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**Problem 01:** This problem considers two coupled ODEs with different timescales $\epsilon_1$ and $\epsilon_2$ :

$$\frac{dx}{dt} = \epsilon_1^{-1}\left[-x(t) + y(t) + I(t)\right]$$

$$\frac{dy}{dt} = \epsilon_2^{-1}\left[-y(t) + x(t)^2 + A\right]$$

**Part (a):**

if $\epsilon_1 \ll \epsilon_2$, it means $\frac{dx}{dt}$ envolves much faster than $dy/dt$. Thus, we assume $x(t)$ rapidly reaches a steady state relative to $y(t)$, Leading to $dx/dt \approx 0$.

From the equation $\epsilon_1^{-1}\left[-x(t) + y(t) + I(t)\right] \approx 0$

$$x(t) \approx y(t) + I(t)$$

Substituting $x(t)$ in the equation for $y(t)$ :

$$dy/dt = \epsilon_2^{-1}\left[-y(t) + (y(t) + I(t))^2 + A\right]$$

This differ from the provided equation (4), suggesting that if there was an error in interpreting the steady-state condition or the simplification.

**Part (b):** if $\epsilon_2 \ll \epsilon_1$ then $dy/dt$ envolves faster than $dx/dt$ suggesting $dy/dt \approx 0$: From the equation $\epsilon_2^{-1}\left[-y(t) + x(t)^2 + A\right] = 0$:

$$y(t) \approx x(t)^2 + A.$$

Substituting $y(t)$ in the equation for $x(t)$ :

$$dx/dt = \epsilon_1^{-1}\left[-x(t) + x(t)^2 + A + I(t)\right]$$

This results in an approximation closer to equation (5) including that simplification based on $\epsilon_2 \ll \epsilon_1$ is consistent.

Part (c): if $\epsilon_1 \ll \epsilon_2$, then none of the approximations leading to equations (4) or (5) would hold because both variables evolve on the same timscale, preventing any simplification based on timescale separation.

Problem 02:

Given the channel equation:

$$dR/dt = \epsilon_1^{-1} [-R(t) + R_0(v(t))]$$

And the influence on the voltage:

$$dv/dt = \epsilon_2^{-1} [-v(t) - V_0 + R(t)^2 A]$$

Part (a): if $\epsilon_2 \ll \epsilon_1$ then $V(t)$ rapidly adjusts, leading to $dv/dt \approx 0$ implying: $v(t) \approx -V_0 + R(t)^2 A$

This dimension reduction allows us to describe $v(t)$ in terms of $R(t)$, making it a consistent simplification if $\epsilon_2 \ll \epsilon_1$.

Part (b): if $\epsilon_1 \approx \epsilon_2$, it becomes impossible to simplify or reduce dimensions based on the timescale because both $R(t)$ and $v(t)$ evolve at similar rates.

Part (c): if $\epsilon_1 \ll \epsilon_2$, $R(t)$ stabilizes quickly:

$$R(t) \approx R_0(v(t))$$

This feedback impacts $dv/dt$, allowing $v(t)$ to be described in terms of $R(t)$, simplifying the system.