Forward Propagation in Neural Networks

1. Introduction to Forward Propagation

Forward propagation is the process through which inputs are passed through a neural network layer-by-layer to compute the output. It involves calculating the weighted sum, applying the bias, and then passing the result through an activation function to generate the output for each neuron.

2. Forward Propagation in a Simple Neural Network

Consider a neural network with one input layer, one hidden layer, and one output layer:

- Input layer: 2 neurons (x_1, x_2)
- Hidden layer: 2 neurons (h_1, h_2) using Sigmoid activation
- Output layer: 1 neuron (y) using Sigmoid activation

Step-by-Step Calculation

Given inputs:

$$x_1 = 0.5, \quad x_2 = 0.2$$

Weights for the hidden layer:

$$W_{h1} = \{0.4, 0.3\}, \quad W_{h2} = \{0.6, 0.1\}$$

Biases for the hidden layer:

$$b_{h1} = 0.1, \quad b_{h2} = 0.2$$

2.1. Calculate the Hidden Layer Outputs

For the first hidden neuron (h_1):

$$h_1 = f(W_{h1} \cdot x + b_{h1}) = f(0.4 \cdot 0.5 + 0.3 \cdot 0.2 + 0.1)$$

$$h_1 = f(0.2 + 0.06 + 0.1) = f(0.36)$$

Using the Sigmoid function:

$$h_1 = \frac{1}{1 + e^{-0.36}} \approx 0.59$$

For the second hidden neuron (h_2):

$$h_2 = f(W_{h2} \cdot x + b_{h2}) = f(0.6 \cdot 0.5 + 0.1 \cdot 0.2 + 0.2)$$

$$h_2 = f(0.3 + 0.02 + 0.2) = f(0.52)$$

Using the Sigmoid function:

$$h_2 = \frac{1}{1 + e^{-0.52}} \approx 0.63$$

2.2. Calculate the Output Layer

The output neuron receives inputs from the hidden layer outputs h_1 and h_2:

$$h_1 = 0.59, \quad h_2 = 0.63$$

Weights for the output neuron:

$$W_o = \{0.7, 0.4\}$$

Bias for the output neuron:

$$b_o = 0.1$$

Calculate the Output Neuron

$$y = f(W_o \cdot h + b_o) = f(0.7 \cdot 0.59 + 0.4 \cdot 0.63 + 0.1)$$

$$y = f(0.413 + 0.252 + 0.1) = f(0.765)$$

Using the Sigmoid function:

$$y = \frac{1}{1 + e^{-0.765}} \approx 0.683$$

3. Summary of Forward Propagation

The final output of the neural network after forward propagation is:

$$y \approx 0.683$$

This completes one forward pass through the network, calculating the output based on the given inputs, weights, and biases.

4. Key Takeaways

- Forward propagation involves computing the weighted sum at each neuron, adding bias, and applying the activation function.
- Each layer's output becomes the input for the next layer.
- The process continues until the output layer produces the final prediction.