Midterm test

Closed book. Attempt all questions. Calculators permitted. 13:00-13:50
Please write your name and student identity number on the front page.

(1) Determine the order of convergence of each of the following sequences (if they converge at all).

$$\text{(a)}\ x_k = \frac{1}{k!}, \quad \text{(b)}\ x_k = 1 + (0.3)^{2^k}, \quad \text{(c)}\ x_k = 2^{-k}, \quad \text{(d)}\ x_k = 1/k$$

[4 marks]

(2) Consider the function on $\mathbb{R}^2$, $f(\boldsymbol{x}) = (x_1^2 + x_2)^2$. Show that the direction $\boldsymbol{p} = (1, -1)^\top$ is a descent direction at $\boldsymbol{x}_0 = (0, 1)^\top$, and determine a step length α that minimizes $f(\boldsymbol{x}_0 + \alpha \boldsymbol{p})$.

[4 marks]

(3) Determine, with justification, which of the following sets is convex.

(a) $\{(x, y) : x > 1,\ y > \log(x)\}$;

(b) $\{(x, y) : x > 0, y > 0, xy < 1\}$;

(c) $\{(x, y, 1) : x^2 + y^2 \leq 2\}$;

(d) $\{\boldsymbol{x} \in \mathbb{R}^n : \|\boldsymbol{x}\|_\infty + \|\boldsymbol{x}\|_1 \leq 1\}$.

Recall that $\|\boldsymbol{x}\|_\infty = \max_i |x_i|$ and $\|\boldsymbol{x}\|_1 = \sum_i |x_i|$.

[4 marks]

(4) Consider the following linear programming problem

$$\begin{aligned} \text{maximize} \quad & x_1 + 2x_2 \\ \text{subject to} \quad & x_1 \geq 0 \\ & x_1 \leq 1 \\ & x_1 + 2x_2 \leq 2 \\ & x_1 + x_2 \geq 1 \end{aligned}$$

(a) Sketch the feasible set and determine the vertices of the polyhedron of feasible points from the diagram or from the inequalities;

(b) Find an optimizer and the optimal value.

[4 marks]

(5) Consider the function

$$f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2.$$

Formulate Newton's method for finding a local minimizer. By inspecting the gradient, show that $(1, 1)^\top$ is the only local minimizer of this function. [4 marks]