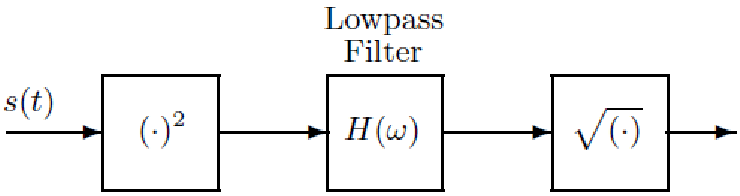
EE 357 – Communication Systems

Square Law Demodulation



Assume the AM signal

$$s(t) = A_c(1 + k_a m(t))\cos(\omega_c t)$$

The message signal has a bandwidth of W (that is $M(\omega) = 0$ for $|\omega| \geq W$).

The squarer output can be written as

$$s^2(t) = A_c^2(1 + k_a m(t))^2\cos^2(\omega_c t)$$

And can be simplified as

$$s^2(t) = \frac{1}{2}A_c^2(1 + k_a m(t))^2 + \frac{1}{2}A_c^2(1 + k_a m(t))^2\cos(2\omega_c t)$$

The first term in the above expression is a low pass signal, however the bandwidth is now increased to $2W$. The second term has a spectrum centered around $\pm 2\omega_c$. For positive frequencies the spectrum spans from $2\omega_c - 2W$ to $2\omega_c + 2W$.

The spectra of the first component and the second component must not overlap. Hence, we have the condition

$$2W < 2\omega_c - 2W \Rightarrow \omega_c > 2W$$

$H(\omega)$ denotes an ideal low pass filter with cut off frequency of $2W$ so that the output becomes

$$\frac{1}{2}A_c^2\left(1 + k_a m(t)\right)^2$$

, the square-rooter output is proportional to $m(t)$ with a dc offset.

Required Sampling Rate

$s^2(t)$ has an upper cut-off frequency of $2(\omega_c + W)$. Hence, according to the Nyquist sampling theorem, the signal must be sampled at a rate of at least $4(\omega_c + W)$. The low pass filter should operate on samples of $s^2(t)$ taken at the rate of $4(\omega_c + W)$. However, the output of the low pass filter has a cut off frequency of $2W$. Therefore, to reduce the complexity of computation, $H(\omega)$ can be implemented by an FIR filter and the output is sampled at a rate of $4W$ (decimation).

Lab Exercise

AM Generation

Write a Matlab program to amplitude modulate a message signal $m(t) = \cos(2\pi f_m t)$. Your program should allow variable values for k_a, f_m and f_c.

Assume $A_c = 1$. Use an appropriate sampling rate and time interval, and plot the message signal, $m(t)$, carrier signal $c(t)$ and AM signal $s(t)$ for

(i) $k_a = 0.5, f_m = 500$ Hz and $f_c = 2$ kHz
(ii) $k_a = 0.7, f_m = 1$ kHz and $f_c = 5$ kHz
(iii) $k_a = 1, f_m = 1$ kHz and $f_c = 5$ kHz

Also in a different figure, plot the magnitude spectrum of $S(\omega)$ of $s(t)$ in Case (i), Case (ii) and Case (iii). The x – axis should be frequency in kHz.

AM Demodulation

Assume that $s(t)$ is now passed through a "square-law demodulator" to extract the message signal $m(t)$. Apply a gain of 2 at the squarer output and subtract 1 at the end of the square-root operation to extract the message. Write a Matlab program.

Plot the original message signal $m(t)$ and the demodulated message signal below as subfigures for Case (i), Case (ii) and Case (iii) respectively. The following web page on Matlab FIR filter design would be useful to implement $H(\omega)$.

https://www.mathworks.com/help/signal/ug/practical-introduction-to-digital-filter-design.html

Choose an appropriate filter type, filter order and other parameters such as passband/stopband frequencies etc. in your code.

In a separate figure, investigate the impact of setting incorrect passband/stopband frequencies on the demodulated message signal for Case (i), Case (ii) and Case (iii) respectively.

Also, discuss about (a) how would you set the correct passband/stopband frequencies according to the message/carrier signal frequencies and (b) the distortion in cases of too low and too high values of your filter bandwidth.

Your report should include all Matlab Code.

E/16 Batch
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