_b & + 1_b & = 4 + 2 + 1 & = 7 \\
 \bullet \quad 110110_b & = 1 \cdot 2^5 & + 1 \cdot 2^4 & + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 & = \\
 & = 100000_b + 10000_b + 100_b + 10_b & = 32 + 16 + 4 + 2 & = 54
 \end{array}$$

## CONVERTING BINARY TO HEXADECIMAL

### **Method**

- Conversion between binary and hex is easy. Simply substitute four-bit groups for the hex digit of the same value. Specifically

### Example

Consider Binary: 1000100100110111 (a 16-bit Byte)

Step 1-Break the Byte into 'quartets' -      1000 1001 0011 0111

Step 2 Use the above table to covert each quartet to its Hex equivalent - **8937**

Therefore ... 1000100100110111 = 8937Hex

## CONVERTING BINARY TO OCTAL

### Method

- Since the octal system is again a power of two ( $2^3$ ), we can take group the bits into groups of 3 and represent each group as an octal digit. The steps are the same for the binary to hexadecimal conversions.

### Examples

Take the binary number  $(10011)_2$  and convert it to octal

010 011

2 3 = **(23)**<sub>8</sub>



## CONVERT HEXADECIMAL TO BINARY

- This conversion is also simplistic. Given a hexadecimal number, simply convert each digit to it's binary equivalent. Then, combine each 4 bit binary number and that is the resulting answer and we also seen the comparisons table of it.

### Example

To convert hexadecimal F8 to binary, write down the binary for F first, then the binary for 8.

F 8

1111 1000

So, the answer is **11111000**.



### CONVERT HEXADECIMAL TO DECIMAL

- Converting hexadecimal to decimal can be performed in the conventional mathematical way, by showing each digit place as an increasing power of 16. Of course, hexadecimal letter values need to be converted to decimal values before performing the math.

#### Example

convert  $(1128)_{16}$  to decimal

$$1 \times (16^3) + 1 \times (16^2) + 2 \times (16^1) + 8 \times (16^0)$$

$$4096 + 256 + 32 + 8 = (4392)_{10}$$

### CONVERT HEXADECIMAL TO OCTAL

- convert from any base to any other base, you repeatedly divide by the second base, truncating down to the nearest integer, and recording the remainders in each step, until the result is zero. Then you write the remainders in reverse order.

#### Example

- converting  $12A_{16}$  to Octal
  - Firstly convert to Decimal:
  - $1 \times 16^2 + 2 \times 16^1 + A \times 16^0$  ( $10 \times 16^0$ ) =  $256 + 32 + 10 = 298$
- $12A \approx 298 / 8$  is 37 remainder 2  
 $37 / 8$  is 4 remainder 5  
 $4 / 8$  is 0 remainder 4

Answer is  $452_8$



## CONVERT OCTAL TO DECIMAL

### Method

- Multiply the last to first digit of the octal number by (8 to the zeroth power) and increment in the power acc to the digits.

### Example

$$\begin{aligned}
 &\text{convert } (632)_8 \text{ to decimal} \\
 &= (6 \times 8^2) + (3 \times 8^1) + (2 \times 8^0) \\
 &= (6 \times 64) + (3 \times 8) + (2 \times 1) \\
 &= 384 + 24 + 2 \\
 &= (410)_{10}
 \end{aligned}$$

## CONVERT OCTAL TO BINARY

### Method

- To go from octal to binary, simply reverse the above algorithm and represent each octal digit in it's three bit binary form

### Example

Octal	=3	4	5	
Binary	=011	100	101	= 011100101 Binary

## CONVERT OCTAL TO HEXADECIMAL

### Method

- When converting from octal to hexadecimal, it is often easier to first convert the octal number into binary and then from binary into hexadecimal. For example, to convert 345 octal into hex:

### Example

Octal =3      4      5

Binary =011   100   101= 011100101

- Drop any leading zeros or pad with leading zeros to get groups of four binary digits (bits):

Binary **0**11100101 = 1110 0101

Binary =1110 0101

Hexadecimal =E5 (E5 hex )

Acc to Table of Convertor and Method as Well.

## BCD or Binary Coded Decimal

**BCD or Binary Coded Decimal** - is that number system or code which has the binary numbers or digits to represent a decimal number.

CodeBCD 8421				Code décimal 1 parmi 10									
D	C	B	A	0	1	2	3	4	5	6	7	8	9
0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0	0	0	0	0	0	0
0	0	1	1	0	0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	1	0	0	0	0	0
0	1	0	1	0	0	0	0	0	1	0	0	0	0
0	1	1	0	0	0	0	0	0	0	1	0	0	0
0	1	1	1	0	0	0	0	0	0	0	1	0	0
1	0	0	0	0	0	0	0	0	0	0	0	1	0
1	0	0	1	0	0	0	0	0	0	0	0	0	1

## REPRESENTING INFORMATION IN COMPUTERS

- All the different types of information in computers can be represented using binary code.
  - Numbers.
  - Letters of the alphabet and punctuation marks.
  - Microprocessor instruction.
  - Graphics/Video.
  - Sound.

## USES IN COMPUTER SYSTEM

1. A switch is just used to turn ON or turn OFF any desired device...using the concept binary system
2. Electronic Circuitry uses the octal system and hexadecimal system
3. Decimal number system uses as Basic calculation and complex Calculation In daily life