Divide and Conquer Approach

Merge Sort, Quick Sort, Binary Search

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Recursion

- Recursion is a method of solving a problem where the solution depends on solutions to smaller instances of the same problem.

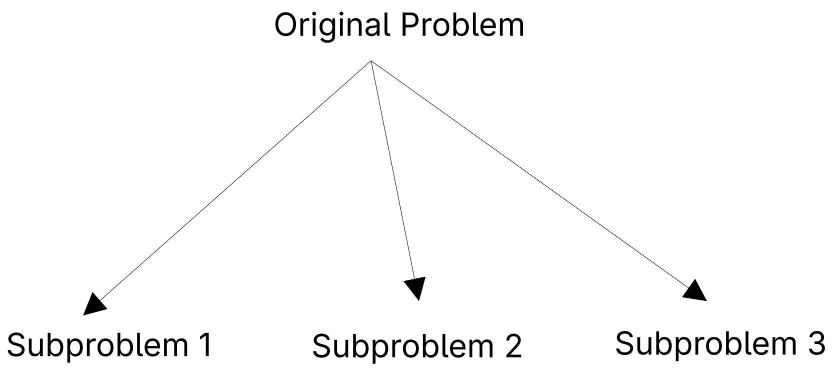
- In programming, this means a function calls itself.

- Example: Calculating a Factorial – $n! = n(n-1)! = n(n-1)(n-2)!$ and so on.

Divide and Conquer Approach

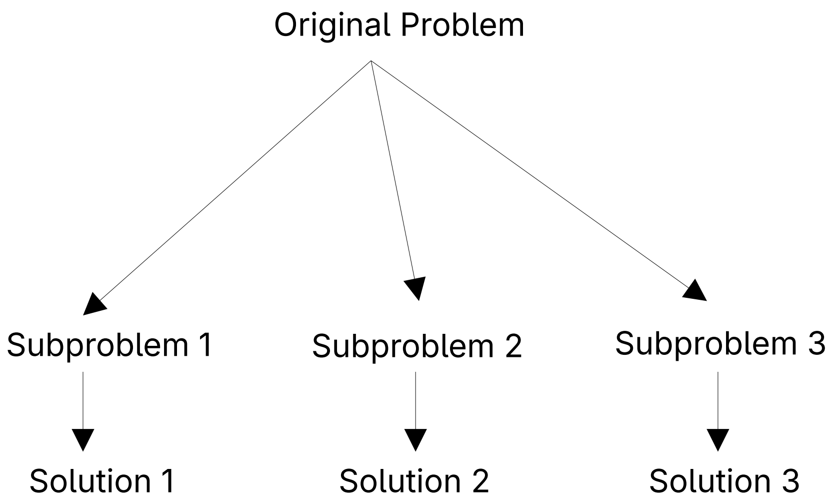
The divide and conquer paradigm involves three steps at each level of the recursion:

- **Divide** the problem into a number of subproblems that are smaller instances of the same problem.



Divide and Conquer Approach

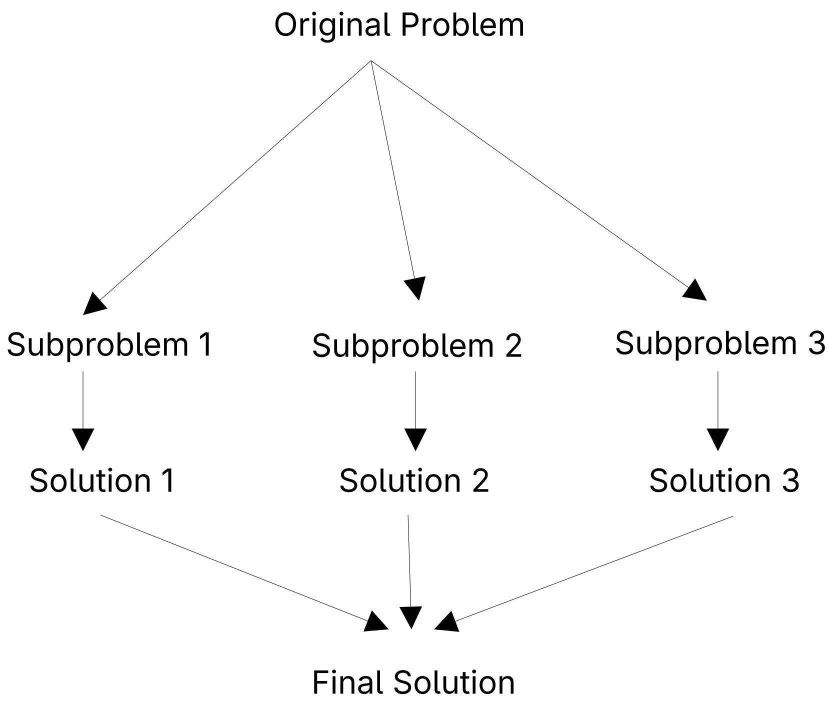
- **Conquer** the subproblems by solving them recursively. If the subproblem sizes are small enough, however, just solve the subproblems in a straightforward manner.



Divide and Conquer Approach

- **Combine** the solutions to the subproblems into the solution for the original problem.

Examples: Merge sort, Quick sort, Binary search.

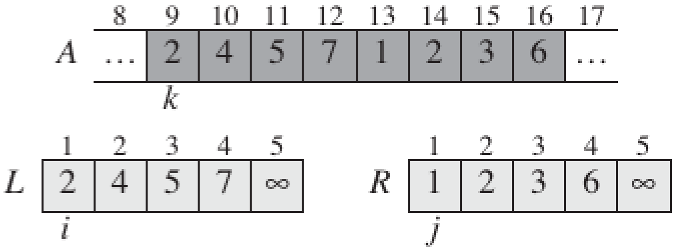


Merge Sort

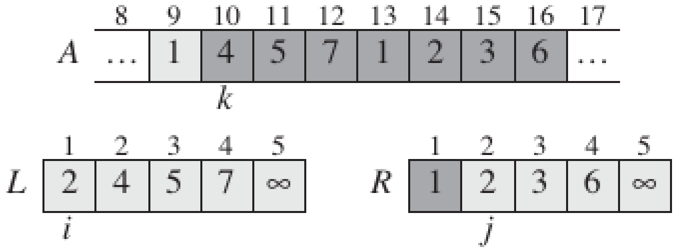
The merge sort algorithm closely follows the divide-and-conquer paradigm. Intuitively, it operates as follows.

- **Divide:** Divide the n-element sequence to be sorted into two subsequences of n/2 elements each.
- **Conquer:** Sort the two subsequences recursively using merge sort.
- **Combine:** Merge the two sorted subsequences to produce the sorted answer.

Merge Sort Simulation



(a)



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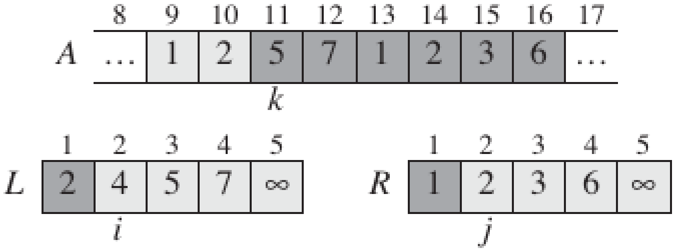

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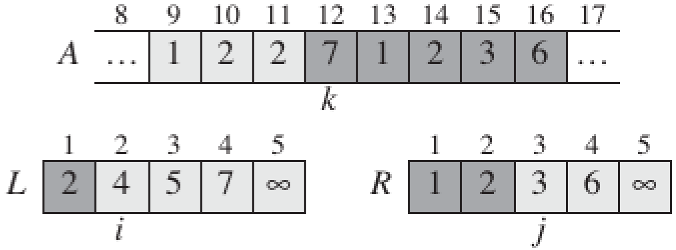

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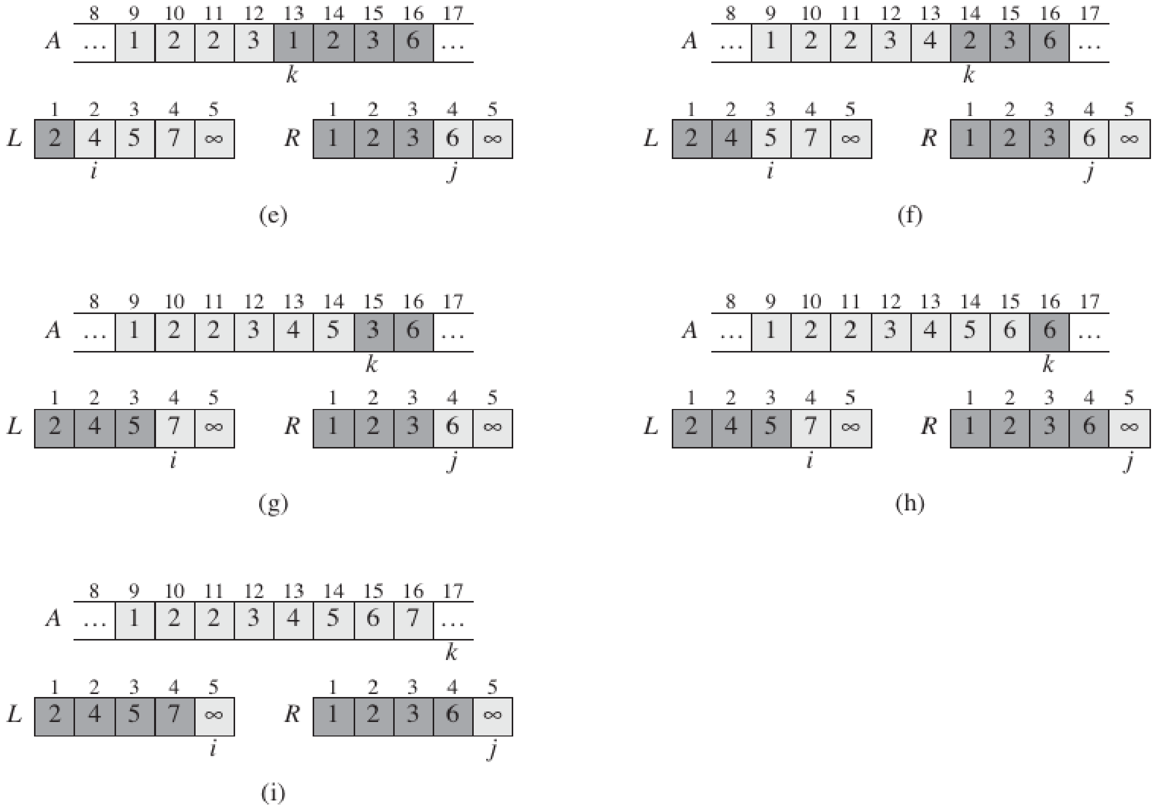
Merge Sort Simulation



Algorithms and Complexities

```
MERGE-SORT(A, p, r)
if p < r
    q = ⌊(p + r)/2⌋
    MERGE-SORT(A, p, q)
    MERGE-SORT(A, q + 1, r)
    MERGE(A, p, q, r)
```

- **Time Complexity:** O(n log n) in all cases. MERGE procedure takes time Θ(n) at each of the log n levels of recursion.

- **Space Complexity:** O(n) because it requires an auxiliary array of size proportional to the input array size, n, to hold the merged elements during the sorting process.

```
MERGE(A, p, q, r)
n1 = q − p + 1
n2 = r − q
let L[1 .. n1 + 1] and R[1 .. n2 + 1] be new arrays
for i = 1 to n1
    L[i] = A[p + i − 1]
for j = 1 to n2
    R[j] = A[q + j]
L[n1 + 1] = ∞
R[n2 + 1] = ∞
i = 1
j = 1
for k = p to r
    if L[i] ≤ R[j]
        A[k] = L[i]
        i = i + 1
    else A[k] = R[j]
        j = j + 1
```

Quick Sort

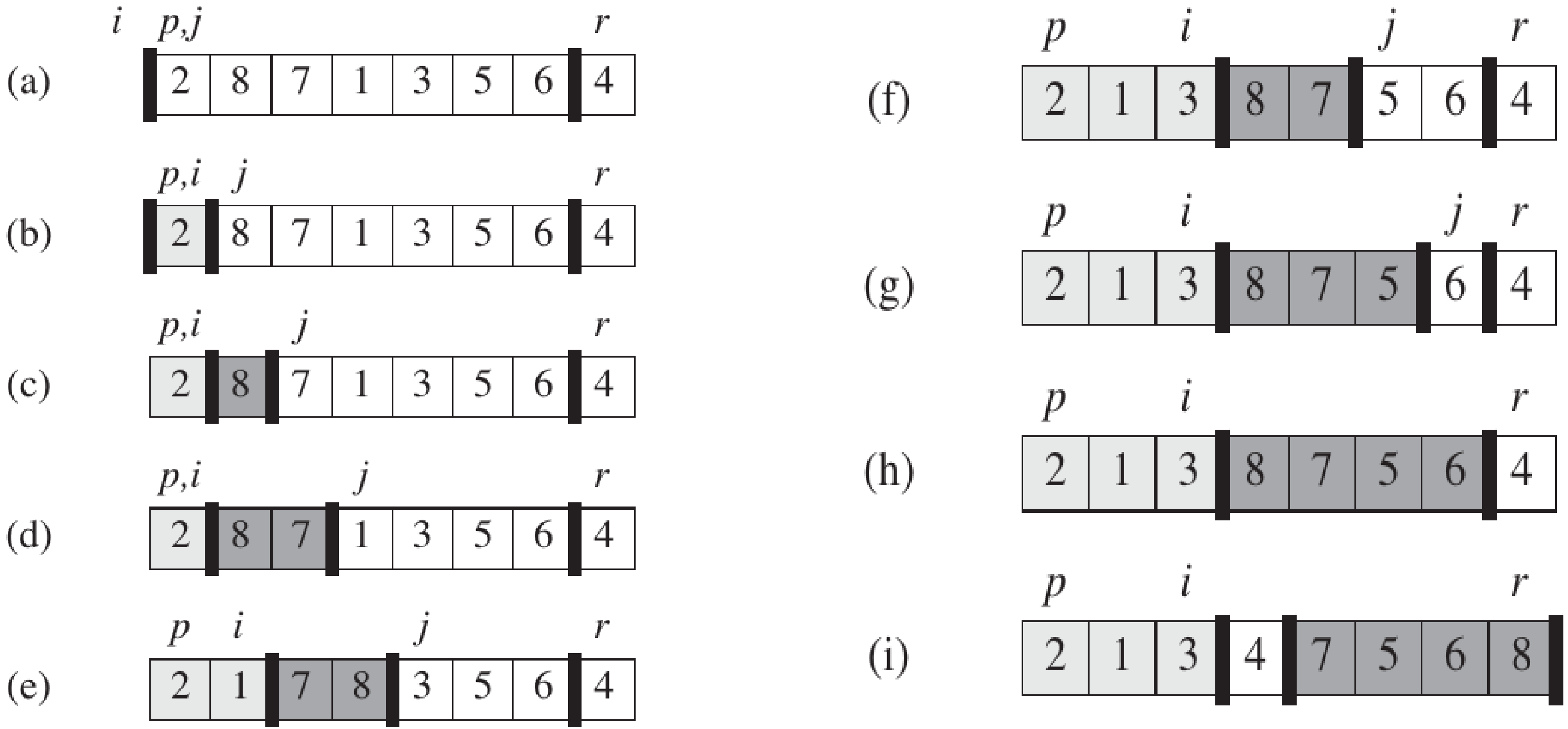
Here is the three-step divide-and-conquer process for sorting a typical subarray A[p ... r]:

- **Divide:** Partition (rearrange) the array A[p ... r] into two (possibly empty) subarrays A[p ... q-1] and A[q+1 ... r] such that each element of A[p ... q-1] is less than or equal to A[q], which is, in turn, less than or equal to each element of A[q+1 ... r]. Compute the index q as part of this partitioning procedure.

- **Conquer:** Sort the two subarrays A[p ... q-1] and A[q+1 ... r] by recursive calls to quicksort.

- **Combine:** Because the subarrays are already sorted, no work is needed to combine them. The entire array A[p ... r] is now sorted.

The Partition Operation



Algorithms and Complexities

```
QUICKSORT(A, p, r)
if p < r
    q = PARTITION(A, p, r)
    QUICKSORT(A, p, q − 1)
    QUICKSORT(A, q + 1, r)
```

```
PARTITION(A, p, r)
x = A[r]
i = p − 1
for j = p to r − 1
    if A[j] ≤ x
        i = i + 1
        exchange A[i] with A[j]
exchange A[i + 1] with A[r]
return i + 1
```

To sort an entire array A, the initial call is QUICKSORT(A, 1, A.length).

- **Time Complexity:** $O(n^2)$ in the worst case when the pivot selection consistently results in highly unbalanced partitions (e.g., always picking the smallest or largest element). Time complexity is O(n log n) in other cases.

- **Space Complexity:** Typically O(log n), because it uses a recursive call stack whose maximum depth is proportional to the level of partitions.

Reference

- *Introduction to Algorithms, Third Edition*
  - *Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein*