Ex-01 Symbol → S_0 S_1 S_2 S_3 S_4

0.4 0.2 0.2 0.1 0.1

S_0	0.4	0.4	0.4	0.6	1
S_1	0.2	0.2	0.4 (00)	0.4 (1)	
S_2	0.2	0.2 (010)	0.2 (10)		
S_3	0.1 (0100)	0.2 (100)			
S_4	0.1 (1100)				

0.4 → 1

0.2 → 10

0.2 → 100

0.1 → 0100

0.1 → 1100

length $L = \sum_{i=1}^{n} p_i (L_i)$

$= 0.4(1) + 0.2(2) + 0.2(3) + 0.1(4) + 0.1(4)$.

1100 – C	0.1 (1100)	0.2 (110)				
1101 – D	0.1 (1101)					
111 – A	0.2	0.2 (111)	0.2 (10)			
10 – E	0.2	0.2	0.4 (11)	0.4 (0)		
0 – B	0.4	0.4	0.4	0.6 (1)	1	

A – 111

B – 0

C – 1100

D – 1101

E – 10

Ex-02 Given Top output path (g_0) = $(1,1,1)$

$$X_i(1) = M_i \oplus M_{i-1} \oplus M_{i-2}$$

Bottom output path (g_1) = (101)

$$X_i(2) = m_i \oplus m_{i-2}$$

message bit sequence, $(m_0, m_1, m_2, m_3, m_4)$
$= (1, 0, 0, 1, 1)$

For, $m_0 = 1$, $i = 0$

$m_0 = 1$, $m_{0-1} = 0$, $m_{0-2} = 0$

$\therefore$ $X_0(1) = m_0 \oplus m_{0-1} \oplus m_{0-2} = 1 \oplus 0 \oplus 0 = 1$

$X_0(2) = m_0 \oplus m_{0-2} = 1 \oplus 0 = 1$

Encoded output $(1,1)$

For, $i = 1$,

$m_0 = 1$, $m_1 = 0$, $M_{-1} = 0$

$X_1(1) = m_1 \oplus m_{1-1} \oplus M_{1-2} = 1 \oplus 0 \oplus 0 = 1$

$X_1(2) = M_1 \oplus M_{1-2} = 0 \oplus 0 = 0$

Encoded output $(1,0)$

For $i = 2$, $m_2 = 0$, $M_{2-1} = 0$, $M_{2-2} = 1$

$X_2(1) = m_2 \oplus m_{2-1} \oplus m_{2-2} = 0 \oplus 0 \oplus 1 = 1$

$X_2(2) = m_2 \oplus m_{2-2} = 0 \oplus 1 = 1$

Encoded output $(1,1)$.

$i = 3,$

$m_3 = 1, \; m_{3-1} = 0, \; m_{3-2} = 0$

$X_3(1) = m_3 \oplus m_{3-1} \oplus m_{3-2} = 1 \oplus 0 \oplus 0 = 1$

$X_3(2) = m_3 \oplus m_{3-2} = 1 \oplus 0 = 1$

Encoded output (1,1)

$i = 4$

$m_4 = 1, \; m_{4-1} = 1, \; m_{4-2} = 0$

$X_4(1) = m_4 \oplus m_{4-1} \oplus m_{4-2} = 1 \oplus 1 \oplus 0 = 0$

$X_4(2) = m_4 \oplus m_{4-2} = 1 \oplus 0 = 1$

Encoded output = (0,1)

Final encoded sequence = 1110111101

Ans

Ex-03 Lempel-ziv [Lossless compression Technique]

|A|AB| ABB| B| ABA| ABAB| BB| ABBA| BB|
1 2 3 4 5 6 7 8 9

A = 0
B = 1
φ = 00

position: 1 2 3 4 5 6 7 8 9

Sequence: A AB ABB B ABA ABAB BB ABBA BB

Numerical rep: φA 1B 2B φB 2A 5B 4B 3A 7

code: 000 11 101 001 0100 1011 1001 0110 0111

more example

X = |0|0 0|1|0 1|1 1|0 0 1|0 1 0|0 1 0 1|
1 2 3 4 5 6 7 8 9

position: 1 2 3 4 5 6 7 8 9

Subseq: 0 1 00 1 01 11 001 010 0101

Numerical rep: 1 12 2 12 22 32 61 92

Ni rep → subseq

code: 0 1 0 10 1 11 101 101 1100 1011

X = |0|1|00|0|0 1|0 11|1 0|0 0 10|1 00|1 01|

3 4 5 6 7 8 9

~~Sub~~ = 0 1

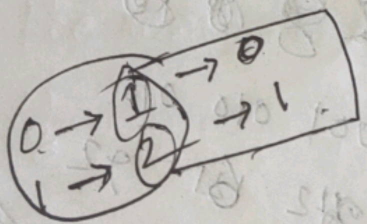
position : 1 2 3 4 5 6 7 8 9

Sub seq : 0 1 00 01 011 10 010 100 101

Numer rep : 1 2 11 12 42 21 41 61 62

code : 0 1 010 011 1001 100 1000 1100 1101

Numeri rep Sub seq
ar binary



or

|0|1| |0 0|0 1|0 1 1|1 0| 0 1 0|1 0 0|1 0 1|

1 2 3 4 5 6 7 8 9

00

1

Ex-4 Hamming code

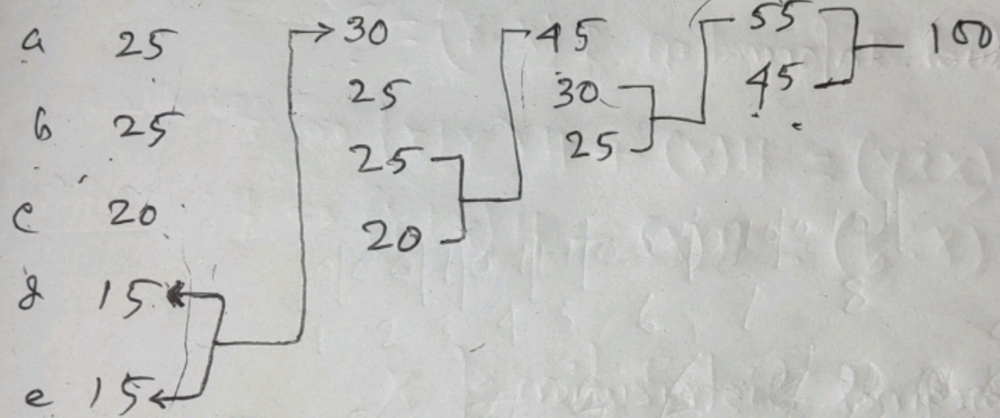
4 bit message, $M = [1010]$

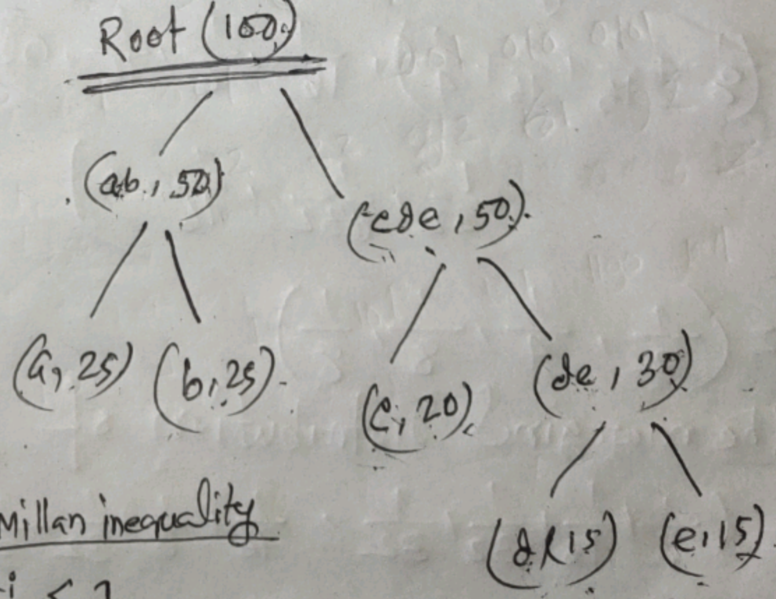
Generator matrix, $G =$

Ex-6

Symbol → a b c d e

Frequency → 25 25 20 15 15.





Length from Root

$l_a \to 2$

$l_b \to 2$

$l_c \to 2$

$l_d \to 3$

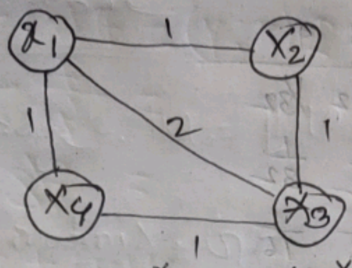
$l_e \to 3$

Kraft-McMillan inequality

$$\sum_{i=1}^{n} 2^{-l_i} \leq 1$$

$$= 2^{-2} + 2^{-2} + 2^{-2} + 2^{-3} + 2^{-3} = 1 \text{ (optimal)}.$$

Ex-07



weight matrix,

$$W = \begin{array}{c|cccc} & x_1 & x_2 & x_3 & x_4 \\ \hline x_1 & 0 & 1 & 2 & 1 \\ x_2 & 1 & 0 & 1 & 0 \\ x_3 & 2 & 1 & 0 & 1 \\ x_4 & 1 & 0 & 1 & 0 \end{array}$$

column sum of weight, $w_1 = 4$

$w_2 = 2$

Total weight $= w_1 + w_2 + w_3 + w_4 = 12$ $\quad w_3 = 4$

$w_4 = 2$

we get stationary distribution,

$$\mu = \left(\frac{4}{12}, \frac{2}{12}, \frac{4}{12}, \frac{2}{12}\right)$$

The entropy rate,

$$H(\alpha) = H\left(\frac{1}{12}, \frac{2}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}, \frac{2}{12}, \frac{1}{12}, \frac{1}{12}, \frac{1}{12}\right) - H\left(\frac{4}{12}, \frac{2}{12}, \frac{4}{12}, \frac{2}{12}\right)$$

$$\left[H\left(\frac{1}{12}\right) = -\frac{1}{12}\log_2\left(\frac{1}{12}\right) + \cdots \right]$$

$= 3.25 - 1.92$

$= 1.33$

EX-08

$Y \backslash X$	1	2	3	4
1	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{32}$
2	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{32}$
3	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
4	$\frac{1}{4}$	0	0	0

conditional entropy $H(X|Y) = ?$

joint entropy $H(X,Y) = ?$

Mutual Information $I(X;Y) = ?$

$H(X;Y) = H(X) + H(Y|X)$ or $H(Y) + H(X|Y)$

$I(X;Y) = H(X) - H(X|Y)$ or $H(Y) - H(Y|X)$

The marginal distribution of X

$= \left(\left(\frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{4}\right), \frac{1}{16}, \frac{1}{8}, \frac{1}{16}, 0\right), \frac{1}{32}, \frac{1}{32}, \frac{1}{16}, 0,$

$\frac{1}{32}, \frac{1}{32}, \frac{1}{16}, 0 \Big)$

$= \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right)$

The marginal distribution of Y,

$= \left(\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{32}, \frac{1}{16} + \frac{1}{8} + \frac{1}{32} + \frac{1}{32}, \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16}, \frac{1}{4} + 0 + 0 + 0\right)$

$= \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$

$$H(X) = -\sum_{i=1}^{4} P(x) \log P(x)$$

$$= -\{P(1)\log P(1) + P(2)\log P(2) + P(3)\log P(3) + P(4)\log P(4).$$

$$= -\{\tfrac{1}{2}\log\tfrac{1}{2} + \tfrac{1}{4}\log\tfrac{1}{4} + \tfrac{1}{8}\log\tfrac{1}{8} + \tfrac{1}{8}\log\tfrac{1}{8}\}$$

$$= 7/4$$

Similarly, $H(Y) = 2$

$$H(X|Y) = \sum_{i=1}^{4} P(y=i)\, H(X|y=i)$$

1st row divided by y

$$P(y=1)H(X|y=1) = \frac{1}{4} H\left(\frac{\frac{1}{8}}{\frac{1}{4}}, \frac{\frac{1}{16}}{\frac{1}{4}}, \frac{\frac{1}{32}}{\frac{1}{4}}, \frac{\frac{1}{32}}{\frac{1}{4}}\right)$$

$$= \frac{1}{4} H\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right)$$

$$= \cancel{40}.$$

Similarly,

2nd row divided by y

$$P(Y=2)\, H(X|Y=2) = \frac{1}{4} H\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{8}, \frac{1}{8}\right)$$

$$P(Y=3)\, H(X|Y=3) = \frac{1}{4} H\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$$

$$P(Y=4)\, H(X|Y=4) = \frac{1}{4} H(1, 0, 0, 0)$$

$$\therefore H(X|Y) = P(y_1)H(X|y=1) + P(y_2)H(X|y=2) + P(y_3)H(X|Y=3) + P(y_4)H(X|Y=4)$$

$$= \frac{1}{4}\times\frac{7}{4} + \frac{1}{4}\times\frac{7}{4} + \frac{1}{4}\times 2 + \frac{1}{4}\times 0$$

$$= \frac{11}{8}$$

Mutual information, $I(X;Y) = H(X) - H(X|Y)$

$$= \frac{7}{4} - \frac{11}{8}$$

$$= 3/8$$

joint entropy $H(X;Y) = H(Y) + H(X|Y)$

$$= 2 + \frac{11}{8}$$

$$= \frac{27}{8}$$