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Sonargaon University [SU]
সোনারগাঁও ইউনিভার্সিটি (এসইউ)

Course Title: Financial Management
Course Code: FIN 312

LECTURE SHEET

Lectured By:
Md. Shahbub Alam
Lecturer, Dept. of Business Administration
Sonargaon University (SU)

Chapter: Risk and Return (Portfolio Theory)

Questions at a glance:

1. What is the relationship between risk and return?
2. What do you understand by the terms expected return and required return?
3. How would you define risk in financial management?
4. Compare and contrast the three main attitudes to risk which investors may exhibit.
5. Explain the function of the statistical concepts of standard deviation and coefficient of variation as measures of investment risk.
6. What is meant by probability distribution?
7. Define correlation. What are three categories of correlation?
8. Describe the correlation coefficient.
9. Compare between covariance and correlation.
10. Explain the effect of the correlation of asset returns on portfolio risk.
11. What is meant by an efficient portfolio?

1. *What is the relationship between risk and return?*

The relationship between risk and return exists in the form of a risk-return trade-off, by which we mean that it is only possible to earn higher returns by accepting higher risk. If an investor wishes to earn higher returns, then the investor must appreciate that this will only be achieved by accepting a commensurate increase in risk. Risk and return are positively correlated. An increase in one is accompanied by an increase in the other.

The implication for the financial manager in evaluating a prospective investment project is that an effective decision about the project's value to the firm cannot be made simply by focusing on its expected level of returns: the project's expected level of risk must also be simultaneously considered. This risk-return trade-off is central to investment decision-making.

2. *What do you understand by the terms expected return and required return?*

Expected return: An investment's expected return usually denoted E(r) is the investment's most likely return and is measured in terms of the future cash flows, positive or negative, it is expected to generate. It represents the investor's best estimate of the investment's future returns.

Expected or actual return on any investment over a defined period of time can be calculated simply as:

$$E(r) = \frac{(\text{Ending value} - \text{Beginning value}) + \text{ Income}}{\text{Beginning value}} \times 100$$

For example, if you bought a security such as a share for £10.00 which one year later was valued at £11.00 and it paid you a £0.50 dividend during the year, your return would be:

$$E(r) = \frac{(£11.00 - £10.00) + £0.50}{£10.00} \times 100$$

$$= \frac{£1.50}{£10.00} \times 100$$

= 15%

Required return: The required rate of return is the minimum rate of return an investor requires an investment to earn, given its risk characteristics, for the investment to be considered worthwhile. The required rate of return is equal to the rate of return given by a risk-free investment—such as a government treasury bill—plus a risk premium. The risk premium is necessary to compensate the investor for undertaking a risky investment.

Required rate of return, R(r) = risk-free return + risk premium

The required return can be used as a benchmark against expected return. An investment's expected return may or may not be the same as the investor's required return. If the expected return is greater than the required return then the investment will be considered worthwhile. If the expected return is less than the required return, then the investor will not consider the investment to be beneficial.

For example, if the required return of and investment is 15 per cent and the expected return is 11.75 per cent, then this investment project would be rejected as its expected return is less than its required return.

3. *How would you define risk in financial management?*

In financial management risk can be defined as: the chance that the actual return will differ from the expected return. Clearly there is a chance that the actual return will be greater than, equal to, or less than the expected return. In financial decision-making it is this potential variability of returns that we call risk.

4. *<u>Compare and contrast the three main attitudes to risk which investors may exhibit</u>*.

Investment decisions will be influenced by the investor's risk propensity or the investor's attitude to risk. The three main attitudes to risk which investors may exhibit are compared and contrasted as follows:

1. Investors who have a low risk propensity, in other words they have a preference for less risk, are said to be **risk-averse**.
2. Investors who have a high risk propensity, or a positive desire for risk, are referred to as **risk-taking or risk-seeking**.
3. Other individuals may be **risk-indifferent or risk-neutral**, that is, for an increase in risk, they do not necessarily require an increase in return.

Shareholders and managers are generally considered to be risk-averse, that is, for an increase in risk they require a commensurate increase in return.

5. *<u>Explain the function of the statistical concepts of standard deviation and coefficient of variation as measures of investment risk.</u>*

Standard deviation: The standard deviation is defined as the square root of the variance. The variance is defined as the weighted average of the squared deviations of possible outcomes from the expected value or mean. It is used to measure an asset's or an investment's total risk.
The variance and standard deviation are expressed mathematically as follow:

$$\text{Variance, Var}_{(r)} = \sum_{i=1}^{n} (r_i - \overline{r})^2 \times P_i$$

$$\text{Standard deviation, } \sigma_r = \sqrt{\sum_{i=1}^{n} (r_i - \overline{r})^2 \times P_i}$$

Where,

$\text{Var}_{(r)}$= the variance of returns
σ_r= the standard deviation of returns
$\overline{r}$ = the expected or mean value of a return
r_i= return for the ith outcome
P_i= probability of occurrence of the ith outcome

The higher the variance and consequently the standard deviation, the greater is the degree of dispersion and therefore the higher is the asset's or investment's total risk.

Coefficient of variation: Expected return and standard deviation are absolute measures, to make a valid comparison between two such investments we need a relative measure of risk and return. This is where the coefficient of variation (CV) is helpful. The coefficient of variation is a relative measure, or ratio, of dispersion and is particularly useful in comparing assets that have different risk-return characteristics.

The coefficient of variation is measured as the ratio of the standard deviation to the expected return.
The coefficient of variation is calculated as follows:

Coefficient of variation, CV = $\frac{\sigma_r}{\bar{r}}$

Where,

σ_r= the standard deviation of returns

$\acute{r}$ = the expected or mean value of a return

Basically the higher the CV, the higher the risk.
For Assets X and Y the coefficient of variation is calculated as:

		Asset X	Asset Y
Expected return, $\bar{r}$		10%	20%
Standard deviation, σ_r		5%	8%
Coefficient of variation (CV)	=	5÷10	8÷20
	=	0.50	0.40

We can see that although Asset Y has the higher absolute risk measure, σ, it has the lower coefficient of variation, which means that it actually has lower risk per unit of return. The returns from Asset X are relatively more volatile (risky) compared to those from Asset Y. For a rational, risk-averse investor the preferred choice in this case would be Asset Y.

6. *What is meant by probability distribution?*

A probability distribution is simply a statistical model of the probabilities assigned to a range of possible outcomes. It indicates the likelihood of each outcome occurring. Probabilities may be determined subjectively (based on opinion and judgement) or objectively (based on observed data).
Probability distributions can be discrete or continuous. A discrete distribution implies that only a limited number of possible outcomes can be identified with probability weightings assigned to them. A continuous distribution in contrast has an infinite number of possible outcomes.

7. Define correlation. What are three categories of correlation?

Correlation is a statistical technique which is used to measure the relationship between two variables or data series. In portfolio management correlation is used to measure the relationship between two assets or investments. Correlation measures both the degree and direction of the relationship.
There are three categories or states of correlation. They are as follows:

1. **Positive correlation:** This is the state which exists when two variables move in the same direction at the same time e.g. sales and profits. Under normal business conditions one would expect sales and profits to be positively correlated. An increase in sales would normally be expected to produce an increase in profits, and a fall in sales a reduction in profits.
2. **Negative correlation:** This state occurs when two variables move in the opposite direction at the same time, that is they are inversely related. Assuming normal business conditions, one would expect the price and demand for a company's products to be negatively correlated. An increase in price is likely to produce a decrease in demand and vice versa.

3. **Zero correlation.** This applies when there is no relationship between variables, a change in one variable is independent of a change in the other.

8. Describe the correlation coefficient.

The degree to which two variables, or the returns from two assets, are correlated is measured by the correlation coefficient, ρ (Greek letter 'rho') which ranges from +1.0 for perfect positive correlation to −1.0 for perfect negative correlation.

The different interpretations of correlation coefficient are given as follows:

ρ = +1.0: Perfect positive correlation exists where two variables move together in exactly the same direction at the same time and by the same relative degree of magnitude. Again the relative change between the variables remains constant over time.

ρ = −1.0: Perfect negative correlation occurs when two variables move in exactly opposite directions at the same time and by the same relative degree of magnitude. Again the proportionate relationship between the variables must remain constant over time.

ρ = +0.8: A correlation coefficient lying between 0 and +1.0 suggests that there is a generally positive, but not necessarily a precise predictable, relationship between variables and the closer ρ is to 0, the weaker the positive relationship.

ρ = −0.8: A correlation coefficient lying between 0 and −1.0 suggests a generally negative, but not necessarily a precise predictable, relationship between variables, and similarly the closer ρ is to 0, the weaker the negative relationship.

ρ = 0: A correlation coefficient of 0 suggests no relationship between variables.

9. Compare between covariance and correlation.

The comparisons between covariance and correlation are described as follows:

1. Covariance (COV) is a measure of how two assets move together in terms of the degree and magnitude of the co-movement. Whereas correlation measures degree and direction of movement.
2. Covariance is an absolute measure of how two variables move together and depends on the units in which the variables are measured. Whereas correlation is a relative measure of co-variability and is completely independent of the units of measurement of either variable.
3. Covariance can assume any value however large or small. Correlation can only assume values ranging between −1.0 and +1.0.
4. The covariance of two securities, Security 1 and Security 2, denoted Cov_{12}, is simply the product of the standard deviations of the two securities $(\sigma_1\sigma_2)$multiplied by their correlation coefficient (ρ_{12}), that is, $(\rho_{12})(\sigma_1\sigma_2)$. Whereas the correlation coefficient of the two securities $(\rho_{12})$is equal to the covariance of the two securities, Cov_{12}, divided by the product of their standard deviations $(\sigma_1\sigma_2)$.
5. The relationship between covariance and correlation can be seen more clearly from the following:

   Covariance, $Cov_{12} = (\rho_{12})(\sigma_1\sigma_2)$

   Correlation coefficient, $\rho_{12} = \frac{Cov_{12}}{(\sigma_1\sigma_2)}$

10. Explain the effect of the correlation of asset returns on portfolio risk.

The effects of the correlation of asset returns on portfolio risk are explained as follows:

1. If there is **perfect positive correlation** of returns between two assets, portfolio risk is equal to the weighted average of the standard deviations of the two assets. No reduction in risk is achieved.
2. If there is **perfect negative correlation** of returns between two assets, portfolio risk may be virtually eliminated when the optimum combination of assets is achieved.
3. If the correlation of returns between two assets is less than 1.0, portfolio risk can be reduced by **diversification.** The less the degree of positive correlation, the greater will be the risk reduction effects. However, combining assets which are negatively correlated will reduce risk further.

11. ***What is meant by an efficient portfolio?***

An **efficient portfolio** is one which will maximize return for a certain level of risk or alternatively minimize risk for a required level of return.

Formula

- **Expected return**

$$r_t = \frac{V_{t+1} - V_t + CF}{V_t}$$

Where,

r_t = actual or expected rate of return during period t
V_t = value of asset at time t
V_{t-1} = value of asset at time t−1
CF = cash flow from investment over the period t−1 to t

$$E(r) = \sum_{i=1}^{n} P_i r_i$$

Or, $E(r) = \frac{\sum_{i=1}^{n} r_i}{n}$

Where,

$E(r)$= the expected return
r_i = rate of return for the identified ith outcome
P_i= probability of earning return i for the identified outcome
n = number of possible outcomes

- **Standard deviation**

$$\sigma_r = \sqrt{\sum_{i=1}^{n} P_i (r_i - \overline{r})^2}$$

Or, $\sigma_r = \sqrt{\frac{\sum_{i=1}^{n}(r_i-\bar{r})^2}{n-1}}$

Where,

σ_r= the standard deviation of returns
$\bar{r}$ = the expected or mean value of a return
r_i= return for the ith outcome
P_i= probability of occurrence of the ith outcome
n = number of possible outcomes

- **Coefficient of variation**

$CV = \frac{\sigma_r}{\bar{r}}$

Where,

σ_r= the standard deviation of returns
$\bar{r}$ = the expected or mean value of a return

- **Portfolio return**

$$E(r_p) = \sum_{i=1}^{n} w_i\, r_i$$

Where,

$E(r_p)$ = the expected return of the portfolio
w_i = weights of the individual assets (where i=1,2,…n)
r_i= expected return of the individual assets (where i=1,2,…n)

- **Portfolio risk**

$$\sigma_p = \sum_{i=1}^{n} w_i\, \sigma_i$$

$$\sigma_p = \sqrt{(w_1)^2(\sigma_1)^2+(w_2)^2(\sigma_2)^2 + 2(w_1 w_2 \rho_{12}\sigma_1\sigma_2)}$$

Where,

σ_p= the standard deviation for *a* two-asset portfolio
w_i =weights or proportions of the individual portfolio assets (i=1,2)
σ_i =standard deviations of the individual portfolio assets (i=1,2)
ρ_{12} =correlation coefficient of returns between assets 1 and 2

- **Covariance**

$Cov_{12} = (\rho_{12})(\sigma_1\sigma_2)$

- **Correlation coefficient**

$$\rho_{12} = \frac{Cov_{12}}{(\sigma_1\sigma_2)}$$

Md. Shahbub Alam
Lecturer, Dept. of Business Administration

Problems & Solution

1. Calculate the rate of return earned (realized) on each of the following investments over the past year.

Investment	Opening Value (£)	Closing Value (£)	Cash Flow (£)
1	10,000	11,000	500
2	20,000	19,000	500
3	3,500	3,800	−100
4	123,000	131,000	9,000
5	65,000	63,000	1,100

Answer:

The rate of return, r_t earned on an investment is given by: $r_t = \frac{V_{t+1} - V_t + CF}{V_t}$

Investment	Calculation	r_t
1	[(11,000−10,000)+500]/10,000	15.0%
2	[(19,000−20,000)+500]/20,000	−2.5%
3	[(3,800−3,500)−100]/3,500	5.7%
4	[(131,000−123,000)+9,000]/123,000	13.8%
5	[(63,000−65,000)+1,100]/65,000	−1.4%

2. Calculate the expected return, standard deviation and coefficient of variation on the following investment for FutureSpec Technologies. The expected returns from the project are related to the future performance of the economy over the period as follows:

Economic scenario	Probability of occurrence (p)	Rate of return (r)
Strong growth	0.25	15%
Moderate growth	0.50	12%
Low growth	0.25	8%

Answer:

The expected return is calculated as:

$$E(r) = \sum_{i=1}^{n} P_i r_i$$

$$= P_1 r_1 + P_2 r_2 + P_3 r_3$$

=0.25(15%) + 0.50(12%) + 0.25(8% =11.75%

The standard deviation is calculated as follows:

(1)	(2)	(3)	(2)−(3) (4)	(4)×(4) (5)	(6)	(5)×(6) (7)
i	r_i	$\bar{r}$	$r_i - \bar{r}$	$(r_i - \bar{r})^2$	P_i	$P_i(r_i - \bar{r})^2$
1	15%	11.75%	3.25%	10.56%	0.25	2.64%
2	12	11.75	0.25	0.0625	0.50	0.03
3	8	11.75	-3.75	14.0625	0.25	3.52
						$\sum P_i(r_i - \bar{r})^2$ = 6.19%

$\sigma_r = \sqrt{\sum P_i (r_i - \bar{r})^2}$

$= \sqrt{6.19\%}$

$= 2.49\%$

The coefficient of variation is calculated as follows:

$CV = \frac{\sigma_r}{\bar{r}}$

$= \frac{2.49\%}{11.75\%}$

$= 0.21$

3. (i) Calculate the expected rate of return and the standard deviation for each asset A and B as shown in following table.

Asset A		Asset B	
Rate of return (%)	**Probability of occurrence, P(ri)**	**Rate of return (%)**	**Probability of occurrence, P(ri)**
6	0.05	8	0.25
8	0.10	10	0.50
10	0.20	12	0.25
12	0.30		
14	0.20		
16	0.10		
18	0.05		

(ii) Calculate the coefficient of variation (CV) for assets A and B. Which do you consider to be the riskier investment? Give your reasons.

(iii) Calculate the expected return of the two-asset portfolio A and B assuming funds are invested in the proportions 60 per cent asset A and 40 per cent asset B.

<u>Answer:</u>

(i)

Expected return of Asset A is calculated as follows:

$$E(r) = \sum_{i=1}^{n} P_i r_i$$

$= P_1 r_1 + P_2 r_2 + P_3 r_3 + P_4 r_4 + P_5 r_5 + P_6 r_6 + P_7 r_7$

$= 0.05(6\%) + 0.10(8\%) + 0.10(20\%) + 0.30(12\%) + 0.20(14\%) + 0.10(16\%) + 0.05(18\%)$

$= 0.30\% + 0.80\% + 2.00\% + 3.60\% + 2.80\% + 1.60\% + 0.90\%$

$= 12.00\%$

Expected return of Asset B is calculated as follows:

$$E(r) = \sum_{i=1}^{n} P_i r_i$$

$= P_1 r_1 + P_2 r_2 + P_3 r_3$

$= 0.25(8\%) + 0.50(10\%) + 0.25(12\%)$

$= 10.00\%$

Md. Shahbub Alam
Lecturer, Dept. of Business Administration

The standard deviation of Asset A is calculated as follows:

			(2)−(3)	(4)×(4)		(5)×(6)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
i	r_i	$\bar{r}$	$r_i - \bar{r}$	$(r_i - \bar{r})^2$	P_i	$P_i(r_i - \bar{r})^2$
1	6%	12.00%	−6.00%	36.00%	0.05	1.80%
2	8	12.00	−4.00	16.00	0.10	1.60
3	10	12.00	−2.00	4.00	0.20	0.80
4	12	12.00	0.00	0.00	0.30	0.00
5	14	12.00	2.00	4.00	0.20	0.80
6	16	12.00	4.00	16.00	0.10	1.60
7	18	12.00	6.00	36.00	0.05	1.80
						$\sum P_i(r_i - \bar{r})^2 = 8.40\%$

$$\sigma_r = \sqrt{\sum P_i(r_i - \bar{r})^2}$$
$$= \sqrt{8.40\%}$$
$$= 2.90\%$$

The standard deviation of Asset B is calculated as follows:

			(2)−(3)	(4)×(4)		(5)×(6)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
i	r_i	$\bar{r}$	$r_i - \bar{r}$	$(r_i - \bar{r})^2$	P_i	$P_i(r_i - \bar{r})^2$
1	8	10	−2	4	0.25	1
2	10	10	0	0	0.50	0
3	12	10	2	4	0.25	1
						$\sum P_i(r_i - \bar{r})^2 = 2\%$

$$\sigma_r = \sqrt{\sum P_i(r_i - \bar{r})^2}$$
$$= \sqrt{2\%} \quad = 1.41\%$$

(ii)For Assets A and B the coefficient of variation (CV) is calculated as follows:

		Asset A	**Asset B**
Expected return, E(r)		12%	10%
Standard deviation, σ		2.90%	1.41%
Coefficient of variation, (CV)	=	2.90÷12	1.41÷10
	=	0.24	0.14

Asset B would be the preferred investment as it has the lower coefficient of variation: it is the less risky investment. However the final decision would depend on the investor's attitude to risk.

(iii) The expected return on the portfolio is calculated as follows:

	Asset A	**Asset B**
Expected return, E(r)	12%	10%
Proportion invested	0.6	0.4

$$E(r_p) = \sum_{i=1}^{n} w_i r_i$$
$= w_A r_A + w_B r_B$
$=0.6(12\%) + 0.4(10\%)$
$= 7.2\%+4.0\%$
$= 11.2\%$

4. FutureSpec Technologies decided to invest £10,000 in a two-asset portfolio in the following proportions:

	Security 1	Security 2
Expected return on security, E(r)	20%	15%
Amount invested	£6,000	£4,000
Proportion invested	0.6	0.4
Standard deviation of security, σ	9%	7%

From this information you are required to:
(i) Calculate the expected return on this portfolio.
(ii) Calculate the standard deviation of the portfolio.
(iii) Calculate the portfolio risk (standard deviation) assuming perfect negative correlation between asset returns.
(iv) Calculate the portfolio risk (standard deviation) assuming correlation between the two securities to be zero.

Answer:
(i) The expected return on the portfolio would be:

$$E(r_p) = \sum_{i=1}^{n} w_i r_i$$
$= w_1 r_1 + w_2 r_2$
$=0.6(20\%) + 0.4(15\%)$
$=12\%+6\%$
$=18\%$

(ii) The standard deviation of the portfolio would be:

$$\sigma_p = \sum_{i=1}^{n} w_i \sigma_i$$
$= w_1\sigma_1 + w_2\sigma_2$
$= 0.6\ (9\%) + 0.4\ (7\%)$
$= 5.4\% + 2.8\%$
$= 8.2\%$

(iii) The portfolio risk (standard deviation) assuming perfect negative correlation between asset returns would be:

$$\sigma_p = \sqrt{(w_1)^2(\sigma_1)^2 + (w_2)^2(\sigma_2)^2 + 2(w_1 w_2 \rho_{12}\sigma_1\sigma_2)}$$
$$= \sqrt{(0.6)^2(9\%)^2 + (0.4)^2(7\%)^2 + 2[(0.6)(0.4)(-1.0)(9\%)(7\%)]}$$

$= \sqrt{29.16\% + 7.84\% + 2(-15.12\%)}$
$= \sqrt{37.0\% - 30.24\%}$
$= \sqrt{6.76\%}$
$= 2.6\%$

(iv) The portfolio risk (standard deviation) assuming correlation between the two securities to be zero would be:

$$\sigma_p = \sqrt{(w_1)^2(\sigma_1)^2 + (w_2)^2(\sigma_2)^2 + 2(w_1 w_2 \rho_{12} \sigma_1 \sigma_2)}$$
$$= \sqrt{(0.6)^2(9\%)^2 + (0.4)^2(7\%)^2 + 2[(0.6)(0.4)(0)(9\%)(7\%)]}$$
$$= \sqrt{29.16\% + 7.84\% + 0}$$
$$= \sqrt{37.0\%}$$
$$= 6.1\%$$

Practice Questions

1. Calculate the expected return on the following investment for FutureSpec Technologies.

Probability	**Return**
0.10	−10%
0.20	8%
0.40	10%
0.30	20%

2. Calculate the standard deviation of the two-asset portfolio with the original 60/40 weightings reversed and assuming perfect negative correlation.

	Security 1	**Security 2**
Expected return on security, E(r)	20%	15%
Amount invested	£6,000	£4,000
Proportion invested	0.6	0.4
Standard deviation of security, σ	9%	7%

3. Calculate the expected return and risk (standard deviation) of the following investment. If the return on a risk-free investment (e.g. a Treasury bill) is currently 7 per cent should the following investment be undertaken?

Return	**Probability**
5%	0.10
6%	0.20
7%	0.40
8%	0.30

4. Set out below are the returns and their respective probabilities for two assets A and B.

Asset A		**Asset B**	
Return	**Probability**	**Return**	**Probability**
10	0.05	9	0.10
11	0.25	12	0.15
12	0.40	13	0.50
13	0.25	15	0.15
14	0.05	16	0.10

From this information you are required to:
(i) Compute the expected return and the risk (standard deviation) for each asset.
(ii) Determine the coefficient of variation (CV) for each asset.
(iii) Comment on your findings in (i) and (ii).
(iv) It has been suggested that these two assets should be combined into a portfolio in the proportions 40 per cent Asset A and 60 per cent Asset B. Calculate the expected return for this portfolio and its risk (standard deviation), assuming the correlation coefficient to be 0.6.

5. You have been asked for your advice in selecting a portfolio of assets and have been given the following data:

	Expected return		
Year	**Asset A**	**Asset B**	**Asset C**
2013	12%	16%	12%
2014	14	14	14
2015	16	12	16

You have been told that you can create two portfolios—one consisting of assets A and B and the other consisting of assets A and C—by investing equal proportions (50%) in each of the two component assets.
a. What is the expected return for each asset over the 3-year period?
b. What is the standard deviation for each asset's return?
c. What is the expected return for each of the two portfolios?
d. What is the standard deviation for each portfolio?
e. Which portfolio do you recommend? Why?