Deep Generative Models

Thomas Lucas & Jakob Verbeek

Content of this talk

Focus of this talk: Latent variable deep generative models

- Part I: Generative adversarial networks (GANs)
- Part II: Variational auto-encoders (VAEs)

Latent variable generative models

Reminder:

- A latent variable generative model defines $p_\theta(x) = \int_z p(z)p_\theta(x|z)dz$
- A VAE maximises a lower bound of $\mathbb{E}_{x\sim p^*}(log(p_\theta(x)))$ called the ELBO
- A GAN avoids intractable integrals using a discriminator

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    - Explicitely optimize coverage of training points
    - implicitely optimize sample quality (density is **normalized**)

Training support VS sample quality

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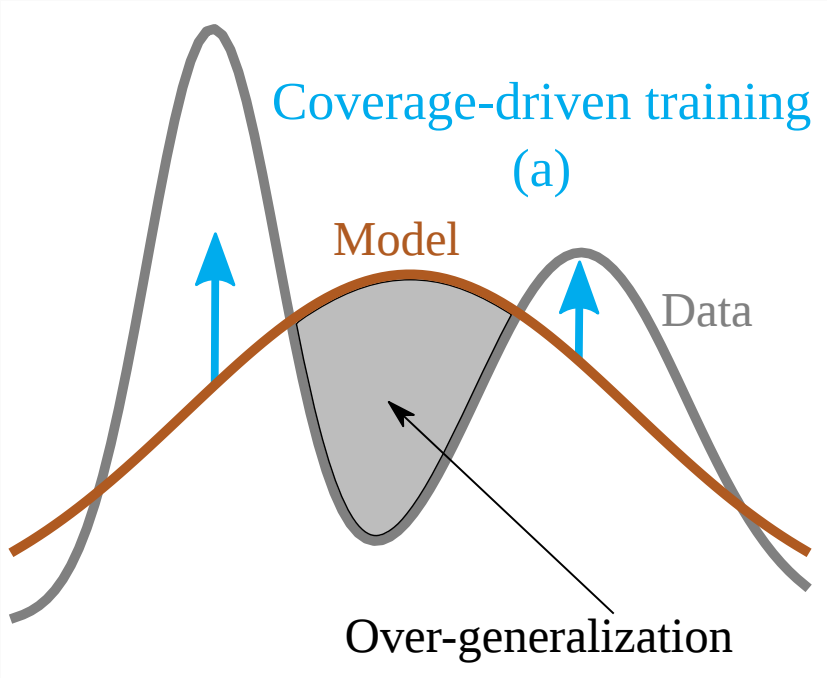
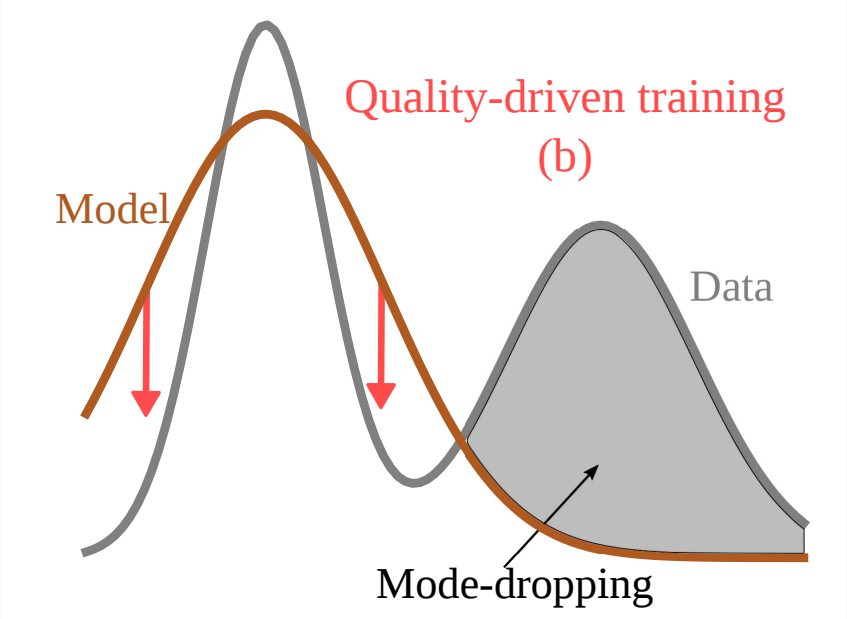
- Maximum-likelihood: sample from p^*, evaluate probability under p_θ.
  - Explicitely optimize coverage of training points
  - implicitely optimize sample quality (density is **normalized**)
- Adversarial training: sample from p_θ, "evaluate" probability under p^*.
  - implicitely optimize coverage of training points (**normalise**)
  - Explicitely optimize sample quality

Different paradigms





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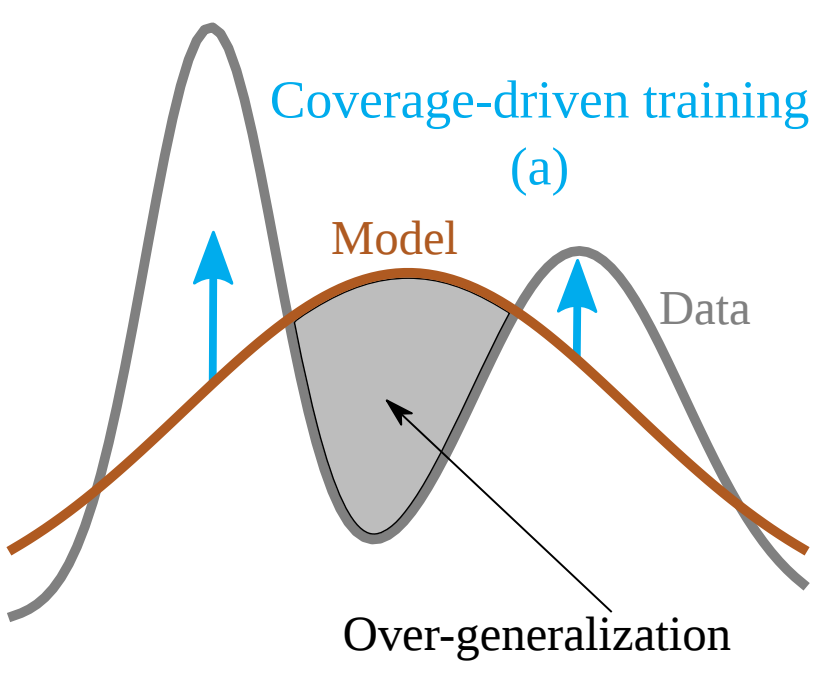

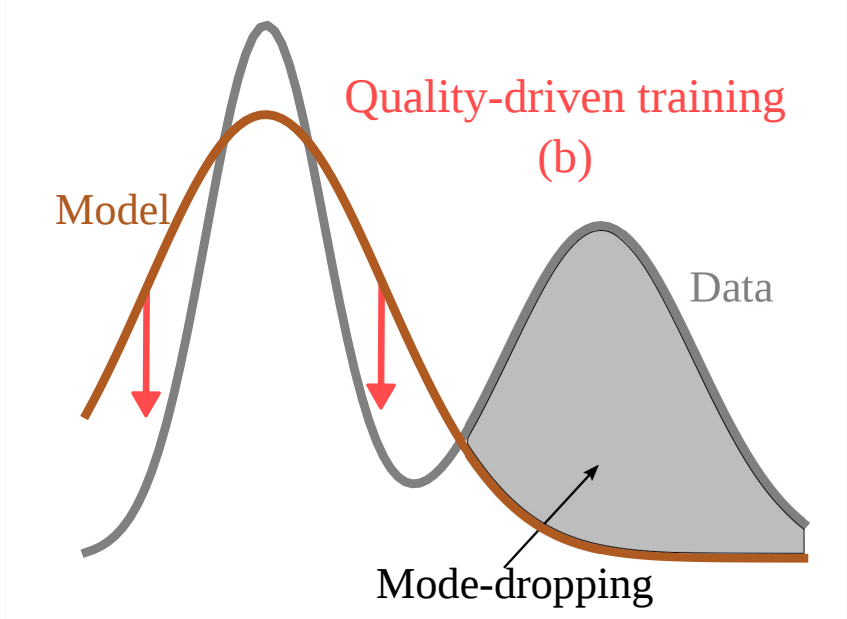

If p_θ is less flexible than p^*:

- Maximum-likelihood: p_θ goes through regions of low density: Over-generalization
- Adversarial: p_θ drops some modes to avoid low density: mode-dropping

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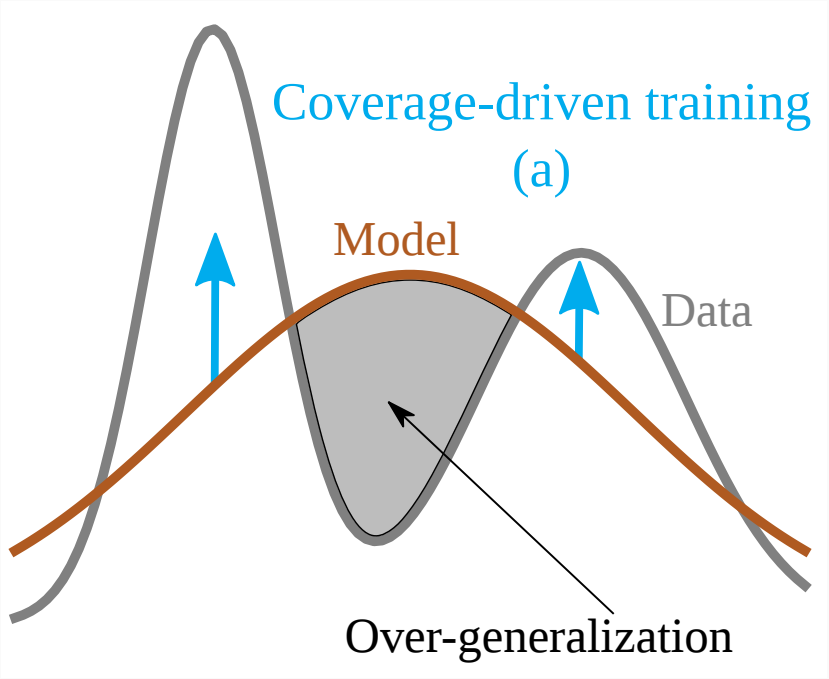

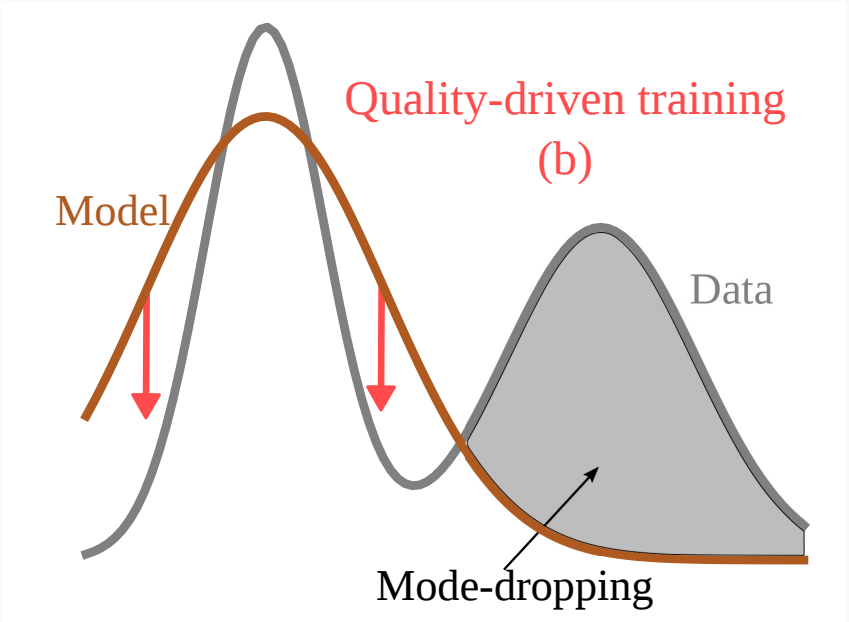

From this perspective:

- You can expect GANs to produce better looking images
- You can expect VAEs to better compress the training data

Some research directions

Improving on VAEs and GANs:

- Better losses for GANs: encourage full support, improve stability.
- Avoiding poor assumptions in VAEs to reduce over-generalization.

Part I

Stabilizing GAN training

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- Jensen-Shannon divergence: idealized loss approximated by the discriminator
  - Symmetric KL to mixture of p and q

$$D_{JS}(p||q) = \frac{1}{2}D_{KL}\left(p\middle|\middle|\frac{p+q}{2}\right) + \frac{1}{2}D_{KL}\left(q\middle|\middle|\frac{p+q}{2}\right) \tag{2}$$

A discussion on the GAN training loss

- Training loss for the Discriminator:

$$V(\phi, \theta) = \mathbb{E}_{x \sim p_{\text{data}}(x)}[\ln D_\phi(x)] + \mathbb{E}_{z \sim p(z)}[\ln(1 - D_\phi(f_\theta(z)))] \quad (3)$$

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- The generator is trained on the red term.

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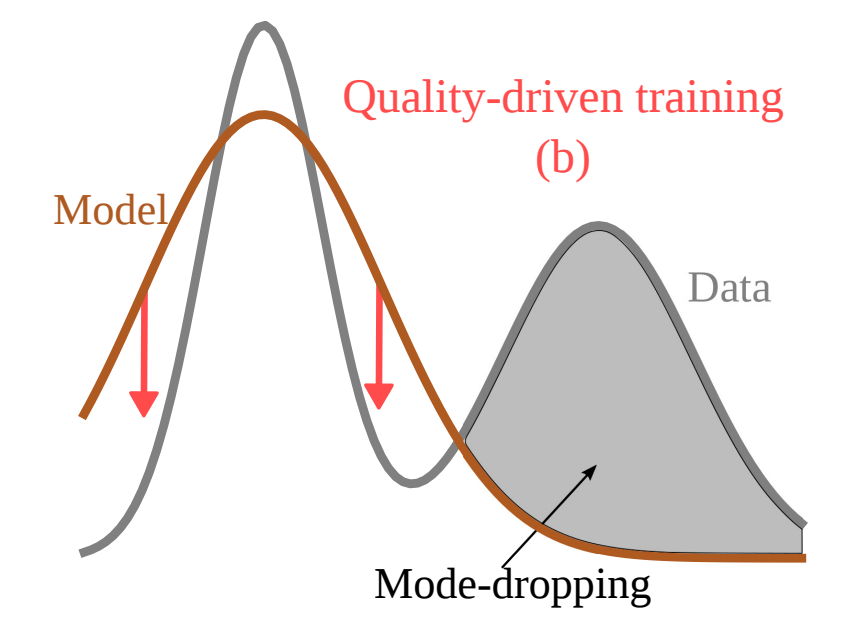

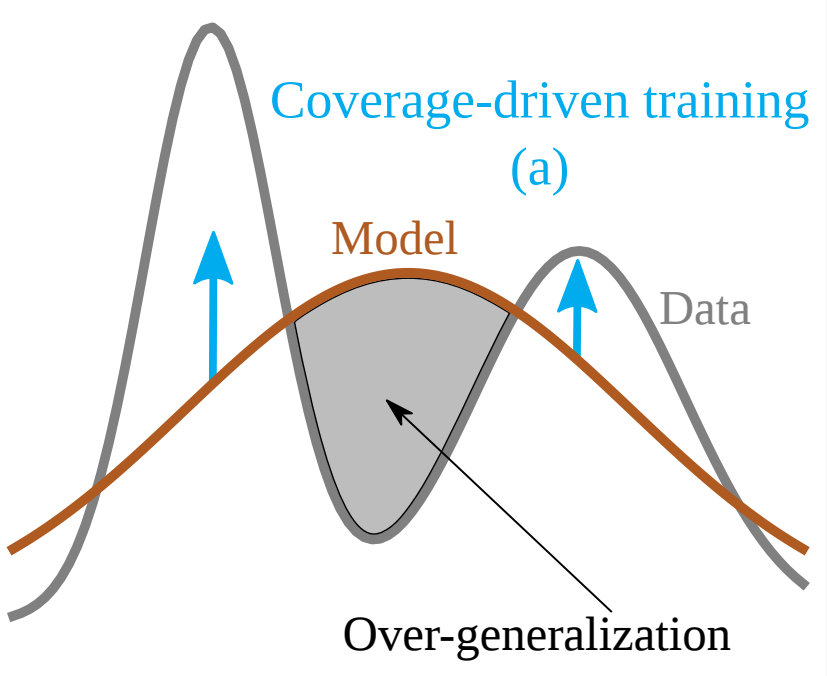

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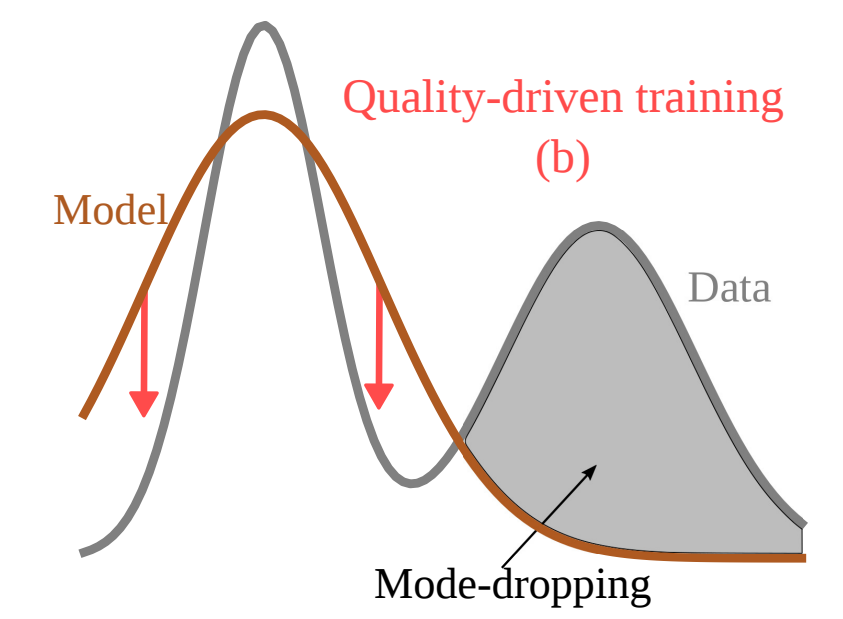

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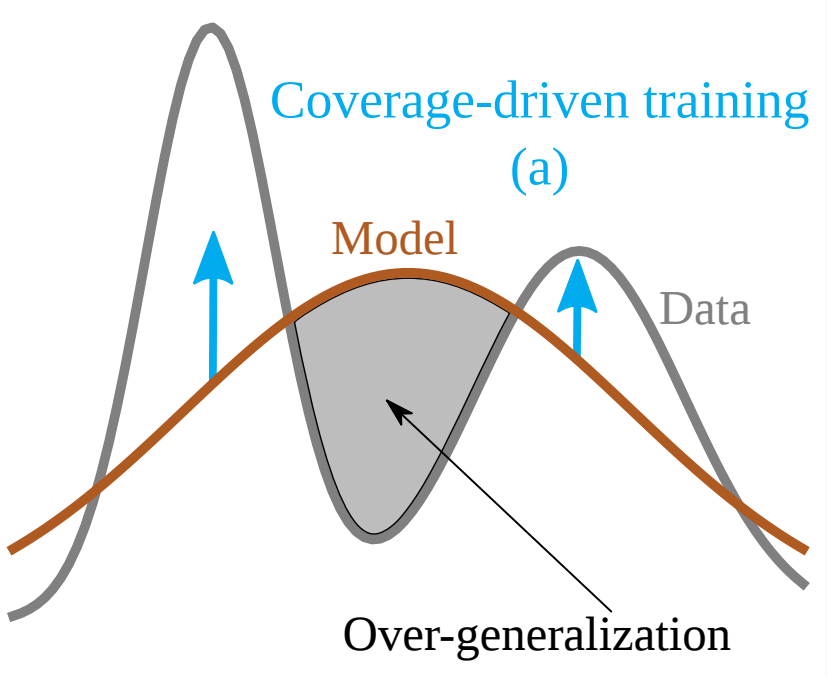

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   - Same stable points in the minimax optimization
   - Helps early, but problem remains: if D_ϕ becomes too good, gradients vanish

Question:

Can we think of a better 'ideal loss'?

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- Wasserstein distance is the cost of optimal transformation

$$D_{WS}(p||q) = \inf_{\gamma \in \Gamma(p,q)} T(\gamma) \tag{5}$$

Distributions with low dimensional support

- Simple example: support on lines in $\mathbb{R}^2$
  - p_0 uniform on $x_2 \in [0, 1]$ for $x_1 = 0$
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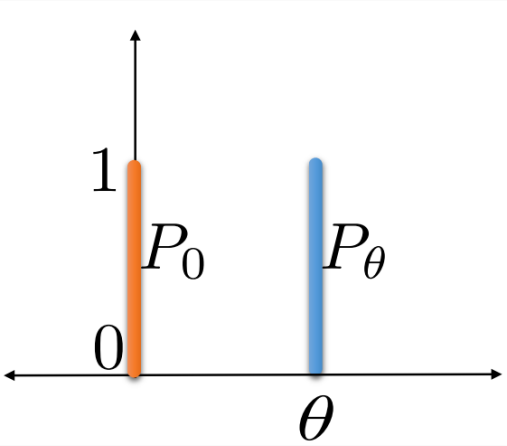

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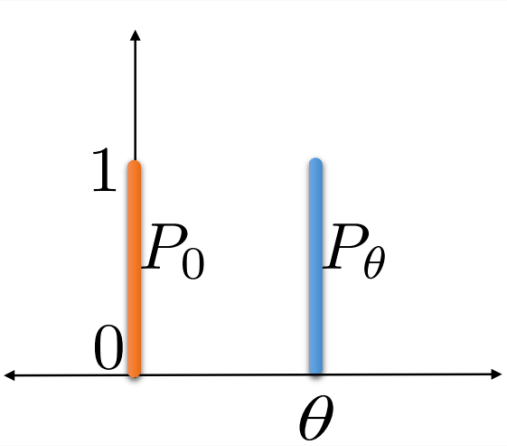

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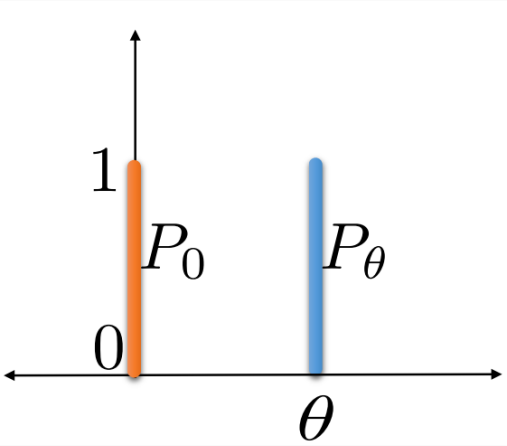

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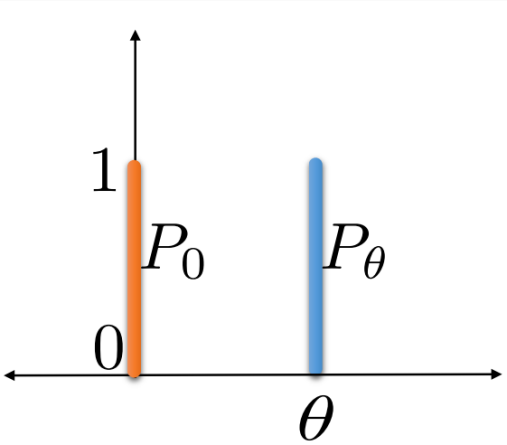

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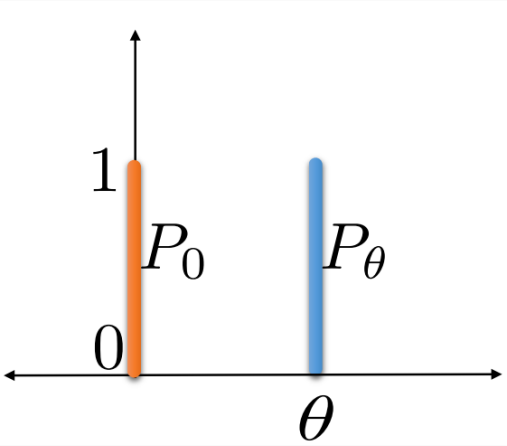

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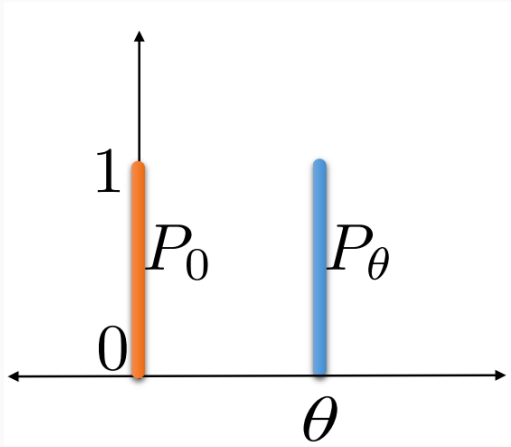

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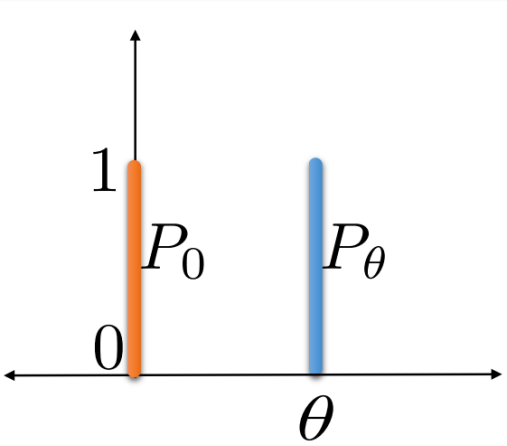

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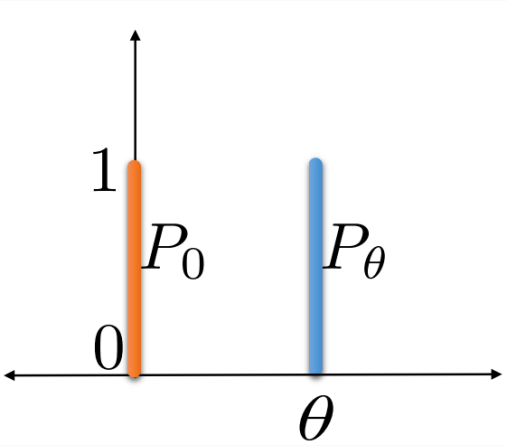

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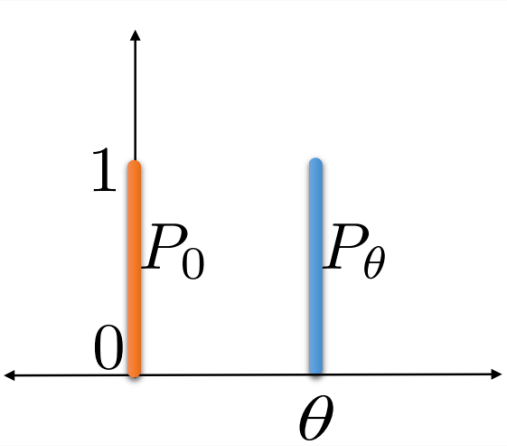

- Wasserstein based on proximity of support
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  - GAN has support with dimension of latent variable **z**

Wasserstein GAN

- Dual formulation of Wasserstein distance

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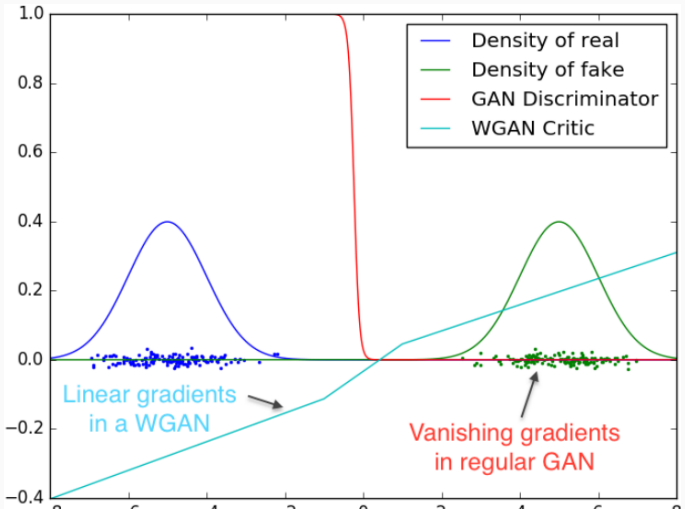
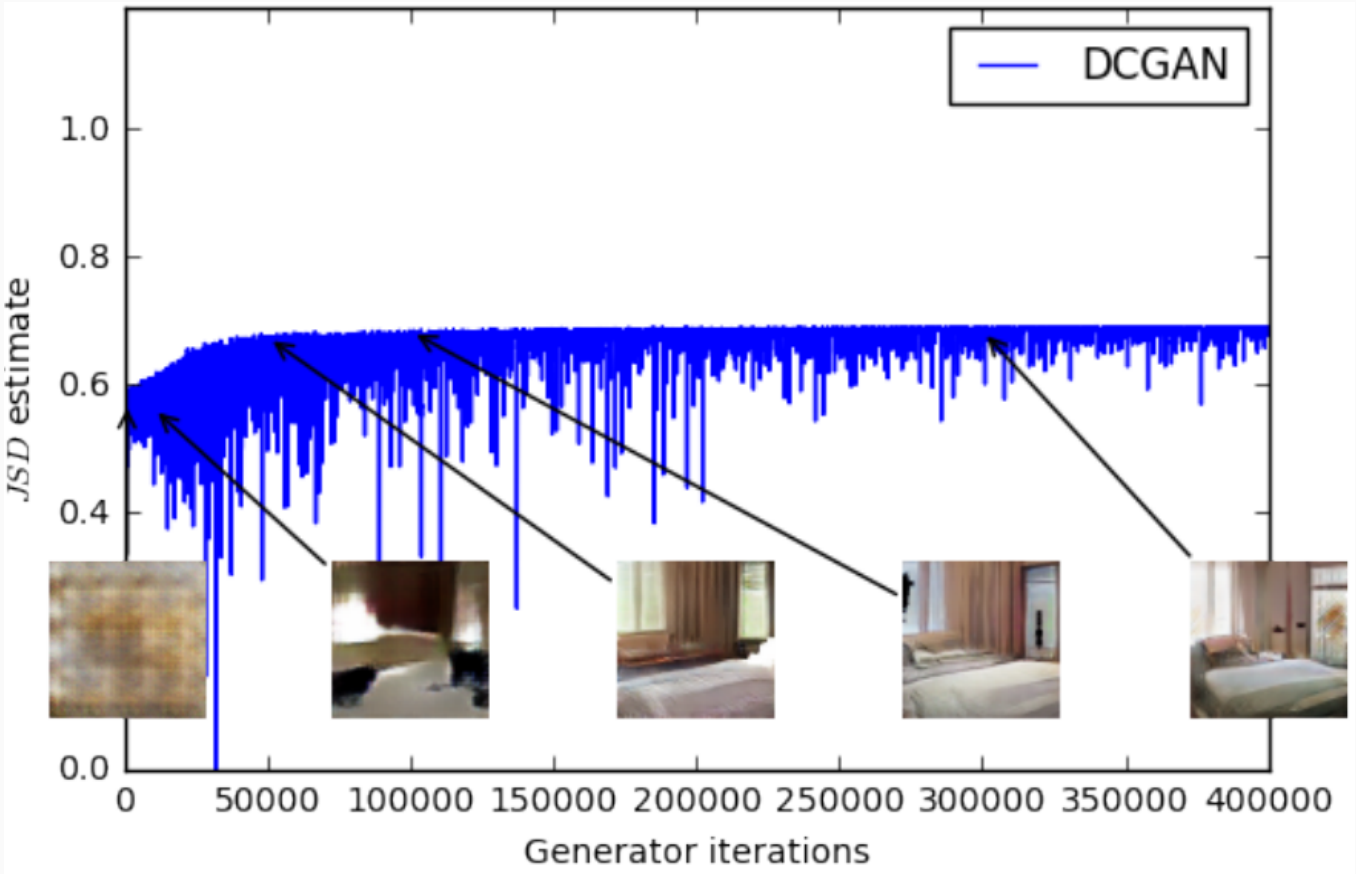


Experimental comparison GAN and WGAN

- WGAN loss may decrease in a more stable manner
- WGAN loss may give better correlation between loss and sample quality



GAN

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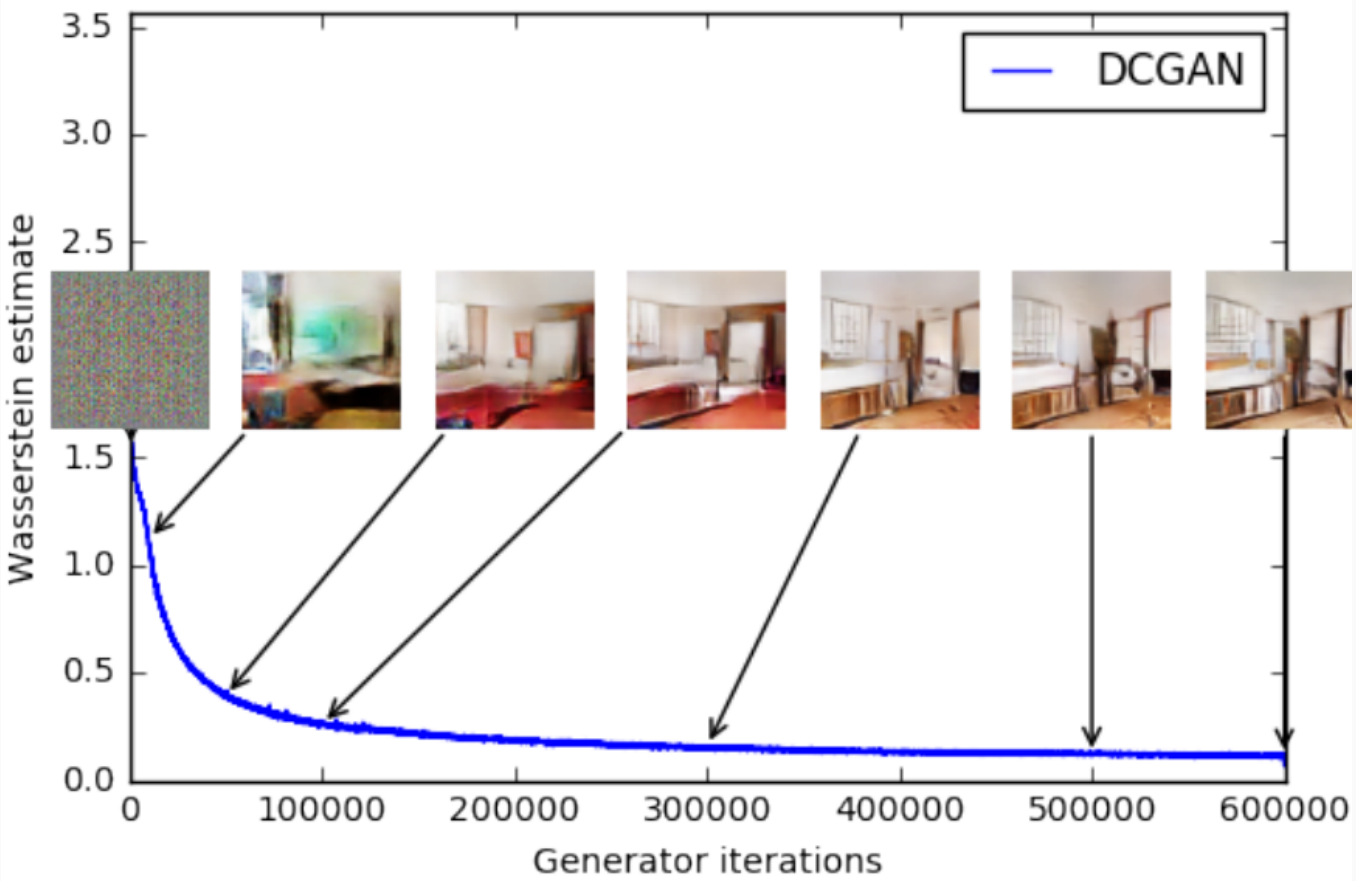

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- Constraining the Lipshitz constant is a good regularizer
- Removing the log avoids vanishing gradients

Lipschitz continuity as a regularizer

- **Reminder:** k-Lispschitz means $|f(x) - f(y)| \leq |x - y|$
- **Reminder:** For linear functions, seek the largest eigenvalue

Lipschitz continuity as a regularizer

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- Gradient penalty [Gulrajani et. al, 2017]
  - Add a penalty to the loss:

$$G_{\text{pen}} = \lambda \mathbb{E}_x[||\nabla_x D(x)||_2 - 1)^2]$$

Wrap up

- A **lot** of other losses have been develloped
- The lipschitz regularization is a widely adopted regularization
- The log is usually avoided to improve gradients when Discriminator is good.

Part II

Adversarial Inference

Latent variable inference in GANs [Donahue et al., 2017]

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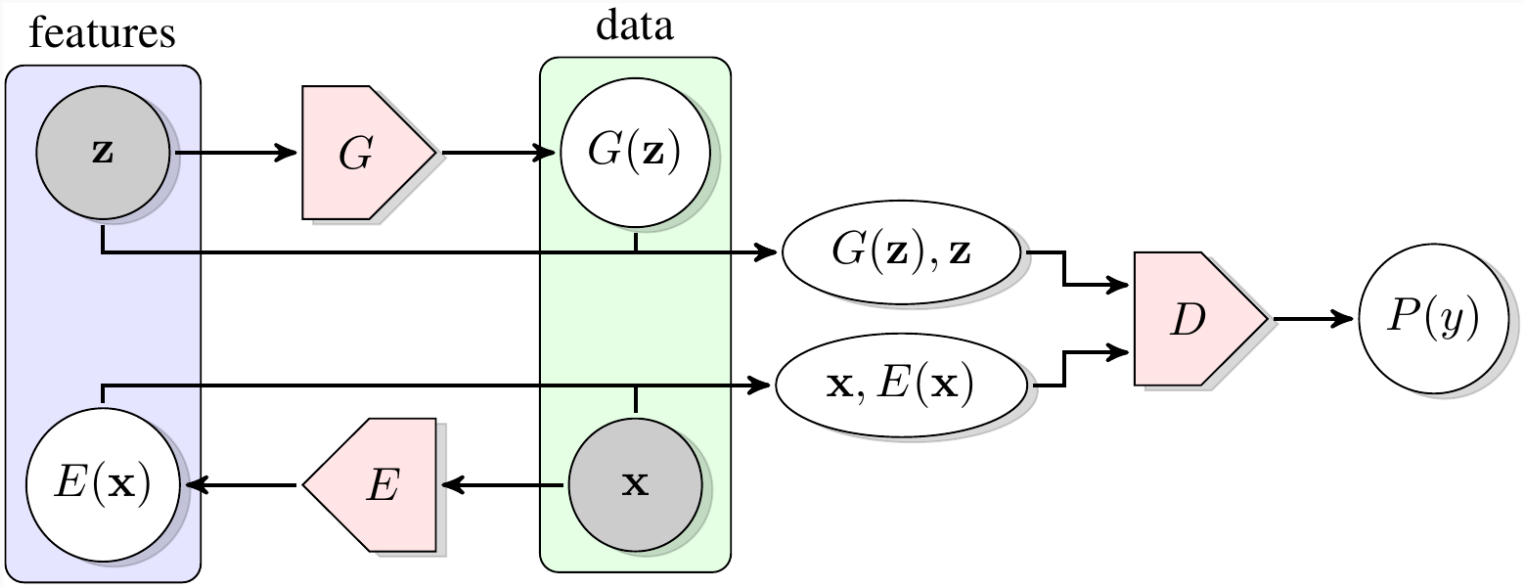
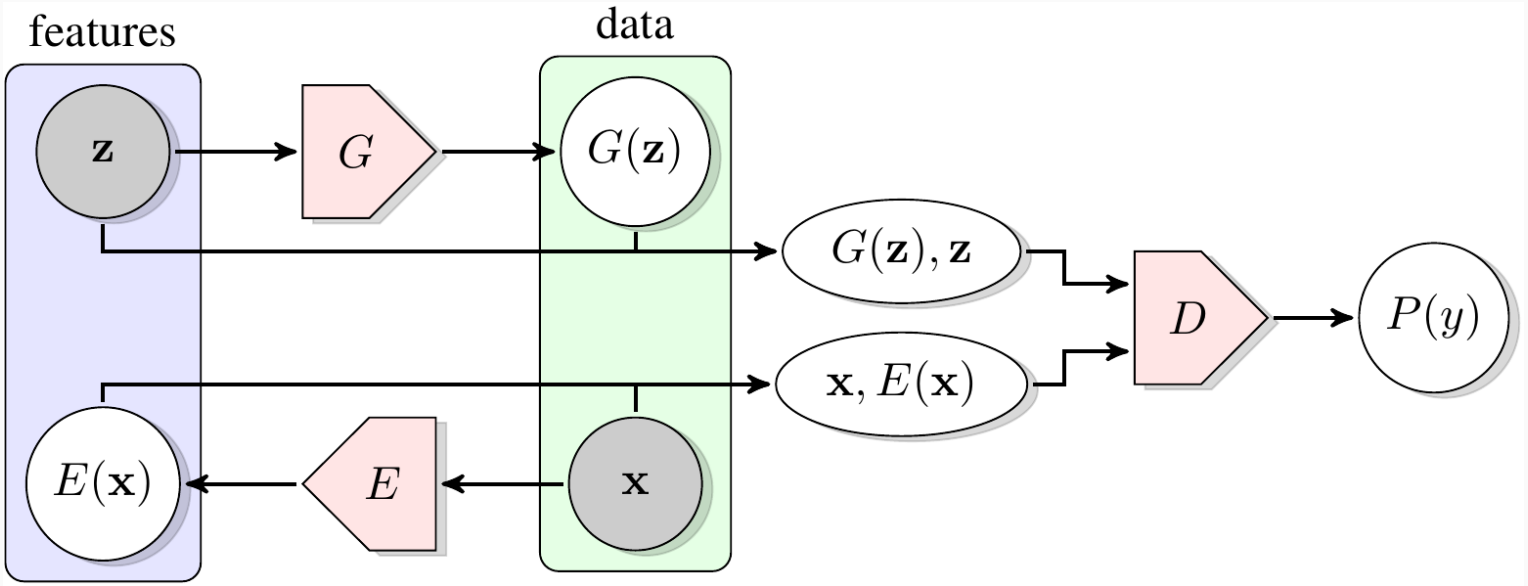


Induced joint distributions over $(\mathbf{x}, \mathbf{z})$



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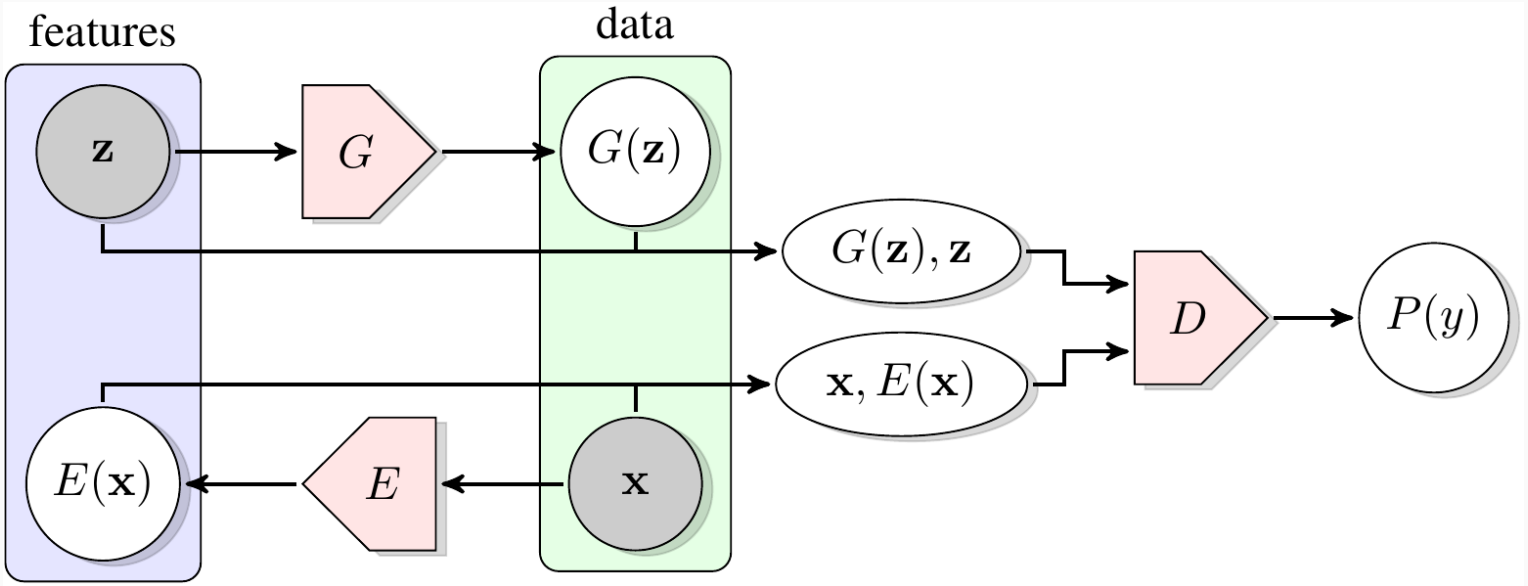

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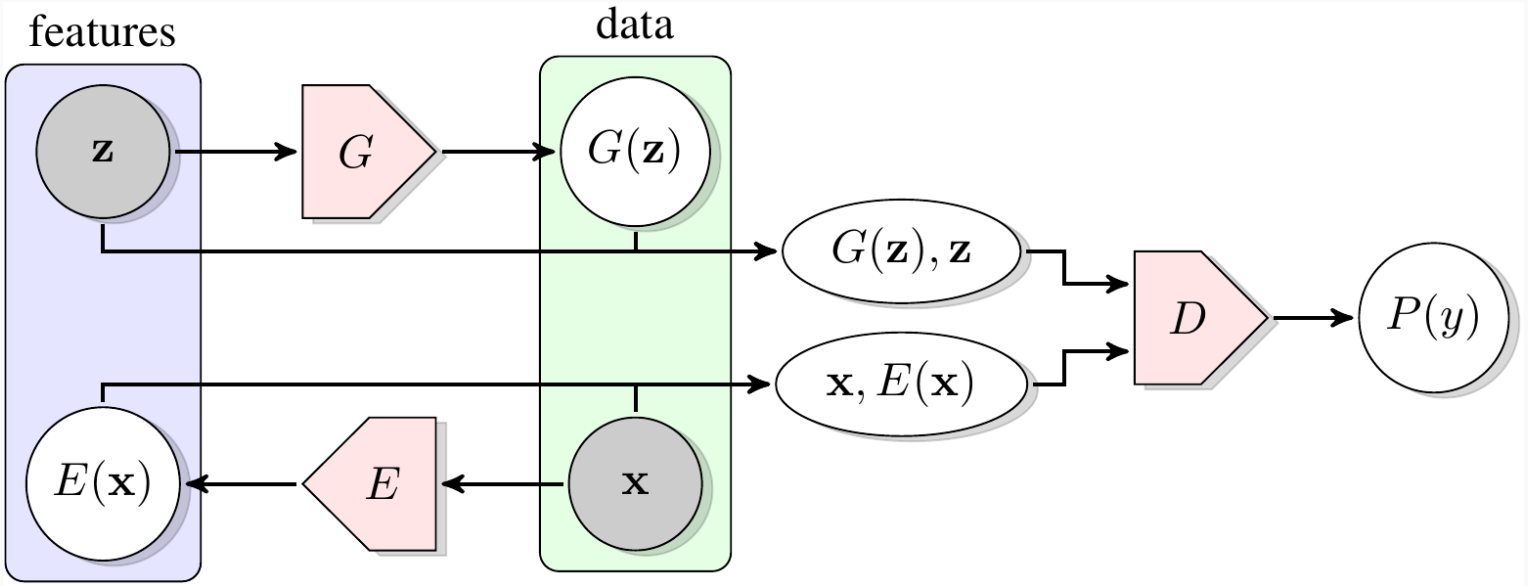

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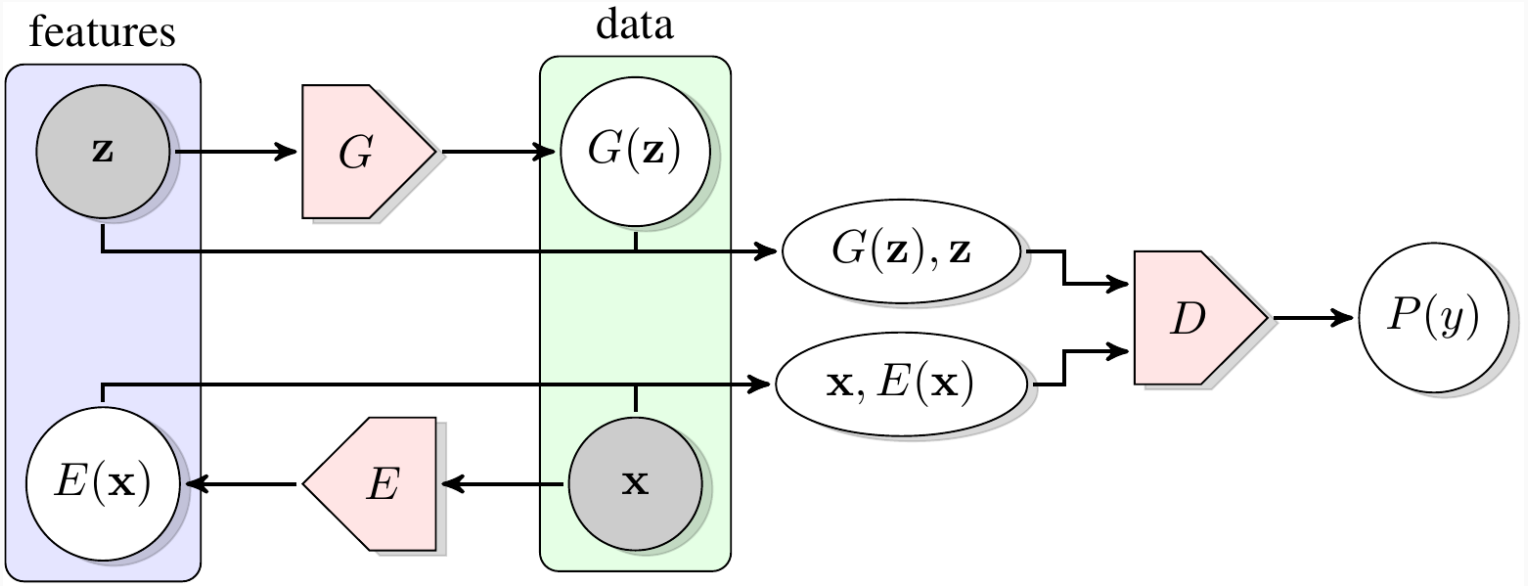

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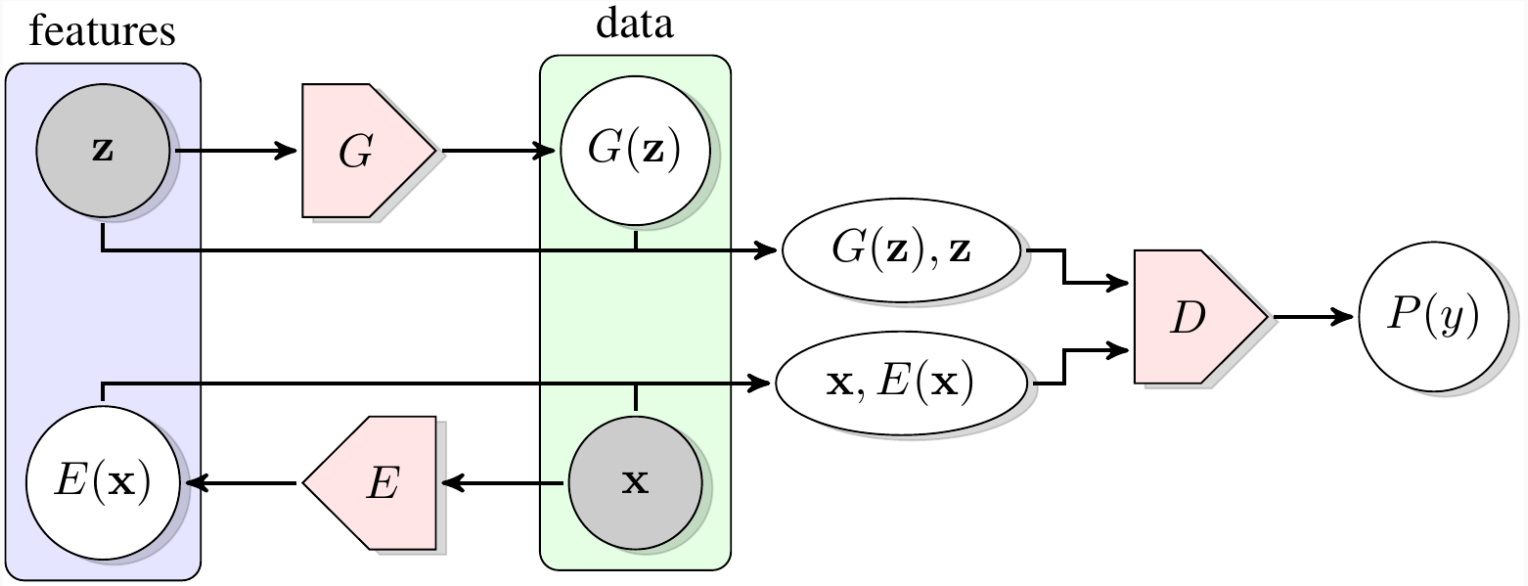
- Discriminator: pair $(\mathbf{x}, \mathbf{z})$ completed by generator or encoder?

Bidirectional GANs [Donahue et al., 2017]



$$V(D, E, G) = \mathbb{E}_{p_{\text{data}}}[\ln D(\mathbf{x}, E(\mathbf{x}))] + \mathbb{E}_{p(\mathbf{z})}[\ln(1 - D(G(\mathbf{z}), \mathbf{z}))]$$

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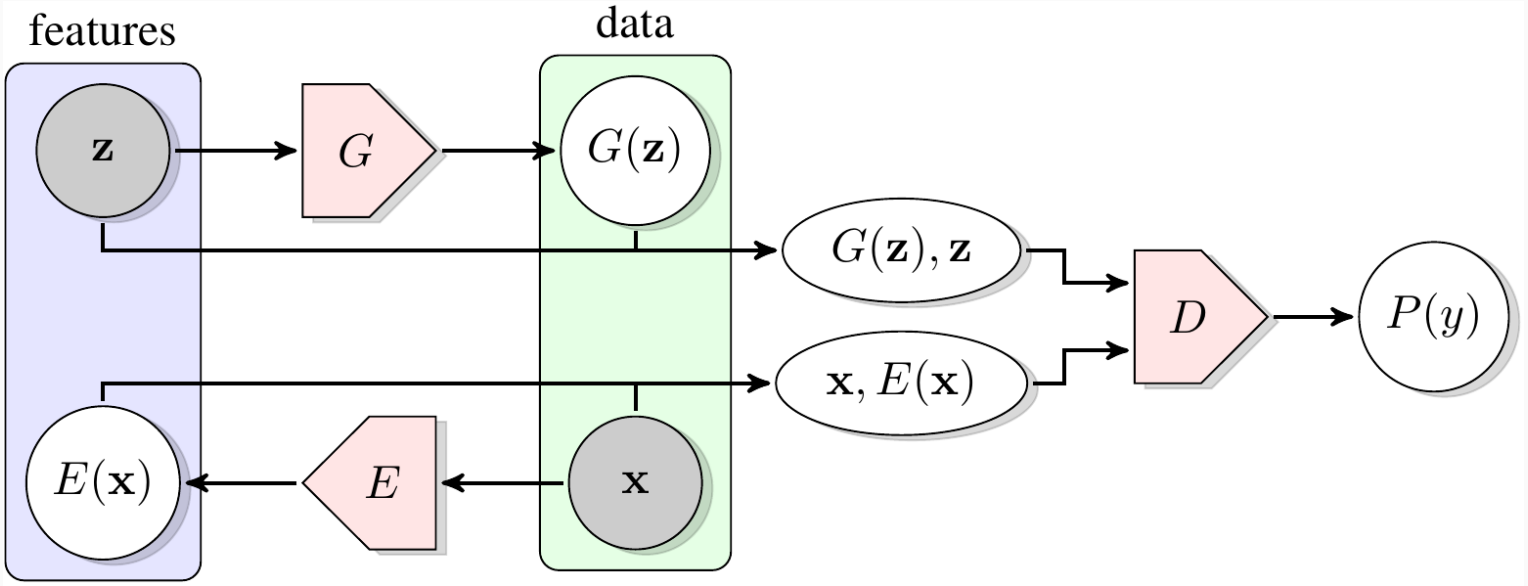

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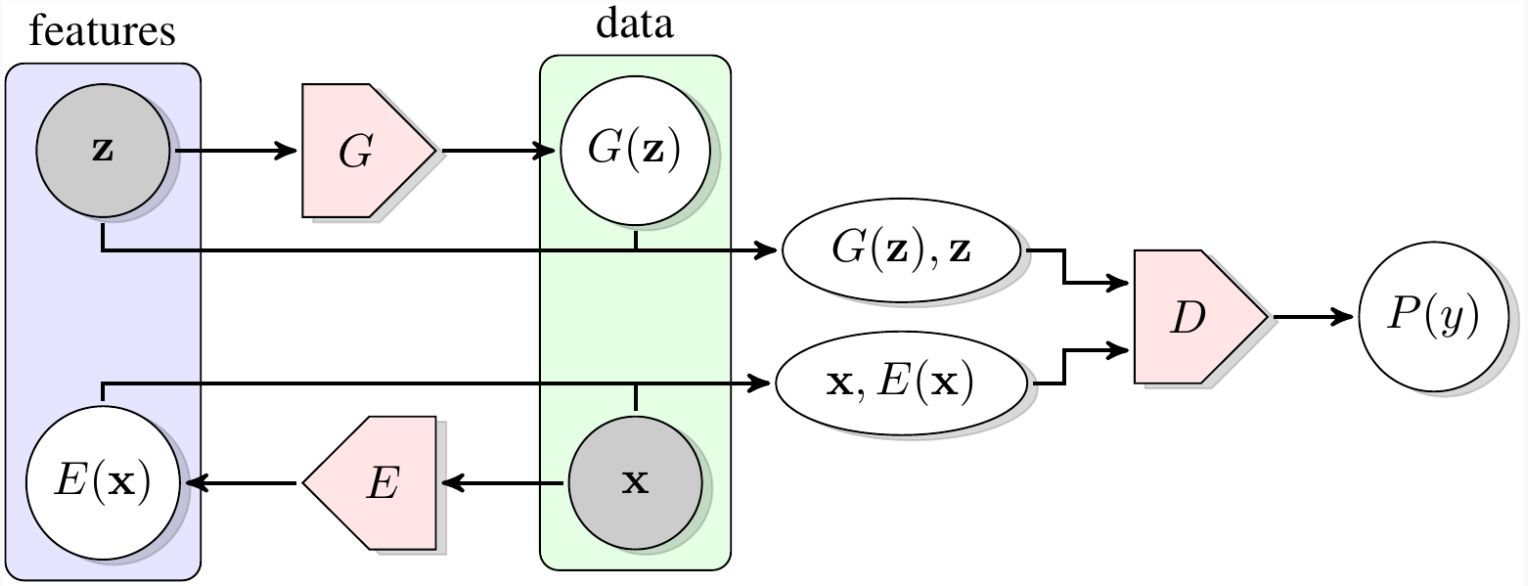

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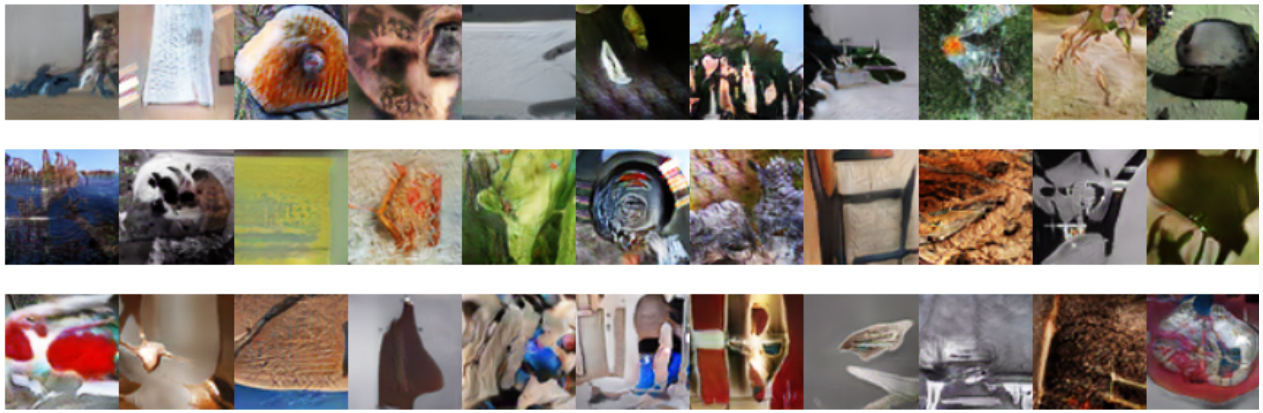
- At optimum G and E are each others inverse

BiGAN samples, ImageNet 64×64

$G(\mathbf{z})$



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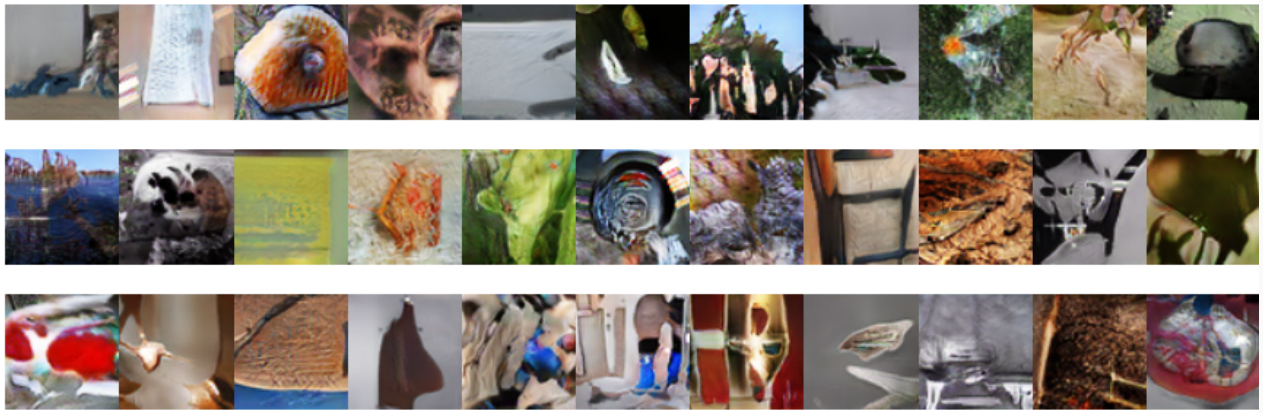

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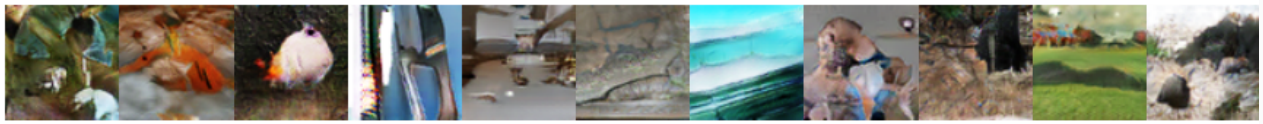

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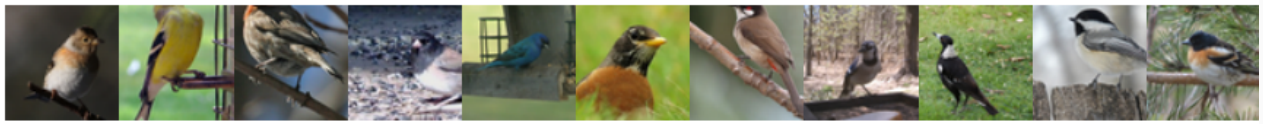

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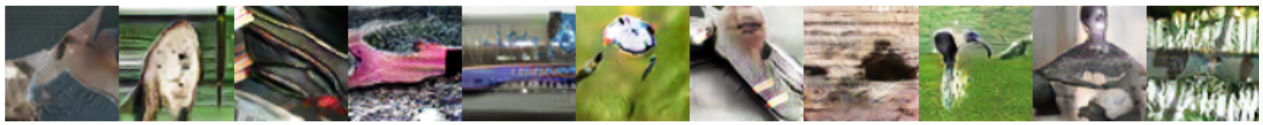
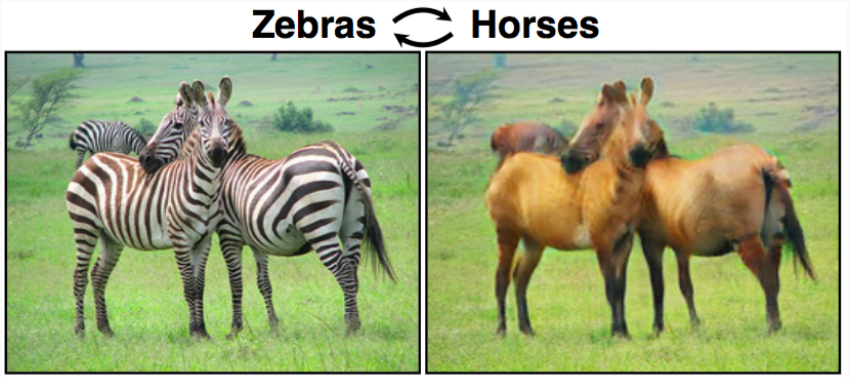


BiGAN: feature transfer to PASCAL VOC'07

	trained layers	Classification (% mAP) fc8	fc6-8	all	*FRCN* Detection (% mAP) all	*FCN* Segmentation (% mIU) all
sup.	ImageNet (Krizhevsky et al., 2012)	**77.0**	**78.8**	**78.3**	**56.8**	**48.0**
self-sup.	Agrawal et al. (2015)	31.2	31.0	54.2	43.9	-
	Pathak et al. (2016)	30.5	34.6	56.5	44.5	**30.0**
	Wang & Gupta (2015)	28.4	**55.6**	63.1	47.4	-
	Doersch et al. (2015)	**44.7**	55.1	**65.3**	**51.1**	-
unsup.	k-means (Krähenbühl et al., 2016)	32.0	39.2	56.6	45.6	32.6
	Discriminator (D)	30.7	40.5	56.4	-	-
	Latent Regressor (LR)	36.9	47.9	57.1	-	-
	Joint LR	37.1	47.9	56.5	-	-
	Autoencoder (ℓ_2)	24.8	16.0	53.8	41.9	-
	BiGAN (ours)	37.5	48.7	58.9	46.2	34.9
	BiGAN, 112×112 E (ours)	**40.7**	**52.3**	**60.1**	**46.9**	**35.2**

- Encoder network used to pre-train/initialize recognition net
- Similar performance of BiGAN and self-supervised methods
  - Self-sup: CNN trained using image structure as supervision
  - Discriminator: uses GAN discriminator layers as features
  - Latent regressor: trains network to invert GAN (jointly)

Unpaired image-to-image translation [Zhu et al., 2017]



- Learn 2-way mapping between different image domains

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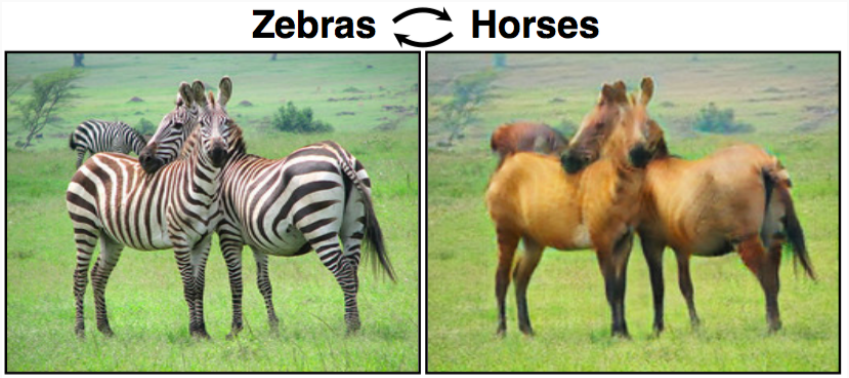

- Learn 2-way mapping between different image domains
- Without using supervised aligned training samples

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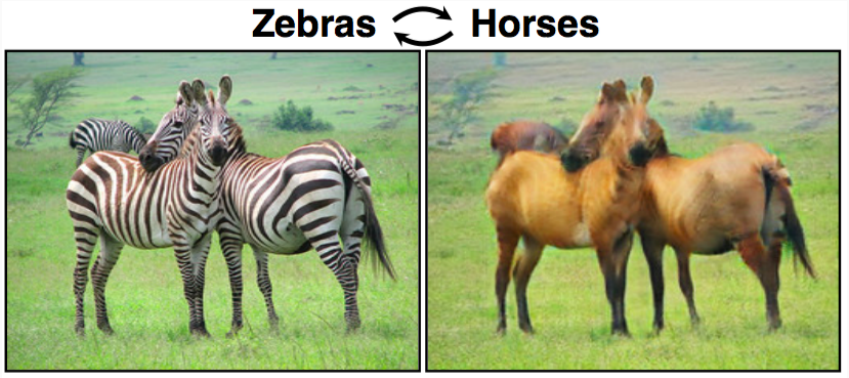

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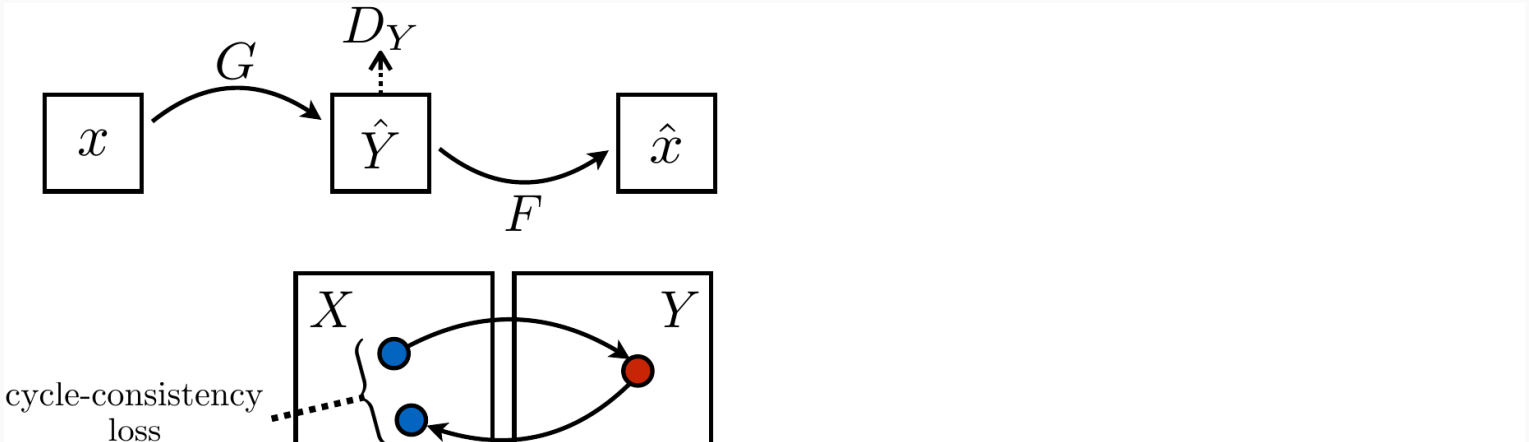

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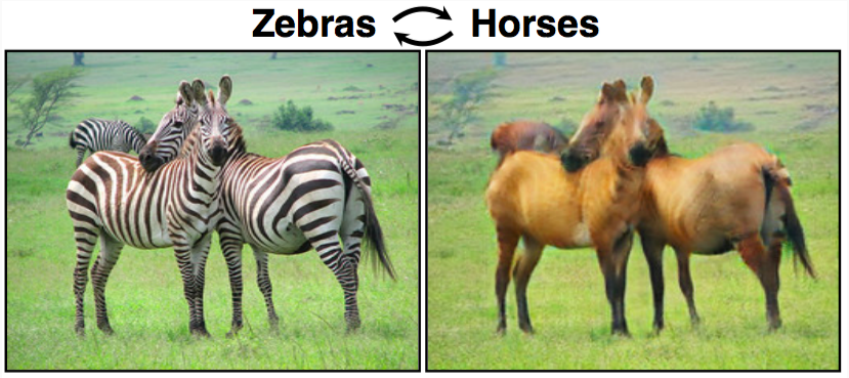

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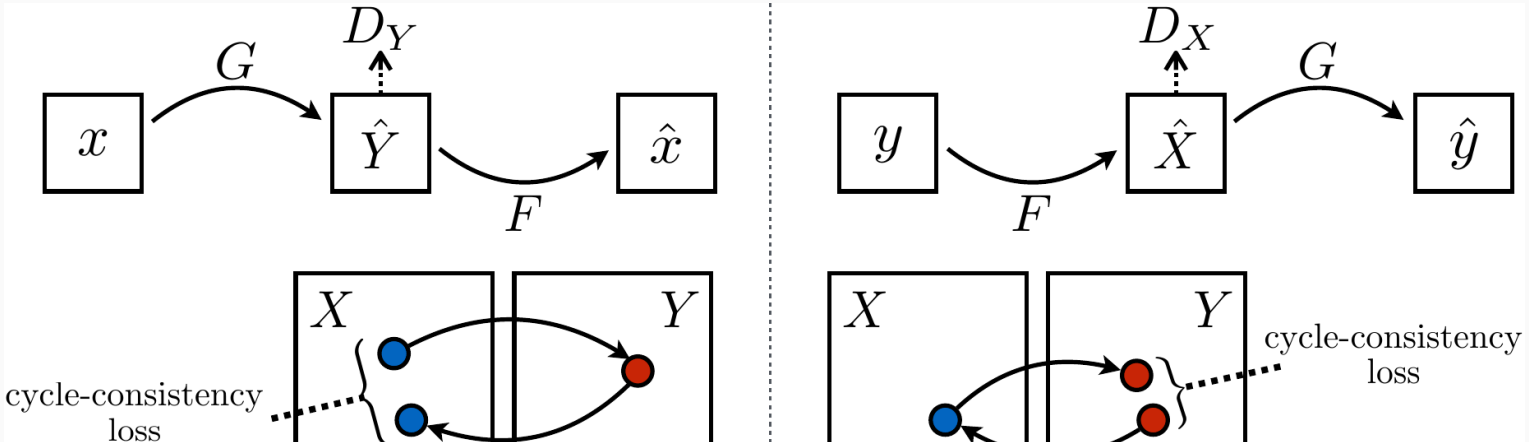
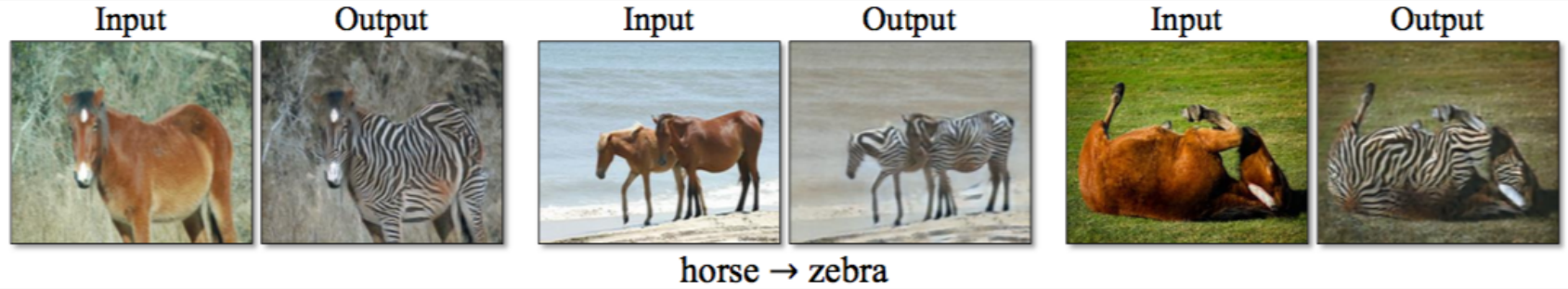
1. Discriminator ensures realistic samples in each domain
2. Cycle-consistency loss ensures alignment

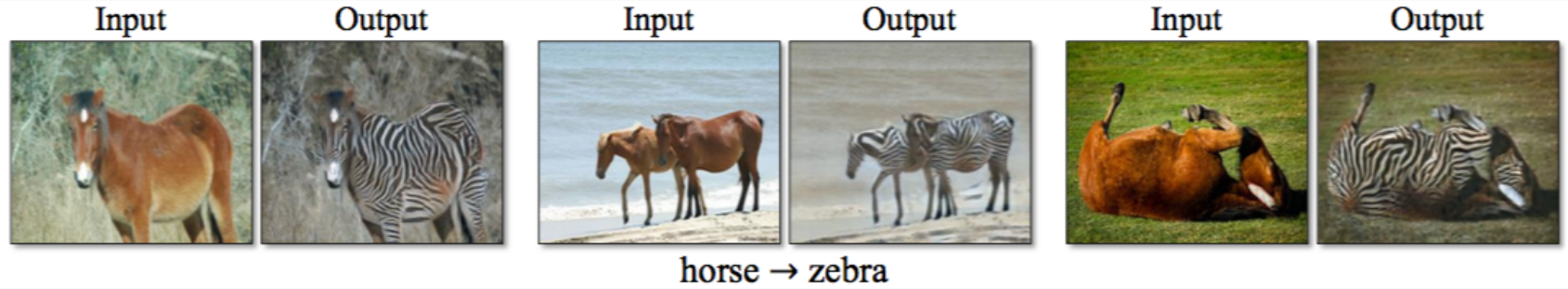


Some successful examples



Some successful examples

- Without using any supervised/aligned examples!



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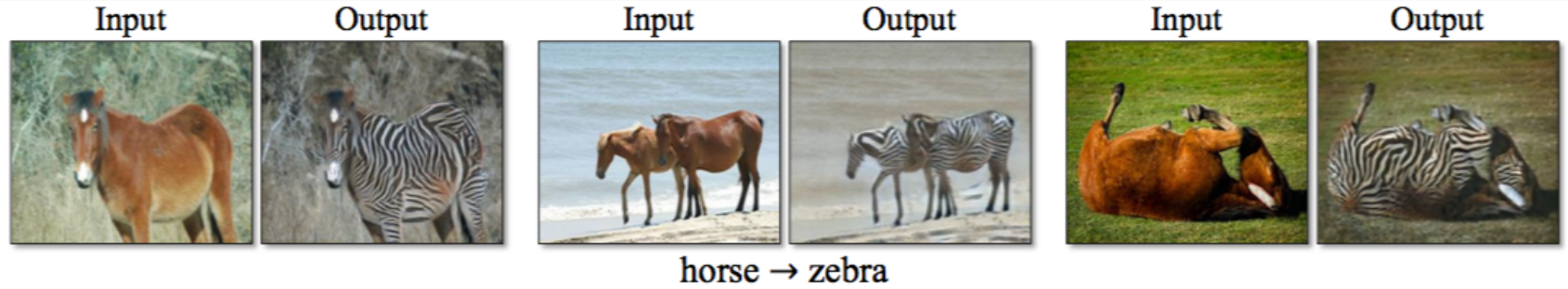

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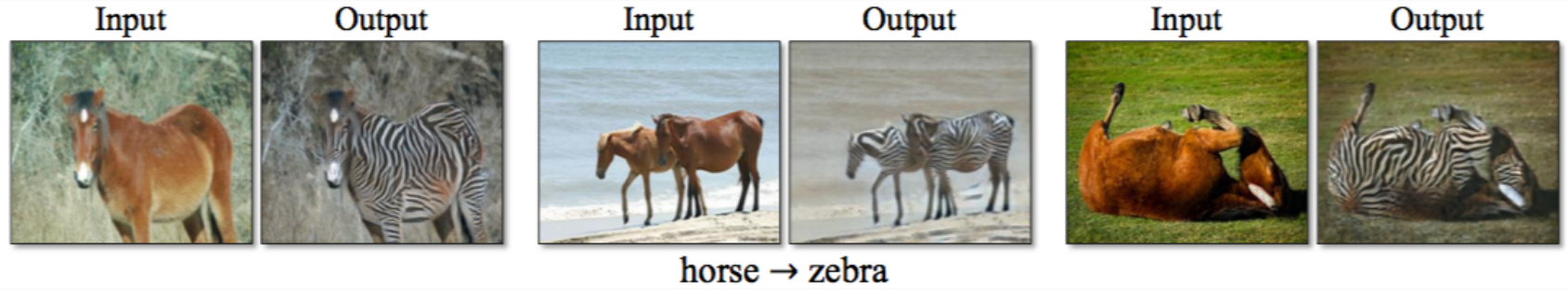


And a failure case

Wrap-up on GANs

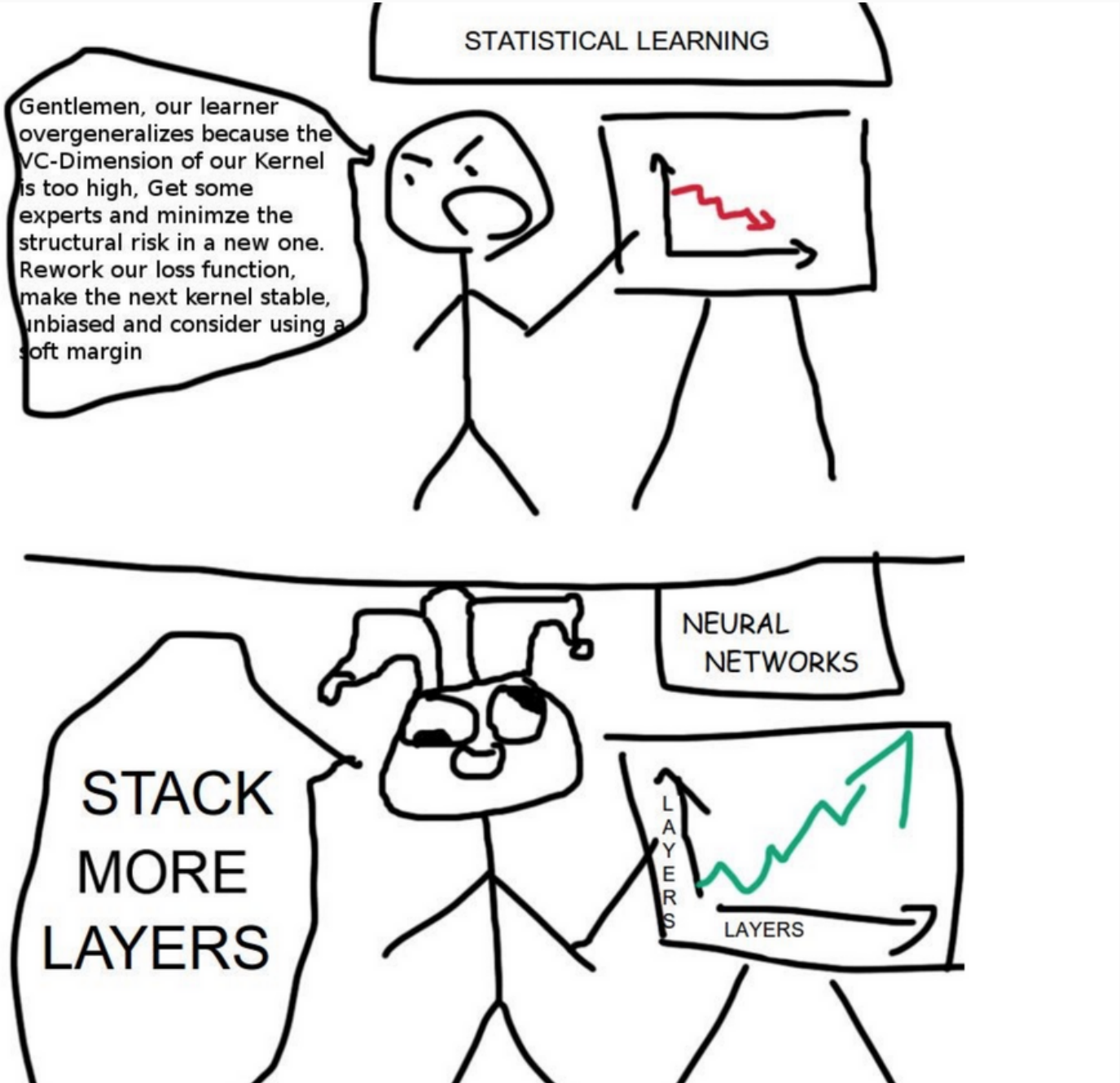
Summary of what we discussed

- Improved losses using lipschitz constraints, inspired by earth-mover distance
- Adversarially trained inference networks.
- Style transfer

Pause

We are done talking about GANs.

It do be like that



Part III

Improving Variational Auto-encoders

Improving variational autoencoders

Improving variational autoencoders

- **Reminder:** VAEs optimize the ELBO, with a KL divergence to bound the data log-likelihood

$$F(\mathbf{x}, \theta, \phi) = \ln p(\mathbf{x}) - D\left(q_\phi(\mathbf{z}|\mathbf{x})||p(\mathbf{z}|\mathbf{x})\right) \tag{8}$$

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  1. More accurate bound for given posterior
  2. Enlarge the family of variational posteriors
     - Hierarchical latent variables
     - Improved flexibility with flows

Importance weighted autoencoders [Burda et al., 2016]

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- Use as likelihood estimator for (IW-)VAE with $k \approx 10^3$

Training procedure importance weighted autoencoders

- Gradients of importance weighted lower bound

$$\nabla F_k(\mathbf{x}) = \mathbb{E}_{\mathbf{z}_{1:k} \sim q_\phi(\mathbf{z}|\mathbf{x})} \left[\sum_{i=1}^{k} \widetilde{w}_i \nabla \big(\ln p(\mathbf{x}, \mathbf{z}_i) - \ln q_\phi(\mathbf{z}_i|\mathbf{x}) \big) \right]$$

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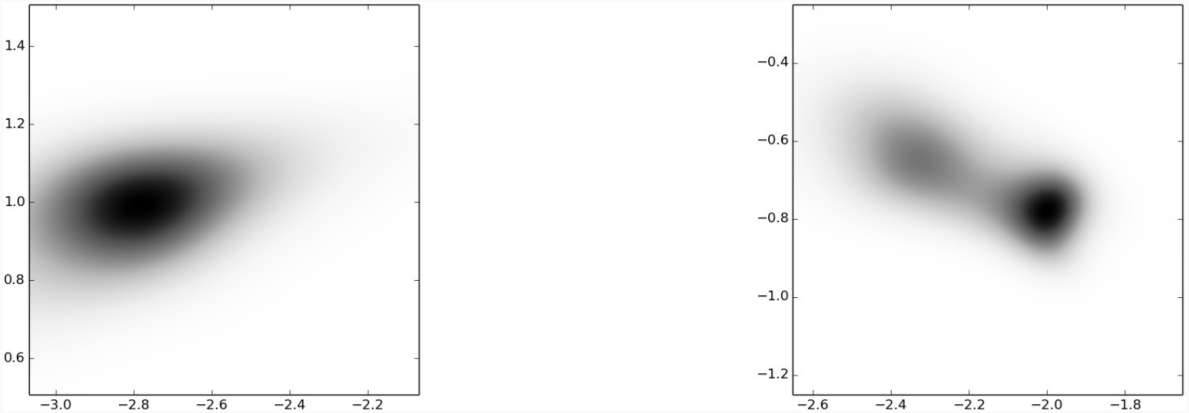
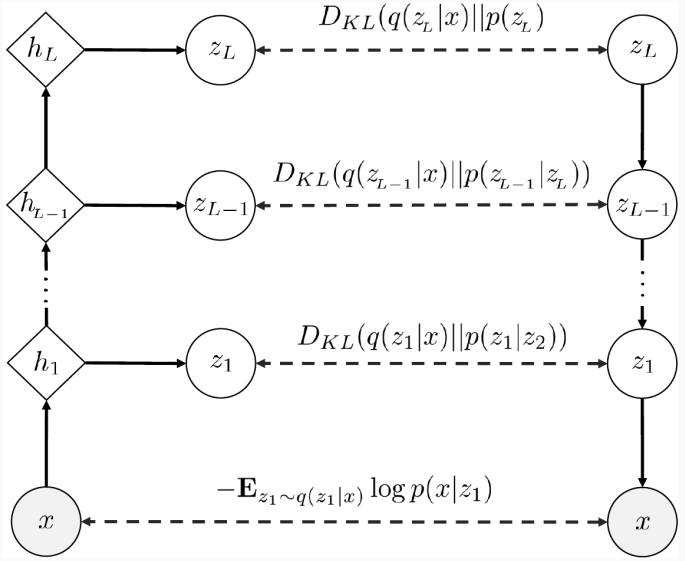
- Allows for more accurate models with complex posteriors



True posterior $p(\mathbf{z}|\mathbf{x})$ VAE (left) and IW-VAE (right)

Top-down hierarchical sampling

- **Multiple levels of latent variables at increasing resolutions**



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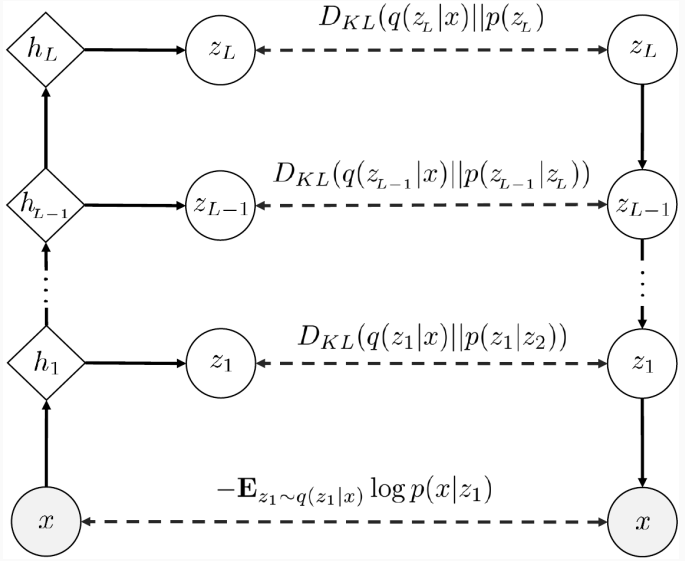

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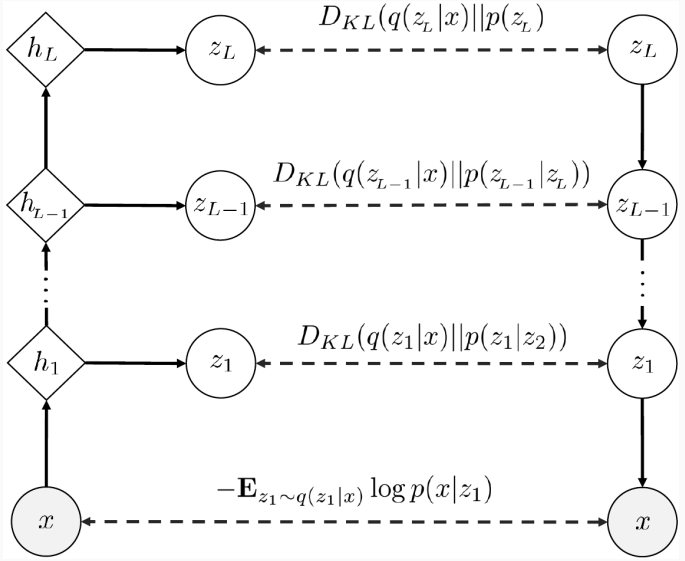

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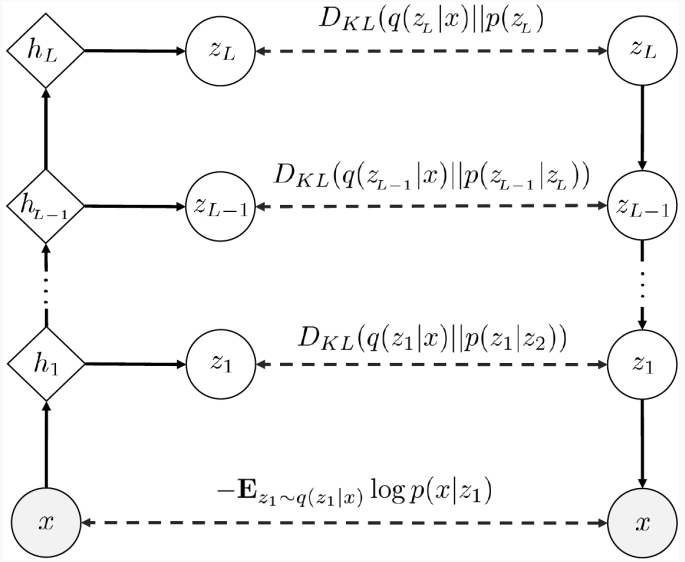
- Extended VAE log-likelihood bound



$$F = \ln p(\mathbf{x}) - D_{KL}(q(\mathbf{z}_{1:L}|\mathbf{x})||p(\mathbf{z}_{1:L}|\mathbf{x})$$

$$= \underbrace{\mathbb{E}_{q(\mathbf{z}_1|\mathbf{x})}[\ln p(\mathbf{x}|\mathbf{z}_1)]}_{\text{Reconstruction}} - \underbrace{\sum_{i=1}^{L} \mathbb{E}_{q(\mathbf{z}_{i+1})}[D_{KL}(q(\mathbf{z}_i|\mathbf{x})||p(\mathbf{z}_i|\mathbf{z}_{i+1})])}_{\text{Regularization}}$$

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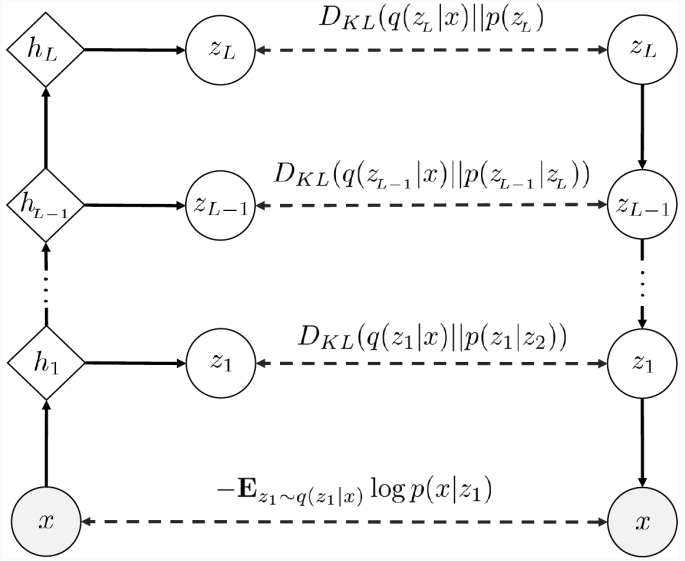

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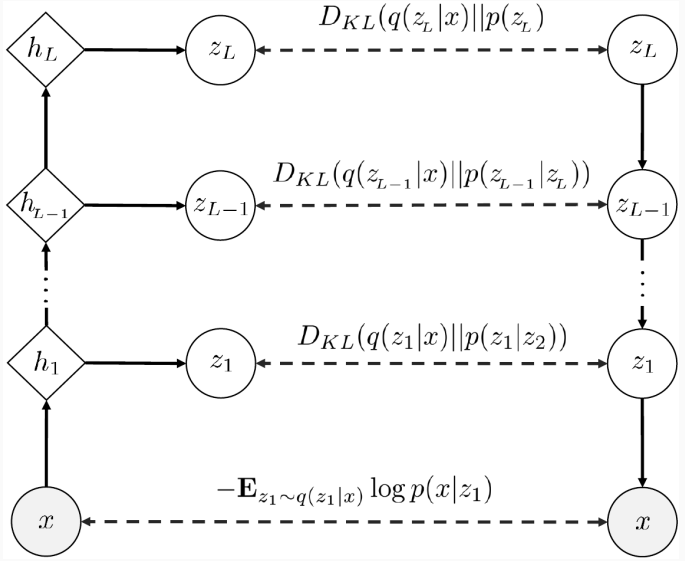
- Autoregressive distribution over latent variables in 2D grid ($p(z_1)p(z_2|z_1)$)
- Latent variables must be sampled in the same order when encoding or when sampling
- Posterior $q(\mathbf{z}|\mathbf{x})$ is no longer gaussian
  - $q(z_2|x) = \int_{z_1} q(z_1|x)q(z_2|x, z_1)$



Variational inference with normalizing flows

[Rezende and Mohamed, 2015]

- Improve posterior with series of invertible transformations

Variational inference with normalizing flows

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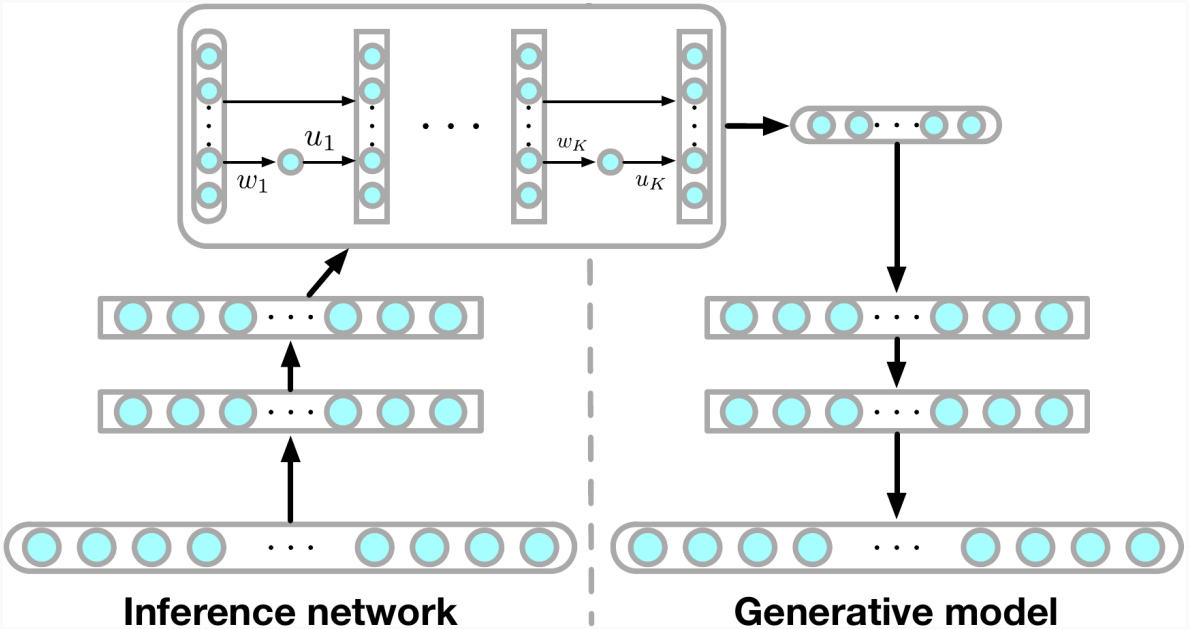
- **Improve posterior with series of invertible transformations**
- Variational inference (in VAE) uses limited class of posteriors
  - For example, Gaussian with diagonal covariance
  - Optimizing loose bound on data log-likelihood



Normalizing flows

- Let density "flow" through set of invertible transformations
- 'Reshape' the density, tractable because invertible

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- Linear-time determinant for planar and radial flows

$$f(\mathbf{z}) = \mathbf{z} + \mathbf{u} h(\mathbf{w}^\top \mathbf{z} + b)$$

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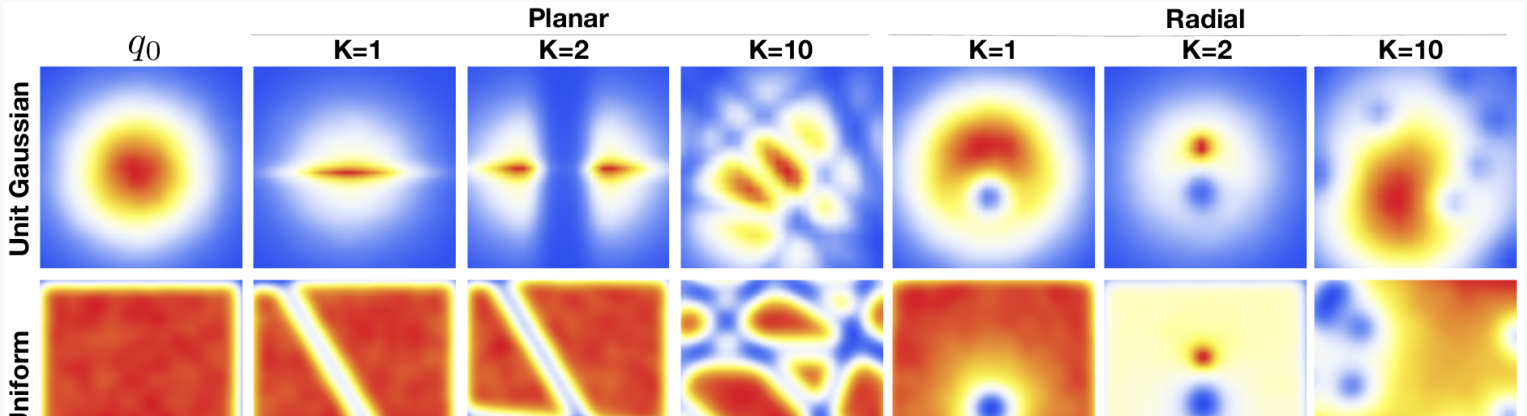
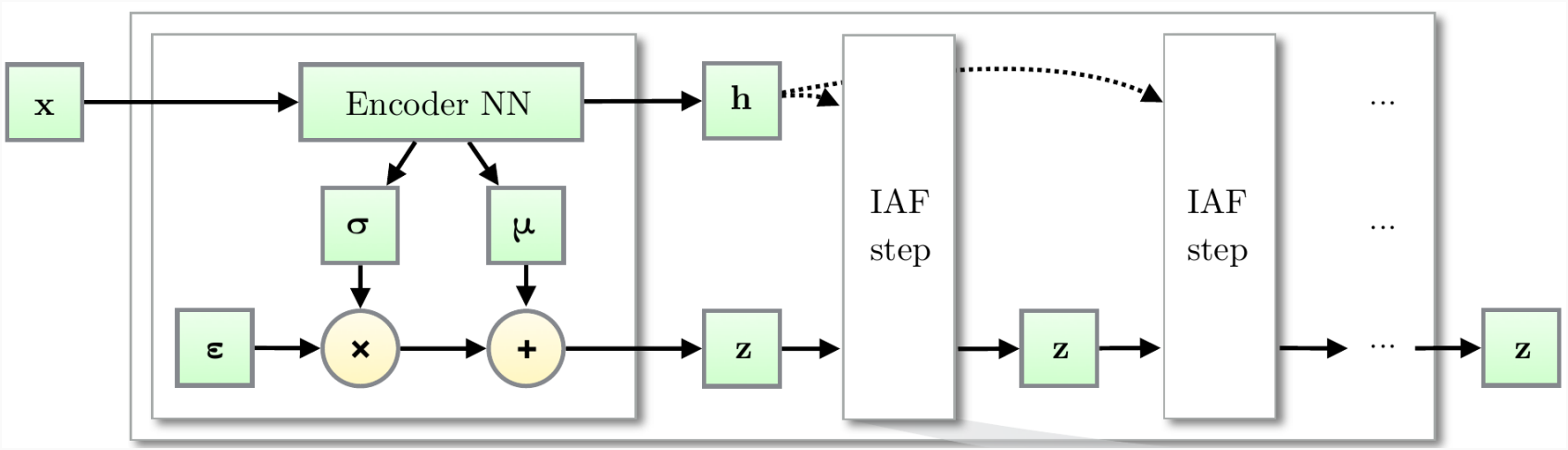


Autoregressive flow [Kingma et al., 2016]

- Restictive flows in [Rezende and Mohamed, 2015]
  - Planar flow similar to MLP with single hidden unit
- Use autoregressive transformations in flow
  - Rich and tractable class of transformations
  - Fewer transformations needed



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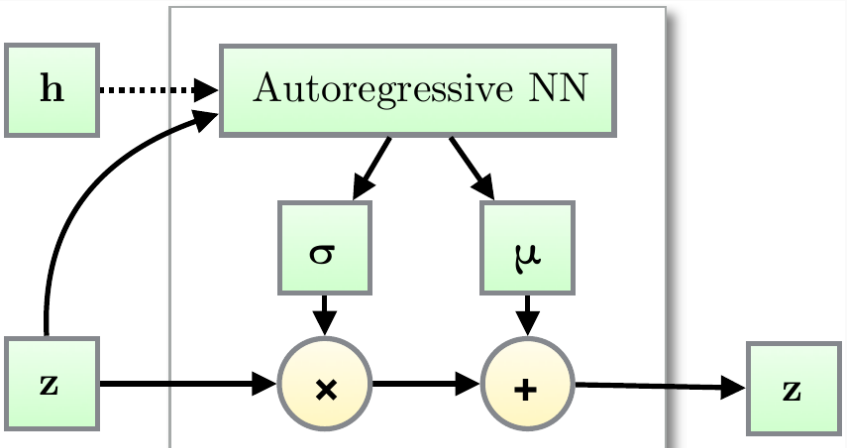
- Free to chose form of autoregressive dependency



Recap

Ways to improve the tightness of the ELBO:

- Importance weighted autoencoder
- Hierarchical top-down sampling
- Flow transormations

Beyond conditional independance assumptions

Standard VAE decoders make a conditional independance on x given z.

- Similar to L_2 regression. Bad metric of image similarity
- Leads to blurry images, and over-generalization

Hybrid VAE autoregressive models [Gulrajani et al., 2017b, Chen et al., 2017]

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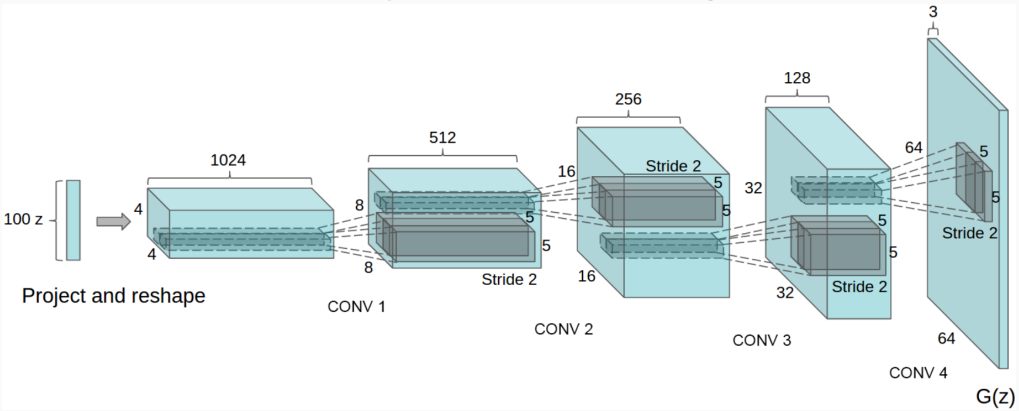
- Variational autoencoder
  - Latent variable **z** generates global dependencies
  - Pixels conditionally independent given code



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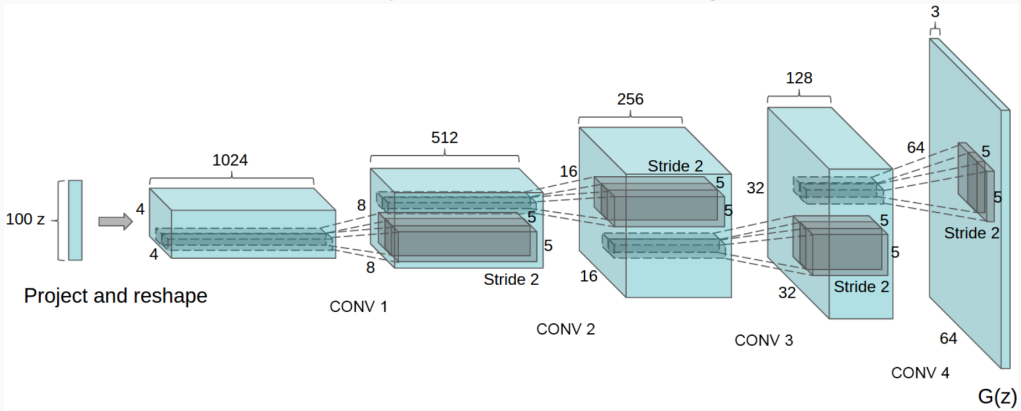
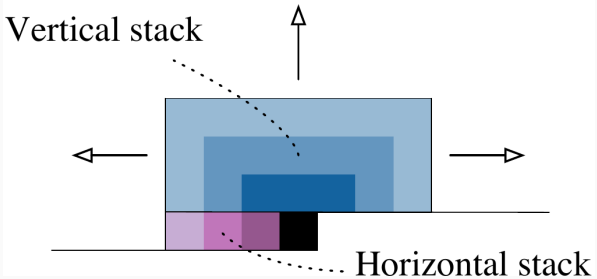
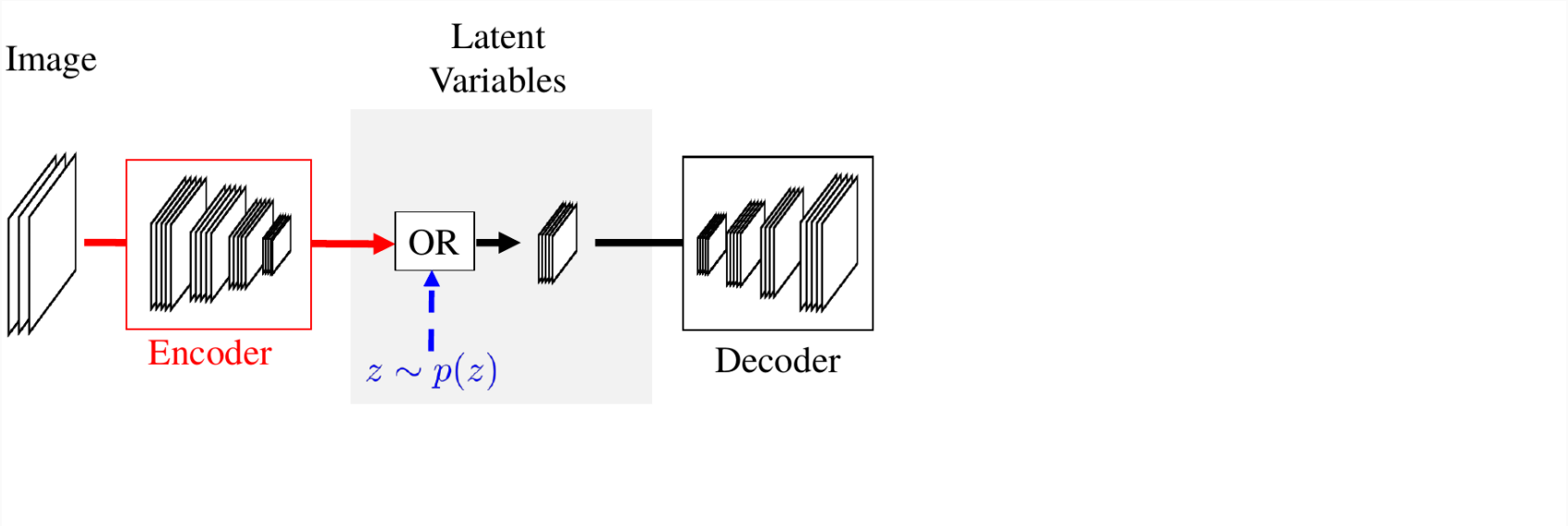


- Autoregressive PixelCNN
  - Needs many layers to induce long-range dependencies
  - Doesn't learn latent representation



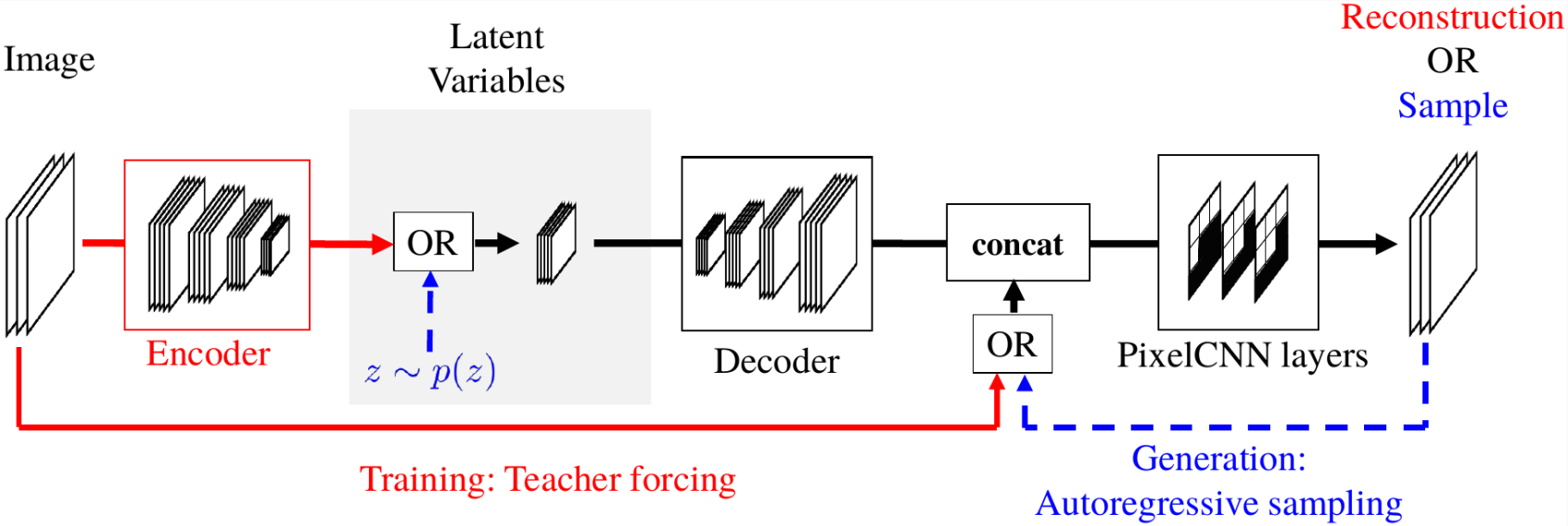
Hybrid PixelVAE model [Gulrajani et al., 2017b]

- Latent var. input to deterministic upsampling decoder $f(\mathbf{z})$



Hybrid PixelVAE model [Gulrajani et al., 2017b]

- Latent var. input to deterministic upsampling decoder $f(\mathbf{z})$
- Pixel-CNN layers induce local pixel dependencies

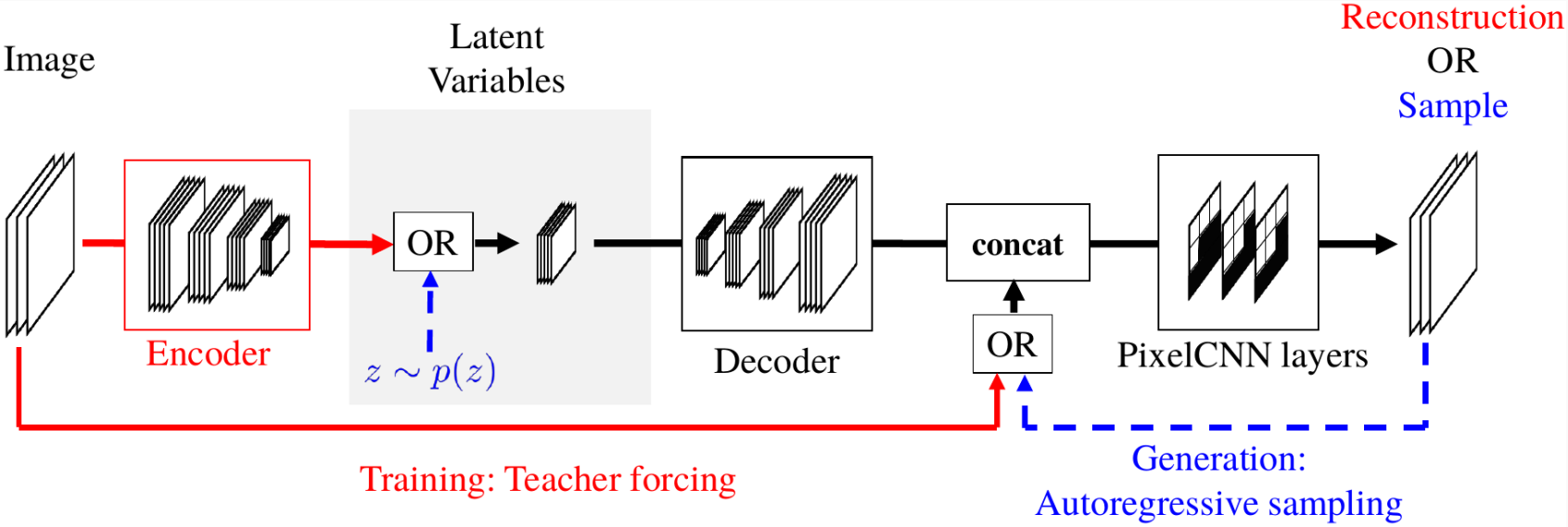


Hybrid PixelVAE model [Gulrajani et al., 2017b]

- Latent var. input to deterministic upsampling decoder $f(\mathbf{z})$
- Pixel-CNN layers induce local pixel dependencies

$$p(\mathbf{z}) = \mathcal{N}(\mathbf{z}; 0, I), \tag{9}$$

$$p(\mathbf{x}) = \int_{\mathbf{z}} p(\mathbf{z}) \prod_i p(x_i | \mathbf{x}_{<i}, f(\mathbf{z})) \tag{10}$$



Samples PixelVAE model LSUN dataset

- Model with three levels of stochasticity
  - Latent variables at 1×1
  - Latent variables at 8×8
  - PixelCNN at 64×64

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Re-sampling PixelCNN only

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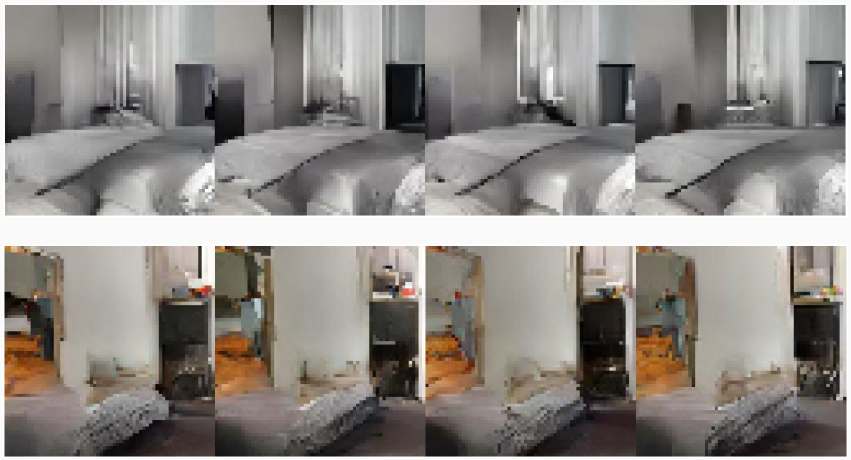

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Samples PixelVAE model LSUN dataset

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  - Latent variables at 8×8
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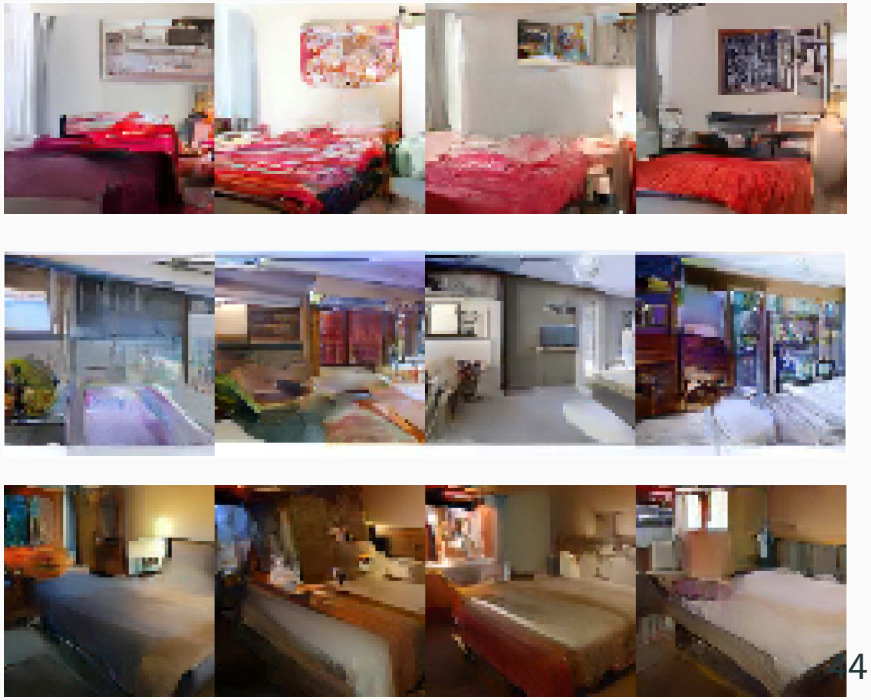
- Hierarchical representation learning

Re-sampling PixelCNN only

Re-sampling 8×8 + PixelCNN



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